An Enhanced PID Controller for Speed Control of Brushless DC Motors Based on Convex Set Optimization

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Abstract: This paper presents an enhanced PID controller design method with an internal feedback PD controller. The proposed controller structure is organized as the convex set between PID controller and PI controller with an internal PD controller. Control parameters are determined through an optimum tuning method to improve the step response characteristics in the controllable set. The new PID structure and optimum tuning algorithm are applied to the speed control of brushless direct-current (BLDC) motors. Computer simulation results show that the proposed controller is more effective in the performance of time domain by comparing with the existence tuning rules of PID controller.

Keywords: Brushless DC motor, Convex set, Optimization, PID controller.

1. INTRODUCTION

The BrushLess Direct Current (BLDC) motors are gaining grounds in the industries, especially in the areas of appliances production, aeronautics, robotics, computer peripherals, consumer and industrial automations and so on. The reason is that BLDC motors offer many advantages over the conventional brushed DC motors, including higher efficiency, reliability, higher starting torque, reduced mechanical and electrical noises, and overall reduction of electromagnetic interference (EMI).

Recently, many modern control methodologies such as nonlinear control (Hemati *et al*., 1990), optimal control (Pelczewski and Kunz, 1990), variable structure control (Lin *et al*., 1999) and adaptive control (Cerruto *et al*., 1995) are applied to the motor control systems of diverse types including BLDC. However, these approaches are either theoretically complex or difficult to implement practically (Lin and Jan, 2002).

For these issues, conventional PID controller is most commonly used in industry owing to there merits of simple structure, high efficiency and easy implementation. But the optimally tuning gains of PID controllers have been quite difficult. Yu *et al*. (2004) have presented a LQR method to optimally tune the PID gains, Lin *et al*. (2003) have proposed Genetic Algorithm based PID control, and Kuo *et al.* (2008) have proposed a novel adaptive sliding mode control with PID tuning method for a class of uncertain systems. A partical swarm optimization (PSO) method for determining the PID controller parameters for speed control of BLDC motor has proposed by Nasri *et al*. (2007), LMI method for obtaining PID controller has introduced by Dobra (2003) and Cai *et al*. (2007). And also, BLDC control system implemented with the speed control and current control has been developed for the high performance of BLDC driver based on a digital incremental PID control algorithm using AVR microcontroller (Xu *et al*., 2008).

This paper proposes an enhanced PID controller design method with an internal feedback PD controller based on Kim *et al.* (2005, 2007) for BLDC motor. The proposed controller structure is organized as the convex set between PID controller and PI controller with an internal PD controller. Control parameters are determined through an optimum tuning method to improve the step response characteristics in the controllable set. The new PID structure and optimum tuning algorithm are applied to the speed control of BLDC motors.

This paper is organized as follows: the proposed enhanced PID control scheme is presented in Section 2. The design method for tuning PID controller parameters is discussed in Section 3. BLDC motor model is reviewed in Section 4. Finally, simulation results are presented in Section 5.

2. MULTI-LOOP PID CONTROL SCHEME

Figure 1 shows the structure of an enhanced PID control system which has the inner feedback compensation.

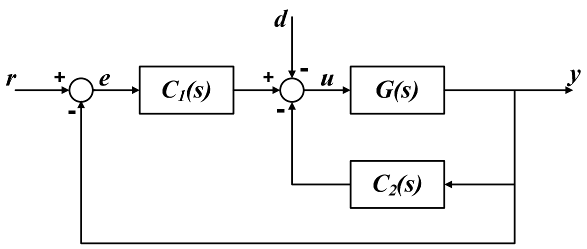

Fig. 1. Structure of Enhanced PID Control System

In Fig. 1, $G(s)$ is the uncertain plant model, $C_1(s)$ and $C_2(s)$ are the controllers, r is the reference input, u is the control command, y is the plant output, d is the disturbance signal applied to the system and e is the error defined as $e = r - y$.

The input-output relation is written in the form as

$$W_{ry}(s) = \frac{C_1(s)G(s)}{1 + \left(C_1(s) + C_2(s)\right)G(s)}. \tag{1}$$

If $C_2(s)$ and the inner feedback loop don't exist, the above structure is equal to the conventional PID control system. In accordance with the forms of $C_1(s)$ and $C_2(s)$, the system can be represent as PID-P, PI-PD or PI-D control system, and so on. It is considered PID and PI-PD controllers in this paper.

Figure 2 shows that the structure of multi-loop control system is equivalent to 2 degree-of-freedom (DOF) control system, when the controller $C(s)$ and the prefilter $F(s)$ are

$$C(s) = C_1(s) + C_2(s), \tag{2}$$

$$F(s) = \frac{C_2(s)}{C_1(s) + C_2(s)}. \tag{3}$$

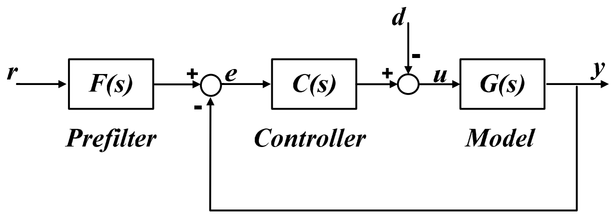

Fig. 2. Two DOF Control Systems

The transfer functions of $C_1(s)$ and $C_2(s)$ are PI controller and PD controller, respectively, as following:

$$C_1(s) = K^*_{p1}(1 + \frac{1}{T^*_{i1}s}), \tag{4}$$

$$C_2(s) = K^*_{p2}(1 + T^*_{d2}s). \tag{5}$$

From (2) ~ (5), controller $C(s)$ of 2 DOF systems is denoted as the PID controller

$$C(s) = K^*_p(1 + \frac{1}{T^*_i s} + T^*_d s), \tag{6}$$

where

$$K^*_p = K^*_{p1} + K^*_{p2}, \tag{7}$$

$$T^*_i = \frac{(K^*_{p1} + K^*_{p2})T^*_{i1}}{K^*_{p1}}, \tag{8}$$

$$T^*_d = \frac{K^*_{p2}T^*_{d2}}{(K^*_{p1} + K^*_{p2})} \tag{9}$$

and prefilter $F(s)$ is

$$F(s) = \frac{K^*_{p1}(T^*_{i1}s + 1)}{T^*_{i1}K^*_{p2}T^*_{d2}s^2 + T^*_{i1}(K^*_{p1} + K^*_{p2})s + K^*_{p1}}. \tag{10}$$

And also, the transfer functions of $C_1(s)$ and $C_2(s)$ is PID controller and PD controller, respectively, as following:

$$C_1(s) = K_{p1}(1 + \frac{1}{T_{i1}s} + T_{d1}s), \tag{11}$$

$$C_2(s) = K_{p2}(1 + T_{d2}s) \tag{12}$$

and the traditional PID controller means that

$$C_1(s) = K_p(1 + \frac{1}{T_i\, s} + T_d\, s), \tag{13}$$

$$C_2(s) = 0. \tag{14}$$

We will disregard the derivation of the controller $C(s)$ and the prefilter $F(s)$ for the respective PID-PD and PID controller because it can be derived from (2) ~ (3) easily.

3. CS-BASED PID-PD PARAMETER DESIGN

3.1 Convex Set of Linear Control System

In geometry of design specifications, H denotes the set of all closed-loop transfer matrices which is satisfying design specifications D and H denotes one element of transfer matrices set H. We think of H as the set of all conceivable candidate transfer matrices for the given plant. With each design specification D_i we associate the set H_i of all transfer matrices that satisfy it:

$$\mathrm{H}_i = \{H \in \mathrm{H} \mid \mathrm{H} \text{ satisfies } \mathrm{D}_i\}. \tag{15}$$

Table 1. Properties of design specifications and the corresponding sets of transfer matrices.

Design specifications	Sets of transfer matrices
H satisfies D	$H \in \mathrm{H}$
D_1 is stronger than D_2	$\mathrm{H}_1 \subseteq \mathrm{H}_2$
D_1 is weaker than D_2	$\mathrm{H}_1 \supseteq \mathrm{H}_2$
$\mathrm{D}_1 \wedge \mathrm{D}_2$	$\mathrm{H}_1 \cap \mathrm{H}_2$
D_1 is infeasible	$\mathrm{H}_1 = 0$
D_1 is feasible	$\mathrm{H}_1 \neq 0$

According the definition 1 of affine, a set of transfer matrices is affine if, whenever an affine combination of two distinct transfer matrices is in the set including these transfer matrices.

***Definition 1**: $\mathrm{H}_1 \subseteq \mathrm{H}$ is affine if for any $H, \widetilde{H} \in \mathrm{H}_1$, and any $\lambda \in R$, $\lambda H + (1-\lambda)\widetilde{H} \in \mathrm{H}_1$*, where if λ is restricted within $[0,1]$, the affine combination $\lambda H + (1-\lambda)\widetilde{H}$ is in the convex set.

***Definition 2**: A functional ϕ on H is affine if for any $H, \widetilde{H} \in \mathrm{H}$ and any $\lambda \in R$, $\phi(\lambda H + (1-\lambda)\widetilde{H}) = \lambda\phi(H) + (1-\lambda)\phi(\widetilde{H})$.*

***Definition 3**: A functional ϕ on H is convex if for any $H, \widetilde{H} \in \mathrm{H}$ and any $\lambda \in [0,1]$, $\phi(\lambda H + (1-\lambda)\widetilde{H}) \le \lambda\phi(H) + (1-\lambda)\phi(\widetilde{H})$.*

Boyd and Barratt (1991) applied the concept of affine and convex set to linear controller design. But it has two difficulties to apply these properties to system control.

One is that the formation or order of the controllers cannot be determined independently but relies on the plants. The unfixable controllers may cause not only the technical trouble in realization of the system controller, but also would became a high order controller unexpectively. Even though both controllers which consist of the convex set of the closed-loop transfer matrices are 3rd order, the system controller may turn out to be 9th order.

The second difficulty is that the system controller, which yields a closed-loop transfer matrix that is the affine combination of H and $\widetilde{H}$, would not have been found by varying the parameters in the controllers of H and $\widetilde{H}$. It means that the formulation itself of the controller should be changed every time to control a new plant.

Thus, we propose the new formulation of the multi-loop PID controller to implement the properties of the affine set and functional to controller design, being independent of the system plants.

In the design scheme, forming affine combination, two transfer functions H and $\widetilde{H}$ are respectively PI-PD and the conventional PID controllers

$$H(s) = \frac{C_{pi}(s)G(s)}{1+(C_{pi}(s)+C_{pd}(s))G(s)}, \qquad (16)$$

$$\widetilde{H}(s) = \frac{C_{pid}(s)G(s)}{1+C_{pid}(s)G(s)} \qquad (17)$$

We use the relative constant β for the proportional factors of PI-PD controller in order to unify denominators of transfer functions H and $\widetilde{H}$. Then, the correlative equation between PID and PI-PD controllers is

$$C_{pid}(s) = C_{pi}(s) + C_{pd}(s) \qquad (18)$$

and correlative coefficients are obtained directly

$$C_{pi}(s) = \frac{\beta}{1+\beta}K_p^*\left(1+\frac{1+\beta}{\beta T_i^* s}\right), \qquad (19)$$

$$C_{pd}(s) = \frac{1}{1+\beta}K_p^*(1+(1+\beta)T_d^* s) \qquad (20)$$

where K_p^*, T_i^* and T_d^* are the parameters of the PID controller $C_{pid}(s)$. To form the affine combination of two transfer functions H and $\widetilde{H}$ including PID and PI-PD controllers, PID-PD controller are found by varying the parameters of PI-PD and PID controllers

$$C_{PID}(s) = \frac{K_p^* + \beta K_p^* - \lambda K_p^*}{1+\beta} \times \left(1 + \frac{(1+\beta)(K_p^* T_d^* - \lambda K_p^* T_d^*)s}{(1+\beta-\lambda)K_p^*} + \frac{1+\beta}{(1+\beta-\lambda)T_i^* s}\right), \qquad (21)$$

$$C_{PD}(s) = \frac{\lambda K_p^*}{1+\beta}(1+(1+\beta)T_d^* s) \qquad (22)$$

3.2 Optimal Tuning of CS Based PID-PD Controller

Before tuning the parameters of the PID-PD controller, design specifications should be considered. Proposed in this paper, the controller forms the convex set which consists of the feasible controllers to meet the design specifications such as overshoot, rising time, and settling time. By the definition 2, the step response of the affine combination, $H_\lambda = \lambda H + (1-\lambda)\widetilde{H}$, is the convex functional ϕ_{sr} of which corresponding points via λ lie on a convex line passing through the points $(0, \phi_{sr}(\widetilde{H})), (1, \phi_{sr}(H))$. By this property, the functional convexities of design specifications are derived. Specially, the overshoot and rise-time functionals $\phi_{OS}(H_\lambda)$, $\phi_{rise}(H_\lambda)$ are convex.

We use two functionals of H for the specifications: the overshoot,

$$\phi_{OS}(H) \overset{\Delta}{=} \sup_{t\ge 0} h(t) - 1, \qquad (23)$$

And the rise-time,

$$\phi_{rise}(H) \overset{\Delta}{=} \inf\{T \mid h(t) > \alpha \ for\, t \ge T\} \qquad (24)$$

where $h(t)$ denotes the unit step response of the transfer function H.

These functionals on H are convex, so the functional inequality specifications

$$\mathrm{D}_{\mathrm{os}}^{(\mathrm{a}_1)} : \phi_{OS}(H) \le \alpha, \qquad (25)$$

$$\mathrm{D}_{\mathrm{rise}}^{(\mathrm{a}_2)} : \phi_{rise}(H) \le T_{\max} \qquad (26)$$

are convex.

When the design specifications are given by convex functional inequalities, we can design the controller by the convex optimization method.

The functional inequality specifications become the constraints of the classical optimization problem, indexed by the parameters $a_1, \cdots, a_L$, given by

$$\mathrm{D}^{(\mathrm{a}_1,\cdots \mathrm{a}_L)} : \mathrm{D}_{\phi_1}^{(\mathrm{a}_1)} \wedge \cdots \wedge \mathrm{D}_{\phi_L}^{(\mathrm{a}_L)}, \qquad (27)$$

which can be expressed as the shaded region of the achievable specifications or the set of feasible controllers in figure 3.

And the objective functional can be formed as a weighted-sum objective.

$$\phi_{obj}(H) = b_1\phi_1(H) + \cdots + b_L\phi_L(H) \tag{28}$$

where b_i are nonnegative numbers, called weights, which assign relative values among the functional ϕ_i .

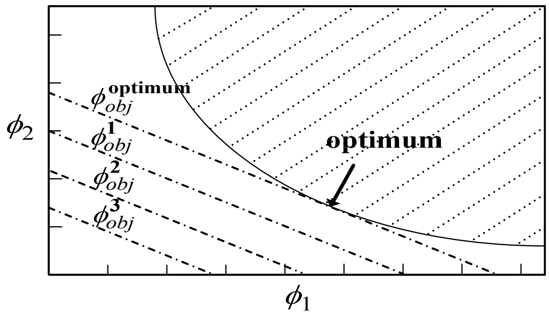

Fig. 3. Lines of constant objective functional and the region of the achievable performances.

In order to design the controller, first we bound the region of the feasible controllers on H to satisfy the constraints of the system specifications, then solve the optimization problem which is to find the optimum value λ of the weighted-sum objective functional.

$$\begin{aligned} &\text{minimize } \ \phi_{obj}(\lambda) = b_1\phi_1(\lambda) + \cdots + b_L\phi_L(\lambda) \\ &\text{subject to } \ \phi_1(H) \le a_1, \ldots, \phi_L(H) \le a_L \\ &\text{bounded to } \ o \le \lambda \le 1 \end{aligned} \tag{29}$$

When the convex set of H and $\widetilde{H}$ is fixed to satisfy the functional inequality specifications, through (21) and (22) the parameters of PID-PD controller are determined.

4. BRUSHLESS DC MOTOR

Mechanical rotors and brushes are commonly used to achieve the commutation for brushed DC motors. The stationary brushes come into contact with different sections of the rotating commutator. The commutator and brush system forms a set of electrical switches, such that electrical-power always flows through the armature coil closest to the stationary stator (permanent magnet). In BLDC motors, the electromagnets however do not move, but the permanent magnets rotate and the armature remains static. Instead of a mechanical commutation system based on brushes, Hall effect sensors are used as non-contact position sensors for BLDC motors. It detects the position of the rotors as the commutating signals.

The dynamic characteristics of BLDC motors are similar to permanent magnet DC motors. The characteristic equations of BLDC motors can be represented as Ong (1998). The electric and mechanical part of the motor are respectively described as

$$u_{app}(t) = Ri(t) + L\frac{di(t)}{dt} + u_{emf}(t) \tag{30}$$

$$J\frac{d\omega(t)}{dt} = \sum \tau_i \tag{31}$$

where $u_{app}(t)$ is the applied voltage, $i(t)$ is the current of the circuit, $u_{emf}(t)$ is the counter electromotive force, L is the inductance of the stator, and R is the resistance of the stator. Equation (31) means that the inertial load J times the angular acceleration is equal to the sum of all torques τ_i . A change in the magnetic environment of a coil causes a voltage (electromagnetic force)to be induced in the coil. This is the same principle for the eddy current sensor so that the back emf , $u_{emf}(t)$, is proportional to the angular velocity $\omega(t)$ and the produced torque $\tau(t)$ is proportional to the current $i(t)$ as the following (32) and (33)

$$u_{emf}(t) = K_b \cdot \omega(t) \tag{32}$$

$$\tau(t) = K_m \cdot i(t) \tag{33}$$

where K_b is the back electromotive force constant and K_m is the armature constant, which are both related to physical properties of the motor, such as magnetic field strength and number of turns of the coil. From (31) and (33), the following expression is derived

$$J\frac{d\omega(t)}{dt} = K_m i(t) - K_f\omega(t) \tag{34}$$

where K_f is the friction coefficient and the viscous friction $K_f\omega(t)$ is the result of friction in the motor.

These two differential equations (30) and (34) describing the system lead the characteristic equations in state-space representation,

$$\frac{di(t)}{dt} = -\frac{R}{L}i(t) - \frac{K_b}{L}\omega(t) + \frac{1}{L}u_{app}(t) \tag{35}$$

$$\frac{d\omega(t)}{dt} = \frac{K_m}{J}i(t) - \frac{K_f}{J}\omega(t) \tag{36}$$

The equations of the state-space model of BLDC motor can be rewritten as a linear combination of the state and input variables, so (35) and (36) become

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} &= \begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} \\ \frac{K_m}{J} & -\frac{K_f}{J} \end{bmatrix}\begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}u_{app}(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} \end{aligned} \tag{37}$$

where the current $i(t)$ and the angular velocity $\omega(t)$ are two states, input variable is the applied voltage $u_{app}(t)$, and output $y(t)$ is the angular velocity .

The transfer function of BLDC motor can be derived from the state-space representation from $u_{app}(t)$ to $\omega(t)$,

$$G(s)=\frac{W(s)}{U(s)}=\frac{K_m}{JLs^2+(RJ+K_fL)s+RK_f+K_bK_m} \quad (38)$$

5. COMPUTER SIMULATIONS

To show the effectiveness of the proposed CS (Convex Set) control method, a comparison is made with the CS-based PID controller and the existing PID controllers for BLDC motor.

The specifications of the BLCD motor are shown in table 2. The transfer function of the BLDC motor is obtained

$$G(s)=\frac{275577.36}{s^2+417.7s+43567.5} \quad (39)$$

Table 2. Specifications of the BLDC motor

Parameters	Values and units
R	$21.2\,\Omega$
K_b	0.1433 V s/rad
K_f	1×10^{-4} kg - m s/rad
L	0.052 H
K_m	0.1433 kg - m/A
J	1×10^{-5} kg - m s^2/rad

To control the speed of the BLDC motor at 1000 rpm, the comparable PID controllers are designed by using the various optimal PID tuning methods. For the optimal PID control using LQR approach, the weighting matrices Q and R are selected as

$$Q=\begin{bmatrix}100 & 0 & 0\\ 0 & 5000 & 0\\ 0 & 0 & 0\end{bmatrix} \quad (40)$$

$$R=1 \quad (41)$$

Through the algebraic Riccati equation, the optimal values k_p, k_i, and k_d are obtained as 70.566, 10 and 0.0212 respectively. Additionally, PSO-PID, pole placement-based LQ-PID (Suh and Yang, 2005), and GA-PID methods are considered for the comparison, the performances of which are listed in table 3.

Table 3. Performances of the comparative PID controllers

	LQR	LQ-PID	GA	PSO
P	70.556	72.423	93.162	190.018
I	10	14.515	38.623	50
D	0.022	0.0311	0.0278	0.0396
Tr(ms)	0.225	0.177	0.185	0.128
Mp(%)	17.9	8.9	15.6	16.8
Ts(ms)	1.1	1.14	0.982	0.69

To implement the proposed CS-based PID control, the convex set of controllers can be constituted to satisfy the requirements of a designer. In this paper, we chose the pole placement-based LQ-PID and PI-PD to meet the design specifications as the subsets $(\{H \mid \phi_{\mathrm{OS}}(H)\le 10\%\})$ and $(\{H \mid \phi_{\mathrm{settling}}(H)\le 0.2\text{ ms}\})$, where the poles of LQ-PID were assigned to be (-2 x 10^{-3}, -8 x 10^{3}, -3 x 10^{3}) and $\beta=80$ of PI-PD controller.

Remark: It is noted that the poles of LQ-PID controller is selected for satisfying the overshoot constraint condition as $\phi_{\mathrm{OS}}(H)\le 10\%$, and the $\beta=80$ of PI-PD controller is chosen in order to meet the settling time constraint condition as $\phi_{\mathrm{settling}}(H)\le 0.2\text{ ms}$.

Equation (42) shows the inequalities and the objective functional to solve the convex optimization problem.

$$\begin{aligned}&\text{minimize } \phi_{obj}(\lambda)=0.2\,\phi_{OS}(\lambda)+10^4\phi_{rt}(\lambda)\\ &\text{subject to } \phi_{OS}(H)\le 10\%,\ \phi_{st}(H)\le 0.2\,ms\\ &\text{bounded to } o\le\lambda\le 1\end{aligned} \quad (42)$$

where the relative weights are chosen to scale the each functionals ϕ_{OS}, ϕ_{rt} by the nominal value:

$$b_i=\frac{1}{\phi_i^{nom}} \quad (43)$$

where ϕ_i^{nom} represents some nominal value of the functional ϕ_i, which can be determined according to the designer's requirements. For this example we chose

$$\phi_{OS}^{nom}=\frac{\phi_{OS}^{\max}-\phi_{OS}^{\min}}{2}=5\%,\quad \phi_{rt}^{nom}=\frac{\phi_{rt}^{\max}-\phi_{rt}^{\min}}{2}=0.1\,\text{ms} \quad (44)$$

so that $b_1=0.2$ and $b_2=10^4$.

The optimal value of PID-PD control parameter λ is 0.2 to make the objective functional to be minimal. Fig. 4 shows the step responses of the CS-based PID-PD control system in accordance with λ, Fig. 5 shows the CS-based PID-PD response in comparison with LQR, GA, LQ-pole placement and PSO methods for PID control and Table 4 lists the performances of the proposed method in time domain and the values of the control parameters.

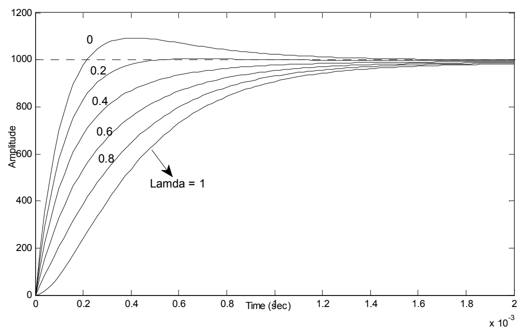

Fig. 4. According to changes, step responses of BLDC motor in CS-based PID-PD control system

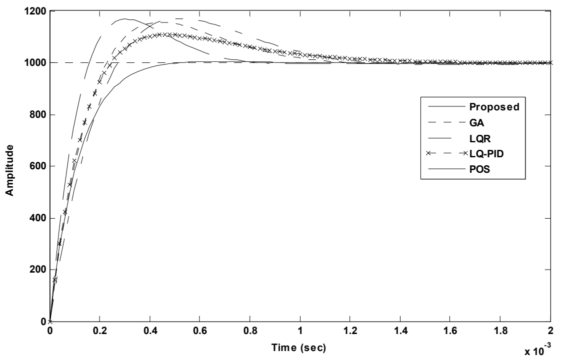

Fig. 5. Comparison between CS, LQR, LQ-pole placement, PSO and GA based PID control in speed control of BLDC motor

Table 4. Performances of the comparative PID controllers

CS-based PID-PD			
P	86.72	**P**	0.2146
I	0.0307	**D**	0.0077
D	0.1742		
Tr(ms)	0.2426		
Mp(%)	0.397		
Ts(ms)	0.3967		

Through the simulation results shown in Fig. 5, it is presented that the proposed CS-based PID-PD controller achieves the better speed performances of time domain comparably with the other methods in this case.

6. CONCLUSIONS

This paper presents a novel tuning formula that shapes the convex set between PID and PI-PD controller to deal with a good time domain performance. While the proposed method uses a more complicated PID controller with an inner PD control loop, it has the following advantages over existing traditional design methods due to useful properties of convex set. Firstly, all the tuning parameters of the optimal PID-PD controller are determined simply and analytically by only one control factor. Secondly, because the time responses of the controlled system are clearly expected according to variable changes in the convex set, it is easy to retune the controller, weights of object functional or the controller set constrained by design specifications. Thirdly, if only boundaries of convex set are stable, there are no specific requirements for closed loop control system to be stable even with nonminimum-phase, high degree or long dead time. It means that during tuning processes in the convex set, the closed-loop system stability is guaranteed. It is clear from the simulation results that the proposed design methodology is superior to the other comparative PID methods in the time domain performance, satisfying the design specifications, and simultaneously minimizing the value of the cost function

In this paper, the frequency properties of convex set which complementary sensitivity functions compose aren't considered. Hence, one of our ongoing research is how to select the optimum control value for extending the controller design from time domain to frequency domain to consider the sensibility and robustness of the system.

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