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Robust Safety Critical Control of Uncertain Nonlinear Systems with DoS Attacks

Yi Dong, *Member, IEEE*, Xiran Cui, Yiguang Hong, *Fellow, IEEE* and Jie Chen, *Fellow, IEEE*

***Abstract*—This paper considers the robust safety critical control problem of an uncertain second-order nonlinear system subject to denial-of-service (DoS) attacks on the sensor-controller and controller-actuator channels. In order to handle the safety constraint described by a general nonlinear function, we first propose a governor function, regarded as a generalized barrier function, with additional properties. Then a safety critical control is designed to guarantee the safety and stability of the closed-loop system. Finally, a combined time- and event-triggered sampling mechanism is further introduced to detect the randomly occurring DoS, and the latest sampled information is used to update the safety critical control once the signal can be received through the sensor-controller channel. Our sampled-data safety critical control strategy can reduce to solve the safety critical control and the security control problems of nonlinear systems with parametric uncertainty.**

***Index Terms*—safety critical control, uncertain nonlinear system, stabilization, DoS attack**

I. Introduction

For a practical system, both the state safety and the security are of paramount importance. The safety control problems usually refer to safety verification problems [1], [2] or safety critical control problems, the latter of which require to design control laws to preliminarily guarantee the safety of the system. The methods for safety critical control problems can mainly be classified into two groups. Most existing works are based on the optimization technique in the context of quadratic programming (QP), and the representative ones can be found in [3]–[11], etc. [3], [4] created the framework for the safety critical control problem with multiple objectives of safety and stabilization, and synthesized control laws by uniting control barrier function (CBF) with control Lyapunov function. Within the CBF-QP framework, [5] further considered the model uncertainty and external disturbances with the output measurement by combining the extended state observer. [6] proposed a robust optimal control to deal with the system stability as well as input and state constraints through QP, while [7] developed an adaptive data-driven safety control method to achieve the forward invariance of a safe set. [8] designed a distributed predictive control for constrained linear multi-agent systems, and [9] considered the safety-critical multi-agent systems based on QP. The main design idea for the second group of the safety critical control is to construct a control Lyapunov barrier function, based on which, a safety and stabilization control law is designed with the aid of Lyapunov-like theorem [12]. [13] proposed a safety critical control law to handle the conflicted safety and tracking purposes, and a tradeoff between keeping the trajectory in the safe area and decreasing the tracking error was achieved based on a constructed performance indicator function.

Networked systems with state/output or safety constraints are more vulnerable to cyber attacks in the sense that the measurement data and the control input can be manipulated by attackers to severely compromise the system performance by inducing the system into the unsafe mode. Commonly seen cyber attacks include deception, replay, DoS attacks and their finite combinations. Deception attacks destroy the trustworthiness of the data, and the adaptive control technique can be applied to achieve the uniform ultimate boundedness of the closed-loop system [14], [15]. DoS disrupts the time line of the information exchange. Great efforts have been made to mitigate DoS attacks ever since [16] established the relationship between the input-to-state stability for a linear system and the frequency and duration conditions of DoS attacks. [17] considered the input-to-state stabilizing problem with DoS in multiple transmission channels. Innovative methods such as predictor-based resilient strategy in [18] and deadbeat controller in [19] have been proposed for linear time-invariant systems against DoS attacks. For multi-agent linear systems, [20] proposed a secure cooperative event-triggered control for the consensus problem, while [21] designed a resilient practical control law for the cooperative output regulation problem with unknown switching leader system. Generally speaking, most control methods for resisting DoS attacks were proposed for linear systems with a few exceptions. A group of Euler-Lagrange systems were studied in [22] based on the Lyapunov approach and multidimensional small-gain technique, and [23] considered a special class of nonlinear systems.

Motivated by the safety and security requirements of practical systems such as railway signaling network [24] and unmanned vehicles [25], it is imperative to consider the safety critical control problem with cyber attacks. Such a cross-layer problem is very challenging, noting that safety critical control requires the continuous monitoring on the system trajectory whereas cyber attacks randomly tamper or prevent the transmission of the information. For example, [26] considered DoS attacks and collision avoidance. The state

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Y. Dong, X. Cui, Y. Hong and J. Chen are with College of Electronic and Information Engineering, State Key Laboratory of Autonomous Intelligent Unmanned Systems, Frontiers Science Center for Intelligent Autonomous Systems, Ministry of Education, Tongji University, Shanghai, China.

Corresponding author: Y. Hong. E-mail: yghong@tongji.edu.cn.

information is denied on the sensor-controller channel and the input is prevented from reaching the actuator, and thus, the system is actually an open-loop system with no control over the system trajectory, let alone to keep the trajectories safe for all the time. Technically, it is difficult to bridge the gap between the safety critical control and security control because the former follows a relatively fixed design procedure based on CBFs, while for detecting and mitigating different cyber attacks, the latter usually adopts a variety of control strategies ranging from predictive control, adaptive control to reinforcement learning technique.

This paper considers the safety critical control problem for an uncertain second-order nonlinear system subject to randomly occurring DoS attacks at both controller and actuator sides, and aims to achieve multiple control objectives including guaranteeing the system safety, mitigating DoS attacks and stabilizing the closed-loop system. Such a problem generalizes the safety critical control problems in [3], [4], [6], [7], [12] and the security control problems in [16], [17], [22], [23]. To solve such a complicated problem, we propose a governor function with its explicit construction, based on which, an effective safety critical control is provided. The contribution of our design includes the following three aspects. First, we define a novel governor function to simultaneously accommodate the safety constraint with a general nonlinear function and the stabilization requirement. It can also be used to solve the state/output constrained problems in [27]–[29]. Then a safety critical control law is designed for a second-order nonlinear system, noting that CBF based control methods in existing works [12], [13] are inadmissible for further fighting against cyber attacks. Finally, it is also technically challenging to solve the security control problem for a nonlinear system under DoS attacks, as opposed to that for the linear ones in [16], [17], [20], [21]. It actually requires that the trajectory of the nonlinear system converges exponentially without DoS and its growth can be accurately evaluated during DoS. By further proposing a combined time- and event-triggered sampling mechanism, we eventually provide a sampled-data safety critical control design along with an algorithm for selecting control parameters.

Throughout the paper, for a vector or a matrix, $\|\cdot\|$ represents its Euclidean ($\mathcal{L}_2$) norm. $\mathbf{0}_n = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}^T_{n\times 1}$ denotes the n-dimensional vector with all element being 0, and $\mathbf{1}_n = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^T_{n\times 1}$.

II. Problem Formulation

Motivated by the safety and security requirements of networked systems, we consider the following nonlinear system with safety constraints and cyber attacks on the sensor-controller and controller-actuator channels.

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x_1,\sigma) \\ \dot{x}_2 &= u + f_2(x_1,x_2,\sigma) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x(t) = \mathrm{col}(x_1(t), x_2(t))$ with $x_1(t) \in \mathbb{R}^n$ and $x_2(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^n$ is the input, $y(t) \in \mathbb{R}^n$ is the output, and $\sigma \in \Omega \subseteq \mathbb{R}^{n_\sigma}$ is an uncertain parameter vector with Ω compact and containing the origin of $\mathbb{R}^{n_\sigma}$. Functions $f_i(\cdot)$, $i = 1, 2$, are globally defined, sufficiently smooth and satisfy $f_i(0,\sigma) = \mathbf{0}_n$ for all $\sigma \in \Omega$.

For the safe operation of system (1), the output is required to satisfy the following constraint,

$$\begin{aligned} S &= \{y \in \mathbb{R}^n :\ h(y) < \epsilon\} \\ \partial S &= \{y \in \mathbb{R}^n :\ h(y) = \epsilon\} \end{aligned} \tag{2}$$

where constant $\epsilon > 0$ and $h : \mathbb{R} \to \mathbb{R}$ is a C^3 function satisfying that $\frac{d^i h(y)}{dy^i}$, $i = 1, 2, 3$, is bounded for all $y(t) \in S$. Assume $h(0) < \epsilon_0$ for some constant $0 < \epsilon_0 < \epsilon$ and $\bar{S} = S \cup \partial S$ is bounded. Note that (2) can describe a variety of constraints including the output constraint in [29], [30] and the configuration constraint for robotic systems in [10].

Remark 2.1: The safety critical control problem of second-order nonlinear system (1) has been considered in the literature. See [4], [10]. (1) can describe benchmark nonlinear systems such as mechanical systems [12], [13], robots [10], unmanned vehicles [25] and aircrafts [11]. Control techniques for system (1) are also applicable to the high-order nonlinear system as follows,

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(\bar{x}_i,\sigma),\ i = 1,\cdots,r-1 \\ \dot{x}_r &= u + f_r(\bar{x}_r,\sigma)\ y = x_1 \end{aligned} \tag{3}$$

where $\bar{x}_i = \mathrm{col}(x_1,\cdots,x_i)$ with $x_j \in \mathbb{R}^n$, $j = 1,\cdots,i$. The details will be provided in Section IV-B.

The state $x(t)$ of the networked system (1) is transmitted through the sensor-controller channel and the input $u(t)$ is through the controller-actuator channel. Both channels are vulnerable to malicious cyber attacks, which motivates us to further consider the security problem of the safety critical system (1). To be specific, the attackers may deny the transmission of the signals $x(t)$ and $u(t)$. We use the sequence $\{\delta_p\}_{p\in\mathbb{N}^+}$ to denote DoS off/on transitions with $\delta_1 > 0$. Define $\Delta_p = \{\delta_p\} \cup [\delta_p, \delta_p + \tau_p)$ to represent the p-th DoS time-interval, and $\tau_p \geq 0$ is the length that the signal $x(t)$ fails to transmit. If $\tau_p = 0$, DoS is a single pulse. For each time interval $[\tau, t]$ with $0 \leq \tau \leq t$, define

$$\Xi(\tau,t) = \bigcup_{p\in\mathbb{N}^+} \Delta_p \cap [\tau,t], \quad \Theta(\tau,t) = [\tau,t] \setminus \Xi(\tau,t), \tag{4}$$

where $\Xi(\tau,t)$ denotes the set of time instants where the signal transmission is denied and the control input $u(t) = \mathbf{0}_n$.

We provide the following conditions for DoS attacks, which capture certain randomness of DoS attacks in the real-world network environments and determine the amount of DoS that a system can tolerate before undergoing instability under the current control technologies as in [16], [17], [20]–[23].

Assumption 2.1: There exist $\kappa_1 \geq 0$ and $T_D > 1$ such that

$$|\Xi(\tau,t)| \leq \kappa_1 + \frac{t-\tau}{T_D} \tag{5}$$

where $|\Xi(\tau,t)|$ denotes the length of the intervals over which the signal transmission is denied, and κ_1 serves to make (5) consistent.

Assumption 2.2: There exist $\kappa_2 \geq 0$ and $\tau_D > 0$ such that

$$P(\tau,t) \leq \kappa_2 + \frac{t-\tau}{\tau_D} \tag{6}$$

where $\tau_D \geq \tau_m > 0$ and $P(\tau, t)$ denotes the number of DoS off/on transitions during $[\tau, t]$.

Remark 2.2: Based on the concept of average dwell-time introduced by [31], Assumption 2.1 requires that the average time instants, over which communication is interrupted, do not exceed a certain fraction of time. The rationality of this assumption lies in that the duration of DoS cannot be unbounded, constrained by the limited energy of the attacker. Also, by imposing a minimum interval between consecutive attacks, Assumption 2.2 restricts the frequency of DoS attacks, which must be sufficiently small compared to the minimum sampling rate [16]. Otherwise, the control update based on the sampled signals cannot compete with the high-frequency occurrence of DoS, resulting in the failure of any adopted control even if Assumption 2.1 holds.

Then the robust safety critical control problem of the uncertain nonlinear system (1) with DoS attacks on both channels is formulated as follows.

Problem 2.1: Consider the second-order nonlinear system (1) with the safety constraint (2) and subject to DoS attacks (4) satisfying Assumptions 2.1 and 2.2. For any $y(0) \in S$, propose a control strategy such that the trajectory of the closed-loop system exists, and satisfies that $y(t) \in S$ for all $t \geq 0$ and $\lim_{t\to+\infty} x(t) = \mathbf{0}_{2n}$.

Remark 2.3: The technical challenges of Problem 2.1 are summarized as follows. First, it is difficult to handle the safety constraint $h(y) < \epsilon$ with $h(\cdot)$ being a general nonlinear function, and existing barrier functions are mainly designed for the state/output constraints, inadmissible for the constraint (2). Second, Problem 2.1 also considers DoS attacks and safety critical control laws generated from CBF-QP framework cannot be applied here since DoS not only prevents the continuous monitoring of the trajectory, but also results in an open-loop system with $u(t) = \mathbf{0}_n$. Finally, DoS occurs randomly, and therefore, it is required to figure out a way to detect DoS, noting that mimicking the zero-order-hold control is impossible since we cannot let $u(t) = \mathbf{0}_n$ for $t \in [\delta_p, \delta_p+\tau_p)$ with δ_p and τ_p unknown.

The safety critical control laws in existing works, e.g., [12], [13], take the following form,

$$u(t) = \phi(x(t)) \tag{7}$$

where $\phi(\cdot)$ is some smooth function with $\phi(0) = \mathbf{0}_n$. However, DoS disrupts the transmission of the signal $x(t)$ over $\Xi(0, t)$ whereas (7) requires the continuous feedback. Thus, any safety critical control in the form of (7), depending on $x(t)$ for all $t \geq 0$, is inapplicable. To further handle the randomly occurring DoS in Problem 2.1, we design a robust safety critical control with sampling mechanism for detecting DoS, which takes the following form,

$$u(t) = \phi(x(t_{k(t)})) \tag{8a}$$

$$x(t_{k(t)}) = \begin{cases} \mathbf{0}_{2n}, & \text{if DoS is detected} \\ x(t_k), & \text{otherwise} \end{cases} \tag{8b}$$

where $t_{k(t)}$ is the latest time instant for sampling the state $x(t)$, $\{t_k\}_{k\in\mathbb{N}}$ is the sequence of sampling time instants and $x(t_{k(t)})$ is the latest sampled data. Under the control (8), the structure of the closed-loop system (1) is given by Fig. 1.

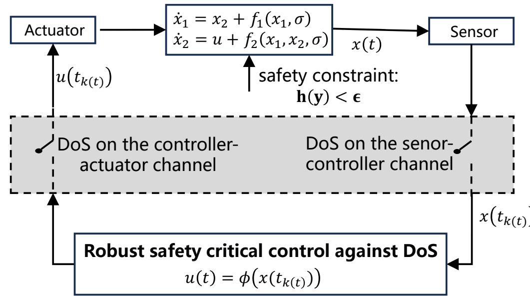

Fig. 1: The structure of the system (1).

III. Governor Function based Safety Critical Control Design

In this section, we specify the form of the sampled-data safety critical control (8).

A. Governor Function Design

Note that existing CBF based safety critical control cannot work effectively in practical situations with some dynamic requirements and/or cyber attacks, and barrier functions cannot deal with the safety constraint (2) with $h(\cdot)$ nonlinear in y. Thus, we generalize barrier functions in [27], [28] and propose a new concept, called the governor function.

Definition 3.1: Function $G : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is called a governor function if it satisfies the following properties:

1) $G = G(h(y), y)$ is C^r for some integer $r \geq 1$;
2) $G \to \mathbf{0}_n$ as $y(t) \to \mathbf{0}_n$;
3) $||G|| \to G_m$ for some $0 < G_m \leq +\infty$ as $h(y) \to \epsilon$;
4) $\frac{dG}{dy} \geq G_0 I_n$ for all $y(t) \in S$ and some constant $G_0 > 0$.

In order to present a suitable governor function for a second-order system, let $r = 2$ and

$$S_m = \{y \in \mathbb{R}^n : \ h(y) < \epsilon_m\}, \ \epsilon_0 < \epsilon_m < \epsilon. \tag{9}$$

Since $h(0) < \epsilon_0 < \epsilon_m$, $y = \mathbf{0}_n$ is an interior point of S_m, and thus, there exists a neighborhood S_0 of $\mathbf{0}_n$ such that $S_0 \subset S_m$. Define S_0 as

$$\begin{aligned} &S_0 = \{y \in \mathbb{R}^n : ||y|| \leq r_0\}, \quad 0 < r_0 < x_m, \\ &x_m \leq \text{dist}(\mathbf{0}_n, \partial S_m). \end{aligned} \tag{10}$$

Define a smooth switching function $\chi : \mathbb{R} \to [0, 1]$, which is nondecreasing, C^3, and positive for all $\tau > 0$. To be specific, $\chi(\cdot)$ satisfies:

i) $\chi(\tau) = 0$ for $\tau \leq 0$, and $0 < \chi(\tau) \leq 1$ for $\tau > 0$;
ii) $\chi(\tau) = 1$ for $\tau \geq x_m - r_0$;
iii) $\frac{d^i\chi(\tau)}{d\tau^i}$, $i = 1, 2, 3$, is bounded for all τ.

Then construct a function of the following form,

$$z_1 = \chi(||y|| - r_0)H(h(y))\mathbf{1}_n + \varsigma y \tag{11}$$

where $\varsigma > 0$ is a constant to be designed, and

$$H(h(y)) = \frac{1}{\epsilon - h(y) + \frac{1}{\beta}} \tag{12}$$

with a constant $\beta > 0$.

We establish the following lemma to show that (11) is a governor function. Define

$$\mu(y)=\frac{dz_1}{dy}=\frac{\frac{d\chi(||y||-r_0)}{d||y||}\mathbf{1}_n\frac{y^T}{||y||}}{(\epsilon-h(y)+\frac{1}{\beta})}+\frac{\chi(||y||-r_0)\mathbf{1}_n\frac{dh(y)}{dy}}{(\epsilon-h(y)+\frac{1}{\beta})^2}+\varsigma I_n. \tag{13}$$

Lemma 3.1: Under the conditions that the set $\bar{S}$ and $||\frac{dh(y)}{dy}||$ for all $y(t)\in S$ are bounded, (11) is a governor function with $0<G_m<+\infty$ if ς satisfies

$$\varsigma\geq\sqrt{n}\beta\chi_m+\sqrt{n}\beta^2h_m+\varsigma_0,\ \varsigma_0>0, \tag{14}$$
$$\Big|\frac{d\chi(||y||-r_0)}{d||y||}\Big|\leq\chi_m,\ h_m=\max_{y(t)\in\bar{S}}\Big\{\Big|\Big|\frac{dh(y)}{dy}\Big|\Big|\Big\}.$$

Proof: (1) From $H(\cdot)$ in (12) and the definition of the smooth switching function $\chi(\cdot)$, z_1 is a C^2 function.

(2) Note that as $h(y)\to\epsilon$, $y(t)\in S\backslash S_m$, and thus, $||y(t)||\geq x_m$ with x_m given by (10). From Property ii) of function $\chi(\cdot)$, $\chi(||y||-r_0)=1$ for $||y||-r_0\geq x_m-r_0$. Also note that $H(h(y))\to\beta$ as $h(y)\to\epsilon$. Thus, $z_1\to\beta\mathbf{1}_n+\varsigma y$. When $h(y)=\epsilon$,

$$||z_1||=G_m\in\Big\{y\in\partial S:\sqrt{n\beta^2+2\beta\varsigma\sum_{j=1}^{n}y_j+\varsigma^2||y||^2}\Big\}, \tag{15}$$

where $y=\mathrm{col}(y_1,\cdots,y_n)$ for $y_i\in\mathbb{R}$, $i=1,\cdots,n$. Since $\bar{S}$ in (2) is bounded, y is bounded for $h(y)\leq\epsilon$, and then there exists some constant $0<G_m<+\infty$ such that $||z_1||\to G_m$ as $h(y)\to\epsilon$.

(3) As $y(t)\to\mathbf{0}_n$, $y(t)\in S_0$, and thus, $\chi(\cdot)\to0$ from Property i) of function $\chi(\cdot)$. $H(h(y))\to\frac{1}{\epsilon-h(\mathbf{0}_n)+\frac{1}{\beta}}$ with $h(\mathbf{0}_n)<\epsilon_0<\epsilon$ as $y(t)\to\mathbf{0}_n$, implying that $z_1\to my\to\mathbf{0}_n$.

(4) From Property iii) of function $\chi(\cdot)$, χ_m exists. And for all $y(t)\in S$,

$$\Big|\Big|\frac{\frac{d\chi(||y||-r_0)}{d||y||}\mathbf{1}_n\frac{y^T}{||y||}}{\epsilon-h(y)+\frac{1}{\beta}}\Big|\Big|\leq\beta\chi_m\Big|\Big|\mathbf{1}_n\frac{y^T}{||y||}\Big|\Big|\leq\sqrt{n}\beta\chi_m.$$

Since $||\frac{dh(y)}{dy}||$ is upper bounded for all $y(t)\in\bar{S}$, h_m also exists. Note that $0<H(h(y))<\beta$ for all $y(t)\in S$ and $\chi(||y||-r_0)\leq1$ for any y. Then for all $y(t)\in S$,

$$\Big|\Big|\frac{\chi(||y||-r_0)\mathbf{1}_n\frac{dh(y)}{dy}}{(\epsilon-h(y)+\frac{1}{\beta})^2}\Big|\Big|\leq\beta\Big|\Big|\frac{dh(y)}{dy}\Big|\Big|\leq\sqrt{n}\beta^2h_m.$$

Thus, for all $y(t)\in S$,

$$||\mu(y)||\leq\mu_m, \tag{16}$$

where $\mu_m=\sqrt{n}\beta\chi_m+\sqrt{n}\beta^2h_m+\varsigma$. Note that $\mu(y)$ in (13) is in the form of (72). By Lemma A.1, if ς satisfies (14), then for all $y(t)\in S$,

$$\mu(y)\geq\varsigma_0I_n. \tag{17}$$

In conclusion, according to Definition 3.1, (11) is a governor function. ■

From Definition 3.1, G_m serves as a flag when $y(t)\to\partial S$. However, from (15), G_m is not unique. Thus, we establish the following lemma to provide a condition to check if $h(y)<\epsilon$ is satisfied.

Lemma 3.2: For $y(T_1)\in S$ with $0\leq T_1<+\infty$, if z_1 satisfies

$$||z_1(t)||<z_m,\ t\in[T_1,T_2],\ T_1<T_2\leq+\infty \tag{18a}$$
$$z_m=\min\{G_m\} \tag{18b}$$

with G_m given by (15), then $y(t)\in S$ for all $t\in[T_1,T_2]$.

Proof: Suppose that there exists some time instant $T_1<\tau_1\leq T_2$ such that $y(\tau_1)\notin S$. For $y(T_1)\in S$, by the continuity of the trajectory $y(t)$, there exists $T_1<\tau_2\leq\tau_1$ such that $y(\tau_2)\in\partial S$. From (15) in Lemma 3.1, $||z_1(\tau_2)||=G_m$, implying that $||z_1(\tau_2)||\geq z_m$, contradicting (18a). Then $y(t)\in S$ for all $t\in[T_1,T_2]$ if $||z_1(t)||<z_m$. ■

Remark 3.1: The governor function in Definition 3.1 is essentially a generalized barrier function, and its design is based on those for the state/output constraint systems [27]–[30] with some improvement. First, the governor function is $G\in\mathbb{R}^n$, while the barrier function is usually one-dimensional and in the form of $B=\frac{y}{h(y)}$. For example, $h(y)=h_1(y)h_2(y)$ for $h_1(y)=y+F_{11}$ and $h_2(y)=F_{12}-y$ in [27]. The second one is the design of G_m. In existing works, $G_m=+\infty$. However, in order to evaluate the growth of $x(t)$ of the uncontrollable system during DoS, we can only design a bounded one, which requires to provide a finite and specific value, depending on G_m, that $||z(t)||$ cannot overstep before $y(t)$ arrives at ∂S. It is nontrivial to do so since $z(t)$ depends on both $h(y)$ and y, and from (15), $||z(t)||$ does not necessarily reach its largest value when $h(y)\to\epsilon$. Finally, Property 4) is autonomously satisfied for barrier functions, while it is very hard to provide the positive value of the lower bound of $\frac{dG(h(y),y)}{dy}$ since the sign of $\frac{dh(y)}{dy}$ is dramatically changing. To overcome this difficulty, we adopt an additive form of a safety term and a stabilization term ςy in (11). In the safety term, the smooth switching function $\chi(\cdot)$ is introduced to realize the smooth transition from the safety purpose to the stabilization once $y(t)$ enters S_0, and the ingenuity of the stabilization term ςy is that it can be used to satisfy Property 2), and meanwhile, the parameter ς is designed from (14) to govern the first two terms in (13) and ensure $\frac{dz_1}{dy}\geq\varsigma_0I_n$.

B. Safety Critical Control Design

Since DoS attacks are launched randomly, δ_p and τ_p, $p\in\mathbb{N}^+$, are unknown for the control design. Thus, we employ the combined time- and event-triggered sampling mechanism for detecting DoS. For this purpose, define two sequences of non-negative numbers $\{\bar{\delta}_p\}_{p\in\mathbb{N}^+}$ and $\{\bar{v}_p\}_{p\in\mathbb{N}^+}$ to store the switch time instants of two sampling methods, and Fig. 2 is provided to show the basic idea.

If the signal $x(t)$ can be received through the sensor-controller channel at $t=t_i$, $i\in\mathbb{N}$, one can infer that there is no DoS. In such a case, update the control with the latest received data $x(t_{k(t)})=x(t_i)$, and use the event-triggered condition to determine the next time instant $t_{i+1}=\vartheta_i$ for

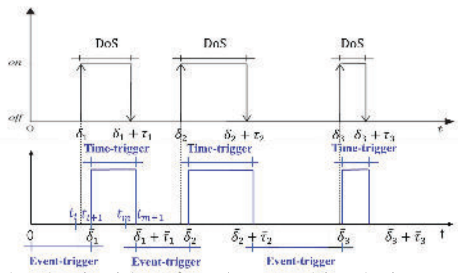

Fig. 2: The basic idea for the combined time- and event-triggered sampling mechanism.

updating the control. At $t = t_{i+1}$, the signal transmission is prevented and we can infer that DoS occurs within the time interval $[t_i, t_{i+1})$, and switch to the time-triggered mechanism at $t = t_{i+1}$. We use $\bar{\delta}_i$ to record the beginning of the time-triggered sampling after DoS is detected. Let $x(t_{k(t)}) = \mathbf{0}_{2n}$ and $u(t) = \mathbf{0}_n$ once DoS is detected. And sample the data with the time period ϑ_t, i.e., $t_{m+1} = t_m + \vartheta_t$, $m \in \mathbb{N}$. The time-triggered sampling is used until $t = t_{m+1}$ when the signal can be received again, and then one can infer that DoS stops within the time interval $[t_m, t_{m+1})$. We also use $\bar{\delta}_i + \bar{\tau}_i$ to record the ending of the time-triggered sampling after DoS stops. Under such combined sampling mechanism, $\{t_k\}_{k\in\mathbb{N}}$ is designed as

$$t_{k+1} = \begin{cases} t_k + \vartheta_t, & k \in \Gamma \\ \vartheta_k, & k \notin \Gamma \end{cases} \tag{19}$$

where $\Gamma = \{k \in \mathbb{N} : t_k \in \cup_{p\in\mathbb{N}^+} \Delta_p\}$ denotes the set of integers associated with the control update attempt during DoS, $\tau_m \leq \vartheta_t \leq \vartheta_m$ is the time sapling period with ϑ_m as the upper bound of inter-sampling rate during DoS, and ϑ_k is the time instant determined by the event-triggered condition

$$\vartheta_k = \inf\{t \in \mathbb{R}_{>t_k} : \ ||e(t)|| \leq \gamma ||z(t)||\} \tag{20}$$

for $e(t) = u(t_{k(t)}) - u(t)$ and $z = \mathrm{col}(z_1, z_2)$ and $\gamma > 0$. Also define $\bar{\Theta}(\tau, t)$ to denote the sets of time instants that the event-triggered mechanism is used during $[\tau, t)$, while the set $\bar{\Xi}(\tau, t)$ is for the time-triggered mechanism.

$$\begin{aligned} &\bar{\Theta}(\tau, t) = \bigcup_{p\in\mathbb{N}^+} W_{p-1} \cap [\tau, t], \quad \bar{\Xi}(\tau, t) = \bigcup_{p\in\mathbb{N}^+} Z_p \cap [\tau, t], \\ &W_{p-1} = \{\bar{\delta}_{p-1} + \bar{v}_{p-1}\} \cup [\bar{\delta}_{p-1} + \bar{v}_{p-1}, \bar{\delta}_p), \\ &Z_p = \{\bar{\delta}_p\} \cup [\bar{\delta}_p, \bar{\delta}_p + \bar{v}_p), \end{aligned} \tag{21}$$

for $\bar{\delta}_0 = 0$ and $\bar{v}_0 = 0$. Note that (20) holds for the set $\bar{\Theta}(0, t)$.

Then based on the governor function z_1, the original system (1) can be converted into the following systems. Over $\bar{\Theta}(0, t)$,

$$\begin{aligned} \dot{z}_1 &= \mu(x_1)(z_2 + \alpha_1) + \mu(x_1) f_1(x_1, \sigma), \\ \dot{z}_2 &= f_2(x_1, x_2, \sigma) + u(t) + e(t) - \dot{\alpha}_1, \end{aligned} \tag{22}$$

where $z_2 = x_2 - \alpha_1$. Over $\bar{\Xi}(0, t)$,

$$\begin{aligned} \dot{z}_1 &= \mu(x_1) z_2 + \mu(x_1) f_1(x_1, \sigma), \\ \dot{z}_2 &= f_2(x_1, x_2, \sigma). \end{aligned} \tag{23}$$

With the aid of Lemma 3.2, the safety control problem of system (1) with the constraint $h(y) < \epsilon$ can be converted into the state constrained problems of (22) and (23) with the constraint $||z(t)|| < z_m$.

For the purpose of satisfying the constraint $||z(t)|| < z_m$ for system (22), inspired by [32], by combining the high gain control and backstepping control techniques, design the following stabilizing controller,

$$\begin{aligned} \alpha_1 &= -\frac{1}{2}(\mu(x_1))^{-1}\Big(k_1 + K_1(x_1)\Big) z_1, \ z_2 = x_2 - \alpha_1, \\ u(t) &= -\frac{1}{2}\Big(k_2 + K_2(x_1, x_2)\Big) z_2 + \dot{\alpha}_1 - \mu^T(x_1) z_1, \end{aligned} \tag{24}$$

where for $j = 1, 2$, $k_j > 0$, and $K_j(\cdot)$ are smooth functions to be designed.

Finally, the sampled-data safety critical control (8a) is given by

$$\begin{aligned} u(t) &= \phi(x(t_{k(t)})) \\ &= -\frac{1}{2}\Big(k_2 + K_2(x_1(t_{k(t)}), x_2(t_{k(t)}))\Big)(x_2(t_{k(t)}) \\ &\quad - \alpha_1(t_{k(t)})) + \frac{d\alpha_1}{dt}\Big|_{x_1 = x_1(t_{k(t)})} - \mu^T(x_1(t_{k(t)})) z_1(t_{k(t)}) \end{aligned} \tag{25}$$

where $t_{k(t)}$ is the latest sampling time instant defined in (8b) with t_k from (19), and α_1 is given by (24).

Remark 3.2: The benefits of the governor function based control strategy (24) are threefold. First, (11) is designed for the safety constraint in a relatively general form with $h(\cdot)$ being some smooth nonlinear function, and (24) is also applicable to high-order nonlinear systems without redefining or sacrificing the safe area. Second, (25) is a bounded control input since we design a bounded governor function (11) with $G_m < +\infty$. In fact, when unbounded barrier functions are involved, the infinite force is required as the trajectory approaches the boundary. Last but not the least, (25) resolves the control feasibility problem in the CBF-based framework by endowing the governor function with Property 4). That is, we do not need Assumption 1 in [13], [33] for defining a control-dependent CBF, and the condition (17) makes sure the virtual controller α_1 can always take effect in (22).

Remark 3.3: Our safety critical control with DoS attacks is given by (25) based on the latest sampled data $x(t_{k(t)})$ given by (8b), the stabilizing controller (24) for system (22), and the combined time- and event-triggered sampling mechanism (19). Compared with the time-triggered method, the main advantage of (19) is to reduce the frequency of updating the control law without sacrificing the control performance.

IV. Solvability of Problems

In this section, we provide the conditions for the solvability of Problem 2.1 and discuss the possible generalizations of our design.

A. *Safety Critical Control Problem With DoS*

We first construct two lemmas for the design of control parameters β, $K_1(x_1)$, and $K_2(x_1, x_2)$ in (24).

Lemma 4.1: Assume

$$\beta > \frac{\sqrt{n}}{\epsilon - \epsilon_m}, \tag{26a}$$

$$\max_{y\in\bar{S}_m}\{||y||\} \leq \min_{y\in\partial S}\{||y||_\infty\}. \tag{26b}$$

If $z_1(t) \geq z_m$ with z_m defined in (18b) for all $t \geq 0$, then $x_1(t) \notin \bar{S}_m$ where $\bar{S}_m = S_m \cup \partial S_m$ and S_m is given by (9).

Proof: We show that if $y(t) \in \bar{S}_m$, $||z_1(t)|| < z_m$. Note that $\chi(\cdot) \leq 1$ and $H(h(y))$ is increasing with $h(y)$ for all $y(t) \in \bar{S}_m$. Then for $h(y) < \epsilon_m$,

$$\chi(||y|| - r_0)H(h(y)) \leq H(\epsilon_m) = \frac{1}{\epsilon - \epsilon_m + \frac{1}{\beta}} < \frac{1}{\epsilon - \epsilon_m}.$$

Then for all $y(t) \in \bar{S}_m$,

$$||z_1(t)|| \leq \frac{||\mathbf{1}_n||}{\epsilon - \epsilon_m} + \varsigma||y|| < \frac{\sqrt{n}}{\epsilon - \epsilon_m} + \varsigma \max_{y \in \bar{S}_m}\{||y||\}. \quad (27)$$

From (15) and (18b),

$$\begin{aligned} z_m = \min\{G_m\} &= \min_{x_1(t) \in \partial S}\{||z_1(t)||\} \quad (28) \\ &\geq \min_{y \in \partial S}\{||z_1(t)||_\infty\} = \beta + \varsigma \min_{y \in \partial S}\{||y||_\infty\}. \end{aligned}$$

Under conditions (26a)-(26b), from (27) and (28), for all $y(t) \in \bar{S}_m$, $||z_1(t)|| < z_m$. As a result, if $||z_1(t)|| \geq z_m$, $y(t) \notin \bar{S}_m$. ■

In order to deal with the norm-bounded nonlinear uncertainty in system (1), we use Corollary 11.1 in [34] to show that functions $f_i(\cdot)$, $i = 1, 2$, can be upper bounded by nonlinear functions and design gain functions $K_i(\cdot)$ in (24) to dominate the upper bounds. To be specific, since functions $f_i(\cdot)$, $i = 1, 2$, are globally defined, sufficiently smooth and satisfy $f_i(0, \sigma) = \mathbf{0}_n$ for all $\sigma \in \Omega$, by Corollary 11.1 in [34], there exist smooth functions $\bar{k}_1(\sigma) \geq 0$, $\hat{k}_1(x_1) \geq 0$, $\bar{k}_{2i}(\sigma) \geq 0$ and $\hat{k}_{2i}(x_i) \geq 0$, $i = 1, 2$, such that

$$\begin{aligned} &||f_1(x_1, \sigma)|| \leq \bar{k}_1(\sigma)\hat{k}_1(x_1)||x_1||, \\ &||f_2(x_1, x_2, \sigma)|| \leq \bar{k}_{21}(\sigma)\hat{k}_{21}(x_1)||x_1|| + \bar{k}_{22}(\sigma)\hat{k}_{22}(x_2)||x_2||. \end{aligned}$$

For $\sigma \in \Omega$ with Ω compact, there exist constants $\check{k}_1 \geq 0$ and $\check{k}_{2i} \geq 0$, $i = 1, 2$, such that $\bar{k}_1(\sigma) \leq \check{k}_1$ and $\bar{k}_{2i}(\sigma) \leq \check{k}_{2i}$. Thus,

$$\begin{aligned} &||f_1(x_1, \sigma)|| \leq \check{k}_1\hat{k}_1(x_1)||x_1||, \quad (29) \\ &||f_2(x_1, x_2, \sigma)|| \leq \check{k}_{21}\hat{k}_{21}(x_1)||x_1|| + \check{k}_{22}\hat{k}_{22}(x_2)||x_2||. \end{aligned}$$

Define

$$\begin{aligned} K_1(x_1) &= \frac{2}{\varsigma}\check{k}_1\hat{k}_1(x_1)||\mu(x_1)||, \quad (30) \\ K_2(x_1, x_2) &= \frac{1}{\varsigma^2}\check{k}_{21}^2\hat{k}_{21}^2(x_1) + \check{k}_{22}^2\hat{k}_{22}^2(x_2)||\bar{\alpha}_1(x_1)||^2 \\ &\quad + 2\check{k}_{22}\hat{k}_{22}(x_2), \end{aligned}$$

where

$$\bar{\alpha}_1(x_1) = \frac{1}{2}(\mu(x_1))^{-1}(k_1 + K_1(x_1)). \quad (31)$$

Lemma 4.2: Functions $K_1(x_1)$ and $K_2(x_1, x_2)$ in (30) are bounded if $||z(t)|| < z_m$.

Proof: By Lemma 3.2, $y(t) \in S$ if $||z_1(t)|| \leq ||z(t)|| < z_m$.

Since $\chi(||y|| - r_0) \geq 0$ and $H(h(y)) > 0$ for all $y(t) \in S$, $\varsigma||x_1|| \leq ||z_1|| < z_m$ from (11), and thus,

$$||x_1|| \leq \frac{||z_1||}{\varsigma} < \frac{z_m}{\varsigma} \triangleq x_{1m}. \quad (32)$$

Since $\hat{k}_1(\cdot)$ is a smooth function determined from (29), $\hat{k}_1(x_1)$ is bounded under (32), i.e., there exists $\hat{k}_{1m} \geq 0$ such that $\hat{k}_1(x_1) \leq \hat{k}_{1m}$. Together with (16),

$$K_1(x_1) \leq \bar{K}_1 = \frac{2}{\varsigma}\check{k}_1\hat{k}_{1m}\mu_m. \quad (33)$$

From (17) in Lemma 3.1, $\mu(x_1) \geq \varsigma_0 I_n$, implying that $||(\mu(x_1))^{-1}|| \leq \frac{1}{\varsigma_0}$. Then from (31) and (24),

$$\begin{aligned} ||\bar{\alpha}_1(x_1)|| &\leq \frac{1}{2}||(\mu(x_1))^{-1}||(k_1 + K_1(x_1)) \\ &\leq \frac{1}{2\varsigma_0}(k_1 + \bar{K}_1) \triangleq \bar{\alpha}_{1m} \\ ||x_2|| &= ||z_2 + \alpha_1|| \leq ||z_2|| + ||\bar{\alpha}_1(x_1)||||z_1|| \\ &\leq \bar{\alpha}_{1m}||z_1|| + ||z_2|| \leq \sqrt{2(\bar{\alpha}_{1m}^2||z_1||^2 + ||z_2||^2)} \\ &\leq \sqrt{2\max\{1, \bar{\alpha}_{1m}^2\}(||z_1||^2 + ||z_2||^2)} \\ &\leq \sqrt{2}\max\{1, \bar{\alpha}_{1m}\}||z|| \leq \sqrt{2}\max\{1, \bar{\alpha}_{1m}\}z_m \\ &\triangleq x_{2m}. \quad (34) \end{aligned}$$

Together with the smoothness of $\hat{k}_{21}(\cdot)$ and $\hat{k}_{22}(\cdot)$, there exist constants $\hat{k}_{21m} \geq 0$ and $\hat{k}_{22m} \geq 0$ such that $\hat{k}_{21}(x_1) \leq \hat{k}_{21m}$ and $\hat{k}_{22}(x_2) \leq \hat{k}_{22m}$. Then for all $y(t) \in S$,

$$\begin{aligned} K_2(x_1, x_2) \leq \bar{K}_2 = \frac{1}{\varsigma^2}\check{k}_{21}^2\hat{k}_{21m}^2 + \frac{1}{4\varsigma_0^2}\check{k}_{22}^2\hat{k}_{22m}^2(k_1 + \bar{K}_1)^2 \\ + 2\check{k}_{22}\hat{k}_{22m}. \quad (35) \end{aligned}$$

■

Based on Lemmas 3.2-4.2, we provide Algorithm 1 for designing control parameters in (25).

It is time to present our main result.

Theorem 4.1: Consider the system (1) with the safety constraint (2) and DoS attacks satisfying Assumptions 2.1 and 2.2 with $\kappa < 1$ in (36). For any $y(0) \in S$ and $||z(0)|| \leq z_0$, Problem 2.1 is solvable under the sampled-data safety critical control (25) where $x(t_{k(t)})$ is given by (8b) with t_k in (19), α_1 is given by (24), and control parameters are determined by Algorithm 1.

Proof: The proof includes three steps.

Step 1: prove $y(t) \in S$ for all $t \geq 0$.

For all $t \in [0, \bar{\delta}_1)$, from Fig. 2, the event-triggered mechanism is used and the triggering condition (20) holds. For $y(0) \in S$, by the continuity of the trajectory $y(t)$, there exists $0 < \bar{\delta}_1^* \leq \bar{\delta}_1$ such that $y(t) \in S$ for all $t \in [0, \bar{\delta}_1^*)$. Let

$$V_i = \sum_{j=1}^{i} z_j^T z_j, \ i = 1, 2. \quad (41)$$

From (29), (30) and (32), for all $t \in [0, \bar{\delta}_1^*)$,

$$\begin{aligned} &2z_1^T\mu(x_1)f_1(x_1, \sigma) \leq 2||z_1||||\mu(x_1)||||f_1(x_1, \sigma)|| \\ &\leq 2||z_1||||\mu(x_1)||\check{k}_1\hat{k}_1(x_1)||x_1|| \quad (42) \\ &= \frac{2}{\varsigma}\check{k}_1\hat{k}_1(x_1)||\mu(x_1)||||z_1||^2 = K_1(x_1)||z_1||^2. \end{aligned}$$

Algorithm 1 Control Parameters Design

0. Initialization:

The initial condition satisfies $y(0) \in S$.

1. Sampling mechanism:

Select $\vartheta_t > \tau_m$ and determine $\vartheta_m \geq \vartheta_t$. Choose $\gamma > 0$. Calculate

$$\kappa = \frac{\vartheta_m}{\tau_D} + \frac{1}{T_D}. \tag{36}$$

If $\kappa \geq 1$, stop, and if otherwise, go to the next step.

2. Governor function:

Choose ϵ_m to satisfy $\epsilon_0 < \epsilon_m < \epsilon$, and determine S_m and x_m from (9) and (10). Choose $r_0 < x_m$ and design the smooth switching function $\chi(\cdot)$ with Properties i)-iii). Calculate χ_m and h_m from (14). Choose a small constant $0 < \bar{m}_0 < 1$ and select sufficiently large

$$\beta > \max\Big\{\frac{\sqrt{n}}{\epsilon - \epsilon_m},\ \frac{1}{h_m}(\frac{1}{r_0} - \chi_m - \bar{m}_0)\Big\}. \tag{37}$$

Let $\varsigma_0 = \bar{m}_0\sqrt{n}\beta$. Then calculate ς from (14) and design the governor function z_1 from (11). Determine μ_m from (16) and z_m from (18b).

3. Control gains:

Calculate $\bar{K}_1$ from (33) and $\bar{K}_2$ from (35). Thus, M_D can be calculated from

$$M_D = \max\{\bar{K}_1 + \mu_m^2 + 2, \bar{K}_2 + 1\} \tag{38}$$

Choose sufficiently large control gains k_1 and k_2 such that M_F, determined from

$$M_F = \min\{k_1 - 2 - 2\gamma, k_2 - 2\gamma\}, \tag{39}$$

satisfies

$$\frac{M_F}{M_D} > \frac{\kappa}{1-\kappa}, \tag{40a}$$

$$\bar{v}_j < \frac{2}{M_D}\ln\frac{z_m}{z_0} + \frac{M_F}{M_D}|\bar{\Theta}(0,\bar{\delta}_j)| - |\bar{\Xi}(0,\bar{\delta}_j)|,\ j \in \mathbb{N}^+. \tag{40b}$$

Check if $z_m > z_0 \geq ||z(0)||$, and increase β and repeat Steps 2 and 3 if otherwise.

Along the trajectory of (24) and (22), for all $t \in [0, \bar{\delta}_1^*)$, the time derivative of V_1 satisfies

$$\begin{aligned}\dot{V}_1 &= 2z_1^T\mu(x_1)z_2 + 2z_1^T\mu(x_1)\alpha_1 + 2z_1^T\mu(x_1)f_1(x_1,\sigma)\\ &\leq 2z_1^T\mu(x_1)z_2 - z_1^T\mu(x_1)(\mu(x_1))^{-1}(k_1 + K_1(x_1))z_1\\ &\quad + K_1(x_1)||z_1||^2\\ &= 2z_1^T\mu(x_1)z_2 - k_1||z_1||^2.\end{aligned}$$

Note that from (24) and (31), $||\alpha_1|| \leq ||\bar{\alpha}_1(x_1)||||z_1||$. From (24), (29), (30) and (32),

$$\begin{aligned}&2z_2^Tf_2(x_1,x_2,\sigma) \leq 2||z_2||||f_2(x_1,x_2,\sigma)||\\ &\leq 2\check{k}_{21}\hat{k}_{21}(x_1)||x_1||||z_2|| + 2\check{k}_{22}\hat{k}_{22}(x_2)||z_2 + \alpha_1||||z_2||\\ &\leq \frac{2}{\varsigma}\check{k}_{21}\hat{k}_{21}(x_1)||z_1||||z_2|| + 2\check{k}_{22}\hat{k}_{22}(x_2)||z_2||^2\\ &+ 2\check{k}_{22}\hat{k}_{22}(x_2)||\bar{\alpha}_1(x_1)||||z_1||||z_2||\\ &\leq 2||z_1||^2 + \Big(\frac{1}{\varsigma^2}\check{k}_{21}^2\hat{k}_{21}^2(x_1) + \check{k}_{22}^2\hat{k}_{22}^2(x_2)||\bar{\alpha}_1(x_1)||^2\Big)||z_2||^2\\ &+ 2\check{k}_{22}\hat{k}_{22}(x_2)||z_2||^2\\ &= 2||z_1||^2 + K_2(x_1,x_2)||z_2||^2.\end{aligned} \tag{43}$$

Then along the trajectory of (24) and (22), for all $t \in [0, \bar{\delta}_1^*)$, the time derivative of V_2 satisfies

$$\begin{aligned}\dot{V}_2 &\leq 2z_1^T\mu(x_1)z_2 - k_1||z_1||^2 + 2z_2^Tf_2(x_1,x_2,\sigma)\\ &\quad + 2z_2^T(u(t) - \dot{\alpha}_1) + 2z_2^Te(t)\\ &\leq -k_1||z_1||^2 + 2||z_1||^2 + K_2(x_1,x_2)||z_2||^2 + 2z_2^T(u(t)\\ &\quad - \dot{\alpha}_1 + \mu^T(x_1)z_1) + 2\gamma||z||^2\\ &\leq -(k_1-2)||z_1||^2 + 2z_2^T(u(t) - \dot{\alpha}_1 + \mu^T(x_1)z_1\\ &\quad + \frac{1}{2}K_2(x_1,x_2)z_2) + 2\gamma||z||^2\\ &\leq -(k_1 - 2 - 2\gamma)||z_1||^2 - (k_2 - 2\gamma)||z_2||^2\\ &\leq -M_FV_2,\end{aligned} \tag{44}$$

where $M_F > 0$ is given by (39) and $||e(t)|| \leq \gamma||z(t)||$ from (20). Then for all $t \in [0, \bar{\delta}_1^*)$,

$$V_2(t) \leq V_2(0)e^{-M_Ft} = V_2(0)e^{-M_F|\bar{\Theta}(0,t)|}e^{M_D|\bar{\Xi}(0,t)|} \tag{45a}$$

$$\begin{aligned}||z(t)|| &\leq ||z(0)||e^{-\frac{M_F}{2}t}\\ &= ||z(0)||e^{-\frac{M_F}{2}|\bar{\Theta}(0,t)|}e^{\frac{M_D}{2}|\bar{\Xi}(0,t)|},\end{aligned} \tag{45b}$$

where $|\bar{\Theta}(0,t)| = t$, $|\bar{\Xi}(0,t)| = 0$ and M_D is given by (38). Then $||z(t)||$ is exponentially decreasing with t, and $||z(t)|| \leq z_0 < z_m$ for all $t \in [0, \bar{\delta}_1)$ with $\bar{\delta}_1 \geq \bar{\delta}_1^*$. By Lemma 3.2, $y(t) \in S$ for all $t \in [0, \bar{\delta}_1)$.

For all $t \in [\bar{\delta}_1, \bar{\delta}_1 + \bar{v}_1)$, the time-triggered mechanism is used, and the trajectory of $z(t)$ is generated from (23). Since z_1 satisfies Property 1) in Definition 3.1, $z(t)$ is continuous by the continuity of $x(t)$. Thus, $||z(\bar{\delta}_1)|| < z_me^{-\frac{M_F}{2}\bar{\delta}_1}$ from (45b) and there exists $\bar{v}_1^* \geq 0$ such that $||z(t)|| < z_m$ for all $t \in [\bar{\delta}_1, \bar{\delta}_1 + \bar{v}_1^*)$. Along the trajectory of (23) and from (42), (43), for all $t \in [\bar{\delta}_1, \bar{\delta}_1 + \bar{v}_1^*)$, the time derivative of V_2 satisfies,

$$\begin{aligned}\dot{V}_2 &= 2z_1^T\mu(x_1)z_2 + 2z_1^T\mu(x_1)f_1(x_1,\sigma) + 2z_2^Tf_2(x_1,x_2,\sigma)\\ &\leq ||\mu(x_1)||^2||z_1||^2 + ||z_2||^2 + K_1(x_1)||z_1||^2\\ &+ 2||z_1||^2 + K_2(x_1,x_2)||z_2||^2\\ &= (K_1(x_1) + ||\mu(x_1)||^2 + 2)||z_1||^2 + (K_2(x_1,x_2) + 1)||z_2||^2\\ &\leq \max\{K_1(x_1) + ||\mu(x_1)||^2 + 2, K_2(x_1,x_2) + 1\}||z||^2.\end{aligned} \tag{46}$$

By Lemma 3.2, $y(t) \in S$, and $\mu(x_1)$, $K_1(x_1)$ and $K_2(x_1,x_2)$ are bounded for all $t \in [\bar{\delta}_1, \bar{\delta}_1 + \bar{v}_1^*)$, i.e., (16), (33) and (35) are satisfied. Thus, for all $t \in [\bar{\delta}_1, \bar{\delta}_1 + \bar{v}_1^*)$, $\dot{V}_2 \leq M_DV_2$ with M_D given by (38). Then from (45),

$$\begin{aligned}V_2(t) &\leq V_2(\bar{\delta}_1)e^{M_D(t-\bar{\delta}_1)},\ \forall t \in [\bar{\delta}_1, \bar{\delta}_1 + \bar{v}_1^*)\\ &\leq V_2(0)e^{-M_F|\bar{\Theta}(0,\bar{\delta}_1)|}e^{M_D|\bar{\Xi}(0,\bar{\delta}_1)|}e^{M_D(t-\bar{\delta}_1)},\end{aligned} \tag{47a}$$

$$||z(t)|| \leq ||z(0)||e^{-\frac{M_F}{2}|\bar{\Theta}(0,\bar{\delta}_1)|}e^{\frac{M_D}{2}(t-\bar{\delta}_1+|\bar{\Xi}(0,\bar{\delta}_1)|)}. \tag{47b}$$

Thus, if $\bar{v}_1 < \bar{v}_1^* = \frac{2}{M_D}\ln\frac{z_m}{z_0} + \frac{M_F}{M_D}|\bar{\Theta}(0,\bar{\delta}_1)| - |\bar{\Xi}(0,\bar{\delta}_1)|$, $||z(t)|| < z_m$ from (47b). By Lemma 3.2, $y(t) \in S$ for all $t \in [\bar{\delta}_1, \bar{\delta}_1 + \bar{v}_1)$.

Now we claim that for all $t \geq 0$,

$$\dot{V}_2(t) \leq \begin{cases} -M_F V_2(t), & t \in [\bar{\delta}_{p-1} + \bar{v}_{p-1}, \bar{\delta}_p) \\ M_D V_2(t), & t \in [\bar{\delta}_p, \bar{\delta}_p + \bar{v}_p) \end{cases} \tag{48a}$$

$$V_2(t) \leq e^{-M_F|\bar{\Theta}(0,t)|} e^{M_D|\bar{\Xi}(0,t)|} V_2(0), \tag{48b}$$

$$||z(t)|| \leq e^{-\frac{M_F}{2}|\bar{\Theta}(0,t)|} e^{\frac{M_D}{2}|\bar{\Xi}(0,t)|} z_0 < z_m. \tag{48c}$$

To verify this claim, assume that (48) holds for all $t \in [0, \bar{\delta}_j + \bar{v}_j)$ with some $j \in \mathbb{N}^+$.

For all $t \in [\bar{\delta}_j + \bar{v}_j, \bar{\delta}_{j+1})$, the event-triggered mechanism is used and (20) holds. Repeat the procedure in the time interval $[0, \bar{\delta}_1)$, and (44) holds. Since (48) holds for all $t \in [0, \bar{\delta}_j + \bar{v}_j)$,

$$\begin{aligned} V_2(t) &\leq e^{-M_F(t-\bar{\delta}_j-\bar{v}_j)} V_2(\bar{\delta}_j + \bar{v}_j) \\ &\leq e^{-M_F(t-\bar{\delta}_j-\bar{v}_j)} e^{M_D(\bar{\delta}_j+\bar{v}_j-\bar{\delta}_j)} V_2(\bar{\delta}_j) \\ &\leq e^{-M_F(t-\bar{\delta}_j-\bar{v}_j)} e^{M_D\bar{v}_j} e^{-M_F|\bar{\Theta}(0,\bar{\delta}_j)|} e^{M_D|\bar{\Xi}(0,\bar{\delta}_j)|} V_2(0) \\ &= e^{-M_F(t-\bar{\delta}_j-\bar{v}_j+|\bar{\Theta}(0,\bar{\delta}_j)|)} e^{M_D(\bar{v}_j+|\bar{\Xi}(0,\bar{\delta}_j)|)} V_2(0). \end{aligned} \tag{49}$$

Note that $|\bar{\Theta}(\bar{\delta}_j, t)| = t - \bar{\delta}_j - \bar{v}_j$. Then for all $t \in [\bar{\delta}_j + \bar{v}_j, \bar{\delta}_{j+1})$,

$$\begin{aligned} &|\bar{\Theta}(0,t)| = |\bar{\Theta}(0,\bar{\delta}_j)| + |\bar{\Theta}(\bar{\delta}_j,t)| = |\bar{\Theta}(0,\bar{\delta}_j)| + t - \bar{\delta}_j - \bar{v}_j, \\ &|\bar{\Xi}(0,t)| = \bar{v}_j + |\bar{\Xi}(0,\bar{\delta}_j)|. \end{aligned} \tag{50}$$

Therefore, from (49), for all $t \in [\bar{\delta}_j + \bar{v}_j, \bar{\delta}_{j+1})$,

$$V_2(t) \leq e^{-M_F|\bar{\Theta}(0,t)|} e^{M_D|\bar{\Xi}(0,t)|} V_2(0), \tag{51}$$

implying that (48b) holds for all $t \in [0, \bar{\delta}_{j+1})$. Since (48c) holds for all $t \in [0, \bar{\delta}_j + \bar{v}_j)$, $||z(t)|| \leq e^{-\frac{M_F}{2}|\bar{\Theta}(0,\bar{\delta}_j+\bar{v}_j)|} e^{\frac{M_D}{2}|\bar{\Xi}(0,\bar{\delta}_j+\bar{v}_j)|} z_0 < z_m$. From (50), $|\bar{\Theta}(0, \bar{\delta}_j + \bar{v}_j)| = |\bar{\Theta}(0, \bar{\delta}_j)|$ and $|\bar{\Xi}(0, \bar{\delta}_j + \bar{v}_j)| = \bar{v}_j + |\bar{\Xi}(0, \bar{\delta}_j)|$. Then from (49),

$$\begin{aligned} ||z(t)|| &\leq e^{-\frac{M_F}{2}(t-\bar{\delta}_j-\bar{v}_j+|\bar{\Theta}(0,\bar{\delta}_j)|)} e^{\frac{M_D}{2}(\bar{v}_j+|\bar{\Xi}(0,\bar{\delta}_j)|)} z_0 \\ &\leq e^{-\frac{M_F}{2}(t-\bar{\delta}_j-\bar{v}_j)} e^{-\frac{M_F}{2}|\bar{\Theta}(0,\bar{\delta}_j+\bar{v}_j)|} e^{\frac{M_D}{2}|\bar{\Xi}(0,\bar{\delta}_j+\bar{v}_j)|} z_0 \\ &< e^{-\frac{M_F}{2}(t-\bar{\delta}_j-\bar{v}_j)} z_m \leq z_m, \end{aligned} \tag{52}$$

implying that $||z(t)||$ decreases with time t, and (48c) holds for all $t \in [0, \bar{\delta}_{j+1})$.

For all $t \in [\bar{\delta}_{j+1}, \bar{\delta}_{j+1} + \bar{v}_{j+1})$ with $\bar{v}_{j+1}$ satisfying (40b), $||z(\bar{\delta}_{j+1})|| < e^{-\frac{M_F}{2}(\bar{\delta}_{j+1}-\bar{\delta}_j-\bar{v}_j)} z_m \leq z_m$ from (52). Using a similar argument to the time interval $[\bar{\delta}_1, \bar{\delta}_1 + \bar{v}_1)$, we obtain

$$\dot{V}_2(t) \leq M_D V_2(t), \quad \forall t \in [\bar{\delta}_{j+1}, \bar{\delta}_{j+1} + \bar{v}_{j+1}). \tag{53}$$

Since (48b) holds for all $t \in [0, \bar{\delta}_{j+1})$, we have

$$V_2(\bar{\delta}_{j+1}) \leq e^{-M_F|\bar{\Theta}(0,\bar{\delta}_{j+1})|} e^{M_D|\bar{\Xi}(0,\bar{\delta}_{j+1})|} V_2(0). \tag{54}$$

Then for all $t \in [\bar{\delta}_{j+1}, \bar{\delta}_{j+1} + \bar{v}_{j+1})$,

$$\begin{aligned} V_2(t) &\leq e^{M_D(t-\bar{\delta}_{j+1})} V_2(\bar{\delta}_{j+1}) \\ &\leq e^{M_D(t-\bar{\delta}_{j+1})} e^{-M_F|\bar{\Theta}(0,\bar{\delta}_{j+1})|} e^{M_D|\bar{\Xi}(0,\bar{\delta}_{j+1})|} V_2(0) \\ &= e^{-M_F|\bar{\Theta}(0,\bar{\delta}_{j+1})|} e^{M_D(t-\bar{\delta}_{j+1}+|\bar{\Xi}(0,\bar{\delta}_{j+1})|)} V_2(0) \\ &= e^{-M_F|\bar{\Theta}(0,t)|} e^{M_D|\bar{\Xi}(0,t)|} V_2(0), \end{aligned} \tag{55}$$

because for all $t \in [\bar{\delta}_{j+1}, \bar{\delta}_{j+1} + \bar{v}_{j+1})$, $|\bar{\Theta}(0,t)| = |\bar{\Theta}(0, \bar{\delta}_{j+1})|$ and $|\bar{\Xi}(0,t)| = t - \bar{\delta}_{j+1} + |\bar{\Xi}(0, \bar{\delta}_{j+1})|$. From (55), we also obtain

$$||z(t)|| \leq e^{-\frac{M_F}{2}|\bar{\Theta}(0,\bar{\delta}_{j+1})|} e^{\frac{M_D}{2}(t-\bar{\delta}_{j+1}+|\bar{\Xi}(0,\bar{\delta}_{j+1})|)} z_0. \tag{56}$$

From (40b), $\bar{v}_{j+1} < \frac{2}{M_D}\ln\frac{z_m}{z_0} + \frac{M_F}{M_D}|\bar{\Theta}(0, \bar{\delta}_{j+1})| - |\bar{\Xi}(0, \bar{\delta}_{j+1})|$, and then for all $t \in [\bar{\delta}_{j+1}, \bar{\delta}_{j+1} + \bar{v}_{j+1})$,

$$||z(t)|| \leq e^{-\frac{M_F}{2}|\bar{\Theta}(0,\bar{\delta}_{j+1})|} e^{\frac{M_D}{2}(v_{j+1}+|\bar{\Xi}(0,\bar{\delta}_{j+1})|)} z_0 < z_m.$$

Thus, for all $t \in [0, \bar{\delta}_{j+1} + \bar{v}_{j+1})$, (48) also holds, which implies (48) holds for all $t \geq 0$. Since $||z(t)|| \leq z_m$ for all $t \geq 0$, by Lemma 3.2, $y(t) \in S$ for all $t \geq 0$.

Step 2: prove $\lim_{t\to\infty} x(t) = \mathbf{0}_{2n}$**.**

Note that $|\bar{\Xi}(0,t)|$ denotes the length of the intervals over which the time-triggered method is used, and it can be upper bounded by the length and the number of DoS attacks, i.e.,

$$\begin{aligned} |\bar{\Xi}(0,t)| &\leq |\Xi(0,t)| + P(0,t)\vartheta_t \\ &\leq |\Xi(0,t)| + P(0,t)\vartheta_m. \end{aligned}$$

Under (5) and (6) in Assumptions 2.1 and 2.2, we have

$$|\bar{\Xi}(0,t)| \leq \kappa_1 + \frac{t}{T_D} + (\kappa_2 + \frac{t}{\tau_D})\vartheta_m = \bar{\kappa} + \kappa t, \tag{57}$$

where $\bar{\kappa} = \kappa_1 + \kappa_2\vartheta_m$ and $\kappa = \frac{1}{T_D} + \frac{\vartheta_m}{\tau_D}$. Since $\bar{\Theta}(0,t)$ and $\bar{\Xi}(0,t)$ are disjoint over the time interval $[0,t]$, $|\bar{\Theta}(0,t)| = t - |\bar{\Xi}(0,t)|$.

Denote $\bar{\kappa}^* = M_F - (M_F + M_D)\kappa$. Then from (48b) and (57),

$$\begin{aligned} V_2(t) &\leq e^{-M_F(t-|\bar{\Xi}(0,t)|)} e^{M_D|\bar{\Xi}(0,t)|} V_2(0) \\ &= e^{-M_F t} e^{(M_F+M_D)|\bar{\Xi}(0,t)|} V_2(0) \\ &\leq e^{-M_F t} e^{(M_F+M_D)(\bar{\kappa}+\kappa t)} V_2(0) \\ &= e^{(M_F+M_D)\bar{\kappa}} e^{(-M_F+(M_F+M_D)\kappa)t} V_2(0) \\ &= e^{(M_F+M_D)\bar{\kappa}} e^{-\bar{\kappa}^* t} V_2(0). \end{aligned} \tag{58}$$

Since $\bar{\kappa}^* > 0$ from (40a), $V_2(x(t)) = V_2(t) \leq e^{(M_F+M_D)\bar{\kappa}} V_2(0) \triangleq \bar{V}_2$, and thus, $x(t)$ belongs to the compact set $\Omega_0 = \{x \in \mathbb{R}^{2n} : V_2 \leq \bar{V}_2\}$. Also from (58), $\lim_{t\to+\infty} V_2(t) = 0$. From (41), $\lim_{t\to+\infty} z(t) = \mathbf{0}_{2n}$. Thus, $\lim_{t\to+\infty} z_1(t) = \mathbf{0}_n$. From (11), as $t \to +\infty$, two cases can happen.

Case 1: $x_1(t) \in S \backslash S_0$ and

$$x_1 = -\frac{1}{\varsigma}\chi(||x_1 - r_0||)\frac{\mathbf{1}_n}{\epsilon - h(x_1) + \frac{1}{\beta}}.$$

Since $\chi(||x_1|| - r_0) \leq 1$ and $\frac{1}{\epsilon - h(x_1) + \frac{1}{\beta}} < \beta$ for all $x_1(t) \in S$, from (14) with $\varsigma_0 = \bar{m}_0\sqrt{n}\beta$ in Algorithm 1, we obtain

$$||x_1|| \leq \frac{\sqrt{n}\beta}{\varsigma} \leq \frac{1}{\chi_m + \beta h_m + \bar{m}_0} < r_0,$$

noting that $\beta > \frac{1}{h_m}(\frac{1}{r_0} - \chi_m - \bar{m}_0)$ from (37). Therefore, $x_1(t) \in S_0$, contradicting $x_1(t) \in S \backslash S_0$ as $t \to +\infty$. Thus, Case 1 is impossible.

Case 2: $x_1(t) \in S_0$ and $z_1 = \varsigma x_1$, noting that $\chi(||x_1|| - r_0) = 0$ for $||x_1|| \leq r_0$. As a result, $\lim_{t\to+\infty} z_1(t) = \varsigma x_1(t) = \mathbf{0}_n$, and thus, $\lim_{t\to+\infty} x_1(t) = \mathbf{0}_n$ and $\lim_{t\to+\infty} \alpha_1(t) = \mathbf{0}_n$

since $\mu(x_1) \geq \varsigma_0 I_n$ from (17) and $K_1(x_1)$ is bounded by Lemma 4.2. Then from (24), $\lim_{t\to+\infty} x_2(t) = \lim_{t\to+\infty}(z_2(t) + \alpha_1(t)) = \mathbf{0}_n$.

Step 3: prove that no Zeno behavior occurs.

Note that from (19), for $k \in \Gamma$, the inter-sampling time during DoS is $\vartheta_t \geq \tau_m > 0$. For $k \notin \Gamma$, we show that no Zeno behavior occurs by contradiction. If Zeno behavior happens, there exists a finite positive constant τ^* such that $\lim_{k\to\infty} t_k = \tau^*$. That is, for each $\epsilon^* > 0$, there exists an integer K_0 such that

$$\tau^* - \epsilon^* < t_k \leq \tau^*, \quad k \geq K_0. \tag{59}$$

Next we consider the evolution of $||e(t)||$ over $[t_k, t_{k+1})$, $k \notin \Gamma$, with the following calculations. From (16), (29), (32) and (34),

$$||\dot{x}_1|| \leq ||x_2|| + ||f_1(x_1, \sigma)|| \leq x_{2m} + \check{k}_1\hat{k}_{1m}x_{1m} \triangleq D_1,$$
$$||\dot{z}_1|| \leq ||\mu(x_1)||||\dot{x}_1|| \leq \mu_m D_1 \triangleq Z_1.$$

Since $x_1(t) \in S$, $H(h(x_1)) < \beta$. From Property iii) of function $\chi(\cdot)$ and the fact that $\frac{d^i h(x_1)}{dx_1^i}$, $i = 1, 2, 3$, is bounded for all $x_1(t) \in S$, $\dot{\mu}(x_1)$ and $\ddot{\mu}(x_1)$ are bounded, i.e., there exist constants $M_1 \geq 0$ and $M_2 \geq 0$ such that $||\dot{\mu}(x_1)|| \leq M_1$ and $||\ddot{\mu}(x_1)|| \leq M_2$. Also note that

$$\frac{d}{dt}(\mu(x_1))^{-1} = -(\mu(x_1))^{-1}\dot{\mu}(x_1)(\mu(x_1))^{-1},$$
$$||\frac{d}{dt}(\mu(x_1))^{-1}|| \leq \frac{M_1}{\varsigma_0^2},$$
$$||\frac{d^2}{dt^2}(\mu(x_1))^{-1}|| \leq 2||\frac{d}{dt}(\mu(x_1))^{-1}||||\dot{\mu}(x_1)||||(\mu(x_1))^{-1}|| + ||(\mu(x_1))^{-1}||||\ddot{\mu}(x_1)||||(\mu(x_1))^{-1}|| \leq \frac{2M_1^2}{\varsigma_0^3} + \frac{M_2}{\varsigma_0^2}.$$

Since $\hat{k}_1(x_1) \geq 0$ is sufficiently smooth, $\frac{d\hat{k}_1(x_1)}{dx_1}$ and $\frac{d^2\hat{k}_1(x_1)}{dx_1^2}$ are continuous functions of x_1. Together with the fact that $x \in \Omega_0$ with Ω_0 being compact, by extreme value theorem, there exist constants $\bar{\beta}_1 \geq 0$ and $\check{\beta}_1 \geq 0$ such that $||\frac{d\hat{k}_1(x_1)}{dx_1}|| \leq \bar{\beta}_1$ and $||\frac{d^2\hat{k}_1(x_1)}{dx_1^2}|| \leq \check{\beta}_1$. Similarly, we have $||\frac{d\hat{k}_{21}(x_1)}{dx_1}|| \leq \bar{\beta}_{21}$ and $||\frac{d\hat{k}_{22}(x_2)}{dx_2}|| \leq \bar{\beta}_{22}$ for constants $\bar{\beta}_{21} \geq 0$ and $\bar{\beta}_{22} \geq 0$. Then

$$|\dot{\hat{k}}_1(x_1)| \leq \left|\left|\frac{d\hat{k}_1(x_1)}{dx_1}\right|\right|||\dot{x}_1|| \leq \bar{\beta}_1 D_1.$$

Together with (16) and (30), we obtain

$$|\dot{K}_1(x_1)| \leq \frac{2}{\varsigma}\check{k}_1|\dot{\hat{k}}_1(x_1)|||\mu(x_1)|| + \frac{2}{\varsigma}\check{k}_1\hat{k}_1(x_1)||\dot{\mu}(x_1)|| \leq \frac{2}{\varsigma}\check{k}_1(\bar{\beta}_1 D_1\mu_m + \hat{k}_{1m}M_1) \triangleq \hat{K}_1.$$

From (17) and (31),

$$||\dot{\bar{\alpha}}_1(x_1)|| \leq \frac{1}{2}||\frac{d}{dt}(\mu(x_1))^{-1}||(k_1 + K_1(x_1)) + \frac{1}{2}||(\mu(x_1))^{-1}|||\dot{K}_1(x_1)| \leq \frac{1}{2}(k_1 + \bar{K}_1)\frac{M_1}{\varsigma_0^2} + \frac{1}{2\varsigma_0}\hat{K}_1 \triangleq \bar{\alpha}_m. \tag{60}$$

Since $\alpha_1 = \bar{\alpha}_1(x_1)z_1$ with $\bar{\alpha}_1(x_1)$ given by (31),

$$||\dot{\alpha}_1|| \leq ||\dot{\bar{\alpha}}_1(x_1)||||z_1|| + ||\bar{\alpha}_1(x_1)||\dot{z}_1|| \leq \bar{\alpha}_m z_m + (k_1 + \bar{K}_1)\frac{Z_1}{2\varsigma_0} \triangleq \hat{\alpha}_m.$$

Then from (24),

$$||u(t)|| \leq \frac{k_2}{2}||z_2|| + \frac{1}{2}K_2(x_1, x_2)||z_2|| + ||\dot{\alpha}_1|| + ||\mu(x_1)||||z_1|| \leq \max\{\mu_m, \frac{1}{2}(k_2 + \bar{K}_2)\}z_m + \hat{\alpha}_m \triangleq u_m.$$

Recalling (29), we have

$$||\dot{x}_2|| \leq ||e(t)|| + ||f_2(x_1, x_2, \sigma)|| + ||u(t)|| \leq ||e(t)|| + \check{k}_{21}\hat{k}_{21m}x_{1m} + \check{k}_{22}\hat{k}_{22m}x_{2m} + u_m \triangleq ||e(t)|| + D_2,$$
$$||\dot{z}_2|| \leq ||\dot{x}_2|| + ||\dot{\alpha}_1|| \leq ||e(t)|| + D_2 + \hat{\alpha}_m,$$

where $D_2 = \check{k}_{21}\hat{k}_{21m}x_{1m} + \check{k}_{22}\hat{k}_{22m}x_{2m} + u_m$. Then from (30),

$$|\dot{K}_2(x_1, x_2)| \leq \frac{2}{\varsigma^2}\check{k}_{21}^2\hat{k}_{21}(x_1)|\dot{\hat{k}}_{21}(x_1)| + 2\check{k}_{22}^2\hat{k}_{22}(x_2)|\dot{\hat{k}}_{22}(x_2)|||\bar{\alpha}_1(x_1)||^2 + 2\check{k}_{22}^2\hat{k}_{22}^2(x_2)||\bar{\alpha}_1(x_1)||||\dot{\bar{\alpha}}_1(x_1)|| + 2\check{k}_{22}|\dot{\hat{k}}_{22}(x_2)|$$
$$\leq \frac{2}{\varsigma^2}\check{k}_{21}^2\hat{k}_{21m}\bar{\beta}_{21}D_1 + 2\check{k}_{22}^2\hat{k}_{22m}\bar{\beta}_{22}(||e(t)|| + D_2)(\frac{k_1 + \bar{K}_1}{2\varsigma_0})^2 + 2\check{k}_{22}^2\hat{k}_{22m}^2\frac{k_1 + \bar{K}_1}{2\varsigma_0}\bar{\alpha}_m + 2\check{k}_{22}\bar{\beta}_{22}(||e(t)|| + D_2) \triangleq \bar{\pi}_1||e(t)|| + \bar{\pi}_2.$$

Also, $f_1(x_1, \sigma)$ is smooth and $||\frac{df_1(x_1,\sigma)}{dx_1}||$ is bounded for all $x_1(t) \in S$, i.e., there exists some constant $\bar{f}_1 \geq 0$ such that $||\frac{df_1(x_1,\sigma)}{dx_1}|| \leq \bar{f}_1$. Then

$$||\ddot{x}_1|| \leq ||\dot{x}_2|| + ||\dot{f}_1(x_1, \sigma)|| \leq ||e(t)|| + D_2 + \bar{f}_1 D_1,$$
$$|\ddot{\hat{k}}_1(x_1)| \leq \left|\left|\frac{d^2\hat{k}_1(x_1)}{dx_1^2}\right|\right|||\dot{x}_1|| + \left|\left|\frac{d\hat{k}_1(x_1)}{dx_1}\right|\right|||\ddot{x}_1|| \leq \check{\beta}_1 D_1 + \bar{\beta}_1(||e(t)|| + D_2 + \bar{f}_1 D_1).$$

From (22),

$$||\ddot{z}_1|| \leq ||\dot{\mu}(x_1)||||x_2|| + ||\mu(x_1)||||\dot{x}_2|| + ||\dot{\mu}(x_1)||||f(x_1, \sigma)|| + ||\mu(x_1)||||\dot{f}(x_1, \sigma)|| \leq M_1 x_{2m} + \mu_m(||e(t)|| + D_2) + M_1\check{k}_1\hat{k}_{1m}x_{1m} + \mu_m\bar{f}_1 D_1 \triangleq \bar{\pi}_3||e(t)|| + \bar{\pi}_4.$$

Since $\dot{K}_1(x_1) = \frac{2\check{k}_1}{\varsigma}(\dot{\hat{k}}_1(x_1)||\mu(x_1)|| + \hat{k}_1(x_1)\frac{d}{dt}||\mu(x_1)||)$,

$$|\ddot{K}_1(x_1)| \leq \frac{2\check{k}_1}{\varsigma}\Big(|\ddot{\hat{k}}_1(x_1)|||\mu(x_1)|| + 2|\dot{\hat{k}}_1(x_1)|||\dot{\mu}(x_1)|| + \hat{k}_1(x_1)||\ddot{\mu}(x_1)||\Big) \leq \frac{2\check{k}_1}{\varsigma}\Big((\check{\beta}_1 D_1 + \bar{\beta}_1(||e(t)|| + D_2 + \bar{f}_1 D_1))\mu_m + 2\bar{\beta}_1 D_1 M_1 + \hat{k}_{1m}M_2\Big) \triangleq \bar{\pi}_5||e(t)|| + \bar{\pi}_6.$$

Then from (60),

$$\begin{aligned}||\ddot{\bar{\alpha}}_1(x_1)|| &\le \frac{1}{2}||\frac{d^2}{dt^2}(\mu(x_1))^{-1}||(k_1+K_1(x_1)) \\ &+ ||\frac{d}{dt}(\mu(x_1))^{-1}|||\dot{K}_1(x_1)| + \frac{1}{2}||(\mu(x_1))^{-1}|||\ddot{K}_1(x_1)| \\ &\le \Big(\frac{M_1^2}{\varsigma_0^3}+\frac{M_2}{2\varsigma_0^2}\Big)(k_1+\bar{K}_1) + \frac{M_1}{\varsigma_0^2}\hat{K}_1 \\ &+ \frac{1}{2\varsigma_0}(\bar{\pi}_5||e(t)||+\bar{\pi}_6) \triangleq \bar{\pi}_7||e(t)||+\bar{\pi}_8.\end{aligned}$$

As a result,

$$\begin{aligned}||\ddot{\alpha}_1|| &\le ||\ddot{\bar{\alpha}}_1(x_1)||||z_1|| + 2||\dot{\bar{\alpha}}_1(x_1)||||\dot{z}_1|| + ||\bar{\alpha}_1(x_1)||||\ddot{z}_1|| \\ &\le (\bar{\pi}_7||e(t)||+\bar{\pi}_8)z_m + \frac{k_1+\bar{K}_1}{2\varsigma_0}(\bar{\pi}_3||e(t)||+\bar{\pi}_4) \\ &+ 2\bar{\alpha}_m Z_1 \triangleq \bar{\pi}_9||e(t)||+\bar{\pi}_{10}.\end{aligned}$$

Then from (24),

$$\begin{aligned}&||\dot{u}(t)|| \le ||\ddot{\alpha}_1(x_1)|| + ||\dot{\mu}^T(x_1)||||z_1|| + ||\mu^T(x_1)||||\dot{z}_1|| \\ &+ \frac{1}{2}|\dot{K}_2(x_1,x_2)|||z_2|| + \frac{1}{2}(k_2+K_2(x_1,x_2))||\dot{z}_2|| \\ &\le \bar{\pi}_9||e(t)|| + \bar{\pi}_{10} + M_1 z_m + \mu_m Z_1 + \frac{1}{2}(\bar{\pi}_1||e(t)||+\bar{\pi}_2)z_m \\ &+ \frac{1}{2}(k_1+\bar{K}_2)(||e(t)||+D_2+\hat{\alpha}_m) \\ &\triangleq \pi_1||e(t)||+\pi_2.\end{aligned}$$

Recalling (20), we have

$$\frac{d||e(t)||}{dt} \le ||\dot{e}(t)|| \le ||\dot{u}(t)|| \le \pi_1||e(t)||+\pi_2.$$

Then $||e(t)|| \le E(t,E_0)$ where $E(t,E_0)$ is the solution to $\dot{E} = \pi_1 E + \pi_2$ with $E_0 = E(0,E_0)$. Since $||e(t_k)|| = ||u(t_{k(t)}) - u(t_k)|| = 0$ with $t_{k(t)}$ given by (8b) and (19),

$$\begin{aligned}||e(t)|| &\le E(t-t_k,0) \\ &\le \frac{\pi_2}{\pi_1}\Big(e^{\pi_1(t-t_k)}-1\Big),\ t\in[t_k,t_{k+1}),\ k\notin\Gamma.\end{aligned} \quad (61)$$

From the definition of $V_2(t)$, (58) and the event-triggering condition (20),

$$||e(t)|| \le \gamma||z(t)|| \le \gamma\sqrt{V_2(t)} \le \gamma\sqrt{e^{(M_F+M_D)\bar{\kappa}}e^{-\bar{\kappa}^* t}V_2(0)}.$$

Thus, the inter-event time $t_{k+1}-t_k$, $k\notin\Gamma$, is lower bounded by the solution τ_D of the equation

$$\frac{\pi_2}{\pi_1}\Big(e^{\pi_1\tau_D}-1\Big) = \gamma_0 e^{-\frac{\bar{\kappa}^*}{2}(\tau_D+t_k)} \quad (62)$$

where $\gamma_0 = \gamma e^{(M_F+M_D)\frac{\bar{\kappa}}{2}}\sqrt{V_2(0)}$. Let $\epsilon^* = \frac{1}{2\pi_1}\ln(1+\frac{\pi_1\gamma_0}{\pi_2}e^{-\frac{\bar{\kappa}^*}{2}\tau^*})$. Then from (62), when $k>K_0$, $\tau_D \ge 2\epsilon^*$. It is noted that from (59), for all $k>K_0$, $t_k>\tau^*-\epsilon^*$, and then $t_{k+1}\ge t_k+\tau_D>\tau^*-\epsilon^*+\tau_D\ge\tau*+\epsilon^*$, contradicting $t_{k+1}\le\tau^*$ for all $k>K_0$ in (59). Thus, Zeno behavior is excluded. ■

Based on Theorem 4.1, we provide two corollaries for robust safety critical control and security control problems.

Corollary 4.1: Consider the system (1) with the safety constraint (2). For any $y(0)\in S$ and $||z(0)||\le z_0$, design $u(t)$ from (24) where $k_1\ge 3$, $k_2\ge 1$, and parameters β,ς,z_m follow Step 2 of Algorithm 1. The robust safety critical control problem of the uncertain system (1) is solvable by (24) in the sense that the trajectory of the closed-loop system satisfies $y(t)\in S$ for all $t\ge 0$ and $\lim_{t\to+\infty}x(t)=\mathbf{0}_{2n}$.

Proof: From Step 1 of the proof for Theorem 4.1 with $\bar{\delta}_1=+\infty$, (44) holds for all $t\ge 0$ with $e(t)=\mathbf{0}_n$ and $\gamma=0$. Then $V_2(t)\le V_2(0)e^{-M_F t}$ and $||z(t)||\le||z(0)||e^{-\frac{M_F}{2}t}\le z_0<z_m$, $\forall t\ge 0$, implying that $\lim_{t\to+\infty}z(t)=\mathbf{0}_{2n}$ and $y(t)\in S$ for all $t\ge 0$ by Lemma 3.2. Follow the analysis of Cases 1 and 2 in Step 2, and we can obtain $\lim_{t\to+\infty}x(t)=\mathbf{0}_{2n}$. ■

Corollary 4.2: Consider the nonlinear system (1) subject to DoS attacks satisfying Assumptions 2.1 and 2.2 with $\kappa<1$. For any $||z(0)||\le z_0$, design the control law (25) where $x(t_{k(t)})$ is defined in (8b) with t_k in (19), $u(t)$ is given by (24) with $z_1=x_1$ and $\mu(x_1)=I_n$, and control parameters are designed by Steps 1 and 3 in Algorithm 1. Then the security control problem of (1) is solvable by (25), i.e., the trajectory of the closed-loop system satisfies $\lim_{t\to+\infty}x(t)=\mathbf{0}_{2n}$, and no Zeno behavior occurs.

The conclusion follows from the proof of Theorem 4.1 with $S\triangleq\{y\in\mathbb{R}^n : ||y||<z_m\}$.

Remark 4.1: Corollary 4.1 provides a class of governor function based robust safety critical control, the procedure of which is completely different from that with QP in [3]–[11]. Corollary 4.2 sheds light on the security control against DoS for a system with strong nonlinearity, instead of linear ones in most existing works such as [16], [17] or special nonlinear systems in [22], [23]. It essentially proposes a semi-global security control with an efficient method to evaluate the growth of the nonlinear trajectory during DoS and ensure the exponential convergence without DoS.

B. Discussion

Our design (25) can be generalized to the high-order nonlinear uncertain system (3) with relative degree $r\ge 2$. Define $z_i=x_i-\alpha_{i-1}$, $i=2,\cdots,r$, and then we can obtain the following transformed system, over $\bar{\Theta}(0,t)$,

$$\begin{aligned}\dot{z}_1 &= \mu(x_1)(z_2+\alpha_1)+\mu(x_1)f_1(x_1,\sigma), \\ \dot{z}_i &= f_i(x_i,\sigma)+z_{i+1}+\alpha_i-\dot{\alpha}_{i-1},\ i=2,\cdots,r-1, \\ \dot{z}_r &= f_r(x_r,\sigma)+u(t)+e(t)-\dot{\alpha}_{r-1}.\end{aligned} \quad (63)$$

The design of the stabilizing controller α_i, $i=1,\cdots,r$, follows the similar procedure of (24), and with the aid of the backstepping technique, it takes the following form,

$$\begin{aligned}\alpha_1 &= -(\mu(x_1))^{-1}(\eta_1+\varrho_1(x_1)) \\ \alpha_2 &= -(\eta_2+\varrho_2(x_2))z_2+\dot{\alpha}_1-\mu^T(x_1)z_1 \\ \alpha_i &= -(\eta_i+\varrho_i(x_i))z_i+\dot{\alpha}_{i-1}-z_{i-1},\ i=3,\cdots,r\end{aligned} \quad (64)$$

for constants $\eta_j>0$, $j=1,\cdots,r$ and some smooth functions $\varrho_j(\cdot)>0$, determined by $f_j(\cdot)$. Then the sampled-data safety critical control against DoS for system (3) can be designed as $u(t)=\alpha_r(x_r(t_{k(t)}))$.

Remark 4.2: It is also possible to accommodate multiple safety constraints in the form of

$$\begin{aligned}S &= \{y\in\mathbb{R}^n:\ \epsilon_1<h(y)<\epsilon_2\}, \\ \partial S &= \{y\in\mathbb{R}^n:\ h(y)=\epsilon_1\ \cup\ h(y)=\epsilon_2\}.\end{aligned}$$

The governor function z_1 is designed by (11) with $H(h(y)) = \frac{1}{h(y)-\epsilon_1+\frac{1}{\beta_0}} + \frac{1}{\epsilon_2-h(y)+\frac{1}{\beta_0}}$ for some $\beta_0 > 0$. Let $\beta = \beta_0 + \frac{1}{\epsilon_2-\epsilon_1+\frac{1}{\beta_0}}$. Instead of (37), β is chosen such that $\beta > \max\{\frac{\sqrt{n}}{\epsilon_m-\epsilon_1} + \frac{\sqrt{n}}{\epsilon_2-\epsilon_m},\ \frac{1}{h_m}(\frac{1}{r_0} - \chi_m - \bar{m}_0)\}$.

V. Example

Consider the mechanical system in [12], [13] as follows,

$$\dot{x}_1 = x_2, \tag{65}$$
$$\dot{x}_2 = u - \begin{bmatrix} (0.8+0.2e^{-100|x_{2x}|})\tanh(x_{2x}) + x_{2x} \\ (0.8+0.2e^{-100|x_{2y}|})\tanh(x_{2y}) + x_{2y} \end{bmatrix} - x_1$$

where $x_1 = \mathrm{col}(x_{1x}, x_{1y}) \in \mathbb{R}^2$ is the position, $x_2 = \mathrm{col}(x_{2x}, x_{2y}) \in \mathbb{R}^2$ is the velocity, and $u = \mathrm{col}(u_x, u_y) \in \mathbb{R}^2$ is the control force.

For the safety requirement of avoiding obstacles, system (65) is subject to the constraint (2) with $\epsilon = 9$ and

$$\begin{aligned} h(y) = 9 - 80(1 - x_{1x}^2 - x_{1y}^2)(0.225^2 - (x_{1x}+0.3)^2 \\ - (x_{1y}-0.15)^2)(0.04 - (x_{1x}-0.3)^2 - (x_{1y}+0.3)^2) \end{aligned} \tag{66}$$

Also, (65) is subject to randomly occurring DoS attacks. In particular, for $p \in \mathbb{N}^+$, the p-th DoS off/on transition δ_p follows a uniform distribution over $[0.25(p-1), 0.25p]$, which implies that the probability of δ_p taking any value within $[0.25(p-1), 0.25p]$ is equal. Similarly, the p-th DoS time-interval τ_p follows a uniform distribution over $[0, 0.25p - \delta_p]$. DoS attacks can be found in Figs. 4 and 5, satisfying Assumptions 2.1 and 2.2 with $\kappa_1 = 0.05$, $T_D = 2$ in (5), and $\kappa_2 = 1$, $\tau_D = 0.25$ in (6). The minimum sampling rate of digital devices is $\tau_m = 0.001$.

The initial conditions are $x_1(0) = \mathrm{col}(-0.6, 0.6)$ and $x_2(0) = \mathrm{col}(-2, -3)$, satisfying $x_1(0) \in S$. Choose $\check{k}_1 = 0$, $\hat{k}_1(x_1) = 0$, $\check{k}_{21} = \check{k}_{22} = 1$, $\hat{k}_{21}(x_1) = 1$, and $\hat{k}_{22}(x_2) = 2$ such that (29) is satisfied. From (30),

$$K_1(x_1) = 0,\ K_2(x_1, x_2) = \frac{1}{\varsigma}^2 + 4||\bar{\alpha}_1(x_1)||^2 + 4. \tag{67}$$

where $\bar{\alpha}_1(x_1) = \frac{k_1}{2}(\mu(x_1))^{-1}$.

Following Step 1 of Algorithm 1, select

$$\vartheta_t = 0.002,\ \gamma = 50. \tag{68}$$

Then $\kappa = 0.508 < 1$.

From Step 2 of Algorithm 1, choose $\epsilon_m = 2.95^2$ satisfying $h(0) < \epsilon_m < \epsilon$. From (9) and (10), $S_m = \{x_1 \in \mathbb{R}^2 : h(x_1) < 2.95^2\}$ and $x_m = 0.14$. Then select $r_0 = 0.13 < x_m$. Design the smooth switching function $\chi(\tau)$ as follows,

$$\chi(\tau) = \begin{cases} 0 & \tau \le 0 \\ (1 + \frac{e^{(x_m-r_0)/\tau}}{e^{(x_m-r_0)/(x_m-r_0-\tau)}})^{-1} & 0 < \tau < x_m - r_0 \\ 1 & \tau \ge x_m - r_0 \end{cases} \tag{69}$$

It can be verified that (69) has the required properties. From (37) and (14), choose

$$\begin{aligned} \chi_m = 40,\ h_m = 245.87,\ \bar{m}_0 = 0.05\sqrt{2} \\ \beta = 5,\ \varsigma_0 = 0.5,\ \varsigma = 9000 \end{aligned} \tag{70}$$

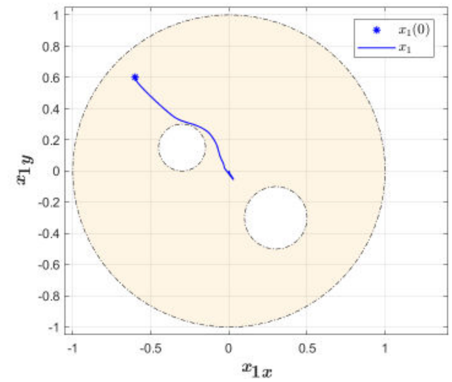

Fig. 3: The trajectory of $y(t)$ under the control (25) in Theorem 4.1.

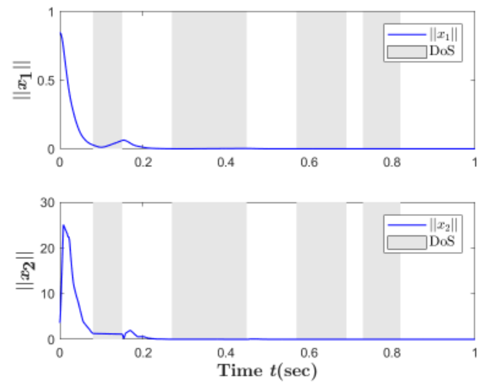

Fig. 4: $||x_1(t)||$ and $||x_2(t)||$ under the control (25) in Theorem 4.1.

Then design z_1 from (11) and $\mu(x_1)$ from (13). From (16) and (18b),

$$\mu_m = 17975.80,\ z_m = 8964.58. \tag{71}$$

From Step 3 of Algorithm 1, choose

$$\bar{K}_1 = 0,\ \bar{K}_2 = 4k_1^2 + 5,\ k_1 = 300,\ k_2 = 300.$$

Then design $u(t)$ from (25). Note that $||z(0)|| = 7637.80$. Select $z_0 = 8000$ satisfying $||z(0)|| \le z_0 < z_m$.

The performance of the control law (25) can be found in Figs. 3-5. Fig. 3 shows that the trajectory of $y(t)$ remains within the safe set, and Fig. 4 demonstrates the convergence of $x(t)$. There is no Zeno behavior, shown in Fig. 5.

Now we compare our design (25) with the safety critical control laws in [12], [13]. Fig. 6 shows the trajectories of system (65) under safety critical control laws in [12], [13], and one can observe that different from Fig. 3, the safety constraint (2) cannot be satisfied for all $t \ge 0$ by the methods in [12], [13] due to the existence of randomly occurring DoS.

According to Corollaries 4.1 and 4.2, our method can reduce to solve the safety critical control and security control problems. Thus, in the following two subsections, we also compare the reduced safety critical control (24) and the security control based on (25) with existing results.

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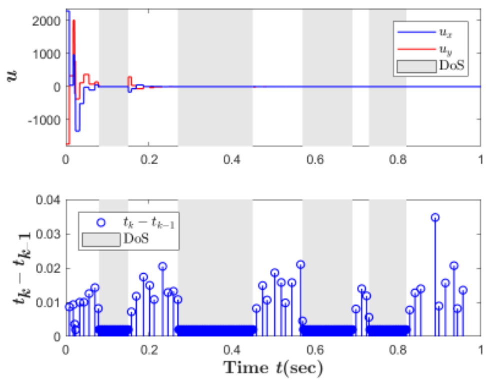

Fig. 5: $u(t)$ and $t_k - t_{k-1}$ under the control (25) in Theorem 4.1.

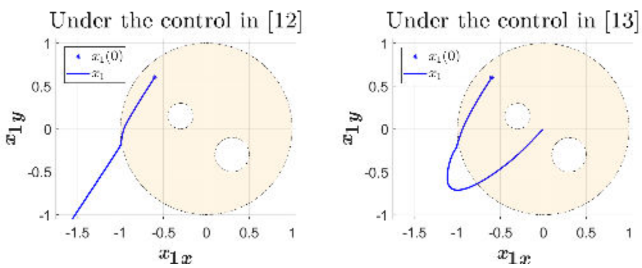

Fig. 6: The trajectories under the safety critical control laws in [12], [13].

A. Solvability of Safety Critical Control Problem

In this subsection, we solve the safety critical control of system (65) subject to safety constraint (66) without DoS, and compare the control (24) with those in [12], [13].

Based on Corollary 4.1 with control parameters in (67), (69), (70) and (71), choose $k_1 = 100$ and $k_2 = 1$ to design $u(t)$ in (24). Note that $||z(0)|| = 7636.87$. Select $z_0 = 8000 \geq ||z(0)||$, satisfying $z_m > z_0$.

From Fig. 7, the trajectory of $y(t)$ remains in the safe set, and $x(t)$ converges to 0 in Fig. 8. Compared with CBF based methods in [12], [13], our governor function based control (24) can solve the safety critical control problem for system (65) without control-dependent conditions for CBF, e.g., Assumption 1 in [13].

B. Solvability of Security Control Problem

In this subsection, we study the security control problem of the system (65) subject to DoS attacks with the reduced control (25) and compare with the classic method against DoS in [16].

According to Corollary 4.2, choose $z_1 = x_1$ and $\mu(x_1) = I_2$. Select

$$\mu_m = 1,\ \bar{K}_1 = 0,\ \bar{K}_2 = k_1^2 + 5,\ k_1 = 15,\ k_2 = 1,$$
$$\vartheta_t = 0.01,\ \gamma = 50$$

Then $\kappa = 0.54 < 1$. Design $u(t)$ from (25) with the initial conditions $x_1(0) = \text{col}(-6, 6)$ and $x_2(0) = \text{col}(-2, -3)$. Note that $||z(0)|| = 63.60$. Select $z_0 = 65 > ||z(0)||$ and $z_m = 70 > z_0$.

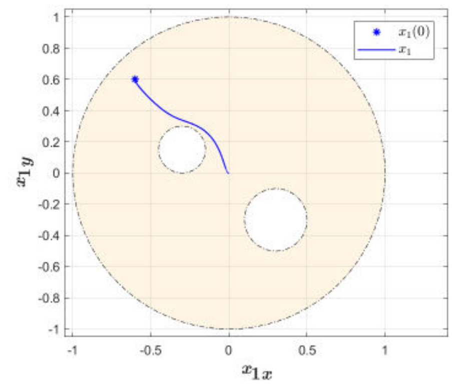

Fig. 7: The trajectory of $x_1(t)$ under the control (24) in Corollary 4.1.

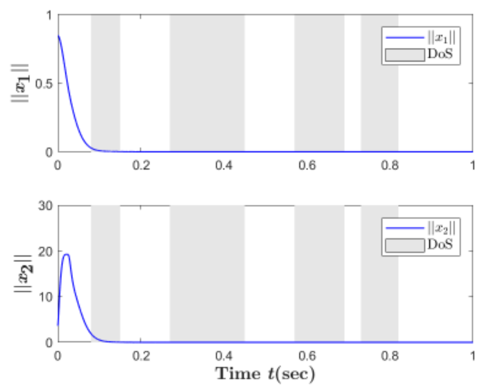

Fig. 8: $||x_1(t)||$ and $||x_2(t)||$ under the control (24) in Corollary 4.1.

Figs. 9 and 10 display the performance of the reduced control (25). In the presence of DoS, $x(t)$ decreases to 0, and no Zeno behavior occurs. The security control in [16] is designed for linear systems, and can only provide a local solution by linearizing system (65) at the origin. As shown in Fig. 11, it is required that the initial condition is chosen in the neighborhood of the origin, and in the simulation, let $x_1(0) = \text{col}(-0.01, 0.01)$ and $x_2(0) = \text{col}(-0.01, -0.02)$.

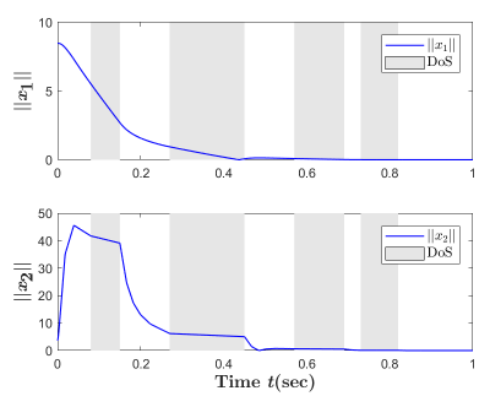

Fig. 9: $||x_1(t)||$ and $||x_2(t)||$ under the control (25) in Corollary 4.2.

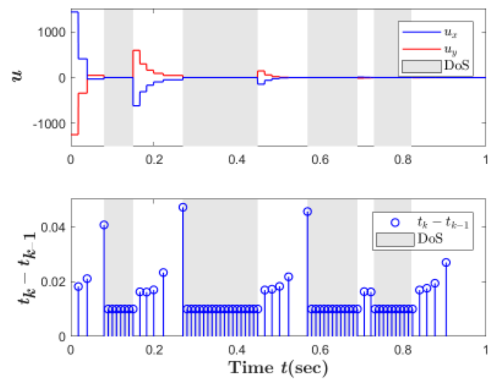

Fig. 10: $u(t)$ and $t_k - t_{k-1}$ under the control (25) in Corollary 4.2.

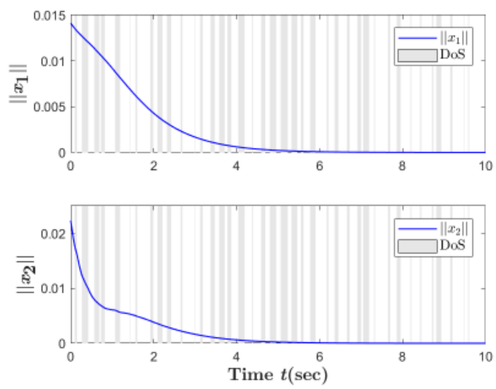

Fig. 11: $||x_1(t)||$ and $||x_2(t)||$ under the security control law in [16].

VI. Conclusion

This paper has designed a sampled-data safety critical control for a second-order nonlinear system with parametric uncertainty, which is capable of detecting and foiling malicious DoS attacks at both controller and actuator ends as well as satisfying the safety constraint as the priority task and stabilizing the closed-loop system. A new concept, called the governor function, has been proposed, and we have provided a specific form to handle the safety constraint denoted by a general nonlinear function. Our governor function based control strategy has increased the robustness of the safety critical control and the resiliency of recovering the control performance from malicious sensor and actuator attacks. Algorithm 1 has also been presented for the selection of control parameters, which are pre-determined, satisfying the real-time implementation requirement. They are mainly influenced by the form of the safety constraint and the intensity of DoS attacks, and potentially bring some conservatism since high gain technique is utilized in the selection procedure. In the future, we will extend our design to the safety critical control of general nonlinear systems with more complex constraints and hybrid attacks such as the random combinations of false data injection, relay and DoS attacks.

Appendix

We provide the following lemma for Lemma 3.1.

Lemma A.1: Define

$$F_0(x) = F_1(x) + F_2(x) + MI_n \tag{72}$$

where $x \in \mathbb{R}^n$, $F_i : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $i = 0, 1, 2$, satisfies $||F_1(x)|| \le M_1$ and $||F_2(x)|| \le M_2$ for $M_i \ge 0$, $i = 1, 2$. If

$$M \ge M_1 + M_2 + M_0, \tag{73}$$

for some $M_0 > 0$, then $F_0(x) \ge M_0 I_n$.

Proof: For simplicity, denote $F_i = F_i(x)$, $i = 0, 1, 2$. Note that

$$\begin{aligned} 0 &\le \left(\frac{1}{\sqrt{M_i}}F_i + \sqrt{M_i}I_n\right)^T\left(\frac{1}{\sqrt{M_i}}F_i + \sqrt{M_i}I_n\right) \\ &= \frac{1}{M_i}F_i^T F_i + F_i + F_i^T + M_i I_n, \ i = 1, 2. \end{aligned} \tag{74}$$

Since $||F_i||^2 = \lambda_{\max}(F_i^T F_i) \le M_i^2$ with $\lambda_{\max}(\cdot)$ denoting the largest eigenvalue of a matrix, for $i = 1, 2$, from (74), $F_i + F_i^T \ge -\frac{1}{M_i}F_i^T F_i - M_i I_n \ge -\frac{1}{M_i}M_i^2 I_n - M_i I_n = -2M_i I_n$. Then by (73),

$$\begin{aligned} &F_0 - M_0 I_n + (F_0 - M_0 I_n)^T \\ &= F_1 + F_1^T + F_2 + F_2^T + 2MI_n - 2M_0 I_n \\ &\ge 2(-M_1 - M_2 + M - M_0)I_n \ge 0. \end{aligned}$$

As a result, $F_0 \ge M_0 I_n$. ■

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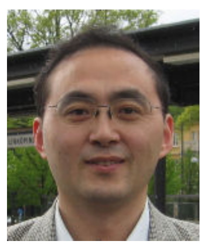
Yi Dong received her Ph.D. degree in the Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong in 2013. Since August 2013, she worked as a Postdoctoral Fellow in the Chinese University of Hong Kong, Hong Kong and then in the Royal Institute of Technology, Stockholm, Sweden. She was a professor in School of Automation, Nanjing University of Science and Technology from July 2015 to December 2019. Since January 2020, she joined College of Electronic and Information Engineering, Tongji University. Her research interests include safety critical control, cooperative control, output regulation, nonlinear control systems, etc.

Xiran Cui a Ph.D. candidate in College of Electronic and Information Engineering, Tongji University, Shanghai, China. His research interests include safety critical control and nonlinear systems.



Yiguang Hong (Fellow, IEEE) received the B.S. and M.S. degrees from Peking University, China, and the Ph.D. degree from the Chinese Academy of Sciences (CAA), China. He is currently a Professor with the Shanghai Institute of Intelligent Science and Technology, Tongji University and was a professor in Academy of Mathematics and Systems Science, CAS. His current research interests include distributed optimization and games, nonlinear control, multi-agent systems, and social networks. He serves as the Editor-in-Chief for Control Theory and Technology. He also serves or served as an Associate Editors for many journals, including the IEEE Transactions on Automatic Control, IEEE Transactions on Control of Network Systems, and IEEE Control System Magazine. He was a recipient of the Guan Zhaozhi Award from the Chinese Control Conference, the Young Author Prize of the IFAC World Congress, the Young Scientist Award of CAS, the Youth Award for Science and Technology of China, and the National Natural Science Prize of China.

Jie Chen (Fellow, IEEE) received the B.Sc., M.Sc., and the Ph.D. degrees in control theory and control engineering from the Beijing Institute of Technology, Beijing, China, in 1986, 1996, and 2001, respectively. From 1989 to 1990, he was a Visiting Scholar with the California State University, Long Beach, CA, USA. From 1996 to 1997, he was a Research Fellow with the School of Engineering, University of Birmingham, Birmingham, U.K. He is the Director of the National Key Laboratory of Autonomous Intelligent Unmanned Systems, Tongji University. His research interests include safety critical control, complex systems, multiagent systems, and multiobjective optimization and decision. Dr. Chen is currently the Editor-in-Chief of Unmanned Systems and Journal of Systems Science and Complexity. He was on the Editorial Boards of several journals, including the IEEE Transactions on Cybernetics, International Journal of Robust and Nonlinear Control, and Science China Information Sciences. He is a Fellow of IFAC, and a Member of the Chinese Academy of Engineering.