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# Nonlinear state feedback control of chaos system of brushless DC motor

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## Abstract

The brushless DC motor (BLDCM) will be suffered the impact of chaos at the certain conditions. Therefore, This paper proposes a nonlinear state control method to suppress and control the emergence of chaotic motion based on the Lyapunov stability theory. Firstly, We analyze the dynamic characteristics of the state equation of BLDCM system, and prove that it will appear chaos phenomenon at some certain parameters. Finally, the nonlinear feedback controller is designed based on the Lyapunov stability theory, and is proved that the control method can make the system asymptotically stable to the equilibrium point. And further, simulation results are given to verify that the proposed method can be successfully used to control a chaotic BLDCM and make BLDCM stable quickly.

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*Keywords:* brushless DC motor; chaotic motion; nonlinear state feedback control; Lyapunov stable theory

## 1. Introduction

Brushless DC motor (BLDCM) not only has a series of advantages such as simple structure, reliable running and easy maintenance, but also has many advantages such as high efficiency of DC motor, no excitation loss and good speed regulation performance. Therefore, BLDCM has been widely applied in various fields of the national economy today, especially in the servo and drive systems of robots, aviation, machine tools, vehicles and other

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fields [1-4]. BLDCM is a typical multi-variable, a highly nonlinear and strong coupling system. Under certain operating parameters, it is often accompanied by irregular operating behaviors such as electromagnetic noise, low-frequency oscillation of torque and speed, and unstable control performance. Therefore, the operating quality and reliability of the BLDCM system are greatly affected by these factors [5]. Because these irregular motions are very similar to chaotic phenomena, they have attracted the attention of many scholars to prove that these irregular motions are chaotic phenomena [3-8]. In order to control the chaotic motion phenomenon in BLDCM and make the system stable quickly, this work has also become a hot issue in the field of motor control. At present, there are many methods of the control of chaos phenomena in mathematical theory research, such as OGY method [6], delayed feedback control method [7], sliding mode control method [8], etc. In these studies, the Control and suppression of the chaos of BLDCM can receive better results.

In this paper, in order to control and suppress the occurrence of the chaotic state of BLDCM, we design a nonlinear state feedback controller and apply it to the BLDCM system. After substituting the controller into the BLDCM system, the state equation of the system can be greatly simplified, and after adding state feedback control to the system, the system state will be able to quickly stabilize to the equilibrium point, to meet requirements of the system for rapid stabilization. Therefore, after the introduction of the controller, it can effectively prevent the system from entering a chaos state while the BLDCM is operating, and can also stabilize the motor movement to a certain value.

## 2. Mathematical model and dynamic characteristics of brushless DC motor

The brushless DC motor (BLDCM) is an electromechanical system. The state equation of the BLDCM consists of the voltage balance equation and the torque balance equation[9]. If the BLDCM air gap is uniform, the BLDCM state equation can be rewritten as a dimensionless mathematical model, then the BLDCM system can be described as follows:

$$\begin{cases} \dot{x}_1 = v_q - x_1 - x_2x_3 + \rho x_3 \\ \dot{x}_2 = v_d - x_2 + x_1x_3 \\ \dot{x}_3 = \sigma(x_1 - x_3) - T_L \end{cases} \quad (1)$$

where the state variables  $x_1$ ,  $x_2$ ,  $x_3$  respectively represent the motor equivalent q-axis current  $i_q$ , d-axis current  $i_d$  and speed  $\omega$ ,  $\rho$  and  $\sigma$  are the system parameters. Considering that the motor has no input voltage and no load,  $v_q = 0$ ,  $v_d = 0$ ,  $T_L = 0$ . This state corresponds to the braking operation process when the motor is powered off without load. Then the state equation of BLDCM can be rewritten as

$$\begin{cases} \dot{x}_1 = -x_1 - x_2x_3 + \rho x_3 \\ \dot{x}_2 = -x_2 + x_1x_3 \\ \dot{x}_3 = \sigma(x_1 - x_3) \end{cases} \quad (2)$$

The parameters  $\rho$  and  $\sigma$  in equation (2) are easily changed by the influence of temperature and noise in the working environment of BLDCM. Hemati [10-11], Wang Mufeng [12] and other have found that when the system parameters are in a certain range, the BLDCM system will appear chaotic motion. The system equation (2) has symmetric invariance about the  $x_2$  axis, that is, the equation remains unchanged under the transformation of  $(x_1, x_2, x_3) \rightarrow (-x_1, x_2, -x_3)$ . Due to  $\nabla \dot{V} = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} = -2 - \sigma < 0$  (where  $\sigma$  is positive), the system (2) is dissipative and converges in the exponential form  $\dot{V} = e^{-(2+\sigma)t}$ , which shows that when  $t \rightarrow \infty$ , each volume element of the system trajectory shrinks to zero at an exponential rate  $-(2 + \sigma)$ . Furthermore, the system will eventually be limited to a set with zero volume, and the progressive motion will be fixed on an attractor, which shows the existence of attractors. When  $\rho = 20$ ,  $\sigma = 5.46$ , and the initial value of the system  $(x_1, x_2, x_3) = (0.1, 10, -1.5)$ , Lyapunov exponent is calculated using literature [13-14],  $LE_1 = 0.463$ ,  $LE_2 = 0$ ,  $LE_3 = -7.923$ , and Lyapunov dimension is calculated,  $D_L = 2.059$ , and the largest Lyapunov exponent is greater than 0. These results indicate that the system (2) is in a chaotic state under these parameter. The phase portrait and timing diagram of the system (2) under the above parameters and initial values are shown in Figure 1, 2.

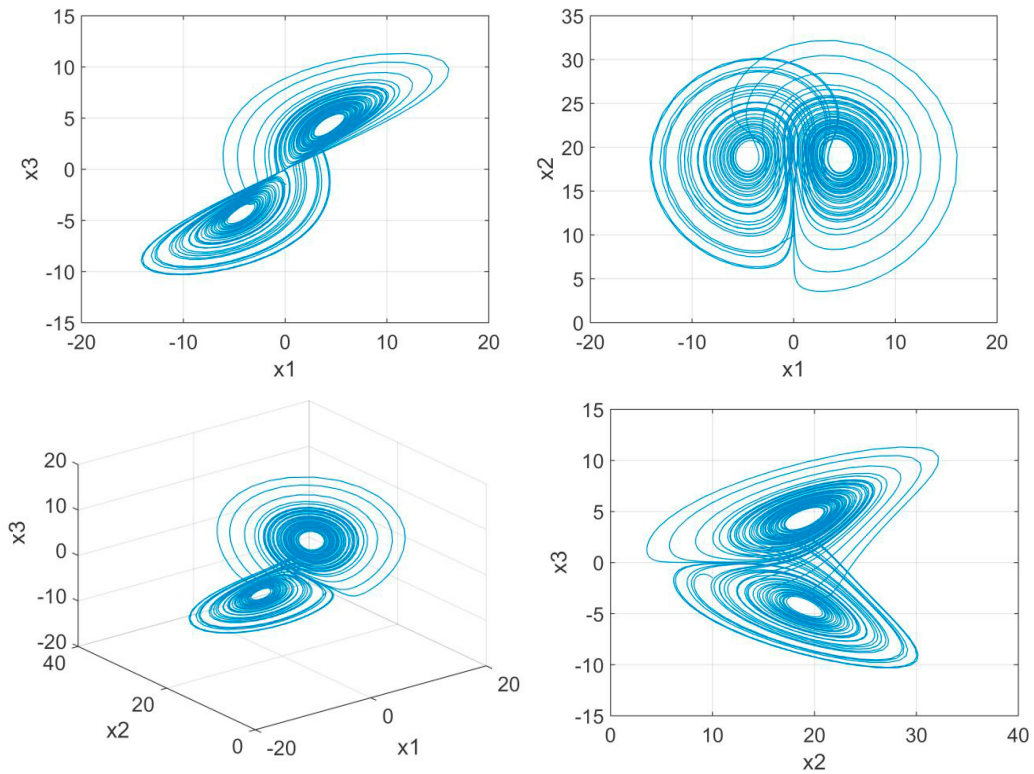


Fig. 1. The phase portrait of BLCDM with  $\rho = 20$ ,  $\sigma = 5.46$ .

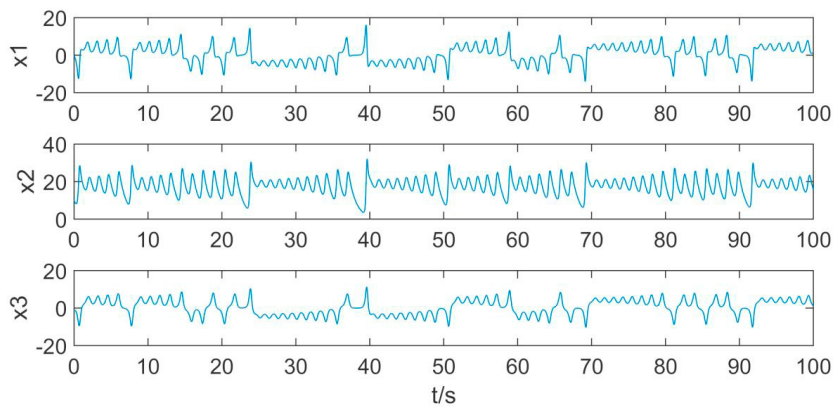


Fig. 2. The timing diagram of BLCDM with  $\rho = 20$ ,  $\sigma = 5.46$ .

### 3. BLCDM feedback control scheme design

BLCDM system will be in a chaos state under certain parameters. Therefore, it is extremely important to control its chaotic state to steady state quickly. So we use a nonlinear state feedback control method to design a chaotic controller. The feedback controller of BLCDM system is designed as shown in equation (3).

$$\begin{cases} u_1 = x_2x_3 - \rho x_3 \\ u_2 = -x_1x_3 \\ u_3 = -\sigma x_1 + kx_3 \end{cases} \tag{3}$$

where  $k$  is the feedback coefficient, the controller (3) is brought into the system state equation (2), and the controlled BLCDM chaotic system is obtained as shown in equation (4).

$$\begin{cases} \dot{x}_1 = -x_1 - x_2x_3 + \rho x_3 + u_1 \\ \dot{x}_2 = -x_2 + x_1x_3 + u_2 \\ \dot{x}_3 = \sigma(x_1 - x_3) + u_3 \end{cases} \tag{4}$$

According to the Lyapunov stability theory, we construct a suitable Lyapunov function  $V(x_1, x_2, x_3)$  based on the chaotic system, and it can make the chaotic system gradually stabilize when  $V(x_1, x_2, x_3)$  satisfy certain conditions. Therefore, for equation (4), we construct the following Lyapunov function:

$$V(x_1, x_2, x_3) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \tag{5}$$

Obviously,  $V(x_1, x_2, x_3) > 0$ , that is,  $V(x_1, x_2, x_3)$  is positive definite. Derivation of (5) can be obtained by

$$\begin{aligned} \dot{V}(x_1, x_2, x_3) &= x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 \\ &= -[x_1 \ x_2 \ x_3] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sigma - k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -X^T P X \end{aligned} \tag{6}$$

To make the matrix  $P$  positive definite, it is necessary to satisfy  $k > \sigma$ . When  $k > \sigma$ ,  $\dot{V}(x_1, x_2, x_3) < 0$ , the feedback controller (3) will inevitably make the system gradually stabilize at zero equilibrium point in the sense of Lyapunov stability.

#### 4. Simulation and result analysis

The parameters are set to  $\rho = 20$ ,  $\sigma = 5.46$ ,  $k = 6$ , and the initial values of  $(x_1, x_2, x_3) = (0.1, 10, -1.5)$  in the controlled BLCDM chaotic system equation (4). In order to observe the effect of nonlinear feedback control, a feedback controller is added at  $t = 25s$ , and the simulation results are shown in Figure 3. Figure 3 is the state response curves of  $x_1$ ,  $x_2$ , and  $x_3$ . It can be seen that after adding the feedback controller, the system quickly converges to the zero balance point, and further, the rapid stabilization of the BLCDM chaotic system was achieved.

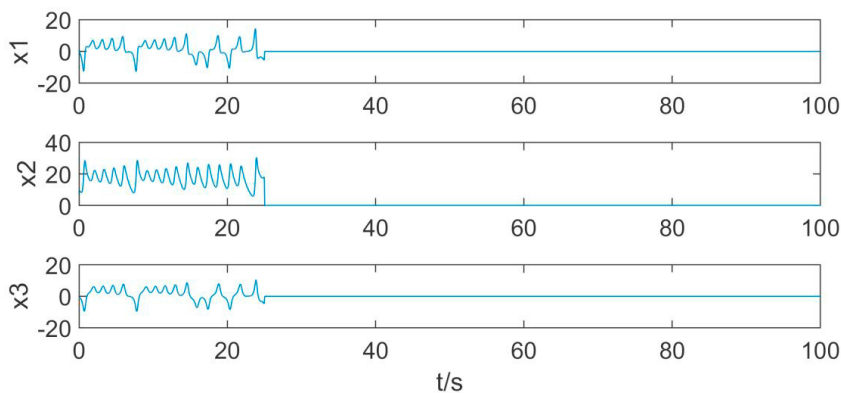


Fig. 3. Response curve of controlled BLCDM chaotic system

## 5. Conclusion

The brushless DC motor (BLCDM) will produce chaos under certain electromechanical parameters, which greatly affects the operation quality and reliability of the BLCDM system. Therefore, it is very important to control and suppress a chaos BLCDM. This paper proposed a nonlinear feedback controller for a chaos BLCDM based on Lyapunov stability theory. Firstly, the chaotic properties of the state equation of the BLCDM system are analyzed at certain parameters. Then, a nonlinear feedback controller is designed based on the Lyapunov stability theory. Theoretical analysis shows that the designed controller can be used to stabilize the system to the equilibrium point. Finally, the nonlinear feedback control system is simulated, and the response curve of the system is obtained. From simulation results, it was definitely verified that the proposed control method gives very robust control performance. The research results have a high reference value for stable operation and chaos control of the BLCDM.

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## References

1. Huang X, Goodman A, Gerada C, et al. a single sided matrix converter drive for a brushless DC motor in aerospace applications. *IEEE Transactions on Industrial Electronics* 2012; 59(9):3542-52.
2. Praveen R.P, Ravichandran M H, Achari V T, et al. A novel slotless halfbach-array permanent-magnet brushless DC motor for spacecraft applications. *IEEE Transactions on Industrial Electronics* 2012; 59(9):3553-60.
3. Melkote H, Khorrami F. Nonlinear adaptive control of direct-drive brushless DC motors and applications to robotic manipulators. *IEEE/ASME Transactions on Mechatronics* 1999; 4(1): 71-81.
4. Huang C C, Li P, Liu C T, Chen C. Design and analysis of a brushless DC motor for applications in robotics. *Electric Power Applications* 2012; 6(7):385-9.
5. Ge Z M, JW Cheng, Chen Y S. Chaos anticontrol and synchronization of three time scales brushless DC motor system. *Chaos Solitons & Fractals* 2004; 22(5):1165-82.
6. Sakai K. Application of OGY Control Method on Chaotic Vibrations for Tractor-Implement Systems. *Asae International Meeting* 1998;1-15.
7. Ren H P, Liu D, Li J. Delay feedback control of chaos in permanent magnet synchronous motor. *proceedings of the csee* 2003; 23(6):175-8.
8. Liu H, M C Y, Chen D Y. Study on sliding mode control of brushless DC motor chaotic system. *Small & Special Electrical Machines* 2012; 40(4):47-50.
9. Zhang X H, Wang D M. State feedback control of chaos system of brushless DC motor. *Micromotors* 2009; 42(11):82-5.
10. Hemati N, Leu M C. A complete model characterization of brushless DC motors. *IEEE Transactions on Industry Applications* 1992; 28(1):172-80.
11. Hemati N. Strange attractors in brushless DC motors. *IEEE Transactions on Circuits & Systems I Fundamental Theory & Applications* 1994; 41(1):40-5.
12. Wang M F, Wei D Q, Luo X S, et al. Chaos control in a brushless DC motor based on finite-time stability theory. *Journal of Vibration and Shock* 2016; 35(13):90-3.
13. Wolf A, Swift J B, Swinney H L, et al. Determining Lyapunov exponents from a time series. *Physica D Nonlinear Phenomena* 1985; 16(3):285-317.
14. Briggs K. An improved method for estimating Lyapunov exponents of chaotic time series. *Physics Letters A* 1990; 151(1):27-32.