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Reliability modeling and optimal random preventive maintenance policy for parallel systems with damage self-healing

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ABSTRACT

Materials with intrinsic self-healing phenomenon possess the ability to heal in response to external random shocks. Introducing a recovery factor to quantitatively measure the damage self-recovery efficiency, this paper designs a self-healing mechanism corresponding to both damage load and shock arrival numbers for a parallel redundant system consisting of multiple non-identical components. From the actual engineering perspective, each shock arriving on the system selectively affects one component or more but not necessarily all units in parallel, and consequently, random shocks are categorized according to their sizes, attributes and affected components. This study investigates novel reliability models and schedules optimal preventive maintenance policies, in which the closed-form reliability quantities are derived analytically and the optimum preventive replacement interval is demonstrated theoretically. In addition, a Nelder-Mead downhill simplex method is introduced to seek the optimal replacement age in minimizing the long-run average maintenance cost rate for the condition system failure distribution is rather complex. A micro-electro-mechanical system (MEMS) whose constitutional materials are integrated by microcrystalline silicon, where polymer binders with self-healing capability are always synthesized, is designed to verify the results we obtained numerically, illustrating the significance of considering damage self-healing phenomena.

1. Introduction

Most industrial systems fail to work due to two competing failure processes incorporating a soft failure process and a hard failure process. The soft failure occurs owing to internal performance degradation whereas the hard failure happens as a result of external random shocks. In general, these two failure processes are dependent as well as competing with each other, i.e., random shocks influence the increments on degradation amount or accelerate the degradation rate or both. That is, systems are suffering from dependent and competing failure processes (DCFP). Degradation modeling and reliability analysis for systems subject to DCFP have sought a lot of attention from researchers in the literature (Caballe & Castro, 2019; Hao & Yang, 2018; Qi, Zhou, Niu, Wang, & Wu, 2018; Wang, Li, Bai, & Zuo, 2020).

For the soft failure process, degradation models such as a general degradation path, a gamma process or a Wiener process are commonly developed in previous works (Cha, Finkelstein, & Levitin, 2018). Peng, Feng, and Coit (2010) considered a linear degradation path for the wear volume due to internal continuous degradation. Different from Peng et al.'s work (2010), Shen, Elwany, and Cui (2018) regarded that the degradation measurement between two adjacent shocks was regulated by a gamma process, and random shocks caused a jump in degradation level and accelerated the degradation rate simultaneously. Wei, Zhao, He, and He (2019) modeled system internal degradation as a two-phase Wiener process, which has a larger shift and diffusion parameter when the system transfers from the normal state to the weakened state. On the other aspect, random shocks resulting in catastrophic failures are considered as seven different types (Rafiee, Feng, & Coit, 2017): (a) extreme shock model, in which a system fails as soon as the first shock exceeds a specified threshold value; (b) standardly cumulative shock model, when hard failure occurs until the cumulative shock damage is beyond a critical level; (c) mixed shock model, where a system fails as soon as an extreme shock failure occurs or a cumulative shock failure occurs, whichever takes place first; (d) m−shock model, where a system experiences failure after shocks whose magnitudes are all larger than a threshold level; (e) δ−shock model, where failure happens when the time lag between two successive shocks is less than a threshold δ; (f) run shock model, in which a system fails if there is a run of n

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consecutive shocks that are larger than a specified threshold; (g) traumatic shock model, in which any shock is regarded as traumatic and a system fails in presence of shocks. These two failure processes affect system reliability and performability mutually. Other researches on systems suffering from DCFP are seen, for example, see Dong, Liu, and Du (2019); An and Sun (2017); Huang, Jin, He, and He (2019); Tang, Chen, and Huang (2019); Zhao, He, He, and Xie (2018); Huynh, Castro, Barros, and Bérenguer (2012). It is noted that the two competing processes can be regarded as a series system containing two dependent components, in which system failure occurs when any component fails (Li & Pham, 2005).

Based on certain applications, it can be argued that it is reasonably realistic and viable that random shocks are categorized into distinct shock sets according to their sizes, functions, specifications and affected components among others. Therefore, shocks with specific sizes or functions may selectively influence one or more components in a system, but not necessarily all the components in the meantime. For example, shocks on an automobile are categorized into mechanical shocks, voltage shocks, thermal shocks and other types according to their attributes. All types of shocks work on the whole system of the automobile, however, one component in the system can only be affected by several types of those shocks belonging to its distinctive shock set. This phenomenon is intensively common, especially for multi-component systems. A parallel redundant system is a typical multi-component system which is widely designed in industrial industries to meet the demand for a high reliability, and it is described that the system fails when all units in parallel have failed. Different from traditional parallel redundant systems, components in parallel in this current research are failure correlated though structure independent because their failure times may be affected by the same kind of random shocks, and all components are not exactly identical.

In order to reduce the occurrence probability of catastrophic failures and to enhance system reliability and availability, maintenance is always adopted as an effective approach. Maintenance policies are typically classified into preventive maintenance (PM) and corrective maintenance (CM) according to the arrangement time of maintenance actions (de Jonge and Scarf, 2020). In practical engineering, it is of significant sense that PM is scheduled on systems before catastrophic failure when system operations reach a certain usage time or a specific usage number. Generally, PM is usually adopted at some continuous measures (e.g., age, damage threshold, operating time), or at some discrete quantities (e.g., usage number, failure number, shock number) or at some bivariate maintenance times (Sheu, Liu, Zhang, & Tsai, 2018; Zhang & Wang, 2019). Besides maintenance policies, researchers have been devoted to designing a self-healing mechanism for systems or replacing components that are sensitive to failure by components with self-healing characteristics in past decades. Damage self-healing behavior should be noted as it enables a system to have resilience to failure and to recover part of system performances by itself without any external maintenance resources.

It should be noted that even though both maintenance actions and self-healing (also called, self-recovery) can recover the system to a better state, the two phenomena are distinct and dissimilarities do exist. Distinct from maintenance behaviors which consume outside resources and may return the system into a brand new state, self-healing ability is an intrinsic character in material itself and most self-healing processes recover system performance partially. In fact, self-healing has been widely designed especially in software integration. For example, built-in self-repair schemes of 3-dimensional memories containing a built-in self-test are introduced to guarantee the quality and yield when designing integrated circuits (Kang, Lee, Lim, & Kang, 2015). Consequently, the manufacturing process can be greatly improved after random shocks resulting from stresses, overloads, or more batch customers as the shock events may disclose some weaknesses during the manufacturing process. Hence, the software can be steadily improved by itself because of the increasing ability of self-adaptiveness.

Damage self-healing exists in a wide range of products and materials. Zhong and Yao (2008) conducted experimental tests by using specimens of normal strength concrete and high strength concrete respectively. They measured the damage degree from decline in ultrasonic pulse velocity (UPV) and compared the self-healing effects of different strength grades concrete, showing that the self-healing of damaged concrete is a process of crack closing with rehydration products of dehydrated or insufficiently-hydrated cementitious particles in damage regions. As the successful crack-healing methods require some forms of manual intervention, White et al. (2001) reported a structural polymeric material that has the ability to heal crack automatically. In their conducted experiments, self-healing is achieved through the incorporation of a microencapsulated healing agent and a catalytic chemical trigger within an epoxy matrix. The damage-induced triggering mechanism provides site-specific autonomic control of repair. Other discussions about self-healing mechanism are seen, for example, see Dong, Liu, Yang, Wang, and Fang (2019); Blaiszik et al. (2010); Frei et al. (2013). Succeeding in the design of self-healing materials significantly influences material safety, product performance and enhanced fatigue lifetime (Khalil, Eldash, Kumar, & Bayoumi, 2019; Psaier & Dustdar, 2011).

In reliability engineering, self-healing mechanism is limitedly noticed by researchers. Cui, Chen, and Gao (2018) introduced the concept of self-healing effects which may be permanent or limited duration on system health by building a class of cumulative damage shock models in terms of integral or counting processes. Motivated by Hawkes processes (Hawkes, 2018), the quantitative measure on self-healing effect in reliability models was firstly proposed. Liu, Yeh, and Cai (2017) built a novel reliability model for systems subject to DCFP while considering the self-healing phenomenon. For each random shock, they proposed the concepts of healing time and healing level to describe the self-healing process. Zhao, Guo, and Wang (2018) proposed a two-stage shock model with self-healing mechanism as an extension of cumulative shock model and $\delta-$shock model. They adopted a change-point to describe the limit of healing ability under the cumulative shock effects and damage is healed immediately when the system meets a certain condition in terms of self-healing mechanism. Shen, Cui, and Yi (2018) studied a system subject to random shocks by considering the self-healing action. They assumed that the system cannot be recovered by itself any more when it is seriously damaged, where the output performance is lower than a predetermined discrete state. Self-healing is the ability to repair damage and restore lost or degraded performance using resources inherently without human intervention. As stated by Blaiszik et al. (2010), the recovery process is triggered by the damage to materials. Hence, damage self-healing is designed with shock loads and shock arrival numbers, which are both considered in this research.

In light of the above surveyed literature, this current research intends to investigate the reliability model and schedule a preventive replacement policy for a parallel system subject to multiple external shocks. Random shocks are categorized into various finite types and not each shock set affects all units in the system at the same time. The significant contribution of this research is that a recovery factor is introduced to quantitatively measure the self-healing efficiency of shock damages, where the self-healing mechanism is sensitive to both the number of affected shocks and shock arrival time. As the redundant system does not fail until all components in parallel have failed, a systematic preventive replacement policy along with the self-healing action is developed to reduce the influence of catastrophic failures, in which the long-run average maintenance cost rate is minimized in order to determine the preventive replacement age. Numerical examples of a micro-electro-mechanical system (MEMS) are considered to demonstrate the effectiveness and viability of the model constructed.

The remainder of this paper is organized as follows. In Section 2, the problem statement is described, as well as some basic assumptions. Reliability models for a parallel redundant system considering shock damage self-healing are built in Section 3. Based on the reliability

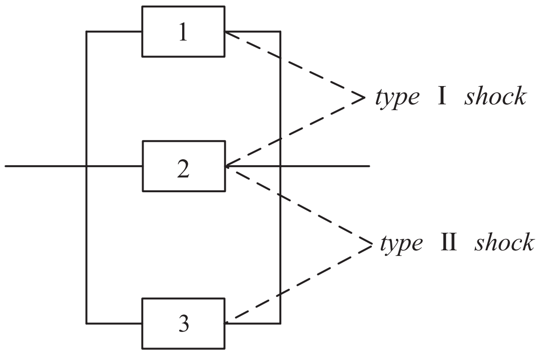

Fig. 1. A possible path of a parallel system consisting of three components.

models developed in Section 3, a preventive replacement maintenance policy is studied in Section 4. In Section 5, illustrative examples are exhibited to verify the results obtained in this paper numerically. Finally, we summarize the contents and point out the future research directions in Section 6.

2. Model description

Consider a parallel redundant system containing n $(n \geq 2)$ non-identical components, each of which is subject to its categorized shocks that arrive on random time epochs. Suppose the number of affected shock type on component i $(i = 1, 2, \cdots, n)$ is $|\phi_i|$ and the $j-th$ $(j = 1, 2, \cdots, |\phi_i|)$ categorized shock set arrives with a non-homogeneous Poisson process (NHPP) $\{N_{ij}(t), t \geq 0\}$. As a simple illustration, we take a parallel system consisting of three components as an example. As shown in Fig. 1, the system is suffered from two different types of shocks and the categorized shock set of each component is $\phi_1 = \{1\}$, $\phi_2 = \{1, 2\}$, and $\phi_3 = \{2\}$, respectively. That is, component 1 and component 2 are sensitive to type I shock while component 2 and component 3 are sensitive to type II shock.

From Fig. 1, components in the parallel system are failure dependent because for example, component 1 and component 2 share the same type of random shock. In addition, we develop an improved cumulative shock damage model to depict the failure process for each component in the parallel system, which is shown in Fig. 2. Without loss of generality, the hard failure thresholds of the three components are assumed to be the same level ℓ $(\ell > 0)$. W_{ijk} $(k = 1, 2, \cdots, N_{ij}(t))$ is the $j-th$ $(j = 1, 2, \cdots, |\phi_i|)$ shock magnitude of the affected shock set for component i $(i = 1, 2, \cdots, n)$ until time t.

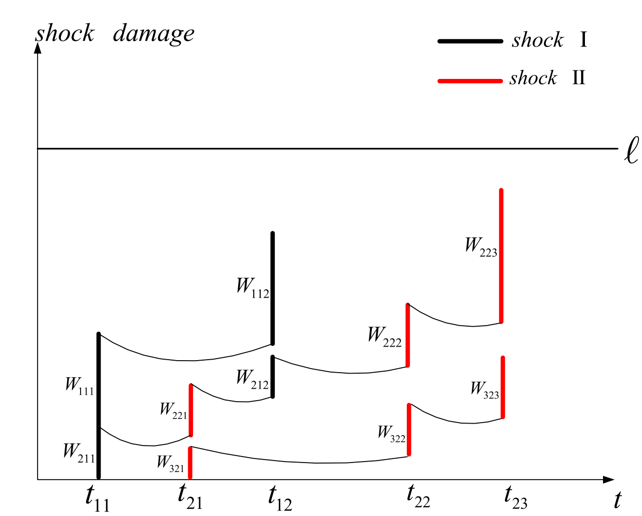

Fig. 2. A possible sample path of shock processes for the three-component system.

In Fig. 2, affected shock damages are not standardly cumulative owing to self-healing mechanism. As a matter of fact, plenty of modified damage models have been developed, e.g., a damage model with imperfect shock where some shocks may produce no damage, a random failure level with a general distribution, the total damage decreases exponentially with time, and the total damage increases with time (Zhao & Nakagawa, 2018). As damage self-healing action should be associated with random shocks, it is more realistic and practical to design a self-healing mechanism corresponding to both damage load and damage arrival numbers. In this paper, a damage recovery factor α $(\alpha \geq 0)$ is introduced to quantitatively measure the self-healing degree in components. Specifically, let $N_{ij}(t)$ be the arrival number of the $j-th$ $(j = 1, 2, \cdots, |\phi_i|)$ affected shock set for component i $(i = 1, 2, \cdots, n)$, shock load W_{ijk} $(1 \leq k \leq N_{ij}(t))$ reduces into $W_{ijk}e^{-(N_{ij}(t)-k)\alpha}$ after its $j-th$ effective shock.

To develop generalized reliability models for a system with N component in parallel and schedule a systematic PM policy, some basic assumptions are summarized as follows.

(1) At time epoch $t = 0$, component i $(i = 1, 2, \cdots, n)$ operates from an as-good-as-new (AGAN) state.
(2) External random shocks are categorized into different shock sets according to their affected components. Component i $(i = 1, 2, \cdots, n)$ is just sensitive to its affected shock set whereas other shocks have no influence on component i itself. When $\phi_d \bigcap \phi_i = \varnothing$ for any $d \neq i$ $(i, d = 1, 2, \cdots, n)$, the self-healing mechanism of component i cannot be triggered. The total number of random shock types on the whole system is

$$m = |\phi_1 \bigcup \phi_2 \bigcup \cdots \bigcup \phi_n|. \tag{1}$$

(3) Let ℓ_i be the failure threshold for component i in the parallel system and ℓ_i is considered to be a constant.
(4) Component i fails to work as soon as the total shock damage $W_i(t)$ considering both cumulative shock and damage self-healing exceeds its failure threshold ℓ_i. The first passage time to failure threshold ℓ_i of component i at time t is

$$T_i = inf\{t: W_i(t) \geq \ell_i | W_i(0) < \ell_i\}. \tag{2}$$

(5) Magnitudes $W_{ij1}, W_{ij2}, \cdots, W_{ijN_{ij}(t)}$ of the $j-th$ $(j = 1, 2, \cdots, |\phi_i|)$ categorized shock set for component i $(i = 1, 2, \cdots, n)$ are independent and identically distributed with a distribution function $F_{W_{ij}}(x) = P(W_{ijk} \leq x)$.
(6) The parallel system needs to operate for a job with a random working cycle Y $(Y > 0)$. Y is assumed to have a general distribution $H(t) = P(Y \leq t)$.
(7) As preventive maintenance, the system is arranged to be replaced at a planned age T $(0 < T \leq \infty)$ or at the completion of its working cycle Y independent with T, whichever occurs first. c_T $(c_T > 0)$ and c_Y $(c_Y > 0)$ are supposed to be the costs when the system is preventively replaced at time T or at the completion of the working cycle Y, respectively. Though the two preventive replacements restore the system to an AGAN state, the costs are not necessarily identical in practice. Since replacement at the fixed age T may interrupt the operating cycle, we assume that $c_T > c_Y$.
(8) A corrective replacement with a maintenance cost c_F is adopted when the system fails. The times to PM and CM are negligible. It is assumed that $c_F > c_T$ and $c_F > c_Y$.

3. Reliability evaluation models

In this section, the generalized system reliability evaluation models as well as special cases under some basic distribution assumptions for a parallel redundant system are derived analytically. It is noted that components in the parallel system are statistically dependent, and the reliability models for each component should be deduced before assessing system reliability.

3.1. Reliability for component *i*

Total shock damage $W_i(t)$ for component containing its affected cumulative shocks and damage self-healing is a random process in terms of t, where $W_i(t)$ is defined as

$$W_i(t) = \sum_{j=1}^{|\phi_i|} (W_{ij1}e^{-(N_{ij}(t)-1)\alpha} + W_{ij2}e^{-(N_{ij}(t)-2)\alpha} + \cdots + W_{ijN_{ij}(t)}) = \sum_{j=1}^{|\phi_i|}\sum_{k=1}^{N_{ij}(t)} W_{ijk}e^{-(N_{ij}(t)-k)\alpha}, \quad (3)$$

in which α ($\alpha \geq 0$) is designed as a damage recovery factor. The distribution function of $W_i(t)$ at a specific time t is

$$\begin{aligned} G_i(x) &= P(W_i(t) \leq x) \\ &= P\left(\sum_{j=1}^{|\phi_i|}\sum_{k=1}^{N_{ij}(t)} W_{ijk}e^{-(N_{ij}(t)-k)\alpha} \leq x\right) \\ &= \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\cdots\sum_{m_{|\phi_i|}=0}^{\infty} P\left(\sum_{j=1}^{|\phi_i|}\sum_{k=1}^{m_j} W_{ijk}e^{-(m_j-k)\alpha} \leq x | N_{i1}(t) = m_1, N_{i2}(t) = m_2, \cdots, N_{i|\phi_i|}(t) = m_{|\phi_i|}\right) \\ &\times P(N_{i1}(t) = m_1, N_{i2}(t) = m_2, \cdots, N_{i|\phi_i|}(t) = m_{|\phi_i|}). \end{aligned} \quad (4)$$

The total expected damage at time t is

$$E[W_i(t)] = \int_0^{\infty} x dG_i(x) = \int_0^{\infty} x dP\left(\sum_{j=1}^{|\phi_i|}\sum_{k=1}^{N_{ij}(t)} W_{ijk}e^{-(N_{ij}(t)-k)\alpha} \leq x\right), \quad (5)$$

and the Laplace-Stiejtjes (LS) transform of $G_i(x)$ is

$$\begin{aligned} G_i^*(s) &= \int_0^{\infty} e^{-sx} dP(W_i(t) \leq x) \\ &= \underbrace{F_{W_{i1}}^*(e^{-(N_{i1}(t)-1)\alpha}s)F_{W_{i1}}^*(e^{-(N_{i1}(t)-2)\alpha}s)\cdots F_{W_{i1}}^*(s)}_{N_{i1}(t)} \\ &\underbrace{F_{W_{i|\phi_i|}}^*(e^{-(N_{i|\phi_i|}(t)-1)\alpha}s)F_{W_{i|\phi_i|}}^*(e^{-(N_{i|\phi_i|}(t)-2)\alpha}s)\cdots F_{W_{i|\phi_i|}}^*(s)}_{N_{i|\phi_i|}(t)} \\ &= \prod_{j=1}^{|\phi_i|}\prod_{k=1}^{N_{ij}(t)} F_{W_{ij}}^*(e^{-(N_{ij}(t)-k)\alpha}s), \end{aligned} \quad (6)$$

where s is a complex number and

$$F_{W_{ij}}^*(s) = \int_0^{\infty} e^{-sx} dF_{W_{ij}}(x) = \int_0^{\infty} e^{-sx} dP(W_{ijk} \leq x), \quad 1 \leq k \leq N_{ij}(t). \quad (7)$$

Hence, in terms of the inversion formula in LS transform, reliability function for component i ($i = 1, 2, \cdots, n$) is derived as

$$\begin{aligned} R_i(t) &= P(T_i \geq t) \\ &= P(W_i(t) \leq \ell_i) \\ &= \lim_{c\to\infty}\frac{1}{2\pi a}\int_{b-ac}^{b+ac}\frac{e^{s\ell_i}}{s}G_i^*(s)ds \\ &= \lim_{c\to\infty}\frac{1}{2\pi a}\int_{b-ac}^{b+ac}\frac{e^{s\ell_i}}{s}\prod_{j=1}^{|\phi_i|}\prod_{k=1}^{N_{ij}(t)} F_{W_{ij}}^*(e^{-(N_{ij}(t)-k)\alpha}s)ds, \end{aligned} \quad (8)$$

where $a = \sqrt{-1}$ is an imaginary unit, $b > max\{0, \varkappa\}$ and $\varkappa$ is a radius of convergence.

3.2. System reliability

Since components in the redundant parallel system are dependent, $R(t) = 1 - \prod_{i=1}^{n}[1 - R_i(t)]$ does not hold in deriving system reliability. Assume that random shocks arrive independently and the intensity of the $q-th$ ($q = 1, 2, \cdots, m$) external shock on the system at time t is $\lambda_q(t)$ ($t \geq 0$), we can approach system reliability $R(t)$ by conditioning on the total shock arrival numbers as

$$\begin{aligned} R(t) &= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\sum_{n_m=0}^{\infty}\left\{1 - \prod_{i=1}^{n}[1 - R_i(t|N_1(t) = n_1, N_2(t) = n_2, \cdots, N_m(t) = n_m)]\right\} \\ &\times \prod_{q=1}^{m}\frac{(\int_0^t \lambda_q(\tau)d\tau)^{n_q}}{(n_q)!}e^{-\int_0^t \lambda_q(\tau)d\tau}. \end{aligned} \quad (9)$$

Substituting Eq.(8) into Eq.(9), system reliability becomes

$$\begin{aligned} R(t) &= \sum_{n_1=0}^{\infty}\cdots\sum_{n_m=0}^{\infty}\{1 - \prod_{i=1}^{n}[1 \\ &- \lim_{c\to\infty}\frac{1}{2\pi a}\int_{b-ac}^{b+ac}\frac{e^{s\ell_i}}{s}\prod_{j=1}^{m}\prod_{k=1}^{n_j} F_{W_{ij}}^*(e^{-(n_j-k)\alpha}s)ds]\} \\ &\times \prod_{q=1}^{m}\frac{(\int_0^t \lambda_q(\tau)d\tau)^{n_q}}{(n_q)!}e^{-\int_0^t \lambda_q(\tau)d\tau}. \end{aligned} \quad (10)$$

The mean time to failure (MTTF) is

$$\begin{aligned} \mu_n &= E[X] \\ &= \int_0^{\infty} R(t)dt \\ &= \sum_{n_1=0}^{\infty}\cdots\sum_{n_m=0}^{\infty}\int_0^{\infty}\{1 - \prod_{i=1}^{n}[1 - \lim_{c\to\infty}\frac{1}{2\pi a}\int_{b-ac}^{b+ac}\frac{e^{s\ell_i}}{s} \\ &\prod_{j=1}^{m}\prod_{k=1}^{n_j} F_{W_{ij}}^*(e^{-(n_j-k)\alpha}s)ds]\} \\ &\times \prod_{q=1}^{m}\frac{(\int_0^t \lambda_q(\tau)d\tau)^{n_q}}{(n_q)!}e^{-\int_0^t \lambda_q(\tau)d\tau}dt. \end{aligned} \quad (11)$$

where X ($X \geq 0$) is the system failure time and $X = \max\{T_1, T_2, \ldots, T_n\}$. Eqs. (10) and (11) are the generalized representations of system reliability and MTTF, respectively. Although the generalized forms of $R(t)$ and μ_n are obtained in Eqs. (10) and (11), it is almost impossible to solve them analytically because n_1 to n_m in the aforementioned formulae reach to infinity. Large numbers N_1 to N_m are set for substitutes when we resort to numerical results, and the approximation error is satisfactory as long as N_1 to N_m are appropriately selected (Hao & Yang, 2019; Huynh, Barros, & Bérenguer, 2012; Liu, Xie, Xu, & Kuo, 2016).

Here we show a special case under the normally distributed shock damage, i.e., $W_{ijk} \sim N(\mu_{W_{ij}}, \sigma_{W_{ij}}^2)$. Because the shock damage should be larger than zero, we introduce a truncated normal distribution to illustrate the CDF of each shock damage by the $k-th$ ($k = 1, 2, \cdots, N_{ij}(t)$) shock magnitude of the $j-th$ ($j = 1, 2, \cdots, |\phi_i|$) affected shock set. Hence,

$$P(W_{ijk} \leq x | W_{ijk} \geq 0) = \frac{P(0 \leq W_{ijk} \leq x)}{P(W_{ijk} \geq 0)} = \frac{\Phi[(x - \mu_{W_{ij}})/\sigma_{W_{ij}}] - \Phi[-\mu_{W_{ij}}/\sigma_{W_{ij}}]}{\Phi[\mu_{W_{ij}}/\sigma_{W_{ij}}]}. \quad (12)$$

The distribution of $W_i(t)$ is

$$
\begin{aligned}
&P\left(\sum_{j=1}^{|\phi_i|} \sum_{k=1}^{N_{ij}(t)} W_{ijk} e^{-(N_{ij}(t)-k)\alpha} \le x | \forall W_{ijk} \ge 0\right) \\
&= \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \cdots \sum_{m_{|\phi_i|}=0}^{\infty} \frac{\Phi\left[\frac{\ell_i - \sum_{j=1}^{|\phi_i|} \frac{1-e^{-m_j\alpha}}{1-e^{-\alpha}} \mu_{W_{ij}}}{\sqrt{\sum_{j=1}^{|\phi_i|} \frac{1-e^{-2m_j\alpha}}{1-e^{-2\alpha}} \sigma_{W_{ij}}^2}}\right] - \prod_{j=1}^{|\phi_i|} \Phi\left[-\frac{\mu_{W_{ij}}}{\sigma_{W_{ij}}}\right]^{m_j}}{\prod_{j=1}^{|\phi_i|} \Phi\left[\frac{\mu_{W_{ij}}}{\sigma_{W_{ij}}}\right]^{m_j}} \\
&\times \prod_{j=1}^{|\phi_i|} \frac{\left(\int_0^t \lambda_j(\tau)d\tau\right)^{m_j}}{(m_j)!} e^{-\int_0^t \lambda_q(\tau)d\tau}.
\end{aligned} \tag{13}
$$

Then the special case of $R(t)$ is

$$
\begin{aligned}
&R(t) \\
&= \sum_0^{N_1} \cdots \sum_0^{N_m} \left\{1 - \prod_{i=1}^{n} \left[1 - \frac{\Phi\left[\frac{\ell_i - \sum_{j=1}^{|\phi_i|} \frac{1-e^{-n_j\alpha}}{1-e^{-\alpha}} \mu_{W_{ij}}}{\sqrt{\sum_{j=1}^{|\phi_i|} \frac{1-e^{-2n_j\alpha}}{1-e^{-2\alpha}} \sigma_{W_{ij}}^2}}\right] - \prod_{j=1}^{|\phi_i|} \Phi\left[-\frac{\mu_{W_{ij}}}{\sigma_{W_{ij}}}\right]^{n_j}}{\prod_{j=1}^{|\phi_i|} \Phi\left[\frac{\mu_{W_{ij}}}{\sigma_{W_{ij}}}\right]^{n_j}}\right]\right\} \\
&\times \prod_{q=1}^{m} \frac{\left(\int_0^t \lambda_q(\tau)d\tau\right)^{n_q}}{(n_q)!} e^{-\int_0^t \lambda_q(\tau)d\tau}.
\end{aligned} \tag{14}
$$

A special case of MTTF is

$$
\begin{aligned}
&\mu_n \\
&= \sum_0^{N_1} \cdots \sum_0^{N_m} \int_0^{\infty} \left\{1 - \prod_{i=1}^{n} \left[1 - \frac{\Phi\left[\frac{\ell_i - \sum_{j=1}^{|\phi_i|} \frac{1-e^{-n_j\alpha}}{1-e^{-\alpha}} \mu_{W_{ij}}}{\sqrt{\sum_{j=1}^{|\phi_i|} \frac{1-e^{-2n_j\alpha}}{1-e^{-2\alpha}} \sigma_{W_{ij}}^2}}\right] - \prod_{j=1}^{|\phi_i|} \Phi\left[-\frac{\mu_{W_{ij}}}{\sigma_{W_{ij}}}\right]^{n_j}}{\prod_{j=1}^{|\phi_i|} \Phi\left[\frac{\mu_{W_{ij}}}{\sigma_{W_{ij}}}\right]^{n_j}}\right]\right\} \\
&\times \prod_{q=1}^{m} \frac{\left(\int_0^t \lambda_q(\tau)d\tau\right)^{n_q}}{(n_q)!} e^{-\int_0^t \lambda_q(\tau)d\tau} dt.
\end{aligned} \tag{15}
$$

It is pointed out that the reliability model we have constructed is not only applied to the parallel connection configuration, but also to other structures. As long as the component junction is defined, system reliability can be evaluated via the above approach. Denote system failure distribution as $F(t) \equiv 1 - R(t) \equiv 1 - \bar{F}(t)$. In order to schedule an optimal preventive replacement maintenance policy, the failure rate function $h(t) = -dR(t)/R(t)dt$ should be addressed. Though it is difficult to prove that $h(t)$ increases strictly with respect to time t analytically, it may be easy to draw this conclusion. As the cumulative damage process for each component is non-decreasing, the failure probability of any component in the time interval $[t, t + dt]$ becomes higher and higher, given that it operates well at time t (Dong et al., 2019). Hence, the failure rate $h(t)$ of the parallel system increases strictly from $h(0)$ to $h(\infty) = \lim_{t\to\infty} h(t)$, i.e., $F(t)$ belongs to an increasing failure rate (IFR) category.

4. Optimal random preventive replacement policy

As stated in Section 2, the parallel system has to operate for its working cycle Y. The system is preventively replaced at time T or at the completion of Y, or correctively replaced at failure, whichever occurs first, where T is a predetermined age and Y is a random variable with distribution $H(t)$, independent with the fixed replacement age T and system failure time X. In this section, we denote $\bar{H}(t) \equiv 1 - H(t)$. The probability that the system is preventively replaced at time T is

$$
p_T = P(T \le X, T < Y) = \bar{F}(T)\bar{H}(T), \tag{16}
$$

the probability that the system is preventively replaced at the completion of Y is

$$
p_Y = P(Y < X, Y \le T) = \int_0^T \bar{F}(t)dH(t), \tag{17}
$$

the probability that the system is correctively replaced at failure is

$$
p_F = P(X \le Y, X < T) = \int_0^T \bar{H}(t)dF(t), \tag{18}
$$

where should note that $p_T + p_Y + p_F \equiv 1$. Thus, the mean time to a systematic replacement is

$$
\begin{aligned}
&T\bar{F}(T)\bar{H}(T) + \int_0^T t\bar{F}(t)dH(t) + \int_0^T t\bar{H}(t)dF(t) \\
&= \int_0^T \bar{H}(t)\bar{F}(t)dt.
\end{aligned} \tag{19}
$$

Let $D(t)$ be the expected cost of the parallel system over time interval $[0, t]$. According to the basic renewal theory, the expected long-run maintenance cost rate is

$$
\begin{aligned}
C(T) &= \lim_{t\to\infty} \frac{D(t)}{t} \\
&= \frac{c_T + (c_Y - c_T)\int_0^T \bar{F}(t)dH(t) + (c_F - c_T)\int_0^T \bar{H}(t)dF(t)}{\int_0^T \bar{H}(t)\bar{F}(t)dt}.
\end{aligned} \tag{20}
$$

Clearly,

$$
\begin{aligned}
&C(0) = \lim_{T\to 0} C(T) = \infty, \\
&C(\infty) = \lim_{T\to\infty} C(T) = \frac{c_F - (c_F - c_Y)\int_0^{\infty} \bar{F}(t)dH(t)}{\int_0^{\infty} \bar{H}(t)\bar{F}(t)dt}.
\end{aligned} \tag{21}
$$

The aim is to seek an optimal T^* which minimizes $C(T)$ in Eq.(20). From Section 3, it should be noted that the failure rate $h(t)$ increases strictly with time t. Firstly, differentiating $C(T)$ with respect to T and setting it equal to zero, we find T^* satisfies

$$
\begin{aligned}
&(c_F - c_T)\left[h(T^*)\int_0^{T^*} \bar{H}(t)\bar{F}(t)dt - \int_0^{T^*} \bar{H}(t)dF(t)\right] \\
&- (c_T - c_Y)\left[r(T^*)\int_0^{T^*} \bar{H}(t)\bar{F}(t)dt - \int_0^{T^*} \bar{F}(t)dH(t)\right] = c_T,
\end{aligned} \tag{22}
$$

where $r(t) = dH(t)/\bar{H}(t)dt$. The minimum maintenance cost rate is

$$
C(T^*) = (c_F - c_T)h(T^*) - (c_T - c_Y)r(T^*). \tag{23}
$$

Let $Q(T^*)$ be the left hand side of Eq. (22), i.e.,

$$
\begin{aligned}
&Q(T^*) = (c_F - c_T)\left[h(T^*)\int_0^{T^*} \bar{H}(t)\bar{F}(t)dt - \int_0^{T^*} \bar{H}(t)dF(t)\right] \\
&- (c_T - c_Y)\left[r(T^*)\int_0^{T^*} \bar{H}(t)\bar{F}(t)dt - \int_0^{T^*} \bar{F}(t)dH(t)\right].
\end{aligned} \tag{24}
$$

It has been proved that $Q(T^*)$ increases strictly from $Q(0) = 0$ to $Q(\infty) = \lim_{T\to\infty} Q(T)$ with respect to T^* (Chen, Zhao, & Nakagawa, 2019). Hence, if $Q(\infty)$ is greater than c_T, there exists a finite and unique T^* which satisfies Eq. (22) and the resulting maintenance cost rate $C(T^*)$ is given as that in Eq. (23). If $Q(\infty) \le c_T$, the optimal preventive replacement age $T^* = \infty$ and $C(T^*) = C(\infty) = \lim_{T\to\infty} C(T)$ is given as that in Eq. (21).

In fact, as system failure distribution $F(t) \equiv 1 - R(t)$ obtained from Eq. (10) is complex, it becomes rather difficult to judge that $Q(T^*)$ in Eq. (24) is greater than c_T or not. In addition, the objective function in Eq. (20) is a nonlinear function, making the problem more laborious. Nelder-Mead downhill simplex method is one of the most well-known direct search approaches to seek the optimum solution of unconstrained nonlinear function, which does not need the difficult calculation of derivatives (Li & Pham, 2005). Hence, we develop a step-by-step algorithm based on the Nelder-Mead downhill simplex method for seeking the optimum decision variable T^* such that the long-run average maintenance cost rate $C(T)$ is minimized.

Nelder-Mead downhill simplex method is an iterative algorithm by comparing the function values at the $n+1$ vertices for $n-$dimensional decision variables. The initial simplex vertices are iterated through reflection, expansion and contraction operations with aims to quest for a better solution. The step-by-step algorithm based on Nelder-Mead downhill simplex method is shown in Algorithm 1.

Algorithm 1. (*Search the termination condition of Eq. (20)*).

Step 1: Let $T^{(1)}$ and $T^{(2)}$ denote the list of vertices in the current simplex.
Step 2: Order and relabel the two vertices from lower function value $C(T^{(1)})$ to higher function value $C(T^{(2)})$, so that $C(T^{(1)}) \le C(T^{(2)})$.
Step 3: Compute the reflected point $Z^{(1)}$ by $Z^{(1)} = T^{(1)} + \rho(T^{(1)} - T^{(2)})$, where the reflection coefficient $\rho > 0$.If $C(T^{(1)}) \le C(Z^{(1)}) < C(T^{(2)})$, then replace $T^{(2)}$ with $Z^{(1)}$ and go to **Step 6**; end
Step 4: If $C(Z^{(1)}) < C(T^{(1)})$, then compute the expanded point by $Z^{(2)} = T^{(1)} + \chi(Z^{(1)} - T^{(1)})$, where the expansion coefficient $\chi > 1$; end
If $C(Z^{(2)}) < C(Z^{(1)})$, then replace $T^{(2)}$ with $Z^{(2)}$ and go to **Step 6**; else replace $T^{(2)}$ with $Z^{(1)}$ and go to **Step 6**; end
Step 5: If $C(Z^{(1)}) \ge C(T^{(2)})$, then calculate $Z^{(3)} = T^{(1)} + \eta(T^{(2)} - T^{(1)})$, where the contraction coefficient $0 < \eta < 1$; end
If $C(Z^{(3)}) < C(T^{(2)})$, then replace $T^{(2)}$ with $Z^{(3)}$ and go to **Step 6**; end
Step 6: Order and relabel the two vertices from the lower function value $C(T^{(1)})$ to the higher function value $C(T^{(2)})$, so that $C(T^{(1)}) \le C(T^{(2)})$.
If $\sqrt{\frac{1}{2}[(C(T^{(1)}) - C(\bar{T}))^2 + (C(T^{(2)}) - C(\bar{T}))^2]} < \varepsilon$, then stop, where ε is a predetermined tolerance and $\bar{T} = \frac{1}{2}(T^{(1)} + T^{(2)})$; else return to **Step 3**.

5. Numerical examples

With the progress of micro-electro-mechanical systems (MEMS) and the further development of interface and conditioning circuitry, the tiny silicon MEMS resonators fabricated on silicon wafers with micro-scale mechanical structures have been proven reliable and stable in providing frequency sources for relevant products. Compared with quartz crystals, MEMS have the features of being cheaper, thinner and easier for system integration.

A MEMS oscillator is a typical multi-component system consisted of multiple comb drive resonators (Song, Coit, & Feng, 2014). The external shocks on MEMS resonators may arise from electrostatic, piezoelectric, optical, mechanical vibration or magnetic stimuli. In addition, a silicon resonator needs to be sealed in a vacuum or hermetic environment to keep its stability. Based on the materials and processes for sealing, wafer level vacuum packaging for MEMS resonators are categorized into two different types: wafer bonding and deposition sealing (Hsu, 2008). The choice of two packaging technologies is based on the resonator design as well as vacuum requirements, final package design, and products specifications. Fig. 3 shows an example of wafer level packaging for MEMS resonators.

To reduce the silicon's dramatic volume change during cycling, a water-soluble polymer binder, poly (acrylic acid)-poly (2-hydroxyethyl acrylate-co-dopamine methacrylate) is always synthesized to keep a better wettability to liquid electrolyte (Xu et al., 2018). Formed with rigid-soft bonds and structured with a multiple network, the polymer binder enables special self-healing capability in the electrode, which provides enough mechanical support and buffers the strain caused by the volume change of Si micro-particle (SiMP).

Consider a redundant MEMS oscillator with five non-identical comb drive resonators in parallel in Fig. 4. Suppose the MEMS operates in stable working environments and there are totally three different kinds of random shocks, in which the arrival intensities are $\lambda_1 = 0.01 month^{-1}$, $\lambda_2 = 0.02 month^{-1}$, and $\lambda_3 = 0.03 month^{-1}$, respectively. Shocks arrive independently, but not every random shock affects all components in the system, as described in Fig. 4.

From Fig. 4, the categorized shock sets for each component are $\phi_1 = \{1\}$, $\phi_2 = \{1, 2\}$, $\phi_3 = \{1, 2, 3\}$, $\phi_4 = \{2, 3\}$ and $\phi_5 = \{3\}$, respectively. The basic parameters for the components are tabulated in Table 1.

According to Eq.(9), system reliability function is

$$R(t) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \sum_{n_3=0}^{N_3} \left\{1 - \prod_{i=1}^{5} [1 - R_i(t|N_1(t) = n_1, N_2(t) = n_2, N_3(t) = n_3)]\right\} \times \prod_{i=1}^{3} \frac{(\lambda_i t)^{n_i}}{(n_i)!} e^{-\lambda_i t}.$$

Which is shown in Fig. 5. In Fig. 5, $\alpha = 0$ means that the polymer binder synthesized in the silicon MEMS resonator does not show any self-healing impact in respond to external random shocks. From Fig. 5, system reliability becomes higher with the increase of damage recovery factor α, showing the self-recovery influence of the synthesized polymer binder to external shocks. When α is small, self-healing ability of the polymer binder is slightly triggered. Hence, system reliability $R(t)$ seldom improves from $\alpha = 0$ to $\alpha = 1$. When α increases from 1 to 1.5, system reliability varies greatly owing to the sufficient stimulation of self-healing materials. When the self-healing ability tends to saturated, $R(t)$ changes little as well, corresponding to the condition from $\alpha = 1.5$ to $\alpha = 2$ in Fig. 5.

Fig. 6 plots the failure rate functions under different α, which illustrates that failure rate functions increase strictly with time t.

Then we discuss the effects of shock intensities on system reliability. Assume the recovery factor $\alpha = 2$, Figs. 7 and 8 illustrate the effects of shock intensities on system reliability and failure rate.

From Figs. 7 and 8, system reliability becomes lower with the increase of shock intensities. As the MEMS resonator is sealed, it is impossible to repair the failed components unless replace the packaged system entirely. Assume that the maintenance costs $c_F = 100USD$, $c_T = 15USD$, and $c_Y = 5USD$. Firstly we discuss the change of maintenance cost rate when $\theta = 0$, which means that the maintenance policy is regarded as a pure age replacement without considering the random working cycle Y. The results are shown in Fig. 9.

Applying the Nelder-Mead downhill simplex method, the search results are presented in Table 2.

As can be seen from Table 2, MTTF and the optimal preventive replacement age of the parallel system increases with the improvement of recovery factor from $\alpha = 0$ to $\alpha = 2$, while at the same time, the long-run maintenance cost rate decreases from $0.3403USD/month$ to $0.0957USD/month$. The conclusion tells that if the damage self-healing is not considered, the failure probability is overestimated while MTTF of the redundant system is underestimated.

Fig. 10 plots the maintenance cost rate under different conditions with the fixed recovery factor $\alpha = 2$. It is clearly seen that the optimum maintenance cost rate $C(T^*)$ becomes higher when θ becomes larger.

The optimization results are tabulated in Table 3 for Fig. 10 with Nelder-Mead downhill simplex method.

From Fig. 10 and Table 3, the optimum maintenance cost rate $C(T^*)$ becomes from $2.5USD/month$ to $10USD/month$ with the increase of θ. At the same time, $T^* = \infty$ for all the cases $\alpha = 2$, which means that we should preventively replace the system at the completion of its random working cycle, or correctively replace it at system failures, without considering the planned replacement age T.

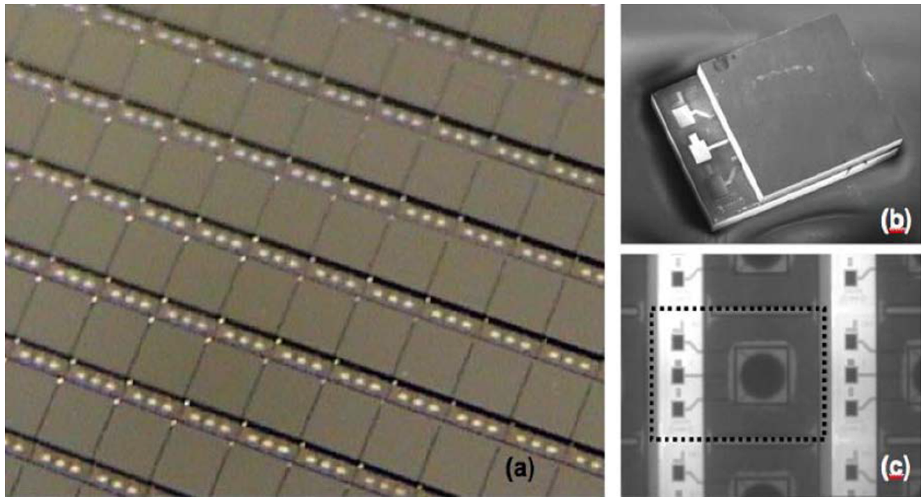

Fig. 3. An example of wafer level packaged silicon MEMS resonators (Hsu, 2008).

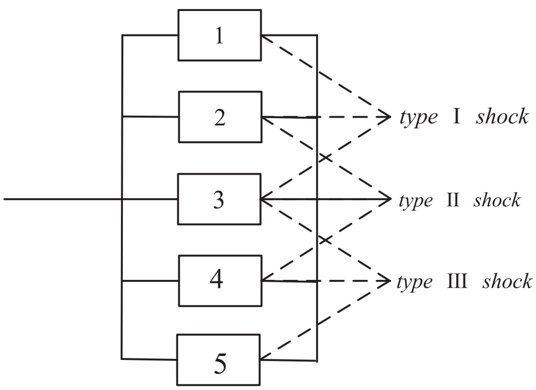

Fig. 4. A five-units parallel system with three kinds of shocks.

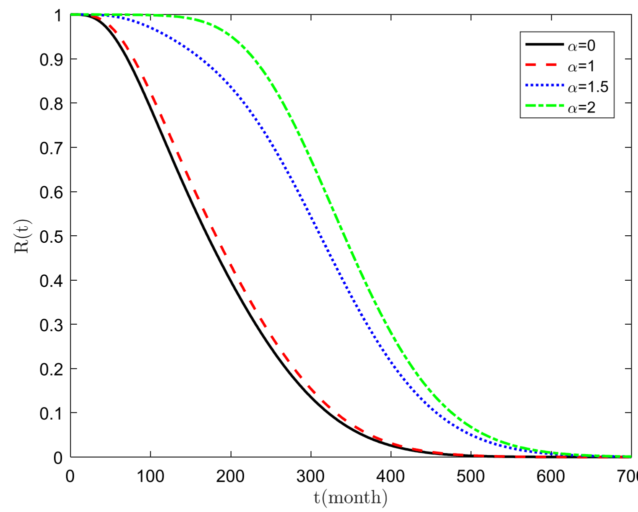

Fig. 5. System reliability under different recovery factors.

6. Concluding remarks

Damage self-healing (or self-recovery) exists widely in materials such as polymers, metals, ceramics and their composites. The design for self-healing action into materials paves way for the greater lifetime use of a product and thus is important not only because of economic concerns but for human safety as well. In this present research, we have attempted to quantify the self-healing efficiency by introducing a damage recovery factor. By quantitatively measuring the self-healing effectiveness, two reliability quantities including reliability function and mean failure time are derived. From the actual engineering point of view, shocks affecting a multi-component system are categorized into distinct types according to their sizes, functions, affected components and so forth. After that, we develop a systematic preventive replacement policy considering that the system needs to operate for a random working cycle. A packaged MEMS oscillator, which is composed of five non-identical redundant comb drive resonators, is designed as an illustrative example to verify the results numerically.

Degradation is another significant factor leading to product failure (Hao & Yang, 2019). Meanwhile, random shocks may accelerate internal degradation rate, changing the product's failure mechanism or shrinking its operation duration, especially shocks stemming from extreme environments. More work on the interaction of damage self-healing and degradation process should be conducted in the future.

Table 1
Basic parameters of the parallel redundant system.

Units	1	2	3	4	5
ℓ_i	91	92	93	94	95
W_{ij}	$N(\mu_{W_{1j}}, \sigma^2_{W_{1j}})$ $\mu_{W_{11}} = 72.6$, $\sigma_{W_{11}} = 6.3$	$N(\mu_{W_{2j}}, \sigma^2_{W_{2j}})$ $\mu_{W_{21}} = 71.6$, $\sigma_{W_{21}} = 5.9$; $\mu_{W_{22}} = 72.3$, $\sigma_{W_{22}} = 6.1$	$N(\mu_{W_{3j}}, \sigma^2_{W_{3j}})$ $\mu_{W_{31}} = 72.1$, $\sigma_{W_{31}} = 5.9$; $\mu_{W_{32}} = 69.9$, $\sigma_{W_{32}} = 5.8$; $\mu_{W_{33}} = 72.8$, $\sigma_{W_{33}} = 6.2$	$N(\mu_{W_{4j}}, \sigma^2_{W_{4j}})$ $\mu_{W_{41}} = 72.2$, $\sigma_{W_{41}} = 6.1$; $\mu_{W_{42}} = 68.9$, $\sigma_{W_{42}} = 6.3$	$N(\mu_{W_{5j}}, \sigma^2_{W_{5j}})$ $\mu_{W_{51}} = 73.2$, $\sigma_{W_{51}} = 6.4$

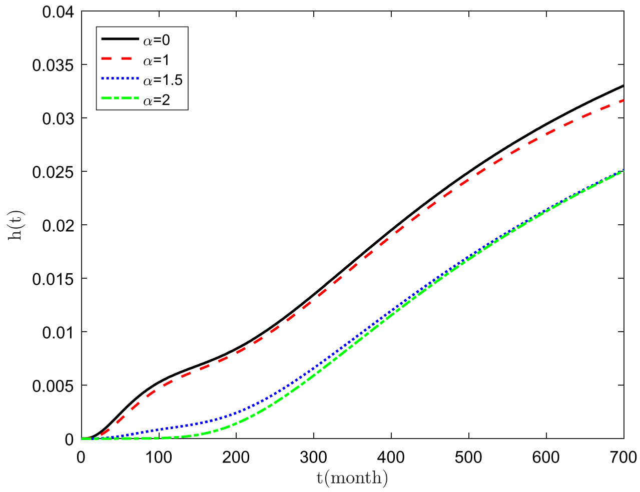

Fig. 6. Failure rate function under different recovery factors.

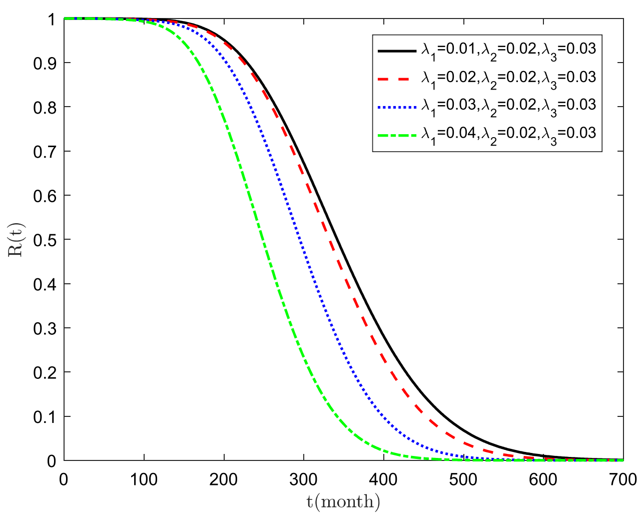

Fig. 7. System reliability under different shock intensities with $\alpha = 2$.

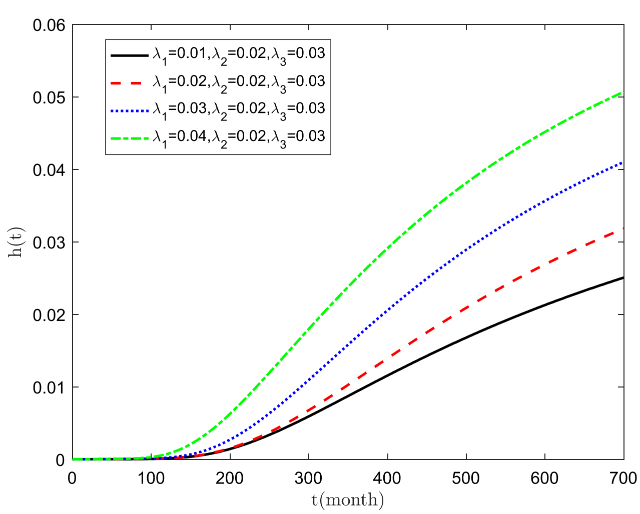

Fig. 8. Failure rate function under different shock intensities with $\alpha = 2$.

CRediT authorship contribution statement

Wenjie Dong: Conceptualization, Methodology, Investigation, Writing - original draft. **Sifeng Liu:** Resources, Supervision, Visualization, Data curation. **Yingsai Cao:** Validation, Formal analysis, Visualization. **Saad Ahmed Javed:** Conceptualization, Validation, Software. **Yangyang Du:** Writing - review & editing.

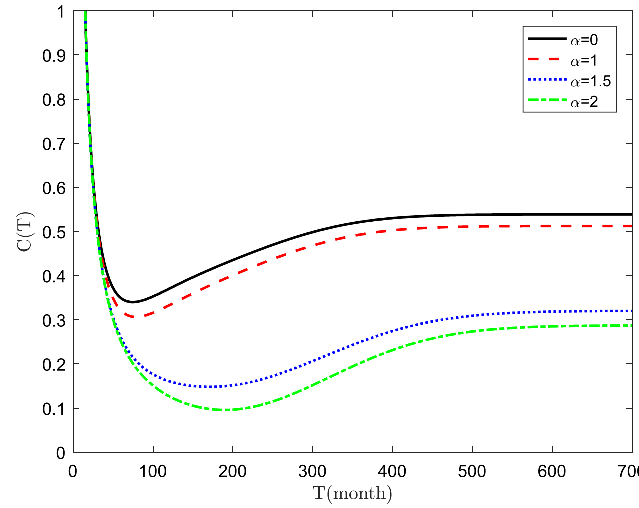

Fig. 9. The average long-run maintenance cost rate with $\theta = 0$.

Table 2
Nelder-Mead algorithm results with $\theta = 0$.

	$\alpha = 0$	$\alpha = 1$	$\alpha = 1.5$	$\alpha = 2$
MTTF (month)	185.58	195.14	312.29	348.19
T^*	73.2	79.4	169.5	190.3
$C(T^*)$	0.3403	0.3065	0.1482	0.0957

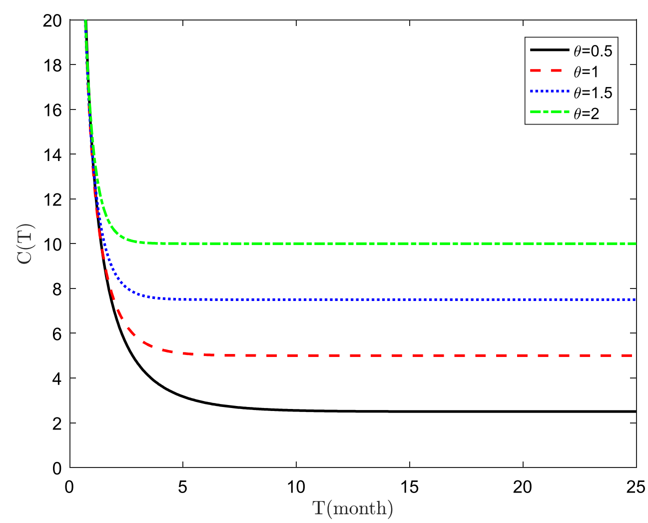

Fig. 10. The average long-run maintenance cost rate with $\alpha = 2$.

Table 3
Nelder-Mead algorithm results with $\alpha = 2$.

	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$
MTTF (month)	348.19	348.19	348.19	348.19
T^*	∞	∞	∞	∞
$C(T^*)$	2.50	5.00	7.50	10.00

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Declaration of Competing Interest

The authors declared that there is no conflict of interest.

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