

Meta-heuristic Search Algorithms for Solving the Economic Load Dispatch Problem

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Abstract—Economic load dispatch (ELD) is a challenge optimization problem to minimize the total cost of the thermally generated power that satisfies a set of equality and inequality constraints. We need to maximize the power network load under several operational constraints to solve this problem. Meanwhile, we need to minimize the cost of power generation and minimize the loss in the network transmission. Traditional optimization methods were used to solve such problems as linear programming. Meta-heuristic search algorithms have shown encouraging performance in solving various real-life engineering problems. This paper attempts to provide a comprehensive comparison between nine meta-heuristic search algorithms, including Genetic Algorithms (GAs), Particle Swarm Optimization (PSO), Crow Search Algorithm (CSA), Differential Evolution (DE), Salp Swarm Algorithm (SSA), Harmony Search (HS), Sine Cosine Algorithm (SCA), Multi-Verse Optimizer (MVO), and Moth-Flame Optimization Algorithm (MFO) for solving the economic load dispatch problem. Our developed results demonstrated that meta-heuristics search algorithms (i.e., CSA and DE) offer the optimal power set for each power station. These computed power fulfill the supply needs and maintain both minimum power costs and power losses in power transmission.

Index Terms—Meta-heuristics Search algorithms, Economic Load Dispatch, Optimization.

I. INTRODUCTION

The Economic Load Dispatch (ELD) problem is broadly known as a challenging optimization problem [1], [2]. Numerous online units (i.e., generators) are accessible in the ELD problem. The goal is to determine the power to be generated by each unit to achieve the mandatory load at minimum total fuel cost. This power planning system can serve the energy distribution for numerous industrial, commercial, and residential sectors to enhance the network system accessibility. It is critical that the allocated power network be reliable and can fulfill the customer's needs and minimize power losses during transmission [3], [4].

In the past, traditional optimization methods were utilized to optimize the power distribution using linear programming (LP), dynamic programming (DP), and Lagrange multiplier (LM). It was reported that these methods suffer a problem when dealing with non-linear constrained optimization problems and are incapable of finding an optimal set of power weight for the network to maintain stability and reliability. In [5], the authors presented a method to solve multi-objective optimization procedures using LP. The LP technique with piecewise linearization provided an overall economic benefit. This was reported in terms of the total cost, time, and load fulfillment accuracy [6]. The dynamic ELD problem was also solved using Artificial Neural Networks trained with Levenberg Marquardt (LM) [7]. Other methods were used to solve the ELD problem such as algebraic method [8], Quadratic Programming (QP) [9]–[11], and Nonlinear Programming (NLP) [9]. It was reported that classical calculus-based methods were unable to perform adequately to solve ELD problems because of their highly non-linearity and several constraints. These traditional techniques were unsuccessful in dealing with the ELD problem with a non-convex fuel cost function.

The computational intelligence society has accepted Meta-heuristic search-based algorithms as a robust optimization method because of their ability to search the complex search space of real-world industrial applications. The main inspiration of meta-heuristic algorithms depends on the successful integration of the randomness and local search mechanisms in their approaches to find the optimal solutions [12]. Thus, they can achieve both exploration and exploitation in searching the space of possible solutions [12], [13].

Various Meta-heuristics algorithms were used to handle the ELD problem. In [13], the author presented a comparative study on solving the ELD problem of power systems using several meta-heuristic search algorithms.

These algorithms can find the best-generated load from each power unit that satisfies the distribution need while minimizing the cost. The bat algorithm was used to minimize the total generator cost from a thermal power plant in [14]. Bat algorithm has a stable convergence performance. In this study, the author shows that the Bat algorithm can save approximately 1.23% compared to the actual cost and 0.12% to the firefly algorithm. Several meta-heuristics search algorithms were used to solve the ELD problem [15]–[18]. Meta-heuristics show effectiveness alongside minimal computational requirements with a given ample search space. These valuable characteristics are often make successful in solving complex optimization problems.

The goal of this research is to provide a comparison between nine meta-heuristic search algorithms in the estimation of the distribution of thermal unit power over a power network to achieve the following 1) fulfill the power demand, 2) minimize the cost of the distributed power, and 3) minimize the loss of power in the transmission power systems. This paper is organized as follows. In Section III, we provide a mathematical formulation of the ELD problem. In Section IV, we provide a solution to the ELD problem for three case studies, and finally, we provide our conclusion.

II. METAHEURISTIC SEARCH ALGORITHMS

Meta-heuristic algorithms gained a lot of popularity over the last two decades. They are population-based approaches, as given in Figure 1. Noteworthy, some of them are well-known not only by computer scientists but also by scientists from other fields as Particle Swarm Optimization (PSO) [19], Ant colony optimization (ACO) [20] and Genetic Algorithm (GA) [21]. Metaheuristic search algorithms have been successfully used to solve a variety of problems in manufacture and control system design [13], [22]–[25]. The following points can summarize the reasons for this popularity:

- 1) Simplicity as they are inspired by simple concepts such as animal behaviors or evolutionary concepts.
- 2) Generality as those algorithms can adapt to any optimization problem without changing the algorithm structure itself, the inputs and outputs are only the essential parts, only you have to know how to redesign the problem for met-heuristics.
- 3) Better exploration-exploitation balancing as those algorithms have the ability to explore the search space heavily based on their stochastic nature, which supports avoiding local optima. They have also shown an outstanding accuracy in exploitation in many problems.

Meta-heuristics may be classified into the main classes:

- Evolutionary Algorithms (EAs)
- Swarm Algorithms (SAs)
- Physics-based Algorithms
- Bio-Inspired-based Algorithms

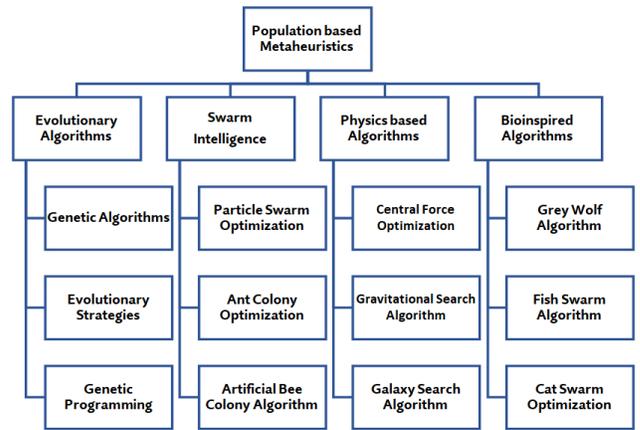


Fig. 1. Population based search algorithms

The concepts of evolution usually inspire EAs in nature. The most popular algorithm in this branch is GA [26]. This algorithm was proposed by Holland in 1992 and simulated Darwinian evolution concepts. Generally, optimization is done by evolving an initial random solution in EAs. Each new population is created by the combination and mutation of the individuals in the previous generation. Since the best individuals have a higher probability of generating the new population, the new population is likely to be better than the previous generation(s).

III. PROBLEM FORMULATION

The mathematical principle of the ELD problem depends on formulating the power cost as a minimization of an optimization function. The primary goal of the ELD problem is to decrease the generation cost of power distribution. For a particular thermal system that consists of n generators, the total generation cost is given in Equation 1.

$$\text{Min } L_T = \sum_{i=1}^n F_i(P_i) \quad (1)$$

where:

L_T	Total cost of power generation
F_i	Fuel cost of the i^{th} generating unit P_i
P_i	Power generated from the i^{th} generator
$\alpha_i, \beta_i, \gamma_k$	Weight coefficients of the i^{th} generation

Equation 1 can be formulated in a quadratic form as shown in Equation 2 [16], [27].

$$\text{Min } L_T = \sum_{i=1}^n F_i(P_i) = \sum_{i=1}^n \alpha_i P_i^2 + \beta_i P_i + \gamma_i \quad (2)$$

In this study, we consider two types of constraints; these constraints are referred to as equality constraints and inequality constraints.

A. Power Balance Constraints

It is critical that the power generator on the network can generate power that matches both the demand power and the expected power loss due to transmission. The power balance can be achieved as given in Equation 3.

$$\sum_{k=i}^n P_k = P_D + P_L \quad (3)$$

Where P_D and P_L are the demand and lost power due to transmission lines, respectively. We followed the guidelines in defining the loss coefficient as provided by *Kron and Kirchmayer* [28], [29]. ζ is a matrix known as *the transmission loss coefficients matrix* and utilized to recognize the power loss. Thus, P_L , the overall transmission lost power as a function of ζ , can be described as provided in Equation 4.

$$P_L = \sum_{i=1}^n \sum_{j=1}^n P_i \zeta_{ij} P_j \quad (4)$$

where:

- P_i Power generated from the i^{th} power unit
- P_j Power generated from the j^{th} power unit
- ζ Matrix of size $n \times n$
- n Number of power unit.

B. Inequality Constraints

Each power generator shall have a capacity such that it can generate maximum and minimum power. These constraints are defined, for the i^{th} power unit as given in Equation 5 [8].

$$P_i^{min} \leq P_i \leq P_i^{max}, i = 1, \dots, n \quad (5)$$

where P_i^{min} and P_i^{max} correspond to the minimum and maximum power of the i^{th} unit, respectively.

C. Cost Function

The meta-heuristic search algorithms adopted in this study shall be used to optimize the cost function of the generated power. This function can be presented as given in Equation 6.

$$L = \sum_{i=1}^n C_i(P_i) + \lambda \times \left[\sum_{i=1}^n P_i - P_D - P_L \right] \quad (6)$$

where $C_i(P_i)$ is the power's cost for the P_i generator, and λ is a penalty constant selected to penalize the losses when any power limits are over the allowable range. λ was arbitrarily chosen as 100 in our case.

IV. RESULT AND DISCUSSION

A. Optimization of Three Units Power System

In this section, we are adopting a three-unit system presented in [8]. Our goal is to find the optimal power generation from each $P_i, i = 1, \dots, 3$. We are using nine meta-heuristic search algorithms to find the optimal powers. The demand power required is 150 megawatts (MW). Table I shows the cost fuel coefficient of the three units system.

TABLE I
COST FUEL COEFFICIENT OF THE THREE UNITS SYSTEM

P_i	α_i (\$/MW ²)	β_i (\$/MW ²)	γ_i (\$/MW ²)	P^{min} (MW)	P^{max} (MW)
P_1	0.0080	7.00	200	10	85
P_2	0.0090	6.30	180	10	80
P_3	0.0070	6.80	140	10	70

$$\zeta = \begin{bmatrix} 0.000218 & 0.000093 & 0.000028 \\ 0.000093 & 0.000228 & 0.000017 \\ 0.000028 & 0.000017 & 0.000179 \end{bmatrix}$$

Table II shows the calculated power P_1, P_2, P_3 for the three-unit thermal system using Gas, PSO, CSA, DE, SSA, HS, SCA, MVO, and MFO. We also show the total processing power $\sum P_i$ compared to the demand load of 150 MW. The calculated cost from each algorithm is provided as \$/hr. From these results, we can see that both CSA and DE provided the minimum cost. The convergence performance of proposed search algorithms are shown for up to 500 iterations in Figure 2. This convergence curve characterizes the calculated fitness for 500 generations with 100 search agents (i.e., population size).

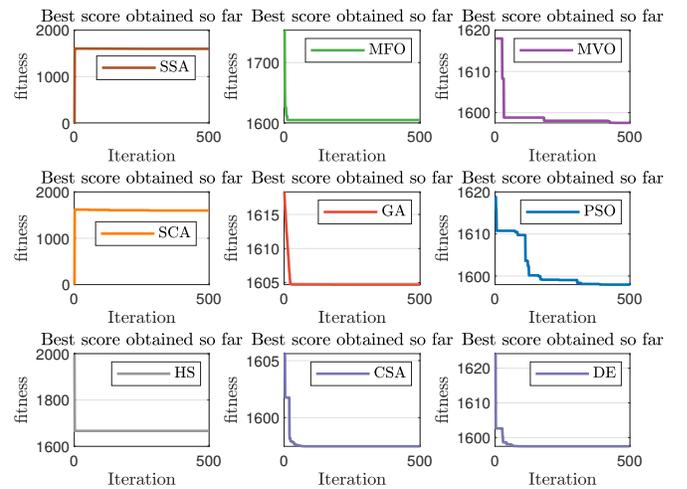


Fig. 2. Three Units System: Convergence of evolutionary process of several meta-heuristic search algorithms

TABLE II
OPTIMAL GENERATIONS POWER OF VARIOUS ALGORITHMS FOR A THREE UNIT SYSTEM

	SSA	MFO	MVO	SCA	GA	PSO	HS	CSA	DE
P_1	35.52	23.9847	32.6064	27.2953	57.3072	25.5111	53.4052	32.8112	32.8101
P_2	59.71	80	65.0719	67.4894	72.084	61.0358	56.4331	64.5944	64.595
P_3	57.07	48.5765	54.6698	57.5698	23.5098	65.7379	43.1074	54.9365	54.9369
$\sum P_i$	152.29	152.56	152.35	152.35	152.90	152.28	152.95	152.34	152.34
P_D	150.00	150.00	150.00	150.00	150.00	150.00	150.00	150.00	150.00
Cost (\$/hr)	1597.83	1600.93	1597.53	1598.12	1612.05	1599.11	1656.28	1597.48	1597.48

B. Optimization of a Six Units Power System

In this case, we studied six thermal power plant units and explored nine meta-heuristics search algorithms to find the optimal power for P_1, \dots, P_6 to minimize the overall generation cost (see Figure 5). The thermal unit cost coefficient is provided in Table III, and the coefficient matrix ($\zeta \times 10^{-3}$).

TABLE III
COST COEFFICIENT OF SIX UNITS SYSTEM

P_i	α_i (\$/MW ²)	β_i (\$/MW ²)	γ_i (\$/MW ²)	P^{min} (MW)	P^{max} (MW)
P_1	0.0070	7.0	240.0	100	500
P_2	0.0095	10	200.0	50	200
P_3	0.009	8.5	220.0	80	300
P_4	0.009	11	200.0	50	150
P_5	0.008	10.5	220.0	50	200
P_6	0.0075	12	190.0	50	120

$$\zeta = \begin{bmatrix} 0.017 & 0.012 & 0.007 & -0.001 & -0.005 & -0.002 \\ 0.012 & 0.014 & 0.009 & 0.001 & -0.006 & -0.001 \\ 0.007 & 0.009 & 0.031 & 0 & -0.010 & -0.006 \\ -0.001 & 0.001 & 0 & 0.024 & -0.006 & -0.008 \\ -0.005 & -0.006 & -0.010 & -0.006 & 0.129 & -0.002 \\ -0.002 & -0.001 & -0.006 & -0.008 & -0.002 & 0.150 \end{bmatrix}$$

The performance of the suggested meta-heuristic search algorithms is shown for 500 generations in Figure 3. Table IV provides the optimal generation power of various algorithms that minimize the distribution cost for the six power generators. CSA and DE provide the best evolutionary power distribution.

C. Planning IEEE 30 Bus System

The IEEE 30 bus is one of the standard power systems that consists of six thermal power plant units. The goal is to evolve the optimal power to be generated from the six thermal units (see Figure 5). The IEEE 30 bus system diagram is shown in Table V and the coefficient matrix ($\zeta \times 10^{-3}$) is also below.

$$\zeta = \begin{bmatrix} 0.1400 & 0.0170 & 0.0150 & 0.0190 & 0.0260 & 0.0220 \\ 0.0170 & 0.0600 & 0.0130 & 0.0160 & 0.0150 & 0.0200 \\ 0.0150 & 0.0130 & 0.0650 & 0.0170 & 0.0240 & 0.0190 \\ 0.0190 & 0.0160 & 0.0170 & 0.0710 & 0.0300 & 0.0250 \\ 0.0260 & 0.0150 & 0.0240 & 0.0300 & 0.0690 & 0.0320 \\ 0.0220 & 0.0200 & 0.0190 & 0.0250 & 0.0320 & 0.0850 \end{bmatrix}$$

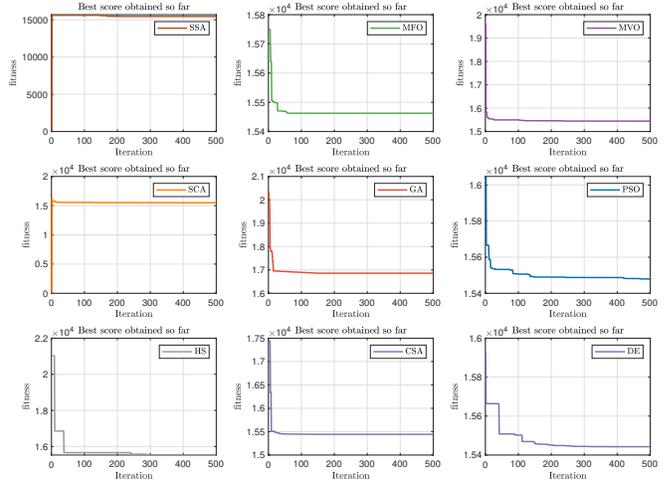


Fig. 3. Six Units System: Convergence of evolutionary process of several meta-heuristic search algorithms

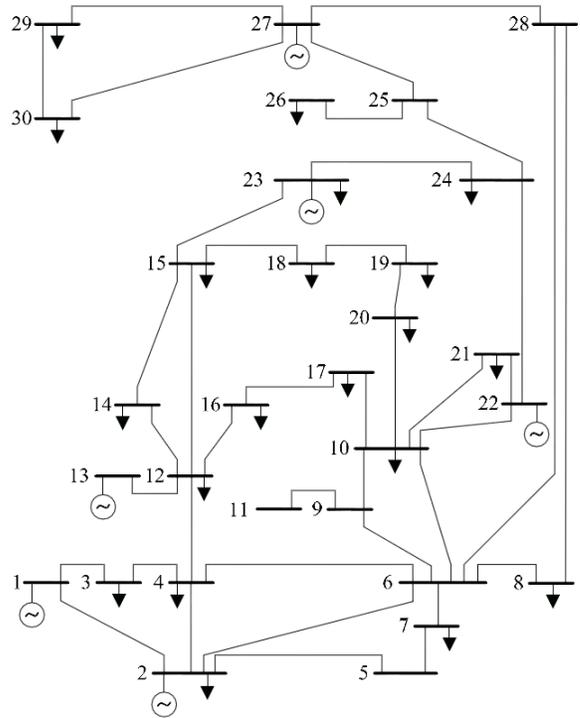


Fig. 4. IEEE 30 Bus consisting of six generators test system

TABLE IV
OPTIMAL GENERATIONS POWER OF VARIOUS ALGORITHMS

Power	SSA	MFO	MVO	SCA	GA	PSO	HS	CSA	DE
P_1	443.5555	500	452.8114	417.6776	288.9704	471.9346	409.6441	446.9736	447.5787
P_2	173.5464	200	182.2327	200	315.0599	187.8771	193.7739	173.319	173.0238
P_3	269.286	236.0095	263.0214	300	116.3195	272.913	285.2534	263.7248	263.9873
P_4	131.7837	150	135.484	150	160.0623	140.4767	147.215	138.9444	139.1728
P_5	182.4648	128.8926	152.5703	134.1542	259.6067	100.0305	124.6418	165.6265	165.0263
P_6	75.15023	60.3902	89.25042	73.43134	137.9263	102.2144	114.7046	86.8287	86.62046
$\sum P_i$	1275.787	1275.292	1275.37	1275.263	1277.945	1275.446	1275.233	1275.417	1275.409
P_D	1263	1263	1263	1263	1263	1263	1263	1263	1263
Cost (\$/hr)	15447.54	15498.2	15445.69	15488.04	16135.5	15494.94	15490.75	15442.66	15442.66

TABLE V
COST COEFFICIENT OF IEEE 30 BUS SYSTEM

P_i	α_i (\$/MW ²)	β_i (\$/MW ²)	γ_i (\$/MW ²)	P^{min} (MW)	P^{max} (MW)
P_1	15.240×10^{-2}	38.53973×10^2	756.79886	10	125
P_2	10.587×10^{-2}	46.15916×10^2	451.32513	10	150
P_3	2.803×10^{-2}	40.39655×10^2	1049.9977	35	225
P_4	03.546×10^{-2}	38.30553×10^2	1243.5311	35	210
P_5	2.111×10^{-2}	36.32782×10^2	1658.5596	130	325
P_6	1.799×10^{-2}	38.27041×10^2	1356.6592	125	315

In Table VI, we provide the optimal generation power P_i of various algorithms that minimize the distribution cost for the six power generators. CSA and DE-based methods offer the best performance results for the six units P_1, \dots, P_6 . The convergence curves of the nine meta-heuristic search algorithms for the power load evolution are shown in Figure 5 over 500 generations and population size of 100.

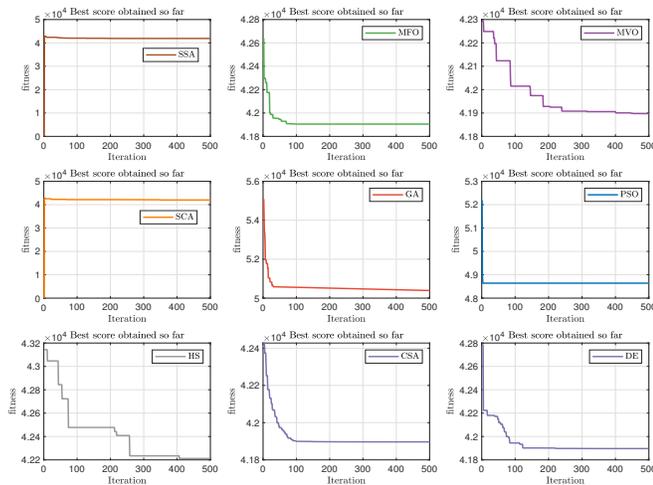


Fig. 5. IEEE 30 Bus: Convergence of evolutionary process of several meta-heuristic search algorithms

V. CONCLUSIONS

In this paper, we compared several meta-heuristic search algorithms to obtain the optimal power distribution of three thermal power systems such that the cost of distribution is minimal. It includes Genetic Algorithms, Particle

Swarm Optimization, Crow Search Algorithm, Differential Evolution, Salp Swarm Algorithm, Harmony Search, Sine Cosine Algorithm, Multi-Verse Optimizer, and Moth-Flame Optimization Algorithm. Our finding was that differential evolution and crow search algorithms demonstrated the best performance in solving the economic load dispatch problems by providing the minimum distribution cost.

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TABLE VI
OPTIMAL GENERATIONS POWER OF VARIOUS ALGORITHMS

Power	SSA	MFO	MVO	SCA	GA	PSO	HS	CSA	DE
P_1	31.3561	27.71748	33.05922	23.49077	120.4477	125	21.56365	32.59279	32.60999
P_2	13.29429	21.9596	13.88867	14.34295	103.4747	125	105.1503	14.49964	14.36192
P_3	144.0152	130.5341	139.9768	135.2147	115.2445	125	204.1934	141.6495	141.6919
P_4	135.9548	125.1346	134.5273	157.3899	124.2421	125	100.7099	136.0354	135.7933
P_5	263.4618	255.0508	262.7322	262.9998	124.5823	125	193.4693	257.5987	257.7318
P_6	237.2559	265.3226	241.2151	231.2804	121.2529	125	196.193	242.9515	243.1477
$\sum P_i$	825.338	825.7192	825.3993	824.7184	709.2442	750	821.2795	825.3275	825.3366
P_D	800	800	800	800	800	800	800	800	800
Cost (\$/hr)	41898.68	41925.3	41897.52	41962.89	50023	48639.58	43143.66	41896.63	41896.63

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