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Reliability modeling and maintenance optimization for the two-unit system with preset self-repairing mechanism

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Abstract

In this article, the reliability model and the opportunistic maintenance optimization model are formulated for the preset self-repairing mechanism which is artificially designed and applied to many engineering systems. The preset self-repairing mechanism is first introduced into the reliability model, and a series system consisting of two units is built to describe the proposed model. One unit in the system is subject to external shocks and has the preset self-repairing mechanism, the other does not have the recovery mechanism and its lifetime distribution follows exponential distribution. For the system, the analytical expression of reliability is derived, and a maintenance optimization model taking the long-run average cost per unit time as objective function is established. The decision parameters of the maintenance policy are preventive and opportunistic degradation levels. Besides, a preventive maintenance policy is proposed for comparison with the opportunistic maintenance policy. Finally, the numerical examples are provided to obtain the optimal decision parameters and demonstrate the effectiveness of opportunistic maintenance policies.

Keywords

Self-repairing mechanism, reliability, opportunistic maintenance, shock model, series system

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Introduction

Since sudden failures will interrupt the normal operation of important systems and cause severe economical or social consequences, it is necessary to devise a recovery action for the system. A suitable recovery action guarantees the operation of system and reduces the loss caused by system failures.[1–3] In fact, the modes of different recovery actions usually have unique characteristics.[4–7]

Cui et al.[8] classified roughly the existing recovery modes into two categories: maintenance action and self-healing effect. Most maintenance actions are artificially designed for solving sudden failures of system and require outside backup source to support even some maintenance durations.[9–12] The mode of self-healing effect is different from maintenance action.[13,14] Commonly, self-healing effect is described to the following characteristics:[15] (a) self-healing effect is an intrinsic property of the system or a mechanism embedded in the system; (b) self-healing effect is infinite until the system reaches a specific state, and it will be destroyed when the state of system is worse than the specific state; (c) self-healing effect cannot restore the failed components.

In addition, there exist other recovery actions with unique characteristics, in which the preset self-repairing mechanism is an important recovery action. The mechanism of embedding the fluid healing material into systems to gain preset self-repairing ability has been widely applied in engineering field.[16–18] A gear with multi-layer composite surface coatings which mixed MoS_2/titanium in the wind turbine gearbox is a typical example for the unit with preset self-repairing mechanism.[19,20] The gear generates crack because of vibration, which increases the risk of system failure.[21] When the length of crack reaches a certain level and sufficient crack stress is produced, the healing material is released automatically to repair the crack. Besides, Cui et al.[18] demonstrated that approaches for fluid secretion, which typically rely on fluid encapsulation and release from a shelled compartment, do not usually allow a fine

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continuous modulation of secretion, and can be difficult to adapt for monitoring purposes. The conclusion indicates that the repairing capability of liquid healing material is uncertain, and the unit can be randomly upgraded to a better state. Commonly, the system loses self-repairing ability if a valid shock interrupts the self-repairing process.[15] Based on the above operation features, three characteristics of preset self-repairing mechanism are summarized: (a) preset self-repairing mechanism is an artificial recovery mechanism, and the self-repairing ability derives from the healing material embedded in the system; (b) preset self-repairing mechanism is triggered immediately when the state of system reaches a certain extent, and the system can be randomly repaired to a better state; (c) the system loses self-repairing ability when the healing material is exhausted or the self-repairing process is interrupted by a valid shock. In reality, the healing material can be released several times before it is exhausted, and the preset self-healing mechanism disappears after several successful executions. To simplify the model, assume that the self-repairing mechanism is disposable, namely all the healing material is released to repair the crack of system during the first self-repairing process, and the preset self-repairing mechanism disappears when the first self-repairing process ends or the self-repairing process is interrupted.

In fact, there are many researches on maintenance action and self-healing effect; for example, Tang et al.[9] devised optimal condition-based maintenance (CBM) policy with a failure rate–based control limit for the age- and state-dependent degrading system. Zhu et al.[10] proposed a new CBM policy for multi-component systems with continuous stochastic deterioration to reduce the high maintenance setup cost. Cui et al.[8] formulated the reliability model for a system with self-healing effect under shock model, and the arrival process of the shock is modeled by some counting processes. Shen et al.[15] established a cumulative shock model to discuss the reliability analytical expressions of a system in the case that the magnitude of external shocks follows exponential distribution. However, no research considers the preset self-repairing mechanism. Most previous researches focus on the innovative system model or the reliability index of other recovery modes. In this article, the preset self-repairing mechanism is first introduced into the reliability model. A two-unit series system is proposed, in which unit A is subject to condition monitoring and external shocks and unit B has an exponential lifetime distribution. Besides, external shocks consist of valid shocks and invalid shocks. In addition to interrupting the preset self-repairing mechanism, valid shocks cause a certain amount of damage to the system, while invalid shocks have no effect on the system. For the two-unit series system, the reliability is discussed based on the relationship between reliability function and probability density function (PDF) of system lifetime.

It is worth noting that the preset self-repairing mechanism has no effect on units that have already failed. A suitable maintenance policy is indispensable for the system containing units with preset self-repairing mechanism to guarantee the normal operation.[22–24] Since different recovery modes can make up for the shortcomings of other modes, the combination of different recovery modes is a topic worth studying, for example, Zhao et al.[14] considered a two-stage shock model to describe the system with self-healing mechanism, and three preventive maintenance policies are proposed. Typically, maintenance strategies are roughly defined as time-based maintenance and CBM.[25–29] In a CBM policy, maintenance decisions depend on the data collected through inspection of systems, and the inspection epoch is often considered as the decision epoch.[30,31] Moreover, the interval of inspection in a CBM policy can be classified as constant or varying.[32,33] In this article, a CBM policy is implemented for unit A, and the inspection interval of the opportunistic maintenance policy is varying. However, the system usually generates downtime cost because of the time interval of maintenance decision, while the downtime cost is less considered in CBM policies.[34–37] Therefore, a CBM policy considering the downtime cost is devised for the system.

The organization of the remainder of this article is as follows. In section "Model formulations," basic model assumptions and different scenarios of preset self-repairing mechanism under external shocks are considered, and then the reliability analytical expressions for unit A and the system are derived. In section "Optimal opportunistic maintenance policy," the opportunistic maintenance policy is applied to the two-unit series system. Aiming at the minimum long-run average cost per unit time, the optimal preventive maintenance and opportunistic degradation levels are determined. In section "Numerical example," numerical examples for the optimal maintenance policy are given. Eventually, section "Summary" summarizes this article and discusses the future research directions.

Model formulations

In this section, different scenarios of the preset self-repairing mechanism are analyzed to illustrate the deterioration process of unit A, and the reliability of the two-unit series system is derived.

Basic assumptions

1. Unit A is subject to external shocks, and the arrival of external shocks follows the Poisson process with parameter λ_1. Let U_i denote the time interval between the $(i-1)$th and the ith external shock, $i = 1, 2, \ldots$ and $U_i \sim Exp(\lambda_1)$. Then, let Z_n denote the time interval between an initiate point and the

nth shock arrival epoch in the Poisson process, that is, $Z_n = U_1 + U_2 + \cdots + U_n$. The PDF of Z_n is as follows

$$f_{Z_n}(t) = \frac{{\lambda_1}^n}{\Gamma(n)} t^{n-1} e^{-\lambda_1 t} = \frac{{\lambda_1}^n}{(n-1)!} t^{n-1} e^{-\lambda_1 t}, n = 1, 2, \ldots \tag{1}$$

2. Let Stages 1, 2, 3 divide the whole lifetime process of unit A to better describe the influence of the preset self-repairing mechanism and valid shocks on the deterioration process of unit A. Stage 1: from the initiate epoch (totally new state) to the first time point (the specific state), the state of unit A deteriorates gradually from totally new to the initiate condition of the preset self-repairing mechanism. Stage 2: from the first time point to the second time point (a better or worse state), the self-repairing ability comes into effect until it disappears. Stage 3: from the second time point to the end epoch (failure), unit A deteriorates gradually to failure.
3. Let n_i denote the number of external shocks arriving in Stage i, $i = 1, 2, 3$. The external shock has two types: valid shock and invalid shock. Each valid shock causes a certain equal damage to unit A, while the invalid shock has no effect on unit A. Let p_i denote the probability that an arriving shock in Stage i is a valid shock, $i = 1, 2, 3$, $p_i + q_i = 1$. Unit A is prone to failure when the self-repairing ability is operating, that is, $p_1 = p_3 < p_2$.
4. Let $\Omega = \{0, 1, ..., s, ..., K\}$ be the state space of unit A, and the state division is based on the degree of accumulated damage. State 0 represents that unit A is totally new without any damage. State K represents that unit A is considered to be failed. State s is the initiate condition of the preset self-repairing mechanism.
5. The probability that unit A is repaired to state i by self-repairing ability is p_{Ri}, $i \in \{0, 1, ..., s-1\}$ and $\sum_{i=0}^{s-1} p_{Ri} = 1$. When $i = 0$, the self-repairing ability can be seen as perfect maintenance; when $i \in \{1, 2, ..., s-1\}$, the ability can be seen as imperfect maintenance. Let random variable W denote the duration of self-repairing process, and $W \sim Exp(\lambda_2)$.
6. Let random variable V denote the lifetime of unit B, and $V \sim Exp(\lambda_3)$.

Model formulation

It is necessary for system operation to analyze the preset self-repairing mechanism. As mentioned in Assumption 2: in Stage 1, unit A deteriorates gradually until the state of unit A reaches s, and the recovery mechanism is triggered. In Stage 2, there are two scenarios for the preset self-repairing mechanism. Scenario I: this mechanism performs successfully, and unit A is upgraded randomly to a better state i, $i \in \{0, 1, ..., s-1\}$. Scenario II: this mechanism performs unsuccessfully, the self-repairing process is interrupted by a valid shock, and the state of unit A degrades to $s + 1$. The self-repairing ability disappears when the self-repairing process ends or Scenario II occurs. Then, unit A enters Stage 3 and deteriorates gradually to failure. Since the self-repairing mechanism is assumed to be disposable in section "Introduction," the self-repairing ability disappears when Stage 2 ends. To better illustrate the two scenarios, suppose that $i = 2$, $s = 4$, $K = 6$. Figures 1 and 2 depict Scenarios I and II, respectively.

As illustrated in Figure 1, there are s valid shocks in Stage 1, and the sth valid shock triggers the preset self-repairing mechanism. In Stage 2, no valid shock arrives during the self-repairing process, and unit A is upgraded to state i. Then, the self-repairing ability disappears. In Stage 3, unit A degrades gradually to failure because of valid shocks.

It is worth mentioning that there are some invalid shocks between two adjacent valid shocks. Because the arrival of external shocks follows the Poisson process, the duration distribution of Stage 1 follows the Erlang distribution. Moreover, the duration distribution of Stage 2 follows exponential distribution because the self-repairing process is not interrupted. Last but not the least, the starting epoch of Stage 3 can be regarded as an initiate point of the Poisson process due to the memoryless property of the exponential distribution, and the duration distribution of Stage 3 follows the Erlang distribution.

As illustrated in Figure 2, there are s valid shocks in Stage 1, and the sth valid shock triggers the preset self-repairing mechanism. In Stage 2, a valid shock arrives during the self-repairing process and interrupts the recovery mechanism, and unit A is degraded to state $s + 1$. In Stage 3, unit A degrades gradually to failure because of valid shocks.

Same to Scenario I, the duration distribution of Stage 1 follows the Erlang distribution. Since both the starting and ending epochs of Stage 2 are the arrival epoch of a valid shock, the duration distribution of Stage 2 follows the Erlang distribution. Similarly, the duration distribution of Stage 3 follows the Erlang distribution.

Reliability analysis

Reliability analysis of the proposed system is carried out in two steps. First, the reliability analysis of unit A is discussed. Second, the reliability of two-unit system is considered.

The reliability of unit A is derived based on the relationship between the reliability function and the PDF

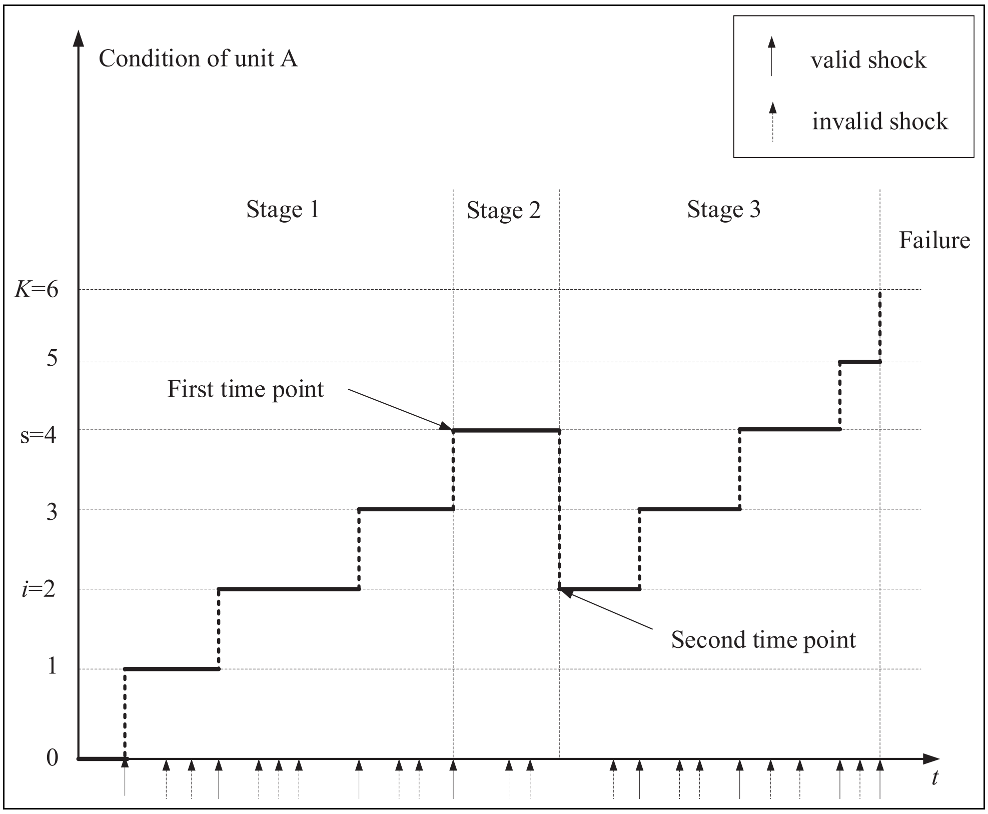

Figure 1. Scenario I of preset self-repairing mechanism.

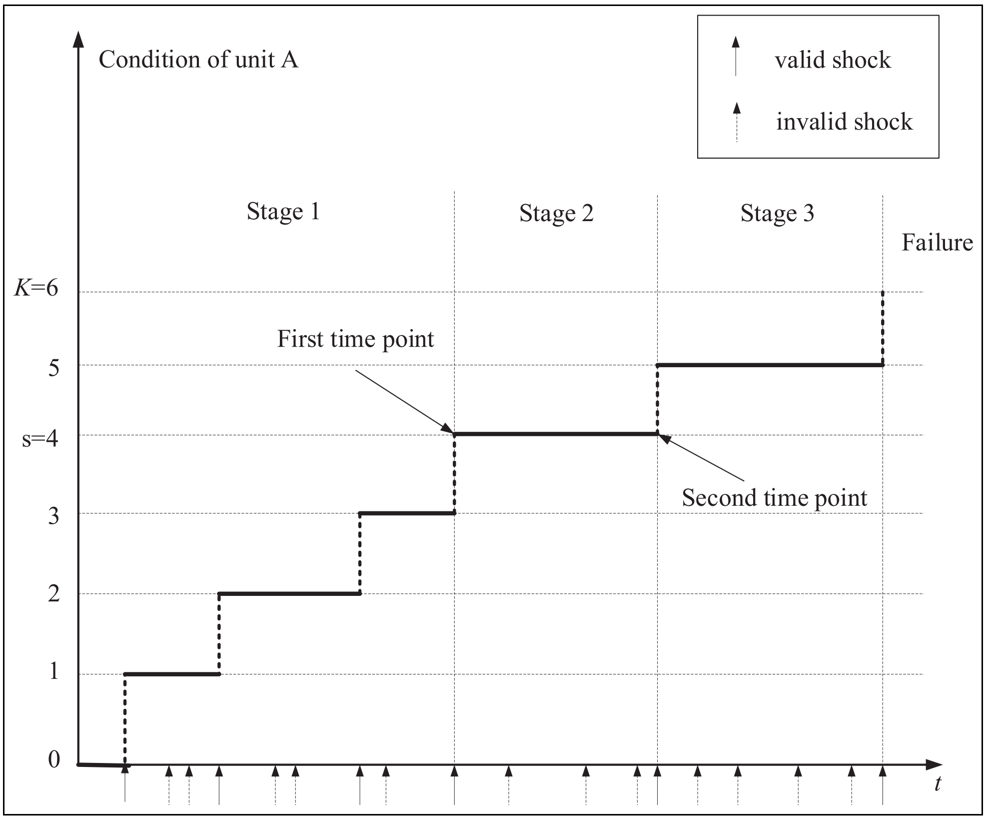

Figure 2. Scenario II of preset self-repairing mechanism.

of unit A lifetime. Let $R_A(t)$ be the reliability function of unit A, and $R_A(t)$ is as follows

$$R_A(t) = 1 - \int_0^t h_A(x)dx \tag{2}$$

where $h_A(t)$ represents the PDF of unit A lifetime.

The two scenarios mentioned above occur when unit A reaches state K from state 0.

Scenario I. Preset self-repairing mechanism performs successfully.

As illustrated in Figure 1, the deterioration process of unit A is divided into three stages. There are s valid shocks in n_1 arriving external shocks in Stage 1. The n_1th external shock triggering the preset self-repairing mechanism is a valid shock. In Stage 2, n_2 invalid shocks arrive during the self-repairing duration. In Stage 3, $K-i$ valid shocks in n_3 arriving external shocks cause the failure of unit A, and the n_3th external shock is a valid shock. Let $h_A^i(t)$ represent the probability that the lifetime of unit A is t if unit A is upgraded to state i in Stage 2, $i = 0, 1, 2, ..., s-1$, and $h_A^i(t)$ is as follows

$$h_A^i(t) = \sum_{n_1=s}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=K-i}^{\infty} C_{n_1-1}^{s-1} {p_1}^s {q_1}^{n_1-s} {q_2}^{n_2} C_{n_3-1}^{K-i-1} {p_3}^{K-i} {q_3}^{n_3-K+i} p_{Ri} \int_0^t\int_0^{t-l} P(Z_{n_1} = l, Z_{n_3} = t-w-l, Z_{n_2} \leqslant w < Z_{n_2} + U_{n_2+1} | W = w) dF(W \leqslant w) dl \tag{3}$$

where

$$\begin{aligned}&\int_0^t\int_0^{t-l} P(Z_{n_1} = l, Z_{n_3} = t-w-l, Z_{n_2} \leqslant w < Z_{n_2} + U_{n_2+1} | W = w) dF(W \leqslant w) dl \\ &= \int_0^t\int_0^{t-l} \lambda_2 e^{-\lambda_2 w} e^{-\lambda_1 t} \frac{{\lambda_1}^{n_1} l^{n_1-1}}{(n_1-1)!} \frac{{\lambda_1}^{n_3}(t-w-l)^{n_3-1}}{(n_3-1)!} \int_0^w \frac{{\lambda_1}^{n_2} z^{n_2-1}}{(n_2-1)!} dz dw dl \\ &= \lambda_2 e^{-\lambda_1 t} {\lambda_1}^{n_1+n_2+n_3} \int_0^t\int_0^{t-l} e^{-\lambda_2 w} \frac{l^{n_1-1} w^{n_2} (t-w-l)^{n_3-1}}{(n_1-1)! n_2! (n_3-1)!} dw dl\end{aligned}$$

Scenario II. Preset self-repairing mechanism is interrupted.

As illustrated in Figure 2, there are also s valid shocks in n_1 arriving external shocks in Stage 1. The n_1th external shock triggering the preset self-repairing mechanism is a valid shock. In Stage 2, the situation different from Scenario I is that one valid shock in n_2 arriving external shocks interrupts the preset self-repairing mechanism. Then, $K-s-1$ valid shocks in n_3 arriving external shocks cause the failure of unit A, and the n_3th external shock is a valid shock. Let $h_A^{s+1}(t)$ represent the probability that the lifetime of unit A is t if unit A is degraded to state $s+1$ in Stage 2, and $h_A^{s+1}(t)$ is as follows

$$h_A^{s+1}(t) = \sum_{n_1=s}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=K-s-1}^{\infty} C_{n_1-1}^{s-1} {p_1}^s {q_1}^{n_1-s} p_2 {q_2}^{n_2-1} C_{n_3-1}^{K-s-2} {p_3}^{K-s-1} {q_3}^{n_3-K+s+1} \int_0^t\int_0^{t-l} P(Z_{n_1} = l, Z_{n_3} = t-x-l, W > x | Z_{n_2} = x) dF(Z_{n_2} \leqslant x) dl \tag{4}$$

where

$$\begin{aligned}&\int_0^t\int_0^{t-l} P(Z_{n_1} = l, Z_{n_3} = t-x-l, W > x | Z_{n_2} = x) dF(Z_{n_2} \leqslant x) dl \\ &= e^{-\lambda_1 t} {\lambda_1}^{n_1+n_2+n_3} \int_0^t\int_0^{t-l} e^{-\lambda_2 x} \frac{l^{n_1-1} x^{n_2-1} (t-x-l)^{n_3-1}}{(n_1-1)!(n_2-1)!(n_3-1)!} dx dl\end{aligned}$$

The sum of the probabilities of two scenarios represents the PDF of unit A lifetime, denoting the probability that the lifetime of unit A is t. According to equations (2)–(4), the $h_A(t)$ is obtained as follows

$$h_A(t) = \sum_{i=0}^{s-1} h_A^i(t) + h_A^{s+1}(t) \tag{5}$$

Then, consider the reliability of the two-unit series system with preset self-repairing mechanism. The reliability function of unit B is $R_B(t)$

$$R_B(t) = e^{-\lambda_3 t} \tag{6}$$

According to equations (2), (5) and (6), the reliability function of system is

$$R(t) = R_A(t) R_B(t) \tag{7}$$

Optimal opportunistic maintenance policy

A suitable maintenance policy is essential for the two-unit series system because the preset self-repairing mechanism cannot restore the failed units. In this section, the probabilities of different situations are analyzed and calculated, as well as the long-run average cost per unit time is obtained.

The opportunistic maintenance policy

In this opportunistic maintenance policy, the preventive and opportunistic degradation levels of unit A are state m and state o, respectively. To the utmost utilization of the preset self-repairing mechanism, let $m > o > s$. Because of the memoryless property of exponential distribution, it is notable that the opportunistic maintenance policy has no effect on unit B. Therefore, there is no need to replace unit B when unit A is correctively or preventively replaced. Besides, assume that the replacement durations of corrective action, preventive action and opportunistic action are a certain fixed time Q, and the replacement actions are performed one by one instead of simultaneous.

If the inspection results indicate that the units of system do not need to be replaced, the next inspection is performed after Δ time interval. If the inspection results indicate that the units of system need to be replaced, the next inspection is performed after the replacement duration is over and Δ time interval is passed. Different situations are summarized as follows:

1. Unit A and unit B do not fail in an inspection interval, and the state of unit A does not reach m. No maintenance actions for unit A and unit B.
2. Unit A and unit B do not fail in an inspection interval, and the state of unit A reaches or exceeds m. unit A is preventively replaced.
3. Unit A fails in an inspection interval, and it is correctively replaced in the next inspection epoch. There is no opportunistic replacement for unit B.
4. Unit B fails in an inspection interval, and it is correctively replaced in the next inspection epoch. The state of unit A does not reach o, and there is no opportunistic replacement for unit A.
5. Unit B fails in an inspection interval, and it is correctively replaced in the next inspection epoch. The state of unit A reaches or exceeds o, and unit A is opportunistically replaced.

Cost analysis

Since the system can be randomly upgraded to a better state by self-repairing ability, the worst repairing capability situation of self-repairing mechanism is considered in the maintenance optimization model. Suppose $p_{Rs-1} = 1$, namely the system can merely be repaired to state $s-1$ by self-repairing ability. Let $P_{ij}, i = 1, 2, 3, 4, 5, j = 1, 2, ..., 9$ be the probability of the jth case in ith situation. a, b denote the initial and terminal state of unit A in an inspection interval, respectively. n_1, n_2, n_3 denote the number of external shocks arriving in Stages 1,2,3, respectively. n denotes the number of external shocks arriving in the lifetime of unit A, that is, $n = n_1 + n_2 + n_3$.

Situation 1. No replacement actions to unit A and unit B.

Case 1: $0 \leqslant a \leqslant s-1$, $a \leqslant b \leqslant s-1$

$$P_{11}(\Delta) = \sum_{n=b-a}^{\infty} C_n^{b-a} {p_1}^{b-a} {q_1}^{n-b+a} A(\Delta) L(\Delta)$$

where $L(\Delta)$ denotes the probability that unit B does not fail in the time interval Δ, and the expression of $A(\Delta)$ is given by equation (11) in Appendix 2. The $L(\Delta)$ is as follows

$$L(\Delta) = e^{-\lambda_3 \Delta}$$

Case 2: $0 \leqslant a \leqslant s-1$, $b = s$

$$P_{12}(\Delta) = \sum_{n_1=s-a}^{\infty} \sum_{n_2=0}^{\infty} C_{n_1-1}^{s-a-1} {p_1}^{s-a} {q_1}^{n_1-s+a} {q_2}^{n_2} B(\Delta) L(\Delta)$$

where the expression of $B(\Delta)$ is given by equation (12) in Appendix 2.

Case 3: $a = s$, $b = s$

$$P_{13}(\Delta) = \sum_{n=0}^{\infty} {q_2}^{n} C(\Delta) L(\Delta)$$

where the expression of $C(\Delta)$ is given by equation (13) in Appendix 2.

When the preset self-repairing mechanism performs successfully,

Case 4: $0 \leqslant a \leqslant s-1$, $s-1 \leqslant b \leqslant m-1$

$$P_{14}(\Delta) = \sum_{n_1=s-a}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=b-s+1}^{\infty} C_{n_1-1}^{s-a-1} {p_1}^{s-a} {q_1}^{n_1-s+a} {q_2}^{n_2} C_{n_3}^{b-s+1} {p_3}^{b-s+1} {q_3}^{n_3-b+s-1} D(\Delta) L(\Delta)$$

where the expression of $D(\Delta)$ is given by equation (14) in Appendix 2.

Case 5: $a = s$, $s-1 \leqslant b \leqslant m-1$

$$P_{15}(\Delta) = \sum_{n_1=0}^{\infty} \sum_{n_2=b-s+1}^{\infty} {q_2}^{n_1} C_{n_2}^{b-s+1} {p_1}^{b-s+1} {q_1}^{n_2-b+s-1} E(\Delta) L(\Delta)$$

where the expression of $E(\Delta)$ is given by equation (15) in Appendix 2.

Case 6: $s-1 \leqslant a \leqslant m-1$, $a \leqslant b \leqslant m-1$

$$P_{16}(\Delta) = \sum_{n=b-a}^{\infty} C_n^{b-a} {p_1}^{b-a} {q_1}^{n-b+a} A(\Delta) L(\Delta)$$

When the preset self-repairing mechanism is interrupted,

Case 7: $0 \leqslant a \leqslant s-1,\ s+1 \leqslant b \leqslant m-1$

$$P_{17}(\Delta) = \sum_{n_1=s-a}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=b-s-1}^{\infty} C_{n_1-1}^{s-a-1} {p_1}^{s-a} {q_1}^{n_1-s+a} p_2 {q_2}^{n_2-1} C_{n_3}^{b-s-1} {p_3}^{b-s-1} {q_3}^{n_3-b+s+1} F(\Delta)L(\Delta)$$

where the expression of $F(\Delta)$ is given by equation (16) in Appendix 2.

Case 8: $a = s,\ s+1 \leqslant b \leqslant m-1$

$$P_{18}(\Delta) = \sum_{n_1=1}^{\infty}\sum_{n_2=b-s-1}^{\infty} p_2 {q_2}^{n_1-1} C_{n_2}^{b-s-1} {p_1}^{b-s-1} {q_1}^{n_2-b+s+1} G(\Delta)L(\Delta)$$

where the expression of $G(\Delta)$ is given by equation (17) in Appendix 2.

Case 9: $s+1 \leqslant a \leqslant m-1,\ a \leqslant b \leqslant m-1$

$$P_{19}(\Delta) = \sum_{n=b-a}^{\infty} C_n^{b-a} {p_1}^{b-a} {q_1}^{n-b+a} A(\Delta)L(\Delta)$$

Situation 2. Unit A is preventively replaced.

When the preset self-repairing mechanism performs successfully,

Case 1: $0 \leqslant a \leqslant s-1,\ m \leqslant b \leqslant K-1$

$$P_{21}(\Delta) = \sum_{n_1=s-a}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=b-s+1}^{\infty} C_{n_1-1}^{s-a-1} {p_1}^{s-a} {q_1}^{n_1-s+a} {q_2}^{n_2} C_{n_3}^{b-s+1} {p_3}^{b-s+1} {q_3}^{n_3-b+s-1} D(\Delta)L(\Delta)$$

Case 2: $a = s,\ m \leqslant b \leqslant K-1$

$$P_{22}(\Delta) = \sum_{n_1=0}^{\infty}\sum_{n_2=b-s+1}^{\infty} {q_2}^{n_1} C_{n_2}^{b-s+1} {p_1}^{b-s+1} {q_1}^{n_2-b+s-1} E(\Delta)L(\Delta)$$

Case 3: $s-1 \leqslant a \leqslant m-1,\ m \leqslant b \leqslant K-1$

$$P_{23}(\Delta) = \sum_{n=b-a}^{\infty} C_n^{b-a} {p_1}^{b-a} {q_1}^{n-b+a} A(\Delta)L(\Delta)$$

When the preset self-repairing mechanism is interrupted,

Case 4: $0 \leqslant a \leqslant s-1,\ m \leqslant b \leqslant K-1$

$$P_{24}(\Delta) = \sum_{n_1=s-a}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=b-s-1}^{\infty} C_{n_1-1}^{s-a-1} {p_1}^{s-a} {q_1}^{n_1-s+a} p_2 {q_2}^{n_2-1} C_{n_3}^{b-s-1} {p_3}^{b-s-1} {q_3}^{n_3-b+s+1} F(\Delta)L(\Delta)$$

Case 5: $a = s,\ m \leqslant b \leqslant K-1$

$$P_{25}(\Delta) = \sum_{n_1=1}^{\infty}\sum_{n_2=b-s-1}^{\infty} p_2 {q_2}^{n_1-1} C_{n_2}^{b-s-1} {p_1}^{b-s-1} {q_1}^{n_2-b+s+1} G(\Delta)L(\Delta)$$

Case 6: $s+1 \leqslant a \leqslant m-1,\ m \leqslant b \leqslant K-1$

$$P_{26}(\Delta) = \sum_{n=b-a}^{\infty} C_n^{b-a} {p_1}^{b-a} {q_1}^{n-b+a} A(\Delta)L(\Delta)$$

Situation 3. Unit A is correctively replaced, and there is no maintenance action to unit B.

Be aware of that unit A is correctively replaced in the next inspection epoch, which means that the system generates downtime cost. Let β denote the normal operation time of system when unit A or unit B fails in an inspection interval.

When the preset self-repairing mechanism performs successfully,

Case 1: $0 \leqslant a \leqslant s-1,\ b = K$

$$P_{31}(\beta) = \sum_{n_1=s-a}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=K-s+1}^{\infty} C_{n_1-1}^{s-a-1} {p_1}^{s-a} {q_1}^{n_1-s+a} {q_2}^{n_2} C_{n_3-1}^{K-s} {p_3}^{K-s+1} {q_3}^{n_3-K+s-1} H(\beta)L(\beta)$$

where the expression of $H(\beta)$ is given by equation (18) in Appendix 2.

Case 2: $a = s,\ b = K$

$$P_{32}(\beta) = \sum_{n_1=0}^{\infty}\sum_{n_2=K-s+1}^{\infty} {q_2}^{n_1} C_{n_2-1}^{b-s} {p_1}^{b-s+1} {q_1}^{n_2-K+s-1} I(\beta)L(\beta)$$

where the expression of $I(\beta)$ is given by equation (19) in Appendix 2.

Case 3: $s-1 \leqslant a \leqslant m-1,\ b = K$

$$P_{33}(\beta) = \sum_{n=K-a}^{\infty} C_{n-1}^{K-a-1} {p_1}^{K-a} {q_1}^{n-K+a} e^{-\lambda_1\beta} \frac{{\lambda_1}^n \beta^{n-1}}{(n-1)!} L(\beta)$$

When the preset self-repairing mechanism is interrupted, Case 4: $0 \leqslant a \leqslant s-1,\ b=K$

$$P_{34}(\beta)=\sum_{n_1=s-a}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=K-s-1}^{\infty}C_{n_1-1}^{s-a-1}{p_1}^{s-a}{q_1}^{n_1-s+a}$$
$$p_2{q_2}^{n_2-1}C_{n_3-1}^{K-s-2}{p_3}^{K-s-1}{q_3}^{n_3-K+s+1}M(\beta)L(\beta)$$

where the expression of $M(\beta)$ is given by equation (20) in Appendix 2.

Case 5: $a=s,\ b=K$

$$P_{35}(\beta)=\sum_{n_1=1}^{\infty}\sum_{n_2=K-s-1}^{\infty}p_2{q_2}^{n_1-1}C_{n_2-1}^{K-s-2}{p_1}^{K-s-1}$$
$${q_1}^{n_2-K+s+1}N(\beta)L(\beta)$$

where the expression of $N(\beta)$ is given by equation (21) in Appendix 2.

Case 6: $s+1 \leqslant a \leqslant m-1,\ b=K$

$$P_{36}(\beta)=\sum_{n=K-a}^{\infty}C_{n-1}^{K-a-1}{p_1}^{K-a}{q_1}^{n-K+a}e^{-\lambda_1\beta}\frac{{\lambda_1}^n\beta^{n-1}}{(n-1)!}L(\beta)$$

Situation 4. Unit B is correctively replaced, and the state of unit A does not reach or exceed state o.

Case 1: $0 \leqslant a \leqslant s-1,\ a \leqslant b \leqslant s-1$

$$P_{41}(\beta)=\sum_{n=b-a}^{\infty}C_n^{b-a}{p_1}^{b-a}{q_1}^{n-b+a}A(\beta)J(\beta)$$

where $J(\beta)$ denotes the probability that the lifetime of unit B is β, and $J(\beta)$ is as follows

$$J(\beta)=\lambda_3 e^{-\lambda_3\beta}$$

Case 2: $0 \leqslant a \leqslant s-1,\ b=s$

$$P_{42}(\beta)=\sum_{n_1=s-a}^{\infty}\sum_{n_2=0}^{\infty}C_{n_1-1}^{s-a-1}{p_1}^{s-a}{q_1}^{n_1-s+a}{q_2}^{n_2}B(\beta)J(\beta)$$

Case 3: $a=s,\ b=s$

$$P_{43}(\beta)=\sum_{n=0}^{\infty}{q_2}^nC(\beta)J(\beta)$$

When the preset self-repairing mechanism performs successfully,

Case 4: $0 \leqslant a \leqslant s-1,\ s-1 \leqslant b \leqslant o-1$

$$P_{44}(\beta)=\sum_{n_1=s-a}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=b-s+1}^{\infty}C_{n_1-1}^{s-a-1}{p_1}^{s-a}$$
$${q_1}^{n_1-s+a}{q_2}^{n_2}C_{n_3}^{b-s+1}{p_3}^{b-s+1}{q_3}^{n_3-b+s-1}D(\beta)J(\beta)$$

Case 5: $a=s,\ s-1 \leqslant b \leqslant o-1$

$$P_{45}(\beta)=\sum_{n_1=0}^{\infty}\sum_{n_2=b-s+1}^{\infty}{q_2}^{n_1}C_{n_2}^{b-s+1}{p_1}^{b-s+1}$$
$${q_1}^{n_2-b+s-1}E(\beta)J(\beta)$$

Case 6: $s-1 \leqslant a \leqslant o-1,\ a \leqslant b \leqslant o-1$

$$P_{46}(\beta)=\sum_{n=b-a}^{\infty}C_n^{b-a}{p_1}^{b-a}{q_1}^{n-b+a}A(\beta)J(\beta)$$

When the preset self-repairing mechanism is interrupted,

Case 7: $0 \leqslant a \leqslant s-1,\ s+1 \leqslant b \leqslant o-1$

$$P_{47}(\beta)=\sum_{n_1=s-a}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=b-s-1}^{\infty}C_{n_1-1}^{s-a-1}{p_1}^{s-a}{q_1}^{n_1-s+a}$$
$$p_2{q_2}^{n_2-1}C_{n_3}^{b-s-1}{p_3}^{b-s-1}{q_3}^{n_3-b+s+1}F(\beta)J(\beta)$$

Case 8: $a=s,\ s+1 \leqslant b \leqslant o-1$

$$P_{48}(\beta)=\sum_{n_1=1}^{\infty}\sum_{n_2=b-s-1}^{\infty}p_2{q_2}^{n_1-1}C_{n_2}^{b-s-1}{p_1}^{b-s-1}$$
$${q_1}^{n_2-b+s+1}G(\beta)J(\beta)$$

Case 9: $s+1 \leqslant a \leqslant o-1,\ a \leqslant b \leqslant o-1$

$$P_{49}(\beta)=\sum_{n=b-a}^{\infty}C_n^{b-a}{p_1}^{b-a}{q_1}^{n-b+a}A(\beta)J(\beta)$$

Situation 5. Unit B is correctively replaced, and unit A is opportunistically replaced.

Case 1: $o \leqslant a \leqslant m-1,\ a \leqslant b \leqslant K-1$

$$P_{51}(\beta)=\sum_{n=b-a}^{\infty}C_n^{b-a}{p_1}^{b-a}{q_1}^{n-b+a}A(\beta)J(\beta)$$

When the preset self-repairing mechanism performs successfully,

Case 2: $0 \leqslant a \leqslant s-1,\ o \leqslant b \leqslant K-1$

$$P_{52}(\beta) = \sum_{n_1 = s-a}^{\infty} \sum_{n_2 = 0}^{\infty} \sum_{n_3 = b-s+1}^{\infty} C_{n_1-1}^{s-a-1} {p_1}^{s-a} {q_1}^{n_1-s+a} {q_2}^{n_2} C_{n_3}^{b-k} {p_3}^{b-k} {q_3}^{n_3-b+k} D(\beta) J(\beta)$$

Case 3: $a = s$, $o \leqslant b \leqslant K-1$

$$P_{53}(\beta) = \sum_{n_1 = 0}^{\infty} \sum_{n_2 = b-s+1}^{\infty} {q_2}^{n_1} C_{n_2}^{b-s+1} {p_1}^{b-s+1} {q_1}^{n_2-b+s-1} E(\beta) J(\beta)$$

Case 4: $s-1 \leqslant a \leqslant o-1$, $o \leqslant b \leqslant K-1$

$$P_{54}(\beta) = \sum_{n = b-a}^{\infty} C_n^{b-a} {p_1}^{b-a} {q_1}^{n-b+a} A(\beta) J(\beta)$$

When the preset self-repairing mechanism is interrupted,

Case 5: $0 \leqslant a \leqslant s-1$, $o \leqslant b \leqslant K-1$

$$P_{55}(\beta) = \sum_{n_1 = s-a}^{\infty} \sum_{n_2 = 1}^{\infty} \sum_{n_3 = b-s-1}^{\infty} C_{n_1-1}^{s-a-1} {p_1}^{s-a} {q_1}^{n_1-s+a} p_2 {q_2}^{n_2-1} C_{n_3}^{b-s-1} {p_3}^{b-s-1} {q_3}^{n_3-b+s+1} F(\beta) J(\beta)$$

Case 6: $a = s$, $o \leqslant b \leqslant K-1$

$$P_{56}(\beta) = \sum_{n_1 = 1}^{\infty} \sum_{n_2 = b-s-1}^{\infty} p_2 {q_2}^{n_1-1} C_{n_2}^{b-s-1} {p_1}^{b-s-1} {q_1}^{n_2-b+s+1} G(\beta) J(\beta)$$

Case 7: $s+1 \leqslant a \leqslant o-1$, $o \leqslant b \leqslant K-1$

$$P_{57}(\beta) = \sum_{n = b-a}^{\infty} C_n^{b-a} {p_1}^{b-a} {q_1}^{n-b+a} A(\beta) J(\beta)$$

In a word, the maintenance cost of system is composed of inspection cost, replacement cost and downtime cost. Let C_k be the inspection cost, and C_d be the average downtime cost per unit time. Meanwhile, C_A^c denotes the corrective replacement cost of unit A, C_B^c denotes the corrective replacement cost of unit B, C_A^p denotes the preventive replacement cost of unit A and C_A^o denotes the opportunistic replacement cost of unit A. The relationship between C_A^c, C_A^p and C_A^o is $C_A^c > C_A^p > C_A^o$.

The long-run average cost per unit time C, which derives from the long-run expected cost C_e and the long-run average period T, is regarded as objective function in the opportunistic maintenance optimization model. As calculated above, the long-run expected cost is as follows

$$\begin{aligned} C_e = {} & C_k \sum_{i=1}^{9} P_{1i}(\Delta) + (C_A^p + C_k) \sum_{i=1}^{6} P_{2i}(\Delta) + \left(C_A^c + C_k\right) \int_0^{\Delta} \sum_{i=1}^{6} P_{3i}(\beta) d\beta \\ & + \int_0^{\Delta} C_d \sum_{i=1}^{6} P_{3i}(\beta)(\Delta - \beta) d\beta + \left(C_B^c + C_k\right) \int_0^{\Delta} \sum_{i=1}^{9} P_{4i}(\beta) d\beta \\ & + \int_0^{\Delta} C_d \sum_{i=1}^{9} P_{4i}(\beta)(\Delta - \beta) d\beta + \left(C_B^c + C_A^o + C_k\right) \int_0^{\Delta} \sum_{i=1}^{7} P_{5i}(\beta) d\beta \\ & + \int_0^{\Delta} C_d \sum_{i=1}^{7} P_{5i}(\beta)(\Delta - \beta) d\beta + C_d Q \sum_{i=1}^{6} P_{2i}(\Delta) \\ & + C_d Q \int_0^{\Delta} \sum_{i=1}^{6} P_{3i}(\beta) d\beta + C_d Q \int_0^{\Delta} \sum_{i=1}^{9} P_{4i}(\beta) d\beta \\ & + 2 C_d Q \int_0^{\Delta} \sum_{i=1}^{7} P_{5i}(\beta) d\beta \end{aligned} \tag{8}$$

The long-run average period T is

$$\begin{aligned} T = {} & \Delta \left(\sum_{i=1}^{9} P_{1i}(\Delta) \right) + (\Delta + 2Q) \int_0^{\Delta} \sum_{i=1}^{7} P_{5i}(\beta) d\beta + (\Delta + Q) \\ & \left(\sum_{i=1}^{6} P_{2i}(\Delta) + \int_0^{\Delta} \sum_{i=1}^{6} P_{3i}(\beta) d\beta + \int_0^{\Delta} \sum_{i=1}^{9} P_{4i}(\beta) d\beta \right) \end{aligned} \tag{9}$$

The long-run average cost per unit time is obtained as follows

$$C = \frac{C_e}{T} \tag{10}$$

Finally, the optimal values of m and o are obtained by minimizing the long-run average cost per unit time.

Numerical example

In this section, a wind turbine gearbox is considered to present some numerical illustrations for the proposed model. The wind turbine gearbox is composed of a gear with MoS_2/titanium coating material and a transmission device, and the preset self-repairing mechanism is

Table 1. Long-run average cost per unit time in different levels.

Preventive replacement level	Opportunistic replacement level		
	$o=4$	$o=5$	$o=6$
$m=5$	405.22	–	–
$m=6$	404.32	403.59	–
$m=7$	406.30	405.30	404.79

Table 2. Long-run average cost per unit time in different levels when $C_d=0$.

Preventive replacement level	Opportunistic replacement level		
	$o=4$	$o=5$	$o=6$
$m=5$	156.41	–	–
$m=6$	155.56	153.17	–
$m=7$	154.87	152.94	150.85

derived from the MoS_2/titanium coating material. Gear and transmission device constitute a two-unit series system, and the failure of each unit will cause the failure of wind turbine gearbox. The length of crack in the gear increases as the arrival of valid shocks. According to the length of crack, the state of gear is classified into eight states. Let $\Omega = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ be the state space of gear. The preset self-repairing mechanism is triggered immediately when the state of gear reaches 3, that is, $s = 3$. If the self-repairing process is not interrupted, the gear is upgraded to at least state 2.

The arrival of external shocks is modeled by the Poisson process with parameter $\lambda_1 = 10/\text{unit} - \text{time}$, and the probabilities of the occurrence of a valid shock in Stages 1,2,3 are $p_1 = 0.3$, $p_2 = 0.5$, $p_3 = 0.3$, respectively. The magnitude of each valid shock is fixed and equal, and one valid shock degrades one state of the gear. Besides, the self-repairing duration follows exponential distribution with parameter $\lambda_2 = 4/\text{unit} - \text{time}$, and the lifetime distribution of transmission device follows exponential distribution with parameter $\lambda_3 = 2/\text{unit} - \text{time}$. It is noticeable that the equidistant inspection interval of monitoring action is one unit time, that is, $\Delta = 1$. The duration of replacement action, such as corrective replacement, preventive replacement or opportunistic replacement, is considered as $Q = 0.1$.

The following cost parameters are considered

$$C_A^c = 150,\ C_A^p = 100,\ C_A^o = 50,\ C_B^c = 150,$$

$$C_d = 400,\ C_k = 20$$

As calculated in section "Optimal opportunistic maintenance policy," the preventive degradation level m and the opportunistic degradation level o are two decision parameters of the opportunistic maintenance policy, and the minimum long-run average cost per unit time is an index for the economics of the maintenance policy. All combinations of the two levels are listed to confirm the optimal maintenance policy. The average cost per unit time in different preventive and opportunistic degradation levels are shown in Table 1.

Based on the results given in Table 1, the optimal opportunistic maintenance policy is obtained. When the system does not fail in the inspection interval and the state of unit A reaches or exceeds $m = 6$, unit A is preventively replaced. An opportunity to replace unit A appears when unit B fails in the inspection interval, and the decision foundation is whether the state of unit A reaches $o = 5$.

Considering the influence of downtime durations on the numerical example, the average downtime cost per unit time is assumed to be 0 to compare with Table 1. The average cost per unit time in different preventive and opportunistic degradation levels is shown in Table 2.

Two phenomena are discovered from Table 2: (a) when the preventive degradation level is a constant, the maintenance cost decreases as the opportunistic degradation level increases and (b) when the opportunistic degradation level is a constant, the maintenance cost decreases as the preventive degradation level increases. These phenomena indicate that two degradation levels of the opportunistic maintenance policy are increased to avoid frequent replacement actions and reduce replacement costs when the average downtime cost per unit time is zero. Besides, the comparison results between Tables 1 and 2 indicate that the optimal values of two decision parameters are decreased (a higher level of prevention actions) to minimize the sum of inspection cost, downtime cost and replacement cost.

In general, each numerical data has an effect on the inspection cost, replacement cost and downtime cost. Furthermore, these costs influence the optimal opportunistic maintenance policy. Mathematically, the reason of this phenomenon is that the long-run average cost per unit time consists of five ingredients: (a) $\sum_{i=1}^{9} P_{1i}(\Delta)$, (b) $\sum_{i=1}^{6} P_{2i}(\Delta)$, (c) $\int_0^{\Delta} \sum_{i=1}^{6} P_{3i}(\beta)d\beta$, (d) $\int_0^{\Delta} \sum_{i=1}^{9} P_{4i}(\beta)d\beta$, and (e) $\int_0^{\Delta} \sum_{i=1}^{7} P_{5i}(\beta)d\beta$. Each numerical data affects one or more of the five ingredients. Therefore, the optimal maintenance decision parameters are determined by all numerical data.

Finally, a preventive maintenance policy is devised to demonstrate the significance of opportunistic maintenance. The preventive maintenance policy has no opportunity to replace unit A when unit B is correctively replaced. The comparison results between the two maintenance policies are given in Table 3.

From Table 3, the results show that the minimum long-run average cost of preventive replacement policy is higher than the opportunistic replacement policy. Meanwhile, the results demonstrate that the existence of opportunistic maintenance can reduce cost.

Table 3. Minimum long-run average cost per unit time and optimal levels in different maintenance policies.

Policy	State m	State o	Minimum long-run average cost per unit time
Opportunistic maintenance	6	5	403.59
Preventive maintenance	6	–	409.31

Summary

In this article, the preset self-repairing mechanism is first introduced and described in reliability model, and a two-unit series system is devised to illustrate the proposed model. For the system, reliability analysis is discussed to evaluate the influence of preset self-repairing recovery mechanism on system. Besides, an opportunistic maintenance policy considering downtime cost is proposed to ensure the normal operation of system. The optimal opportunistic maintenance policy is determined by adopting the enumeration method in section "Numerical example," and the optimal values of two decision parameters are obtained by adjusting inspection cost, replacement cost and downtime cost. Numerical results indicate that the existence of downtime cost leads to preventive actions that are performed at a higher deterioration level to avoid the longer downtime duration. Finally, the significance of opportunistic maintenance is also demonstrated in the comparison results. It is foreseeable that the performance of preset self-repairing mechanism in more intricate systems is deserved to be discussed. There will be many unique recovery modes being discovered, and corresponding researches are indispensable for new recovery modes.

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Appendix 1

Notation

C	long-run average cost per unit time
C_d	average downtime cost per unit time
C_k	inspection cost
C_i^c	corrective replacement cost of unit $i, i \in \{A, B\}$
C_A^o	opportunistic replacement cost of unit A
C_A^p	preventive replacement cost of unit A
$h_A(t)$	PDF of unit A lifetime
$h_A^i(t)$	probability that the lifetime of unit A is t if unit A is upgraded to state i in Stage 2, $i = 0, 1, 2, ..., s-1$
$h_A^{s+1}(t)$	probability that the lifetime of unit A is t if unit A is degraded to state $s+1$ in Stage 2
n_i	number of external shocks arriving in Stage $i, i = 1, 2, 3, n = n_1 + n_2 + n_3$
p_i	probability that an arriving shock in stage i is a valid shock, $i = 1, 2, 3, p_i + q_i = 1$
p_{Ri}	probability that unit A is repaired to state i by the preset self-repairing mechanism, $i = 0, 1, 2, ..., s-1$
Q	duration of corrective replacement, preventive replacement and opportunistic replacement
$R(t)$	reliability function of the two-unit series system
$R_A(t)$	reliability function of unit A
$R_B(t)$	reliability function of unit B
T	long-run average period
U_i	time interval between the $(i-1)$th and the ith external shock, $i = 1, 2, ...$
V	lifetime of unit B
W	duration of self-repairing process
Z_n	time interval between an initiate point and the nth shock arrival epoch in the Poisson process, and $Z_n = U_1 + U_2 + \cdots + U_n$
Δ	inspection interval
Ω	state space of unit A

Appendix 2

In this appendix, the mathematical expressions of some formulas in section "Cost analysis" are given as follows

$$
\begin{aligned}
A(x) &= P(Z_n \leqslant x \leqslant Z_n + U_{n+1}) \\
&= \int_0^x P(x - t \leqslant U_{n+1} | Z_n = t) dF(Z_n \leqslant t) \\
&= \int_0^x e^{-\lambda_1 (x-t)} e^{-\lambda_1 t} t^{n-1} \frac{\lambda_1{}^n}{(n-1)!} dt \\
&= \int_0^x e^{-\lambda_1 x} t^{n-1} \frac{\lambda_1{}^n}{(n-1)!} dt \\
&= e^{-\lambda_1 x} \frac{\lambda_1{}^n x^n}{n!}
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
B(x) &= \int_0^x P(Z_{n_1} = x - t, Z_{n_2} < t < Z_{n_2} + U_{n_2+1} | W = t) dF(W \leqslant t) \\
&= \int_0^x e^{-\lambda_1 (x-t)} \frac{\lambda_1{}^{n_1} (x-t)^{n_1 - 1}}{(n_1 - 1)!} \int_0^t P(t - x \leqslant U_{n_2+1} | Z_{n_2} = x) dF(Z_{n_2} \leqslant x) \lambda_2 e^{-\lambda_2 t} dt \\
&= \lambda_2 e^{-\lambda_1 x} \lambda_1{}^{n_1 + n_2} \int_0^x e^{-\lambda_2 t} \frac{(x-t)^{n_1 - 1} t^{n_2}}{(n_1 - 1)! n_2!} dt
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
C(x) &= P(Z_n \leqslant x < Z_n + U_{n+1}) P(W \geqslant x) \\
&= \int_0^x P(x - t \leqslant U_{n+1} | Z_n = t) dF(Z_n \leqslant t) e^{-\lambda_2 x} \\
&= \int_0^x e^{-(\lambda_1 + \lambda_2) x} \frac{\lambda_1{}^n t^{n-1}}{(n-1)!} dt \\
&= e^{-(\lambda_1 + \lambda_2) x} \frac{\lambda_1{}^n x^n}{n!}
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
D(x) &= \int_0^x \int_0^{x-l} P(Z_{n_1} = l, Z_{n_3} \leqslant x - t - l < Z_{n_3} + U_{n_3+1}, Z_{n_2} \leqslant t < Z_{n_2} + U_{n_2+1} | W = t) dF(W \leqslant t) dl \\
&= \int_0^x \int_0^{x-l} e^{-\lambda_1 l} l^{n_1 - 1} \frac{\lambda_1{}^{n_1}}{(n_1 - 1)!} \int_0^{x-t-l} e^{-\lambda_1 (x-t-l)} x^{n_3 - 1} \frac{\lambda_1{}^{n_3}}{(n_3 - 1)!} dx \int_0^t e^{-\lambda_1 t} z^{n_2 - 1} \frac{\lambda_1{}^{n_2}}{(n_2 - 1)!} dz \lambda_2 e^{-\lambda_2 t} dt dl \\
&= \lambda_2 e^{-\lambda_1 x} \lambda_1{}^{n_1 + n_2 + n_3} \int_0^x \int_0^{x-l} e^{-\lambda_2 t} \frac{l^{n_1 - 1} t^{n_2} (x - t - l)^{n_3}}{(n_1 - 1)! n_2! n_3!} dt dl
\end{aligned}
\tag{14}
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\begin{aligned}
E(x) &= \int_0^x P(Z_{n_2} \leqslant x - t < Z_{n_2} + U_{n_2+1}, Z_{n_1} \leqslant t < Z_{n_1} + U_{n_1+1} | W = t) dF(W \leqslant t) \\
&= \int_0^x \int_0^{x-t} P(x - t - x \leqslant U_{n_2+1} | Z_{n_2} = x) dF(Z_{n_2} \leqslant x) \lambda_2 e^{-\lambda_2 t} \\
&\quad \int_0^t P(t - z \leqslant U_{n_1+1} | Z_{n_1} = z) dF(Z_{n_1} \leqslant z) dt \\
&= \lambda_2 e^{-\lambda_1 x} \lambda_1{}^{n_1 + n_2} \int_0^x e^{-\lambda_2 t} \frac{(x-t)^{n_2}}{n_2!} \frac{t^{n_1}}{n_1!} dt
\end{aligned}
\tag{15}
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$$
\begin{aligned}
F(x) &= \int_0^x \int_0^{x-l} P(Z_{n_1} = l, Z_{n_3} \leqslant x - t - l < Z_{n_3} + U_{n_3+1}) P(t \leqslant W | Z_{n_2} = t) dF(Z_{n_2} \leqslant t) dl \\
&= \int_0^x \int_0^{x-l} e^{-\lambda_2 t} e^{-\lambda_1 l} \frac{\lambda_1{}^{n_1} l^{n_1-1}}{(n_1-1)!} \int_0^{x-t-l} e^{-\lambda_1 (x-t-l)} \frac{\lambda_1{}^{n_3} x^{n_3-1}}{(n_3-1)!} dx e^{-\lambda_1 t} \frac{\lambda_1{}^{n_2} z^{n_2-1}}{(n_2-1)!} dt dl \\
&= e^{-\lambda_1 x} \lambda_1{}^{n_1+n_2+n_3} \int_0^x \int_0^{x-l} e^{-\lambda_2 t} \frac{l^{n_1-1} t^{n_2-1} (x-t-l)^{n_3}}{(n_1-1)!(n_2-1)! n_3!} dt dl
\end{aligned}
\tag{16}
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$$
\begin{aligned}
G(x) &= \int_0^x P(Z_{n_2} \leqslant x - t < Z_{n_2} + U_{n_2+1}, t < W | Z_{n_1} = t) dF(Z_{n_1} \leqslant t) \\
&= \int_0^x e^{-\lambda_2 t} \int_0^{x-t} P(x - t - x \leqslant U_{n_2+1} | Z_{n_2} = x) dF(Z_{n_2} \leqslant x) dF(Z_{n_1} \leqslant t) \\
&= e^{-\lambda_1 x} \lambda_1{}^{n_1+n_2} \int_0^x e^{-\lambda_2 t} \frac{(x-t)^{n_2}}{n_2!} \frac{t^{n_1-1}}{(n_1-1)!} dt
\end{aligned}
\tag{17}
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$$
\begin{aligned}
H(x) &= \int_0^x \int_0^{x-l} P(Z_{n_1} = l, Z_{n_3} = x - t - l, Z_{n_2} \leqslant t < Z_{n_2} + U_{n_2+1} | W = t) dF(W \leqslant t) dl \\
&= \int_0^x \int_0^{x-l} e^{-\lambda_1 l} \frac{\lambda_1{}^{n_1} l^{n_1-1}}{(n_1-1)!} e^{-\lambda_1 (\beta - t - l)} \frac{\lambda_1{}^{n_3} (x-t-l)^{n_3-1}}{(n_3-1)!} \int_0^t e^{-\lambda_1 t} \frac{\lambda_1{}^{n_2} z^{n_2-1}}{(n_2-1)!} dz \lambda_2 e^{-\lambda_2 t} dt dl \\
&= \lambda_2 e^{-\lambda_1 x} \lambda_1{}^{n_1+n_2+n_3} \int_0^x \int_0^{x-l} e^{-\lambda_2 t} \frac{l^{n_1-1} t^{n_2} (x-t-l)^{n_3-1}}{(n_1-1)! n_2! (n_3-1)!} dt dl
\end{aligned}
\tag{18}
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$$
\begin{aligned}
I(x) &= \int_0^x P(Z_{n_2} = x - t, Z_{n_1} \leqslant t \leqslant Z_{n_1} + U_{n_1+1} | W = t) dF(W \leqslant t) \\
&= \int_0^x \lambda_2 e^{-\lambda_2 t} e^{-\lambda_1 (x-t)} \frac{(x-t)^{n_2-1}}{(n_2-1)!} \int_0^t P(t - z \leqslant U_{n_1+1} | Z_{n_1} = z) dF(Z_{n_1} \leqslant z) dt \\
&= \lambda_2 e^{-\lambda_1 x} \lambda_1{}^{n_1+n_2} \int_0^x e^{-\lambda_2 t} \frac{t^{n_1} (x-t)^{n_2-1}}{n_1! (n_2-1)!} dt
\end{aligned}
\tag{19}
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$$
\begin{aligned}
M(x) &= \int_0^x \int_0^{x-l} P(Z_{n_1} = l, Z_{n_3} = x - t - l) P(t \leqslant W | Z_{n_2} = t) dF(Z_{n_2} \leqslant t) dl \\
&= \int_0^x \int_0^{x-l} e^{-\lambda_2 t} e^{-\lambda_1 l} \frac{\lambda_1{}^{n_1} l^{n_1-1}}{(n_1-1)!} e^{-\lambda_1 (x-t-l)} \frac{\lambda_1{}^{n_3} (x-t-l)^{n_3-1}}{(n_3-1)!} e^{-\lambda_1 t} \frac{\lambda_1{}^{n_2} z^{n_2-1}}{(n_2-1)!} dt dl \\
&= e^{-\lambda_1 x} \lambda_1{}^{n_1+n_2+n_3} \int_0^x \int_0^{x-l} e^{-\lambda_2 t} \frac{l^{n_1-1} t^{n_2-1} (x-t-l)^{n_3-1}}{(n_1-1)!(n_2-1)!(n_3-1)!} dt dl
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
N(x) &= \int_0^x P(Z_{n_2} = x - t, t < W | Z_{n_1} = t) dF(Z_{n_1} \leqslant t) \\
&= \int_0^x e^{-\lambda_2 t} e^{-\lambda_1 (x-t)} \frac{(x-t)^{n_2}}{n_2!} dF(Z_{n_1} \leqslant t) \\
&= e^{-\lambda_1 x} \lambda_1{}^{n_1+n_2} \int_0^x e^{-\lambda_2 t} \frac{t^{n_1-1} (x-t)^{n_2-1}}{(n_1-1)!(n_2-1)!} dt
\end{aligned}
\tag{21}
$$