Reliability-Constrained Power System Expansion Planning: A Stochastic Risk-Averse Optimization Approach

Luiz Carlos da Costa Jr., Fernanda Souza Thomé, Joaquim Dias Garcia, *Student Member, IEEE,*
Mario V. F. Pereira, *Fellow, IEEE,*

Abstract—**This work presents a methodology to incorporate reliability constraints in the optimal power systems expansion planning problem. Besides Loss Of Load Probability (LOLP) and Expected Power Not Supplied (EPNS), traditionally used in power systems, this work proposes the use of the risk measures VaR (Value-at-Risk) and CVaR (Conditional Value-at-Risk), widely used in financial markets. The explicit consideration of reliability constraints in the planning problem can be an extremely hard task and, to minimize computational effort, this work applies the Benders decomposition technique splitting the expansion planning problem into an investment problem and two subproblems to evaluate the system's operation cost and the reliability index. The operation subproblem is solved by Stochastic Dual Dynamic Programming (SDDP) and the reliability subproblem by Monte Carlo simulation. The proposed methodology is applied to the real problem of optimal expansion planning of the Bolivian power system.**

Index Terms—**System expansion planning, Benders decomposition, Power systems, Reliability, Stochastic programming, Risk measures, Mixed Integer Programming, Monte Carlo, Stochastic Dual Dynamic Programming.**

I. INTRODUCTION

THE goal of power system expansion planning (SEP) is to determine necessary changes in the system due to load growth, new technologies and policy-related constraints. In this sense, new generators should be constructed to satisfy the new system's needs and the planning process decisions are associated with the selection of the best set of equipment (such as generators, transmission lines, transformers, etc.) to accomplish this task. This decision process originates from a complex optimization problem, where the objective is to plan the future power system minimizing the investment and operation costs subject to a pre-established set of constraints. Cases of SEP are generation expansion planning (GEP) [1], [2] and transmission expansion planning (TEP) [3], [4] that can be combined and integrated [5], [6] generally called SEP or GTEP.

This planning process constitutes an extremely complex problem that cannot be solved without simplifications. Depending on the goals of the planner different aspects and details of power systems can be considered in general SEP. Many possible constraints are described in [7], some specific aspects include: carbon capture and storage [8]; unit commitment in the operation [9]; flexible demand and electric vehicles [10];

Authors are with PSR, Rio de Janeiro, Brazil, www.psr-inc.com, e-mail: {luizcarlos, fernanda, joaquim, mario}@psr-inc.com

aggressive wind power penetration [11]. A common aspect in most SEP models is the representation of uncertainties, although each model typically focuses on sources of randomness, like renewable energy and load [12], outages or contingencies [13], [14]. Frequently used frameworks to deal with uncertainties in SEP are Stochastic Optimization [15], [16] and Robust Optimization [12], [17], both of which can also be combined [18].

Many techniques were proposed to solve the large-scale problems that arise from SEP modelling. Heuristics like Particle Swarm Optimization and GRASP were proposed in [8] and [19]. Many decomposition methods were presented due to natural scenario-wise and/or stage-wise structure: [20] [21] and [22] apply progressive hedging, Dantzig Wolfe decomposition was applied in [23] and Benders decomposition, perhaps the most used one, was applied in [12], [13], [15], [24]–[28].

Reliability is an important aspect of power systems that can be considered in SEP. The simplest way to taking reliability into account is adopting a hierarchical approach, in which the expansion plan is at first elaborated under the economic focus (first step), in other words, aiming the minimization of the investment costs plus the cost of load supply (operation cost). Hereafter, the necessary additional investments to meet a minimum criterion of security (reliability reinforcements) are evaluated (second step).

Among the first approaches for the solution of the SEP with reliability constraints is the work [29] where linear approximations of a reliability function were obtained from a non-linear formulation. Afterward, [30] presented a similar model, but able to generate Benders cuts from a probabilistic simulation model. However, this last model was non-convex which led to convergence problems. Robust optimization was used to induce expansion plans with better reliability indexes in [6]. In [15], reliability requirements were modeled by including a cost of load loss in the objective function.

In general, SEP can be formulated as a minimum total cost function (investment and operation) subject to the operation and reliability constraints that depend directly upon the investment decisions. This formulation contains a very opportune structure for the application of decomposition techniques and this characteristic was first explored by [31]. Thereafter, [32] presented a model for load-peak capacity expansion considering Expected Power Not Supplied (EPNS) constraints, but its application was restricted to the second step problem, in other words, the evaluation of additional investments to

attend the reliability requisites. Reliability constraints were also applied in [33], [34], but the problem was solved by a genetic algorithm without optimality proofs. More recently, [13], [28] considered explicit constraints on reliability indexes via Monte Carlo simulation and Benders decomposition with feasibility cuts. [35] considered Loss Of Load Probability (LOLP) constraint, originally modeled as a Mixed Integer Nonlinear Program (MINLP) and approximated by a Mixed Integer Linear Program (MIP).

This work proposes an integrated methodology for the solution of the SEP, where the economic and reliability analyses are carried out on an integrated problem. Therefore, it is possible to assess the benefit of each project both in terms of reduction of the operative cost as well as the increase in the overall system reliability. Additionally, we compare the obtained results from the proposed integrated approach with the simplification made by using the hierarchical approach for the solution of the SEP.

We use the Benders decomposition technique to split the original problem in investment, operation and reliability modules. This partition allows each subproblem to be solved by a specialized algorithm, for instance, the investment master problem (a mixed integer programming problem) is solved by *Branch-and-Bound* (B&B), the operation subproblem by *Stochastic Dual Dynamic Programming* (SDDP) [36] and the reliability subproblem by *Monte Carlo simulation* (MC) [37]. Instead of focusing on classical measures like LOLP and EPNS [28] we propose the use of the two risk measures VaR_α and $CVaR_\alpha$, commonly used in the financial area, but also applied in different contexts in SEP [27], [38], [39].

The main contributions of this paper are as follows. First, a general description of SEP that can be solved by Benders decomposition. This includes a simple master problem that can be broken into three parts: investment, operation and reliability problems, such description allows the development of specialized methods for each piece, as described in this paper. Secondly, we present various reliability metrics that could be considered in the reliability subproblem and show the explicit formulation of the reliability constraint as mathematical programming deterministic problems equivalent to the MC version of the constraint evaluation, which is fundamental for deriving the reliability cuts. In third, we present the CVaR as an alternative reliability metric, which, to our knowledge, was not done before in a decomposition scheme. Although CVaR is not a classic reliability metric, it is gaining more attention recently [40], therefore deserves attention as well as the development of methods that incorporate it in the planning tools. Finally, the fourth contribution is the presentation of a realistic case study in which we combine the specialized methods SDDP for power system operation, MC for reliability assessment and MIP for investment, although the combination of two of these was previously done, we have no information of the combination of the three methods for solving any problem.

The remainder of this work is organized as follows. In the next section, II, a generic formulation of a reliability constrained system expansion problem is presented. Section III describes reliability metrics and their corresponding formulations as optimization problems. In section IV, we describe the decomposition procedure that combines the plan choice, cost and reliability evaluation, after that, we present two case studies in section V. Finally, conclusions are drawn in section VI.

II. POWER SYSTEMS EXPANSION PLANNING

The reliability constrained SEP can be formulated as the following mixed integer programming problem:

$$\text{Min} \quad I(x) + O(x) \tag{1a}$$
$$\text{s.t.:} \quad R(x) \leq \bar{R} \tag{1b}$$
$$x \in \mathcal{X} \tag{1c}$$

where x is the vector of investment decisions, $I(x)$ the investment cost, $O(x)$ the operation cost and $R(x)$ the risk measure function, $\bar{R}$ the reliability criterion and $\mathcal{X}$ the set of decisions that meet planning constraints.

A. Economic Planning (EP)

The first step of an hierarchical planning process is a simplification of problem (1), in which the explicit representation of the reliability aspects and the risk constraints (1b) are disregarded, as shown in problem (2).

$$\text{Min} \quad I(x) + O(x) \tag{2a}$$
$$\text{s.t.:} \quad x \in \mathcal{X} \tag{2b}$$

In this sense, the explicit representation of the uncertainty associated to the state of each generating unit is, in general, simplified by applying a reduction factor in every power plants' capacity corresponding to their average availability rate. This approximation avoids the representation of all power system states and the SEP of generation systems. A simplified formulation of EP is given by:

$$\text{Min} \quad \sum_{j \in \mathcal{G}} c_j x_j + \sum_{j \in \mathcal{G}} d_j g_j + hr - \delta D \tag{3a}$$
$$\text{s.t.:} \quad \sum_{j \in \mathcal{G}} g_j + r = D \tag{3b}$$
$$g_j \leq \tilde{g}_j x_j \qquad\qquad j \in \mathcal{G} \tag{3c}$$
$$x \in \mathcal{X} \tag{3d}$$

where $\mathcal{G}$ is the set generators, c_j and d_j represent, respectively, the investment cost and variable operation cost of generating unit j. The load shedding is represented by the variable r and associated unit cost h, $\tilde{g}_j$ is the available capacity of generator calculated in terms of average availability as $\tilde{g}_j = (1-p_j) \times \bar{g}_j$, p_j is the average failure rate, $\bar{g}_j$ is the installed capacity and x_j is the investment decision for generator j. Finally, demand is represented by decision variable D and its associated willingness to pay δ[1], which can represent Demand Side Response (DSR).

The objective function (3a) is the minimization of the total cost (investment and operation), subject to demand supply in

---

[1]It is possible to have multiple segments in order to better represent demand sensitivity to energy price.
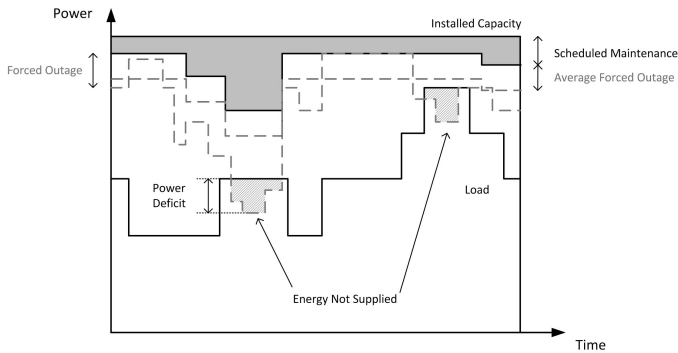
Fig. 1. Random behavior of total system capacity

each time step (3b), limits on generation (3c), and investment constraints (3d).

The operation problem is usually much more complex that the illustrative model presented above. As mentioned in the introductory section, many details might be taken into consideration such as renewable generation, demand response, storage equipment and time coupling, uncertainty, competition, unit commitment, energy network representation and many others.

Although this model has been frequently used, the use of a generation capacity based on average availability may not be enough to capture the true exposure/risk of load shedding events, as illustrated in Figure 1. Even if the system can meet the load on average, it might exist one or more states of failure in which the remaining capacity is not sufficient to supply the demand (hatched regions). Hereof it might be necessary to model the system in probabilistic terms to better represent the reliability aspects and this requires modeling the operating state of all generators and, consequently, the total capacity of the system as a random variable (r.v.).

III. RELIABILITY ANALYSIS OF POWER SYSTEMS

Power system reliability analysis is a vast area. Multiple methods with different levels of detail were presented throughout the years. More detailed methods include sequential Monte Carlo (MC) techniques such as [41], while simpler methods include standard MC, in which state transitions are not taken into account, instead, the system is analyzed in a steady-state fashion as in [42] and [43]. There are also intermediary methods such as pseudo-sequential MC [44], which tries to lighten the burden of the more complex sequential method.

The selection of the most appropriate method of course depends on the purpose of the model. In planning models, it is reasonable to apply standard MC [42] [43] and successful applications in real systems can be found [40]. In this work, we will not consider state transitions and their duration. Instead, we use the steady-state analysis based on sampling states of the system to compute the traditional reliability metrics, such as LOLP and EPNS. This method is especially good for capturing peaking issues, although longer, consequently more expensive, events are not explicitly captured, this problem is usually approximated because many events will be seen in the snapshot analysis.

A power system is composed of elements such as generators, transmission lines, transformers and load, each element may be in a state among a set of possible states. For example, the operating state of a generating unit can be either (a) 0 if the equipment is not working or (b) 1 if it is working. Other elements, such as demand and combined thermal cycle plants, may require a multi-state representation.

A state of a power system with J generators is represented by the random vector $\xi = (\xi_1, \xi_2, \ldots, \xi_J)$ where ξ_j is a r.v. representing the state of the j-th generator. Let S represent the set of states of the power system, given by the combination of all possible states of each element, each state of the power system is denoted by ξ^s, $s \in S$. For each state of generator j there is an associated probability of occurrence $p_j = P(\xi_j)$ and, once the state of each generator for the system state $s \in S$ is given, it is possible to calculate the probability of this system state as $p^s = P(\xi^s)$.

In turn, the demand D can be represented in several ways. For the reliability analysis we consider only the inelastic load and therefore shedding it results in a loss of load event. Additionally, demand variation over the hours and throughout the year is captured by using a stratified sampling strategy considering representative snapshots for each season/month and period of the day.

The total system capacity is denoted by the r.v. $\bar{G} = \sum_{j=1}^{J} \xi_j \bar{g}_j$ and the load shedding is given by the r.v. $R = \max(D - \bar{G}, 0)$, which is the insufficient generation capacity. Since ξ has finite support distribution, each realization of system state ξ^s is associated to a total capacity of $\bar{G}^s$ and a respective load shedding R^s.

The performance of a given investment plan x can be measured with risk indexes based, in general, on the probability distribution of the load shedding. Since the objective of the SEP is to determine which and when generators should be constructed, the probability distribution of the load shed, $P(R)$, must also be a function of the investment decision vector x. Therefore, the objective of the reliability-constrained SEP becomes to find the plan that minimizes the investment and operation costs and present a "controlled" load shedding distribution function in the sense that its associated reliability index meets some given pre-established planning criterion.

The next section presents the reliability indexes LOLP and EPNS, traditionally used in power systems, and then introduces in the context of power system reliability the indexes VaR_α and $CVaR_\alpha$, frequently used in the financial sector. Moreover, the calculation of these indexes will be formulated as optimization problems aiming for their incorporation into the SEP.

A. Typical Reliability Indexes

1) LOLP: The most straightforward approach to measure the risk of failure of supplying the power system load is to assess the number of insufficient states among all of them. The LOLP (*Loss Of Load Probability*) is the probability of load shedding, as illustrated in Figure 2(a), and is given by

$$\text{LOLP} = P(R > 0) = \sum_{s \in \Omega} p^s \qquad (4)$$
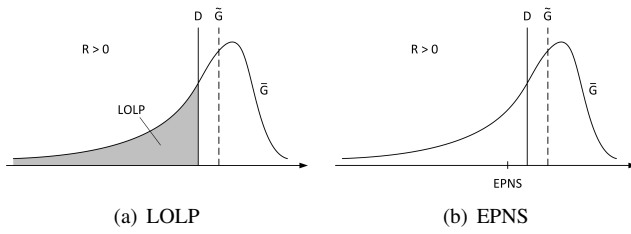
(a) LOLP   (b) EPNS

Fig. 2. Typical Reliability Indexes

where $\Omega = \{s \in S | r^s > 0\}$ is the set of power system states where there is load shedding.

The calculation of LOLP can be formulated as a (possibly large) mixed integer linear programming problem, using a binary variable ϕ^s, for each state s to indicate whether it leads to a load shedding. Then, the LOLP is calculated as the average of these indicator variables weighted by the state probability. By explicitly incorporating this formulation into the problem, the LOLP-constrained SEP model can be defined as:

$$\text{Min} \quad \sum_{j \in \mathcal{G}} c_j x_j + O(x) \tag{5a}$$

$$\text{s.t.:} \quad \sum_{s \in S} p^s \phi^s \leq \overline{\text{LOLP}} \tag{5b}$$

$$r^s \geq D - \sum_{j \in \mathcal{G}} \xi_j^s \bar{g}_j x_j \qquad \forall s \in S \tag{5c}$$

$$\phi^s \geq \frac{1}{D} r^s \qquad \forall s \in S \tag{5d}$$

$$r^s \geq 0 \qquad \forall s \in S \tag{5e}$$

$$\phi^s \in \{0, 1\} \qquad \forall s \in S \tag{5f}$$

$$x \in \mathcal{X} \tag{5g}$$

where $\overline{\text{LOLP}} \in [0, 1]$ is the accepted level of reliability adopted as the planning criterion. Constraint (5c) relates the load shedding to the system's capability of load supply in each state and constraint (5d) ensures that the indicator variable ϕ_s will be equal to 1 for the states with load shedding, and it will be 0 otherwise. Note that constraint (5b) limits the value of LOLP and, consequently, restricts the set of possible investment plans.

One characteristic of LOLP, which is also the most common source of criticism, is that the depth of the load shedding is disregarded since "bad" states are equally labeled with $\phi^s = 1$, independent of the amount of load being shed. This can mislead the SEP to find investment plans with a small probability of failure but exposed to states with a high level of severity.

2) EPNS: The *Expected Power Not Supplied* is the average value of the load shedding of all system states, as illustrated in Figure 2(b). It can be defined as

$$\text{EPNS} = E[R] = \sum_{s \in S} p^s r^s \tag{6}$$

where E is the expected value operator.

The EPNS can also be formulated as an optimization problem and explicitly incorporated into the SEP, resulting in the EPNS-constrained SEP model (7).

$$\text{Min} \quad \sum_{j \in \mathcal{G}} c_j x_j + O(x) \tag{7a}$$

$$\text{s.t.:} \quad \sum_{s \in S} p^s r^s \leq \overline{\text{EPNS}} \tag{7b}$$

$$r^s \geq D - \sum_{j \in \mathcal{G}} \xi_j^s \bar{g}_j x_j \qquad \forall s \in S \tag{7c}$$

$$r^s \geq 0 \qquad \forall s \in S \tag{7d}$$

$$x \in \mathcal{X} \tag{7e}$$

where $\overline{\text{EPNS}}$ is the pre-established planning criterion to EPNS.

Although the EPNS captures the severity of the load shedding in average terms, this index also considers all the "good" states (i.e. without load shedding) and, thus, results on a "diluted" index, not reflecting the real exposure of the system to states with failure.

B. Risk Measures Used in Financial Area

It would be interesting if the reliability index could capture both the characteristics of LOLP and EPNS, i.e., both the number of states with load shedding and the severity of these states. To accomplish this task, we propose the risk measures used in portfolio optimization problems in the financial sector. In the next sections, the risk measures VaR_α and CVaR_α will be presented and introduced in the context of reliability analysis for power systems expansion planning.

1) VaR$_\alpha$: The *Value-at-Risk* [45] is a risk index that aims to measure the lowest load shedding associated with a probability of occurrence α or, similarly, the maximum load shedding within a specified level of confidence $1 - \alpha$, as illustrated in Figure 3(a). For example, VaR$_{5\%}$ answers the question "what is the maximum possible load shedding considering the 95% *best* states"?

As in the case of LOLP and EPNS, it is possible to define the SEP using $R(x) = \text{VaR}_\alpha(x)$ as the reliability index. Its formulation can be explicitly incorporated into the problem (1), resulting in problem (8).

$$\text{Min} \quad \sum_{j \in \mathcal{G}} c_j x_j + O(x) \tag{8a}$$

$$\text{s.t.:} \quad r^s - D\phi^s \leq \overline{\text{VaR}} \qquad \forall s \in S \tag{8b}$$

$$\sum_{s \in S} p^s \phi^s \leq \alpha \tag{8c}$$

$$r^s \geq D - \sum_{j \in \mathcal{G}} \xi_j^s \bar{g}_j x_j \qquad \forall s \in S \tag{8d}$$

$$r^s \geq 0 \qquad \forall s \in S \tag{8e}$$

$$\phi^s \in \{0, 1\} \qquad \forall s \in S \tag{8f}$$

$$x \in \mathcal{X} \tag{8g}$$

where $\overline{\text{VaR}}_\alpha$ is the limit determined by the system planner. Note that, as in the case of LOLP, an integer variable for each state is also required, what makes its representation more difficult.
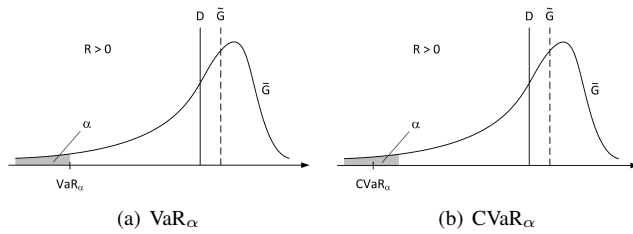
Fig. 3. Financial Risk Measures

Rearranging the constraint (8b), leads to the the inequality: $\phi^s \geq D^{-1}\left(r^s - \overline{\text{VaR}}\right)$ which indicates that when the load shedding r^s exceeds the limit $\overline{\text{VaR}}$ the variable ϕ^s should assume value 1, characterizing the states in the tail of the probability distribution function. Even though it incorporates the parameter α that allows this index to focus on the set of states in the tail of the distribution, the VaR_α is the smallest value in this set and, therefore, cannot detect the severity of the states with load shedding greater than VaR_α, which is a drawback similar to the one associated to LOLP.

2) CVaR$_\alpha$: The *Conditional Value-at-Risk* measures the expected value of the α worst states, i.e., the average of the states that comprise the tail of the probability distribution function of the load shedding and is defined as

$$\text{CVaR}_\alpha(R) = E[R : R \geq \text{VaR}_\alpha(R)] \tag{9}$$

The CVaR_α index has several interesting properties [46] and [47] demonstrated that it is possible to formulate the CVaR_α calculation as a *linear* programming problem independent of VaR_α, which needs integer variables in its formulation.

As defined for the measures presented above, the SEP with CVaR_α constraint is shown in (10).

$$\text{Min} \quad \sum_{j \in \mathcal{G}} c_j x_j + O(x) \tag{10a}$$

$$\text{s.t.:} \quad b + \alpha^{-1} \sum_{s \in S} p^s y^s \leq \overline{\text{CVaR}} \qquad \forall s \in S \tag{10b}$$

$$r^s \geq D - \sum_{j \in \mathcal{G}} \xi_j^s \bar{g}_j x_j \qquad \forall s \in S \tag{10c}$$

$$r^s \geq 0 \qquad \forall s \in S \tag{10d}$$

$$y^s \geq r^s - b \qquad \forall s \in S \tag{10e}$$

$$y^s \geq 0 \qquad \forall s \in S \tag{10f}$$

$$x \in \mathcal{X} \tag{10g}$$

where $\overline{\text{CVaR}}$ is a limit pre-established by the planner, b is the variable that represents the implicitly calculated VaR_α while y^s is the amount of load shedding that exceeds b, calculated by equation (10e). Therefore, the CVaR_α can be calculated as the sum of b plus the expected value of y^s conditioned to the states that exceed the VaR_α, as shown in left-hand side of equation (10b).

IV. CONSIDERING RISK CONSTRAINTS IN DECOMPOSITION SCHEMES

As noted in previous sections, the reliability-constrained SEP problem is a large scale mixed-integer optimization problem. The incorporation of reliability constraints into the SEP requires the representation of a (possibly large) set of additional variables and constraints for *every* state of the system and the number of states grows combinatorially with the number of generating units of the system.

The main objective of the design of mathematical decomposition techniques is to solve very complex or large problems through the repeated solution of a series of easier or smaller problems. From model (1) it is possible to notice that it has a block structure and the problems are coupled by the investment decision vector x. This structure is opportune for the application of such techniques.

In this work, we use the Benders decomposition technique [48] to split the original problem into three subproblems, intuitively reproducing the expansion planning process, which may consist of the following steps:

- First, the *investment subproblem* (master) is solved, aiming for a new trial investment plan x^μ, based on information obtained until iteration μ: an approximation of the total cost function (investment plus approximate operation cost) and the approximation of the reliability "feasible region" (constraints representing the set of plans that meet the reliability criterion);
- Given the proposed plan, x^μ, the *operation subproblem* (slave) is solved and we check if the approximation of the cost function represented in the investment subproblem is appropriate. If this function has not the adequate accuracy, a sensitivity analysis is conducted to build a new Benders optimality cut and improve the approximation of the cost function in the master problem;
- For the same proposed plan x^μ, the *reliability subproblem* (another slave) is solved to verify if the proposed solution is feasible concerning the selected reliability index. If the solution is not feasible, a sensitivity analysis in this problem is carried out and a new Benders feasibility cut is obtained, improving the representation of the feasible region in the master problem.

In brief, at each iteration a trial solution is obtained from the master problem and sent to both slave subproblems. Each subproblem evaluates the decision x^μ in terms of its cost and reliability and also return Benders cuts for improving the representation of approximated operation cost and reliability index in the master problem. This procedure is repeated iteratively until a feasible solution with minimal total cost is found.

A. Investment Subproblem

The investment subproblem can be formulated as the following mixed integer linear programming problem:

$$\text{Min} \quad \sum_{j \in \mathcal{G}} c_j x_j + \gamma \qquad (11a)$$

$$\text{s.t.:} \quad \gamma \geq O(x^i) + \sum_{j \in \mathcal{G}} \frac{\partial O(x^i)}{\partial x_j^i}(x_j - x_j^i) \qquad i \in \mathcal{A} \quad (11b)$$

$$R(x^i) + \sum_{j \in \mathcal{G}} \frac{\partial R(x^i)}{\partial x_j^i}(x_j - x_j^i) \leq \bar{R} \qquad i \in \mathcal{R} \quad (11c)$$

$$x \in \mathcal{X} \qquad (11d)$$

where $\mathcal{A}$ and $\mathcal{R}$ are the sets of iterations where a cut has been added and x^i is the trial solution vector found at iteration i. Here, γ is an additional variable used to model the epigraph of $O(x)$. Constraints (11b) are called optimality cuts and are a first order approximation of the operation cost function $O(x)$ constructed from the value of $O(x)$ and its partial derivatives evaluated at x^i. Similarly, constraints (11c) are called feasibility cuts and are also a linear approximation of the feasibility region associated to the reliability index represented by the function $R(x)$. Feasibility cuts are, therefore, responsible for removing expansion plans that do not satisfy the reliability criterion, consequently, in order to keep the investment problem always feasible it is necessary to have enough candidate projects available, which is usually the case for real-world problems. Problem infeasibility indicates that the pre-established reliability criterion is to tight or the set of candidate projects is insufficient (or both). In this case, the input data should be revised (by lowering the criterion or adding more projects).

B. Operation subproblem

Given a trial investment decision x^μ, the operation subproblem can be formulated as

$$O(x^\mu) = \text{Min} \quad \sum_{j \in \mathcal{G}} d_j g_j + hr \qquad (12a)$$

$$\text{s.t.:} \quad \sum_{j \in \mathcal{G}} g_j + r = D \qquad (12b)$$

$$g_j \leq \bar{g}_j x_j^\mu \qquad : \pi_j^{\bar{g}} \quad j \in \mathcal{G} \quad (12c)$$

where $\pi_j^{\bar{g}}$ corresponds to the dual variable of the maximum generation constraint of generator j.

From linear programming theory, it is known that $\pi_j^{\bar{g}}$ is the derivative of the objective function with respect to the right-hand side of constraints (12c). Applying the chain rule, we have

$$\frac{\partial O(x^\mu)}{\partial x_j^\mu} = \frac{\partial O(x^\mu)}{\partial(\bar{g}_j x_j^\mu)} \times \frac{\partial(\bar{g}_j x_j^\mu)}{\partial x_j^\mu} = \pi_j^{\bar{g}} \times \bar{g}_j \qquad (13)$$

which is the derivative of the operation cost with respect to the investment decision x^μ, used to construct the optimality Benders cuts (11b).

In this section we considered the simplest single-stage deterministic representation of the system's operation. However, the construction of the Benders cut is easily generalized to the

multistage stochastic setting with all operative details [25], [26]. Note also that the feasibility of operation problems is ensured because of the variables, r, that represent energy load shedding.

C. Reliability subproblem

Given a trial investment decision x^μ, it is possible to calculate the value of the risk measure associated to this plan, as well as the derivative of the reliability function $R(x^\mu)$ with respect to the investment decision, needed for the construction of the Benders cuts (11c) that approximate the feasible region for the adopted reliability criterion in the master problem.

The feasibility Benders cuts consist of cutting planes that are tangent to the original feasibility region. Because of this reason, one of the requirements for the Benders decomposition method to be successfully applied is that the generated cuts cannot eliminate feasible solutions and this fact cannot be guaranteed if the subproblem is non-convex. This same decomposition scheme cannot be implemented for SEP with LOLP and VaR_α constraints because they require the use of integer variables, characterizing a non-convex problem. It is still possible to use alternative and less efficient decompositions that remove infeasible integer solutions one by one. Therefore, this work considered only the SEP with EPNS or CVaR_α constraints as reliability criterion[2].

1) EPNS Criterion: This subproblem considers $R(x) = \text{EPNS}(x)$ and the solution can be obtained without the explicit representation of the load shedding variable r_s because, given x^μ, it is possible to calculate the EPNS as

$$\text{EPNS}(x^\mu) = \sum_{s \in \Omega} p_s \times \left(D - \sum_{j \in \mathcal{G}} \xi_j^s \bar{g}_j x_j^\mu \right) \qquad (14)$$

Deriving the equation (14) with respect to the investment variable x_j^μ, we have

$$\frac{\partial \text{EPNS}(x^\mu)}{\partial x_j^\mu} = -\sum_{s \in \Omega} p_s \xi_j^s \bar{g}_j \qquad \forall j \in \mathcal{G} \quad (15)$$

which consists in the coefficients of the feasibility Benders cuts (11c). For each investment plan, the $\text{EPNS}(x^\mu)$ and the derivative with respect to each x_j^μ can be obtained by solving the states obtained from the Monte Carlo simulation independently. Then, a single cut is calculated by properly aggregating the results.

2) $CVaR_\alpha$ Criterion: The value for $\text{CVaR}_\alpha(x^\mu)$ if given by the solution of following linear programming subproblem:

$$\text{Min} \quad b + \alpha^{-1} \sum_{s \in S} p^s y^s \qquad (16a)$$

$$\text{s.t.:} \quad r^s \geq D - \sum_{j \in \mathcal{G}} \xi_j^s \bar{g}_j x_j^\mu \quad : v^s \quad \forall s \in S \quad (16b)$$

$$r^s \geq 0 \qquad \forall s \in S \quad (16c)$$

$$y^s \geq r^s - b \qquad : w^s \quad \forall s \in S \quad (16d)$$

$$y^s \geq 0 \qquad \forall s \in S \quad (16e)$$

---

[2]Actually, it can be shown that is possible to approximate LOLP, EPNS and VaR_α criteria by just changing the parameters of CVaR_α planning model

TABLE I
SAMPLE CASE: GENERATOR DATA

Gen.	Inv. Cost M$	Oper. Cost M$/MW	Capacity [MW] Scen. 1 / 2	Out. Prob. %
G1	25	0	7 / 11	5
G2	20	1	10	20
G3	-	4	20	10

TABLE II
SAMPLE CASE: OPTIMAL OPERATION SOLUTION FOR TRIAL INVESTMENT
DECISION $x = [1, 0]$

Scen.	G1 MW	G2 MW	G3 MW	D MW	Oper. Cost M$
1	7	-	1	8	8.0
2	11	-	0	11	0.5
Aver.	9	-	0.5	9.5	4.25

TABLE III
SAMPLE CASE: RELIABILITY ANALYSIS FOR TRIAL INVESTMENT
DECISION $x = [1, 0]$.

Scen.	ξ	Prob. %	G1	G2	G3	D	PNS
1	[0, 0, 0]	0.04	0	-	0	8	8.0
1	[0, 0, 1]	0.34	0	-	12	8	0.0
1	[0, 1, 0]	0.21	0	-	0	8	8.0
1	[0, 1, 1]	1.91	0	-	12	8	0.0
1	[1, 0, 0]	0.71	7	-	0	8	1.0
1	[1, 0, 1]	6.41	7	-	12	8	0.0
1	[1, 1, 0]	4.04	7	-	0	8	1.0
1	[1, 1, 1]	36.34	7	-	12	8	0.0
2	[0, 0, 0]	0.04	0	-	0	8	8.0
2	[0, 0, 1]	0.34	0	-	12	8	0.0
2	[0, 1, 0]	0.21	0	-	0	8	8.0
2	[0, 1, 1]	1.91	0	-	12	8	0.0
2	[1, 0, 0]	0.71	11	-	0	8	0.0
2	[1, 0, 1]	6.41	11	-	12	8	0.0
2	[1, 1, 0]	4.04	11	-	0	8	0.0
2	[1, 1, 1]	36.34	11	-	12	8	0.0

where the decision variables b, r^s and y^s are non-negative and v^s and w^s are the dual variables of constraints (16d) and (16b), respectively. The same solution process used for EPNS is applied for CVaR_α to obtain the feasibility cut.

V. CASE STUDY

A. Small test system

In this section we will illustrate the proposed methodology considering a small system composed of 3 generators and 1 demand. The first generator is a variable renewable energy (VRE) plant (for example, wind), the second is a thermal plant with low production cost and the third is also a thermal plant with higher production cost. For ease of presentation, we will consider that generators 1 and 2 are candidate projects and, thus, will have associated investment decisions, and that generator 3 is an existing one. Additionally, we will also consider in this setting only 2 equally likely scenarios to represent the variability of VRE resources. The investment costs and data required for operation and reliability analysis are presented in table I. As shown in the table, the capacity of generator G1 depends on the scenario.

On the other side, Demand Side Response (DSR) is represented by considering price sensitivity with 3 segments, described as follows: the first 8 MW is inelastic[3], the second additional 2 MW if energy price is lower than 2 M$/MW and an additional 2 MW for prices lower than 1 M$/MW.

To illustrate one iteration between investment, operation and reliability subproblems, lets consider the trial investment decision vector $x = [1, 0]$ that represents generator 1 being constructed and generator 2 not.

In the operation subproblem we will have to solve problems for the 2 scenarios, as shown in table II. In this setting, the solution can be found by dispatching the plants by merit order until the demand is met. In the first scenario, we dispatch the cheapest generator available G1 at its maximum available capacity (7 MW) and then G3 at 1 MW to meet the 8 MW of inelastic demand, since G3's cost is higher than the maximum willingness to pay of the elastic demand. The operating cost is given by the sum of generating costs: $0.5 \times 7 + 4.5 \times 1 = 8M\$$. In the second scenario, G1 is dispatched at its maximum available capacity 11 MW since its cost is lower than the demand willingness to pay price of the third segment. The total operation cost for this scenario is given by the generating costs minus the revenue from the elastic segments: $0.5 \times 11 - 2 \times 2 - 1 \times 1 = 0.5M\$$. The average operation cost for this investment plan is 4.25 M$.

[3]Inelastic segment can represent an amount that is contracted so its variable cost is not represented in the problem.

The reliability subproblem for this plan is illustrated by solving all possible states as shown in table III. For each state, we calculate the corresponding probability based on the outage probability of each generator and of the VRE scenario and obtain the available capacity of each plant that depends on the investment plan and the generator's state, in MW. The demand considered is the inelastic segment since the objective of this analysis is to calculate load shedding. The power not supplied (PNS) is calculated by the difference of the total capacity and the demand, in MW.

Table IV shows the costs and reliability indexes for all investment plans. EPNS, $\text{VaR}_{2\%}$ and $\text{CVaR}_{2\%}$ and are calculated as a percentage of the inelastic demand. The most economical plan is $[0, 1]$ with total cost of 31 M$. However, it is interesting to notice that depending on the selected reliability index this plan might not be the best one. For instance, both LOLP and $\text{VaR}_{2\%}$ indexes are lower than that of the $[1, 0]$ plan but we must recall that the main drawback of these indexes is the fact that they are not sensitive to the depth of the load shedding for the extreme scenarios. This becomes very clear by looking to the $\text{CVaR}_{2\%}$ which indicates that the most economical plan is exposed to a very high load shedding of 75% of the load in the 2% worst cases. On the other hand, in the second most economical plan the $\text{CVaR}_{2\%}$ is 34.4% which is much lower. This reinforces the idea that the adoption of reliability constraints in the SEP problem can be a very interesting solution.

TABLE IV
SAMPLE CASE: SOLUTION FOR ALL INVESTMENT DECISIONS

	[0, 0]	[1, 0]	[0, 1]	[1, 1]
Inv.Cost	0 M$	30.0 M$	20 M$	50 M$
Exp. Ope.Cost	36 M$	4.3 M$	11 M$	3 M$
Total Cost	36 M$	34.3 M$	31 M$	53 M$
LOLP	10%	5.25%	1.5%	0.79%
EPNS	10%	1.10%	1.5%	0.20%
$VaR_{2\%}$	100%	12.50%	0.0%	0.00%
$CVaR_{2\%}$	100%	34.40%	75.0%	8.20%

B. Bolivia

This case study consists of the expansion planning of the Bolivian (BO) generation system for a 7-years horizon. The system is composed of 28 existing generating hydro plants and 25 thermal plants with a total installed capacity of about 850MW (in the first year). Besides, 30 thermal projects are available as investment alternatives for the expansion plan. Note that the maximum number of plants that may be operating in the system is 83 and, thus, the maximum number of states in the reliability subproblem is 2^{83} (approximately 10^{25} states).

Initially, we perform a comparative analysis for the expansion plans obtained with a hierarchical against an integrated methodology. Additionally, a comparison between the results obtained for the EPNS-constrained SEP and the $CVaR_{\alpha}$-constrained SEP is carried out.

We adopted a reliability limit for the EPNS equals to 1% of the demand for each month over the horizon of study. The reliability subproblem is solved by Monte Carlo simulation and it was considered that the convergence is reached when the coefficient of variation for the estimator is lower than 5% [49]. Checking the coefficient of variation is a common stopping criterion for MC also used in [50] and [37].

The operation problem is solved by the SDDP algorithm to find the optimal dispatch under uncertainty, typically on inflow, demand, fuel costs, etc. All real-world details considered by the Bolivian ISO are represented.

1) Hierarchical Planning with EPNS criterion (HP-EPNS): The hierarchical planning approach consists of obtaining an initial investment plan (first step) considering only the economic aspects, i.e., we aim to find the investment vector that minimizes the investment and operating costs (EP), as presented in section II-A. Table V shows the additional capacity added in each year of the planning horizon and table VI its respective total cost. It can be seen in Figure 4 that the resulting EPNS is not feasible according to the reliability criterion pre-established.

The second step of HP-EPNS is accomplished by considering the investment vector obtained in the first stage and solving the problem again to obtain reinforcements due to reliability requirements. Observe in table V that it was necessary to invest in an additional 44.1MW in Year 2 (the year that occurs the first violation of EPNS) to ensure feasibility for the reliability criterion. The associated total cost for this investment plan is also shown in table VI. This highlights that the EPNS and
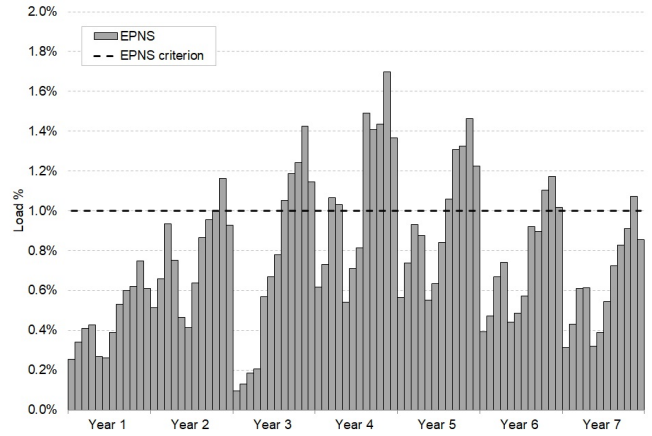

Fig. 4. Resulting EPNS for plan obtained with EP

TABLE V
HIERARCHIC X INTEGRATED PLANNING: ADDITIONAL CAPACITY [MW]

Methodology	Y1	Y2	Y3	Y4	Y5	Y6	Y7
EP	0	**0.0**	65.8	44.1	285.6	44.1	44.1
HP-EPNS	0	**44.1**	65.8	44.1	285.6	44.1	44.1
IP-EPNS	0	44.1	**44.1**	44.1	285.6	44.1	**44.1**
IP-CVaR	0	44.1	44.1	44.1	285.6	44.1	**0.0**

TABLE VI
HIERARCHICAL X INTEGRATED PLANNING: COSTS

Methodology	Investment Cost M$	Operation Cost M$	Total Cost M$	# of months violating criteria
EP	98.42	146.66	245.08	**22**
HP-EPNS	117.80	145.02	262.82	0
IP-EPNS	100.06	152.17	252.23	0
IP-CVaR	98.05	153.66	251.71	0

CVaR criteria are intrinsically different and capture diverse characteristics of the system reliability and adequacy.

2) Integrated Planning with EPNS criterion (IP-EPNS): In the integrated planning methodology both operation and reliability subproblems are solved for each trial decision of the investment problem. Since the economic solution is not fixed when solving the reliability subproblem as in the HP-EPNS, it is possible to consider in an integrated manner both the economic benefits and the contribution for the attendance of the reliability criterion and then seek for the global optimal solution.

Table V shows that, in comparison with HP-EPNS, it is possible to invest in 44.1MW in Year 3 instead of 65.8MW and still get a reliability feasible plan. Note that compared to the methodology HP-EPNS, the IP-EPNS methodology obtains a lower cost investment plan but with higher operation cost. This fact illustrates that there is a benefit to invest in a generator with lower construction cost and higher operating cost because the reliability criterion could be met and total cost is lower, as shown in table VI.

3) Integrated Planning with $CVaR_{\alpha}$ criterion (IP-CVaR): To compare the risk measures EPNS and $CVaR_{\alpha}$, the $CVaR_{5\%}$
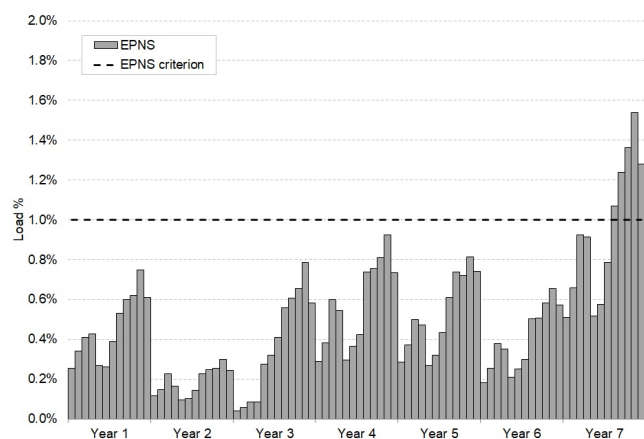
Fig. 5. Resulting EPNS for plan obtained with IP-CVaR. Although EPNS violations are seen here, no CVaR violations were found in this plan.

was calculated for the optimal investment plan obtained from the IP-EPNS and its maximum value (approximately 10% of total load) was used as the $CVaR_{5\%}$ limit for the expansion planning.

It is possible to observe the final costs obtained with this plan in table VI and the additional capacity in table V. Note that the violations shown in the table VI correspond to the reliability criterion considered in the optimization, that is EPNS for the first three plans and CVaR for the last plan. The difference from the IP-EPNS is that the IP-CVaR methodology did not consider necessary to invest in 44.1MW in the last year, obtaining an investment plan at a lower cost. Furthermore, observing the EPNS for the plan in Figure 5, note that, as the criterion of IP-CVaR methodology are related to the average of worst 5% states and not the average of all states (EPNS), it was possible to find a more economic investment plan at a cost of EPNS exceeding 1% of the load in the last year (but, as expected, the plan meets the $CVaR_{5\%} \leq 10\%$ of load criterion).

VI. Conclusions

This work presented a methodology to incorporate reliability constraints in the optimal power systems expansion planning. Through a real case example, it is shown that the simple application of an economic planning criterion is not enough to guarantee that the reliability criterion will be satisfied. It is also showed that the application of a hierarchical two-step procedure to solve the expansion planning problem with the reliability criterion does not lead to the least-cost solution.

It was shown that the integrated reliability constrained expansion planning problem can be modeled as a large-scale MIP. In particular, we presented the MIP formulation for multiple reliability indexes. Benders decomposition can be used to exploit the problem structures and decouple the problem in an investment problem solved by standard MIP techniques; an operation problem solved SDDP; and a reliability problem solved by Monte Carlo simulation.

The advantage of using the integrated approach is to identify projects that contribute both in economic terms and in terms of

improving overall system reliability, which might not possible by using the hierarchical procedure.

In addition to the traditional reliability measures commonly used in electrical systems, LOLP and EPNS, this work also illustrated how to incorporate the risk measures VaR_{α} and $CVaR_{\alpha}$, widely used in the financial area, into the power system expansion planning. It was showed that the use of the $CVaR_{\alpha}$ criterion allows the control of the depth of the probability distribution function of the system load shedding. This index can be a powerful tool of interest for system planners once it allows them to shape the expansion plan considering the prevention of disastrous events to the desired level of reliability by stimulating the diversification of the power plants "portfolio". This difference also points to the direction of considering multiple reliability metrics at the same time for a single expansion plan, which can be done within the framework proposed in this paper, simply by considering multiple reliability subproblems.

Moreover, this tool can be used to test and create policies for system expansion and to assess the individual contribution of each project in both economic and reliability aspects. This methodology is implemented in the software OptGen [51] by PSR.

References

[1] J. Zhu and M.-y. Chow, "A review of emerging techniques on generation expansion planning," *IEEE Transactions on Power Systems*, vol. 12, no. 4, pp. 1722–1728, 1997.

[2] V. Oree, S. Z. S. Hassen, and P. J. Fleming, "Generation expansion planning optimisation with renewable energy integration: A review," *Renewable and Sustainable Energy Reviews*, vol. 69, pp. 790–803, 2017.

[3] S. Lumbreras and A. Ramos, "The new challenges to transmission expansion planning. survey of recent practice and literature review," *Electric Power Systems Research*, vol. 134, pp. 19–29, 2016.

[4] R. Hemmati, R.-A. Hooshmand, and A. Khodabakhshian, "Comprehensive review of generation and transmission expansion planning," *IET Generation, Transmission & Distribution*, vol. 7, no. 9, pp. 955–964, 2013.

[5] D. Pozo, E. E. Sauma, and J. Contreras, "A three-level static milp model for generation and transmission expansion planning," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 202–210, 2013.

[6] A. Moreira, D. Pozo, A. Street, and E. Sauma, "Reliable renewable generation and transmission expansion planning: Co-optimizing system's resources for meeting renewable targets," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 3246–3257, 2017.

[7] J. D. Jenkins and N. A. Sepulveda, "Enhanced decision support for a changing electricity landscape: The genx configurable electricity resource capacity expansion model," MIT Energy Initiative, Tech. Rep., 2017.

[8] H. Saboori and R. Hemmati, "Considering carbon capture and storage in electricity generation expansion planning," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 4, pp. 1371–1378, 2016.

[9] B. Hua, R. Baldick, and J. Wang, "Representing operational flexibility in generation expansion planning through convex relaxation of unit commitment," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 2272–2281, 2018.

[10] P. J. Ramírez, D. Papadaskalopoulos, and G. Strbac, "Co-optimization of generation expansion planning and electric vehicles flexibility," *IEEE Transactions on Smart Grid*, vol. 7, no. 3, pp. 1609–1619, 2016.

[11] Y. Zhan, Q. P. Zheng, J. Wang, and P. Pinson, "Generation expansion planning with large amounts of wind power via decision-dependent stochastic programming," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 3015–3026, 2017.

[12] R. Jabr, "Robust transmission network expansion planning with uncertain renewable generation and loads," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4558–4567, 2013.

[13] J. H. Roh, M. Shahidehpour, and L. Wu, "Market-based generation and transmission planning with uncertainties," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1587–1598, 2009.

[14] A. Moreira, G. Strbac, R. Moreno, A. Street, and I. Konstantelos, "A five-level milp model for flexible transmission network planning under uncertainty: A min–max regret approach," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 486–501, 2018.

[15] P. Jirutitijaroen and C. Singh, "Reliability constrained multi-area adequacy planning using stochastic programming with sample-average approximations," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 504–513, 2008.

[16] J. Á. López, K. Ponnambalam, and V. H. Quintana, "Generation and transmission expansion under risk using stochastic programming," *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1369–1378, 2007.

[17] D. Mejia-Giraldo and J. McCalley, "Adjustable decisions for reducing the price of robustness of capacity expansion planning," *IEEE Transactions on Power Systems*, vol. 29, no. 4, pp. 1573–1582, 2014.

[18] L. Baringo and A. Baringo, "A stochastic adaptive robust optimization approach for the generation and transmission expansion planning," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 792–802, 2018.

[19] S. Binato, G. C. De Oliveira, and J. L. De Araújo, "A greedy randomized adaptive search procedure for transmission expansion planning," *IEEE Transactions on Power Systems*, vol. 16, no. 2, pp. 247–253, 2001.

[20] Y. Liu, R. Sioshansi, and A. J. Conejo, "Multistage stochastic investment planning with multiscale representation of uncertainties and decisions," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 781–791, 2018.

[21] F. D. Munoz and J.-P. Watson, "A scalable solution framework for stochastic transmission and generation planning problems," *Computational Management Science*, vol. 12, no. 4, pp. 491–518, 2015.

[22] Y. H. Wu, "A stochastic mathematical program with complementary constraints for market-wide power generation and transmission expansion planning," Master's thesis, Iowa State University, Industrial and Manufacturing Systems Engineering, 2014.

[23] K. J. Singh, A. B. Philpott, and R. K. Wood, "Dantzig-wolfe decomposition for solving multistage stochastic capacity-planning problems," *Operations Research*, vol. 57, no. 5, pp. 1271–1286, 2009.

[24] S. Binato, M. V. F. Pereira, and S. Granville, "A new benders decomposition approach to solve power transmission network design problems," *IEEE Transactions on Power Systems*, vol. 16, no. 2, pp. 235–240, 2001.

[25] B. Gorenstin, N. Campodonico, J. Costa, and M. Pereira, "Power system expansion planning under uncertainty," *IEEE Transactions on Power Systems*, vol. 8, no. 1, pp. 129–136, 1993.

[26] N. Campodónico, S. Binato, R. Kelman, M. Pereira, M. Tinoco, F. Montoya, M. Zhang, and F. Mayaki, "Expansion planning of generation and interconnections under uncertainty," in *3rd Balkans Power Conference*, 2003.

[27] L. Baringo and A. J. Conejo, "Risk-constrained multi-stage wind power investment," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 401–411, 2013.

[28] S. Dehghan, N. Amjady, and A. J. Conejo, "Reliability-constrained robust power system expansion planning," *IEEE Transactions on Power Systems*, vol. 31, no. 3, pp. 2383–2392, 2016.

[29] G. Coté, "Reliability aspects of optimal generation planning models for power systems," Ph.D. dissertation, Thesis (Ph. D.)–University of London, 1979.

[30] J. A. Bloom, "Solving an electricity generating capacity expansion planning problem by generalized benders decomposition," *Operations Research*, vol. 31, no. 1, pp. 84–100, Jan-Feb 1983.

[31] M. V. F. Pereira, "Application of sensitivity analysis on generation-transmission system expansion planning," Ph.D. dissertation, Federal University of Rio de Janeiro - COPPE/UFRJ, 1985.

[32] G. C. Oliveira, "Metodologia de expansão da capacidade de ponta em sistemas interligados de geração hidrotermoelétrica," Ph.D. dissertation, COPPE/UFRJ, 1987.

[33] J. Sirikum and A. Techanitisawad, "Power generation expansion planning with emission control: a nonlinear model and a ga-based heuristic approach," *International Journal of Energy Research*, vol. 30, no. 2, pp. 81–99, 2006.

[34] J.-B. Park, Y.-M. Park, J.-R. Won, and K. Y. Lee, "An improved genetic algorithm for generation expansion planning," *IEEE Transactions on Power Systems*, vol. 15, no. 3, pp. 916–922, 2000.

[35] S. A. Rashidaee, T. Amraee, and M. Fotuhi-Firuzabad, "A linear model for dynamic generation expansion planning considering loss of load probability," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 6924–6934, 2018.

[36] M. V. F. Pereira and L. Pinto, "Multi-stage stochastic optimization applied to energy planning," *Mathematical Programming*, vol. 52, pp. 359–375, 1991.

[37] M. Pereira, L. Pinto, S. Cunha, and G. Oliveira, "Monte-carlo based composite reliability evaluation: Modeling aspects and computational results," *Reliability Assessment of Composite Generation and Transmission Systems*, 1989.

[38] M. E. P. Maceira, L. Marzano, D. D. J. Penna, A. Diniz, and T. Justino, "Application of cvar risk aversion approach in the expansion and operation planning and for setting the spot price in the brazilian hydrothermal interconnected system," in *Power Systems Computation Conference (PSCC), 2014*. IEEE, 2014, pp. 1–7.

[39] Y. Tohidi, M. R. Hesamzadeh, and F. Regairaz, "Modified benders decomposition for solving transmission investment game with risk measure," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1936–1947, 2018.

[40] NERC, "Probabilistic adequacy and measures - technical reference report," NERC, Tech. Rep., 2018. [Online]. Available: https://www.nerc.com/comm/PC/Probabilistic%20Assessment%20Working%20Group%20PAWG%20%20Relat/Probabilistic%20Adequacy%20and%20Measures%20Report.pdfvisitedonApril2nd2020

[41] A. Sankarakrishnan and R. Billinton, "Sequential monte carlo simulation for composite power system reliability analysis with time varying loads," *IEEE Transactions on power Systems*, vol. 10, no. 3, pp. 1540–1545, 1995.

[42] S. Dehghan, N. Amjady, and A. J. Conejo, "Reliability-constrained robust power system expansion planning," *IEEE Transactions on Power Systems*, vol. PP, no. 99, pp. 1–10, 2015.

[43] S. Bahramirad, W. Reder, and A. Khodaei, "Reliability-constrained optimal sizing of energy storage system in a microgrid," *IEEE Transactions on Smart Grid*, vol. 3, no. 4, pp. 2056–2062, 2012.

[44] J. Mello, M. Pereira, and A. L. Da Silva, "Evaluation of reliability worth in composite systems based on pseudo-sequential monte carlo simulation," *IEEE Transactions on Power Systems*, vol. 9, no. 3, pp. 1318–1326, 1994.

[45] P. Jorion, *Value at Risk*. New York, NY: McGraw Hill, 2000.

[46] C. Acerbi and D. Tasche, "Expected shortfall: a natural coherent alternative to value at risk," *arXiv*, May 2001.

[47] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *Journal of Risk*, vol. 2, pp. 21–41, 2000.

[48] J. F. Benders, "Partitioning procedures for solving mixed-variables programming problems," *Numerische Mathematik*, vol. 4, pp. 238–252, 1962, 10.1007/BF01386316. [Online]. Available: http://dx.doi.org/10.1007/BF01386316

[49] M. V. F. Pereira, M. E. P. Maceira, G. C. Oliveira, and L. M. V. G. Pinto, "Combining analytical models and monte-carlo techniques in probabilistic power system analysis," *IEEE Transactions on Power Systems*, vol. 7, no. 1, pp. 265–272, Feb 1992.

[50] S. Dehghan, A. Kazemi, and N. Amjady, "Multi-objective robust transmission expansion planning using information-gap decision theory and augmented ε-constraint method," *IET Generation, Transmission & Distribution*, vol. 8, no. 5, pp. 828–840, 2014.

[51] PSR-Inc, "OptGen." [Online]. Available: http://www.psr-inc.com/softwares-en/?current=p4040,visitedonApril10th2020.