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Optimal financing and investment strategies under asymmetric information on liquidation value

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ABSTRACT

This study develops the real options model to explore how asymmetric information at the time of liquidation ex-post affects a firm's financing (capital structure) and investment decisions ex-ante. Accordingly, it is found that asymmetric information at the time of liquidation delays investment and reduces the amount of debt issuance. When the degree of asymmetric information is high, the firm cannot take a mixed financing consisting of risky debt and equity. The firm with collateral takes a mixed financing consisting of risk-free debt and equity, whose debt issuance amount equals the collateral value, whereas the firm without collateral takes all-equity financing. This result contrasts with that under symmetric information, where the firm always issues a mix of risky debt and equity. Moreover, when the degree of asymmetric information is substantial, an increase in cash inflow volatility decreases debt issuance. This result also contrasts with that under symmetric information, where an increase in volatility increases debt issuance.

1. Introduction

The term "irreversibility" is among the most important keywords in the real options approach to investment. Suppose a firm installs capital goods (production facilities, like a plant or similar building). If the firm cannot resell the installed capital goods at any price when operations cease, the investment is completely irreversible; if the firm can resell it at a discounted price, the investment is partially irreversible. Arrow (1968) argues that all investments are at least partially irreversible because "... from a realistic point of view, there will be many situations in which the sale of capital goods cannot be accomplished at the same price as their purchases." Shleifer and Vishny (1992), Dixit and Pindyck (1994), and Abel et al. (1996) posit that "facility specificity" and "asymmetric information (lemon effect)" cause the partial irreversibility of investment.[1] Chirinko and Schaller (2009) estimate the "irreversible premium" as a cost of asset illiquidity that is economically and statistically significant. Based on such findings and discussions, the investment is regarded to be partially irreversible because of facility specificity and asymmetric information.

In the past three decades, some studies of real options have examined the effects of partial irreversibility of investment that stem from facility specificity. Such studies include Abel and Eberly (1994), Abel and Eberly (1996), Abel et al. (1996), Hartman and Hendrickson (2002), and Shibata and Nishihara (2018). Further, others examine the effects of asymmetric information. They include Grenadier and Wang (2005), Shibata and Nishihara (2010), Grenadier and Malenko (2011), Morellec and Schurhoff (2011), and Koskinen and Maeland (2016). These studies assume that the source of asymmetric information is the firm's cash flow or investment cost. That is, the effects of partial irreversibility of investment arising from (or with) asymmetric information have not been examined. Thus, the literature analyzes the partial irreversibility of investment and asymmetric information in isolation in the real options model.

In this study, we explore how the partial irreversibility of investment arising from (or with) asymmetric information affects corporate financing and investment decisions. Accordingly, we develop the following hypotheses. The first hypothesis is that the shareholders delegate the corporate operation to the managers, taking advantage of managers' expertise, as in most modern corporations. The second hypothesis is that the investment is partially

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[1] Facility specificity incurs more costs by changing an industry-specific facility to a general one. In addition, asymmetric information discounts the repricing of the installed capital goods.

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irreversible. That is, it is divided into reversible and irreversible investments. The amount of reversible investment becomes a positive residual value at liquidation (ceasing operations). The third hypothesis is that the (realized) residual value at liquidation is privately observed by managers, not shareholders (and creditors if the firm issues debt).[2] That is, managers have an informational advantage over shareholders, thus inducing a manager–shareholder conflict. Hence, uninformed shareholders should induce informed managers to reveal the realized residual value (i.e., a screening game). Otherwise, shareholders will suffer some losses.[3] Thus, shareholders make an optimal contract with managers to reveal managers' private information. Such a contract, though unable to remove distortion completely, enables shareholders to minimize asymmetric information distortion. Notably, renegotiation is not allowed after making a contract. While commitment induces inefficiency ex-post, it increases the option values of the project exante. This notion is identical to that in the screening game of Grenadier and Wang (2005), Shibata and Nishihara (2010), and Koskinen and Maeland (2016). Therefore, this study employs the screening game equilibrium to examine how asymmetric information at the time of liquidation affects corporate financing and investment decisions.

This study differs from the noted studies in the existing literature in three ways: First, Grenadier and Wang (2005), Grenadier and Malenko (2011), and Koskinen and Maeland (2016) assume an all-equity-financed firm. They show that asymmetric information distorts the firm's investment decision. However, they fail to consider the effects of asymmetric information on the firm's financing (capital structure) strategies. Therefore, an investigation of how asymmetric information affects financing strategies is warranted.

Second, Shibata and Nishihara (2010) assume that investment is completely irreversible and the source of asymmetric information is the investment cost, given that the firm can issue a mix of equity and debt. They show that asymmetric information at the time of investment (initiating operations) distorts the firm's financing and investment decisions but does not affect the leverage and credit spreads.[4] The invariant effects of asymmetric information on the leverage and credit spreads do not match the empirical results of Fama and French (2005). Thus, it is worthwhile to consider whether asymmetric information at the time of liquidation affects the leverage and credit spreads.

Third, Morellec and Schurhoff (2011) allow a firm to issue debt or equity, limiting the firm to one type of financing instrument.[5] They examine shareholder–creditor conflict over issuing debt or the conflict between an (existing) shareholder and a new shareholder over issuing equity, where the firm attempts to exercise the growth option. In this situation, (existing) shareholders are informed agents, whereas creditors or new shareholders are uninformed agents. The informed shareholders signal their private information to the uninformed creditors (or new shareholders) to minimize a loss of asymmetric information (i.e., a signaling game). This study clarifies the difference and similarity of the economic mechanism between the signaling game of a shareholder–creditor conflict and the screening game of a manager–investor conflict.

The analyses highlight two novel results. First, asymmetric information at the time of liquidation (ceasing operations) delays investment and reduces the optimal amount of debt issuance. The delayed investment is identical to that in the screening games of Grenadier and Wang (2005), Shibata and Nishihara (2010), and Koskinen and Maeland (2016) but different from the signaling games of Grenadier and Malenko (2011) and Morellec and Schurhoff (2011), where asymmetric information speeds up investment. Despite different results, the mechanism is identical across the noted five studies, in that investment triggers between good and bad types are enlarged under asymmetric information, as in Section 4.1. The reduced debt issuance given asymmetric information at liquidation is a novel result. It differs from those of the existing (real options) studies of Morellec and Schurhoff (2011) and Shibata and Nishihara (2010). This outcome can be attributed to the following reasons: Morellec and Schurhoff (2011) assume that the amount of debt issuance is fixed, while Shibata and Nishihara (2010) show that asymmetric information increases the optimal amount of debt issuance because they assume that the source of asymmetric information exists at the time of investment (initiating operations). Additionally, in this study, asymmetric information reduces the credit spread and leverage, which contrasts the findings of Shibata and Nishihara (2010). Thus, this study finds that asymmetric information at the time of liquidation has a different effect on debt issuance. The difference stems from whether the distortion of asymmetric information affects the firm's bankruptcy strategies, as in Section 3.2. Moreover, asymmetric information at the time of liquidation reduces the credit spread and leverage, which contrasts with the invariant result of Shibata and Nishihara (2010).

The second novel result regards the effects of cash inflow volatility. When the degree of asymmetric information is small, an increase in volatility increases the optimal amount of debt issuance, which is identical to the case under the symmetric information of Leland (1994) and Sundaresan and Wang (2007). However, when the degree of asymmetric information is significant, an increase in volatility decreases the optimal amount of debt issuance. Thus, asymmetric information may change the effect of volatility on debt issuance. Following this novel finding, the credit spread has an inverse U-shaped relationship with the volatility, as the degree of asymmetric information is significant. The nonmonotonic relationship stems from risk-free debt issuance with increasing volatility.[6]

The remainder of the paper is organized as follows. Section 2 describes the study model and formulates the financing and investment decision problem of a firm. Section 3 provides the solution to the problem and analyzes its properties. Section 4 discusses the economic implications of the model under asymmetric information. Section 5 concludes the paper. Three appendices furnish the technical developments. Appendix A details the derivation of the value functions after the firm initiates operations. Appendix B provides proof of the four lemmas and two propositions in the study. Appendix C demonstrates the properties of the solution for related studies, differentiating this study.

2. Model

This section describes the model in three steps. First, we describe the model's setup. Second, we provide the value functions after the firm initiates operations (investment), given the financing and investment strategies. Finally, we formulate the financing and investment decisions problem under asymmetric information.

2.1. Model setup

Consider a firm with an option to install a production facility and initiate operations (e.g., sell the commodity produced).

[2] In practice, it is reasonable in that managers and shareholders, at least, observe the realized value with time delays.

[3] Otherwise, the creditor cannot permit the firm to issue debt.

[4] See Appendix C for details.

[5] They note that a mix of debt and equity would be a significant extension.

[6] This study's results differ from those of prior studies like Shibata and Nishihara (2010), where the credit spread always increases with the volatility, even in a sufficiently significant degree of asymmetric information.

Throughout the analysis, we assume capital markets are frictionless, with a constant risk-free interest rate $r > 0$, and all agents are risk-neutral and aim to maximize their expected payoff.

When the firm initiates operations, it incurs a one-time fixed cost, $I > 0$, to install the production facility. Thereafter, the firm receives an instantaneous cash inflow $X(t)$, which follows the geometric Brownian motion given by

$$dX(t) = \mu X(t)dt + \sigma X(t)dz(t), \quad X(0) > 0, \tag{1}$$

where μ and σ are constants, and $z(t)$ denotes the Brownian motion defined by a risk-neutral probability space $(\Omega, \mathcal{F}, \mathbb{Q})$.[7] For convergence, we assume $r > \mu$.[8] In this study, the firm issues a mix of debt and equity in initiating operations. This notion of mixed financing is identical to that in Sundaresan and Wang (2007) and Shibata and Nishihara (2010).[9] Importantly, debt benefits from the tax shield in that the firm faces a constant tax rate $\tau > 0$ on income after servicing the interest payment on the debt. For analytical convenience, this study limits the condition to perpetual debt (i.e., maturity is infinite). We denote an instantaneous coupon payment of the debt by $c \geq 0$. This assumption, as in Black and Cox (1976) and Leland (1994), simplifies the analysis without substantially altering the key economic insights. Thus, if the firm issues a mix of debt and equity, its instantaneous cash flow is $(1-\tau)(X(t) - c)$.

We assume the firm has the option to cease operations and sell the production facility at the resale price of kI at any time t after initiating operations. Here, $k \in [0, 1]$ indicates the proportion of investment reversibility. This assumption means the investment is partially reversible. That is, at the time of ceasing operations, kI can be converted to cash, whereas $(1-k)I$ cannot. Economically, kI and $(1-k)I$ are regarded as tangible (liquid) and intangible (nonliquid) assets, respectively. Moreover, we assume the proportion k takes one of two possible values: k_H or k_L, where $1 \geq k_H \geq k_L \geq 0$. Here, k_H represents a "high proportion" of the reversible asset and k_L represents a "low proportion." The probabilities of drawing $k = k_H$ and $k = k_L$ are exogenous, and $\mathbb{P}(k_H) = q \in (0, 1)$ and $\mathbb{P}(k_L) = 1 - q$. Following Leland (1994) and Lambrecht and Myers (2008), we assume the firm incurs a bankruptcy cost αkI at liquidation, where $\alpha \in (0, 1)$. Thus, in this study, the liquidation value is defined as $(1-\alpha)kI \geq 0$, regarded as the collateral value for a levered firm.[10]

In this study, there are two kinds of debt: risky and risk-free debt. The face value of perpetual debt is c_j/r, where $c_j := c(k_j)$ for $k = k_j$ $(j \in \{H, L\})$. If the face value of the debt c_j/r is larger than the liquidation value $(1-\alpha)k_jI$, that is,

$$c_j > \theta_{j1} := r(1-\alpha)k_jI, \tag{2}$$

then the debt is risky. Otherwise, the debt is risk-free, where its face value is guaranteed at liquidation. The notion is identical to that of Lambrecht and Myers (2008).

We denote the values of the debt, equity, and total firm as $D_{jm}(X(t), c_j)$, $E_{jm}(X(t), c_j)$, and $V_{jm}(X(t), c_j) := D_{jm}(X(t), c_j) + E_{jm}(X(t), c_j)$, for $k = k_j$ $(j \in \{H, L\})$. Here, the subscript "m" is defined as

$$m := \begin{cases} 0, & c_j = 0, \\ 1, & c_j \in (0, \theta_{j1}], \\ 2, & c_j > \theta_{j1}. \end{cases} \tag{3}$$

The subscripts "0," "1," and "2" denote "all-equity financing," "mixed financing consisting of risk-free debt and equity," and "mixed financing consisting of risky debt and equity," respectively. Further, we denote the time of initiating operations (investment, indicated by superscript "i"), default (indicated by superscript "d"), and ceasing operations (shutdown or liquidation, indicated by superscript "s") as T_j^{i}, T_j^{d}, and T_{jm}^{s}, respectively. Mathematically, these times are defined as

$$\begin{aligned}
T_j^{\mathrm{i}} &:= \inf\left\{t \geq 0 \,\middle|\, X(t) \geq x_j^{\mathrm{i}}\right\}, \\
T_j^{\mathrm{d}} &:= \inf\left\{t \geq T_j^{\mathrm{i}} \,\middle|\, X(t) \leq x_j^{\mathrm{d}}\right\}, \\
T_{jm}^{\mathrm{s}} &:= \inf\left\{t \geq T_j^{\mathrm{i}} \,\middle|\, X(t) \leq x_{jm}^{\mathrm{s}}\right\}, \quad m \in \{0, 1\} \\
T_{j2}^{\mathrm{s}} &:= \inf\left\{t \geq T_j^{\mathrm{d}} \,\middle|\, X(t) \leq \min\left\{x_{j2}^{\mathrm{s}}, x_j^{\mathrm{d}}\right\}\right\},
\end{aligned}$$

respectively, where $x_j^{\mathrm{i}} \geq 0$, $x_j^{\mathrm{d}} \geq 0$, and $x_{jm}^{\mathrm{s}} \geq 0$ denote the associated investment, default, and liquidation triggers, respectively. In this model, default represents operating concern bankruptcy. At the time of default, the right of management ownership is transferred from equity–holders to debt–holders, implying that the equity value is zero, and managers and equity–holders leave the firm. Moreover, default can occur only when the firm issues risky debt. By contrast, liquidation represents ceasing operations. Liquidation can occur in all three types of financing. The meaning of $\min\{x_{j2}^{\mathrm{s}}, x_j^{\mathrm{d}}\}$ in T_{j2}^{s} is that the firm issuing risky debt is never liquidated before default, though it may be liquidated concurrently with default.[11]

2.2. Value functions after investment

This subsection provides the value functions after the firm initiates operations. Value functions are derived for three different types of financing strategies ($m \in \{0, 1, 2\}$). In the derivations of value functions, we use the following parameters:

$$\begin{aligned}
\beta &:= \tfrac{1}{2} - \tfrac{\mu}{\sigma^2} + \left\{\left(\tfrac{\mu}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}\right\}^{1/2} > 1, \\
\gamma &:= \tfrac{1}{2} - \tfrac{\mu}{\sigma^2} - \left\{\left(\tfrac{\mu}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}\right\}^{1/2} < 0, \\
\nu &:= \tfrac{1-\tau}{r-\mu} > 0, \\
\varepsilon &:= \tfrac{\gamma}{\gamma-1} > 0, \\
\eta &:= \tfrac{\nu}{\varepsilon}\tfrac{r}{1-\tau} > 0, \\
h &:= \left(1 - \gamma\left\{1 + \alpha\tfrac{1-\tau}{\tau}\right\}\right)^{-1/\gamma} \geq 1, \\
\psi &:= \left(1 + \tfrac{\tau}{h(1-\tau)}\right)^{-1} \leq 1.
\end{aligned}$$

2.2.1. Value functions for $m \in \{0, 1\}$

In this subsection, we derive the value functions for $m \in \{0, 1\}$.

Given $k = k_j$ $(j \in \{H, L\})$, once the cash inflow $X(t)$, starting at $X(0) = x$, increases and arrives at x_j^{i} from the below, we assume that the firm issues a mix of risk-free debt and equity and initiates operations. Thereafter, the firm obtains an instantaneous cash flow $(1-\tau)(X(t) - c_j)$ as long as $X(t)$ keeps a high level. However, if $X(t)$ decreases and arrives at x_{j1}^{s} from the above, the firm is liquidated. Note that there is no default under risk-free debt financing because the liquidation value is larger than the face value of the debt. The value of the total firm after initiating operations and the optimal liquidation trigger, $V_{j1}(X(t), c_j)$ and $x_{j1}^{\mathrm{s}}(c_j)$, are given as

$$V_{j1}(X(t), c_j) = \nu X(t) + \frac{\tau c_j}{r} + \left\{(1-\alpha)k_jI - \nu x_j^{\mathrm{s}}(c_j) - \frac{\tau c_j}{r}\right\}\left(\frac{X(t)}{x_j^{\mathrm{s}}(c_j)}\right)^{\gamma}, \tag{4}$$

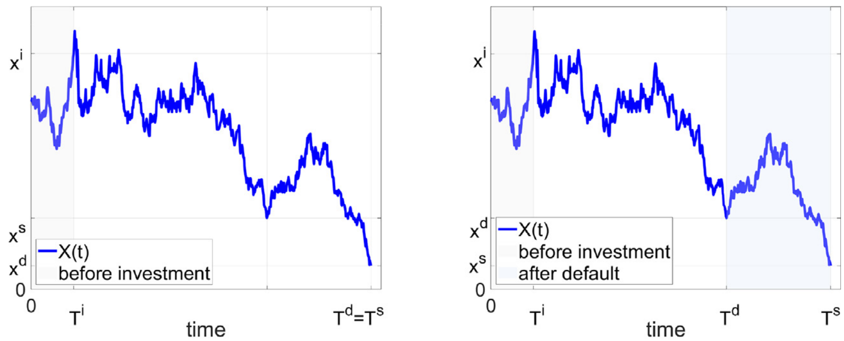
[7] This assumption is identical to that in Goldstein et al. (2001) and Sundaresan and Wang (2007).

[8] The assumption $r > \mu$ ensures that the value of the firm is finite. See Dixit and Pindyck (1994) and Hugonnier et al. (2015) for details.

[9] Morellec and Schurhoff (2011) restrict attention to the situation in which firms issue only one type of financing instrument. They note that a mix of debt and equity is a significant extension. The authors thank the anonymous reviewer for the discussion on this point.

[10] In this study, the liquidation value does not depend on the value of $X(t)$, which is identical to that in Mella-Barral and Perraudin (1997) and Shibata and Nishihara (2018).

[11] Subsection 2.2.2. explains $\min\{x_{j2}^{\mathrm{s}}, x_2^{\mathrm{d}}\}$ of T_{j2}^{s} in-depth.



(a) $x^{\mathrm{s}} \geq x^{\mathrm{d}}$ (b) $x^{\mathrm{d}} > x^{\mathrm{s}}$

Fig. 1. Simultaneous and sequential bankruptcy strategies. Suppose that the firm issues a mix of risky debt and equity when it initiates operations. In Panel (a) of $x^{\mathrm{s}} \geq x^{\mathrm{d}}$, the firm takes default and liquidation simultaneously at x^{d}. In Panel (b) of $x^{\mathrm{d}} > x^{\mathrm{s}}$, the firm employs default and liquidation sequentially at x^{d} and x^{s}, respectively.

$$x^{\mathrm{s}}_{j1}(c_j) = \frac{\varepsilon}{\upsilon}\Big\{(1-\alpha)k_j I - \frac{\tau c_j}{r}\Big\} \geq 0, \tag{5}$$

respectively. See Appendix A for derivations of (4) and (5). We summarize the properties of V_{j1} and x^{s}_{j1} in the following lemma. See Appendix B for the proof.

Lemma 1. *Suppose that the firm issues a mix of risk-free debt and equity ($m=1$), for $k=k_j$ ($j \in \{H, L\}$). The total firm value, $V_{j1}(x, c_j)$, is a positively linear function of c_j. The optimal liquidation trigger, $x^{\mathrm{s}}_{j1}(c_j)$, is a negatively linear function of c_j.*

In Lemma 1, the firm chooses $c_j = \theta_{j1}$ to maximize V_{j1} if it is allowed to issue a risk-free debt ($c_j \in (0, \theta_{j1}]$).

Then, we assume that the firm employs all-equity financing ($m=0$) when it initiates operations. Substituting $c_j = 0$ into (4) and (5) yields

$$V_{j0}(X(t)) = V_{j1}(X(t), 0), \quad x^{\mathrm{s}}_{j0} = x^{\mathrm{s}}_{j1}(0), \tag{6}$$

respectively. Here, V_{j0} and x^{s}_{j0} indicate the value of the equity and the liquidation trigger under all-equity financing.

2.2.2. Value functions for $m=2$

In this subsection, we derive the value functions for $m=2$.

Given $k = k_j$ ($j \in \{H, L\}$), we assume that the firm issues a mix of risky debt and equity ($m=2$). After the firm initiates operations, it obtains an instantaneous cash flow, $(1-\tau)(X(t)-c_j)$, as long as $X(t)$ maintains a high level. However, if $X(t)$ decreases and reaches a low level, it is challenging to pay the coupon $c_j > 0$, implying that the firm must consider the default (operating concern bankruptcy). Following Black and Cox (1976) and Leland (1994), the equity-holders decide the default trigger x^{d}_j to maximize the equity value (before default). Following the absolute priority rule, once the firm defaults, the right of corporate ownership is transferred from the equity-holders to the debt-holders. The new equity-holders determine the liquidation trigger x^{s}_{j2} to maximize the equity value after default.

According to the magnitude of x^{d}_j and x^{s}_{j2}, the following two scenarios emerge. Figure 1(a) depicts the scenario of $x^{\mathrm{d}}_j = \min\{x^{\mathrm{d}}_j, x^{\mathrm{s}}_{j2}\}$. Recall that the firm is never liquidated before it defaults because the debt-holders cannot obtain the right of corporate ownership unless the equity-holders leave at default. Once $X(t)$ reaches x^{d}_j from the above, the firm defaults and is liquidated simultaneously. Figure 1(b) depicts the scenario of $x^{\mathrm{s}}_{j2} = \min\{x^{\mathrm{d}}_j, x^{\mathrm{s}}_{j2}\}$. Once $X(t)$ reaches x^{d}_j from the above, the firm defaults, and the new equity holders continue operations. Thereafter, if $X(t)$ decreases and reaches x^{s}_{j2} from the above, the firm ceases the operation at x^{d}_j. The light blue shaded region indicates the period of operations by the new equity-holders. Thus, the firm employs the *sequential* default-liquidation bankruptcy at x^{d}_j and x^{s}_{j2}, respectively.

Given $k = k_j$ ($j \in \{H, L\}$), the value of the total firm after initiating operations and before default, $V_{j2}(X(t), c_j)$, is given by

$$V_{j2}(X(t), c_j) = \upsilon X(t) + \frac{\tau c_j}{r} + \Big\{W_j(x^{\mathrm{d}}(c_j)) - \frac{\tau c_j}{r} - \upsilon x^{\mathrm{d}}(c_j)\Big\}\Big(\frac{X(t)}{x^{\mathrm{d}}(c_j)}\Big)^{\gamma}, \tag{7}$$

where the equity value after default $W_j(X(t))$, optimal default trigger $x^{\mathrm{d}}(c_j)$, and optimal liquidation trigger x^{s}_{j2} are

$$W_j(X(t)) = (1-\alpha)\Big\{\upsilon X(t) + \big(k_j I - \upsilon \min\{x^{\mathrm{d}}(c_j), x^{\mathrm{s}}_{j2}\}\big)\Big(\frac{X(t)}{\min\{x^{\mathrm{d}}(c_j), x^{\mathrm{s}}_{j2}\}}\Big)^{\gamma}\Big\}, \tag{8}$$

$$x^{\mathrm{d}}(c_j) = \frac{\varepsilon}{\upsilon}\frac{1-\tau}{r}c_j, \tag{9}$$

$$x^{\mathrm{s}}_{j2} = \frac{\varepsilon}{\upsilon}k_j I. \tag{10}$$

See Appendix A for derivations of (7)–(10). In (8), the firm incurs a bankruptcy cost after default, consistent with Black and Cox (1976) and Leland (1994). In their models, given that the liquidation value is assumed to be zero ($k_j = 0$ and $x^{\mathrm{s}}_{j2} = 0$), the residual value at default is $(1-\alpha)\upsilon x^{\mathrm{d}}(c_j)$. That is, this study's model converges to that of Black and Cox (1976). In (9), $x^{\mathrm{d}}(c_j)$ does not involve k_j. In (10), x^{s}_{j2} differs from $x^{\mathrm{s}}_{j1}(c_j)$. From (9) and (10), we have $x^{\mathrm{d}}(c_j) > x^{\mathrm{s}}_{j2}$ if

$$c_j > \theta_{j2} := \frac{r}{1-\tau}k_j I. \tag{11}$$

That is, if $c_j \in (\theta_{j1}, \theta_{j2}]$, the firm takes a simultaneous bankruptcy strategy. If $c_j > \theta_{j2}$, the firm takes a sequential bankruptcy strategy. As c_j/r (debt issuance) is smaller (larger), the firm is more likely to employ simultaneous (sequential) bankruptcy. The simultaneous and sequential default-liquidation strategies can be regarded as Chapters 7 and 11 of U.S. bankruptcy codes, respectively. The result that the firms with significant debt employ a sequential bankruptcy strategy accords with the empirical finding of Bris et al. (2006). We summarize the properties of V_{j2} and x^{d} in the following lemma. See Appendix B for the proof.

Lemma 2. *Suppose that the firm issues a mix of risky debt and equity ($m=2$), for $k=k_j$ ($j \in \{H, L\}$). The total firm value, $V_{j2}(X(t), c_j)$, is a concave function of c_j. The optimal default trigger, $x^{\mathrm{d}}(c_j)$, is a linear function of c_j.*

In Lemma 2, there exists an optimal coupon that uniquely maximizes V_{j2}, because of its concavity. Moreover, $x^{\mathrm{d}}(c_j)$ is a linear function of c_j, identical to that in Black and Cox (1976).

2.2.3. Properties of total firm value

This subsection shows the properties of the total firm value for $k = k_j$ $(j \in \{H, L\})$. We obtain the following lemma. See Appendix B for the proof.

Lemma 3. *The total firm value and trigger are continuous at $c_j = \theta_{j1}$, for $k = k_j$ $(j \in \{H, L\})$, that is,*

$$V_{j1}(X(t), \theta_{j1}) = \lim_{c_j \downarrow \theta_{j1}} V_{j2}(X(t), c_j), \quad x^{\mathrm{s}}_{j1}(\theta_{j1}) = \lim_{c_j \downarrow \theta_{j1}} x^{\mathrm{d}}(c_j). \quad (12)$$

Further, we have

$$\frac{\partial V_{j1}}{\partial c_j}(X(t), \theta_{j1}) = \frac{\partial V_{j2}}{\partial c_j}(X(t), c_j)\Big|_{c_j \downarrow \theta_{j1}} > 0,$$

which implies $V_{j2}(x, c_j)$ is an increasing function of c_j for the regions around θ_{j1}.

Lemma 3 implies that there exists a coupon payment c_j to maximize the total firm value such that $c_j > \theta_{j1}$.

2.3. Asymmetric information problem

In this subsection, we consider the contract under manager-shareholder conflict due to asymmetric information.

Consider that the shareholders (equity-holders) delegate the corporate decision to managers, taking advantage of their unique expertise. We assume that only the managers privately observe whether the realized value of k is k_H or k_L, whereas the shareholders and creditors (debt-holders) cannot.[12] Thus, the managers have an informational advantage over the shareholders and creditors. In this situation, shareholders induce the managers to reveal the realized value. Otherwise, investors suffer some losses. Suppose, for instance, the managers observe $k = k_H$ as the realized value. Once $X(t)$ declines to $x^{\mathrm{d}}(c)$ in operations, the firm would default, and the managers would be fired. Thus, the managers have an incentive to report a false value $k = k_L$ intentionally and divert the difference ΔkI to themselves where $\Delta k := k_H - k_L \geq 0$, whereas the shareholders suffer the loss of ΔkI. Hence, the shareholders should induce the managers to reveal their private information. Moreover, the creditors would refuse the financing to the firm unless the managers' private information was clarified.

In this study, the shareholders make a contract with the managers at time zero. The components of the contract are

$$\{x^{\mathrm{i}}(\tilde{k}_j),\ c(\tilde{k}_j),\ w(\tilde{k}_j)\},$$

which may be contingent on a reported value $\tilde{k}_j$ $(j \in \{H, L\})$. Here, the idea of introducing only the positive incentive $w(\tilde{k})$ is identical to that of Grenadier and Wang (2005).[13] The mechanism of introducing only positive incentive simplifies the analysis substantially without altering the key economic insights. Given that the revelation principle ensures that the shareholders induce the managers to clarify a true value k_j as private information, we make no distinction between k_j and $\tilde{k}_j$. Hence, we can drop the suffix tilde on the reported $\tilde{k}_j$ and simply write the contract as $\{x^{\mathrm{i}}_j, c_j, w_j\}$.

Given $k = k_j$ $(j \in \{H, L\})$, the option value of the project is given by[14]

$$\sup_{x^{\mathrm{i}}_j, c_j, w_j} \Big(\frac{x}{x^{\mathrm{i}}_j}\Big)^{\beta} \Big(E_{jm}(x^{\mathrm{i}}_j, c_j) - \big\{I - D_{jm}(x^{\mathrm{i}}_j, c_j)\big\} - w_j\Big), \quad (13)$$

where $X(0) = x > 0$. See Appendix A for the derivation of (13). Here, the total firm value is defined by $V_{jm} := E_{jm} + D_{jm}$. The subscript "$m$" is then decided per the magnitude of $c_j \geq 0$ $(m \in \{0, 1, 2\})$.

Under asymmetric information, the firm's optimization problem is formulated as

$$O^{**}(x) := \max_{x^{\mathrm{i}}_j, c_j, w_j} \sum_{j \in \{H, L\}} \mathbb{P}(k_j) \Big(\frac{x}{x^{\mathrm{i}}_j}\Big)^{\beta} \big\{V_{jm}(x^{\mathrm{i}}_j, c_j) - I - w_j\big\}, \quad (14)$$

subject to

$$\Big(\frac{x}{x^{\mathrm{i}}_H}\Big)^{\beta} w_H \geq \begin{cases} \Big(\frac{x}{x^{\mathrm{i}}_L}\Big)^{\beta} \Big\{w_L + \Big(\frac{x^{\mathrm{i}}_L}{, x^{\mathrm{s}}_L(c_L)}\Big)^{\gamma} \Delta kI\Big\}, & c_L \leq \theta_{L1}, \\ \Big(\frac{x}{x^{\mathrm{i}}_L}\Big)^{\beta} \Big\{w_L + \Big(\frac{x^{\mathrm{i}}_L}{x^{\mathrm{d}}(c_L)}\Big)^{\gamma} \Delta kI\Big\}, & c_L > \theta_{L1}, \end{cases} \quad (15)$$

$$\Big(\frac{x}{x^{\mathrm{i}}_L}\Big)^{\beta} w_L \geq \begin{cases} \Big(\frac{x}{x^{\mathrm{i}}_H}\Big)^{\beta} \Big\{w_H - \Big(\frac{x^{\mathrm{i}}_H}{x^{\mathrm{s}}_H(c_H)}\Big)^{\gamma} \Delta kI\Big\}, & c_H \leq \theta_{H1}, \\ \Big(\frac{x}{x^{\mathrm{i}}_H}\Big)^{\beta} \Big\{w_H - \Big(\frac{x^{\mathrm{i}}_H}{x^{\mathrm{d}}(c_H)}\Big)^{\gamma} \Delta kI\Big\}, & c_H > \theta_{H1}, \end{cases} \quad (16)$$

$$M(x) := \sum_{j \in \{H, L\}} \mathbb{P}(k_j) \Big(\frac{x}{x^{\mathrm{i}}_j}\Big)^{\beta} w_j \geq 0, \quad (17)$$

$$w_j \geq 0, \quad (18)$$

where the superscript "$**$" represents the optimum under *asymmetric information*.

The objective function (14) is the option value of the project. Constraints (15) and (16) are incentive-compatible constraints for managers in $k = k_H$ and $k = k_L$, respectively. Consider, for example, Constraint (15), which ensures that managers who observe $k = k_H$ have no incentive to intentionally report the false value $k = k_L$. This can be attributed to the following reason: The value for managers who observe $k = k_H$ is given by the left-hand side of (15) if they report the true value $k = k_H$; however, it is given by the right-hand side of (15) if they report the false value $k = k_L$. Thus, if Constraint (15) is satisfied, managers who observe $k = k_H$ have no incentive to report the false value $k = k_L$ intentionally. Constraint (16) follows similarly. Constraint (17) is a participation constraint, in that the managers' option value $M(x)$ is larger than zero. Constraints (18) are limited-liability constraints. Their basis is identical to that of Grenadier and Wang (2005) and Shibata and Nishihara (2010).

3. Model solutions

In this section, we derive the solution to the asymmetric information problem. Just before its derivation, we briefly begin with reviewing the symmetric information problem as a benchmark.

3.1. Symmetric information

We assume all agents (managers, shareholders, and creditors) observe the realized value of k. This problem is identical to when there is no delegation to managers. Thus, we have $w^*_H = w^*_L = 0$. We use the superscript "$*$" to represent the optimum under *symmetric information*.[15] We obtain the following lemma. See Appendix B for the proof.

Lemma 4. *Under symmetric (full) information, the option value of the project is*

$$O^*(x) := q\Big(\frac{x}{x^{\mathrm{i}*}_H}\Big)^{\beta} \big\{V_{H2}(x^{\mathrm{i}*}_H, c^*_H) - I\big\} + (1 - q)\Big(\frac{x}{x^{\mathrm{i}*}_L}\Big)^{\beta} \big\{V_{L2}(x^{\mathrm{i}*}_L, c^*_L) - I\big\}. \quad (19)$$

[12] This assumption follows Shleifer and Vishny (1992) and Abel et al. (1996). In Shleifer and Vishny (1992), given that most assets are specialized, they are sold at prices below the values in best use when liquidated (via asymmetric information). Abel et al. (1996) argue that an underpriced liquidation value can be attributed to lemon effects.

[13] See Shibata and Nishihara (2010) for a discussion of negative incentives (e.g., a penalty).

[14] See the Appendix for the derivation.

[15] Note that $M^*(x) = 0$ under symmetric information.

where $c_H^* > \theta_{H1}$ and $c_L^* > \theta_{L1}$; that is, the firms in H and L issue a mix of risky debt and equity ($m = 2$).

Here, if $c_j^* \in (\theta_{j1}, \theta_{j2}]$ for $k = k_j$ ($j \in \{H, L\}$), we obtain $x_j^{\mathrm{i}*}$ and c_j^* by solving $f_{j1}(x_j^{\mathrm{i}}, c_j) = 0$ and $f_{j2}(x_j^{\mathrm{i}}, c_j) = 0$, where f_{j1} and f_{j2} are given in (B.11). If $c_j^* > \theta_{j2}$, we obtain $x_j^{\mathrm{i}*}$ by solving $f_{j5}(x_j^{\mathrm{i}}) = 0$, where f_{j5} is given in (B.14). Thus, we obtain $c_j^* = (\eta/h)x_j^{\mathrm{i}*}$.

3.2. Asymmetric information

This subsection derives the solution under asymmetric information.

We show that only two of the five constraints (15)–(18) are binding at the equilibrium, in three steps. First, Constraint (17) is satisfied automatically because Constraint (18) imply Constraint (17). Second, managers who observe $k = k_L$ have no incentive to pretend they observe $k = k_H$; thus, Constraint (16) is satisfied automatically, and we obtain $w_L = 0$. Finally, suppose Constraint (15) holds as a strict inequality, then a decrease in w_H increases the option value of the objective function, which is a contradiction. Thus, Constraint (15) is binding, which leads to $w_H > 0$.

Following the above three steps, only Constraints (15) and (18) are binding at the equilibrium; that is,

$$w_H = \begin{cases} \left(\dfrac{x_H^{\mathrm{i}}}{x_L^{\mathrm{i}}}\right)^\beta \left(\dfrac{x_L^{\mathrm{i}}}{x_L^{\mathrm{s}}(c_L)}\right)^\gamma \Delta k I, & c_L \le \theta_{L1}, \\ \left(\dfrac{x_H^{\mathrm{i}}}{x_L^{\mathrm{i}}}\right)^\beta \left(\dfrac{x_L^{\mathrm{i}}}{x^{\mathrm{d}}(c_L)}\right)^\gamma \Delta k I, & c_L > \theta_{L1}, \end{cases} \tag{20}$$

$$w_L = 0. \tag{21}$$

Here, $\Delta k \ge 0$ indicates the degree of asymmetric information because an increase in $\Delta k \ge 0$ increases the managers' option value.

We simplify the optimization problem by substituting (20) and (21) into (14). We then obtain the following proposition, using Lemmas 1–3 (see Appendix B for the proof).

Proposition 1. *Under asymmetric information, we obtain the option value of the project as*

$$\begin{aligned} O^{**}(x) :=\; & q\left(\frac{x}{x_H^{\mathrm{i}*}}\right)^\beta \left\{V_{H2}\left(x_H^{\mathrm{i}*}, c_H^*\right) - I\right\} \\ & + (1-q)\left(\frac{x}{x_L^{\mathrm{i}**}}\right)^\beta \left\{V_{Lm}\left(x_L^{\mathrm{i}**}, c_L^{**}\right) - I - a_m\left(x_L^{\mathrm{i}**}, c_L^{**}\right)\right\}, \end{aligned} \tag{22}$$

where the distortion of asymmetric information, $a_m(x_L^{\mathrm{i}}, c_L) \ge 0$, is

$$\begin{aligned} a_1(x_L^{\mathrm{i}}, c_L) &:= \left(\frac{x_L^{\mathrm{i}}}{x_L^{\mathrm{s}}(c_L)}\right)^\gamma \frac{q}{1-q}\Delta k I, \\ a_2(x_L^{\mathrm{i}}, c_L) &:= \left(\frac{x_L^{\mathrm{i}}}{x^{\mathrm{d}}(c_L)}\right)^\gamma \frac{q}{1-q}\Delta k I, \end{aligned} \tag{23}$$

and $a_0(x_L^{\mathrm{i}}) = a_1(x_L^{\mathrm{i}}, 0)$. We obtain $O^{**}(x) \le O^*(x)$. Interestingly, we do not always obtain $c_L^{**} > \theta_{L1}$, whereas we obtain $c_H^{**} = c_H^* > \theta_{H1}$; that is, the firm in L cannot always employ mixed financing consisting of risky debt and equity, whereas the firm in H does employ mixed financing consisting of risky debt and equity ($m = 2$). Particularly, when the distortion of asymmetric information $a_m(x_L^{\mathrm{i}}, c_L)$ is great, the firm in L with collateral $k_L > 0$ takes mixed financing of risk-free debt and equity ($m = 1$) whereas the firm in L without collateral $k_L = 0$ does take all-equity financing ($m = 0$). The solution of $x_L^{\mathrm{i}**}$ and c_L^{**} is given in Appendix B.

Proposition 1 has three important properties. First, the distortion of asymmetric information, $a_m(x_L^{\mathrm{i}}, c_L) \ge 0$, reduces the option value of the project. Second, the distortion $a_m(x_L^{\mathrm{i}}, c_L)$ is in $k = k_L$, not $k = k_H$. That is, it is less costly to distort $(x_L^{\mathrm{i}**}, c_L^{**})$ from $(x_L^{\mathrm{i}*}, c_L^*)$ than to distort $(x_H^{\mathrm{i}**}, c_H^{**})$ from $(x_H^{\mathrm{i}*}, c_H^*)$. This result is identical to those in Grenadier and Wang (2005) and Shibata and Nishihara (2010). Third, when the distortion $a_m(x_L^{\mathrm{i}}, c_L)$ is significant, the firm in L cannot issue a mix of risky debt and equity.[16] Note that an increased distortion of asymmetric information $a_m(x_L^{\mathrm{i}}, c_L)$ is caused by an increased degree of asymmetric information Δk.

Table 1
Baseline parameter values.

r	μ	σ	τ	α	I	$X(0)$
0.06	0.01	0.3	0.15	0.4	100	2

We derive the properties of the asymmetric information solution. In order to so, the option value in L, $O_L^{**}(x)$, is given as

$$O_L^{**}(x) := \max_{x_L^{\mathrm{i}}, c_L} \left(\frac{x}{x_L^{\mathrm{i}}}\right)^\beta \{V_{Lm}(x_L^{\mathrm{i}}, c_L) - I - a_m(x_L^{\mathrm{i}}, c_L)\}. \tag{24}$$

First, we assume $m = 2$. An increase in Δk increases $x_L^{\mathrm{i}**}$ and decreases c_L^{**}. The reason is as follows. Here, to maximize $O_L^{**}(x)$, the firm in L minimizes $a_2(x_L^{\mathrm{i}}, c_L)$, which is equivalent to minimizing its component, $(x_L^{\mathrm{i}}/x^{\mathrm{d}}(c_L))^\gamma$, where $\gamma < 0$. In order to do so, the firm in L increases x_L^{i} and decreases $x^{\mathrm{d}}(c_L) = c_L/\eta$. Next, we assume $m = 1$. An increase in Δk has an ambiguous effect on $x_L^{\mathrm{i}**}$ and c_L^{**}. We summarize the properties of $x_L^{\mathrm{i}**}$ and c_L^{**} in the next proposition. See Appendix B for the proof.

Proposition 2. *Suppose asymmetric information. When the firm in L issues a mix of risky debt and equity, $x_L^{\mathrm{i}**}$ increases with Δk (degree of asymmetric information), whereas c_L^{**} decreases with Δk. When the firm in L issues a mix of risk-free debt and equity, $x_L^{\mathrm{i}**}$ is not monotonic with Δk, whereas c_L^{**} does not depend on Δk, but on k_L.*

From Proposition 2, an increased degree of asymmetric information reduces debt issuance (c_L^{**}/r). The result implies that asymmetric information may not translate into a pecking-order hypothesis. Empirically, Shyan-Sunder and Myers (1999) and Bharath et al. (2009) conclude that the pecking order is a good descriptor of board financing patterns, unlike Frank and Goyal (2003) and Fama and French (2005). Thus, there is no consensus on the divergence of conclusions. This study's result does not match the pecking order hypothesis, similar to Morellec and Schurhoff (2011).

The next section discusses the economic mechanism of the equilibrium under asymmetric information.

4. Model implications

This section derives a numerical solution to the asymmetric information problem and discusses the implications of the study model. In the numerical calculation, we set the baseline parameter values in Table 1. Section 4.1 examines the firm's delayed investment under asymmetric information. Section 4.2 considers the mechanism in which the firm is likely to be induced to issue a mix of risk-free debt and equity when the degree of asymmetric information is large. Section 4.3 investigates the firm's financing (capital structure) decisions under asymmetric information. Particularly, as the degree of asymmetric information is large, the firm with collateral (positive liquidation value) issues a mix of risk-free debt and equity, whereas the firm without collateral issues only equity. Section 4.4 considers how asymmetric information affects default and liquidation strategies. Section 4.5 examines the effects

[16] The next section explains the mechanism in-depth, in that the firm is induced to issue risk-free debt.

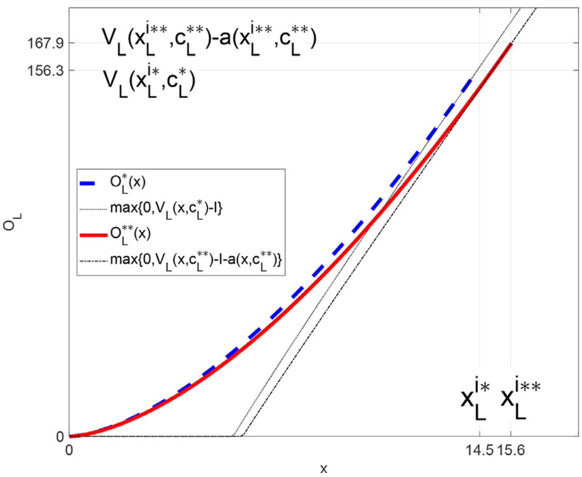


Fig. 2. Option values and investment triggers. This figure depicts the option values and investment triggers. We see $O_L^{**}(x) < O_L^*(x)$ and $x_L^{i**} > x_L^{i*}$. We assume $k = 0.5$ in addition to the baseline parameter values.

on credit spreads and leverage. Section 4.6 investigates the conflicts between the managers and the investors. Section 4.7 considers the social loss from asymmetric information. Section 4.8 examines the effects of the volatility.

4.1. Delayed investment under asymmetric information

This subsection considers how asymmetric information affects the interaction between the option values and investment triggers.

Figure 2 depicts O_L (option value) and x_L^{i} (investment trigger). We assume $k_L = 0.5$ in addition to the baseline parameter values. Here, $O_L^{**}(x)$ is defined as (24) and $O_L^*(x) := (x/x_L^{\mathrm{i}*})^{\beta}\{V_{L2}(x_L^{\mathrm{i}*}, c_L^*) - I\}$. We see that $O_L^{**}(x) < O_L^*(x)$, and $x_L^{\mathrm{i}**} > x_L^{\mathrm{i}*}$, implying that asymmetric information decreases the option value and increases investment trigger (delays investment).

Importantly, while Morellec and Schurhoff (2011) show that asymmetric information speeds up investment, this study's analysis demonstrates that asymmetric information delays investment. The difference in these investment strategies is not surprising. In the screening game for the shareholder–creditor (or shareholder–new shareholder) conflict of Morellec and Schurhoff (2011), the firm in L (the bad type) aims to mimic the firm in H (the good type) to extract more informational rent. The equilibrium is achieved by making mimicking more costly for the firm in L by speeding up investment for the firm in H and rendering investment for the firm in L unchanged. Thus, the distance of the investment triggers for the firms in H and L is enlarged under asymmetric information. In the screening game for the manager-shareholder conflict, the firm in H aims to mimic the firm in L to extract more informational rent. The equilibrium is achieved by making mimicking more costly for the firm in H by delaying investment for the firm in L and rendering investment for the firm in H unchanged. Thus, at the equilibrium, the distance of the investment triggers under asymmetric information, $x_L^{\mathrm{i}**} - x_H^{\mathrm{i}*}$, is larger than under symmetric information, $x_L^{\mathrm{i}*} - x_H^{\mathrm{i}*}$. Hence, the mechanism by which asymmetric information induces the enlarged distance of the investment triggers between the firms in H and L is identical to that in Morellec and Schurhoff (2011).

4.2. Mechanism of issuing risk-free debt

This subsection clarifies the mechanism in which a firm with collateral ($k_L > 0$) issues a mix of risk-free debt and equity whereas the firm without collateral ($k_L = 0$) issues only equity when the degree of asymmetric information is significant.

Figure 3 (a)–(c) depict O_L (option value) with c_L, according to $\Delta k \in \{0.2, 0.7, 1\}$ (we assume $k_L \in \{0.8, 0.3, 0\}$ and $k_H = 1$). In Fig. 3(a), we assume $\Delta k = 0.2$, implying a small degree of asymmetric information. The optimal coupon is $c_L^{**} = 5.7 > \theta_{L1} = 2.88$. Thus, the firm in L issues a mix of risky debt and equity even under asymmetric information. However, in Fig. 3(b), we assume $\Delta k = 0.7$ under $k_L = 0.3$, implying a significant degree of asymmetric information and a positive-liquidation value, which enables the firm in L to provide the amount of money as collateral to the creditor at the time of liquidation. The optimal coupon is $c_L^{**} = \theta_{L1} = 1.08 > 0$, which implies that the firm in L issues a mix of risk-free debt and equity. In Fig. 3(c), we assume $\Delta k = 1$ under $k_L = 0$, implying a significant degree of asymmetric information and a zero-liquidation value, which do not enable the firm in L to provide any money at the time of the liquidation. The optimal coupon is $c_L^{**} = \theta_{L1} = 0$; thus, the firm in L does not issue debt. The following corollary summarizes these results.

Corollary 1. *Suppose the degree of asymmetric information is significant. The firm in L with collateral issues a mix of risk-free debt and equity, whose debt issuance amount equals the collateral value. However, the firm in L without collateral issues only equity.*

We now explain the mechanism by which the firm in L with collateral is induced to issue a mix of risk-free debt and equity when the degree of asymmetric information is significant. Suppose, as a benchmark, the symmetric information case. Figure 3(d) provides the close-up of O_L^* in Fig. 3(b). Here, O_{L1}^* (O_{L2}^*) is an increasing (concave) function of $c_L \le 1.08$ ($c_L > 1.08$) with $\partial O_{L1}^*/\partial c_L|_{c_L=1.08} = \partial O_{L2}^*/\partial c_L|_{c_L \downarrow 1.08} > 0$. Thus, the optimal coupon to maximize O_L^* is $c_L^* = 8.1 > \theta_{L1} = 1.08$, implying that the firm in L issues a mix of risky debt and equity. Suppose the asymmetric information case. Figure 3(e) depicts the close-up of O_L^{**} in Fig. 3(b). Then, O_{L1}^{**} is an increasing function of $c_L \le 1.08$. Interestingly, however, O_{L2}^{**} is a decreasing function of $c_L > 1.08$ because the degree of asymmetric information is significant.[17] Hence, the optimal coupon to maximize O_L^{**} is $c_L^{**} = \theta_{L1} = 1.08$, implying that the firm in L issues a mix of risk-free debt and equity. Moreover, if the firm in L does not have collateral ($k_L = 0$), we have $\theta_{L1} = 0$, implying that the firm in L without collateral is induced to issue equity only.

4.3. Capital and debt structure

In this subsection, for asymmetric information, the study considers the financing strategies of the firm in L, depending on the magnitudes of the parameters k_H and k_L. Here, there are three different financing strategies: a mix of risky debt and equity, a mix of risk-free debt and equity, and all-equity.

Figure 4 depicts the regions where one of the three types of strategies is preferred in the space (k_H, k_L). The line from $(k_H, k_L) = (0, 0)$ to $(k_H, k_L) = (1, 1)$ indicates the boundary of $k_H = k_L$. Given the assumption that $k_H \ge k_L$, we consider only the lower-right triangular region to the boundary of $k_H = k_L$. Under the symmetric information benchmark, firms in H and L issue a mix of risky debt and equity for the entire lower-right triangular region. See Lemma 4 for details.

Under asymmetric information, we consider the preference of capital and debt structure for the firm in L.[18] The firm in L issues

[17] As in (B.18), $a_2(x_L^{\mathrm{i}**}, c_L)$ is an increasing function of c_L, whereas $V_{L2}(x_L^{\mathrm{i}**}, c_L)$ is a concave function of c_L. When c_L increases from θ_{L1} for a significantly large Δk, an increase in $a_2(x_L^{\mathrm{i}}, c_L)$ dominates an increase in $V_{L2}(x_L^{\mathrm{i}}, c_L)$. Thus, $V_{L2}(x_L^{\mathrm{i}}, c_L) - a_2(x_L^{\mathrm{i}}, c_L)$ decreases with c_L, implying that O_{L2}^{**} decreases with c_L.

[18] The firm in H always issues a mix of risky debt and equity even under asymmetric information.

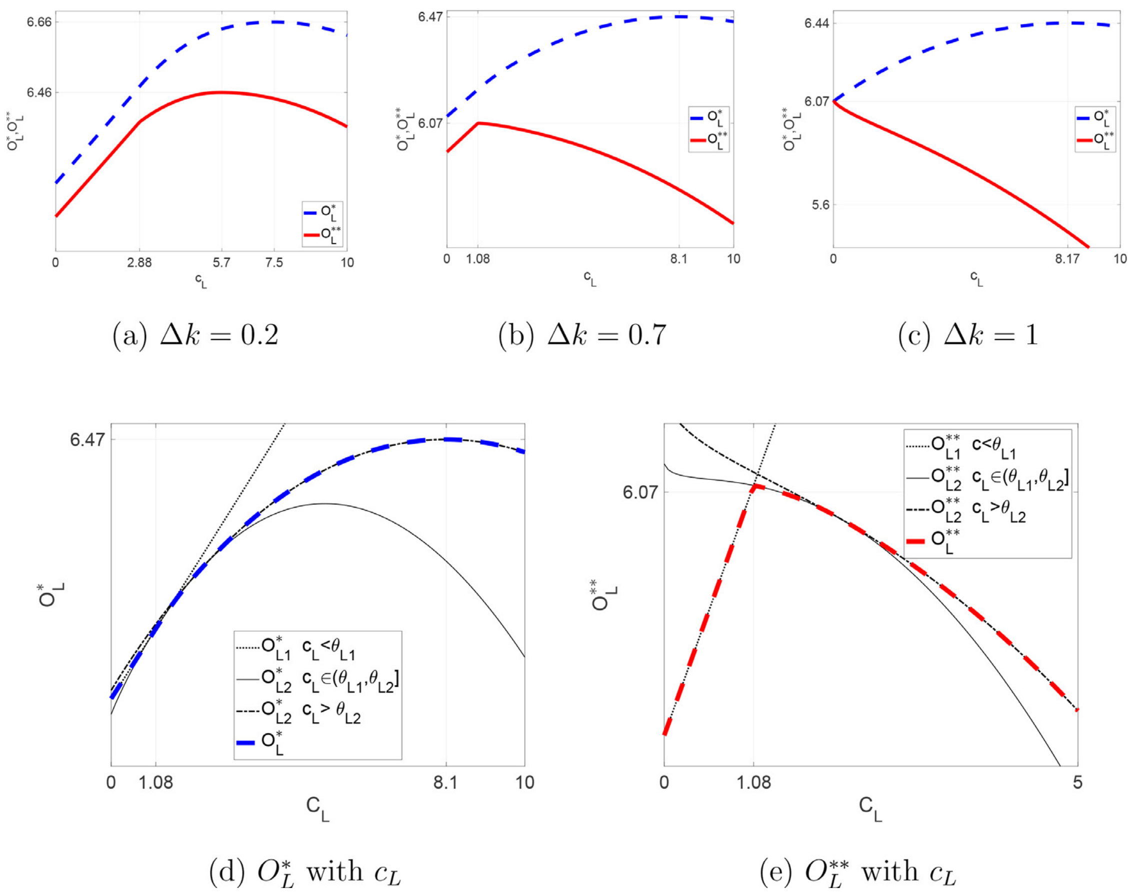


Fig. 3. Values with c_L. In Panels (a)–(c), (k_H, k_L) are assumedly (1,0.8), (1,0.3), and (1,0), respectively. Panels (d) and (e) depict the close-ups of O^*_L and O^{**}_L in Panel (b). The other parameters are the baseline parameter values.

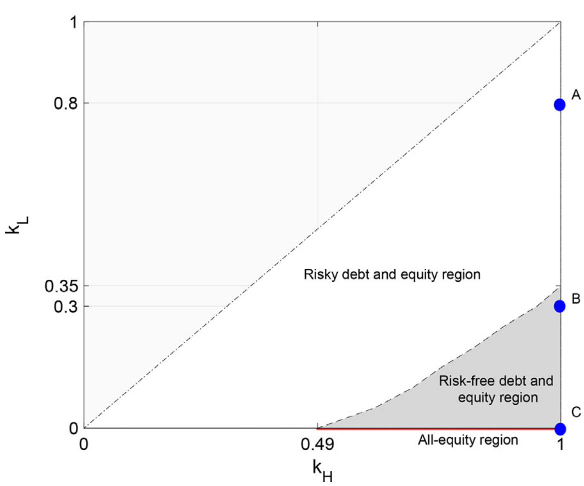


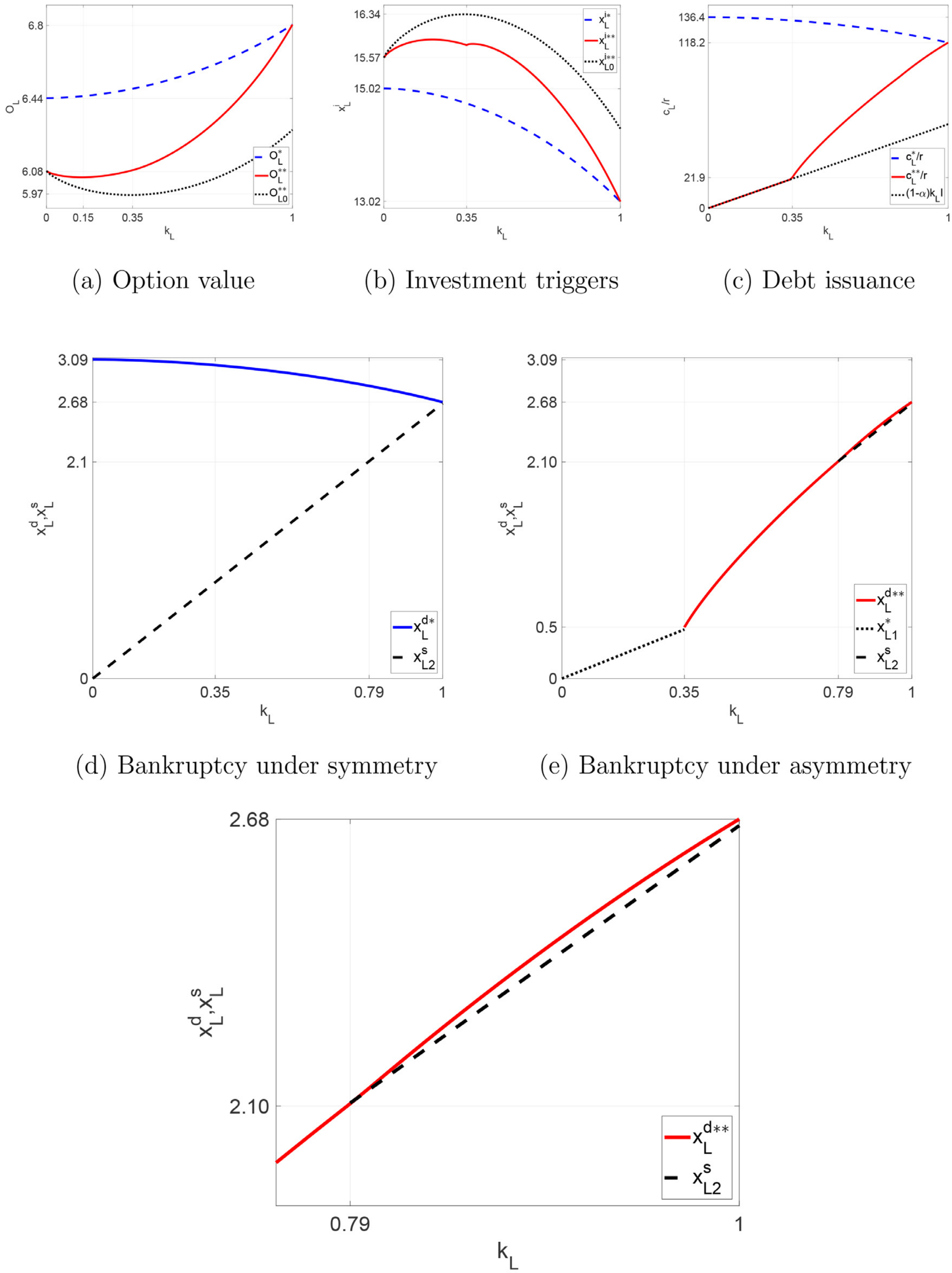
Fig. 4. Capital structures under asymmetric information. This figure depicts the financing strategy for the firm in L in space (k_H, k_L). The other parameters are the baseline values.

a mix of risky debt and equity, a mix of risk-free debt and equity, and all-equity financing at Points A, B, and C, respectively. See Fig. 3(a)–(c). The line from $(k_H, k_L) = (0.49, 0)$ to $(k_H, k_L) = (1, 0.35)$ indicates the boundary of $O^{**}_{L1} = O^{**}_{L2}$. The upper-left region to the boundary of $O^{**}_{L1} = O^{**}_{L2}$ is the region of mixed financing consisting of risky debt and equity, even under asymmetric information, because $\Delta k \geq 0$ in this region is not significant. The lower-right region to the boundary of $O^{**}_{L1} = O^{**}_{L2}$, other than the line from $(k_H, k_L) = (0.49, 0)$ to $(k_H, k_L) = (1, 0)$, is the region of mixed financing consisting of risk-free debt and equity. The line from $(k_H, k_L) = (0.49, 0)$ to $(k_H, k_L) = (1, 0)$ is the region of all-equity financing.

In summary, when the degree of asymmetric information is significant, the firm in L with collateral issues a mix of risk-free debt and equity, whose debt issuance amount equals the collateral value. The mechanism is induced by reducing debt issuance via increasing the degree of asymmetric information. To the best of our knowledge, prior relevant studies do not show such results. For example, in Morellec and Schurhoff (2011), the firm allows for debt or equity financing and issues a fixed amount of debt, which is equal to the investment cost. Shibata and Nishihara (2010) do not consider a risk-free debt financing because the investment is completely irreversible (i.e., no collateral). Additionally, we derive the extreme result that the firm in L without collateral takes all-equity financing at the equilibrium. This theoretical result accords with those of the theoretical studies by Stiglitz and Weiss (1981), DeMarzo and Duffie (1999), Hennessy et al. (2010), Morellec and Schurhoff (2011), and Piskorski and Westerfield (2016), where the H-type (L-type) firms issue debt (equity).

4.4. Default and liquidation strategies

In this subsection, we consider how asymmetric information affects default and liquidation triggers. In Fig. 5, we assume $k_H = 1$ and $k_L \in [0, 1]$ in addition to the base line parameters. Note that an increase in Δk corresponds to a decrease in k_L. When the degree of asymmetric information is significant ($k_L \in (0, 0.35]$), it induces



(a) Option value (b) Investment triggers (c) Debt issuance

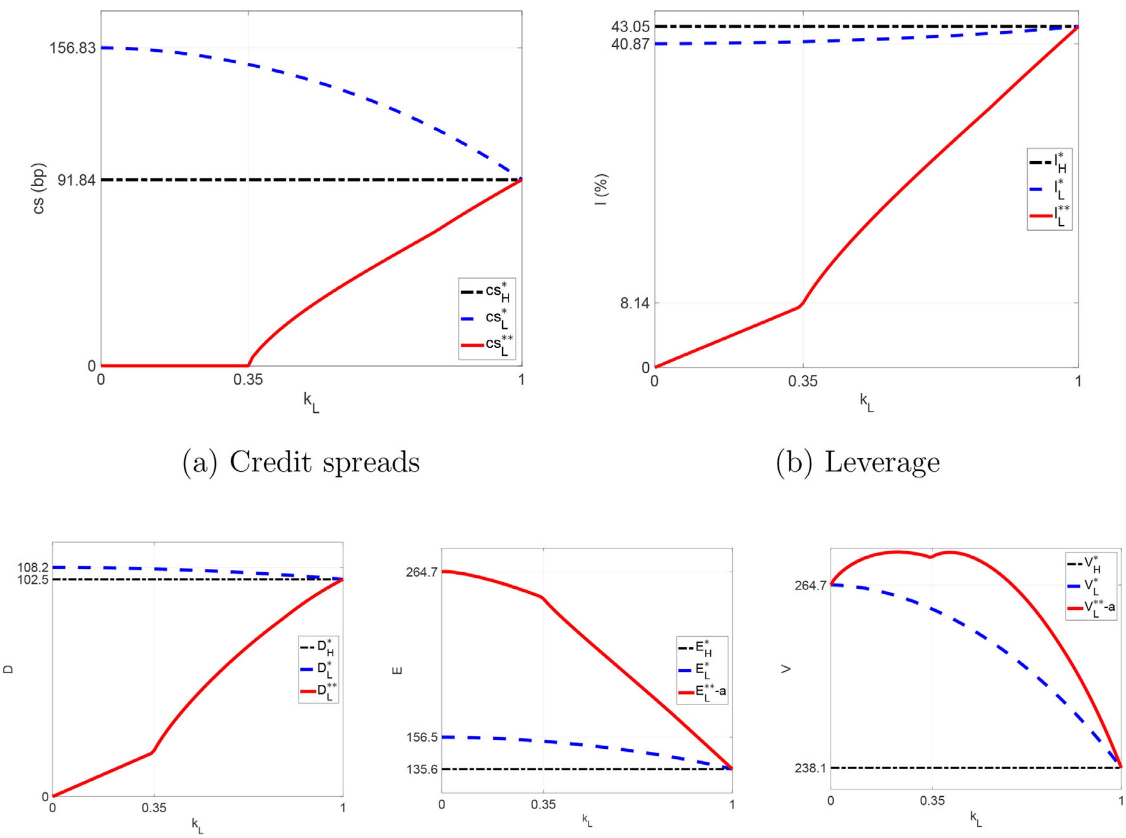
(d) Bankruptcy under symmetry (e) Bankruptcy under asymmetry

(f) Close-up for $k_L \in [0.79, 1]$

Fig. 5. Default and liquidation triggers. We assume $k_H = 1$ in addition to the baseline parameter values. Panels (d) and (e) depict the default and liquidation triggers under symmetric and asymmetric information, respectively. Panel (f) depicts a close-up of triggers in Panel (e).

the firm in L to issue a mix of risk-free debt and equity. When the degree of asymmetric information is small ($k \in (0.35, 1]$), the firm in L then issues a mix of risky debt and equity, even under asymmetric information.

Just before investigating x_L^{d**} (default trigger) and x_{Lm}^s (liquidation trigger), we begin with reviewing the corresponding results of O_L^{**}, x_L^{i**}, and c_L^{**}/r in Fig. 5(a)–(c). These three reconsiderations help us to understand x_L^{d**} and x_{Lm}^s in more detail.



(a) Credit spreads (b) Leverage

(c) Debt value (d) Equity value (e) Total firm value

Fig. 6. Credit spreads and leverage. We assume $k_H = 1$ in addition to the baseline parameter values.

First, Fig. 5 (a) depicts O_L^{**} (option value in L). We have $O_L^{**} = O_{L0}^{**}$ for $k_L = 0$ and $O_L^{**} > O_{L0}^{**}$ for $k_L \in (0, 1]$, where O_{L0}^{**} indicates the option value in L under all-equity financing. Thus, the firm in L issues debt as long as it possesses collateral ($k_L > 0$). As in Proposition 1, $O_L^{**} \leq O_L^{*}$, where asymmetric information reduces the value. As in Proposition 2, $O_L^{**} = O_{L2}^{**}$ decreases with Δk (a decrease in $k_L \geq 0.35$). Second, Fig. 5 (b) depicts x_L^{i**} (investment trigger). Here, x_{L0}^{i**} represents the trigger for the firm in L under all-equity financing. We see $x_L^{i**} = x_{L0}^{i**}$ for $k_L = 0$. As in Proposition 2, $x_L^{i**} \geq x_L^{i*}$, where asymmetric information delays investment. Additionally, for $k_L \in (0.35, 1]$, x_L^{i**} increases monotonically with Δk (a decrease in k_L). However, for $k_L \in (0, 0.35]$, x_L^{i**} has an inverse U-shaped relationship with Δk. This result theoretically accords with Martin (2009), where the relationship between the amounts of investment and collateral need not be monotonic under adverse selection in a static approach. Third, Fig. 5 (c) depicts c_L^{**}/r (debt issuance). As in Proposition 2, $c_L^{**}/r \leq c_L^{*}/r$. Additionally, $c_L^{**} = \theta_{L1}$ for $k_L \in [0, 0.35]$, whereas we have $c_L^{*} > \theta_L$ for $k_L \in (0.35, 1]$.

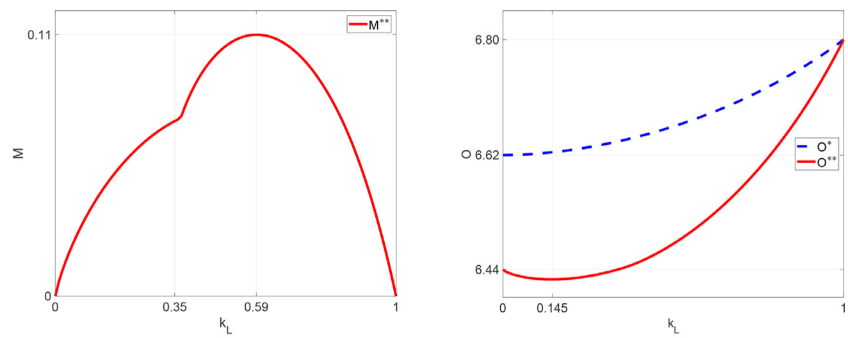
We consider the default and liquidation triggers. Figure 5(d) shows the symmetric information bankruptcy triggers: x_L^{d*} and x_{L2}^{s}, as a benchmark. Recall that the firm in L defaults (liquidates) once $X(t)$ reaches x_L^{d} (x_{L2}^{s}) from the above. We obtain $x_L^{d*} > x_{L2}^{s}$ for $k_L \in [0, 1]$, implying that the firm in L always employs the default and liquidation *sequentially* at x_L^{d*} and x_{L2}^{s}, respectively. Figure 5(e) depicts the asymmetric information bankruptcy triggers: $x_L^{d**} = x^{d}(c_L^{**})$, $x_{L1}^{s}(\theta_{L1})$, and x_{L2}^{s}. For $k_L = 0$, the firm in L issues equity only. For $k_L \in (0, 0.35]$, the firm in L issues a mix of risk-free debt and equity. Thus, for k_L $[0, 0.35]$, it exercises only the liquidation at x_{L1}^{s} because there is no default under all-equity and mix of risk-debt and equity financing. As in Lemma 3, we see $x_{L1}^{s} = x_L^{d**}$ at $c_L = 0.35$. For $k_L > 0.35$, the firm in L issues a mix of risky debt and equity. Figure 5(f) provides a close-up of x_L^{d**} and x_{L2}^{s} for $k_L \in [0.75, 1]$ in Fig. 5(e). From two these figures, we obtain

$$\min\{x_L^{d**}, x_{L2}^{s}\} = \begin{cases} x_L^{d**}, & k_L \in (0.35, 0.795], \\ x_{L2}^{s}, & k_L \in (0.795, 1]. \end{cases}$$

Thus, for $k_L \in (0.35, 0.795]$, the firm in L exercises default and liquidation *simultaneously* at x_L^{d**}. For $k_L \in (0.795, 1]$, the firm in L exercises default and liquidation *sequentially* at x_L^{d**} and x_{L2}^{s}, respectively. Note that $x_L^{d*} = x_L^{d**}$ for $k_L = k_H = 1$ because there does not exist asymmetric information. From Fig. 5(d)–(f), consequently, for $k_L \in (0.35, 0.795]$, asymmetric information changes the default-liquidation strategy from sequential to simultaneous. We summarize these results in the following observation.

Observation 1. Suppose that the firm in L issues a mix of risky debt and equity even under asymmetric information. An increased degree of asymmetric information induces the firm in L to employ a simultaneous rather than a sequential default-liquidation strategy via a change in the capital structure to reduce debt issuance.

Observation 1 implies that an increased degree of asymmetric information is more likely to induce the firm to file for Chapter 7 liquidation than Chapter 11 reorganization because asymmetric information reduces the optimal amount of debt issuance. This result is consistent with the empirical finding of Bris et al. (2006) and the theoretical finding of Nishihara and Shibata (2017).



(a) Managers' values (b) Investors' values

Fig. 7. Managers' and investors' values. We assume $k_H = 1$ in addition to the baseline parameter values.

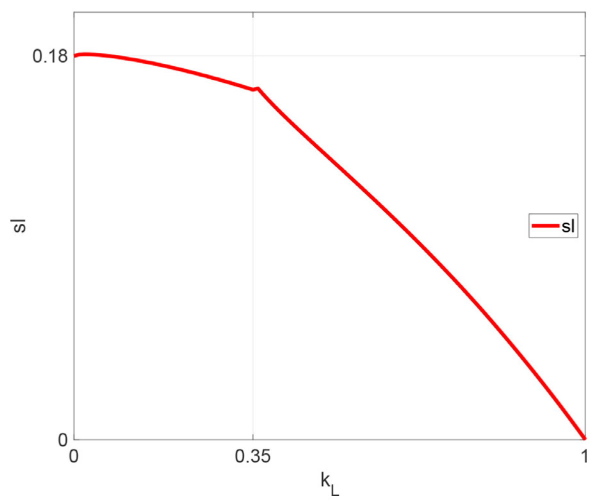


Fig. 8. Social losses. We assume $k_H = 0$ in addition to the baseline parameter values.

4.5. Credit spreads and leverage

In this subsection, we consider how asymmetric information affects credit spread and leverage. For the firm in L under asymmetric information, the equity value is defined as $E_{Lm}(x_L^{\mathrm{i}}, c_L) - a_m(x_L^{\mathrm{i}}, c_L)$, whereas the debt value remains unchanged as $D_{Lm}(x_L^{\mathrm{i}}, c_L)$. Here, the distortion, $a_m(x_L^{\mathrm{i}}, c_L) \geq 0$, is given in (23) and derived via the wage incentive to the managers from the investors. The total firm value is the sum of two values; that is, $V_{Lm}(x_L^{\mathrm{i}}, c_L) - a_m(x_L^{\mathrm{i}}, c_L)$.

The credit spread (in basis points) is defined as

$$cs_L := \Big(\frac{c_L}{D_{Lm}(x_L^{\mathrm{i}}, c_L)} - r\Big) \times 10^4, \quad m \in \{1, 2\}.$$

We obtain $cs_L = 0$ for $m = 1$ because of $D_{L1} = c_L/r$, but $cs_L > 0$ for $m = 2$ because of $D_{L2} < c_L/r$. Moreover, the leverage (percentage) is defined as

$$l_L := \frac{D_{Lm}(x_L^{\mathrm{i}}, c_L)}{V_{Lm}(x_L^{\mathrm{i}}, c_L) - a_m(x_L^{\mathrm{i}}, c_L)} \times 10^2.$$

Figure 6 (a) and (b) depict cs_L (credit spread) and l_L (leverage), respectively. We see $cs_L^{**} \leq cs_L^*$ and $l_L^{**} \leq l_L^*$. Additionally, cs_L^{**} and l_L^{**} decrease monotonically with Δk (a decrease in k_L). The following observation summarizes these results.

Observation 2. Suppose that the firm in L issues a mix of risky debt and equity. Then, an increased degree of asymmetric information decreases the credit spread and leverage monotonically.

We explain the mechanism of the two results as follows. First, we show why asymmetric information reduces credit spreads. As in Figs. 5(c) and 6(c), asymmetric information reduces c_L/r (face value) and D_L (debt value). Importantly, the decrease in c_L/r is more significant than in D_L. Thus, asymmetric information reduces cs_L (credit spreads). Second, we state why asymmetric information reduces leverage. As in Fig. 6(d), asymmetric information rises $E_L - a$ (equity value) to compensate for a decrease in D_L. Further, as in Fig. 6(e), asymmetric information rises $V_L - a$ (total firm value) because the magnitude of increase in $E_L - a$ is more significant than the magnitude of decrease in D_L.[19] Thus, asymmetric information reduces l_L (leverage).

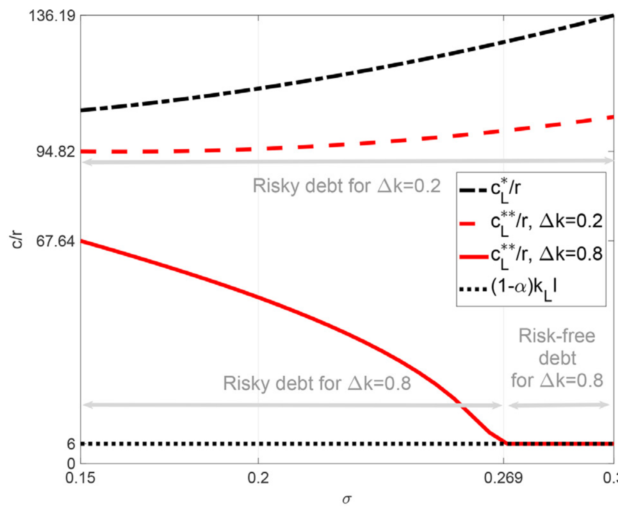
The result in observation 2 may not be counter-intuitive, because this study's results may not match the pecking order hypothesis, where information asymmetry increases credit spreads and leverages. For example, Bharath et al. (2009) show empirically that credit spread increases with the degree of asymmetric information, on the condition that the debt capacity is not considered. However, as noted, there is no consensus on the pecking order hypothesis. Leary and Roberts (2010) state that the divergence is driven primarily by the predictive ability of the capital structure. They show that such predictive ability improves significantly only when we allow firms' debt capacity to vary. In this study, as in Proposition 2, asymmetric information reduces debt issuance, reducing credit spreads and leverage. Based on Leary and Roberts (2010), the result makes sense. Thus, we provide a testable hypothesis that asymmetric information reduces credit spreads and leverage by decreasing debt issuance, whereas asymmetric information increases credit spreads and leverage by increasing debt issuance.

4.6. Conflicts between well-informed managers and less-informed investors

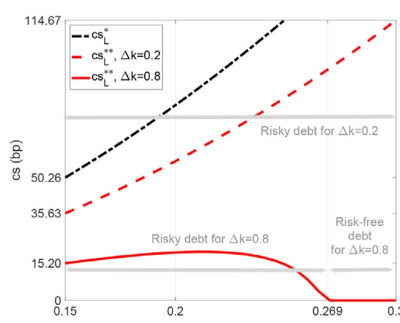
We now compare the option values of the managers (insiders) and investors (outsiders).

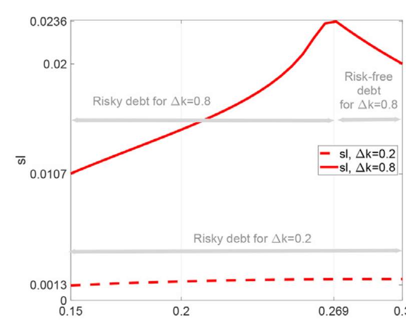
Figure 7 (a) shows $M^{**}(x)$ (managers' option value). The managers' option values have an inverse U-shaped relationship with Δk (a decrease in k_L). We have $M^{**}(x) = 0$ for $k_L = 0$ (given zero-liquidation value) and $M^{**}(x) = 0$ for $k_L = 1$ (given no informational rent). Importantly, $M^{**}(x)$ decreases with Δk for $k_L \in (0, 0.59]$ but increases with Δk for $k_L \in (0.59, 1]$. Here, two conflicting effects drive the shape of the managers' option value. First, an increase in Δk (a decrease in k_L) increases $M^{**}(x)$ (the "informational advantage effect"). Second, an increase in Δk decreases $M^{**}(x)$ via a decrease in c_L^{**} (the "leverage effect"). Thus, when Δk

[19] Under asymmetric information, we have $V_{Lm}(x_L^{\mathrm{i}**}, c_L^{**}) - a_m(x_L^{\mathrm{i}**}, c_L^{**}) > V_L(x^{\mathrm{i}*}, c_L^*)$ as in Fig. 2.



(a) Debt issuance





(b) Credit spreads (c) Social losses

Fig. 9. Effects of volatility. We consider two cases—$\Delta k = 0.8$ and $\Delta k = 0.2$—by assuming $(k_H, k_L) = (0.9, 0.1)$ and $(k_H, k_L) = (0.3, 0.1)$. The other parameters are the baseline parameter values. The former and latter cases correspond to those for large and small degrees of asymmetric information, respectively.

is small ($k_L \in (0.59, 1]$), the informational advantage effect dominates the leverage effect. Hence, $M^{**}(x)$ increases with Δk. However, when Δk is significant ($k_L \in [0, 0.59]$), the leverage effect dominates the informational advantage effect. Thus, $M^{**}(x)$ decreases with Δk. Therefore, conflicting effects induce the inverse U-shaped curve.

Figure 7 (b) shows $O^{**}(x)$ (investors' option value), where $O^{**}(x)$ is defined by (22). O^{**} decreases with Δk (a decrease in k_L) for $k_L \in (0.145, 1]$ but increases with Δk for $k_L \in [0, 0.145]$. Precisely, when the firm issues risky debt, O^{**} decreases with Δk. However, when the firm issues risk-free debt, O^{**} has an inverse U-shaped curve with Δk.

From Fig. 7(a) and (b), an increase in Δk (a decrease in k_L) transfers the wealth from the investors to managers when the degree of asymmetric information is relatively small ($k_L \in (0.59, 1]$). However, an increase in Δk does not always transfer the wealth from investors to managers when the degree of asymmetric information is relatively significant ($k_L \in [0, 0.59]$). Particularly, for $k_L \in (0.14, 0.59]$, an increase in Δk reduces investors' and managers' values. We summarize these results as follows.

Observation 3. Suppose that the firm in L issues a mix of risky debt and equity. An increased degree of asymmetric information likely shifts the wealth from the less-informed investors to the well-informed managers ("asset substitution"). However, suppose that the firm in L issues a mix of risk-free debt and equity. An increased degree of asymmetric information does not always shift the wealth.

It is common knowledge that asset substitution does not always occur. An increase in Δk always reduces the total welfare as the sum of the investors' and managers' values (i.e., the social loss increases), as in the next subsection.

4.7. Social losses of information asymmetry

We examine the effects of asymmetric information regarding social loss; the social loss $sl(x)$ is defined as

$$sl(x) := O^{*}(x) - O^{**}(x) - M^{**}(x) \geq 0,$$

where $x \leq x_H^{i*}$. The notion of social loss here is identical to that of Grenadier and Wang (2005).

Figure 8 depicts sl (social loss). We see sl increases with Δk (a decrease in k_L). Moreover, when a firm is induced to employ all-equity financing or risk-free debt financing, social loss increases. We summarize these results as follows.

Observation 4. An increased degree of asymmetric information increases social loss. In particularly, the social loss is more significant under mixed financing of risk-free debt and equity (or all-equity financing) than under mixed financing of risky debt and equity.

The results in observation 4 highly relate to the firm's benefit from the tax shield. An increased degree of asymmetric information induces the firm to reduce the amount of debt issuance, which induces a decrease in the firm's benefit from the tax shield. Thus, the more significant the degree of asymmetric information, the more significant the social loss.

4.8. Effects of volatility

We examine the effects of σ (volatility). Here, σ changes from 0.15 to 0.3. We consider two cases—$\Delta k = 0.8$ and $\Delta k = 0.2$—by assuming $(k_H, k_L) = (0.9, 0.1)$ and $(k_H, k_L) = (0.3, 0.1)$ in addition to the baseline parameter values. The former and latter cases correspond to those for the large and small degrees of asymmetric information.

Figure 9 (a) depicts c_L/r (optimal amount of debt issuance). Under symmetric information benchmark, c_L^*/r increases with σ, as in Leland (1994), Sundaresan and Wang (2007), and Shibata and Nishihara (2015). Consider asymmetric information. When the degree of asymmetric information is small ($\Delta k = 0.2$), the red dotted line, c_L^{**}/r, increases with σ. This property is identical to that under symmetric information. Interestingly, however, when the degree of asymmetric information is significant ($\Delta k = 0.8$), the red solid line, c_L^{**}/r, decreases with σ. This result accords with the theoretical findings of studies such as Myers (1984), Myers and Majluf (1984), and DeMarzo and Duffie (1999)[20] and the empirical findings of studies such as Fama and French (2005). Moreover, the debt is risky for $\sigma \in [0.15, 0.269)$, whereas it is risk-free for $\sigma \in [0.269, 0.3]$.

In summary, under asymmetric information, volatility effects under $\Delta k = 0.2$ are different from those under $\Delta k = 0.8$ because of two conflicting effects: positive and negative effects. Under $\Delta k = 0.2$, a positive effect dominates a negative effect, implying that an increase in σ increases debt issuance. However, under $\Delta k = 0.8$, a negative effect dominates a positive effect, indicating that an increase in σ decreases debt issuance. The following observation summarizes this result.

Observation 5. The volatility effects under a significant degree of asymmetric information differ from those under a small degree. When the degree of asymmetric information is large (small), an increase in volatility decreases (increases) debt issuance.

Figure 9 (b) shows cs_L (credit spread). Under symmetric information, as a benchmark, cs_L^* increases with σ. Consider asymmetric information cases. When the degree of asymmetric information is small ($\Delta k = 0.2$), the red dotted line, cs_L^{**}, increases with σ. This property is identical to that under symmetric information. When the degree of asymmetric information is significant ($\Delta k = 0.8$), the red solid line, cs_L^{**}, has an inverse U-shaped relationship with σ. The nonmonotonic relationship is induced by changing the debt from risky to risk-free with increasing σ.

Figure 9 (c) depicts sl (social loss). When the degree of asymmetric information is small ($\Delta k = 0.2$), the red dotted line increases monotonically with σ. However, when the degree of asymmetric information is significant ($\Delta k = 0.8$), the red solid line has a Λ-shaped relationship with σ. The peak of the Λ-shaped curve is at the change of the debt from risky to risk-free.

5. Conclusion

This study examines how asymmetric information about the liquidation value between well-informed managers and less-informed investors affects a firm's financing (capital structure) and investment strategies.

It yields two novel results. First, asymmetric information at the time of liquidation delays investment and reduces the optimal amount of debt issuance. Second, when the degree of asymmetric information is significant, an increase in volatility decreases debt issuance under asymmetric information, unlike symmetric information.

The result of reduced debt issuance given asymmetric information has three economic implications. First, a decrease in debt issuance may induce a firm to issue a mix of risk-free debt and equity. That is, the firm must prepare a perfect guarantee for its debt's face value when a degree of asymmetric information is significant, which is quite different from that under symmetric information, where the firm always issues a mix of risky debt and equity. In other words, a firm with (without) collateral issues a mix of risk-free debt and equity (only equity). Thus, this theoretical result provides a new testable implication. Second, a decrease in debt issuance may change the firm's bankruptcy strategies because such a decrease delays the default (operating concern bankruptcy) but does not affect the liquidation bankruptcy (shutdown). For example, by delaying default given asymmetric information, a firm's optimal bankruptcy strategies are changed from sequential default-liquidation to simultaneous default-liquidation. In practice, defaults and liquidations are regarded as Chapter 11 reorganizations and Chapter 7 liquidations, respectively, of the U.S. bankruptcy code. Hence, as the degree of asymmetric information increases, a firm becomes more likely to file for Chapter 7 rather than Chapter 11. This result is consistent with the empirical finding of Bris et al. (2006). Third, a decrease in debt issuance decreases credit spread and leverage. The reason for the former is that the decrease in coupon payments dominates the decrease in the market value of the debt. The reason for the latter is that the decrease in the market value of the debt dominates the decrease in the market value of the equity. These results differ from those of Shibata and Nishihara (2010), where leverage and credit spreads are invariant of asymmetric information. Asymmetric information at the time of liquidation affects the leverage and credit spreads, although asymmetric information at the time of investment does not affect them.

Data availability

No data was used for the research described in the article.

CRediT authorship contribution statement

Takashi Shibata: Conceptualization, Methodology, Writing – original draft, Investigation, Writing – review & editing, Supervision, Validation. **Michi Nishihara:** Conceptualization, Methodology, Writing – original draft, Investigation, Supervision, Validation.

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[20] In DeMarzo and Duffie (1999), the optimal face value of debt decreases with volatility.

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Appendix A. Derivation of value functions

Given x_j^{i} and c_j for $k = k_j$ ($j \in \{H, L\}$), we derive the value functions for $m = 1$ (mixed financing consisting of risk-free debt and equity) and $m = 2$ (mixed financing consisting of risky debt and equity). We then derive the option value of the project.

Value functions for $m = 1$

Suppose $t > T_j^{\mathrm{i}}$ and $m = 1$. Under the assumption of perpetual debt, the face value of debt equals c_j/r. When the debt is risk-free, its market value is given by $D_{j1}(X(t), c_j) = c_j/r \geq 0$, independent of $X(t)$. The value of the equity, $E_{j1}(X(t), c_j)$, is given by

$$E_{j1}\big(X(t), c_j\big) : = \sup_{T_{j1}^{\mathrm{s}}(\geq t)} \mathbb{E}^{X(t)}\Bigg[\mathrm{e}^{rt}\bigg(\int_t^{T_{j1}^{\mathrm{s}}} \mathrm{e}^{-ru}(1-\tau)\big(X(u) - c_j\big)\mathrm{d}u + \mathrm{e}^{-rT_{j1}^{\mathrm{s}}}\Big\{(1-\alpha)k_j I - \frac{c_j}{r}\Big\}\bigg)\Bigg],$$

where $\mathbb{E}^{X(t)}$ denotes the expectation operator, conditional on $X(t)$. Here, the first term represents the cumulative cash flow from t to T_{j1}^{s} (the time of liquidation), and the second term represents the residual value at T_{j1}^{s}. Using the standard arguments of Dixit and Pindyck (1994), $E_{j1}(X(t), c_j)$ is given by

$$E_{j1}(X(t), c_j) = \upsilon X(t) - (1-\tau)\frac{c_j}{r} + \Big\{(1-\alpha)k_j I - \upsilon x_{j1}^{\mathrm{s}}(c_j) - \tau\frac{c_j}{r}\Big\}\Big(\frac{X(t)}{x_{j1}^{\mathrm{s}}(c_j)}\Big)^{\gamma}, \tag{A.1}$$

where the optimal liquidation trigger, $x_{j1}^{\mathrm{s}}(c_j)$, is obtained by

$$\begin{aligned} x_{j1}^{\mathrm{s}}(c_j) &:= \operatorname*{argmax}_y \Big\{(1-\alpha)k_j I - \upsilon y - \tau\frac{c_j}{r}\Big\}\Big(\frac{X(t)}{y}\Big)^{\gamma} \\ &= \frac{\varepsilon}{\upsilon}\Big\{(1-\alpha)k_j I - \tau\frac{c_j}{r}\Big\} \geq 0. \end{aligned} \tag{A.2}$$

Thus, we obtain $V_{j1}(X(t), c_j) = D_{j1}(X(t), c_j) + E_{j1}(X(t), c_j)$ in (4).

Value functions for $m = 2$

Suppose $t > T_j^{\mathrm{i}}$ and $m = 2$. We derive the value functions by working backward in two steps. We first derive the value function after default and then derive the value functions before default. These values are similar to those in Shibata and Nishihara (2018).

Suppose that the firm defaults. The value of the equity after default, $W_j(X(t))$, is defined as

$$W_j(X(t)) := \sup_{T_{j2}^{\mathrm{s}}(\geq t)} \mathbb{E}^{X(t)}\Big[\mathrm{e}^{rt}(1-\alpha)\Big\{\int_t^{T_{j2}^{\mathrm{s}}} \mathrm{e}^{-ru}(1-\tau)X(u)\mathrm{d}u + \mathrm{e}^{-rT_{j2}^{\mathrm{s}}}k_j I\Big\}\Big].$$

Note that $W_j(X(t))$ is the value of the original debt–holders the right of corporate ownership is transferred to after default. Particularly, the first term represents the cumulative cash flow from t (any time after default) to T_{j2}^{s} (the time of liquidation), and the second term represents the residual value at T_{j2}^{s}. As in the derivation of (A.1), $W_j(X(t))$ is given by

$$W_j(X(t)) := \begin{cases} (1-\alpha)k_j I, & x_j^{\mathrm{d}} \leq x_{j2}^{\mathrm{s}}, \\ (1-\alpha)\Big(\upsilon X(t) + \big\{k_j I - \upsilon x_{j2}^{\mathrm{s}}\big\}\Big(\frac{X(t)}{x_{j2}^{\mathrm{s}}}\Big)^{\gamma}\Big), & x_j^{\mathrm{d}} > x_{j2}^{\mathrm{s}}. \end{cases}$$

We then derive the value functions before default. The values of the debt and equity, $D_{j2}(X(t), c_j)$ and $E_{j2}(X(t), c_j)$, are given by

$$\begin{aligned} D_{j2}(X(t), c_j) &:= \mathbb{E}^{X(t)}\Big[\mathrm{e}^{rt}\Big\{\int_t^{T^{\mathrm{d}}(c_j)} \mathrm{e}^{-ru}c_j\mathrm{d}u + \mathrm{e}^{-rT^{\mathrm{d}}(c_j)}W_j(x^{\mathrm{d}}(c_j))\Big\}\Big] \\ &= \frac{c_j}{r} + \Big\{W_j(x^{\mathrm{d}}(c_j)) - \frac{c_j}{r}\Big\}\Big(\frac{X(t)}{x^{\mathrm{d}}(c_j)}\Big)^{\gamma}, \end{aligned} \tag{A.3}$$

$$\begin{aligned} E_{j2}(X(t), c_j) &:= \sup_{T_j^{\mathrm{d}}(\geq t)} \mathbb{E}^{X(t)}\Big[\int_t^{T_j^{\mathrm{d}}} \mathrm{e}^{-r(u-t)}(1-\tau)(X(u) - c_j)\mathrm{d}u\Big], \\ &= \upsilon X(t) - (1-\tau)\frac{c_j}{r} + \Big\{(1-\tau)\frac{c_j}{r} - \upsilon x^{\mathrm{d}}(c_j)\Big\}\Big(\frac{X(t)}{x^{\mathrm{d}}(c_j)}\Big)^{\gamma}, \end{aligned} \tag{A.4}$$

respectively. The optimal default and liquidation triggers, $x^{\mathrm{d}}(c_j)$ and x_{j2}^{s}, are

$$\begin{aligned} x^{\mathrm{d}}(c_j) &:= \operatorname*{argmax}_y \Big\{\upsilon X(t) - (1-\tau)\frac{c_j}{r} + \Big\{(1-\tau)\frac{c_j}{r} - \upsilon y\Big\}\Big(\frac{X(t)}{y}\Big)^{\gamma}\Big\} \\ &= \frac{\varepsilon}{\upsilon}\frac{1-\tau}{r}c_j = \frac{c_j}{\eta} > 0, \end{aligned} \tag{A.5}$$

$$\begin{aligned} x_{j2}^{\mathrm{s}} &:= \operatorname*{argmax}_y \Big\{\upsilon X(t) + \big\{k_j I - \upsilon y\big\}\Big(\frac{X(t)}{y}\Big)^{\gamma}\Big\} \\ &= \frac{\varepsilon}{\upsilon}k_j I \geq 0. \end{aligned} \tag{A.6}$$

Thus, we obtain $V_{j2}(X(t), c_j) = D_{j2}(X(t), c_j) + E_{j2}(X(t), c_j)$ in (7).

Option value of the project

Given x_j^{i} and c_j for $k = k_j$ ($j \in \{H, L\}$), the option value of the project is given by

$$\sup_{T_j^{\mathrm{i}}, c_j, w_j} \mathbb{E}^x\Big[\mathrm{e}^{-rT_j^{\mathrm{i}}}\Big(E_{jm}(X(T_j^{\mathrm{i}}), c_j) - \big\{I - D_{jm}(X(T_j^{\mathrm{i}}), c_j)\big\} - w_j\Big)\Big], \tag{A.7}$$

where $x := X(0) > 0$. Using standard arguments, the discounted factor of (A.7) is rewritten as

$$\mathbb{E}^x[\mathrm{e}^{-rT_j^{\mathrm{i}}}] = \Big(\frac{x}{x_j^{\mathrm{i}}}\Big)^{\beta}.$$

Thus, we obtain the option value of the project for $k = k_j$ in (13).

Appendix B. Proofs of lemmas and propositions

Proof of Lemma 1

Suppose $x := X(t) > x_{j1}^{\mathrm{s}}(c_j) > 0$ and $m = 1$. The total firm value, $V_{j1}(x, c_j)$, is given as

$$V_{j1}(x, c_j) = \upsilon x + \tau\frac{c_j}{r} + \Big\{(1-\alpha)k_j I - \upsilon x_{j1}^{\mathrm{s}}(c_j) - \tau\frac{c_j}{r}\Big\}\Big(\frac{x}{x_{j1}^{\mathrm{s}}(c_j)}\Big)^{\gamma}, \quad c_j \leq \theta_{j1}. \tag{B.1}$$

Differentiating $V_{j1}(x, c_j)$ with respect to c_j yields

$$\begin{aligned} \frac{\mathrm{d}V_{j1}\big(x, c_j\big)}{\mathrm{d}c_j} &= \frac{\partial V_{j1}\big(x, c_j\big)}{\partial c_j} + \underbrace{\frac{\partial V_{j1}\big(x, c_j\big)}{\partial x_{j1}^{\mathrm{s}}\big(c_j\big)}}_{=0}\frac{\partial x_{j1}^{\mathrm{s}}\big(c_j\big)}{\partial c_j} \\ &= \frac{\tau}{r}\left\{1 - \left(\frac{x}{x_{j1}^{\mathrm{s}}\big(c_j\big)}\right)^{\gamma}\right\} \geq 0, \end{aligned} \tag{B.2}$$

where we use the envelope theorem, i.e., $\partial V_{j1}/\partial x_{j1}^{\mathrm{s}} = \partial E_{j1}/\partial x_{j1}^{\mathrm{s}} = 0$. The positive sign of (B.2) stems from $\gamma < 0$. Thus, $V_{j1}(x, c_j)$ increases with c_j.

Proof of Lemma 2

Suppose $x := X(t) > \max\{x^{\mathrm{d}}(c_j), x^{\mathrm{s}}_{j2}\} > 0$ and $m = 2$. Note that we cannot use the envelope theorem because $\partial V_{j2}/\partial x^{\mathrm{d}} \neq \partial E_{j2}/\partial x^{\mathrm{d}}$. Substituting $x^{\mathrm{d}}(c_j) = c_j/\eta$ into $V_{j2}(x, c_j)$ yields

$$V_{j2}(x,c_j) = \begin{cases} \upsilon x + \tau\dfrac{c_j}{r} + \underbrace{(1-\alpha)k_j I\left(\frac{\eta x}{c_j}\right)^{\gamma}}_{=:\Psi_{j11}} - \underbrace{\frac{\varepsilon(1-\tau)+\tau}{r} c_j\left(\frac{\eta x}{c_j}\right)^{\gamma}}_{=:\Psi_{j21}}, & c_j \in (\theta_{j1}, \theta_{j2}], \\ \upsilon x + \tau\dfrac{c_j}{r} + \underbrace{\frac{(1-\alpha)k_j I}{1-\gamma}\left(\frac{x}{x^{\mathrm{s}}_{j2}}\right)^{\gamma}}_{=:\Psi_{j12}} - \underbrace{\frac{\alpha\varepsilon(1-\tau)+\tau}{r} c_j\left(\frac{\eta x}{c_j}\right)^{\gamma}}_{=:\Psi_{j22}}, & c_j > \theta_{j2}. \end{cases} \tag{B.3}$$

Differentiating $V_{j2}(x, c_j)$ with respect to c_j, we have

$$\frac{\mathrm{d}V_{j2}(x,c_j)}{\mathrm{d}c_j} = \begin{cases} \frac{\tau}{r}\left(1 - \left(\frac{\eta x}{c_j}\right)^{\gamma}\left\{1 + \frac{\gamma}{\tau}\underbrace{\left(\frac{\theta_{j1}}{c_j} - 1\right)}_{<0}\right\}\right), & c_j \in (\theta_{j1}, \theta_{j2}], \\ \frac{\tau}{r}\left(1 - \left(\frac{\eta x}{c_j}\right)^{\gamma}\underbrace{\left\{1 - \gamma\left(1 + \alpha\frac{1-\tau}{\tau}\right)\right\}}_{>0}\right), & c_j > \theta_{j2}, \end{cases} \tag{B.4}$$

and

$$\frac{\mathrm{d}^2V_{j2}(x,c_j)}{\mathrm{d}c_j^2} = \begin{cases} \frac{\tau\gamma}{rc_j}\left(\frac{\eta x}{c_j}\right)^{\gamma}\left\{1 + \frac{\gamma}{\tau}\underbrace{\left(\frac{\theta_{j1}}{c_j} - 1\right)}_{<0} + \frac{\theta_{j1}}{\tau c_j}\right\} < 0, & c_j \in (\theta_{j1}, \theta_{j2}], \\ \frac{\tau\gamma}{rc_j}\left(\frac{\eta x}{c_j}\right)^{\gamma}\underbrace{\left(1 - \gamma\left\{1 + \alpha\frac{1-\tau}{\tau}\right\}\right)}_{>0} < 0, & c_j > \theta_{j2}. \end{cases} \tag{B.5}$$

From (B.5), $V_{j2}(x, c_j)$ is a concave function of c_j.

Proof of Lemma 3

First, substituting $c_j = \theta_{j1}$ into (A.2) and (A.5) gives $x^{\mathrm{s}}_{j1}(\theta_{j1}) = \lim_{c_j \downarrow \theta_{j1}} x^{\mathrm{d}}(c_j)$. Second, substituting $c_j = \theta_{j1}$ into (B.1) and the upper equation of (B.3) gives $V_{j1}(x, \theta_{j1}) = \lim_{c_j \downarrow \theta_{j1}} V_{j2}(x, c_j)$. Finally, substituting $c_j = \theta_{j1}$ into (B.2) and the upper equation of (B.4) gives

$$\frac{\mathrm{d}V_{j1}}{\mathrm{d}c_j}(x, \theta_{j1}) = \frac{\tau}{r}\left\{1 - \left(\frac{x}{x^{\mathrm{d}}(\theta_{j1})}\right)^{\gamma}\right\} = \frac{\mathrm{d}V_{j2}}{\mathrm{d}c_j}(x, c_j)\Big|_{c_j \downarrow \theta_{j1}} > 0. \tag{B.6}$$

Proof of Lemma 4

The problem under symmetric information is the same as the problems that arise when there is no delegation to the managers, as the managers have no private information. The firm then sets $w^*_H = w^*_L = 0$.

Using Lemmas 1–3, there exists $c(x)$ such that $c(x) > \theta_{j1}$, where $c(x)$ is given as

$$c(x) = \underset{c_j}{\operatorname{argmax}}\, V_{j2}(x, c_j). \tag{B.7}$$

That is, the firm issues a mix of risky debt and equity under symmetric information.

Next, we provide $x^{\mathrm{i}*}_j$ and c^*_j for $k = k_j$ $(j \in \{H, L\})$. For $c_j > \theta_{j1}$, dividing the objective function by x^{β}, the problem is defined as

$$\max_{x^{\mathrm{i}}_j, c_j} \; G(x^{\mathrm{i}}_j, c_j), \tag{B.8}$$

where

$$G(x^{\mathrm{i}}_j, c_j) := (x^{\mathrm{i}}_j)^{-\beta}\left\{V_{j2}(x^{\mathrm{i}}_j, c_j) - I\right\}.$$

Here, $V_{j2}(x^{\mathrm{i}}_j, c_j)$ is given by (B.3). Differentiating (B.8) with respect to x^{i}_j and c_j gives

$$\frac{\partial G}{\partial x^{\mathrm{i}}_j} = (x^{\mathrm{i}}_j)^{-\beta-1}\left\{(-\beta)\left\{V_{j2}(x^{\mathrm{i}}_j, c_j) - I\right\} + \upsilon x^{\mathrm{i}}_j + \gamma\left[\Psi_{j1q} - \Psi_{j2q}\right]\right\}, q \in \{1, 2\},$$

$$\frac{\partial G}{\partial c_j} = (x^{\mathrm{i}}_j)^{-\beta}\frac{\partial V_2(x^{\mathrm{i}}_j, c_j)}{\partial c_j}. \tag{B.9}$$

Differentiating (B.9), we obtain

$$\frac{\partial^2 J}{\partial {x^{\mathrm{i}}_j}^2} < 0, \quad |S| > 0, \quad S = \begin{pmatrix} \dfrac{\partial^2 G}{\partial {x^{\mathrm{i}}_j}^2} & \dfrac{\partial^2 G}{\partial x^{\mathrm{i}}_j \partial c_j} \\ \dfrac{\partial^2 G}{\partial c_j \partial x^{\mathrm{i}}_j} & \dfrac{\partial^2 G}{\partial c_j^2} \end{pmatrix}, \tag{B.10}$$

implying that the second-order conditions are satisfied numerically. Arranging (B.9), we obtain $x^{\mathrm{i}*}_j$ and c^*_j for $c_j \in (\theta_{j1}, \theta_{j2}]$ by solving $f_{j1}(x^{\mathrm{i}}_j, c_j) = 0$ and $f_{j2}(x^{\mathrm{i}}_j, c_j) = 0$, where f_{j1} and f_{j2} are

$$\begin{aligned} f_{j1}(x^{\mathrm{i}}_j, c_j) &:= -(\beta-1)\upsilon x^{\mathrm{i}}_j - \beta\left(\frac{\tau c_j}{r} - I\right) - (\beta-\gamma)\left[\Psi_{j11} - \Psi_{j21}\right], \\ f_{j2}(x^{\mathrm{i}}_j, c_j) &:= \frac{\tau}{r}\left(1 - \left(\frac{\eta x^{\mathrm{i}}_j}{c_j}\right)^{\gamma}\left\{1 + \frac{\gamma}{\tau}\left(\frac{\theta_{j1}}{c_j} - 1\right)\right\}\right), \end{aligned} \tag{B.11}$$

respectively. Moreover, we obtain $x^{\mathrm{i}*}_j$ and c_j such that $c_j > \theta_{j2}$ by solving $f_{j3}(x^{\mathrm{i}}_j, c_j) = 0$ and $f_{j4}(x^{\mathrm{i}}_j, c_j) = 0$, where f_{j3} and f_{j4} are

$$\begin{aligned} f_{j3}(x^{\mathrm{i}}_j, c_j) &:= -(\beta-1)\upsilon x^{\mathrm{i}}_j - \beta\left(\frac{\tau c_j}{r} - I\right) + (\beta-\gamma)\left[\Psi_{j12} - \Psi_{j22}\right], \\ f_{j4}(x^{\mathrm{i}}_j, c_j) &:= \frac{\tau}{r}\left(1 - \left(\frac{\eta x^{\mathrm{i}}_j}{c_j}\right)^{\gamma}\left\{1 - \gamma\left(1 + \alpha\frac{1-\tau}{\tau}\right)\right\}\right), \end{aligned} \tag{B.12}$$

respectively. Further, arranging $f_{j4}(x^{\mathrm{i}}_j, c_j) = 0$ gives

$$c(x^{\mathrm{i}}_j) := \frac{\eta}{h}x^{\mathrm{i}}_j. \tag{B.13}$$

Note that $c(x^{\mathrm{i}}_j)$ is a linear function of x^{i}_j, as originally obtained by Leland (1994). Substituting $c_j = c(x^{\mathrm{i}}_j)$ into $f_{j3}(x^{\mathrm{i}}_j, c_j) = 0$ yields $f_{j5}(x^{\mathrm{i}}_j) = 0$, where f_{j5} is

$$f_{j5}(x^{\mathrm{i}}_j) := -(\beta-1)\frac{\upsilon}{\psi}x^{\mathrm{i}}_j - (\beta-\gamma)\Psi_{j22} + \beta I, \tag{B.14}$$

and we use the following:

$$\begin{aligned} &-(\beta-1)\upsilon x^{\mathrm{i}}_j - \beta\tfrac{\tau}{r}\underbrace{\frac{\eta}{h}x^{\mathrm{i}}_j}_{=c_j(x^{\mathrm{i}}_j)} + (\beta-\gamma)\tfrac{\alpha\varepsilon(1-\tau)+\tau}{r}h^{\gamma}\underbrace{\frac{\eta}{h}x^{\mathrm{i}}_j}_{=c_j(x^{\mathrm{i}}_j)} \\ &-(\beta-1)\upsilon x^{\mathrm{i}}_j - \underbrace{\left(\frac{\beta}{\varepsilon} - (\beta-\gamma)\overbrace{\left\{\alpha\frac{1-\tau}{\tau} + \frac{1}{\varepsilon}\right\}}^{=-(h^{\gamma}\gamma)^{-1}}h^{\gamma}\right)}_{=\beta-1}\tfrac{\tau}{1-\tau}\tfrac{1}{h}\upsilon x^{\mathrm{i}}_j \\ &= -(\beta-1)\underbrace{\left(1 + \frac{\tau}{1-\tau}\frac{1}{h}\right)}_{=:\psi^{-1}}\upsilon x^{\mathrm{i}}_j. \end{aligned}$$

Thus, we obtain $x^{\mathrm{i}*}_j$ by solving $f_{j5}(x^{\mathrm{i}}_j) = 0$ and obtain $c^*_j = c(x^{\mathrm{i}*}_j)$.

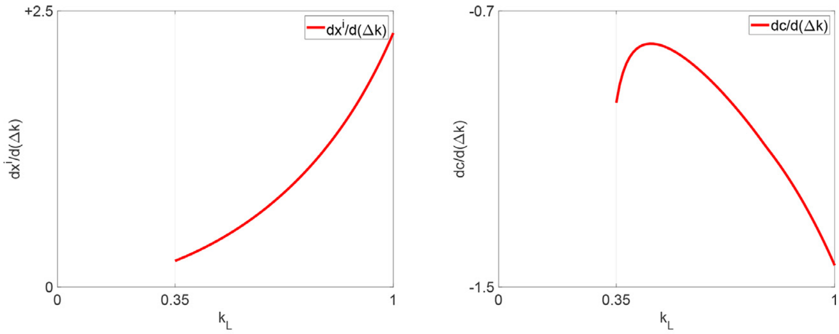
Proof of Proposition 1

First, we show that the firm in L issues a mix of risk-free debt and equity for significant $\Delta k > 0$ under $k_L > 0$ and all-equity for significant $\Delta k > 0$ under $k_L = 0$. To consider the property of $O^{**}_L(x)$ in (24), we define $\phi_{Ln}(x^{\mathrm{i}}_L, c_L)$ as

$$\phi_{Ln}(x^{\mathrm{i}}_L, c_L) := V_{Ln}(x^{\mathrm{i}}_L, c_L) - a_n(x^{\mathrm{i}}_L, c_L), \; n \in \{1, 2\}. \tag{B.15}$$

We examine the property of ϕ_{Ln} with respect to c_L in three steps. First, as in Lemma 1, V_{L1} increases with c_L, and x^{s}_L decreases with c_L. Differentiating (B.15) with respect to c_L gives

$$\frac{\mathrm{d}\phi_{L1}(x^{\mathrm{i}}_L, c_L)}{\mathrm{d}c_L} = \underbrace{\frac{\mathrm{d}V_{L1}(x^{\mathrm{i}}_L, c_L)}{\mathrm{d}c_L}}_{>0 \text{ in (B.2)}} - \underbrace{\frac{\mathrm{d}a_1(x^{\mathrm{i}}_L, c_L)}{\mathrm{d}c_L}}_{<0 \text{ in (B.17)}} > 0, \tag{B.16}$$



(a) $\dfrac{\mathrm{d}x_L^{\mathrm{i}}}{\mathrm{d}(\Delta k)} > 0$ (b) $\dfrac{\mathrm{d}c_L}{\mathrm{d}(\Delta k)} < 0$

Fig. 10. Comparative statics with Δk. We assume $k_H = 1$ in addition to the baseline parameter values.

where

$$\frac{da_1(x_L^{\mathrm{i}}, c_L)}{dc_L} = a_1(x_L^{\mathrm{i}}, c_L)\frac{\gamma}{x_L^{\mathrm{s}}(c_L)}\frac{\varepsilon}{\upsilon}\frac{\tau}{r} < 0. \tag{B.17}$$

Second, as in Lemma 2, V_{L2} increases with c_L for the regions around θ_{L1}, and x^{d} increases with c_L. Differentiating (B.15) yields

$$\frac{d\phi_{L2}(x_L^{\mathrm{i}}, c_L)}{dc_L}\Big|_{c_L\downarrow\theta_{L1}} = \underbrace{\frac{dV_{L2}(x_L^{\mathrm{i}}, c_L)}{dc_L}\Big|_{c_L\downarrow\theta_{L1}}}_{>0 \text{ in (B.6)}} - \underbrace{\frac{da_2(x_L^{\mathrm{i}}, c_L)}{dc_L}\Big|_{c_L\downarrow\theta_{L1}}}_{>0 \text{ in (B.19)}}. \tag{B.18}$$

where

$$\frac{da_2(x_L^{\mathrm{i}}, c_L)}{dc_L} = -a_2(x_L^{\mathrm{i}}, c_L)\frac{\gamma}{c_L} > 0. \tag{B.19}$$

Third, as shown in Lemma 3, we have $V_{L1}(x_L^{\mathrm{i}}, \theta_{L1}) = \lim_{c_L\downarrow\theta_{L1}} V_{L2}(x_L^{\mathrm{i}}, c_L)$, $x_{L1}^{\mathrm{s}}(\theta_{L1}) = \lim_{c_L\downarrow\theta_{L1}} x^{\mathrm{d}}(c_L)$, and $\partial V_{L2}(x_L^{\mathrm{i}}, c_L)/\partial c_L|_{c_L\downarrow\theta_{L1}} > 0$. From the three steps, $\phi_{L1}(x_L^{\mathrm{i}}, \theta_{L1}) = \phi_{L2}(x_L^{\mathrm{i}}, \theta_{L1})$, and whether the sign of (B.18) is positive or negative depends on the magnitudes of the first and second terms on the right-hand side of (B.18).

If $\Delta k \geq 0$ is small, the sign of (B.18) is positive because the first term dominates the second term. In this case, the firm chooses risky debt financing. If $\Delta k \geq 0$ is significant, the sign of (B.18) is negative, because the second term dominates the first term. In this case, the firm chooses risk-free debt financing or all-equity financing. See Fig. 3 for an illustration of the proof.

Next, we can derive $x_L^{\mathrm{i}**}$ and c_L^{**}. For $c_L^{**} = \theta_{L1} \geq 0$ (i.e., $m \in \{0, 1\}$), the problem is defined as

$$\max_{x_L^{\mathrm{i}}} \ (x_L^{\mathrm{i}})^{-\beta}\left\{V_{L1}(x_L^{\mathrm{i}}, \theta_{L1}) - I - a_1(x_L^{\mathrm{i}}, \theta_{L1})\right\}. \tag{B.20}$$

By differentiating (B.20) with respect to x_L^{i} and arranging, we obtain $x_L^{\mathrm{i}**}$ by solving $f_{L0}(x_L^{\mathrm{i}}) + (\beta - \gamma)a_1(x_L^{\mathrm{i}}, \theta_{L1}) = 0$, where f_{L0} is

$$f_{L0}(x_L^{\mathrm{i}}) := -(\beta - 1)\upsilon x_L^{\mathrm{i}} - \beta\left\{\frac{\tau\theta_{L1}}{r} - I\right\} - (\beta - \gamma)\frac{1-\tau}{1-\gamma}(1-\alpha)k_L I\left(\frac{x_L^{\mathrm{i}}}{x_{L1}^{\mathrm{s}}}\right)^{\gamma}, \tag{B.21}$$

and $x_{L1}^{\mathrm{s}} := x_{L1}^{\mathrm{s}}(\theta_{L1}) \geq 0$. For an extreme case of $k_L = 0$, we have $c_L = \theta_{L1} = 0$, $x_{L0}^{\mathrm{s}} = 0$, and $a_0(x_L^{\mathrm{i}}, 0) = 0$, leading to $x_L^{\mathrm{i}**} = \beta I/((\beta - 1)\upsilon)$, which is identical to that of Dixit and Pindyck (1994).

For $c_L^{**} > \theta_{L1}$ (i.e., $m = 2$), the problem is defined as

$$\max_{x_L^{\mathrm{i}}, c_L} \ J(x_L^{\mathrm{i}}, c_L), \tag{B.22}$$

where

$$J(x_L^{\mathrm{i}}, c_L) := (x_L^{\mathrm{i}})^{-\beta}\left\{V_{L2}(x_L^{\mathrm{i}}, c_L) - I - a_2(x_L^{\mathrm{i}}, c_L)\right\}.$$

Similar to the derivations in Lemma 4, differentiating (B.22) with respect to x_L^{i} and c_L yields

$$\frac{\partial J(x_L^{\mathrm{i}}, c_L)}{\partial x_L^{\mathrm{i}}} = 0, \quad \frac{\partial J(x_L^{\mathrm{i}}, c_L)}{\partial c_L} = 0. \tag{B.23}$$

Differentiating (B.23), we obtain numerically

$$\frac{\partial^2 J}{\partial {x_L^{\mathrm{i}}}^2} \leq 0, \quad |A| > 0, \quad A = \begin{pmatrix} \dfrac{\partial^2 J}{\partial {x_L^{\mathrm{i}}}^2} & \dfrac{\partial^2 J}{\partial x_L^{\mathrm{i}}\partial c_L} \\ \dfrac{\partial^2 J}{\partial c_L \partial x_L^{\mathrm{i}}} & \dfrac{\partial^2 J}{\partial c_L^2} \end{pmatrix}, \tag{B.24}$$

implying that the second-order conditions are satisfied. Arranging (B.23), we obtain the optimal trigger and coupon payment, $x_L^{\mathrm{i}**}$ and c_L^{**}, by solving

$$f_{L1}\left(x_L^{\mathrm{i}}, c_L\right) + (\beta - \gamma)a_2\left(x_L^{\mathrm{i}}, c_L\right) = 0,$$
$$f_{L2}\left(x_L^{\mathrm{i}}, c_L\right) + \gamma a_2\left(x_L^{\mathrm{i}}, c_L\right)c_L^{-1} = 0, \tag{B.25}$$

for $c_L \in (\theta_{L1}, \theta_{L2}]$,

$$f_{L3}\left(x_L^{\mathrm{i}}, c_L\right) + (\beta - \gamma)a_2\left(x_L^{\mathrm{i}}, c_L\right) = 0,$$
$$f_{L4}\left(x_L^{\mathrm{i}}, c_L\right) + \gamma a_2\left(x_L^{\mathrm{i}}, c_L\right)c_L^{-1} = 0, \tag{B.26}$$

for $c_L > \theta_{L2}$.

Proof of Proposition 2

Suppose that the firm issues risky debt ($c_L^{**} > \theta_{L1}$). We then obtain $x_L^{\mathrm{i}**}$ and c_L^{**} by solving (B.23). Differentiating (B.23) with $\mathrm{d}x_L^{\mathrm{i}}$, $\mathrm{d}c_L$, and $\mathrm{d}(\Delta k)$ yields

$$A\begin{pmatrix} \mathrm{d}x_L^{\mathrm{i}} \\ \mathrm{d}c_L \end{pmatrix} = \begin{pmatrix} -{x_L^{\mathrm{i}}}^{-1}(\beta - \gamma) \\ -\gamma c_L^{-1} \end{pmatrix} \underbrace{x^{\mathrm{i}-\beta}\left(\frac{\eta x_L^{\mathrm{i}}}{c_L}\right)^{\gamma}\frac{q}{1-q}I}_{=:B\geq 0}\mathrm{d}(\Delta k), \tag{B.27}$$

where A is defined in (B.24). Solving (B.27) with $\mathrm{d}x_L^{\mathrm{i}}$ and c_L yields

$$\frac{\mathrm{d}x_L^{\mathrm{i}}}{\mathrm{d}(\Delta k)} = \frac{1}{|A|}\Big\{\underbrace{-{x_L^{\mathrm{i}}}^{-1}(\beta - \gamma)\frac{\partial^2 J}{\partial c_L^2}}_{\geq 0} + \underbrace{\gamma c_L^{-1}\frac{\partial^2 J}{\partial x_L^{\mathrm{i}}\partial c_L}}_{\leq 0}\Big\}B, \tag{B.28}$$

$$\frac{\mathrm{d}c_L}{\mathrm{d}(\Delta k)} = \frac{1}{|A|}\Big\{\underbrace{-\gamma c_L^{-1}\frac{\partial^2 J}{\partial {x_L^{\mathrm{i}}}^2}}_{\leq 0} + \underbrace{{x_L^{\mathrm{i}}}^{-1}(\beta - \gamma)\frac{\partial^2 J}{\partial x_L^{\mathrm{i}}\partial c_L}}_{\geq 0}\Big\}B, \tag{B.29}$$

where we employ $(\partial^2 J/\partial c_L^2) \leq 0$ and $(\partial^2 J/\partial x_L^{\mathrm{i}}\partial c_L) \geq 0$. It is challenging to show the signs of (B.28) and (B.29) analytically. As long as we have solved (B.28) and (B.29) numerically, the signs of (B.28) and (B.29) are positive and negative, respectively. See Fig. 10 for numerical calculations.

Suppose the firm issues risk-free debt ($c_L^{**} = \theta_{L1}$). We then obtain $x_L^{\mathrm{i}**}$ by solving $f_{L0}(x_L^{\mathrm{i}}) + (\beta - \gamma)a_1(x_L^{\mathrm{i}}, \theta_{L1}) = 0$. Differentiating with x_L^{i} and Δk yields

$$\left\{ -(\beta-1)\nu \underbrace{-(\beta-\gamma)\left(\frac{x_L^{\mathrm{i}}}{x_{L1}^{\mathrm{s}}}\right)^{\gamma}\left(\frac{1-\tau}{1-\gamma}(1-\alpha)k_L I - \frac{q}{1-q}\Delta k I\right)}_{=(\beta-1)\nu x_L^{\mathrm{i}}+\beta\left(\tau(1-\alpha)k_L-1\right)I} \frac{\gamma}{x_L^{\mathrm{i}}} \right\} \mathrm{d}x_L^{\mathrm{i}} = -\frac{(\beta-\gamma)a_1\left(x_L^{\mathrm{i}}, \theta_{L1}\right)}{\Delta k}\mathrm{d}(\Delta k), \tag{B.30}$$

leading to

$$\frac{\mathrm{d}x_L^{\mathrm{i}}}{\mathrm{d}(\Delta k)} = -\frac{\overbrace{(\gamma-1)(\beta-1)\nu}^{<0}\overbrace{-\beta I(\tau(1-\alpha)k_L-1)}^{>0}}{\underbrace{(\beta-\gamma)a_1(x_L^{\mathrm{i}}, \theta_{L1})}_{>0}}\Delta k, \tag{B.31}$$

where $\tau(1-\alpha)k_L < 1$. For a small k_L, the sign of (B.31) is positive, because the first term of the numerator dominates the second term. For a large k_L, the sign of (B.31) is negative, because the second term of the numerator dominates the first term. Furthermore, $c_L^{**} := \theta_{L1} = r(1-\alpha)k_L I$ decreases with Δk because an increase in Δk corresponds to a decrease in k_L.

Appendix C. Related study of Shibata and Nishihara (2010)

As a benchmark, we derive the solution in Shibata and Nishihara (2010). They assume a source of asymmetric information at the time of investment rather than liquidation. Under this assumption, asymmetric information does not affect the leverage and credit spread.

They assume complete irreversibility of investment (i.e., $k = 0$). Debt is risky because $c \geq \theta_1 = 0$. This assumption simplifies the debt and total firm values as

$$D(X(t), c) = \frac{c}{r} + \left\{(1-\alpha)\nu x^{\mathrm{d}}(c) - \frac{c}{r}\right\}\left(\frac{X(t)}{x^{\mathrm{d}}(c)}\right)^{\gamma}, \tag{C.1}$$

$$V(X(t), c) = \nu X(t) + \frac{\tau c}{r} - \left\{\alpha \nu x^{\mathrm{d}} + \frac{\tau c}{r}\right\}\left(\frac{X(t)}{x^{\mathrm{d}}(c)}\right)^{\gamma}. \tag{C.2}$$

Moreover, they assume the source of asymmetric information is the investment cost I, which takes I_1 (low cost) or I_2 (high cost), where $\Delta I := I_2 - I_1 > 0$. Under this assumption, the managers in I_1 aim to mimic the managers in I_2. Thus, to prevent the mimicking, the shareholders impose the incentive-comparative constraint as

$$\left(\frac{x}{x_1^{\mathrm{i}}}\right)^{\beta} w_1 \geq \left(\frac{x}{x_2^{\mathrm{i}}}\right)^{\beta}\left(w_2 + \Delta I\right), \tag{C.3}$$

which corresponds to (15) in this study. We omit the description of another incentive-compatible constraint, which corresponds to (16) because it is automatically satisfied at the equilibrium. Using similar arguments, the option value of the project under asymmetric information is

$$O^{**}(x) = q\left(\frac{x}{x_1^{\mathrm{i}*}}\right)^{\beta}\{V(x_1^{\mathrm{i}*}, c_1^*) - I_1\} + (1-q)\left(\frac{x}{x_2^{\mathrm{i}**}}\right)^{\beta}\{V(x_2^{\mathrm{i}**}, c_2^{**}) - I_2 - \frac{q}{1-q}\Delta I\}. \tag{C.4}$$

We compare (22) and (C.4). In (22), a distortion, $a_m(x^{\mathrm{i}}, c)$ depends on x^{i} and c. However, in (C.4), a distortion, $(q/(1-q))\Delta I$, is independent of x_2^{i} and c_2. These properties simplify the optimal investment trigger and coupon as

$$x_1^{\mathrm{i}**} = x_1^{\mathrm{i}*} = \xi I_1, \qquad c_1^{**} = c_1^* = \frac{\eta}{h}x_1^{\mathrm{i}*},$$
$$x_2^{\mathrm{i}**} = \xi\left(I_2 + \frac{q}{1-q}\Delta I\right) > x_2^{\mathrm{i}*} = \xi I_2, \qquad c_2^{**} = \frac{\eta}{h}x_2^{\mathrm{i}**} > c_2^* = \frac{\eta}{h}x_2^{\mathrm{i}*}, \tag{C.5}$$

where $\xi = (\beta\psi)/((\beta-1)\nu) > 0$. Substituting x and $c = (\eta/h)x$ into $D(x, c)$ and $V(x, c)$ gives

$$D(x, (\eta/h)x) = \left\{1 + \left\{(1-\alpha)\varepsilon(1-\tau) - 1\right\}h^{\gamma}\right\}\frac{\eta}{rh}x, \tag{C.6}$$

$$V(x, (\eta/h)x) = \psi^{-1}\nu x. \tag{C.7}$$

From (C.6) and (C.7), asymmetric information does not affect the leverage and credit spread (i.e., $l_2^{**} = l_2^*$ and $cs_2^{**} = cs_2^*$), which contrast with $l_L^{**} < l_L^*$ and $cs_L^{**} < cs_L^*$ in this study.

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