Scientific African 16 (2022) e01164

Contents lists available at ScienceDirect

Scientific African

journal homepage: www.elsevier.com/locate/sciaf

Mathematical modeling of inventory cost in a 3-tier supply chain with horizontal cooperation

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ARTICLE INFO

Article history:
Received 6 October 2020
Revised 12 September 2021
Accepted 19 March 2022

Editor DR B Gyampoh

Keywords:
Supply chain
Inventory
Horizontal cooperation
Linear programming

ABSTRACT

Inventory together with the transport, location of the production factory, and storage represent important factors that influence the performance of a supply chain. It is thus important for organizations to maintain inventory for efficient and smooth running of their operations. Through cooperation, supply chain players at the same level of the market can work jointly to plan and execute supply chain operations, and achieve greater success than when acting in isolation. Accordingly, this work considers a 3-tier supply chain – Supplier, Manufacturer, and Distributor, and it develops a model for minimizing the cost of inventory across the 3-tiers with horizontal cooperation at the post-production inventory end of manufacturing tier only. A constrained mixed integer mathematical program was developed with the objective of minimizing total inventory cost across the three supply chain tiers. A test problem consisting of 3 suppliers, 3 manufacturers, 3 distributors, and 4 products was used to test the model; the resulting mathematical problem was solved using linear programming with and without post-production inventory horizontal cooperation scenarios. It was found that horizontal cooperation was of immense benefit to supply chain players, and offers great cost savings as against individual operation.

Introduction

Traditionally, most organizations did little to manage their supply chain. Instead, they tend to concentrate on their own operations and their immediate suppliers. However, a number of factors make it desirable for business organizations to proactively manage their supply chain. These factors include the need to manage growing inventory costs, the need to improve operations, the complexity of supply chain, increasing effect of information technology, and competitive pressures etc.

For successful operation of a supply chain, there is need to manage the cost of inventory across the chain. A supply chain is a network of organizations, people, technology, activities, information, and resources involved in the production and

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https://doi.org/10.1016/j.sciaf.2022.e01164

movement of products or services from the pre-production stages, through the production and distribution, to the eventual consumers of the products or service [1].

It is a sequence that begins with the basic suppliers of materials to the factories, warehouses, processing centers, distribution centers, retail outlets, and offices. Other functions and activities such as forecasting, purchasing, inventory management, information management, quality assurance, scheduling, production, distribution, delivery, and customer service help to transform the raw materials and components into a product that is eventually delivered to a consumer or user [2]. In a 3–tier supply chain described as the supplier, producer (manufacturer), and distributor, firms (suppliers) at the supply stage provide the raw materials required by the manufacturers to produce the products that are shipped as finished goods to the distributors (distribution centers). Studies in the area of supply chain have been approached from different angles. This is because one model cannot capture the wide spectrum of a supply chain processes. Perspectives such as information given in [3], inventory [4], routing [5,6] collaboration in a supply chain [7], and supply chain player reliability [8] have all been discussed at various times. In general, supply chain management deals with the management of the network of facilities that produce raw materials, transform them into intermediate goods and then final products, and deliver the products to customers through a distribution system. Supply chain collaboration involves two or more chain members working together to create a competitive advantage through information exchange, joint decision making, and sharing benefits which result from greater profitability of satisfying end customer needs than acting alone [9]. Authors in [10] opine that cooperation promotes tailored business relationships based on mutual trust, openness, shared risk and rewards that yields competitive advantage, and greater business performance than the firms acting in isolation. Authors in [10] affirms that through cooperation, a company can aim at obtaining a competitive advantage in a market, reduce response time in the quickest way, and obtain first mover advantages to enter a new market. Co-operation occurs at various tiers of the chain and it is described by its structure as horizontal, vertical, or lateral. Specifically, vertical cooperation aims at installing beneficial partnerships and seamless linkages between multiple parties operating at different levels of the supply chain to avoid unnecessary logistics costs, or 'waste' while lateral cooperation is a cooperation aimed at gaining more flexibility by combining and sharing capabilities in both vertical and horizontal manners [11]. Of particular interest to this research work is horizontal cooperation. In general, horizontal partnerships are a type of collaboration existing between companies operating at the same level of the market [9]. It includes concerted practices between companies operating at the same level(s) in the market". These can be either competing or unrelated companies that share private information, facilities or resources to reduce costs or improve service. Some examples of horizontal cooperation in logistics are Manufacturers Consolidation Centers (MCCs), joint route planning, and purchasing groups. Horizontal cooperation between logistic service providers (LSPs) can be an effective way to achieve higher capacity utilization by exchanging loads and equipment between the geographically dispersed partners [10]. Horizontal logistics cooperation can either be competitive or non-competitive. Non-competitive horizontal cooperation occurs when transportation companies servicing different industries (e.g. tank transport, express services, removal services) start a joint knowledge platform. If the partners are servicing the same industries, they are direct competitors and the cooperation can be referred to as competitive horizontal cooperation. Identified benefits of horizontal cooperation include improved customer service, exchange of knowledge about products and process, reduced cost of production, operational flexibility, and market domination.

More often than not, supply chain firms accrue more costs as a result of a lack of cooperation between the firms in the area of inventory management. This increase stems from the fact that warehouses, production facilities, and distribution centers are usually not in the same vicinity, thus increasing the overall cost associated with inventory. Cost such as storage cost, ordering cost, holding/carrying cost, and shortage cost tend to multiply as the factors of logistics, facility location, government policy, taxation, geographical market etc. begin to set in. A viable method of reducing these costs is to effectively implement a cooperation network across the supply chain i.e. from the supplier to the manufacturer and distributor to retailers and consumers, hence, the justification for the central theme of this research study.

In order to manage inventory in a collaborative supply chain, it is imperative to design a cost-effective model that considers the inventory of raw materials, work in progress, finished products required at various stages of the network (supply, manufacturing, and distribution). At each point in the chain, requisite levels of raw materials, products, suppliers, manufacturers, distributors must be defined in order to optimize the overall cost of production [12]. This entails determining the amount of inventory needed by each supply chain player at each point in time. In contrast to existing results, in this work, a cost-effective model that optimizes the cost of inventory with and without horizontal cooperation in a three-tier supply chain system is developed. This work seeks to provide answers to challenges of setting baseline levels of the raw materials, products at different tiers of a supply chain. Participating companies in a supply chain simply have to galvanize their efforts together into a cooperation system that help each player of the chain by providing information, increased capacity, better utilization of existing infrastructure, and ease of access to materials and products thus reducing costs of storage, holding and carrying inventory.

Materials and methods

The subject-matter of this research work is to develop and analyze a three-tier supply chain system that minimizes total inventory cost, and permits horizontal cooperation for product inventory. In order to describe a supply chain system that fully optimizes the inventory cost, a 3-tier (multi-supplier, multi-factory, multi-distributor) model is developed. A distinct feature of this model is the incorporation of horizontal cooperation at the production tier of the supply chain. This simply

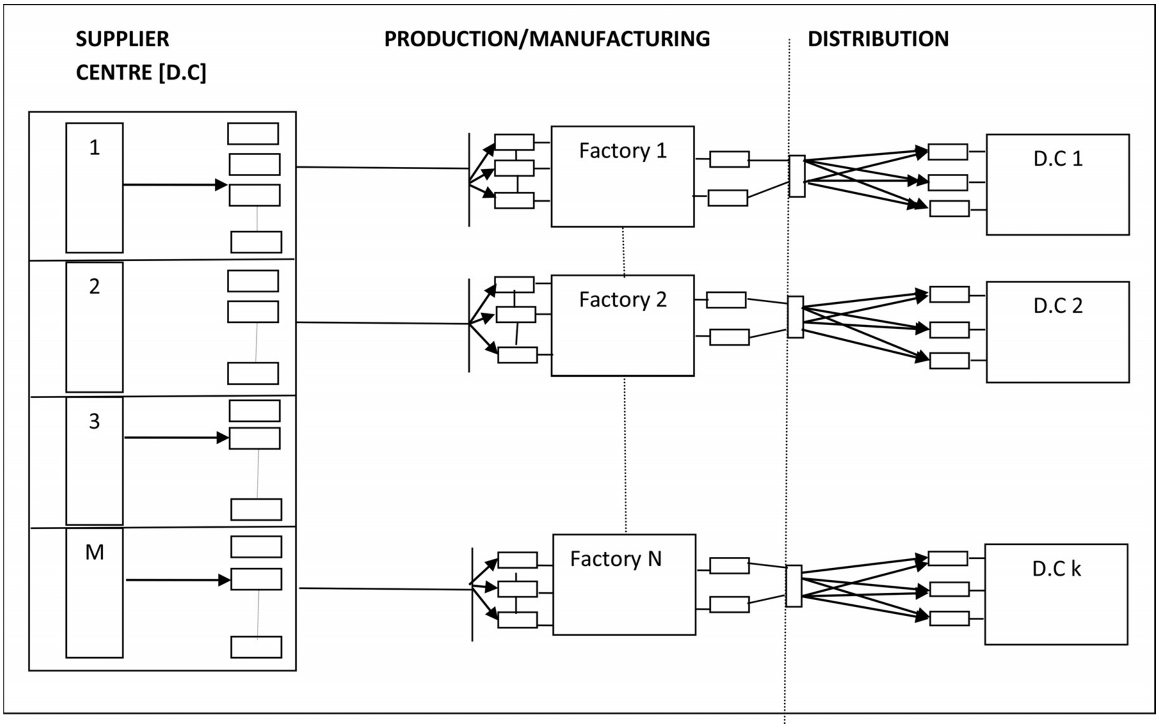

Fig. 1. Connectivity between the supply, production and distribution tiers.

aid distributors to be able to pick any of the products at any of the factories (considering the fact that not all factories produce all products). The objective of the model is to minimize the total cost of inventory at each of the 3-tiers of a supply chain (Supply, Production and Distribution).

The assumptions built into this model are:

- The total logistics cost (total cost of logistics to supply material m from supplier s to factory j) are built into inventory cost for this work.
- The inventory cost here also includes the cost of ordering, holding, carrying, purchasing, shortage, and capital.
- All factories use all the types of materials supplied for production.
- All factories in the supply chain system do not produce all products.
- All factories must be able to supply all products demanded by the distributor.

For the proposed mathematical model of the supply chain system, a linear mixed integer constrained single objective mathematical programming is thus developed.

Horizontal cooperation in product inventory

The model built in this study provides for horizontal cooperation on inventing products of the factory alone. The horizontal inventory cooperation is within tier strategy of inventing. The complementary vertical cooperation which entails cooperation between tiers of the system is not considered. The essence of horizontal cooperation at the production stage is simply to enable distributors to be able to pick any product from any factory. Its modus operandi entails a cooperation network between factories such that factories that do not produce a particular kind of product are able to source that product from another factory in order to have it in stock.

The generalized structure of the proposed supply chain system

Fig. 1 shows a generalized structure of the 3-tier supply chain consisting of a supplier, producer and distributor. It comprises of M number of material suppliers each with assorted material types they supply as the Supply tier. There are N number of factories with assorted products at the production tier and K number of distributors each with assorted products they distribute at the distribution tier. The model assumes that material inventories are kept at the fore side of the supply tier while at the production tier, material inventories are kept at the rear side, and product inventories at the fore side of the structure. By rule too, product inventories are kept at the rear end of the distribution tier. It is worth noting that inventories may also be kept at the foreside of the distribution tier but that is not captured in this presentation as we are not connecting the distribution tier with the usually complex retail tier.

The generalized model parameters and decision variables

There are 3 tiers (supply, manufacturing/production, and distribution) of the supply chain being considered. Each of the tiers has specific parameter symbols assigned to them. In particular, parameters assigned to suppliers-s, factory-j, material-m, distributor k, and product-p are shown below.

a Supply Tier Parameters:

The generalized supply chain structure is modeled with M total number of types of materials $m = 1, 2...$ M, where number of suppliers S_m represents supply for each material.

- $a_{sm}^{(s)}$: This represents the capacity of storage available for the supplier where $m = 1, 2, ...,$ M and supplier s, $s = 1, 2, ...,$ S.
- $C_{sm}^{(s)}$: This is the inventory cost per unit of base level of material where $m = 1, 2, ...,$ M and supplier s, $s = 1, 2, ...,$ S.
- $h_{sm}^{(s)min}$ This is the base level of material m where $m = 1, 2, ...,$ M and supplier s, $s = 1, 2, ...,$ S.
- The inventory cost includes costs for storage, handling, and other logistics required for the materials.
- The inventory infrastructure at the supply tier is limited hence, the need to specify an upper bound quantity in storage.
- The base level of stock is specified in order to avoid stock-out.
- A binary decision parameter is introduced since there is no guarantee that all suppliers have the ability to supply all types of materials. Mathematically, the decision parameter is expressed as

$$f_{sm}^{(s)} = \begin{cases} 1 & If \ \sup pliers \sup plies \, m \, material \\ 0 & if \, otherwise, \end{cases} \tag{1}$$

for each supplier for $s = 1, 2, ...,$ S, and material $m = 1, 2, ...,$ M, for the model to determine which supplier supplies what materials.

a Manufacturing/Production Tier Parameters:

The manufacturing/production tier keeps inventories at both pre-production (material side of production), and post production (at the product side ends).

- The model parameters at the material end of production are $\mathrm{a}_{\mathrm{mj}}^{(\mathrm{pm})}$ ($\mathrm{C}_{\mathrm{mj}}^{(\mathrm{pm})}$, r_{mp}, $h_{pm}^{(pm)min}$) which, respectively, represents the material storage capacity (material inventory cost per unit, material required to produce unit product p, and material base level stock) of material m, $m = 1, 2, ...,$ M of product p, for $p = 1, 2, ...,$ P at factory j, for $j = 1, 2, ...,$ J at the entry end.
- At the product end of the production tier are the product storages. The model parameters used are $\mathrm{a}_{\mathrm{pj}}^{(\mathrm{pp})}$ ($\mathrm{C}_{\mathrm{pj}}^{(\mathrm{pp})}$,$h_{pj}^{(pj)min}$) which, respectively, represents the product storage capacity (product inventory cost per unit, base stock level) of material m, for $m = 1, 2...$ M of product p, for $p = 1, 2...$ P at factory j, for $j = 1, 2...$ J at that end.
- At the supply end, the material inventory cost include all storage, handling and other required logistics cost at their respective end of the production tier.
- Since it is possible that all materials may not be needed to produce all types of products, a binary parameter is introduced to capture this viz:

$$\mathrm{f}_{\mathrm{mp}} = \begin{cases} 1 & \text{if product p requires material type m} \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

for each material m, for $m = 1, 2, ...,$ M, and product p, for $p = 1, 2, ...,$ P.

Also, since it is possible that all factories do not produce all types of product (but may source for and invent products not produced through horizontal cooperation) two binary parameters are introduced viz:

$$f_{pj}^{(pp)} = \begin{cases} 1 & If \, the \, factory \, j \, produce \, product \, p \\ 0 & otherwise \end{cases} \tag{3}$$

$$\bar{f}_{pj}^{(pp)} = \begin{cases} 1 & If \, the \, factory \, j \, does \, \, not \, produce \, product \, p \\ 0 & otherwise \end{cases} \tag{4}$$

a Distribution Tier Parameters

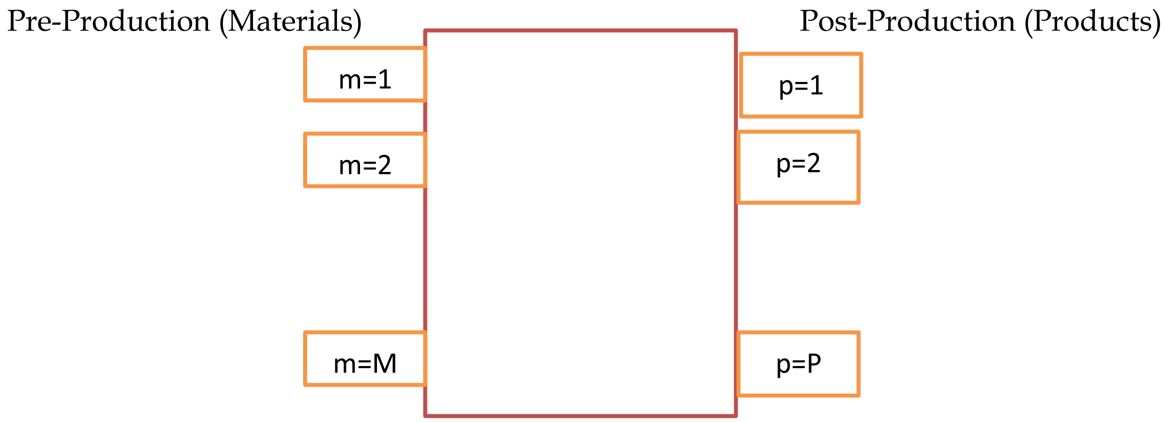

Fig. 2. Material and Products inventory in production.

At the distribution tier, the parameters follow the same pattern used for the two other tiers. Here, $a_{pj}^{(pj)min}(C_{pk}^{(d)}, h_{pk}^{(d)min}, D_{pk})$, respectively, represent the product storage capacity (inventory cost, base level stock, and product demand) of product p, for $p = 1, 2...$ P, distribution center k, for $k = 1, 2...$ K. Again, the inventory cost includes cost of storage, handling and other logistic requirements for the product from the factory and at the distribution centers. Base stocks are also needed here to prevent stock-out.

The generalized model decision variables whose optimal values are to be determined are x_{sm} (y_{pj}, V_{pj}, z_{pjk}) which, respectively, represents the quantity of materials to be supplied (quantity of product types to be produced, quantity of product types not to be produced but to be invented on behalf of another factory for horizontal cooperation, and quantity of products to be distributed) by supplier s, for $s = 1, 2, ...,$ S, material m, for $m = 1, 2, ...,$ M, product type p, for $p = 1, 2, ...,$ P, at or from factory j, for $j = 1, 2, ...,$ J, and to distributor k, for $k = 1, 2, ...,$ K.

The problem on hand is to minimize inventory cost across all tiers of the supply chain incorporating horizontal cooperation at the post production end storage of the production tier only. The following functions are the inventory cost functions at the various tiers of the supply chain system using the parameters and decision variables defined earlier.

Accordingly, the objective function for the supply tier is given by:

I Supply tier

$$F_s(x) = \sum_{s=1}^{S} \sum_{m=1}^{M} (C_{sm}^{(s)} \left[h_{sm}^{min} + \sum_{j} x_{smj}^{(s)} \right]). \tag{5}$$

This function implies the sum of all inventory cost for all suppliers and all materials whose cost are individually computed as the product of the inventory cost and the sum of the base stock level and the optimal quantity to be supplied by each supplier s for each material m and to each factory j. The constraint can be expressed as:

$$\text{Constraint}: C1a_{sm}^{(s)} - h_{sm}^{min} - \sum_{j} x_{smj}^{(s)} \geq 0 \tag{6}$$

For $s = 1, 2...$ S, and $m = 1, 2...$ M

Constraint C1 represented by Eq. (6) ensures the sum of the base stock and the optimal quantity of material m to be supplied from supplier s must not exceed the storage capacity available.

I Production tier:

This consists of the pre-production inventory end (production material storage end) of the tier, and post-production inventory end (product storage end) of the tier.

a Pre-Production Inventory (Production Material Storage) side:

$$\text{Objective}: F_{mp}(y) = \sum_{j} \sum_{m} \sum_{p} f_{mp}^{(mp)} C_{mj}^{(mp)} \left[h_{mj}^{(mp)} + r_{mp} y_{pj} \right] \tag{7}$$

This function implies the sum of all inventory cost for all factories and all materials whose cost are individually computed as the product of the inventory cost and the sum of the base stock level and the optimal quantity to be produced of each product p from each material m and at each factory j. Fig. 2 describes the material and product inventory of a typical production tier of a supply chain. The materials supplied $m = 1, 2 \ldots M$ are invented in each factory j producing products $p = 1, 2 \ldots P$.

Pre-Production (Materials) Post-Production (Products)

$$\text{ConstraintsC2}: \sum_{s} \mathrm{x}_{\mathrm{smj}}^{(\mathrm{s})} = \sum_{m} r_{mp} y_{pj,} \tag{8}$$

for $m = 1, 2 \ldots M$, $p = 1, 2, \ldots P$ and $j = 1, 2 \ldots J$. This constraint stipulates that the total optimal amount of materials type m from supplier s to be supplied to factory j is equal to the product of the quantity of materials m needed to produce one unit of product p and the optimal quantity of products p to be produced at factory j summed over all types of materials.

$$\text{ConstraintC3}: \mathrm{a}_{\mathrm{pj}}^{(\mathrm{pm})} \geq \mathrm{h}_{\mathrm{mj}}^{(\mathrm{pm})} + \mathrm{r}_{\mathrm{mp}}\left(\mathrm{f}_{\mathrm{pj}}^{\mathrm{mp}} \mathrm{y}_{\mathrm{pj}} + \mathrm{r}_{\mathrm{mp}} \mathrm{V}_{\mathrm{pj}}\right) \tag{9}$$

Constraint (C3) stipulates that the sum of the base stock, quantity of material required to produce a product (for those produced at the factory) must not exceed the capacity of the storage at the product material end of factory j, for product p and each material m.

Post-production inventory (Product inventory end) with horizontal cooperation

$$\text{Objective}: \sum_{\mathrm{p}} \sum_{\mathrm{j}} \mathrm{f}_{\mathrm{pj}}^{\mathrm{pp}} \mathrm{C}_{\mathrm{pj}}^{(\mathrm{pp})} \left[y_{pj} + h_{\mathrm{pj}}^{(\mathrm{pp})} \right] \tag{10}$$

To include the quantity v_{pj} *for inventing at the fatory it is not produced*, the cost of keeping the inventory and a base stock for the products is written as:

$$\sum_{p} \sum_{j} \left[\mathrm{f}_{\mathrm{pj}}^{(\mathrm{pp})} \mathrm{C}_{\mathrm{pj}}^{(\mathrm{pp})} \left(y_{pj} + \mathrm{h}_{\mathrm{pj}}^{(\mathrm{pp})} \right) + \bar{f}_{pj}^{(pp)} \bar{C}_{pj}^{(pp)} \left(V_{pj} + \bar{h}_{pj} \right) \right] \tag{11}$$

$$\text{ConstraintC4}: \mathrm{a}_{\mathrm{pj}}^{(\mathrm{pp})} \geq \mathrm{h}_{\mathrm{pj}}^{(\mathrm{pp})} + y_{pj} + v_{pj}, \tag{12}$$

for $p = 1, 2, \ldots, P$ and $j = 1, 2, \ldots, J$. The quantity v_{pj} is the quantity of a product invented at a factory where it was not produced. Accordingly, constraint C4 states that the sum of the base stock, the optimal quantity of products (produced at the factory) and the optimal quantity of products (not produced but invented at the factory) must not exceed the storage capacity of factory j for product p.

I Distribution tier

D_{pk} = Quantity of product p demanded at distribution center k

$$\text{Objective}: \mathrm{F}_{d}(z) = \sum_{\mathrm{p}} \sum_{\mathrm{k}} \mathrm{f}_{\mathrm{pk}}^{(\mathrm{d})} \mathrm{C}_{\mathrm{pk}}^{(\mathrm{d})} \Big(\sum_{\mathrm{j}} z_{pjk} + \mathrm{h}_{\mathrm{pk}}^{(\mathrm{d})} \Big) \tag{13}$$

This function depicts the product of the inventory cost and the sum of the optimal quantity to be distributed, and the base stock of the product p at distribution center k produced over all factories j, $j = 1, 2, \ldots, J$.

$$\text{ConstraintsC5}: \sum_{\mathrm{j}} \Big(\mathrm{f}_{\mathrm{pj}}^{(\mathrm{pp})} y_{pj} + \bar{f}_{pj}^{(pp)} \mathrm{V}_{\mathrm{pj}} \Big) \geq \sum_{\mathrm{k}} D_{pk}, \tag{14}$$

for $p = 1, 2, \ldots, P$. $j = 1, 2, \ldots, J$ and $k = 1, 2, \ldots K$. This constraint stipulates that the sum of the quantity of products produced at the factory and the quantity of products not produced but only invented at the factory for each product p summed over all product types should not be less than the demand for a product p summed over all distribution center k, for $k = 1, 2, \ldots, K$.

$$\text{Constraint C6}: \mathrm{a}_{\mathrm{pk}}^{(\mathrm{d})} \geq \mathrm{h}_{\mathrm{pk}}^{(\mathrm{d})} + \sum_{\mathrm{j}} z_{pjk,} \tag{15}$$

for $p = 1, 2, \ldots, P$ and for $j = 1, 2, \ldots, J$. This constraint stipulates that the sum of the base stock at the distribution center and the optimal quantity to be distributed summed over all types of products j where $j = 1, 2, \ldots, J$ is not greater than the capacity of storage for product p, and distribution center k. z_{pjk} denotes the amount of product p produced at factory j to be given to distributor k

Horizontal cooperation

Product side of production

$$\text{Objective}: F_{pj} = \left[\sum_{\mathrm{p}} \sum_{\mathrm{j}} \mathrm{f}_{\mathrm{pj}}^{(\mathrm{pp})} \mathrm{C}_{\mathrm{pj}}^{(\mathrm{pp})} \left(y_{pj} + \mathrm{h}_{\mathrm{pj}}^{(\mathrm{pp})} \right) + \bar{f}_{pj}^{(pp)} \bar{C}_{pj}^{(pp)} \left(V_{pj} + \bar{\mathrm{h}}_{\mathrm{pj}} \right) \right] \tag{16}$$

$$\text{ConstraintC7}: \mathrm{f}_{\mathrm{pj}}^{(\mathrm{pp})} y_{pj} = \sum_{j} \mathrm{f}_{\mathrm{pj}}^{(\mathrm{pp})} y_{pj} \tag{17}$$

Material side of production

$$\text{Objective}: \sum_j \sum_m \sum_p \mathrm{f}_{\mathrm{pj}}^{\mathrm{mp}} \mathrm{C}_{\mathrm{mj}}^{(\mathrm{mp})} \left[\mathrm{h}_{\mathrm{mj}}^{(\mathrm{mp})} + \mathrm{r}_{\mathrm{mp}} y_{pj}\right] \tag{18}$$

The integrated mathematical model

Across all tiers, the model comprising of all objective functions and constraints at all the three tiers modeled is given as:

$$\begin{aligned} Minimize: F = & \sum_s^S \sum_m^M \left[C_{sm}^{(s)} \left[h_{sm}^{\min} + \sum_j x_{smj}^{(s)} \right] \right] + \sum_j \sum_m \sum_p f_{mp}^{(mp)} C_{mj}^{(mp)} \left[h_{mj}^{(mp)} + r_{mp} \left(y_{pj} + v_{pj} \right) \right] \\ & + \left[\sum_p \sum_j f_{pj}^{(pp)} C_{pj}^{(pp)} \left(y_{pj} + h_{pj}^{(pp)} \right) + \bar{f}_{pj}^{(pp)} \bar{C}_{pj}^{(pp)} (V_{pj} + \bar{h}_{pj}) \right] + \sum_p \sum_k f_{pk}^{(d)} C_{pk}^{(d)} \left[\sum_j z_{pjk} + h_{pk}^{(d)} \right]. \end{aligned} \tag{19}$$

Subject to the following constraints:

$$\begin{aligned} & a_{sm}^{(s)} - h_{sm}^{\min} - \sum_j x_{smj}^{(s)} \geq 0 \, for\, each\, s, ms = 1, 2, \ldots, S\, and\, m = 1, 2, \ldots, M, (C1), \sum_s x_{smj}^{(s)} = \\ & \sum_p r_{mp} (y_{pj} + v_{pj}) \, for\, each\, m, p, m = 1, 2, \ldots, M, p = \\ & 1, 2, \ldots P(C2), a_{mj}^{(pm)} - \sum_p \left(h_{mj}^{(pm)} + f_{pj}^{mp} r_{mp} y_{pj} \right) \geq 0 \, for\, each\, m, jm = 1, 2 \ldots M, j = 1, \ldots, J(C3), a_{pj}^{(pp)} - f_{pj}^{(pp)} \\ & \left(y_{pj} + h_{pj}^{(pp)} \right) + \bar{f}_{pj}^{(pp)} \left(v_{pj} + \bar{h}_{pj} \right) \geq 0 \, for\, each\, p, jj = 1, 2..Jp = 1, 2 \ldots P(C4), \sum_j (f_{pj}^{(pp)} y_{pj} + \\ & \bar{f}_{pj}^{(pp)} V_{pj}) - \sum_k D_{pk} \geq 0 \, for\, each\, p, kp = 1, 2, \ldots, Pk = 1, 2, \ldots, K(C5), a_{pk}^{(d)} \geq \\ & h_{pk}^{(d)} + \sum_j z_{pjk} \, for\, each\, p, kp = 1, 2, \ldots, Pk = 1, 2, \ldots, K(C6) \end{aligned} \tag{20}$$

Now, by fitting in the values of the parameters into the objective function and constraints of the generalized model, a test problem can be used to solve and analyze the model. A test case is used to demonstrate (19). A 3-tier supply chain comprising of three (3) suppliers each supplying three (3) materials to three (3) factories or manufacturing centers that produce four (4) products that are distributed by three (3) distributors, i.e. $S = 3$, $M = 3$, $P = 3$, $J = 4$, $K = 3$.

Sample data (containing various inventories, costs and storage capacities) is obtained from a beverage producing company with a similar supply chain.

Using the sample data from Table 1, the combined mathematical model is given by

$$\begin{aligned} & F = 43411 + 44x_{111} + 44x_{112} + 44x_{113} + 57x_{121} + 57x_{122} + 57x_{123} + 46x_{131} + 46x_{132} + \\ & 46x_{133} + 48x_{211} + 48x_{212} + 48x_{213} + 53x_{221} + 53x_{222} + 53x_{223} + 49x_{231} + 49x_{232} + 49x_{233} + \\ & 38x_{311} + 38x_{312} + 38x_{313} + 37x_{321} + 37x_{322} + 37x_{323} + 37x_{331} + 37x_{332} + 37x_{333} + 265y_{11} + \\ & 273y_{12} + 263y_{13} + 347y_{31} + 360y_{22} + 425y_{23} + 309y_{31} + 311y_{32} + 305y_{33} + 265y_{41} + 286y_{42} + \\ & 304y_{43} + 20z_{111} + 25z_{112} + 25z_{113} + 20z_{121} + 25z_{122} + 25z_{123} + 20z_{131} + 25z_{132} + 25z_{211} + \\ & 30z_{212} + 25z_{213} + 25z_{221} + 30z_{222} + 25z_{223} + 30z_{231} + 25z_{232} + 25z_{233} + 4z_{311} + 35z_{312} + \\ & 50z_{313} + 4z_{321} + 35z_{322} + 50z_{323} + 4z_{331} + 35z_{332} + 50z_{333} + 30z_{411} + 25z_{412} + 35z_{413} + \\ & 30z_{421} + 30z_{422} + 35z_{423} + 30z_{431} + 30z_{432} + 35z_{433} + 323V_{13} + 387V_{21} + 485V_{23} + 354V_{31}. \end{aligned} \tag{21}$$

Results and discussions

The equation for each constraint is developed for each of the material and their suppliers, products and their producers as well as the distributors, these constraints amount to 55.

A linear mixed integer constrained single objective mathematical program in (21) was solved using the linear programming. In order to fully see the effect of the horizontal cooperation on the model, the behavior of the test problem was studied with and without horizontal cooperation at the post-production inventory end as modeled. Without horizontal cooperation, the model produced an average of 267.9419, 213.755, 248.8767 with tonnes of material to order, products to produce and to distribute, respectively. On the other hand, with horizontal cooperation, the model gives optimal results of 684.86, 684.86 and 986.72 with tonnes for quantities of products 1, 2 and 3 *not manufactured but invented* at factories 1 and 3. This clearly argues a strong reason for horizontal cooperation when compared with the quantities of y_{11}, y_{21},y_{31}without horizontal cooperation. Also, without cooperation, the model produces no result for product 1 being invented at factory 3. This means it is better to produce product 1 at factory 3 rather than source and invent it.

Table 1
Sample data from a beverage producing company with a similar supply chain.

Inventory storage cost unit costs C_{sm} (Storage capacity a_{sm}) of material m at supplier s [Unit Costs × ₦100,000]	$C_{11}(a_{11})$ 44(130)	$C_{12}(a_{12})$ 57(120)	$C_{13}(a_{13})$ 46(100)	$C_{21}(a_{21})$ 48(180)	$C_{22}(a_{22})$ 53(150)	$C_{23}(a_{23})$ 49(200)	$C_{31}(a_{31})$ 38(125)	$C_{32}(a_{31})$ 37(135)	$C_{33}(a_{31})$ 31(100)			
Inventory storage cost C_{mj} (base level h_{mj}) of material m at factory j [Unit Costs × ₦100,000]	$C_{11}(h_{11})$ 16(15)	$C_{12}(h_{12})$ 20(18)	$C_{13}(h_{13})$ 20(19)	$C_{21}(h_{21})$ 18(20)	$C_{22}(h_{22})$ 15(25)	$C_{23}(h_{23})$ 20(15)	$C_{31}(h_{31})$ 25(12)	$C_{32}(h_{32})$ 18(10)	$C_{33}(h_{33})$ 15(20)			
Baseline material inventory at supplier s h_{sm} (Tonnes)	h_{11} 15	h_{12} 16	h_{13} 10	h_{21} 20	h_{22} 17	h_{23} 25	h_{31} 15	h_{32} 10	h_{33} 12			
Unit material required to produce p r_{mp}	r_{11} 8	r_{12} 6	r_{13} 5	r_{14} 9	r_{21} 4	r_{22} 7	r_{23} 3	r_{24} 2	r_{31} 1	r_{32} 5	r_{33} 7	r_{34} 2
Storage cost C_{pj} (base level h_{pj}) for product p produced at factory j [Unit Costs × ₦100,000]	$C_{11}(h_{11})$ 40(35)	$C_{12}(h_{12})$ 35(40)	$C_{22}(h_{22})$ 45(25)	$C_{32}(h_{32})$ 40(45)	$C_{33}(h_{33})$ 35(35)	$C_{41}(h_{41})$ 35(25)	$C_{42}(h_{42})$ 40(30)	$C_{43}(h_{43})$ 45(20)				
Storage capacity (a_{mj}) of material m at factory j (Tonnes)	a_{11} 120	a_{12} 120	a_{13} 110	a_{21} 100	a_{22} 100	a_{23} 105	a_{31} 115	a_{32} 120	a_{33} 90			
Storage capacity a_{pj} of product p at factory j, (Tonnes)	a_{11} 225	a_{12} 220	a_{13} 240	a_{21} 200	a_{22} 200	a_{23} 220	a_{31} 210	a_{32} 200	a_{33} 210	a_{41} 230	a_{42} 220	a_{43} 250
Baseline product inventory at distributor k h_{pk} (Tonnes)	h_{11} 15	h_{12} 20	h_{13} 25	h_{21} 30	h_{22} 25	h_{23} 20	h_{31} 40	h_{32} 20	h_{33} 45	h_{41} 25	h_{42} 30	h_{43} 40
Storage cost of product p at distributor k C_{pk} [Unit Costs × ₦100,000]	C_{11} 20	C_{12} 25	C_{13} 25	C_{21} 30	C_{22} 25	C_{31} 40	C_{32} 35	C_{33} 50	C_{41} 30	C_{42} 30	C_{43} 35	
Storage capacity (Demand) of product p at distributor k $a_{pk}(D_{pk})$	$a_{11}(D_{11})$ 350(115)	$a_{12}(D_{12})$ 325(100)	$a_{13}(D_{13})$ 350(125)	$a_{21}(D_{21})$ 330(140)	$a_{22}(D_{22})$ 300(110)	$a_{23}(D_{23})$ 350(150)	$a_{31}(D_{31})$ 310(160)	$a_{32}(D_{32})$ 290(140)	$a_{33}(D_{33})$ 340(200)	$a_{41}(D_{41})$ 350(120)	a_{42} (D_{42}) 370(125)	a_{43} (D_{43}) 500(180)

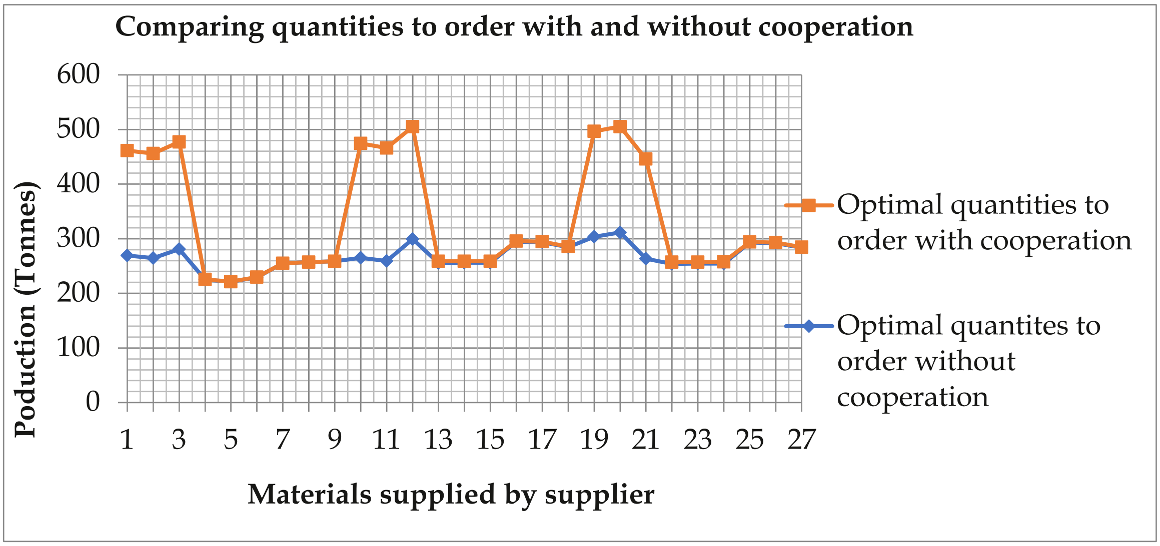

Fig. 3. Graph comparing the optimal quantities to order with and without cooperation.

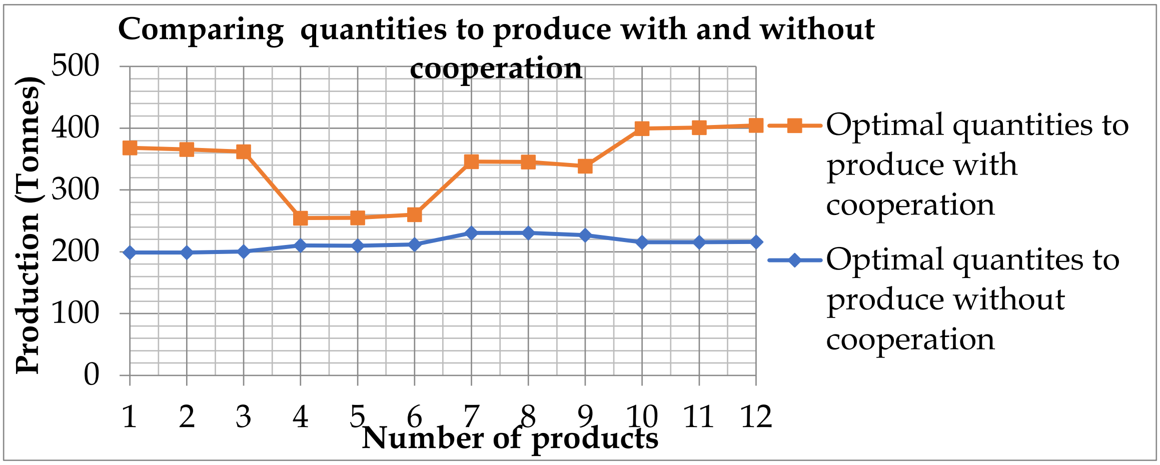

Fig. 4. Graph comparing optimal quantities to produce with and without cooperation.

Objective function (Cost of inventory)

Notably, the overall cost of inventory across the supply without horizontal cooperation is higher than the case of horizontal cooperation. The horizontal cooperation between the factories for sourcing products 1 and 3 produces a significant effect on the model.

(a) Horizontal cooperation affects the number of optimal quantities of materials (products) to supply (produce). Without cooperation, the quantities are significantly higher than with cooperation.
(b) Thus, without cooperation, the total costs of inventory across the supply chain become higher and significantly lower with horizontal cooperation at the factories.

Figs. 3 and 4 are graphs comparing the optimal quantities to order for the two cases (With horizontal cooperation and without horizontal cooperation). The two graphs clearly shows the difference in the optimal quantities of the materials to order from each of the suppliers (with and without cooperation), the quantities to produce at the factories (with and without cooperation) and the quantities of products each distributor takes from the factory.

Analysis of the results obtained by comparing the model optimization without post-production inventory horizontal cooperation and the case with the cooperation shows that there are no statistical differences between the optimal values of the decision variables obtained for supply, production and distribution. Coupled with the fact that the difference in the optimal costs obtained is also marginal, evidently incorporating horizontal cooperation does not worsen optimal decision variable and objective; rather it performs comparatively well with the cooperation. In addition, if we consider the significant and enormous advantages of the incorporation of horizontal cooperation, a good case is made in this work for horizontal cooperation incorporation. Thus, it can be concluded that horizontal cooperation in a supply chain (at the manufacturing centers) drastically reduces the cost of inventory across the supply chain.

Conclusions and future work

The contribution of this paper is twofold. First, the model developed reduces the cost of inventory across the supply chain through the effective implementation of horizontal cooperation into a supply chain design. It can thus be fine-tuned to meet the needs of each organization participating in the supply chain. Players in a supply chain can also recognize the role of cooperation with other players in improving the performance of the overall supply chain. By adopting various cooperation methodologies (information sharing, joint decision making, benefits sharing etc.,) with other facilities, organizations can improve products distribution in a supply chain by optimizing the use of production facilities.

Also, the analysis of the result shows that horizontal cooperation significantly affects the outcome of the model. It is thus safe to assume the horizontal cooperation can significantly improve the overall performance of a supply chain if it is well implemented. This is subject to management buy-in into the ideas, framework and methodologies behind cooperation in a supply chain. Other factors which can affect the implementation of cooperation in a supply chain include identification of performance metrics, effective information management system, identification of other stakeholders in the supply chain, excellent reward system, and employee training.

Further research areas related to this work can include studying a supply chain with horizontal cooperation at all tiers of the chain i.e. the supplier's supplier, suppliers, manufacturers, warehouses and distribution centers, retailers and consumers, examining the role of vertical cooperation in a supply chain, managing inventory in a supply chain with vertical cooperation, and implementation of both horizontal and vertical cooperation into a supply chain among others.

Funding opportunities

This research work has not received any funding.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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