Optimal Requirements of Spare Transformers and Mobile Units for Distribution Substations via Genetic Algorithm and Monte Carlo Techniques

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***Abstract*—This paper proposes a new optimization method based on enhanced genetic algorithm (GA) and Monte Carlo simulation (MCS) techniques, which are simultaneously applied to size regular spare transformer (RST) and mobile unit substations (MUS) stocks for distribution substations. The aim is to serve a group of electrical energy distribution substations to mitigate possible losses caused by load curtailments due to major failures that affect the substation transformers. The proposed method includes the use of resources such as MUS and load transfer, in addition to representing the expansion of the transformers group in operation and the increase in power demand, over a specified planning horizon, considering all waiting times inherent to system actions, e.g.,: RST installation, MUS connection, stock replenishment, etc. Two real systems with different characteristics are used to illustrate the proposed method, allowing the analysis of results obtained from different scenarios and parameters.**

***Index Terms*—Distribution substations, genetic algorithm, mobile unit substations, Monte Carlo simulation, reliability assessment, spare transformer, stock sizing.**

I. Introduction

THE power transformer used in electric power distribution substations draws the attention of system planners for it is a large-scale piece of equipment, with a high acquisition cost and long manufacturing time, e.g., 12 months [1]. As it is not immediate replacement equipment, its failure can affect a large number of electricity consumers for several days, causing financial losses for power distribution companies and their customers. Despite all efforts to identify problems and define preventive maintenance actions [2], [3], failures are inevitable and occur randomly.

In general, substations are planned with transformers in parallel, so that the power system continues to operate even if one transformer fails. Although safe, this strategy requires a high execution cost from the substation arrangement point of view [4], [5], [6]. An economically feasible alternative adopted by electric power companies consists of properly handling their inventories. In this case, regular spare transformers (RSTs) and mobile unit substations (MUS) are shared by groups of substations of the same company that use transformers with identical or similar electrical characteristics [6], [7].

RSTs are used to replace those that suffer major (sometimes called catastrophic) failures, while MUS supply the load during repair of a transformer in operation, or its replacement by a spare [7], [8]. MUS are complete, flexible and dynamic modules that can be moved around, serving the system during emergencies, planned maintenance actions and grid expansion [9]. Their usefulness in reducing the impact created by the loss of a transformer in operation comes from the possibility of being installed faster than a RST, e.g., between 8 and 24 hours after the occurrence of the failure [10].

The sizing of spare inventories is determined by factors such as the required level of reliability, investment costs, storage and maintenance of RSTs, and costs associated with operating the system, such as interruptions to the power supply and penalties provided by the regulatory agency [11]. Sizing a stock of transformers is not a simple task, as a large amount of spares may imply unnecessary costs without producing positive results in system availability, while an insufficient number of spares may endanger system reliability, resulting in high cost values of operation and financial compensation. Therefore, the right sizing of inventories must provide for a balance between investment and operating costs, in order to ensure an adequate level of reliability, at the lowest possible cost.

The first papers on RST inventory sizing applied the Poisson distribution [12], [13], [14]. This distribution, used in [4], [5], [15], and [16], only considers the failure rate to calculate the probability of a certain number of transformer failures occurring in a specified period of time, e.g., 1 year. For simplicity, it is assumed that the spare transformer, when available, is instantly installed, i.e., its installation time is ignored. Poisson-based approaches, although very simple to be utilized, are very

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limited bearing in mind all characteristics and practical details of handling power transformer failures.

Models based on continuous Markov processes [12], [13], [14] have also been developed for sizing RST stocks. Under the assumption of exponential waiting times, Markov-based models found in the literature allow representing failure, repair and installation rates of RSTs and MUS [7], [8], [9], [10], [17], [18], [19], [20], [21], [22], [23], [24]. Thus, performance indices such as availability, frequency and average duration of failure can be straightforwardly calculated. Moreover, Markov-based models also allow considering load transfers to neighboring substations [25].

The sequential or chronological Monte Carlo simulation (MCS) is a very robust technique that allows representing the most relevant characteristics of managing power transformers, including the consideration of non-exponential times not supported by the Poisson and Markov models. The flexibility of the chronological MCS is illustrated in [1], [9], [16], [26] and [27]. Besides individually modeling any equipment in terms of failure, repair and installation rates, non-exponential distributions can be handled by the chronological MCS, if necessary. That means equipment aging processes can be duly captured. Some of these references illustrate the similarities and limitations of Poisson and Markov processes in relation to chronological MCS techniques: see, for instance, [9], [16], [26].

In [9], a simple chronological MCS is used to separately assess the stocks of RST and MUS, considering catastrophic and repairable failures. Afterwards, the assessment is done by combining the two groups of RST and MUS equipment. Decision making is carried out by analyzing reliability and cost indices, but no optimization strategy is used to simultaneously size both RST and MUS stocks. In [27], the chronological MCS is applied to the sizing of the RST stock. The model considers load transfer, RST and MUS installation times, and transformer aging, combining loss of life due to thermal effect (Arrhenius Theory) with additional reductions caused by short circuits and lightning. In [28], two strategies are applied in a simulation environment to mitigate damage caused by catastrophic failures: proactive replacement of older transformers or transformers with a higher probability of failure, and; supply of RST to replace faulty transformers.

The optimal sizing of inventories over a planning period characterizes a problem with a large number of possible combinations between acquisitions of RSTs and MUS. In this context, the application of optimization techniques has been fundamental, as demonstrated in few innovative works [6], [15], [16], [26], [28], [29], [30], [31]. For instance, in [29] and [30], a methodology based on the evolution strategies metaheuristic is applied to size only RST stocks, where reliability and cost indices are calculated by chronological MCS techniques. However, the time required for installing the RST, the possibility of load transfer and the use of MUS are ignored. In [31], the Poisson distribution is applied to dimension the stock of reserves. Independently, this information is used to define the best location for spare transformers through a classical optimization algorithm.

The present work proposes a new method to simultaneously size RST and MUS stocks through the combination of an enhanced genetic algorithm [32], [33] and chronological MCS techniques. Both parameters, the number and acquisition time of RSTs and MUS, are optimized over a pre-established planning horizon. Several practical aspects are considered such as: load transfer, demand growth and increase of the number of transformers, specific times inherent to system actions (e.g., RST installation, MUS connection, stock replenishment, etc.). Heuristic strategies are incorporated into the proposed algorithm, aiming to expand the search space of a set of feasible solutions and reduce processing time. All these aspects ensure the originality of the proposed method in relation to the previously described approaches in the cited literature. More specific details will be addressed in the next sections.

This paper is structured as follows: Section II describes the basic chronological MCS technique for the reliability assessment of the RST and MUS inventory composition; Section III presents the optimization tool based on the enhanced genetic algorithm; Section IV shows and discusses the results obtained with two real systems: a Canadian one with 60 transformers of 115 kV/15 MVA and a Brazilian one with 177 transformers of 138 kV/25 MVA; and Section V reports the main conclusions of the work.

II. Reliability Assessment via Chronological MCS

Monte Carlo simulation is a statistical support tool that reproduces the random operation of any system, allowing estimation of various reliability and cost indices [13], [14]. MCS can be classified in different types including non-sequential and sequential, the latter also called chronological. The non-sequential model is based on the representation of system states, sampled according to their probability, usually assuming a Markov model [34]. Chronological MCS performs the selection and analysis of a large number of samples of operation and repair times of components of a system [34], extracted from its random variables. As the dimensioning of RST and MUS stocks depends on the sequence of random events and the duration of corrective actions to represent their operational behavior, the chronological simulation is the most robust model to deal with this problem.

Fig. 1 illustrates the previous idea in a hypothetical system with three transformers, named $\mathrm{TR_a}$, $\mathrm{TR_b}$ and $\mathrm{TR_c}$. Each transformer can be found in one of the following states at any moment: in operation (up), in the process of being replaced by a new transformer (down), in stock (spare) or with the mobile substation installed (mus). The chronological analysis makes it possible to establish, at any time: the availability of the MUS, the number of reserves, the deficit of equipment in the substations and the number of units in operation, designated by $\mathrm{n_{mus}(t)}$, $\mathrm{n_r(t)}$, $\mathrm{n_d(t)}$ $\mathrm{n_{op}(t)}$, respectively.

At the beginning of the timeline, transformers $\mathrm{TR_a}$ and $\mathrm{TR_b}$ are in operation, while $\mathrm{TR_c}$ and MUS are available in stock, until instant $\mathrm{t_1}$ when $\mathrm{TR_a}$ fails. At this moment, the MUS is sent to the station where the failure occurred to mitigate the outage time, while the transport and installation of $\mathrm{TR_c}$ to replace $\mathrm{TR_a}$ begins. The spare equipment must be replaced with the purchase of a new unit, which will become part of the stock as soon as it becomes available. At $\mathrm{t_2}$, a similar sequence begins with the failure of $\mathrm{TR_b}$. At time $\mathrm{t_3}$ a fault strikes $\mathrm{TR_c}$, and there is no reserve in stock to replace it. Thus, the point remains de-energized until a new

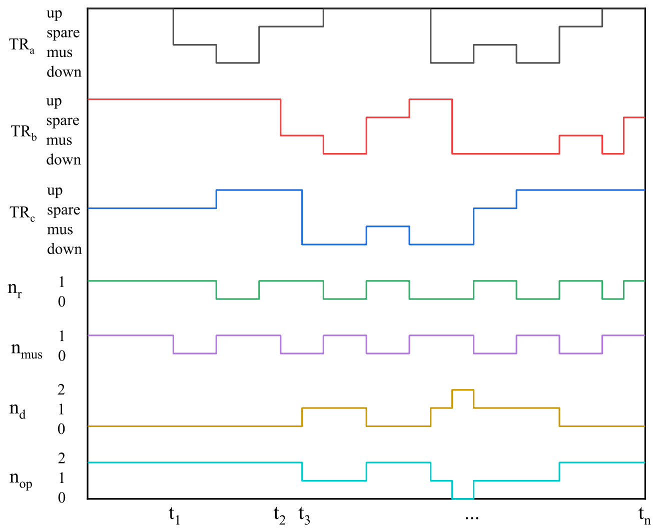

Fig. 1. Illustrative sample of the history generated by the chronological MCS.

reserve unit is acquired. The chronological MCS can properly reproduce this time process.

Once the actions to be taken and the times for carrying them out have been defined, the algorithm identifies the moment of the next event (load transfer, mobile substation connection, transformer installation, stock replacement or other transformer failure) and evaluates performance of the system between the previous and the current instant, accounting for the time of failure and the energy not supplied for the subsequent calculation of reliability indices, as discussed in the following sections. Moreover, if there is no RST available, the MUS is not used to replace the faulty transformer. Clearly, this is a study premise, not an operational practice. This is because, in addition to having a much higher acquisition cost than the transformer, the use of MUS would cancel out the pre-sized stock. Besides, this long-term action would decrease system reliability, given the positive input that MUS provides in mitigating load loss time. For more details of this chronological simulation process, see [1], [9], [16], [26] and [27].

A. Test Functions and Convergence

The estimate of reliability indices and costs related to load curtailments is associated with the evaluation of a set of test functions. Each index is estimated as the expected value of a specific test function *G* [34], i.e.,

$$\tilde{E}[G] = \frac{1}{N_{sim}} \sum_{k=1}^{N_{sim}} G(\mathbf{Y}_k) \tag{1}$$

where N_{sim} is the number of simulations performed (e.g., 100000 repetitions of the period of analysis, $T = 10$ years), $\boldsymbol{Y}_k$ is the sequence of system states in period *k*. The uncertainty of the estimate is given by the sample variance of the estimator, i.e.,

$$V(\tilde{E}[G]) = V(G)/N_{sim} \tag{2}$$

where *V(G)* is the variance of the test function.

The convergence of the MCS is checked using the coefficient of variation defined by [34]:

$$\beta = \sqrt{V(\tilde{E}[G])} \Big/ \tilde{E}[G]. \tag{3}$$

The relative uncertainty or coefficient of variation corresponds to the quotient between the sample standard deviation of the index of interest and its sample mean. In the proposed approach, the convergence of the simulation is measured by the value of β_{EENS}, calculated for the *EENS* index (expected energy not supplied). Therefore, the MCS may be interrupted when the number of simulated periods (N_{sim}) is large enough that β_{EENS} is less than the acceptable limit (e.g., $\beta_{\max} = 1\%$) or when N_{sim} reaches its maximum limit $(N_{sim\text{-}max})$.

B. Reliability Indices

In order to assess the main reliability indicators, the previous variables described in the illustrative example of Fig. 1 are properly used to calculate the total success and failure times, i.e., $t_{success}$ and $t_{failure}$, respectively. Considering the analysis period *T* (e.g., a decade) defined by the planner, the following reliability indices are evaluated:

- Availability *(A)* is the probability that the system will have all transformers in operation during *T*, estimated as the ratio between the total success time (e.g., decades) and the number of simulations performed (e.g., decades), i.e.,:

$$A = t_{success}/N_{sim}. \tag{4}$$

- Unavailability *(U)* is equal to "1−*A*" and it can be interpreted as the expected number of hours in period *T* during which the system presents a deficit in the number of transformers in use. It is the ratio between the total failure time (in hours) and the number of simulations performed (e.g., decades), in h/T:

$$U = t_{failure}/N_{sim}. \tag{5}$$

- Expected frequency of failures *(F)* denotes the average number of failures per period, estimated as the ratio between the total number of failures that occurred during the simulation and the number of simulated periods, in f/T:

$$F = N_{failures}/N_{sim}. \tag{6}$$

- Expected duration of failures *(D)* is the average interruption time per occurred failure. It is calculated by the ratio between the total failure duration and the number of failures in the simulation, in hours or days:

$$D = t_{failure}/N_{failures}. \tag{7}$$

- Expected energy not supplied (EENS) is the average value of energy not supplied in period T, i.e., the ratio between the total energy not supplied and the number of simulated periods, in MWh/T:

$$EENS = ENS/N_{sim}. \tag{8}$$

where, *ENS* corresponds to the total energy (MWh) not supplied in all sampled analysis periods. The *ENS* is given

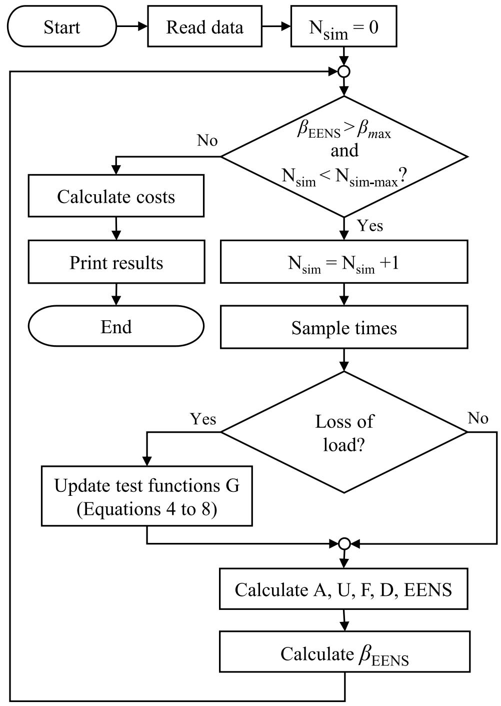

Fig. 2. Simplified flowchart for the MCS.

by the accumulation of all amounts of energy not supplied when a transformer fails. It is important to note that at each transformer failure, the loading of the damaged transformer (interrupted power) and the possibility of load transfer and MUS connection must be checked, as the duration of these emergency actions defines the interruption time of the power supply and, therefore, the energy not supplied as a result of the transformer failure. Moreover, the *EENS* index will be the connection between the MCS stochastic tool and the GA-based optimization process, through the number of RST and MUS equipment defined per year, to be described in Section III.

C. Simplified MCS Flowchart

Fig. 2 shows the simplified flowchart of the chronological MCS for the inventory sizing probabilistic evaluation problem. The first step is to read the data and initialize the count variables and test functions. Auxiliary variables are also created and initialized to estimate the variance, used in the calculation of the coefficient of variation β_{EENS}. Then, the MCS starts the sampling process by reproducing the system stochastic behavior, and it is only interrupted when some stopping criterion is reached. The values of unsupplied energy, failure time and number of failures throughout the simulation are monitored and accumulated in auxiliary variables, which are created for later estimations of reliability indices and expected costs.

III. Optimization via Enhanced Genetic Algorithm

The enhanced genetic algorithm (EGA) [32] via Monte Carlo simulation has as main objective to define the number and the proper moment (e.g., the year) to include new RSTs and MUS to the system. To implement the EGA, the following evolution mechanisms based on Darwin's principles [33] are considered: population initialization, roulette selection, uniform crossing, mutation and elitism. The basic evolution process consists of generating and evaluating individuals.

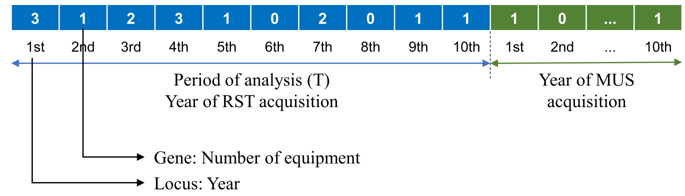

Fig. 3. Chromosomal representation for sizing RST and MUS.

A. Problem Representation

Each possible solution is mathematically represented by a vector, called individual or chromosome. The value of each gene is an integer and positive number corresponding to the number of equipment (RST or MUS) to be acquired in the respective year, according to its position in the vector (locus).

As shown in Fig. 3, the chromosome representation for the proposed problem is divided into two parts. The first part (in blue color) indicates the number of RSTs to be acquired in each year of the analysis period, which can comprise up to 10 years. The second part (in green color) corresponds to the number of MUS to be acquired in each year of the 10 year period, or to the number of years in which the planner wishes to acquire MUS. This can be the complete period, i.e., 10 years, interspersed years, or even admit that the system will not make use of this type of equipment.

B. Objective Function

The main goal is to find a set of feasible solutions with minimum investment costs that positively impact the system, raising its reliability level and reducing interruption costs during the analysis period *T*. Feasible solutions are the number of RSTs (N_{RST}) and MUS (N_{MUS}) per year, ensuring the allowed limits, all over *T*. For this, the solutions found over generations [33] must be evaluated using the following function (9):

$$
\begin{aligned}
&\text{Minimize}: C_{Total}\\
&= \sum_{i=1}^{T} [(N_{RST_i} \times C_{RST} + N_{MUS_i} \times C_{MUS}) \times PV_i]\\
&+ [(C_E + C_I) \times EENS_i(N_{RST_i}, N_{MUS_i})]\\
&\text{where}: PV_i = \frac{\sum_{k=i}^{T} (1 + j_a)^{-k}}{\sum_{k=1}^{T_{AP}} (1 + j_a)^{-k} (1 + j_a)^{year_i - year_c}}\\
&\text{subject to}:\\
&0 \le N_{RST_i} \le N_{RST_{\max}}\\
&0 \le N_{MUS_i} \le N_{MUS_{\max}}.
\end{aligned}
\tag{9}
$$

The first part of the total cost, C_{Total}, corresponds to the investment in RST and MUS, multiplied by *PV*, which represents the present value of the capitalization of the invested amount amortized over the useful life of the equipment. If the lifetime is the same for both RST and MUS, the denominator will be unique. The second part represents the system operating cost estimated by MCS. In (9), *i* defines the year associated with the parameters $N_{RST_{\max}}$ and $N_{MUS_{\max}}$, which are the maximum number of RST and MUS that can be acquired each year, respectively; C_{RST} is the cost of acquiring an RST in \$; C_{MUS} is the cost of acquiring an MUS in \$; C_E is the energy price in \$/MWh (no billing); C_I is the unit interruption cost in \$/MWh; j_a the annual interest rate; T_{AP} is the amortization period in years over the useful life of the RST and MUS; "$year_i - year_c$" is the number of years to be translated from the acquisition date ($year_i$) to the current year ($year_c$); *EENS* is the expected energy not supplied in MWh/year.

The objective function assigns to each individual a measure called "fitness", which in this case corresponds to the total cost that guides the search process. As illustrated in Fig. 3, the optimization provides the input variables of possible chronological acquisitions, i.e., N_{RSTi} and N_{MUSi}, to the chronological MCS, which, in turn, provides the *EENS* index that allows calculating the operating cost caused by the interruptions to the scenario proposed by the EGA. The sum of the investment and operating costs returns to the main program, to feed back into the optimization process. This procedure is better detailed in Subsection III-E.

In general, some particular loads are more relevant and this can be accounted by the unit interruption cost C_I used in (9). These costs are defined for each consumer class (e.g., industrial, commercial, residential, office buildings, rural etc.) and different loads with different class compositions can be specified in (9), so that, parameter C_I will also be decomposed to handle these distinctions. This parameter depends on several characteristics, such as duration, frequency, time of occurrence, warning time, depth of curtailment and geographical coverage; see, for instance, [35], [36], for the Canadian and Brazilian systems, respectively. If there are enough data available for the C_I costs per load/transformer substation, there will be no restriction to input these data in objective function (9) and those more critical loads, i.e., with higher C_I costs, will receive more relevance in terms of energy interruption costs to guide the search process established in (9). In the examples in Section IV, an average C_I value will be used for all load points in order to simplify the analysis.

C. Basic Principles of EGA

The enhanced genetic algorithm is a new model that proposes to optimize the computing time, while expanding the search space. For this, EGA performs a series of iterations of the genetic algorithm, called "internal evolutionary runs" (IER), from different seeds [32].

The algorithm performs $\mathrm{N_{IER}}$-1 internal runs based on an initial coefficient of variation β_{ini} that provides a faster running of the MCS. For the last final run, it uses $\beta_{\mathrm{end}} < \beta_{\mathrm{ini}}$, which provides results with a higher precision. The model also proposes the use of two values for the stopping criterion per repetition, (i.e., $\mathrm{n_{rep\text{-}ini}}$ and $\mathrm{n_{rep\text{-}end}}$). In $\mathrm{N_{IER}}$-1 internal runs, a higher stopping criterion should be used (e.g., $\mathrm{n_{rep\text{-}ini}} = 10$), and in the last final run, a smaller number of repetitions (e.g., $\mathrm{n_{rep\text{-}end}} = 5$).

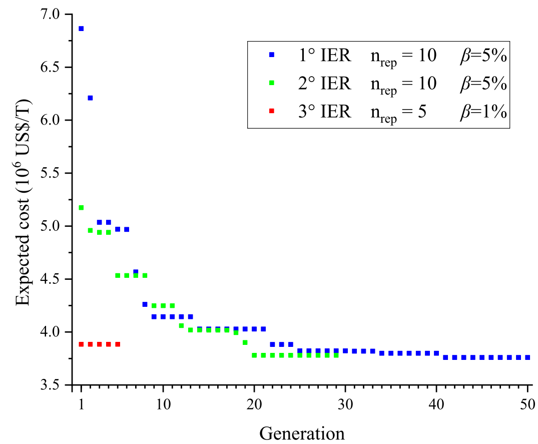

Fig. 4. Evolution of the best individual over generations, during a test with three internal evolutionary runs.

The graph in Fig. 4 illustrates the application of EGA to a system of [9]. The evolution of the best individual is performed through three IER, where two are made from a random initial population, with $\beta_{\mathrm{ini}} = 5\%$ and $\mathrm{n_{rep\text{-}ini}} = 10$, and the last one, from a selected population, with $\beta_{\mathrm{end}} = 1\%$ and $\mathrm{n_{rep\text{-}end}} = 5$. The first run needed more generations to converge via the repeat stopping criterion. Differently, the second run performed 21 generations less due to the better quality of the initial population.

If the genetic algorithm is run without the proposed heuristics, with $\beta = 1\%$, $\mathrm{n_{rep}} = 10$ and a random seed, it would take approximately two hours to complete the evolutionary run, performing an average of 30 generations. The EGA spent, on average, thirty minutes to reach a set of feasible and good quality results. It is concluded, therefore, that the strategy proposed by the EGA is important to reduce the processing time and obtain good solutions for this stochastic optimization problem.

D. EGA Performance Statistical Indices

As EGA is a stochastic optimization tool, its performance can be evaluated through numerical indices. These indices can be calculated after performing "*n*" complete runs of the genetic algorithm, called "external evolutionary runs" (EER) or more simply "tests", using different seeds for the pseudorandom number generator. According to [32], the proposed statistical indices for the performance analysis are defined as:

- $\mathrm{NR_{Best}}$: Number of runs of the algorithm in which better solutions or equal to the known best solution for the problem are identified;
- $\mathrm{N_{Top10}}$: Average number of identified solutions that belong to the set of the 10 known best solutions to the problem, or that have smaller investment;
- $\mathrm{T_M}$: Average time required to run the algorithm;
- $\mathrm{D_{Best}}$: Average percentage deviation between the best investment found in each test and the best known;

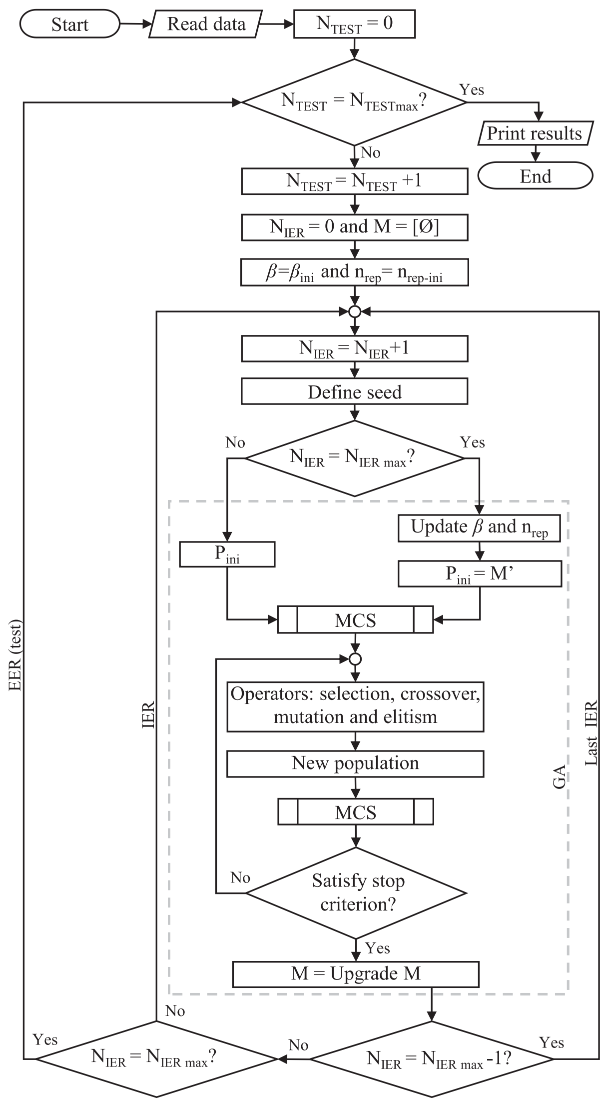

Fig. 5. Simplified flowchart for EGA-MCS and performance statistical tests.

- D_{10Best}: Average percentage deviation of investments from the top 10 solutions found in each test and the best (smallest) known investment.

In order to obtain the indices, it is necessary to inform, in the data input of the program, the ten known best solutions.

E. Simplified EGA-MCS Flowchart

The simplified flowchart in Fig. 5 illustrates the general running of the EGA coupled with the MCS, including the performance statistical indices. The first step is to read the data that define the system, simulation and GA parameters. Then, the count of the number of tests that are performed is started until the predetermined maximum number is reached.

For each external evolutionary run (EER), the algorithm must check if the number of tests has already reached its maximum limit, if not, the test count variable (N_{TEST}) is incremented, the internal evolutionary runs count variable (N_{IER}) is initialized together with matrix M, which stores the non-repeated final solutions of each internal run of the GA. Then, the internal runs begin (i.e., the EGA-MCS itself) incrementing the count variable and checking whether its maximum number has already been reached.

If the maximum number of IER has not been reached, the initial population (P_{ini}) is randomly generated from a different seed drawn at each internal run, for the generation of pseudorandom numbers involved in the evolution process. Then, the initial solutions are evaluated through the chronological MCS and the population evolution process enters a loop, going through the process of selection, crossover, mutation, elitism and evaluation, until a stopping criterion is reached and when this happens solutions not yet found are stored in matrix M. This process is repeated over N_{IER}-1 runs.

When the penultimate inner run comes to an end, the count variable is incremented and the next check is positive for the last inner run. Then, the values of β and n_{rep} are updated and a set of solutions of dimension equal to the pre-established population number is selected (M') to form the initial population ($P_{ini} =$ M') of the last IER, whose results are those presented in the final report.

When the last inner run reaches the stopping criterion, the next check is negative, followed by a positive check. At this moment, the evaluation of the statistical indices is carried out and these partial results are properly stored. Also, the reliability indices from the MCS and the sizing determined by the EGA for the respective test are printed in the final report.

This process repeats until the maximum number of tests is reached. When this happens, the final calculation of the algorithm's performance indices is concluded and the output data is printed. These data delivered by the tool are: reliability indices and expected costs for each test performed, from the SMC; best solutions captured throughout all EER; and EGA performance statistical indices.

Finally, it is possible to replace the chronological MCS tool by a Markov-based model [8], [9], in order to evaluate each gene in Fig. 3. Besides being more complex to manage the structure changes associated to the Markov chains, due to new acquisitions of RSTs and MUS, the residence or waiting times will be limited to the exponential distribution assumptions, inherent to these models. These acquisitions and many other time-dependent aspects, related to the problem being solved, are much easier to be handled through a chronological MCS.

IV. Results

A. Applications to Real Systems

The EGA-MCS is applied to two real systems to size the RST and MUS stocks in a planning period of $T = 10$ years. It is assumed that stocks are empty at the beginning of T. The Canadian system has 60 transformers of 115 kV with rated power of 15 MVA and a failure rate of 0.007 failures/year [10]. The Brazilian system has 177 transformers of 138 kV with 25 MVA and a failure rate of 0.0135 failures/year [29]. The system loads are: 450.0 MW (Canadian) and 2247.9 MW (Brazilian).

TABLE I
BEST SOLUTIONS FOUND – CANADIAN SYSTEM

Type	Year of Inclusion										Total Cost (10^3×US$/T)
	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	
RST	3	1	0	0	0	0	0	0	0	0	3883.43
MUS	1	0	0	0	0	0	0	0	0	0	
RST	3	0	1	0	0	0	0	0	0	0	3894.11
MUS	1	0	0	0	0	0	0	0	0	0	
RST	3	1	0	0	0	0	0	0	0	1	3896.83
MUS	1	0	0	0	0	0	0	0	0	0	
RST	3	1	0	0	0	0	0	0	1	0	3905.56
MUS	1	0	0	0	0	0	0	0	0	0	
RST	3	0	1	0	0	0	0	0	0	1	3906.23
MUS	1	0	0	0	0	0	0	0	0	0	

The RST installation, RST acquisition and MUS installation times are, respectively: from 12 to 16 days with uniform distribution (Canadian system) and from 9 to 11 days with uniform distribution (Brazilian system); from 11 to 13 months with uniform distribution (both systems), and 1 day (both systems). In the EGA, the following parameters are used: maximum number of generations equal to 100 (both systems); parent/offspring population of 40 individuals (Canadian system) and 60 individuals (Brazilian system); mutation rate equal to 0.1 and crossover rate equal to 0.7 (both systems).

Acquisition limits are defined as: up to 4 RSTs per year (Canadian system) and up to 5 RSTs per year (Brazilian system) throughout the period of analysis; up to 2 MUS per year in the first three years (both systems). The economic parameters, i.e., interest rate, investment amortization period, energy price, interruption cost and investment costs, for both RST and MUS, are defined, respectively, with the following values: 10% per year, 420 months, 100 US$/MWh, 800 US$/MWh, US$ 500.0 $\times$ 10^3 and US$ 2800.0 $\times$ 10^3, for both systems.

Most of the parameters mentioned above are in fact values used in both systems, but some of them, mainly those related to economy, are assumptions in order to preserve the autonomy of the analyses accomplished by the companies.

1) Canadian System: Considering constant load all over period T, Table I shows the five best solutions (options) found by the EGA-MCS tool. In fact, any number of ordered options by the total cost is available from the proposed tool and this is one of the major advantages of using EGA-based algorithms in the optimization process. Note that no solution suggested more than 3 RSTs and 1 MUS in the first year. Comparing options 1 and 2, it is noted that, although both suggest the acquisition of 4 RSTs, Option 1 is better, as it suggests that the fourth unit be acquired in the second year, implying lower operating costs. Option 2, in turn, has a lower investment cost, since the analysis period window becomes eight years. However, the investment is not offset by the operating cost. The same reasoning can be used in other comparisons.

Table II shows that all top five options provided a high level of reliability to the system. The average failure duration is less than 1.4 days and availability is greater than 0.998. Analyzing the average costs shown in Table III, it is possible to observe

TABLE II
RELIABILITY INDICES – CANADIAN SYSTEM

Option	*A*	*U* (h/T)	*F* (f/T)	*D* (days)	*EENS* (MWh/T)
1	0.99854	128.03	4.1815	1.28	969.28
2	0.99846	135.22	4.1811	1.35	1028.74
3	0.99855	127.42	4.1851	1.27	961.95
4	0.99857	125.69	4.1820	1.25	947.22
5	0.99847	134.41	4.1835	1.34	1020.00

TABLE III
EXPECTED COSTS (10^3 $\times$ US$/T) – CANADIAN SYSTEM

Option	Investment	Interruption	No billing	Total
1	3011.08	775.42	96.93	3883.43
2	2968.24	822.99	102.87	3894.11
3	3031.07	769.56	96.19	3896.83
4	3053.06	757.78	94.72	3905.56
5	2988.23	816.00	102.00	3906.23

TABLE IV
PERFORMANCE STATISTICAL INDICES – CANADIAN SYSTEM

No. of Tests	NR_{Best}	N_{Top10}	D_{Best} (%)	D_{Best10} (%)	T_M (minutes)
10	9	7.50	0.03	0.71	27.15

that there is a balance between investment and operating costs for the analyzed period, showing that the EGA-MCS found a set of very good solutions.

Based on a combinatorial strategy, the reference results (template) of the whole solution space for the Canadian system are separately assessed. Comparing these results with those in Table I, it is observed that the first five are exactly the same, which allows concluding that the EGA-MCS found the five known best solutions for the problem.

To evaluate the statistical performance of the proposed tool, it is run ten successive times (EER = 10) to create the indices in Table IV. The ten known best solutions, taken from the template, are used as a reference. Thus, the known best solution is not captured only once in 10 repetitions. In this case, the best captured solution corresponds to the second best solution in the template, resulting in a deviation of 0.03% from the best solution. Among the tests carried out, it is also verified that 75% of the ten best solutions found belong to the template, resulting in a $D_{10\mathrm{Best}}$ deviation of 0.71% in relation to the known best solution. The other 25% belong up to the 22nd position of the template. This shows that the EGA-MCS performed very well for the system and that the solutions found form a set of feasible, optimal or suboptimal alternatives.

2) Brazilian System: Considering constant load all over period T, the five best solutions captured by the EGA-MCS are shown in Table V, where it can be seen that all options suggest the acquisition of 5 RSTs in the first year and 2 MUS in the first two years. Comparing the first two options, which still include 4 RSTs in the second year, it is noted that the first one is more advantageous, as it acquires the tenth RST a year earlier. This slightly raises the investment cost, which is offset by the reduced operating cost. Reliability and cost indices are shown

TABLE V
BEST SOLUTIONS FOUND – BRAZILIAN SYSTEM

Type	Year of Inclusion										Total Cost (10^3×US$/T)
	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	
RST	5	4	0	0	0	0	1	0	0	0	14082.99
MUS	1	1	0	0	0	0	0	0	0	0	
RST	5	4	0	0	0	0	0	1	0	0	14115.10
MUS	1	1	0	0	0	0	0	0	0	0	
RST	5	4	0	0	1	0	0	0	0	3	14134.09
MUS	1	1	0	0	0	0	0	0	0	0	
RST	5	3	1	0	0	0	0	1	0	0	14136.95
MUS	1	1	0	0	0	0	0	0	0	0	
RST	5	3	1	0	0	0	0	1	0	1	14150.83
MUS	1	1	0	0	0	0	0	0	0	0	

TABLE VI
RELIABILITY INDICES – BRAZILIAN SYSTEM

Option	*A*	*U* (h/T)	*F* (f/T)	*D* (days)	*EENS* (MWh/T)
1	0.99233	671.74	23.6537	1.18	8897.44
2	0.99229	675.44	23.6920	1.19	8962.68
3	0.99234	670.62	23.6406	1.18	8819.30
4	0.99221	682.47	23.6964	1.20	9034.56
5	0.99221	682.18	23.6973	1.20	9027.77

TABLE VII
EXPECTED COSTS (10^3 × US$) – BRAZILIAN SYSTEM

Option	Investment	Interruption	No billing	Total
1	6075.30	7117.95	889.74	14082.99
2	6048.69	7170.14	896.27	14115.10
3	6196.72	7055.44	881.93	14134.09
4	6005.85	7227.65	903.46	14136.95
5	6025.84	7222.21	902.78	14150.83

TABLE VIII
PERFORMANCE STATISTICAL INDICES – BRAZILIAN SYSTEM

No. of Tests	NR_{Best}	N_{Top10}	D_{Best} (%)	D_{Best10} (%)	T_M (minutes)
10	5	2.80	0.06	0.36	49.54

in Tables VI and VII, respectively. Values close to each other indicate that the options have the same reliability level.

To evaluate the solutions found by the EGA-MCS, two extra tests are performed. The first one increases the purchase limit to 7 RSTs per year. The second one creates a highly redundant system, with 100 RSTs and 100 MUS. In both cases, the availability saturates at 0.993, due to the small probabilities of higher order contingencies. Thus, it is not possible to increase the availability beyond this limit with new additions of RSTs and MUS. Such improvement can be obtained by enabling load transfers and/or reducing RST and MUS installation times.

A performance analysis is also carried out and the results of are shown in Table VIII. It is verified that among the runs carried out, of the 100 best solutions found, 28 belong to the known best solutions, resulting in a $D_{10\text{Best}} = 0.36\%$ in relation to the best solution. Regarding the achieved alternatives, it is noted that the known best solution is captured in five tests; the second best in three, and the seventh in one test. The results obtained imply $D_{\text{Best}} = 0.06\%$. Although no more than 5 tests identified the best solution, the others presented costs very close to the best solution and, therefore, D_{Best} is very low.

Inventory sizing is a study carried out over periods of less than ten years, and therefore, the acquisitions suggested by the EGA-MCS in the last years of the study period can be considered residual and ignored by planners. In addition, the reliability and cost indices obtained allow stating that the solutions found promote high system reliability at the lowest possible cost and indicate good convergence, with a small deviation from the known best solution. Therefore, it is up to the system manager to decide on the option to be implemented.

In the next section, a sensitivity analysis is carried out to establish the robustness of the proposed EGA-MCS method. Several cases are simulated with both systems, where different scenarios are considered: system expansion and load increase, possibility of load transfer for some points and new acquisition of equipment in specific years of the study period. In case of load transfers, it is assumed that the total load of the failing transformer can be duly transferred to another point via a suitable feeder. The accurate representation of load transfers would need a full reliability assessment of the distribution network (see, for instance [37]) involved in the optimization process, which will require a huge amount of simulation time.

TABLE IX
SENSITIVITY ANALYSIS CASES – CANADIAN SYSTEM

Case	Description
1	Initial condition (empty stocks)
2	Case 1 + system expansion and load increase (2025 and 2027)
3	Case 1 + load transfer at 10 points
4	Case 1 + possible acquisition of MUS 2022, 2023 and 2024
5	Case 1 + all previous case considerations

TABLE X
BEST SOLUTIONS: CASES 1 TO 5 – CANADIAN SYSTEM

Case	Type	Year of Inclusion										Total Cost (10^3×US$/T)
		2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	
1	RST	3	1	0	0	0	0	0	0	0	1	10757.00
	MUS	0	0	0	0	0	0	0	0	0	0	
2	RST	3	1	0	0	0	0	0	0	0	0	12290.34
	MUS	0	0	0	0	0	0	0	0	0	0	
3	RST	4	0	0	0	0	0	0	0	0	0	9266.48
	MUS	0	0	0	0	0	0	0	0	0	0	
4	RST	3	1	0	0	0	0	0	0	0	0	3883.43
	MUS	1	0	0	0	0	0	0	0	0	0	
5	RST	3	0	0	1	0	0	0	0	0	0	3946.54
	MUS	1	0	0	0	0	0	0	0	0	0	

B. Sensitivity Analysis

1) Canadian System: Table IX presents the description of simulated cases in the analysis of scenarios. The best stock composition solution for each case is shown in Table X. Case 1 corresponds to the original system, previously described. For this case, the EGA-MCS is applied only to dimension RST.

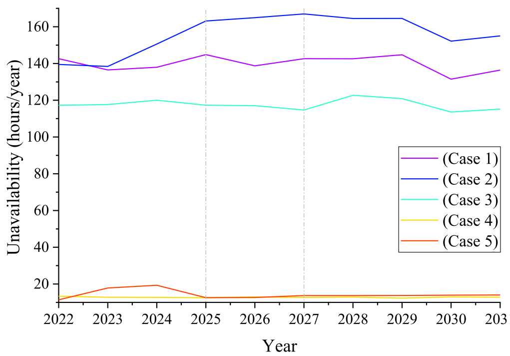

Fig. 6. Cases 1 to 5 Effect of MUS installation time – Canadian system.

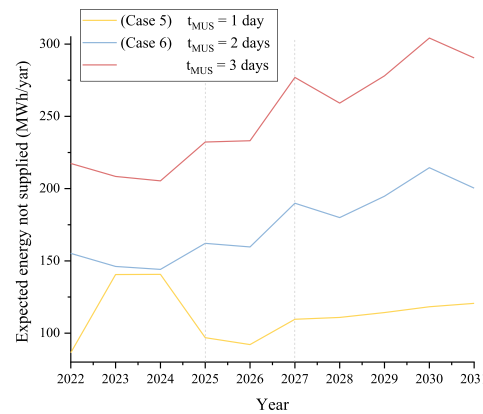

Fig. 7. Case 6: EENS indices – Canadian system.

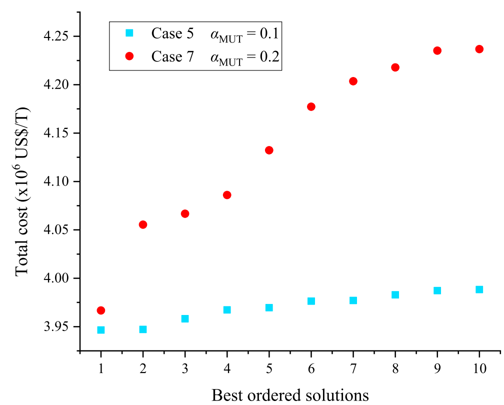

Fig. 8. Case 7: Mutation rate (α_{MUT}) – Canadian system.

In Case 2, it is considered the expansion of the group of transformers in operation with 10 new equipment, 5 in 2025 and 5 in 2027. It is also admitted the load growth of 2% per year from 2025 in all substations, so that the total load goes from 450 to 525 MW. Despite the increase, load changing does not have a large impact on the system, as seen in Table X.

Case 3 assumes the possibility of load transfer at 10 points and, as observed, there is an improvement in the indices and a reduction of almost 1.5 million dollars in the total cost. If in these 10 points only 50% of the load can be transferred, then the results become: 3 RST in 2022, 1 RST in 2023, 1 RST in 2031 and 0 MUS, with a total cost of 9942.00 $\times$ 10^3 US$/T. As expected, by reducing the transfer capacity in these points, the total costs increase by 675.52 $\times$ 10^3 US$/T. Case 4 assumes the acquisition of MUS, which makes the system more robust against failures and drastically reduces its total cost, with a solution already shown in Table I.

Case 5 combines all previous cases, considering the expansion of the group of transformers in operation, the increase in system load, the possibility of acquiring RST and MUS, and load transfer capacity at 10 points. The best stock composition obtained suggests the acquisition of 3 RSTs and 1 MUS in the first year and 1 RST in the fourth year (and not more in the first or second year as in the previous cases). With more emergency actions available, the system becomes more reliable, and thus the fourth RST is only needed to support the system in the face of expansion and increased load. If in these 10 points only 50% of the load can be transferred, then the results become: 3 RST in 2022, 1 RST in 2023 and 1 MUS in 2022, with a total cost of 3992.23 $\times$ 10^3 US$/T. As expected, by reducing the transfer capacity in these points, the total costs slightly increase by 45.69 $\times$ 10^3 US$/T. Differently from Case 3, the presence of MUS reduces the impact of load transfer limitations due to the faster response of these equipment; i.e., 1 day on average.

Fig. 6 illustrates system unavailability in hours per year over the period. The drop in this index stands out with the inclusion of 1 MUS in Cases 4 and 5, which reduces the total cost by more than 6.8 million dollars. This reaffirms the importance of using the MUS during the installation of the backup transformer.

The influence of MUS connection time on system reliability (Case 6) is now analyzed, taking Case 5 as a reference. Assuming connection times ($\mathrm{T_{MUS}}$) of 1, 2 and 3 days, the average duration of failures are 1.28, 1.99 and 2.85 days, respectively, and the EENS values are presented in Fig. 7. Note that the faster the MUS is installed, the greater the benefit it provides to the system.

The mutation amplitude is now varied, which represents an important parameter of the genetic algorithm, since it causes a "disturbance" in the search space. The ten best results found for a mutation rate equal to 0.1 (Case 5) and 0.2 (Case 7) are shown in Fig. 8, where it is possible to notice that, by increasing the mutation frequency, the results achieved are slightly worse.

TABLE XI
SENSITIVITY ANALYSIS CASES – BRAZILIAN SYSTEM

Case	Description
1	Initial condition (empty stocks)
2	Case 1 + system expansion + load increase + load transfer + possible acquisition of MUS in 2022, 2023 and 2024

2) Brazilian System: For this system, the cases in Table XI are analyzed. Case 1 represents the initial system, in which only RST acquisition is allowed. In Case 2, it is considered the expansion of the transformer group in operation with 10 new equipment, 5 in 2025 and 5 in 2027. It is assumed a load growth of 2% per year in all substations from 2025, so that the total system load goes from 2247.90 MW to 2347.90 MW. Besides, it is assumed that the system can perform load transfer at 20

TABLE XII
BEST FOUND SOLUTIONS – CASES 1 AND 2 – BRAZILIAN SYSTEM

Case	Type	Year of Inclusion										Total Cost (10^3×US$/T)
		2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	
1	RST	5	4	0	0	0	0	0	0	0	0	69339.47
	MUS	0	0	0	0	0	0	0	0	0	0	
2	RST	5	4	0	0	1	0	0	0	1	1	14031.06
	MUS	1	1	0	0	0	0	0	0	0	0	

TABLE XIII
BEST FOUND SOLUTIONS – CASES 3 TO 6 – BRAZILIAN SYSTEM

Case	Type	Year of Inclusion										Total Cost (10^3×US$/T)
		2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	
3	RST	5	4	0	0	0	0	1	0	0	1	13904.64
	MUS	2	0	0	0	0	0	0	0	0	0	
4	RST	5	3	1	1	0	0	0	0	0	1	13997.39
	MUS	2	0	0	0	0	0	0	0	0	0	
5	RST	5	4	0	0	0	0	1	0	0	1	13904.64
	MUS	2	0	0	0	0	0	0	0	0	0	
6	RST	5	4	0	0	0	0	0	0	0	0	14191.98
	MUS	1	1	0	0	0	0	0	0	0	0	

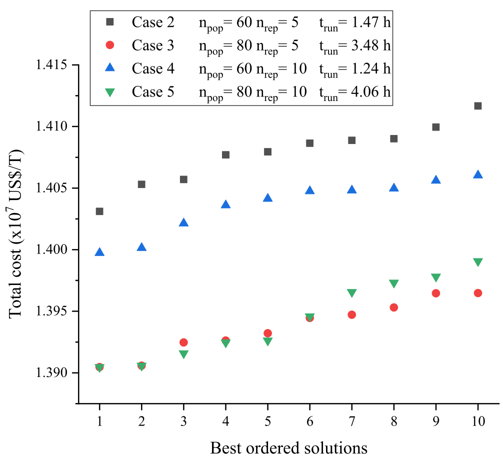

Fig. 9. Cases 2 to 5: Top ten found solutions – Brazilian system.

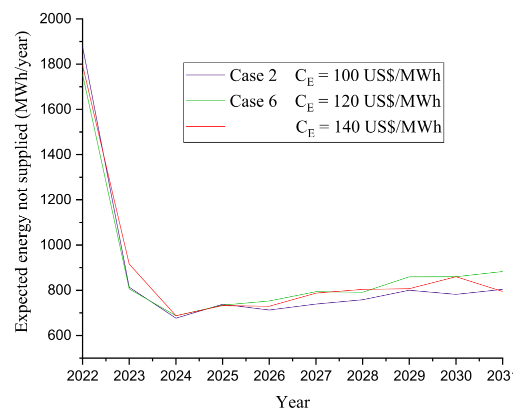

Fig. 10. Cases 2 and 6: EENS – Brazilian system.

points, and the EGA-MCS tool is applied to size the RST and MUS stocks. Table XII shows the best solutions found for these cases.

Notice that in both solutions, the acquisition of 9 RSTs by the second year is suggested. In Case 2, the acquisition of 2 MUS in the first two years and the tenth RST in 2026 is suggested. This strategy is expected, since the system expands and has a high load in 2025. In Case 2, there is a saving of more than 55 million dollars compared to Case 1, caused by the inclusion of the two MUS, which can drastically mitigate the time of loss of load.

The population size and the stopping criterion are important parameters in the performance of the genetic algorithm regarding the quality of the set of solutions reached. Thus, a combination of these parameters is made, taking Case 2 as a reference. The results for the ten best solutions found are shown in Fig. 9. Note that by increasing the population size by 20 individuals (Case 3), the results of the EGA-MCS show a slight improvement. However, the computing time is 2.36 times greater than that of Case 2. In Case 4, the stop criterion is increased from 5 to 10 repetitions also in the last internal run, which causes a slight improvement in the results. In Case 5, the population size is increased to 80 individuals, but keeping the stopping criterion of Case 4. Although the results are similar to those of Case 3, the algorithm requires more computing time. Lastly, the proposed EGA-MCS tool can carry out as many internal runs as necessary, according to the size of the problem, in order to better encompass the solution space and increase the diversity of the population. The adjustment in the EGA and MCS parameters, as discussed in Section III, allow very good solutions to be found and local convergence to be avoided.

It is therefore defined that the best configuration for this system is the one used in Case 3 ($n_{pop} = 80$ and $n_{rep} = 5$). In addition, with these settings, the algorithm is able to capture better solutions, where it is possible to notice that the acquisition of 2 MUS in the first year causes an even greater impact on the system costs, as shown in Table XIII.

With regard to economic parameters, the cost of energy corresponds to the company's tariff and allows estimating losses due to no billing for interruptions in the supply of electricity to customers. In Case 6, the energy cost is increased from 100 US$/MWh (Cases 2) to 120 and 140 US$/MWh. In the best solutions found in Case 6, with 120 US$/MWh (Table XIII), the costs of investment, interruption and no billing are, respectively, 42.15%, 50.30% and 7.54% of the total cost. Because the energy cost represents a small portion, the cost of no billing has a low impact on the total cost. Analyzing Fig. 10, it is possible to observe that Case 2 and Case 6 are similar in terms of expected energy not supplied results during the period of analysis, given the low impact of the cost of non-billing.

Finally, the EGA-MCS tool is developed in MATLAB programming language and all simulations are run with an Intel Core™ i5, 1.8 GHz processor. Therefore, the analysis of the Canadian system takes, on average, 27.15 minutes while the Brazilian system takes, on average, 49.54 minutes. The hypothetical results that would be obtained with the respective templates (reference results for all possible combinations) would take approximately 4.04 years ($5^{10} \times 3^3$ combinations - Canadian system) and 30.31 years ($6^{10} \times 3^3$ combinations - Brazilian system). Surely, to assess the most relevant combinations to be used as references, several computational tricks were considered and, thus, several weeks of computation were needed. Moreover, all these figures will depend on the specified simulation parameters.

V. Conclusion

This paper presents a new methodology for the optimal planning of stocks of regular spare transformers (RSTs) and mobile unit substations (MUS) over time, aiming to serve groups of electrical energy distribution substations.

The chronological MCS made it easier and possible the modeling of the times required for some remedial actions such as: installing spare transformers, carrying out load transfers to neighboring substations, connecting mobile substations and acquiring regular spare transformers for replacement or expansion of stocks. The proposed method allows the estimate of reliability indices and investment and operating costs over a specified planning horizon, even considering the expansion of the system and the increase in its load over this period.

The enhanced genetic algorithm via Monte Carlo simulation (EGA-MCS) allowed simultaneously optimizing the sizing of RST stocks and MUS, which had not yet been described in the literature. The heuristics incorporated into the model allowed expanding the search space and reducing the processing time with the use of different seeds and convergence criteria. The proposed model is applied to two real systems and the results showed that the algorithm has very good performance, being able to find a set of optimal or suboptimal feasible solutions to compose the stock.

Modeling the transformer's useful life, which considers the deterioration of the insulating material, effect of short circuits and other events capable of influencing the aging process, can be easily included in the proposed EGA-MCS method and will be the subject of future work. Specific unit interruption costs per class of consumers associated to load points can also be considered in these new studies. Furthermore, the flexibility of the proposed EGA-MCS tool allows other constraints to be included in the optimization process, such as a specific reliability measure, which can be useful in some applications.

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