Three-phase Distribution OPF in Smart Grids: Optimality versus Computational Burden

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***Abstract*—Existing Mixed Integer Non-linear Programming (MINLP) solution methods and commercially available solvers lack computational efficiency and robustness in solving three-phase Distribution Optimal Power Flow (DOPF) programs, given the large number of continuous and integer variables encountered in practical sized systems. A heuristic approach to solve this problem was proposed by the authors, in which a compromise is made on optimality in order to reduce the computational burden. In the present work, a Genetic Algorithm (GA) based method is applied to determine the optimal solution to the three-phase DOPF problem, and is compared with the heuristic solution in terms of both optimality and computational burden. Two distribution feeders, namely, the IEEE 13-node feeder and a practical feeder from Hydro One are used for these comparisons. The results show that the GA-based method yields superior solutions in terms of optimality but at a rather large computational cost, making it unsuitable for practical implementation. The heuristic method is shown to yield solutions reasonably close to the global optima at a significantly reduced computational burden, demonstrating that the heuristic solution method has the potential to improve distribution system operation in practical real-time applications.**

***Index Terms*—Unbalanced distribution systems, real-time operation, optimal power flow, smart grids, genetic algorithms.**

I. Nomenclature

α, β, γ	Scalar weights of the objective function components.
ω	Set of integer variables considered as continuous.
C	Set of controllable capacitor banks.
cap	Number of capacitors switched in capacitor banks.
h	Hours, $h = 1, 2...24$.
J, J'	Objective functions.
K	A constant multiplier.
n	Set of integer variables.
p	Set of phases a, b, c.
$Psub$	Power drawn from substation.
t	Set of controllable tap changers.
tap	Tap position.

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II. Introduction

THE IMPLEMENTATION of real-time information systems, Advanced Metering Infrastructure (AMI), improved communication capabilities, and improved infrastructure for control systems is envisaged to transform the existing distribution systems into "Smart Grids" [1]. These Smart Grids, because of environmental concerns and incentives from regulators, are expected to accommodate high levels of penetration of Distributed Energy Resources (DERs) and Plug-in Hybrid Electric Vehicles (PHEVs) [2]. Furthermore, dynamic pricing schemes and incentives from utilities will encourage the implementation of Energy Hub Management Systems (EHMSs) at customers' premises [3], which can facilitate their participation in utilities' demand side management, demand response, and demand control programs.

Figure 1 presents the schematics of a distribution feeder in Smart Grids with its various components and features, illustrating the scope of the present work [4]. In such environment, real-time information systems will allow customers access to information such as energy price, emissions, incentive signals, and weather. These data are essential components of the customers' EHMSs to optimize electricity usage while fully accounting for their preferences. With AMI and communication technologies, Local Distribution Companies (LDCs) can gather information on distribution system status and customers' load profiles optimized by their EHMSs. Such an environment in Smart Grids would allow flexibility in distribution system operation via coordinated and centralized control of its components such as Load Tap Changers (LTCs), Switched Capacitors (SCs), switches, controllable loads, DERs, PHEVs, etc. [5]. Moreover, this infrastructure allows real-time control of distribution system components with a variety of operational objectives related to economy, efficiency, reliability, environmental concerns, etc. [6], which is an improvement over predominantly existent volt/var control schemes based on local measurements. A generic three-phase DOPF is at the core of such centralized optimization-based control for improved operation of distribution feeders [4].

Accurate and comprehensive modeling of components and efficient computational methods are critical requirements for real-time control of distribution feeders. The large number of nodes, components and measurements encountered in practical distribution systems lead to significant data handling requirements and extensive computational burden,

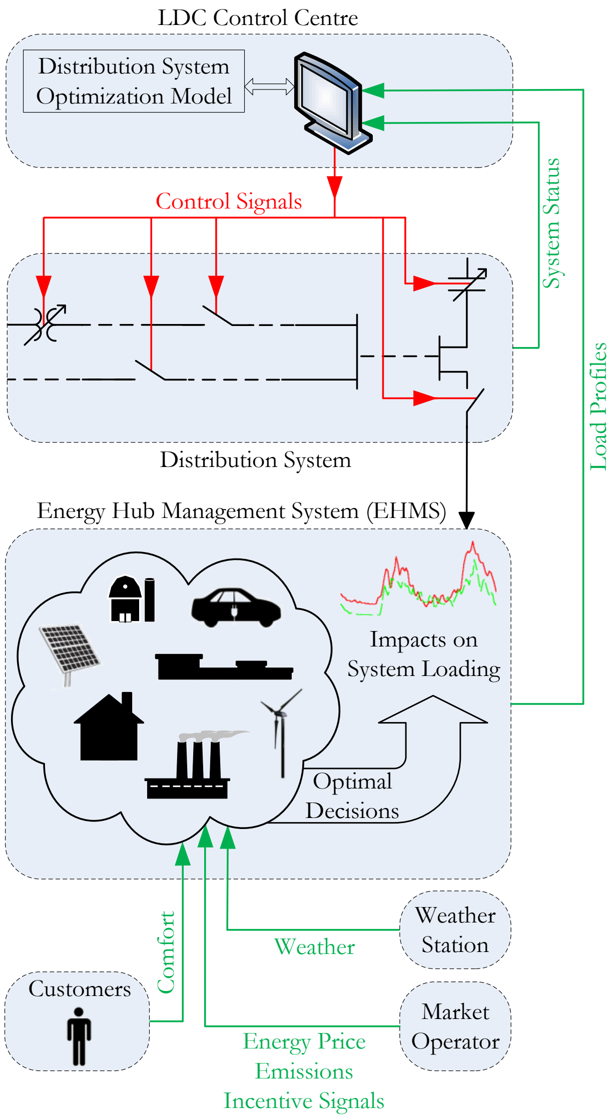

Fig. 1. Schematic diagram of a distribution system in Smart Grids.

rendering the real-time continuous control of distribution system components practically impossible [6]. However, a real-time analysis at 15 to 30 minute intervals at normal operating conditions, are more manageable from a practical stand-point [6], [7]. Computational burden in most of the distribution system analyses is reduced by assuming the distribution system to be a balanced three-phase system, and hence considering a single-phase equivalent model [8]–[10]. However, these models are not suitable for precise real-time operation and control applications, because of existence of untransposed three-phase feeders, single-phase laterals, and single-phase loads. A comprehensive three-phase feeder model with phase specific and voltage dependent load models need to be considered, as is the case of [4] and this paper.

In distribution system optimization problems, a distribution load flow (DLF) [11], [12], and any of the MINLP solution methods discussed in [13]–[18], can be readily implemented to solve the three-phase DOPF problems. Although the computational time for three-phase DLF problems reported in [11] and [12] are promising for real-time applications, issues pertaining to computational robustness and burden of MINLP solution methods are the main challenges for solving the three-phase DOPF for real-time control purposes, as discussed in [4]. This complexity increases substantially for a 24-hour horizon because of the increased number of variables and presence of inter-temporal constraints. The commercially available solvers (particularly BARON and DICOPT) were found to be computationally inefficient in [4], both in terms of robustness and CPU time, making them unsuitable for practical applications. The heuristic solution method proposed in [4] results in a sub-optimal solution in which a compromise is made between optimality and computational cost to render the solution process suitable for real-time applications.

Based on the aforementioned discussions, the main objectives of the present work can be summarized as follows:

- A GA-based three-phase DOPF solution method is developed, implemented and tested to evaluate its performance in terms of the optimal solution and computational burden based on case studies carried out on two distribution systems: IEEE 13-node test feeder and a Hydro One distribution feeder.
- To compare the performance of the GA based method with the previously proposed heuristic solution method proposed in [4], in terms of computational burden and optimality of the solution.

The rest of the paper is organized as follows: Section III describes the three-phase DOPF model, the heuristic solution method, and the GA-based solution method. Section IV presents and discusses the results of the various case studies carried out for two distribution systems to evaluate the heuristic method in terms of real-time applications. A summary of the presented work and the main contributions of this paper are discussed in Section V.

III. Optimization Model

A. Distribution System Component Models

In this work, distribution system components such as conductors/cables, transformers, LTCs, and switches are modeled using ABCD parameters. Single-phase, two-phase, three-wire three-phase, and four-wire three-phase conductors and cables are also represented. Various common distribution system transformers are modeled, such as single-phase and three-phase wye grounded-wye grounded, delta-wye grounded, and open wye-open delta connections. Single-phase LTCs and wye-connected three-phase LTCs are modeled with individual phase control and group control options [4], [19].

Shunt components (loads and capacitors) are modeled for individual phases separately to represent unbalanced three-phase loads, since single-phase loads and single phase capacitors are common in distribution feeders. A polynomial load model is adopted, where each load is modeled as a mix of constant impedance, constant current, and constant power components. Capacitors are modeled as constant impedance loads. Capacitor banks are modeled as multiple units of SCs. Wye-connected and delta-connected loads and capacitors are represented [4], [19].

B. Three-phase Distribution Optimal Power Flow Model

The developed model is a generic optimization model, where any objective function can be selected for distribution system operations. However, to demonstrate its applicability in distribution system optimal operation, the novel objective function proposed in [4] is considered in this work as well, which is a weighted function to minimize energy drawn from the substation and the number of switching operations of the LTCs and SCs. This function is defined as follows, based on the nomenclature defined in Section I:

$$J = \alpha \sum_{h} Psub_h + \sum_{p}\sum_{t}\left(\beta_t \sum_{h=2}^{24}\left|tap_{p,t,h} - tap_{p,t,h-1}\right|\right) + \sum_{p}\sum_{C}\left(\gamma_C \sum_{h=2}^{24}\left|cap_{p,C,h} - cap_{p,C,h-1}\right|\right) \quad (1)$$

The parameters α, β_t, and γ_C are the weights attached to energy drawn from the substation, LTC switchings and SC switchings, respectively. Selection of these weights depends on the level of priority attached to these components by the distribution system operator.

The mathematical models of distribution system components described in the previous section constitute the equality constraints of the three-phase DOPF. In addition to these component models, voltage and current balance equations at each node and in each phase are additional equality constraints required in the modeling of three-phase DOPF. Distribution system operating limits such as voltage limits, feeder current limits, transformer capacity limits, etc., constitute the inequality constraints. The detailed mathematical modeling of three-phase DOPF is discussed in [4].

C. Heuristic Solution Method

This heuristic method for solution of the three-phase DOPF proposed by the authors [4], is based on penalty functions and local searches to reduce the computational burden of the solution process. Thus, the three-phase DOPF with integer variables is transformed into a non-linear programming (NLP) problem using an additional quadratic penalty term in the objective function, as in [9], [10]. This transforms the objective function (1) as follows:

$$J' = J + \sum_{n} K_n \left(\omega_n - round(\omega_n)\right)^2 \quad (2)$$

Hence, the quadratic term adds a high penalty to the objective function for non-integer solutions, and thus drives the variable ω_n towards its closest integer solution $round(\omega_n)$. The parameter K_n needs to be carefully selected to obtain an optimal integer solution to the NLP problem [9], [10].

Commercially available NLP solvers do not guarantee reaching a feasible solution of the NLP-DOPF problem

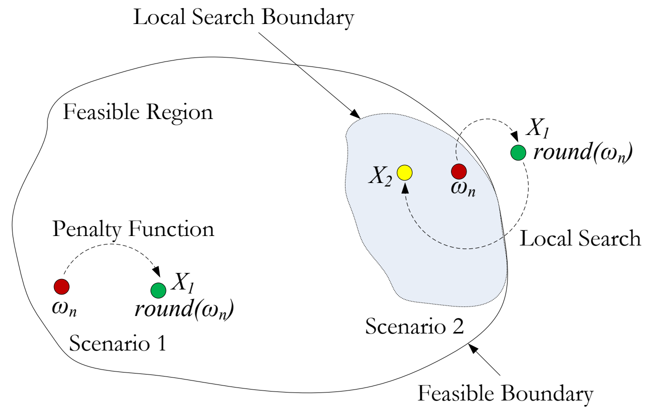

Fig. 2. Optimal and infeasible cases encountered in the solution process based on a quadratic penalty function method.

because of the presence of the discontinuous quadratic penalty term in (2). In Fig. 2, Scenario 1 depicts a case when the optimal integer solution X_1 is obtained using the quadratic penalty function; to obtain an optimal integer solution, both ω_n and $round(\omega_n)$ must lie inside the feasible region of the optimization problem. It is possible that $round(\omega_n)$ may lie outside the feasible region, as depicted in Scenario 2 in Fig. 2; this could happen, in particular, when ω_n is close to the boundary of the feasible region. To address this problem, a local search technique is proposed in [4] to ensure that integer solution X_2 is in the feasible space of the optimization problem.

The proposed three-phase DOPF, with the NLP approximation and local search procedure, is still computationally intensive because of the size of the search space resulting from the 24-hour time-frame. To reduce computation time, an hourly local search approach is implemented which reduces the search space substantially. In this process, mathematical precision is somewhat compromised for the sake of reducing computational burden. In practical applications this is a reasonable sub-optimal approach that allows for the implementation of the proposed technique in real-time. A detailed flowchart and discussion of the proposed heuristic solution method is provided in [4].

D. Genetic Algorithm based Solution Method

A GA-based method, similar to the one discussed in [14], was implemented to solve the three-phase DOPF, so that comparisons of optimal solutions and associated CPU times can be made with the heuristic method. Figure 3 depicts a pseudo-code of the solution method, the parameters and steps of which are briefly discussed next. The details of a generic GA-based solution method can be found in [20]. In Fig. 3, the following are the main parameters:

- *Generations (G):* The proposed GA-based method is set for 100 generations.
- *Chromosome (X):* The controllable variables *tap* and *cap* associated with LTCs and SCs in the three-phase DOPF

```
begin
define X, F
choose G, S, CR, MR
choose individuals of S
solve three-phase DLF for individuals in S
evaluate F for individuals in S
  repeat
  rank individuals of S based on F
    repeat
    create a pool of individuals, based on CR, for mating
    choose parents based on two random numbers
    apply cross-over operator to reproduce offspring
    apply mutation operator to offspring based on MR
    solve three-phase DLF for offspring
    evaluate F for offspring
    discard unfit offspring
    until S number of offspring are reproduced
  choose best S fit individuals among parents and offspring
  until generation G is reached
end
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Fig. 3. A pseudo-code illustrating the GA-based solution method for the three-phase DOPF.

model are integer variables, each variable is represented by a chromosome which is a 6-bits binary number.

- *Population Size (S):* A population size of 25 individuals are considered, where individuals mean binary values assigned to the chromosomes representing all *tap* and *cap* variables for 24 hours. To start, 25 individuals that satisfy the equality and inequality constraints of the three-phase DOPF are chosen arbitrarily.
- *Fitness Function (F):* The objective function in (1) is used to evaluate the fitness of initial population and offsprings. To avoid infeasible cases, any offspring that does not satisfy the equality and inequality constraints is assigned a very high value of F and considered unfit.
- *Cross-over and Mutation:* A cross-over rate (CR) of 80% and a mutation rate (MR) of 1% are used. Figure 4 depicts a two-point cross over (after 2^{nd} and 4^{th} bits) and the mutation operators employed.

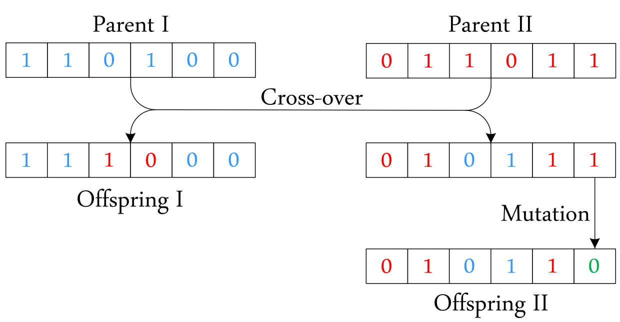

Fig. 4. A two-point cross-over and mutation operator in GA.

IV. Case Studies

The three-phase DOPF model, the heuristic method, and the GA-based method were implemented in GAMS [21], a high-level optimization modeling tool. Both the heuristic and GA-based methods require solutions to the three-phase DLF, which is an NLP problem; for this purpose, commercially available MINOS solver was used [22].

A. IEEE 13-node Test Feeder

The IEEE 13-node test feeder, as shown in Fig. 5 [23], is considered first to compare the performance of the heuristic, and the GA-based methods in solving three-phase DOPF problems. To demonstrate their applicability in a 24-hour time-frame, the load data provided in [23] are assumed to be peak loads and the load profile reported in [24] is used.

The capacitors available in the IEEE-13 node test feeders are single units with fixed values. Hence, to demonstrate the applicability of the proposed method considering SCs, the given capacitor data are modified, assuming that five blocks of 100 kVar capacitors are connected at node 675 in each phase, and five blocks of 50 kVar capacitors are connected at node 611 in phase c. The LTC and the two capacitor banks are considered to be controllable. In the proposed objective function (1), equal weights are assigned to the switching operations of LTC and capacitors and are considered complementary to the weight attached to the energy drawn from the substation, i.e.,

$$\beta_1 = \gamma_1 = \gamma_2 = 1 - \alpha \tag{3}$$

The GA-based solution method, which uses the complete MINLP model of the IEEE-13 node test feeder in a 24-hour timeframe, involves 9,792 continuous and 168 controllable integer variables. The NLP model used in the heuristic method requires solutions to 9,960 continuous variables. In the heuristic solution method, the hourly search technique narrows down the search space to 192 combinations from 4.72×10^{21}, which would have otherwise been required in case of a 24-hour search.

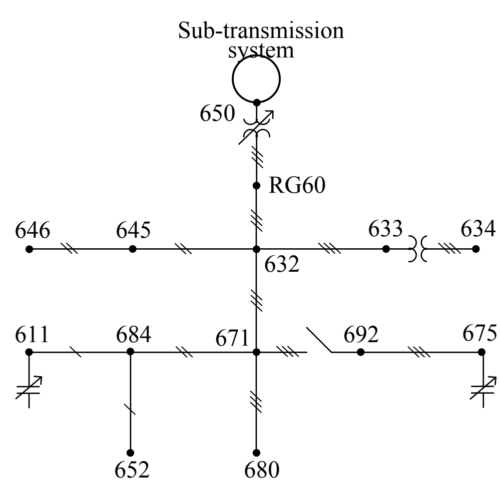

Fig. 5. IEEE 13-node test feeder [23].

TABLE I
SIMULATION RESULTS IN IEEE 13-NODE TEST FEEDER

		Heuristic Solution Method				GA-based Solution Method					% Difference
Case	α	Energy (MWh)	No. of Switchings	Objective Function	Solution Time	Generations	Energy (MWh)	No. of Switchings	Objective Function	Solution Time	in Objective Function
1	1	62.89	30	62.89	4m46s	1	68.88	24	68.88	3m52s	9.53
						25	62.20	35	62.20	53m47s	-1.09
						50	61.81	47	61.81	1h47m32s	-1.71
						100	61.23	88	61.23	3h44m51s	-2.63
2	0.2	67.08	20	29.42	3m39s	1	62.55	73	70.91	3m46s	141.06
						25	65.09	42	46.62	53m14s	58.48
						50	67.86	26	34.37	1h46m58s	16.85
						100	68.18	18	28.04	3h28m36s	-4.69
3	0	67.73	12	12.00	3m11s	1	62.93	76	76.00	3m44s	533.33
						25	63.75	51	51.00	54m19s	325.00
						50	66.81	28	28.00	1h46m32s	133.33
						100	67.85	12	12.00	3h28m39s	0.00

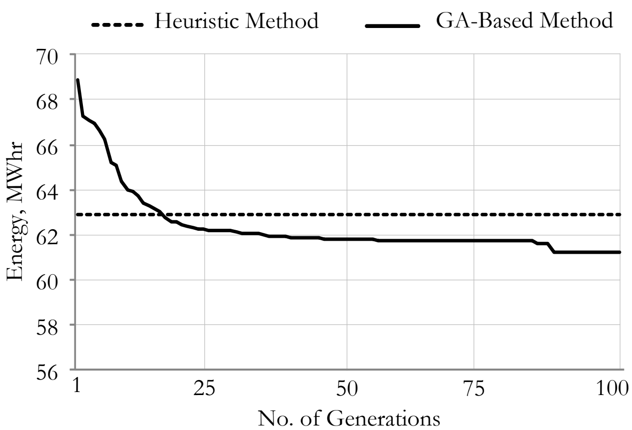

Fig. 6. Comparison of the heuristic and GA-based solution methods in IEEE 13-node test system for α = 1.

First, α is set to 1, which represents minimization of the energy drawn from the substation. The optimal energy drawn from the substation obtained using the heuristic method is 62.89 MWh, while in the GA based method, improved solutions are obtained over the generations, starting from 68.88 MWh after the 1^{st} generation to 61.23 MWh at the end of the 100^{th} generation. Observe in Fig. 6 that the GA-based method starts to yield better solutions after the 16^{th} generation compared to the heuristic method, but requires 39m 17s to complete these 16 generations, which is not a suitable timeframe for real-time applications. Over the subsequent generations, the GA based method yields better solutions as compared to the heuristic method but at rather large computational cost. For example, after the 50^{th} generation, the optimal solution is improved by 1.71%, as compared to the heuristic method, but requires 1h 47m 32s to arrive at this solution. On the contrary, the heuristic method yields a reasonable solution in 4m 46s, which is suitable for real-time applications, and the solution is close to that obtained from the GA based method, since the difference in optimality is only 2.63%.

Table I summarizes the results from the simulation cases considering different value of α. For α = 0.2, which represents a weighted sum of energy and switching operations, the GA-based method requires 3h 28m 36s to arrive at the best solution but the difference in optimal value is only 4.69%, compared to the heuristic solution. For α = 0, which represents the minimization of switching operations, the GA-based method does not yield any solution better than the one obtained from the heuristic method. These results show that the solutions are obtained in much less time (not more than 5m) using the heuristic method, and the solutions are reasonably close to the GA-based method, since the differences in optimality are not more than 4.69%. It is to be noted that the performance of the GA-based method depends on various factors such as the selection of the initial pool of population, cross-over rate, mutation rate, stopping criteria, etc. In general the GA-based method indeed yields a global optimal solution, but because of the tremendous computational effort involved, such solutions are not useful from a practical stand-point, particularly for real-time operational and control purposes.

B. Hydro One Distribution Feeder

Simulations are also carried out considering a practical 41-node unbalanced distribution feeder, which is part of the distribution network of Hydro One Inc. [24]. The system configuration is shown in Fig. 7.

The available load data are considered to be peak loads, and 24-hour load profiles at each node are defined using the same procedure used in the previous test system. In this case, a constant impedance load model is considered, determined from available active and reactive power demands at nominal voltage.

The system has three, three-phase transformers and a single phase transformer. It is assumed that all the three-phase transformers are equipped with LTCs, and are the only controllable devices in the network. As in the previous example, equal weights are attached to the

TABLE II
SIMULATION RESULTS IN HYDRO ONE DISTRIBUTION FEEDER

		Heuristic Solution Method				GA-based Solution Method					% Difference
Case	α	Energy (MWh)	No. of Switchings	Objective Function	Solution Time	Generations	Energy (MWh)	No. of Switchings	Objective Function	Solution Time	in Objective Function
1	1	286.98	46	286.98	10m5s	1	293.80	123	293.80	5m7s	2.38
						25	291.26	164	291.26	1h8m54s	1.49
						50	291.03	147	291.03	2h37m51s	1.41
						100	283.63	139	283.63	5h17m29s	-1.17
2	0.6	293.79	42	193.08	9m4s	1	292.39	136	229.83	5m38s	19.04
						25	292.39	136	229.83	1h21m50s	19.04
						50	292.03	114	220.82	2h42m22s	14.37
						100	292.19	42	192.11	5h34m14s	-0.50
3	0	293.99	32	32.00	7m9s	1	292.39	136	136.00	5m50s	325.00
						25	293.39	128	128.00	1h27m41s	300.00
						50	292.84	57	57.00	2h46m10s	78.13
						100	295.67	30	30.00	5h55m57s	-6.25

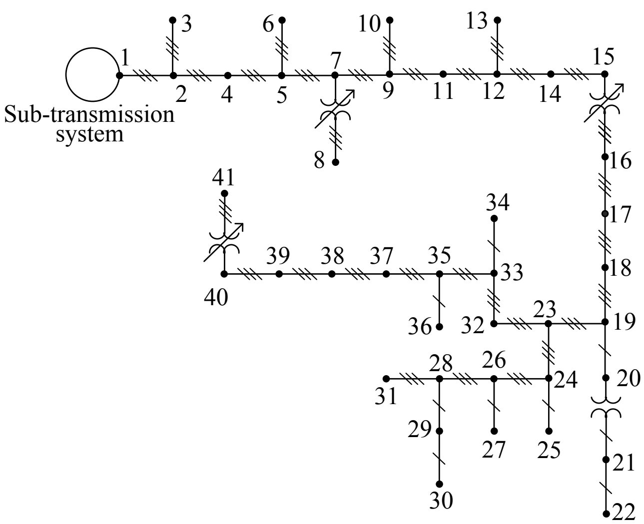

Fig. 7. Hydro One distribution feeder.

switching operations of LTCs and are assumed to be complementary to the weight attached to the energy drawn from substation, as follows:

$$\beta_1 = \beta_2 = \beta_3 = 1 - \alpha \tag{4}$$

The complete MINLP model of the Hydro One feeder in a 24-hour timeframe involves 26,784 continuous and 216 controllable integer variables in the GA based method, while the NLP model used for the heuristic method involves 27,000 continuous variables. By applying the hourly search technique in the heuristic method, the search space is narrowed down to 192 combinations from the 4.72×10^{21} required in case of a 24-hour search.

Table II presents the results from the simulation cases considering different values of α. The heuristic method requires larger computational times to arrive at the optimal solution as compared to the IEEE 13-node test feeder; however, the maximum computational time required is about 10m, which is significantly less compared to the GA-based method and within a reasonable timeframe for real-time applications. Also, the optimal solutions obtained using both the solution methods are reasonably close, with the differences in optimal values being no more than 6.25%.

The simulation results presented in Tables I and II are based on an Intel machine with eight 2.83 GHz, 32-bit, virtual processors, and 3GB memory, running Windows Server 2003. It can be seen that in all cases, the computational time required for the heuristic method is such that the real-time application of the proposed methodology is feasible considering that non-optimized, "over-the-counter" software tools are used. Also, despite only yielding sub-optimal solutions, the results show that the heuristic solution would certainly improve feeder operation.

V. CONCLUSIONS

In this paper, a GA-based solution method is implemented to determine the optimal solutions of a three-phase DOPF problem. The previously proposed heuristic method is compared with the GA results, in terms of both optimality and computational burden, for two distribution feeders. A comparison of the two approaches shows that the GA-based method yields superior solutions in terms of optimality, but at a large computational cost. The heuristic method is shown to yield solutions quite close to the global optima at a significantly reduced computational burden. Despite these sub-optimal solutions, the results obtained using the heuristic methods are such that it would certainly improve feeder operation in Smart Grids, with the solution times that are suitable for real-time applications.

REFERENCES

[1] A. Bose, "Smart Transmission Grid Applications and Their Supporting Infrastructure," *IEEE Trans. Smart Grid*, vol. 1, no. 1, pp. 11-19, June 2010.

[2] R. E. Brown, "Impact of Smart Grid on distribution system design," in *Proc. IEEE PES General Meeting*, 2008, pp. 1-4.

[3] M. C. Bozchalui, H. Hassen, S. A. Hashmi, C. A. Cañizares, and K. Bhattacharya, "Optimal Operation of Residential Energy Hubs in Smart Grids —Part I: The Concept and Mathematical Modeling," *IEEE Trans. on Smart Grid*, submitted.
[4] S. Paudyal, C. Cañizares, and K. Bhattacharya, "Optimal Operation of Distribution Feeders in Smart Grids," *IEEE Trans. Industrial Electronics*, to appear.
[5] E. Santacana, G. Rackliffe, L. Tang, and X. Feng, "Getting Smart," *IEEE Power and Energy Magazine*, vol. 8, no. 2, pp. 41-48, Mar-Apr. 2010.
[6] S. Jim, W. Carr, and S. E. Collier, "Real time distribution analysis for electric utilities," in *Proc. IEEE Rural Electric Power Conference*, 2008, pp. B5-B5-8.
[7] S. Bruno, S. Lamonaca, G. Rotondo, U. Stecchi, and M. L. Scala, "Unbalanced Three-phase Optimal Power Flow for Smart Grids," *IEEE Trans. Industrial Electronics*, to appear.
[8] I. Roytelman, B. K. Wee, and R. L. Lugtu, "Volt/var control algorithm for modern distribution management system," *IEEE Trans. Power Systems*, vol. 10, no. 3, pp. 1454-1460, Aug. 1995.
[9] M. B. Liu, C. A. Cañizares, and W. Huang, "Reactive Power and Voltage Control in Distribution Systems with Limited Switching Operations," *IEEE Trans. Power Systems*, vol. 24, no. 2, pp. 889-899, May 2009.
[10] M. Liu, S. K. Tso, and Y. Cheng, "An extended nonlinear primal-dual interior-point algorithm for reactive-power optimization of large-scale power systems with discrete control variables," *IEEE Trans. Power Systems*, vol. 17, no. 4, pp. 982-991, Nov. 2002.
[11] M. Abdel-Akher, K. M. Nor, and A. H. A. Rashid, "Improved Three-Phase Power-Flow Methods Using Sequence Components," *IEEE Trans. Power Systems*, vol. 20, no. 3, pp. 1389-1397, Aug. 2005.
[12] C. S. Cheng and D. Shirmohammadi, "A three-phase power flow method for real-time distribution system analysis," *IEEE Trans. Power Systems*, vol. 10, no. 2, pp. 671-679, May 1995.
[13] A. Merlin and H. Back, "Search for a minimal-loss operating spanning tree configuration in an urban power distribution system", in *5th Power System Computing Conference*, 1975, pp. 1-18.
[14] K. Nara, A. Shiose, M. Kitagawa, and T. Ishihara, "Implementation of Genetic Algorithm for distribution systems loss minimum re-configuration" *IEEE Trans. Power Systems*, vol. 7, no. 3, pp. 1044-1051, Aug. 1992.
[15] S. Sundhararajan and A. Pahwa, "Optimal selection of capacitors for radial distribution systems using a genetic algorithm," *IEEE Trans. Power Systems*, vol. 9, no. 3, pp. 1499-1507, Aug. 1994.
[16] G. Levitin, A. Kalyuzhny, A. Shenkman, and M. Chertkov, "Optimal capacitor allocation in distribution systems using a genetic algorithm and a fast energy loss computation technique," *IEEE Trans. Power Delivery*, vol. 15, no. 2, pp. 623-628, Apr. 2000.
[17] P. Nallagownden, L. T. Thin, N. C. Guan, and C. M. H. Mahmud, "Application of Genetic Algorithm for the Reduction of Reactive Power Losses in Radial Distribution System," in *Proc. IEEE International Power and Energy Conference (PECon)*, 2006, pp. 76-81.
[18] M. L. Whei and H. C. Chin, "Preventive and corrective switching for feeder contingencies in distribution systems with fuzzy set algorithm", *IEEE Trans. Power Delivery*, vol. 12, no. 4, pp. 1711 -1716, Oct. 1997.
[19] W. H. Kersting, *Distribution System Modeling and Analysis*, 2nd ed. Boca Raton : CRC Press, 2006.
[20] S. S. Rao, *Engineering Optimization: Theory and Practice*, 3rd ed. New York : John Wiley & Sons Inc., 1996.
[21] "GAMS Release 2.25," A Users Guide, GAMS Development Corporation, 1998.
[22] Stanford Business Software Inc. [online]. http://sbsi-sol-optimize.com/asp/sol_product_minos.htm
[23] W. H. Kersting, "Radial distribution test feeders," in *Proc. IEEE PES Winter Meeting*, 2001, vol. 2, pp. 908-912.
[24] "Integration of Storage in Electrical Distribution Systems and its Impact on the Depth of Penetration of DG". [online]. Available: http://canmetenergy-canmetenergie.nrcan-rncan.gc.ca/fichier.php/codectec/En/2009-174/2009-174.pdf