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Robustness and resilience of energy systems to extreme events: A review of assessment methods and strategies

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A R T I C L E I N F O

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A B S T R A C T

This comprehensive review investigates the robustness and resilience of energy systems in response to extreme events, emphasizing the need for enhanced methodologies and strategies to mitigate the growing vulnerabilities in modern power grids. The study clarifies conceptual definitions and relationships between key resilience metrics, reliability, robustness, and flexibility while analyzing their applications across multi-energy systems. The review identifies critical gaps in resilience assessment, including a lack of unified definitions, insufficient documentation of financial and social costs, and challenges in integrating resilience into infrastructure planning. This work provides a framework for advancing energy system robustness and offers recommendations for future research to ensure sustainable and reliable energy delivery amidst increasing uncertainties. Despite the extensive development of resilience robustness measurements since the 1950s, only a limited number of centrality metrics, including degree, betweenness, and clustering coefficient, have been widely used. This study presents an overview of robustness resilience metrics and examines their usefulness in networks. The computational complexity of the resilience indicators examined in this study is also explored. The comprehensive examination of the evaluated robustness metrics promotes their use in addressing diverse network computing and engineering challenges. It emphasizes the need for collaborative efforts and the necessity for equitable solutions across the phases of energy planning, design, and operation to enhance energy resilience.

1. Introduction

The investigation of power system resilience to natural disasters has gained traction due to the growing impact of significant outages on the electrical grid [1]. It is predicted that global urban coverage will grow by 40 % in 2050, accompanied by a significant increase in population [2]. Therefore, service disruptions can vary from a few days to a few weeks, which is a significant amount [3]. To reduce or prevent the expenses associated with more frequent and longer service outages, a substantial investment is needed to enhance the resilience of the electrical system. Power system resilience plans may encompass measures such as reinforced infrastructure, redundant systems, battery energy storage (BESS) optimum capacity integrated with the network, or enhanced flexibility [4,5]. Thus, strengthening the sustainability of cities significantly contributes to improving resiliency indices.

The definition of "long-duration outages" varies, frequently failing to distinguish between short- and long-duration outages [6–8]. Additionally, there is a lack of comprehensive documentation regarding the expenses associated with extended power outages, particularly concerning location, duration, and magnitude differences. The definition of resilience is often unclear and is sometimes confused with reliability and mitigation, yet it is crucial in power system design [4]. Reliability in power systems pertains to consistently meeting service standards over an extended period, while resilience refers to the system's ability to effectively adjust and bounce back from sudden and severe interruptions [9]. The fundamental elements of resilience are robustness, adaptability, and speed [9]. Mitigation involves proactive or reactive measures, with associated costs incurred before or after an event to reduce event size or occurrence rate. Resilience, conversely, refers to the system's capacity to endure and recover from a catastrophic event [10]. Resilience measures encompass pre- and post-event efforts and affect the time and speed of recovery from an event [11].

Investing in resilient power infrastructure to minimize prolonged power outages is difficult due to uncertainties and inconsistencies. This endeavor aims to bridge the gap between decision-makers needs and current modeling solutions. Government and utility authorities face

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hurdles in assessing risks and costs, as seen in state resilience evaluations like those in Oregon and North Carolina [12,13]. Solutions could be better understood, but they have shortcomings, including model suitability, uncertainty evaluation, and plans to address societal changes. Stakeholders seek a deeper understanding of potential models for a conclusive resolution.

Climate change-induced extreme weather events have a profound impact on the energy ecosystem [14], with their frequency and intensity increasing significantly over the past decade and projected to continue rising [15]. These events affect the energy ecosystem in various ways [16]. For instance, a recent severe cold spell in Texas increased heating energy demand in buildings [17] but also disrupted gas and wind turbine operation, leading to widespread power outages and significant financial loss [17,18]. Enhancing resilience in the face of such challenges is complex due to the cumulative impact of significant climate events on the entire energy ecosystem [14,19]. Thus, implementing targeted strategies to improve resilience and enhance interconnectivity is essential. Prioritizing sustainability, interconnectivity, and resilience can effectively address the pressing issue of climate change mitigation and adaptation facing human civilization.

Various strategies can enhance the resilience of energy systems against different threats by leveraging the inherent redundancy of the grid and its infrastructure. In this strategy, energy providers, often operating with limited staff and resources, are responsible for ensuring the resilience of critical network loads. Additionally, it is essential to examine the balance between energy resilience and other objectives, such as energy efficiency, in real-world applications while acknowledging potential trade-offs.

Several measures can be implemented to increase grid resilience and achieve these aims. One physical hardening strategy planning is vegetation management, which involves planting and trimming trees to prevent interference with electrical lines. This proactive approach reduces risks such as damage to wires and short circuits, thereby directly enhancing grid resilience [20,21].

Another crucial strategy is improving operating capacities, which includes deploying emergency generators. A common method during crises caused by extreme weather events is to provide mobile or portable power generators or substations. These temporary solutions ensure a continuous electricity supply for remote yet critical users. Emergency diesel generators or battery systems can sustain essential power supplies in affected areas. Even when substations or distribution lines are severely damaged, these measures offer partial restoration of grid operation until full repairs can be made [22,23].

Additionally, renewable energy grids and storage systems are pivotal in enhancing resilience. These systems offer improved economic efficiency and operational productivity while mitigating the intermittencies associated with integrating renewable energy sources into the grid. These technologies can enhance grid reliability by incorporating substantial energy storage capacity and implementing robust Energy Management Systems [24–26]. The ability of energy storage devices to smooth out fluctuations ensures a more stable and resilient energy supply, which is critical in increasing climate variability [27].

Implementing such approaches helps mitigate the spread of disruptions and improve energy efficiency. However, a comprehensive evaluation of the network's resilience is necessary to guide these efforts effectively.

The focus on improving energy sustainability, resilience, and interconnectedness has been evident Since 2013, as shown in Fig. 1. It is crucial to evaluate the current state of the field and identify areas for further investigation. Numerous review articles have explored resilience, but comprehensive evaluations fail to provide a precise understanding of its effect on a networked energy system. Effectively managing the study by incorporating both comprehensive and extensive aspects is a challenging task.

The study of resilience and robustness metrics spans nearly a century. Over the past two decades, numerous comprehensive surveys have aimed to classify and understand these metrics. For instance, a survey conducted in 2002 explored metrics used in networks to measure graph properties, retrieval capabilities, usage characteristics, and information-theoretic properties [28]. Other recent efforts have examined resilience metrics applicable across various domains. One survey analyzed a limited set of resilience metrics to assess their impact on network performance, specifically regarding topic propagation originating from those central nodes. This study focused on information diffusion and utilized well-known centrality metrics [29]. Another extensive survey concentrated on resilience metrics and demonstrated their effectiveness across different types of networks, including directed, undirected, bipartite, and weighted networks. This analysis examined how centrality metrics can help detect failure risks within these applications [30]. Surveys have recently reviewed resilience metrics in network contexts, discussing the connections between various measures [31]. Additionally, another survey provided a limited overview of robustness and vulnerability metrics for different networks, common network attack strategies, and a summary of attacks occurring in various network topologies. However, this work also focuses on limited robustness metrics in networks [32].

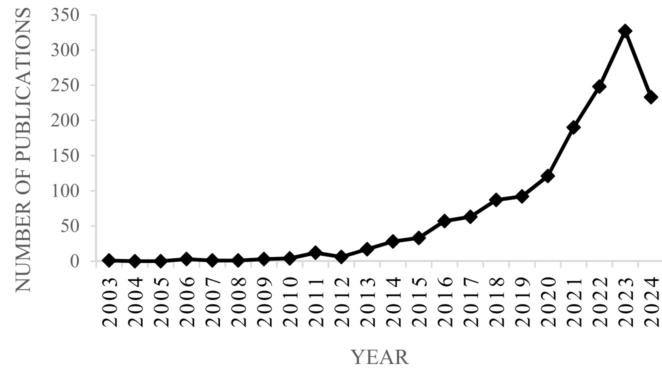

Fig. 1. Publications indexed in Scopus since 2003 with keywords energy system, resilience, and Robustness.

Despite significant progress in developing resilience and robustness metrics across several scientific disciplines over the years, their advantages and uses have not been fully recognized. The following outlines research gaps identified in the literature review.

- While resilience, reliability, robustness, and flexibility are critical components of power system design, there is a notable lack of clear and consistent definitions. The existing literature often fails to adequately distinguish between "short-duration" and "long-duration" outages, resulting in confusion and inconsistent applications of resilience strategies. Therefore, there is a pressing need for more precise and unified definitions of these terms and quantification methods to accurately assess their impact on energy systems.
- There is a significant lack of detailed documentation on the financial and social costs associated with long-duration outages, particularly concerning their locations, durations, and magnitudes. Understanding these costs is essential for guiding policy decisions and investment strategies that enhance resilience. Additionally, current models do not adequately explain how to incorporate these variations into their risk assessments.
- Integrating resilience strategies into long-term infrastructure planning and investment is a significant challenge. While current models consider factors such as redundancy, infrastructure reinforcement, and energy storage, they frequently overlook the complexities of climate change and other external threats. There is a clear need for models that more accurately reflect real-world conditions, emphasizing the importance of flexible, adaptive, and robust solutions that can address existing vulnerabilities and future uncertainties.

- Evaluating the risks, uncertainties, and costs associated with improving resilience in power infrastructure presents considerable challenges. Current models do not provide a comprehensive framework to evaluate these uncertainties and their effects on investment and policy decisions. Exploring decision-making tools that can effectively connect theoretical concepts with practical applications is crucial. This exploration is particularly important for guiding resilience investments in areas vulnerable to extreme weather events or natural disasters.
- Despite significant advancements in analysing and evaluating robustness metrics within systems, research on assessing the resilience of energy networks remains limited and fragmented. Various robustness metrics and resilience measurement methods in energy systems have been studied in isolation across different studies; however, there is no comprehensive and unified synthesis of these metrics and methods in a single work. This fragmentation underscores the need for an integrated and systematic framework to assess the robustness and resilience of energy networks.

The contributions and novel perspective of the paper are summarized below.

- The boundaries between reliability, robustness, resilience, and flexibility are defined, and relevant conceptual definitions, quantification methods, and interconnected relationships in multi-energy systems are clarified.
- A thorough examination is carried out to bolster energy resilience through extensive analysis of diverse threat sources, system recovery mechanisms, and defense measures tailored to different phases.
- Robustness metrics have been explored from diverse scientific perspectives, focusing on their historical evolution and practical applications in engineering domains.
- Integrating energy resilience into the planning, design, and operation stages of district communities aims to strike a harmonious equilibrium among energy efficiency, robustness, and flexibility. This integration provides a clear path forward for bolstering the resilience of energy communities, considering challenges like climate change, heat waves, and military conflicts.

The convergence of catastrophic events and the durability of resilience results in a broad research perspective that cannot be adequately covered in a single publication. The robustness of resilience often entails resilient networks and flows, resilient infrastructure and structure, resilient governance and networking, and resilient socioeconomics and dynamics (Fig. 2). In this scenario, we employed different combinations of the aforementioned keywords and performed an extensive review of the literature using Scopus. These keywords encompass climate change, severe occurrences, resilience, network cascade failures, infrastructure, energy systems, robustness, vulnerability, wildfires, hurricanes, energy resilience, extreme heat events, and interconnections.

The paper is organized as follows: Section 2 delves into resilience concepts and their applications across various fields, focusing on identifying a suitable network resilience measure for designing, optimizing, or controlling network resilience to achieve specific goals. Section 3 Advancing Energy Reliability: Challenges, Solutions, and Resilience Studies. Section 4 explores the interrelation between reliability, robustness, resilience, and flexibility within multi-energy systems. Section 5 delves into robustness metrics, organized into five catagories. Section 6 provides a research limitations and recommendations. Section 7 presents areas of interest for future research activities aimed at improving the resilience of energy systems. Finally, conclusions are presented in Section 8.

2. Resilience concepts across diverse domains

The frequency of significant blackouts in the United States has not decreased since the establishment of resilience standards and technological advancements by NERC in 1984 [34]. Events like the US blackout in August 2003 and the Italian blackout in September 2003 have highlighted vulnerabilities in transmission infrastructures attributed to changes in the electricity industry structure [35]. Despite ongoing debates about restructuring electricity markets, system resilience remains crucial, defined as the ability to withstand and recover from disruptions [36]. This resilience involves the capacity to endure cascading failures, with numerous studies conducted on network resilience [37,38].

Researchers have proposed numerous resilience metrics and analytical frameworks to assess resilience [39–43]. They've conducted robustness assessments at the community level and developed strategies to enhance resilience using optimization algorithms [44–46]. Resilience

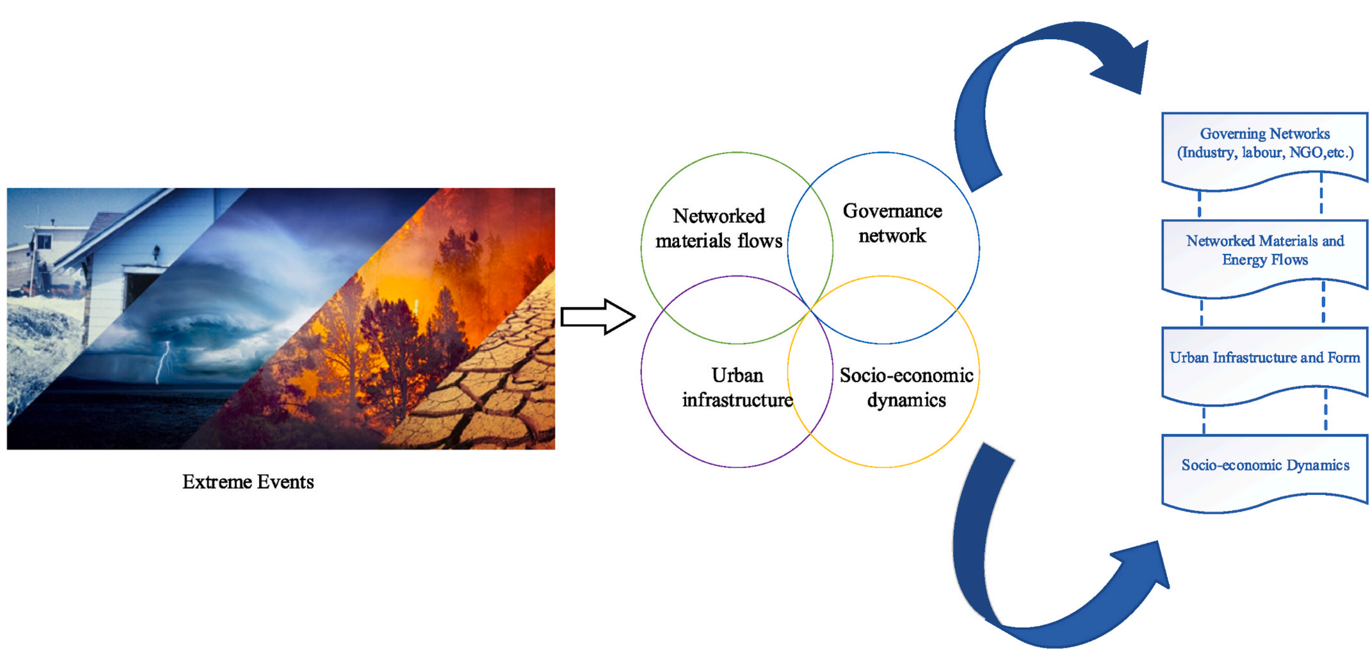

Fig. 2. Expansive scope of resilience and extreme events [33].

assessment has been approached through various perspectives, including a system's structural vulnerability function, quantification based on actual versus expected performance curves, and maximizing resilience by minimizing restoration activity time and cost [47–49]. While existing studies focus on selecting influential nodes through algorithms or improving network structure robustness, they often overlook the application of propagation models in directed networks and the role of network robustness in information transmission. To address this gap, a study introduces a robust influence evaluation factor to measure the influence of nodes in directed networks and proposes a Memetic algorithm to solve the robust influence maximization problem. Empirical analysis on synthetic and real-world networks demonstrates that Memetic algorithm outperforms baseline methods in selecting robust and influential seed nodes [50]. In addition, frameworks have been developed for resilience assessment under different seismic hazard scenarios, considering both direct and indirect losses [51,52]. Resilience analysis has been conducted, incorporating recovery costs, particularly focusing on the smart grid and utilizing Bayesian networks [53–57]. Determining the weights of various factors through Bayesian network analysis enhances resilience improvement [58]. Additionally, dependency analysis techniques such as PCA, fault trees, and Copula functions have been used to describe network damages in failure scenarios [59–62]. Scholars have analyzed the impact of specific emergencies or multiple emergencies on systems [63–67]. Furthermore, resilience assessment frameworks have been constructed under multiple hazards based on fragility functions [68,69].

This paper comprehensively analyzes energy resilience within energy systems, as depicted in Fig. 3. It delves into energy resilience's definition and measurement methods, examining its intricate relationship with reliability, robustness, and flexibility. The study explores various sources of threats, mechanisms for resilience-driven recovery, and technical strategies to enhance energy resilience. Furthermore, it showcases the application of energy resilience in energy systems. Finally, the paper offers research perspectives and recommendations for future endeavors, including a comprehensive approach to quantifying energy resilience, addressing energy resilience during extreme events, and improving energy efficiency during regular operational periods.

Energy resilience remains a term without a universally agreed-upon definition. Essentially, it revolves around the capability to maintain regular operational services [71]. Metrics for assessing resilience in power systems are established and classified across all stages [72]. The resilience of a power system denotes its ability to minimize the likelihood of blackouts or widespread power failures stemming from significant but infrequent events [73]. Perera et al. [14] investigated the impact of climate change on the utilization of renewable energy and its interactions with other variables. According to their findings, climate change is projected to increase the integration of renewable energy into the grid by 34 % while decreasing electricity supply by 16 %.

The time-evolving resilience curves during a disruptive event are shown in Fig. 4, which juxtaposes the performance of resilient and conventional microgrids under random disturbances. Fig. 4 shows that energy system resilience broadly includes operational and infrastructure resilience during extreme events. The term "operational" refers to the system's ability to withstand the effects of severe incidents through effective response, recovery, and service restoration, while "infrastructure" refers to mitigating the consequences by implementing restorative and adaptive measures.

Table 1 presents a detailed compilation of energy resilience measurement indicators, each offering a unique perspective on assessing and enhancing system resilience. These indicators are grouped based on their conceptual foundations and practical applications.

- Absorptive, Adaptive, and Recoverability Indicators: These indicators evaluate a system's ability to absorb disruptions, adapt to

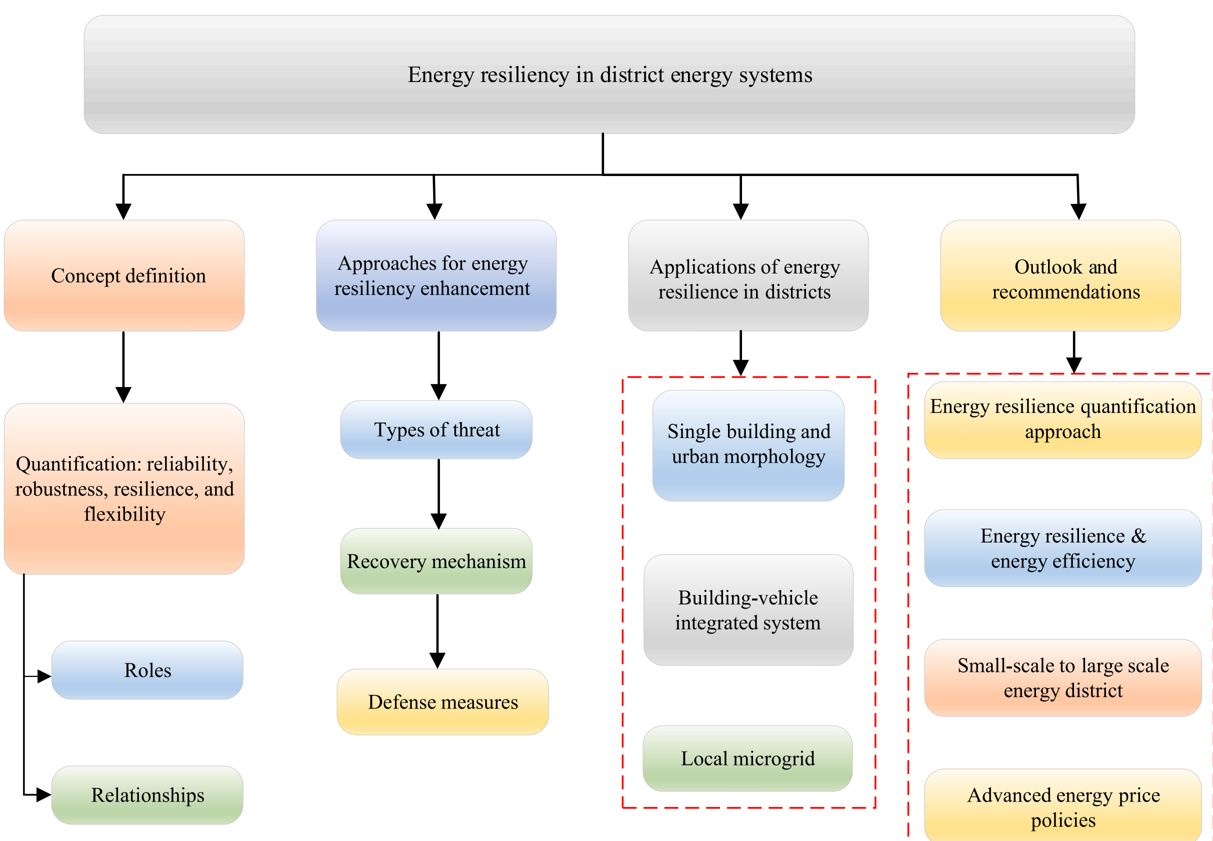

Fig. 3. Comprehensive look at energy resilience in energy systems [70].

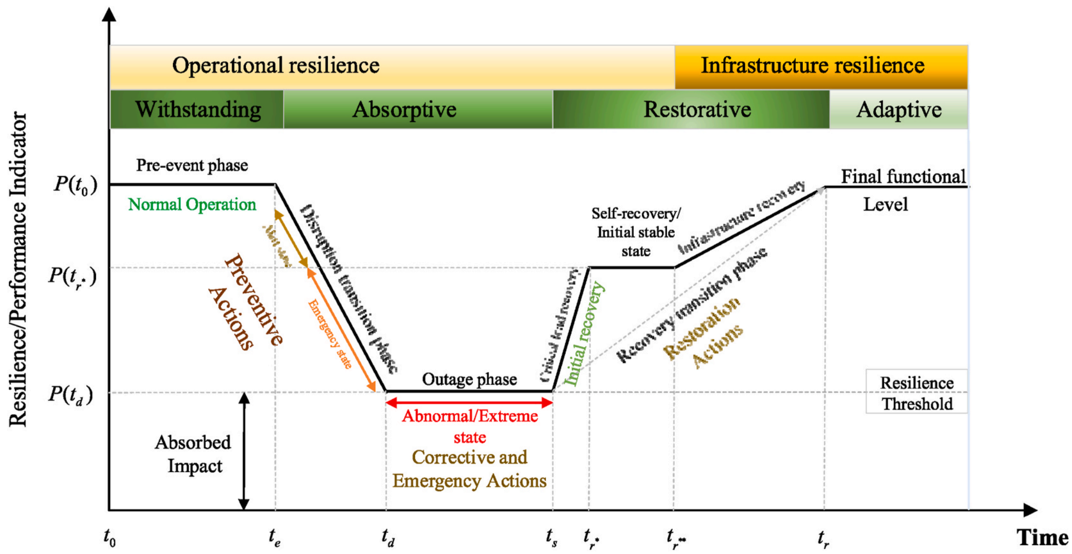

Fig. 4. Visualizing Time-Dependent Resilience Curves During Disruption Events [72].

Table 1
Overview of energy resilience measurement.

Plan	Ref. No.	Definition	Equation
Adaptive, absorbtive, recoverability	[74]	Absorptive	$R = \frac{F(t)_D}{F(t)_E}\frac{F(t)_R}{F(t)_E}S_p;\ S_p = \frac{t_D - t_E}{t_R - t_E}$
Costs for System Reduction and Recovery in Resilience	[34]	Resilience costs	$RC = \frac{SIC + MREXC + TREC}{TMV}$
Costs of Resilience Actions and Losses Due to Inability to Recover	[36]	Total cost of resilience measures and cost of loss due to incapacity	$R = \frac{\text{recovery}_{(t)}}{\text{loss}_{(t)}} = \frac{F(t) - F(t)_D}{F(t)_E - F(t)_D};$ $C = \text{Cost}_{\text{resiliency action}} + \text{L}_{\text{system disruption}}$
Recoverability	[75]	Performance change ratio following restoration attempts during disruption compared to the target performance	$R(t) = \frac{\int_0^T F(t)dt}{\int_0^T F(t)_E dt}$
Adaptability	[76]	The proportion of system parameters to their pre-disruption levels compared to those before the disruptive event	$\text{Adaptability} = \frac{\Phi_{enhanced}}{\Phi_{pre-event}}$
Difference in performance under normal and disruption conditions	[77]	Aggregate performance disparity between standard and disruptive circumstances	$R = \int_0^{T_D} (F(t)_E - F(t))dt$
Aggregate of dependent variables with assigned weights	[79]	The aggregated sum of unmet energy demand, representing energy requirements exceeding supply, and the increase in energy expenses due to a specific outage	$ESC = \exp(-\alpha_1 . UE . \exp(Y_s) * -\alpha_2 . ECI . \exp(Y_s))$ $ECI = \frac{FC - BC}{BC};\ UE = 1 - \frac{P}{D}$
Stability index	[78]	The stability of a system is influenced by the presence or absence of redundancy and diversity in its energy systems, both within and outside the system boundary	$S = \rho_i \log \rho_i$
Resilience index	[81]	The relative percentage indicates the ratio of the highest potential expenses to the actual incurred costs during the failure mode	$R = \frac{IC^{max} - IC}{IC^{max}} IC = OC + \sum_K FL_K XPC_K$
Remaining network utilization	[80]	The mean ratio of available capacity post-satisfying the present demand, compared to the overall network capacity at any specific time	$aru = \frac{\sum_{t=1}^T ru_t}{T}$

(Notations: $F(t)_D$: System performance after the interruption; $F(t)_R$: Performance of the first stable system; t_E: Time when the incident started; t_R: Time when full restoration occurred; t_D: System in the most severely damaged state; SIC: Stands for Systemic Impact Cost; MREC: Market Recovery Effort Cost; TREC: Stands for Transportation Recovery Effort Cost; TMV: Target Market Value of Production.; $\text{Cost}_{\text{resiliency action}}$: Resilience action implementation cost function; $\text{L}_{\text{system disruption}}$: Financial losses incurred as a result of the system's inability to function properly after a disruption; $F(t)$: Effect of restoration attempts on performance during interruption; $F(t)_E$: Evaluation of the first stable system; $\Phi_{enhanced}$: System parameters during restoration; $\Phi_{pre-event}$: Pre-disruption event system parameters; *UE*: Unfulfilled energy requirements; *ECI*: Rise in energy expenditure; *P*: Generation of energy; *D*: Energy consumption; α_1 and α_2: Calculating weighting factors to determine the probability of energy transfer to any predator; *IC*: Expense incurred in the event of a breakdown or malfunction; *PC*: Cost incurred as a result of a penalty; *aru*: Mean residual network utilization index.).

changing conditions, and recover to pre-event performance levels. For example, absorptive capacity quantifies the system's resilience during initial disruptions, while recoverability measures the restoration of functionality over time [74–76].

- Cost-Based Indicators: Indicators such as "resilience costs" and "total cost of resilience measures" focus on the economic dimensions of resilience. They encompass the costs of implementing resilience actions, recovering from disruptions, and financial losses due to system incapacity during extreme events. These metrics are crucial for guiding investment in resilience-enhancing strategies [34,36].
- Performance and Stability Indicators: Metrics like "difference in performance under normal and disruption conditions" and "stability index" assess the system's operational stability and performance during and after disruptions. These indicators emphasize the importance of maintaining critical functionality and reducing performance disparities [77,78].

- Energy Supply and Demand Metrics: "Aggregate of dependent variables with assigned weights" and "remaining network utilization" address the balance between energy supply and demand during disruptions. These indicators quantify unmet energy needs and the system's ability to maintain energy delivery under stress [79,80].
- Resilience and Redundancy Metrics: Metrics such as "resilience index" and "stability influenced by redundancy" emphasize the role of diverse and redundant energy sources in enhancing system robustness. These indicators promote the design of systems capable of withstanding and recovering from severe disruptions [78,81].

These indicators provide a multi-faceted framework for evaluating energy resilience, covering technical, economic, and operational aspects. Their integration into resilience planning and design can significantly enhance the ability of energy systems to withstand, adapt to, and recover from extreme events.

Researchers emphasize the importance of resilience in district energy systems, as it prevents fragility and failures in dangerous situations [82]. Ahmadi et al. [83] thoroughly examined energy system resilience, examining concept definition, characteristics, states, modeling features, methodologies, and solutions. Fig. 5 contrasts traditional and resilient energy systems, highlighting superior operation and quicker recovery time.

3. Concept definition and quantification approach on smart district energy systems

Investing in a robust power infrastructure to mitigate power outages is challenging due to uncertainties and inconsistencies. The aim is to address the disparity between decision-makers needs and current modeling solutions. Government and utility authorities encounter difficulties assessing the risks and costs of prolonged power outages, as demonstrated by state resilience assessments in Oregon and North Carolina [85,86]. Key challenges include identifying suitable models, evaluating uncertainty, and devising plans to address societal changes like technological advancements. Stakeholders are actively seeking a conclusive solution to tackle these concerns.

Methods for enhancing energy reliability often involve integrating renewable energy storage with smart grid communication and control networks [87], incorporating electric vehicles [88], establishing fair power markets [89], and implementing peer-to-peer energy sharing [90], among other strategies. For instance, Jafari et al. [89] investigated a power market strategy to enhance economic returns, customer satisfaction, and microgrid reliability. At the same time, Bayat et al. [90] explored the effectiveness of networked and individual operations in enhancing energy reliability during emergencies. Additionally, researchers have analyzed the integration of energy reliability into economic, environmental, and risk factors using weighted optimization techniques to determine efficient designs for renewable systems [91].

The power grid's instability stems from renewable energy's intermittent nature and unpredictable energy consumption patterns. Interconnecting buildings, renewable energy sources, and power networks with limited renewable energy integration exacerbate this instability. Solutions to align renewable energy generation with demand include demand-side management [92] and power supply regulation [93,94]. Researchers have focused on enhancing energy flexibility through demand-side management and power plant operations. Demand-side flexibility, as measured by Zhou and Cao [95], can cover 48.1 % of the basic electric load.

Research has explored the energy adaptability of power plants, focusing on enhancing flexibility to accommodate high photovoltaic (PV) penetration and coal-fired combined heat and power (CHP) facilities. Hou et al. [96] highlighted the benefits of retrofitting coal-fired units to improve system flexibility. Zhao et al. [97] investigated methods to enhance energy flexibility in CHP facilities, emphasizing power-to-heat conversion and auxiliary heat sources. Additionally, studies have explored improving energy flexibility in power plants by integrating multi-scale steam turbine energy storage [98] and carbon capture and storage [99]. However, economic incentives are necessary to invest in generation units.

Carpinelli et al. [100] analyzed critical distance methods and fault position approaches to improve electrical power network resilience. Gabrielli et al. [101] developed a robust optimum design using uncertainty to minimize annual expenses and CO_2 emissions. Lu et al. [102] found that stochastic optimization had minimal impact on resilience, but reducing the uncertainty level of input parameters was more beneficial. Picard et al. [103] conducted a quantitative analysis to examine how occupant behavior affects the energy performance stability of a zero-energy home. The findings indicate that climate change is projected to lead to a 15 % rise in energy use. In addition, when compared to weather or climatic change, the conduct of occupants has a more evident influence.

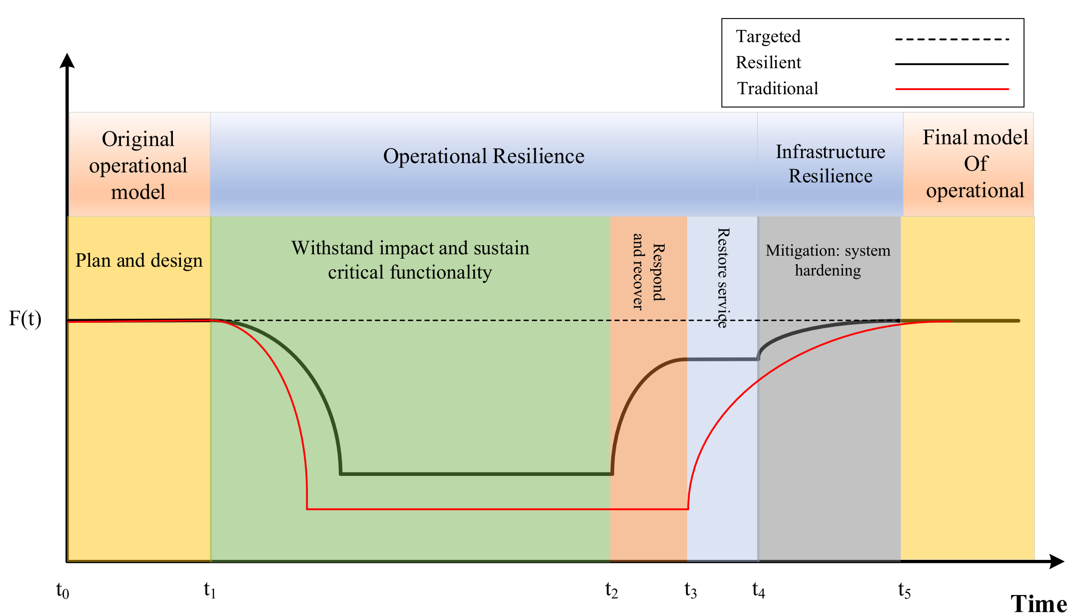

Fig. 5. Comparative analysis of the performance of resilient and traditional networks in the face of unpredictable disturbances [84].

Energy resilience studies have emerged as a significant area of research. This paper comprehensively reviews relevant studies published in reputable databases, including Scopus. The analysis and visualization of this data is conducted using VOSviewer software. The number of papers published in this field from the 1950s to 2024 is examined to assess the frequency of resilience robustness indicators, with the results illustrated in Fig. 6. The findings reveal that most studies have focused on developing centrality metrics, such as degree and betweenness centrality. While the most commonly used centrality metrics, such as closeness centrality, were proposed before the 1950s, substantial research on these indicators has emerged since the 2000s. These trends emphasize enhancing and developing robustness metrics, highlighting their critical role in energy resilience studies.

4. Resilience, reliability, flexibility, and robustness

The differences between reliability and resilience primarily concern their temporal dependency, intensity, frequency, and scale of threats [82,104–107]. Reliability refers to a system's ability to function correctly when faced with likely dangers, while resilience pertains to its capacity to operate effectively despite unlikely threats. Robustness in energy systems involves maintaining performance despite cascading failures caused by random attacks or uncertainties in parameter values [108]. Energy flexibility entails adjusting demand to prevent overproduction and maintain stability in energy networks. Uncertainty-based robustness optimization ensures system functionality in structural breakage or damage [95], involving components like quantifying uncertainty, model development, optimization function definition, and robust analysis [109,110]. Wang et al. [111] employed Monte Carlo simulation to assess integrated energy system resilience through multi-objective optimization, achieving superior performance in total cost, carbon emissions, and failure probability compared to alternatives. Majewski et al. [112] proposed a two-stage strategy balancing robustness and cost-effectiveness to ensure energy supply security while minimizing additional expenses. Kotireddy et al. [113] introduced a non-probabilistic approach to energy system planning resilience evaluation, mitigating significant changes in consumption, cost, and reliability due to uncertainties in external factors and facilitating resilient design implementation.

Fig. 7 illustrates the interconnectedness of reliability, robustness, resilience, and flexibility within multi-energy systems. Ensuring energy supply reliability and power grid stability requires a comprehensive assessment of diverse indicators. Abedi et al. [114] present a structured method for identifying vulnerabilities in power systems and enhancing their resilience and rapid recovery capacity. Shandiz et al. [115] introduce a resilience strategy to maintain energy community stability during persistent challenges and severe events. Nik and Moazami [116] utilize collective intelligence and automated shock absorption methods to enhance demand flexibility and climate resilience, achieving significant reductions in heating demand during extreme weather events. Gracceva and Zeniewski [117] propose a systematic approach to assess the energy security of district communities, considering stability, flexibility, adequacy, resilience, and robustness, thereby contributing to reduced carbon emissions and ensuring reliable energy supply amid climate change challenges.

5. Robustness metrics

Robustness, distinct from reliability, refers to a system's ability to maintain functionality even when specific components fail or unexpected disruptions occur [118,119], focusing on events with high impact but low probability, like extreme natural disasters [120,121]. Understanding and modeling these events are crucial due to their significant consequences [122–125]. Graph theory provides a suitable framework for assessing disturbance propagation and quantifying structural robustness [126–128], identifying critical assets and events triggering cascading failures [129–133]. Research into power system robustness involves modeling and analyzing cascading failures [122,127,128,133–135], recently focusing on leveraging deep learning for efficient measurement [136–139]. Fig. 8 categorizes robustness metrics into five main groups: Priori measures, Posteriori measures, Structural features, Time dependency, and Network area. Each category reflects a specific aspect of network robustness and contributes uniquely to its assessment.

- Priori Measures: These metrics evaluate network robustness based on inherent topological properties, such as node connectivity and degree centrality, without simulating specific attack scenarios. They provide an initial insight into the network's structural integrity and

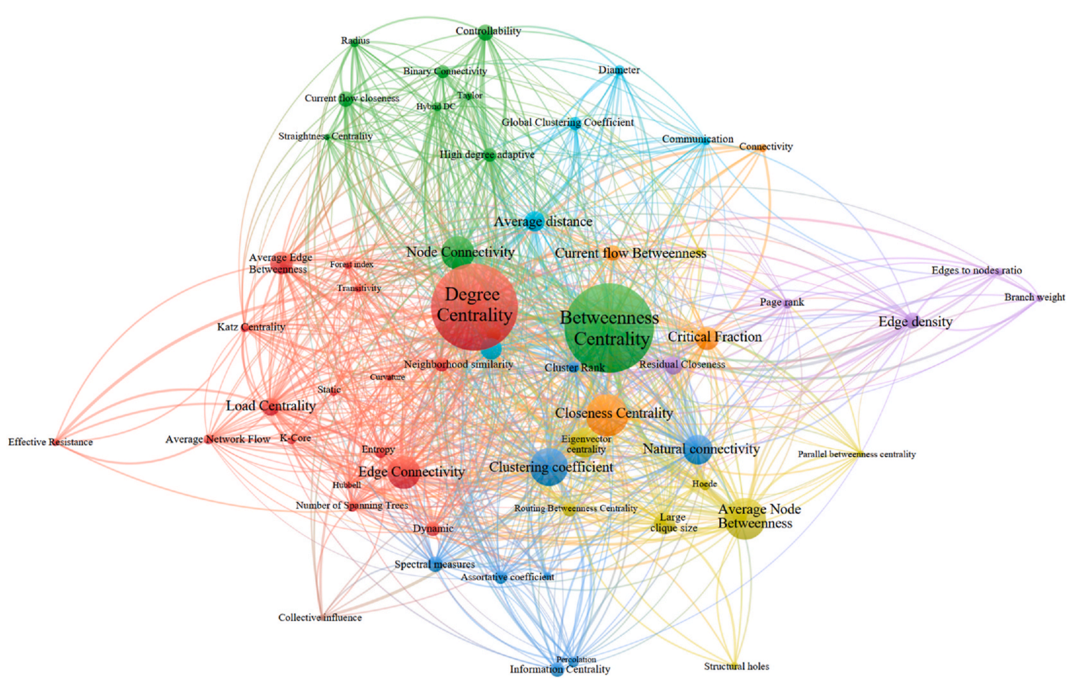

Fig. 6. Robustness metrics.

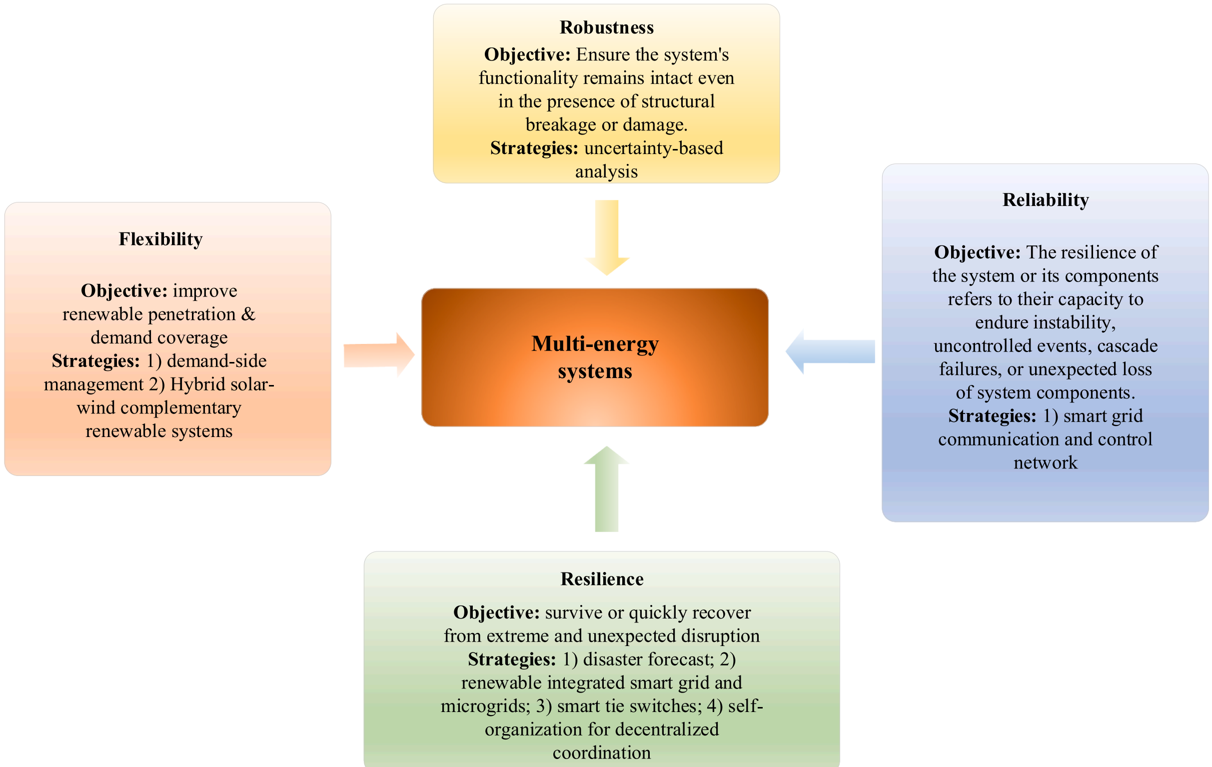

Fig. 7. The interplay between dependability, robustness, resilience, and adaptability in multi-energy systems [70].

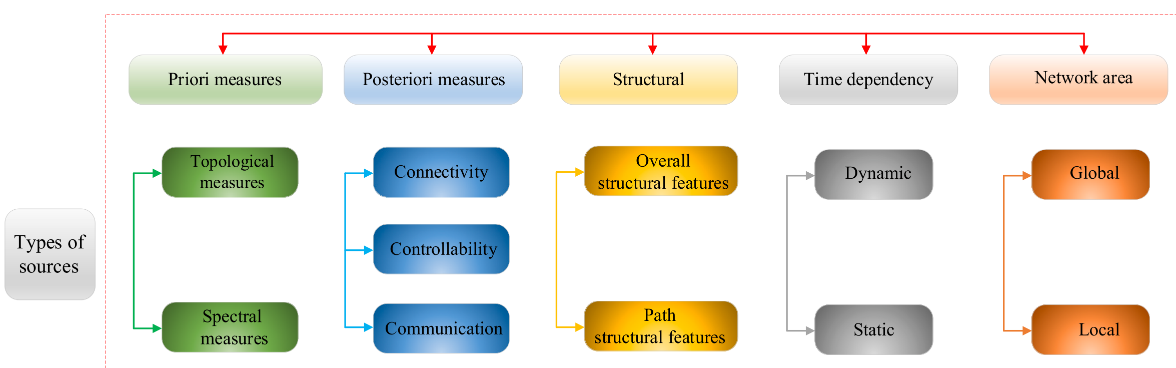

Fig. 8. Quantifying robustness: Categorizing the metrics.

potential vulnerabilities. These features include topological measures and spectral measures.

- Posteriori Measures: This category focuses on the network's ability to sustain functionality after simulated disruptions or attacks. Metrics like the largest connected component (LCC) and controllability robustness are used to determine how well the network retains its operational capacity under failure events. These metrics indicate the network's connectivity, controllability, and communication capabilities.
- Structural Features: Structural metrics examine both overall and path-related attributes of the network, such as clustering coefficients, edge density, and average path length. These features are crucial for understanding the redundancy and connectivity patterns that ensure efficient information or energy flow.
- Time Dependency: Time-based metrics analyze the resilience of networks over time, focusing on how quickly and effectively they can recover from failures. This static and dynamic perspective helps identify weaknesses in recovery mechanisms and guides improvements for faster system restoration.
- Network Area: This category evaluates robustness based on local or global functions segmentation of the network, offering insights into how specific sub-networks or regions contribute to the overall resilience.

5.1. Priori measures

An a priori measure is determined by network features that can be calculated without conducting attack simulations. These features include topological measures and spectral measures. A priori measures require a one-time calculation and typically have lower time and computational complexities than posteriori measures [32,140].

5.1.1. Topological measures

The study comprehensively considers the network's topology and the efficiency of nodes. It also sheds light on the practical utility of advanced deep learning in aiding optimal maintenance prioritization to minimize the intensity and extent of cascading failures effectively [141–143].

5.1.1.1. Degree centrality. *Degree centrality* is a basic measure that evaluates the significance of a node by considering the number of edges connected to it [144,145]. This measure detects failures and attacks in the network graph topology. Degree centrality of node v is defined as:

$$C_{\deg}(v) = \#of\ \ edge\ \ incident\ \ to\ \ v = \sum_{u\in v} A_{vu} = \sum_{u\in v} A_{uv} \tag{1}$$

where A_{vu} represents the number of direct connections between node v and node u.

5.1.1.2. Betweenness centrality. Betweenness centrality measures how often a node connects two other nodes along the shortest paths within the graph. It quantifies the node's influence on network flow, under the assumption that information predominantly travels through these shortest paths in the graph. The betweenness centrality value of a node can be calculated as follows [146,147]:

$$C_i = \sum_{\substack{i\neq j\\ i\neq k}} \frac{g_{jk}(i)}{g_{jk}} \tag{2}$$

The geodesic g_{jk} represents the total number of shortest paths between vertices j and k, where $j \neq k$. $g_{jk}(i)$ denotes the number of these paths that pass through node i.

5.1.1.3. Closeness centrality. The closeness value indicates the average shortest distance from a node to all other nodes in the network. This value is typically denoted as C'_i and is mathematically expressed as [144, 148]:

$$C'_i = \frac{n-1}{\sum_{\substack{i=1\\ i\neq j}}^{n} g_{ij}} \tag{3}$$

where n is number of total nodes and g_{ij} is the geodesic distance between nodes i and j.

Closeness centrality is a measure used to evaluate nodes in a network that offer the most straightforward access to all other nodes. This metric is often used when determining the optimal placement of power stations. Fig. 9 illustrates the node in the graph with the highest centrality value, considering degree, betweenness, and closeness.

5.1.1.4. Binary connectivity. This graph measure serves to assess and involves investigating if all pairs of vertices in the graph have a connecting path. If there exists at least one pair of vertices without a connecting path, the graph is considered unconnected. In simpler terms, this measure allows us to check if every vertex in the graph is reachable from every other vertex, providing valuable insights into the overall structure and connectivity of the graph.

5.1.1.5. Node connectivity. Node connectivity builds upon the concept of binary connectivity and measures the structural robustness of a graph. It is defined as the minimum number of nodes that need to be removed to disconnect the graph, effectively partitioning it into two or more separate components [149]. This metric provides valuable insights into the resilience of a network, highlighting its vulnerability to targeted node failures.

5.1.1.6. Edge connectivity. Edge connectivity, an extension of binary connectivity, quantifies the resilience of a graph by measuring the minimum number of edges that must be removed to disconnect it [150]. This metric highlights the network's robustness against edge failures, offering insights into its structural integrity.

5.1.1.7. Average node betweenness. The average node betweenness captures the overall centrality of nodes within a graph. It is determined by summing the betweenness values of all nodes i in the set v, offering valuable insights into their roles as intermediaries in the network. This metric highlights how nodes contribute to the network's connectivity and the flow of information or resources, as defined in Refs. [146,147]:

$$\bar{b}_v = \sum_{i\in V} C_i \tag{4}$$

The node betweenness C_i for node i is defined in Eq. (2). as the count of shortest paths passing through u out of the total potential shortest paths.

5.1.1.8. Global clustering coefficient. The global clustering coefficient C provides insight into how nodes tend to cluster together in a graph by examining the number of triplets of nodes. This coefficient measures how tightly interconnected nodes are in the graph, mainly focusing on the presence of closed triplets or triangles within the network structure [151]. Global clustering coefficient can be calculated as follows:

$$C = \frac{1}{n} \sum_{n\in V} \frac{2.N_v}{\delta_v(\delta_v - 1)} \tag{5}$$

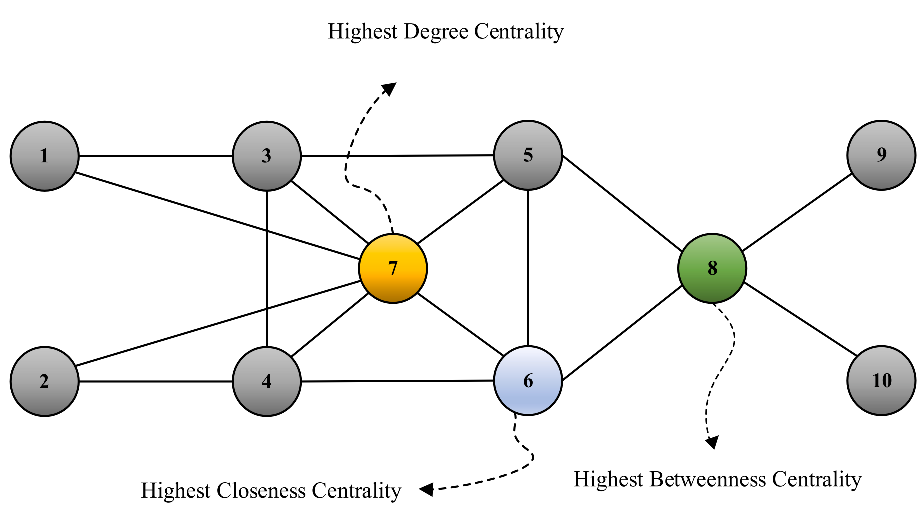

Fig. 9. Nodes with the highest importance based on degree and betweenness metrics.

N_v denotes the number of edges connecting neighboring vertices of v, whereas δ_v indicates the degree of node v.

5.1.1.9. Katz centrality. Katz centrality gauges the significance of a node in a network by considering its number of immediate neighbors (first-degree nodes) and all other nodes linked to it through these immediate neighbors [152]. Katz centrality can be calculated as follows:

$$C_{Katz}(v) = \alpha \sum_{i \in v} A_{uv} C_{Katz}(i) + \beta \tag{6}$$

α represents a coefficient that quantifies the influence of a node's neighbors' centrality on other nodes in the network, taking into account their distances. On the other hand, β denotes a constant credit that is uniformly assigned to all nodes.

5.1.1.10. Hubbell. The Hubbell metric focuses on interpersonal edges within a network topology, emphasizing their role in facilitating bidirectional influence transmission. These edges contribute to cohesion indices, which are vital for detecting cliques and understanding the underlying structure of a network. Based on factor-analytic reasoning, the approach highlights the similarity between incoming and outgoing choice patterns, offering valuable insights into the intricate dynamics of network connectivity and reliability. By analyzing the distribution of edges among nodes, the Hubbell metric provides a deeper understanding of the structural characteristics of a network and its resilience to disruptions or failures [153].

5.1.1.11. Hoede. The Hoede metric evaluates a graph's resistance to node removal, assessing its robustness by measuring the impact on overall connectedness when nodes are removed. This metric provides valuable insights into a network's vulnerability to disturbances or attacks, aiding in evaluating structural soundness and robustness [154].

5.1.1.12. Taylor. The Taylor metric in graph theory evaluates the similarity between nodes based on their neighborhood structures, examining shared neighbors to determine similarities. It facilitates tasks such as node grouping, community detection, and network similarity analysis by assessing neighborhood overlap. This metric aids in understanding the structural properties and organization of complex networks [155].

5.1.1.13. Neighborhood similarity. The neighborhood similarity measure in graph theory assesses node similarity by considering their relative neighborhoods within a graph. It quantifies similarity or dissimilarity by considering shared neighbors, edge density, and other structural aspects. This measure is useful for clustering vertices, detecting communities, and conducting network similarity-based research. It provides insights into connection patterns and linkages, helping understand the structural features of complex networks [156].

5.1.1.14. Edge weights. Edge weights are numerical values assigned to edges in graph theory, indicating the strength or distance of links between nodes. They are crucial for graph-based studies like shortest path computation, network traffic optimization, and clustering analysis. They provide a deeper understanding of interactions and connections, maximize network efficiency, and identify important paths [157].

5.1.1.15. Structural holes. Structural holes quantify the extent to which removing a specific node would disrupt the connections between other nodes in a network. This concept is closely associated with betweenness centrality, as structural holes are often identified using measures derived from it [158]. By highlighting gaps in the network, this metric helps identify critical nodes that facilitate communication and maintain cohesion.

5.1.1.16. Collective influence (CI). The Collective Influence (CI) measure in graph theory quantifies the combined impact of a group of nodes on a network, accounting for both direct and indirect connections. It assists in comprehending information flow dynamics and predicting network behavior, offering insights into resilience and robustness. This metric supports strategic decision-making, resource allocation, and network optimization [159].

5.1.1.17. High degree adaptive (HDA). This metric overlooks scenarios where the network sustains significant damage without a complete collapse. Hence, HDA factors in the size of the largest component throughout all potential malicious attacks, providing a more inclusive evaluation of the network's resilience [160].

5.1.1.18. Page rank (PR). PageRank (PR) is a crucial tool in network analysis that assigns a numerical score to each node within a graph. This score is calculated recursively, considering the quantity and quality of links pointing to a specific node. Higher PR scores indicate more influence or centrality within the network. PR aids in prioritizing resources, optimizing network navigation, and understanding information flow [161].

5.1.1.19. Eigenvector centrality (EC). The Eigenvector Centrality metric in graph theory gauges a node's impact in a network, considering its connections' quality and popularity. Nodes with higher scores are more connected to influential ones. This metric aids in identifying influential nodes, revealing central positions and understanding network functioning. It provides insights into information, influence, and resource flow within the network [162].

5.1.1.20. K-core. The k-core measure in graph theory identifies densely interconnected subgraphs within a network, revealing areas with robust connections. It defines the largest subgraph with at least k connections and assists in identifying central nodes, communities, and clusters. This metric offers insights into the network's hierarchical structure and connectivity patterns, facilitating the discovery of significant structures and relationships [163].

5.1.1.21. Forest index. The forest index is a dependable measure of a network's robustness because it encapsulates certain essential qualities. In a straightforward sense, this index explicitly reflects how well the network can maintain alternative paths between nodes, emphasizing redundancy. Additionally, the forest index considers the number of different pathways of varying lengths between all pairs of nodes, providing a comprehensive view of the network's resilience and connectivity [164–166].

5.1.1.22. Current flow betweenness. The current flow Betweenness metric, derived from a unitary electrical current, considers the spread of current across all potential paths in a circuit. It is not limited to the shortest paths but also considers various pathways for current, providing a broader understanding of the network's response rather than focusing solely on the shortest routes [167,168].

5.1.1.23. Parallel betweenness centrality. The Parallel Betweenness Centrality measure in graph theory employs parallel computing techniques to compute node centrality, reducing computation time, especially in large-scale networks. This metric is valuable for identifying critical nodes, assessing network resilience, and uncovering structural patterns. Its introduction marks a notable advancement in graph theory, offering opportunities for more efficient network analysis [169].

5.1.1.24. Average network flow (ANF). The Average Network Flow is a robustness statistic that measures the capacity of a network with unit edge capacity. It exhibits traits such as compatibility with worldwide network structures, steady growth when more links are added, and quadratic complexity in relation to the number of nodes in networks with few connections. These properties make it a valuable measure for evaluating network resilience [170].

5.1.1.25. Current flow closeness. Current-flow closeness centrality, a variant of closeness centrality, is based on the notion of effective resistance between nodes within a network. Sometimes referred to as information centrality, this metric assesses the efficiency with which information or "current" can traverse between nodes in the network, providing an understanding of their respective significance and interconnectedness [167].

5.1.1.26. Critical fraction (CF). The Critical Fraction (CF), derived from Percolation Theory, assesses network robustness by evaluating the connectivity through its Largest Connected Component (LCC) size. CF represents the fraction of nodes that, when randomly removed, causes the LCC to disappear, with a value approaching zero nearly. This calculation offers insights into network resilience and interconnectedness [171–173]. The critical fraction can be calculated as follows:

$$CF = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1} \tag{7}$$

k represents the average node degree, while k^2 denotes the average square of degrees of all nodes.

5.1.1.27. Number of spanning trees (NST). Kirchhoff's matrix-tree theorem [174] states that the normalized spanning tree count (NST) equals the product of all positive eigenvalues of the Laplacian matrix L*(G)* for a connected graph *G*.

This measure is particularly valuable in network design and analysis, as it reflects the number of alternative paths or configurations available for maintaining connectivity in the presence of node or edge failures. A higher NST indicates a more robust network with greater flexibility in communication or flow between its nodes. The NST also has applications in fields such as electrical circuit analysis, reliability engineering, and biological network modeling, where understanding the structural integrity of a system is critical. It serves as a versatile metric for evaluating the robustness and adaptability of complex networks. The NST can be calculated as follows:

$$NST = \frac{1}{n} \prod_{i=2}^{n} \lambda_i \tag{8}$$

In a network with probabilistic link losses, the probability that a path exists between any pair of nodes is equal to the probability of the existence of a spanning tree, where λ represents an eigenvalue associated with the principal eigenvector.

5.1.1.28. Effective Graph Resistance (EGR). Effective Graph Resistance (EGR) is computed by summing the effective resistances across all pairs of nodes in a graph *G*. It is established that EGR can be computed using the eigenvalues of *L(G)* as follows [175]:

$$EGR = n \sum_{i=2}^{n} \frac{1}{\lambda_i} \tag{9}$$

In simpler terms, when the Effective Graph Resistance (EGR) is higher, the circuit resembles a series configuration rather than a parallel one, suggesting that the network is less robust. Therefore, EGR and network robustness exhibit an inverse relationship. EGR has been applied in various studies, such as [176], to improve network robustness.

5.1.1.29. Natural connectivity. The measure of natural connectivity resilience signifies the structural implications within a network. This metric is linked to the connectivity attributes of a network and discerns alternative routes by considering the weighted count of closed walks. Closed walks represent the total number of complete route traversals. The natural connectedness of the network is calculated as the mean eigenvalue of its adjacency matrix, which is defined as follows [177]:

$$\bar{\lambda}(G) = \ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{\lambda_i}\right) \tag{10}$$

Greater natural connection leads to an increase in the existence of alternate paths, which in turn enhances the resilience of the network. Natural connectedness is beneficial in several domains, such as information technology, and computer science [177]. Moreover, it showcases improved resilience to disconnection in assortative networks, making it a reliable measure even while facing attacks.

Node centrality in a network depends on factors like influence, impact, and popularity, often measured by its connections with neighboring nodes. Local measures, like node degree, are easy to compute and crucial in network analysis, representing the number of neighbors a node has. Metrics often aggregate neighbor-of-neighbor concepts and degree functions. Network density, reflecting edge ratios, can be assessed locally using metrics like clustering coefficient and ClusterRank. Metrics like hybrid degree and clustering coefficient show varying complexity based on the average network degree, assessing centrality by examining the influence of neighboring nodes.

In Table 2, we have condensed the asymptotic complexity of the local centrality metrics discussed in this section using the big-O notation.

In complex networks, where nodes have numerous connections, computation complexity tends to increase sharply with network size. However, in practical scenarios, the degree distribution of many real networks often follows a pattern where most nodes have low degrees, simplifying the calculation of local centrality for these nodes. The term "local" in local centrality refers to its confinement to the immediate vicinity or neighborhood. While local centrality's significance lies in its simplicity, it may only partially capture complex interconnections over extended periods.Table 3 displays topological robustness measures presented in recent years in terms of various parameters.

5.1.2. *Spectral measures*

The "spectral gap of G'' refers to the difference between the two leading eigenvalues (λ_1 and λ_2) of the network's adjacency matrix, essential in assessing network robustness. The symbol $\delta\lambda(G)$ represents the spectral gap and is defined as follows [226]:

$$\delta\lambda(G) = \lambda_1 - \lambda_2 \tag{11}$$

The spectral gap ($\delta\lambda(G)$) indicates network resilience, suggesting optimal interconnection of nodes. This enhances efficiency, mitigates failure impacts, and improves network structural resilience [226]. Graph spectral methods, including spectral gap analysis, are used for network resilience assessment, data clustering, sensor placement optimization, and identifying vulnerable nodes [227].

5.2. *Posteriori*

The Posteriori measure evaluates a network's resilience after multiple attacks, using metrics like the largest connected component (LCC) [160], driver nodes [160,228], and node pairs [229]. These metrics indicate the network's connectivity, controllability, and communication capabilities. A network is considered more robust if it consistently maintains higher LCC node or communicable node pairs, while maintaining lower driver node fractions for control [230].

5.2.1. *Connectivity*

Connectivity means at least one path between two nodes in an undirected network. In contrast, a directed network is considered strongly connected if at least one path exists between nodes in both directions. A

Table 2
Interpretations, calculations, and complexities associated with local point centrality metrics.

Metrics	Definition	Complexity	Ref.
Degree Centrality	Popularity	$O\ (n+m)$ *or* $O\ (n^2)$	[178,179]
Betweenness Centrality	The impact of a node acting as a broker Applying the Floyd-Warshall algorithm Utilizing either Johnson's method or Brandes' technique on a network that has edges with assigned weights Utilizing either Johnson's algorithm or Brandes' technique on an unweighted graph	$O(n^3)$ $O(n^2 \log n + mn)$ $O(mn)$	[146]
Hybrid Degree	A fusion of degree and an adjusted semi-local centrality	$O\left(n\langle k\rangle^2\right)$	[180]
Clustering coefficient	Probability of the neighbors of a node being connected to each other	$O\left(n\langle k\rangle^2\right)$	[181]
Redundancy	Social capital refers to the usefulness or value of a link	$O\left(n\langle k\rangle^2\right)$	[182]
ClusterRank	Weighted semi-local centrality based on clustering coefficient	$O\left(nd_{\max}^2 + n^2\right)$	[183]
K-core	The hierarchical arrangement denotes the node's affiliation within the network	$O(n+m)$	[163]
Mixed degree	Combination of K-core and degree	$O(n+m)$	[184]
Neighborhood coreness	Calculating the combined K-core indices of nearby nodes	$O(n^2+m)$	[185]
Eigenvector	The significance of nearby nodes in determining the relevance of a node	$O(n^3)$	[186]
Katz	Implementing a dampening effect on remote nodes in the calculation of eigenvector centrality	$O(n^3)$	[152]
Page Rank	Google's algorithm incorporates Ktaz centrality, which measures influence based on the number of outgoing links	$O(n^3)$	[161]
Flow betweenness	The quantity of network traffic passing via a node	$O(m^2n)$	[167]
Closeness	The inverse of the sum of distances from a node to all other nodes	$O(mn)$	[144]
Information	The relevance of a node is determined by considering all potential paths that pass through the node	$O(n^3)$	[187]
Current-flow betweenness & closeness	The relevance of a node is determined by the quantity of information it disseminates within a network, analogous to an electric current	$O(n^3)$ for $m < n^2$	[188]
Generalized degree & shortest paths	The significance of a node is evaluated by integrating its degree, closeness, and betweenness metrics, with each metric weighted accordingly	$O(n^2)$ for degree, $O(n^3)$ for closeness and betweenness	[189]
Weight neighborhood	The evaluation of a node's participation in the diffusion process may be done by using a benchmark centrality measure (ϕ), such as degree, betweenness, or K-core	$O(nm)$	[190]
Eccentricity	The influence of a node is determined by its greatest distance from other nodes, with lesser eccentricity indicating higher impact. The Floyd-Warshall algorithm is employed to find a path between two nodes	$O(n^3)$	[191]

(Notations: n represents the total number of nodes, m indicates the number of edges, k signifies the mean degree of nodes, and $d_{\max}$ denotes the maximum degree.).

weakly connected network behaves like an undirected network [32,160,231]. The Largest Connected Component (LCC) assesses network connectivity robustness, as follows [160]:

$$R = \frac{1}{N}\sum_{i=0}^{N-1} n_L(i) = \frac{1}{N}\sum_{i=0}^{N-1}\frac{N_L(i)}{N} \tag{12}$$

$n_L(i)$ and $N_L(i)$ represent the ratio and quantity of nodes in the remaining Largest Connected Component (LCC) after i nodes have been attacked. The measure illustrated in Equation (13) for network robustness under node attacks can be extended to edge attacks [230] as follows:

$$R_1^e = \frac{1}{M+1}\sum_{i=0}^{M} n_L^e(i) = \frac{1}{M+1}\sum_{i=0}^{M}\frac{N_L^e(i)}{N} \tag{13}$$

In the context of edge-attacks denoted by the superscript e, the denominator remains N, assuming a constant number of nodes during edge-attacks. Plotting the values of $N_L(i)$ or $n_L^e(i)$ results in a curve known as the connectivity curve. A higher R_1^e value signifies superior overall connectivity robustness against attacks.

5.2.2. *Controllability*

Controllability robustness shows how effectively a networked system can maintain or regain its controllable state [228,232]. Specifically, removing neighbors of nodes with low degrees (nodes with only 1 or 2 connections) can greatly diminish the network's ability to maintain control effectively. Therefore, during the construction of a network, safeguarding these low-degree nodes and their neighbors can result in networks that are more resilient in controllability [233]. The controllability of robustness can be calculated as follows:

$$Q = \frac{1}{N}\sum_{i=0}^{N-1} q(i) = \frac{1}{N}\sum_{i=0}^{N-1}\frac{N_D(i)}{N-i} \tag{14}$$

The controllability of network is assessed through two metrics: $q(i)$, representing the fraction of driver nodes required, and $N_D(i)$, indicating the number of driver nodes needed to maintain network controllability after removing i nodes. Plotting these values against the fraction of removed nodes creates a controllability curve. A lower Q value on this curve signifies better controllability robustness against node-removal attacks. The term $N_D(i)$ can signify either the state controllability or the structural controllability of the remaining network [228,232,234,235]. In contrast to evaluating the density of directed networks (DNs), control centrality focuses on assessing the control ability of an individual node within a directed network [234,236].

5.2.3. *Communication*

The communication metric in graph theory measures the efficiency

Table 3
Robustness metrics [192–225].

References	Topological measures																							Years
	Degree Centrality	Betweenness Centrality	Closeness Centrality	Binary Connectivity	Node Connectivity	Edge Connectivity	Average Node Betweenness	Global Clustering Coefficient	Katz Centrality	Neighborhood similarity	Edge weights	Collective influence (CI)	High degree adaptive (HDA)	Page rank (PR)	Eigenvector centrality (EC)	K-Core	Forest	Current flow Betweenness	Parallel betweenness centrality	Average Network Flow (ANF)	Current flow closeness	Critical Fraction (CF)	The Number of Spanning Trees (NST)	Effective Graph Resistance	
[166]	✓	✓	×	×	✓	×	✓	×	×	×	✓	×	×	×	✓	×	✓	×	×	×	×	✓	✓	×	2023
[168]	✓	✓	×	×	×	×	×	✓	×	×	✓	✓	×	×	×	×	×	✓	×	×	×	×	×	×	2022
[170]	✓	×	×	×	✓	✓	×	✓	×	✓	✓	×	✓	×	×	×	×	×	×	✓	×	✓	✓	✓	2022
[192]	✓	×	×	×	✓	✓	×	×	×	✓	✓	×	×	×	×	✓	✓	✓	✓	×	×	✓	✓	×	2023
[193]	✓	✓	✓	×	✓	✓	✓	✓	✓	✓	×	✓	×	✓	×	×	×	×	×	✓	×	✓	×	×	2023
[194]	✓	✓	✓	×	✓	×	×	✓	×	×	✓	✓	✓	×	×	×	×	×	×	✓	×	✓	×	×	2023
[195]	✓	✓	✓	×	×	×	×	✓	×	✓	×	✓	✓	✓	×	✓	×	×	×	×	×	×	×	×	2023
[196]	✓	✓	✓	×	×	×	×	×	×	✓	✓	✓	×	×	✓	×	✓	×	×	×	×	×	✓	×	2023
[197]	×	×	×	✓	✓	×	×	×	×	×	✓	×	✓	×	×	×	×	×	×	✓	×	✓	×	×	2023
[198]	✓	✓	✓	×	×	×	✓	✓	×	×	✓	✓	×	✓	✓	✓	×	✓	×	×	×	✓	×	×	2023
[199]	✓	×	×	✓	×	×	×	×	×	×	×	✓	×	✓	✓	×	×	×	×	×	×	×	×	×	2013
[200]	✓	×	×	×	×	×	×	×	×	✓	×	×	×	×	×	×	×	×	×	×	×	✓	×	×	2021
[201]	✓	✓	✓	×	✓	✓	✓	×	×	×	✓	✓	×	✓	×	×	×	×	×	×	×	×	×	×	2023
[202]	✓	✓	×	×	×	×	×	✓	×	✓	×	✓	×	✓	×	×	×	×	×	✓	×	×	×	×	2023
[203]	✓	×	✓	×	×	×	✓	×	×	✓	✓	✓	×	✓	✓	×	×	×	×	×	×	×	×	×	2023
[204]	✓	×	×	×	×	×	✓	×	×	✓	✓	✓	×	✓	×	×	×	×	×	×	×	×	✓	×	2023
[205]	✓	×	×	×	×	×	×	✓	×	✓	×	✓	×	✓	✓	×	×	×	×	×	×	×	×	×	2023
[206]	✓	×	×	×	×	×	×	×	×	✓	×	✓	×	✓	×	×	✓	×	×	×	×	✓	✓	×	2023
[207]	✓	×	×	×	✓	×	×	✓	×	✓	×	×	×	✓	×	×	×	×	×	×	×	×	×	✓	2023
[208]	✓	×	×	×	×	×	×	×	×	×	✓	✓	×	✓	×	×	×	×	×	×	×	×	×	×	2023
[209]	✓	✓	×	×	×	×	×	×	×	✓	✓	✓	✓	✓	×	×	×	×	×	×	×	×	×	×	2024
[210]	✓	×	×	×	✓	✓	×	✓	×	✓	×	✓	✓	✓	✓	✓	×	×	×	×	×	×	×	×	2023
[211]	✓	×	×	✓	×	×	×	×	×	✓	✓	✓	×	×	×	✓	×	×	×	×	×	✓	✓	×	2023
[212]	✓	✓	✓	×	✓	✓	✓	×	×	×	✓	×	×	×	×	×	✓	×	×	×	×	✓	✓	✓	2023
[213]	✓	✓	×	×	×	×	×	×	×	✓	×	✓	×	✓	×	✓	×	✓	×	×	×	✓	×	×	2022
[214]	✓	×	×	×	×	×	×	✓	×	✓	✓	✓	×	✓	✓	×	×	✓	×	×	✓	×	×	×	2023
[215]	✓	×	×	✓	×	×	×	✓	×	✓	✓	✓	×	×	×	×	×	×	✓	✓	×	×	×	×	2022
[216]	×	×	×	✓	×	×	×	✓	×	×	×	×	×	×	×	✓	×	×	✓	✓	×	×	×	×	2022
[217]	✓	×	×	✓	✓	×	×	×	×	✓	✓	×	×	×	×	✓	✓	×	×	✓	×	×	✓	×	2024
[218]	✓	×	×	×	×	×	×	×	×	×	×	✓	✓	×	×		×	×	×	×	×	×	×	×	2023
[219]	✓	×	×	×	×	×	×	×	×	×	×	✓	×	×	×	✓	×	×	×	×	×	✓	×	×	2021
[220]	✓	✓	×	×	✓	✓	×	×	×	×	✓	×	✓	×	×	×	×	×	×	×	×	✓	×	×	2022
[221]	×	×	×	×	×	×	×	×	×	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×	×	2023
[222]	✓	×	×	✓	×	×	×	✓	×	✓	×	✓	×	×	×	×	×	×	×	×	×	×	×	✓	2023
[223]	✓	✓	×	×	×	×	×	×	×	✓	✓	✓	×	×	×	×	×	×	×	✓	×	×	×	✓	2022
[224]	✓	×	×	×	×	×	×	×	×	✓	×	×	×	×	×	×	✓	×	×	×	×	×	✓	×	2023
[225]	✓	×	×	×	×	×	×	×	×	✓	×	✓	✓	×	×	×	×	×	×	×	×	✓	×	×	2023

and effectiveness of communication within a network, evaluating the transmission of information, messages, or signals between nodes. These metrics are crucial for network designers and analysts to enhance topologies and configurations for effective communication and information exchange [237–241].

The CNP-based robustness measure is a widely employed method for evaluating communication robustness, and it is defined as follows [229]:

$$R = \frac{1}{N}\sum_{i=0}^{N-1}\sum_{j=1}^{\Gamma(i)}\frac{\binom{S_j}{2}}{\binom{N}{2}} \tag{15}$$

In this context, $\Gamma(i)$ represents the number of connected components in the remaining network after an attack on a total of i nodes. S_j denotes the number of nodes in the j-th connected component, and $\binom{S_j}{2}$ signifies the count of communicable node pairs, with $\binom{N}{2}$ representing the total number of possible node pairs.

5.3. Structural

The essence of robustness in structural metrics involves conducting a comprehensive evaluation of network structures, focusing on overall structure [242] and pathway-related structure attributes [243]. Complex industrial networks pose challenges in comparison due to their intricacy, leading to the development of various methods to assess specific network properties using comprehensive features and summary statistics [244]. The graph topology generator framework addresses these challenges by flexibly evaluating complex industrial network structures, considering attributes like node density, connectivity, and path-related features such as edge distances and global efficiency metrics, as illustrated in Fig. 10. Analyzing industrial network topology is crucial for optimizing performance, ensuring efficiency, and enhancing resilience against failures or disruptions. As industrial networks grow in complexity, the need for reliable analysis frameworks increases, making this comprehensive study a valuable contribution to developing more dependable industrial networks capable of efficiently adapting to evolving demands.

5.3.1. *Overall structural features*

The overall structural features of a network offer critical insights into the degree of connectivity between its nodes. These features are instrumental in evaluating the network's structure, particularly in terms of redundancy and the classification of its edges. By analyzing these attributes, one can better understand the network's robustness, efficiency, and overall organization.

5.3.1.1. Clustering coefficient. The clustering coefficient offers insights into the connectivity between a specific node and its neighboring nodes. It quantifies the ratio of the number of edges present among the neighbors of node i to the maximum possible number of edges within that group [246]. The clustering coefficient for a node i is computed as follows:

$$C_i(G) = \frac{2e_i}{k_i(k_i - 1)} \tag{16}$$

The variable k_i represents the total number of nodes connected to node i, while ei represents the number of edges linking the neighboring nodes of node i. A node with a high clustering coefficient indicates strong connections with its neighboring nodes [247].

5.3.1.2. Edge density. Edge density is like a measure of how crowded or sparse a network is—think of it as the ratio of actual connections to the maximum possible connections in that network [248]. In simpler terms, edge density is a way of telling us how packed or spread out the connections are in a network, ranging from completely empty (0) to fully packed (1). The edge density can be calculated as follows:

$$g(G) = \frac{2M}{N(N-1)} \tag{17}$$

M represents the number of edges, and N is the total number of nodes. A higher edge density value indicates a greater number of connections between nodes in a network, resulting in more paths and more redundancy.

5.3.1.3. Assortative coefficient. The assortative coefficient is a metric closely related to Pearson's correlation coefficient. It measures the degree of correlation between the attributes of connected node pairs within a network. In essence, it quantifies the extent to which nodes with similar properties tend to be connected. This measure is formally defined as follows [69,249]:

$$r(G) = \frac{\sum jk\left(e_{jk} - q_j q_k\right)}{\sigma_q^2} \tag{18}$$

e_{jk} denotes the joint-excess degree probability for excess degrees j and k, while σ_q^2 represents the variance of q_k.

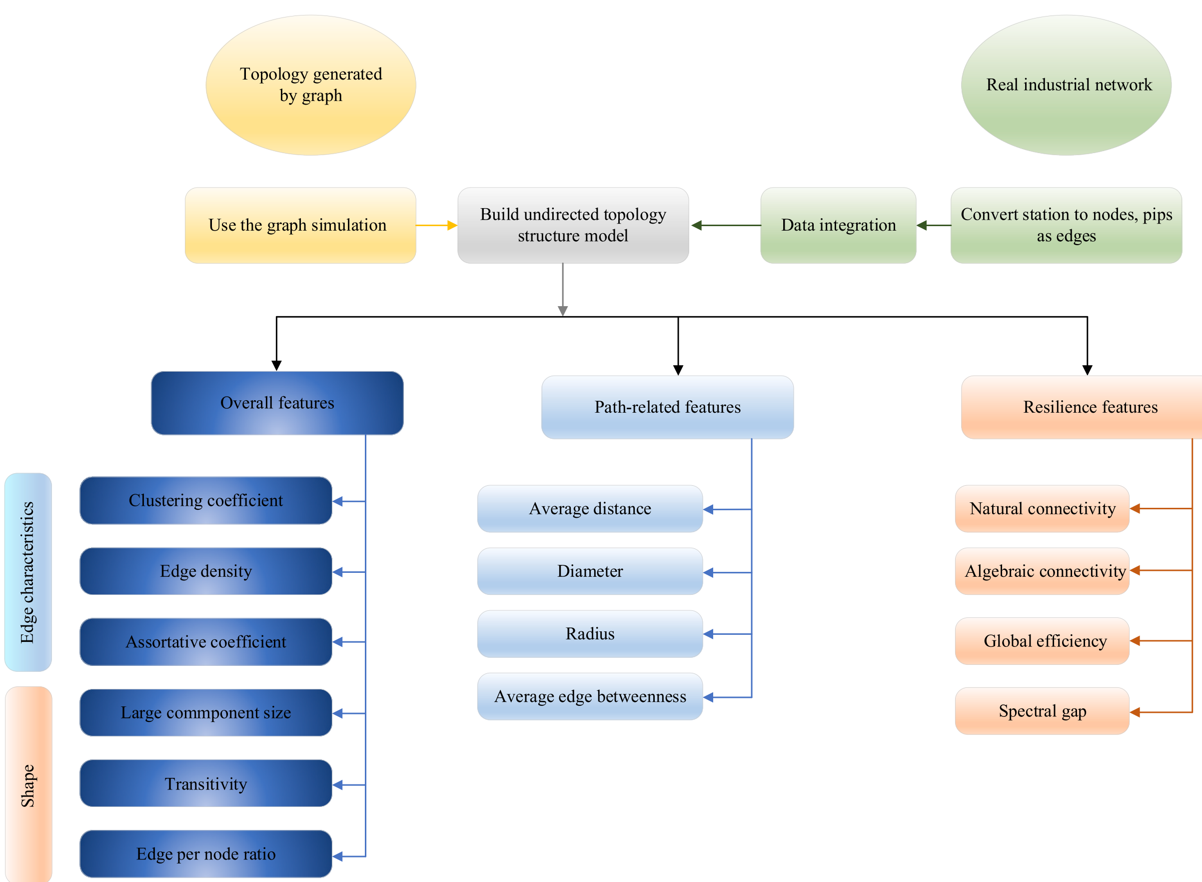

Fig. 10. Overall structure, path-related structure, and topological structure resilience [245].

5.3.1.4. Large clique size. network can consist of several interconnected components, each forming a distinct cluster of nodes. The network is connected if at least one of its components is linked. The largest connected component is the cluster with the largest number of individual nodes, and the more connected nodes within it, the more robust the network's connectivity [250,251]. This information is crucial for network design, ensuring smooth data transfer and identifying dense traffic areas [252]. Equation (19) can be used to assess system performance and determine network resilience.

$$LCC = \frac{N_F}{N_I} \tag{19}$$

where N_F final size of connected nodes in the graph when the failure occurs and N_I initial size of several connected nodes before the failure event.

5.3.1.5. Transitivity. Transitivity measures the probability of all adjacent nodes in a network being linked. It is calculated by comparing the number of triangles (three vertices and edges) to the number of linked triplets of nodes. The triplets can have two undirected edges for open connections or three undirected edges for closed connections [253]. Using this method, the transitivity of a graph can be calculated as follows:

$$T(G) = \frac{3\Delta(G)}{t(G)} \tag{20}$$

$\triangle(G)$ represent the count of triangles in a graph, and $t(G)$ represent the count of triplets. When $T(G) = 1$, it means that the graph has all conceivable edges. This also suggests that the graph has well-defined clusters or cliques with robust and extensive connections between nodes.

5.3.1.6. Edges to nodes ratio. The edges-to-nodes ratio measures a network's interconnectedness, comparing the number of edges to the number of nodes. It is a valuable metric commonly employed to characterize the interconnectedness of a graph, providing insights into its overall structure and complexity. High-density graphs have more edges than nodes, indicating a dense network, while low-density graphs have fewer edges than nodes, indicating a sparser structure [254]. The edges-to-nodes ratio is calculated as follows:

$$e(G) = \frac{M}{N} \tag{21}$$

M denotes the total number of edges, and N represents the total number of nodes in the network. When the value of e(G) is nearly equal to one, the network is likelier to demonstrate a tree-like or multi-star topology. On the other hand, as the value becomes close to two, it is more probable for the network topology to resemble a mesh structure [255].

5.3.2. Path structural features

Understanding the structural features of a path is crucial for evaluating networks, as they reveal the efficiency of information transmission by identifying the shortest path between nodes. Researchers analyze factors like average distance, diameter, radius, and average edge betweenness to understand the network's effectiveness and general functioning [245].

5.3.2.1. Average distance. The mean shortest path length, denoted as D (G), represents the average distance between any pair of nodes within a network [256]. It offers insights into the overall characteristics of the network, indicating the degree of separation between nodes. Moreover, a decrease in the mean distance corresponds to enhanced efficiency and faster transmission within the network. The formula for calculating D(G) is as follows:

$$D(G) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} d_{ij} \tag{22}$$

d_{ij} represents the shortest route between node i and j. Dijkstra's algorithm [257] is commonly used to determine this shortest path.

5.3.2.2. Diameter. The diameter of a network signifies the maximum distance between any pair of nodes, employing the geodesic distance where all edges are assigned the same weight [258]. It quantifies the network's furthest reach, representing the longest potential path between nodes. In simpler terms, it offers insights into the most distant connections within the network. The calculation of the diameter, denoted as *d(G)*, is determined by follows:

$$d(G) = Max(d_{ij}) \tag{23}$$

The diameter metric is essential to a network's size and complexity. A larger diameter generally indicates a more intricate and interconnected structure. In contexts such as supply chains, a large diameter suggests the involvement of multiple intermediaries, meaning that information or resources must pass through several stages before reaching their final destination.

5.3.2.3. Radius. The radius of a graph corresponds to the minimum eccentricity within the network. It represents the shortest distance from a node to the farthest node. A smaller radius signifies a more condensed network, where nodes are closer to each other [259].

5.3.2.4. Average edge betweenness. The edge betweenness is computed by dividing the total count of shortest paths from node i to node j passing through edge e by the overall number of shortest paths between nodes i and j in the graph [260,261]. This can be expressed as:

$$b(G) = \frac{1}{|E|} \sum_{e \in E} b_e \tag{24}$$

$|E|$ denotes the total number of edges in the graph. The average edge betweenness of a graph gauges its connectivity level. A higher value of *b (G)* suggests that the graph is more susceptible to edge removal [262].

5.4. Network area

Network robustness research can be categorized into topology-based and operational robustness. Topology-based research examines network structure, focusing on nodes and links without weights. Operational robustness identifies influential nodes in the network's core using metrics like degree and betweenness. More sophisticated studies aim to algorithmically break down a network, identifying subsets of nodes crucial for its integrity. Both types of research are essential for understanding network integrity.

5.4.1. Global

Global metrics offer a holistic perspective of the entire graph, providing valuable insights into the overall network state and enabling comparisons across different structures. Global robustness measures encompass various metrics such as the number of spanning trees [263], the Kirchhoff index [69,264–266], the spectrum radius of the adjacency matrix [267], and the Randic index [268–270]. The Randic index indicates network robustness, computed as the average square difference of eigenvalues for the normalized Laplacian matrix from their mean. Essentially, a lower Randic index suggests higher network robustness. Moreover, leveraging the eigenvalues of the normalized Laplacian matrix aids in assessing network vulnerability [271].

5.4.1.1. L-betweenness centrality. Ercsey-Ravasz and Toroczkai [272] introduced the concept of L-betweenness as a formalized measure to minimize the computational expenses associated with betweenness calculation. This centrality, initially proposed by Borgatti and Everett [273], involves considering the shortest paths with a length not exceeding L as follows:

$$C_{L-betweenness}(v) = \sum_{s,t|s \neq v \neq t, d(s,t) \leq L} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{25}$$

If the value of L is greater than or equal to the diameter of the network, then the L-betweenness measure corresponds to the betweenness centrality metric. Ercsey-Ravasz and Toroczkai [272] describe this measure explicitly as the cumulative total of betweenness centralities across all nodes, considering just the shortest paths of a certain length.

5.4.1.2. Load centrality. Load centrality is a concept introduced by Goh et al. [274] to calculate the total number of data packets passing through a node. This computation occurs after each node dispatches a solitary packet to other nodes using the shortest available route. If many shortest pathways exist between two nodes, the burden is evenly distributed at each point of divergence along the paths. The process of calculating the load Centrality for a node v is described as follows:

$$C_{load}(v) = \sum_{s,t|s \neq v \neq t} \theta_{st}(v) \tag{26}$$

$\theta_{st}(v)$ represents the count of units that traversed from node s to node t through node v. This quantity is evenly distributed at each junction encountered along the shortest paths from s to t.

5.4.1.3. Routing betweenness centrality. An approach to calculating betweenness centrality, which integrates the predicted traffic load across the network, is introduced by Dolev et al. [275]. The predicted quantity of data packets passing through a particular node is calculated using the routing betweenness centrality measure. The process of calculating the routing betweenness for a node v is described as follows:

$$C_{routing-bet}(v) = \sum_{s,t \in v} \sigma_{st}(v).T(s,t) \tag{27}$$

$\sigma_{st}(v)$ represents the probability of a packet successfully traversing through node v when it is transferred from s to t. The function T(s,t) represents the overall count of possible routes from point s to point t. The probability is influenced by the specific routing protocol utilized.

5.4.1.4. Information centrality. Stephenson and Zelen [276] introduced information centrality, a centrality measure that calculates potential paths connecting nodes and integrates potential information transfer. This metric considers noise during signal transmission, assessing signal fluctuations and diminished information with distance. Node v's information centrality is established by calculating the harmonic mean of information exchanged between nodes, with each link having a variance of one.

$$C_{\inf ormation}(v) = \frac{n}{\sum_{u \in v} \frac{1}{I_{uv}}} \tag{28}$$

I_{uv} reflects the aggregated information from all connections from node u to node v, with each path's length considered as a weight.

5.4.1.5. Residual closeness. Dangalchev [277] proposed the concept of residual closeness as a metric for evaluating the susceptibility of a network to disconnection when certain nodes or edges are removed. This metric is defined as follows:

$$C_{residual-closeness}(v) = \sum_{u \neq v} \left(\frac{1}{2}\right)^{d(u,v)} \tag{29}$$

Residual closeness utilizes a weighting technique instead of adopting the reciprocal of the sum of distances. While a similar concept has been generalized in the existing literature [278], it was only later explicitly formulated as a centrality metric [279].

5.4.1.6. Straightness centrality. Crucitti et al. [280] enhanced conventional metrics to account for the heterogeneity in distances between neighboring network nodes. They suggested modified versions of closeness and betweenness centralities [281], considering the actual distance or weight between nodes as the basis for calculation. Additionally, they introduced a new metric called straightness centrality [280]. The process of calculating the Straightness Centrality for a node v is described as follows:

$$C_{straightness}(v) = \frac{1}{n-1} \sum_{u \in v, u \neq v} \frac{d_{Eucludean}(u,v)}{d(u,v)} \tag{30}$$

$d_{Eucludean}(u,v)$ signifies the Euclidean distance in either the real or embedded space.

5.4.1.7. Analytic Hierarchy Process Centrality. Bian et al. [282] presented the Analytic Hierarchy Process Centrality (AHP) as a decision-making method specifically designed for finding nodes that have a significant impact. The process to calculate it involves the following steps:

$$C_{AHP} = s \times w^T \tag{31}$$

The matrix s has dimensions n × 3, with each column s_j (for j = 1, 2, 3) representing a different component. Additionally, w^T denotes the transpose of the vector w, which consists of weights w_j (where j = 1, 2, 3) arranged sequentially. The core idea is that the SI scores in this method are based on short-term periods, while the results from the AHP could be relevant over longer timeframes. The Analytic Hierarchy Process (AHP) integrates three commonly used centrality metrics and assigns weights to them using a method derived from a short-run epidemic compartmental model.

5.4.1.8. Percolation centrality. Influence centrality has been introduced to represent changes in network structure as the influence process unfolds by Piraveenan et al. [283]. The process of calculating the Percolation Centrality for a node v is described as follows:

$$C_{percolation}(v,t) = \frac{1}{n-2} \sum_{r \neq v \neq s} \frac{\sigma_{rs}(v)}{\sigma_{rs}} \frac{x_r(t)}{\sum_{u \in v} x_u(t) - x_v(t)} \tag{32}$$

σ_{rs} denotes the overall count of shortest paths between nodes r and s, while σ_{rs} represents the total number of these shortest paths passing through node v. When all nodes undergo (partial) percolation uniformly, leading to all shortest paths becoming percolated paths, it leads to a scenario where percolation centrality becomes directly correlated with betweenness centrality.

5.4.2. Local

Local robustness measures evaluate network robustness by focusing on more localized information, relying on factors like degree, triangles, and shortest paths. Many of these measures emphasize network centrality, which includes metrics such as degree centrality and betweenness centrality. Commonly utilized local measures involve assessing the semi-local centrality, the hybrid degree centrality, volume centrality, entropy-based measures of a node or edge.

5.4.2.1. SEMI-LOCAL centrality. A hub node, which connects numerous neighbors, may be located at the network's periphery, causing a lack of impact or information transmission capacity. Chen et al. [284] introduced semi-local centrality, or local centrality, to balance hub node prominence with significant betweenness. This approach considers neighboring node levels and quantifies the importance of a node within its local neighborhood.

$$C_{semi-local}(v) = \sum_{u \in N(v)} Q(u) \tag{33}$$

Q(u) exhibits a more favorable comparison to rankings derived from a susceptible removal procedure.

5.4.2.2. Hybrid degree centrality. The probability of spreading, denoted as p, is crucial in determining the impact of local and near-local neighborhood structures during information transfer processes. A lower value favors degree centrality, while a higher value prioritizes a more inclusive measure. Ma & Ma [180] integrated the magnitude of p into centrality by merging degree centrality with semi-local centrality [284] to define the hybrid degree centrality of a node v, denoted as:

$$C_{hybrid}(v) = (\beta - p) \,.\, \alpha \,.\, C_{\deg}(v) + p \,.\, C_{m-local}(v) \tag{34}$$

The equation includes the following variables: p (spreading probability), $C_{m-local}$ (modified local centrality), α (normalizing factor for adjusting degree centrality), and β (optimization parameter for balancing degree and modified local centrality).

5.4.2.3. Volume centrality. If the spread of a process terminates within a specific range from its origin or includes a time-out element, it's reasonable to assume that this can be effectively represented by the topology surrounding the source node. Wehmuth and Ziviani [285] introduced a definition for the volume centrality of a node within a certain radius h:

$$C_{volume}(v) = \sum_{u \in N_h(v)} d(u) \tag{35}$$

This is an adapted rendition of the initial concept introduced by Wehmuth and Ziviani [285]. When $h = 0$, volume centrality is equivalent to degree centrality. Nonetheless, this aspect is already taken into account when computing the degrees of nodes in $N_h(v)$. However, with the increase in h, the complexity of the approach also grows.

5.4.2.4. Entropy-based measures. Entropy quantifies the amount of information lacking in a specific process within the context of information theory as a measure of system disorder. Entropy concepts are employed in networks to characterize systems or processes [286]. Nie et al. [287] extended the notion of entropy to encompass centrality, introducing two variations for its quantification: local entropy, which assesses how a node influences network entropy, and mapping entropy, which takes into account the neighboring nodes of a specific node.

$$\begin{aligned} C_{local-entropy}(v) &= -\sum_{u \in N(v)} d(u) \log d(u) \\ C_{mapping-entropy}(v) &= -d(v) \sum_{u \in N(v)} \log d(u) \end{aligned} \tag{36}$$

5.4.2.5. Cluster Rank. High clustering can impede information propagation or dissemination. In light of this, Chen et al. [183] introduced the idea of Cluster Rank, which considers both the degree of a node and the connections among its neighbors via the clustering coefficient. The Cluster Rank of a node v is a metric that measures the degree of clustering or interconnectivity of the node inside a network as follows:

$$C_{clusterrank}(v) = f\left(C_{clustering}(v)\right) \sum_{u \in N^{out}(v)} \left(C_{out-\deg}(u) + 1\right) \tag{37}$$

Chen et al. [183] used the function $f(x) = 10^{-x}$, where $C_{clustering}(v)$ denotes the local. The clustering coefficient, tailored for directed networks, incorporates the out-degree of node v in its summation. This coefficient serves as a damping mechanism, imposing a penalty on heightened clustering in scenarios where there are fewer distinct connections to various parts of the network. The influence of this dampening weight is diminished when a significant percentage of v's neighbors have a high quantity of extra neighbors.

5.4.2.6. Curvature. The efficacy of hyperbolic models in faithfully capturing real-world network phenomena [288,289] has sparked interest in quantifying the inherent geometry of complex networks. Measuring the degree of curvature in networks is intriguing since models often assume a uniform curvature, although real-world data seldom adheres to such simplicity. Eckmann and Moses [290] introduced a curvature formulation that corresponds to Watts and Strogatz's local clustering coefficient [181]. This formulation uncovers a correlation between elevated curvature and shared subjects. Another widely used method includes calculating curvature using Gaussian curvature on planar graphs [291,292]. This technique has also been extended to complicated networks [293] as:

$$C_{Gauss-curv}(v) = \sum_{k \geq 0} (-1)^k \frac{S_v^{k+1}}{k+1} \tag{38}$$

S_v^k quantifies the amount of k-cliques associated with node v [294].

Table 4 summarizes the meanings, equations, and computational complexities of global point centrality metrics, typically computed using metrics like betweenness and load. Global centrality metrics offer insights into clustering patterns among network components and identify pivotal nodes promoting connections between different areas. However, calculating these metrics is challenging due to the complex process of establishing distances or shortest paths for each pair of nodes, with computational complexity increasing with network density. Compared

Table 4
Interpretations, calculations, and complexities associated with local point centrality metrics.

Metrics	Definition	Complexity	Ref. No.
Semi-local	The popularity of a node is determined by the sum of its own popularity and the popularity of its neighboring nodes	$O\left(n\langle k\rangle^2\right)$	[284]
Volume	The magnitude of a sphere with a radius of *h*, centered at the node	$O\left(n\langle k\rangle^{h+1}\right)$	[295]
Entropy-based measures	Level of information deficit within the node's local network	$O(n^2)$	[287]
Curvature	Node's local geometry measurement	$O(2^n)$	[293]
Contribution	Calculating eigenvector centrality with weights based on structural dissimilarity	$O(n^3)$	[296]
Subgraph	Frequency of nodes appearing in walks, taking into account the length of the walks	$O(n^3)$	[297]
Dynamical influence	The dynamic initial state is incorporated into the eigenvector centrality	$O(n^3)$	[298]
L-betweenness	A refined iteration of betweenness centrality that specifically considers pairs with distances smaller than L	$O(mn)$	[272]
Load	Determining the number of packets that pass through a node when each node sends a packet to every other node via the shortest path can be achieved using the Dijkstra method	$O(mn)$	[275]
Residual closeness	A modified closeness metric was employed to assess the vulnerability in the graph, utilizing the Floyd-Warshall algorithm to calculate path distances	$O(n^3)$	[277]
AHP-based	The influence of a node can be assessed by ranking nodes according to several centrality measures, such as degree, betweenness, or closeness	$O(n^3)$	[282]
Percolation	Vulnerability of a node to being percolated	$O(n^3)$	[283]

(Notations: n represents the total number of nodes, m indicates the number of edges, k signifies the mean degree of nodes.).

to iterative point centrality measures, global centrality metrics incur higher costs, necessitating networks with a minimum number of edges proportional to the order of nodes. To mitigate complexity costs, limiting the length of pathways considered, such as using the L-betweenness metric and balancing centrality approximation and associated costs, is advisable.

5.5. Time dependency

Current research on network robustness focuses on understanding how the arrangement of connections affects network connectivity, often from a static topology perspective. However, studies frequently overlook the nuanced interactions and dependencies between network nodes. The dependency link, describing the characteristics of interactions between nodes, is crucial for understanding network robustness and resilience. When nodes fail, the burdens they carried shift to other nodes, leading to dynamic spread of loads and secondary cascading failures. This dynamic propagation adds complexity to our understanding of network robustness. The unexpected resilience in the face of cascading failures suggests a more adaptive and robust nature of nodes in real-world networks [231, 299].

5.5.1. Dynamical influence

Klemm and colleagues [298] introduced dynamic influence as a metric of centrality, aiming to assess how a node's dynamic state influences the overall behavior of a complex network, taking into account both the node's dynamic condition and the network's structure. In systems characterized by n time-dependent real variables, linear dynamics are represented by an n × n real matrix *M*. The largest eigenvalue μmax of *M* is utilized to categorize the dynamics. If the maximum growth rate, μmax, is negative, the vector x(t) will converge to a null vector, indicating a stable fixed solution. On the other hand, if μmax is positive, the vector x(t) will expand endlessly from its original state x(0). Given the presence of a conserved quantity, the equation allows for the determination of the end state, denoted as x(t), based on the starting state, *x(0)* as:

$$C_{dynamic-\inf luence} = x(\infty) = \lim_{t\to\infty} x(t) = \frac{c\ .\ x(0)}{c\ .\ e} e \tag{39}$$

e represents a specific eigenvector of matrix *M* corresponding to the maximum eigenvalue μmax. The phrase suggests that $C_{dynamic-\inf luence}$ is calculated by projecting using *x(0)*, where ci denotes the influence of *x (0)* on the ultimate state.

5.5.2. Static influence

The threshold function represents the operator's ability to endure harm in relation to the percentage of components. The evaluation of the resilience of each feasible topological configuration in the distribution system, taking into account the important issue of power flow feasibility, is combined to estimate the overall resilience of the system. The concept of holistic resilience is mathematically characterized utilizing a stochastic, static, energy-based, and operational measure, as represented by Equation (40) [300].

$$R = r + (1 - r) \sum_{u=1}^{u-1} w_u R_u \tag{40}$$

The vector R = [R_1, R_2, …, R_U] represents the resilience scores or values linked to u possible configurations inside the distribution network. The normalized weights allocated to resilient configurations are denoted as wu and r represents the greatest value among [R_1, R_2, …, R_U]. The vector Ru is an ordered arrangement of vectors containing composite resilient values for networks lacking maximum resilience. The rise in the total resilience score signifies an increase in the number of pathways linking functional components to critical loads. Hence, this research provides a comprehensive analysis of the metrics discussed in several studies, including those listed in Table 5, by evaluating them based on different factors.

6. Limitations and discussion

We surveyed 59 robustness of resilience metrics in this work in terms of group selection. We introduced the quantitative and functional measurement methods of each metric. The key limitations and implications are summarized as follows.

- Resilience metrics may be used across diverse fields for distinct objectives. Moreover, a substantial array of robustness measures exists and may be used for diverse design objectives. For instance, we may seek to examine methods for balancing traffic loads, establishing connections between nodes to enhance network resilience against failures or assaults, devising specific targeted attacks, identifying critical nodes based on diverse criteria, or determining the least influential or most vulnerable node within a particular network.
- Energy resilience may be analyzed across energy systems for distinct objectives. Moreover, many strategies exist and may be employed to enhance resilience. We defined and quantified resilience explored evaluation methods, examined its relationship with reliability, robustness, and flexibility, and tried introducing all the quantitative assessment methods for resilience robustness metrics.
- Energy resilience often adopts a top-down approach within the energy community. However, bridging this perspective with a broader, multidisciplinary discourse on resilience is essential for a more nuanced understanding. Addressing the technological and economic aspects comprehensively involves numerous parameters, which adds significant complexity. While many publications concentrate on specific parameters, they overlook finer temporal and spatial resolutions.
- The current state and future potential of energy sharing have not been sufficiently explored. Key aspects such as infrastructure development, a comprehensive analysis of techno-economic-environmental performance, and strategies for enhancing energy resilience require deeper investigation. Furthermore, the willingness of various stakeholders to engage in cross-border energy initiatives has received limited attention despite its critical role in fostering collaboration and ensuring the success of such systems.
- Integrating advanced technologies, such as smart grids, renewable energy sources, and energy storage systems, is crucial for enabling efficient cross-border energy exchange. A robust framework that balances economic feasibility, environmental sustainability, and social acceptance must be developed to overcome existing barriers and facilitate long-term cooperation.

7. Future perspectives

- Numerous resilience metrics can adequately fulfill particular tasks while maintaining lower complexity. These metrics can be utilized as they are or developed to enhance their efficiency and effectiveness. Certain measures, such as communicability and controllability, provide a more comprehensive understanding of centrality, extending beyond simple connectedness. However, their high complexity limits their applicability in other fields.
- The size of robustness measures is commonly used as a standard metric for assessing network resilience. However, future research should aim to develop more comprehensive measures that assess network resilience in greater detail. This should include factors such as fault tolerance, recovery capability, and other characteristics that are specifically tailored to the needs of different systems.
- Centrality metrics may be improved as an innovative measure of network resilience. Centrality metrics assess certain attributes of a network, including node distances, neighbor relationships, and redundant pathways between nodes. We can enhance the current

Table 5
Organizing resilience metrics in scientific papers based on various perspectives.

References	Posteriori measures			Structural		Time dependency		Spectrum	
	Connectivity	Controllability	Communication	Overall	path	dynamic	static	planning	operational
[228]	×	✓	×	✓	×	✓	×	×	×
[229]	×	×	✓	✓	×	✓	×	×	✓
[230]	✓	×	×	✓	×	×	✓	×	✓
[231]	✓	×	×	✓	✓	×	✓	×	✓
[234]	×	✓	×	×	✓	✓	×	×	✓
[232]	×	✓	×	✓	×	✓	×	×	✓
[247]	×	×	✓	✓	×	✓	×	×	✓
[249]	✓	×	×	✓	×	✓	×	×	✓
[255]	✓	×	×	✓	×	×	✓	×	✓
[245]	✓	×	✓	×	✓	✓	×	✓	×
[256]	×	×	✓	×	✓	×	✓	✓	×
[258]	✓	×	×	×	✓	✓	×	✓	×
[259]	✓	×	✓	×	✓	✓	×	✓	×
[260]	×	×	✓	×	✓	✓	×	✓	×
[261]	✓	×	×	×	✓	✓	×	✓	×
[262]	×	×	✓	×	✓	✓	×	✓	×
[301]	×	✓	×	×	×	✓	×	×	✓
[246]	×	✓	×	✓	×	✓	×	×	×
[302]	×	✓	×	×	×	×	✓	✓	✓
[237]	×	×	✓	×	✓	✓	×	×	×
[238]	×	×	✓	×	✓	✓	×	×	✓
[239]	×	×	✓	×	✓	✓	×	×	✓
[233]	×	✓	×	✓	×	✓	×	✓	×
[240]	×	×	✓	×	✓	×	✓	✓	×
[241]	×	×	✓	×	✓	✓	×	✓	×
[248]	✓	×	×	✓	×	✓	×	✓	×
[303]	✓	×	×	✓	×	✓	×	✓	×
[250]	✓	×	×	✓	×	✓	×	✓	×
[251]	×	×	✓	✓	✓	✓	×	×	✓
[252]	✓	×	×	✓	×	×	✓	✓	×
[254]	×	✓	×	✓	×	✓	×	✓	×
[304]	×	✓	×	×	✓	✓	×	×	✓
[305]	✓	✓	×	×	✓	×	✓	×	✓
[306]	×	×	✓	✓	×	×	✓	✓	×
[307]	×	✓	×	×	✓	×	✓	✓	✓
[308]	×	✓	×	×	✓	×	✓	×	✓
[309]	×	✓	×	✓	×	×	✓	×	✓
[310]	✓	×	✓	✓	×	×	✓	×	✓
[311]	×	✓	×	✓	×	✓	×	✓	✓
[312]	✓	×	×	✓	×	✓	×	✓	✓
[313]	×	×	✓	✓	×	✓	×	✓	✓
[314]	×	✓	×	✓	×	✓	×	✓	✓
[315]	×	✓	×	×	✓	✓	×	×	✓
[316]	✓	×	×	✓	×	✓	×	×	✓
[317]	✓	×	×	✓	×	✓	×	×	✓
[318]	✓	×	×	×	✓	✓	×	×	✓
[319]	×	×	✓	×	✓	×	✓	✓	×

centrality metrics or develop new indicators pertinent to the essential characteristics of network resilience.

- A comprehensive examination of network resilience across diverse network conditions is essential. Owing to spatial limitations, we have not presented analyses to examine the impact of employing resilience metrics within the network. We can analyze network resilience by employing robustness metrics or other resilience metrics such as vulnerability and recovery. Furthermore, diverse network topologies may be examined to derive more significant insights that yield generalizable criteria for choosing effective robustness metrics in a specific application.

8. Conclusion

This study provides a comprehensive analysis of energy resilience, focusing on ensuring system survival during extreme events. It covers various aspects, such as defining the concept, quantifying it mathematically, analyzing potential threats, and studying robustness mechanisms. Emphasis is placed on addressing resilience beyond energy efficiency or robustness, particularly for high-impact, low-probability events like extreme weather or wartime scenarios. The interconnectedness of resilience with factors like reliability, robustness, and flexibility in multi-energy systems is highlighted, along with challenges and trade-offs in energy system planning and operation. Major findings include.

1) Concentrating solely on energy planning and design tailored to low-impact, high-probability (LIHP) events may not guarantee resilience in the face of high-impact, low-probability (HILP) events. Energy resilience empowers systems to endure unforeseen and severe disruptions, facilitating rapid recovery thereafter. Nonetheless, the tension between energy resilience and energy efficiency underscores the need for nuanced solutions that strike a balance between the two.
2) A range of threat sources, including vulnerabilities in infrastructure, operational hurdles, challenges in data management, government regulations, and policies, contribute to a diverse set of potential risks. In addressing these diverse threat sources, mechanisms for system recovery through energy resilience primarily entail preparedness for resilience, rapid adaptability, and service restoration. Defensive strategies usually involve forecasting natural disasters during the planning phase focused on resilience, deploying decentralized power supply units and additional tie switches during the response phase

centered on resilience, and implementing targeted reinforcement strategies during the restoration phase emphasizing resilience.

3) Addressing uncertainties, especially in long-term planning, is vital in enhancing resilience. Difficulties integrating future climate data due to low resolution and multiple climate model complexities arise. Moreover, technology maturity and market conditions introduce uncertainties often overlooked in planning processes. While some models attempt to tackle these uncertainties, many overlook them entirely. Transitioning to stochastic models could address uncertainties but might surpass existing models' capabilities to handle them.
4) Assessing system robustness against cascade failures, especially in the electricity sector, is crucial for enhancing resilience. While progress has been made in evaluating N-1 security and addressing wildfires' impact on power outages, studies on extreme events are limited. Additionally, assessments considering interconnected energy infrastructure are scarce. Despite exploring the impact of extreme events on coupled cyber-physical systems, resilience analysis of interconnected infrastructure within the energy system still needs to be improved. Closing these gaps requires holistic approaches to effectively understanding energy infrastructure's robustness to extreme events.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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