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Reliability and opportunistic maintenance for a series system with multi-stage accelerated damage in shock environments

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ABSTRACT

In this paper, a new shock model is proposed to fit the situation that the damage process of the component is accelerated with the increase of the degree of damage in shock environments. Shocks with the same magnitude may have more serious consequences as the component gets worse and component states are divided into different stages according to the degree of damage. Reliability indexes of a series system which consists of two components with the above characteristics are derived by using a Markov renewal process. Then, an opportunistic maintenance strategy is proposed for the system and an optimization model is constructed to obtain the optimal maintenance solutions. Finally, a numerical example for a two-rolling bearing system of a wind turbine is presented to illustrate the proposed shock model and maintenance strategy. Sensitivity analysis of maintenance costs is also discussed in order to improve the reliability of the proposed model. This study is of reference value and application significance for similar series systems.

1. Introduction

Multi-stage accelerated damage shock models systems are very common in engineering applications. Taking the rolling bearings system as an example, rolling bearing is one of the most important and vulnerable parts of wind turbines (Li, Jiang, & Xiong, 2019; Chen & Qu, 2019). Due to small internal clearances needed for high-speed operation, the friction at the roller end, cage pocket and roller end and guiding flange would result in extremely high contact pressure, sliding velocity, and instantaneous temperature in the bearing, which can easily exceed the endurance limit (Mao, Wang, & Zhang, 2018; Alvarez & Ribaric, 2018). In addition, sandstones and other foreign matters often enter the raceway. Those shocks may lead to abnormal friction between the raceway and the rolling body, resulting in increased clearance and surface roughness of bearings, reducing the running accuracy of bearings, even leading to rolling bearing failures and the shutdown of the entire wind turbine. Besides, as the damage degree of rolling bearings increases, rings and shaft walls are becoming thinner due to size and weight constraints, the elastic ring deformation occurs on the rolling bearings, its ability of heat dissipation becomes weaker and weaker, contact pressure becomes larger and larger and risk of bearing failure due to shocks also becomes higher and higher (Cavallaro & Nelias, 2005; Shi & Wang, 2015). Therefore, it is of great practical significance to study the above shock model and propose a reasonable maintenance strategy.

Traditionally, scholars distinguished the classical shock models into five categories. A cumulative shock model refers to that the system fails when its cumulative damage exceeds a threshold (e.g., Zhu, Fouladirad, & Berenguer, 2015; Wang, Wang, & Peng, 2017). If the system fails when a shock exceeds its maximum bearing threshold, it is called extreme shock model (e.g., Mallor & Omey, 2001; Hao, Yang, Ma, & Zhao, 2017). A continuous shock model means that the system fails when the number of continuous shocks reaching a certain threshold (e.g., Eryilmaz, 2017a; Zhao, Cai, Wang, & Song, 2018). The shock model is called δ-shock model (e.g., Eryilmaz, 2017b; Zhao, Guo, & Wang, 2018; Wang, Zhao, & Sun, 2019), when system fails since the interval between two consecutive shocks to the system is less than a given threshold δ. A mixed shock model is obtained by mixing any two or more of shock models described above, which has been researched in Rafiee, Feng, and Coit (2017) and Zhao, Wang, Wang, and Fan (2020). A mixed shock model composed of extreme shock model and cumulative shock model is studied in this paper.

In recent years, multi-stage shock models have been also proposed in many studies. It is very popular to divide the stages based on the failure process. For example, Li and Pham (2005) developed a generalized multi-state degraded system reliability model. It subjects to

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multiple competing failure processes, including two degradation processes and random shocks. The distribution functions of degradation of shock model are different in different stages. Yang, Peng, Zhai and Zhao (2017) developed a single-unit system whose failure has two competing and dependent causes. The system is subject to a two-stage deterioration process, i.e., from new to the initial point of a defect and from that point to failure. In that way, three possible stages are involved for the system, namely, normal, defective and failed. A defective stage incurs a greater degradation rate than a normal stage. Moreover, Chen and Li (2008) and Eryilmaz (2015) studied the situation that stages are divided by the ability of system to withstand shocks.

Although Zhao, Wang, Wang and Cai (2018) classified the shocks into three types according to its magnitude, they failed to consider the situation that as the degree of damage to the component increases, its working performance becomes worse and more fragile and the ability to withstand shocks becomes weaker and weaker. Shocks with the same magnitude may have more serious consequences as the component gets worse, which is common in reality but ignored in their paper. Therefore, it is necessary to consider the current damage degree to describe the effect of the random shocks on the component. In this paper, the situation that the damage process of the component is accelerated with the increase of the degree of damage is considered and the component states are divided into multiple stages according to the degree of damage.

Meanwhile, a maintenance measures is vitally significant for shock models. Common maintenance strategies include corrective maintenance, preventive maintenance, opportunistic maintenance, replacement and so on (Liu, Xie, Xu, & Kuo, 2016; Su & Cheng, 2018). The preventive maintenance is widely studied, for example, see Eryilmaz (2017a) and Zhao, Guo and Wang (2018). The opportunistic maintenance is more and more popular in recent years. Opportunistic maintenance has been studied in order to simultaneously maintain other components meanwhile for multi-components or complex system. Cui and Li (2006) considered about opportunistic maintenance on a multicomponent cumulative damage shock model with stochastically dependent components, it shows that a shock model with opportunistic maintenance experiences less failures stochastically at various components than a similar shock model without such a maintenance. Different from studies which only consider the replacement (eg., Shen, Cui, & Yi, 2018) or the preventive maintenance (eg., Zhao, Cai, et al., 2018) for the proposed shock model, a maintenance strategy composed of the replacement, the preventive maintenance and the opportunistic maintenance is creatively introduced into a series system which consists of two components with multi-stage accelerated damage in this paper.

The major challenges and contribution of this paper are summarized in the following. First, a novel shock model with multi-stage accelerated damage is proposed firstly in order to accommodate some real situations. Shocks with the same magnitude may have more serious consequences as the component gets worse. Second, the system performance indexes are derived by using a Markov renewal process. The state space and the semi-Markov kernel are redefined and constructed according to the characteristics of the model proposed in this paper. Third, an optimal opportunistic maintenance is determined for the proposed shock model, which can reduce failures stochastically and maintenance cost effectively.

The organization of the remainder of this paper is as follows. In Section 2, a multi-stage accelerated damage shock model is proposed and assumptions involved in this shock model are listed. In Section 3, a single component is studied by a Markov renewal process and the corresponding semi-Markov kernel of the Markov renewal process is derived. Besides, reliabilities of the component and the probability of the component state at time t are defined and calculated in Section 3. In Section 4, an opportunistic maintenance strategy is presented for the proposed series system in order to minimize the long run expected average cost per unit time. Section 5 presents a numerical example for the proposed model and the sensitivity analysis of maintenance costs is discussed. The whole paper is summarized in Section 6.

2. Model descriptions

Assumptions about the model in accordance with the motivating example of rolling bearings are described as follows.

(a) At $t = 0$ both components are new and the system is also perfect.
(b) Components are subject to shocks that follow the Poisson process $\{N(t), t \geq 0\}$ with a parameter λ. The distribution of the interval between the i - th shock and $(i+1)$ - th shock is exponential distribution. The interval between shocks does not affect the type of the next shock. The results of each shock are independent of each other.
(c) The component can be divided into different states according to the performance of the component. Let $E = \{1, 2, ..., N\}$ note the state space of the component, working states of the components are $W = \{1, 2, ..., N-1\}$. State 1 indicates that the component is as good as new. State N indicates that the component fails. The component is worse in state i than it is in state j, if $i > j$, $i, j \in E$.
(d) Component states are divided into L stages according to the degree of damage. If $l_2 > l_1$, when the component is in stage l_2, the amplification factor of shocks is larger than that in stage l_1. Let $\alpha(l)$ denote the amplification factor of shocks in stage l and the values of $\alpha(l)$ belong to [1, 2].
(e) Transitions among component states caused by shocks are not only on the magnitude of shocks, but also on the state of the component when the shock arrives. The random variable d_i represents the magnitude of the i - th shock, d_i are independent with identical distribution $G(d_i)$. Then, if the component transfers from state i $(i \in W)$ to state j, after it suffers a shock with the magnitude of d, the state j meets the following conditions,

$$j = \begin{cases} i, & 0 \leq \alpha(l)d < D_{i,i} \\ k, & D_{i,k-1} \leq \alpha(l)d < D_{i,k} \quad k = i+1, i+2, ..., N-1, \\ N, & \alpha(l)d > D_{i.N-1} \end{cases}$$

where $D_{i,i}$, $D_{i,i+1}$, ..., $D_{i,N-1}$ are pre-determined thresholds.

Assumption (b) is based on the practical experience that a defective or a wear-out system will be more susceptible to the environmental impact such as shocks (Chen & Li, 2008; Yang, Ma, & Zhao, 2017). Based on the fact that the system is subject to shocks of random magnitudes and cause different damages on the system, which has been discussed in Shen et al. (2018), assumption (e) is proposed in this paper. These two assumptions suggest that the worse the component is, the more likely it is to transfer to a worse state after a shock.

A possible sample path realization of the component is depicted in Fig. 1. Here a three-stage accelerated damage shock model is shown. It is assumed that component states can be divided into 1 to 6, where state 1 means the component is new and state 6 means the component fails. Let $L_1 = 2$, $L_2 = 4$ and $L_3 = 6$. In Fig. 1, the 4 - th, 6 - th and 7 - th shocks are invalid shocks, which cannot cause the transfer of states; the magnitudes of the 2 - nd and 3 - rd shocks are $D_{1,1} < \alpha(1)d_2 \leq D_{1,2}$ and $D_{2,2} < \alpha(1)d_3 \leq D_{2,3}$, which lead to one-step shift in component states; the magnitude of the 5 - th shock is $D_{3,4} < \alpha(2)d_5 \leq D_{3,5}$, which leads to two-step shift in component states; the 8 - th shock is a fatal shock, which directly leads to component failure.

3. Reliability and performance evaluation

In this section, the reliability of the individual component is calculated by using a homogeneous Markov renewal process and the reliability of the proposed series system is analyzed.

Let T_i and X_i represent the epoch of the i - th state transition and the system state after time T_i respectively. Then the homogeneous Markov renewal process $\{(X_n, T_n), n = 0, 1, ...\}$ is constructed. Let

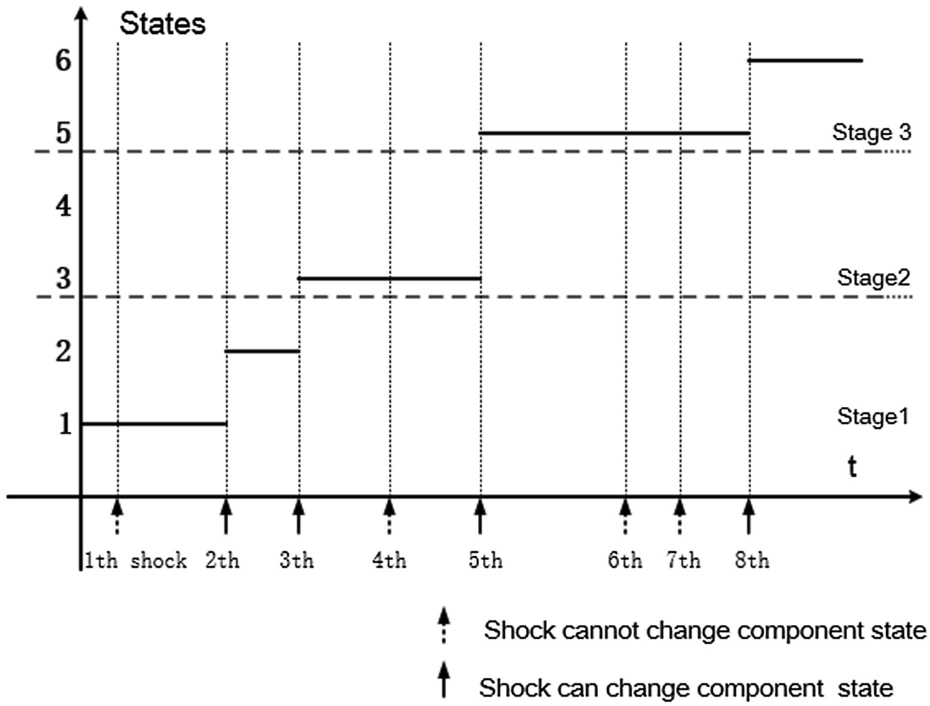

Fig. 1. A possible sample path realization of the component with external shocks.

$\boldsymbol{Q}(t) = \{Q_{i,j}(t),\, i, j = 1, 2,\ldots, N\}$ be the semi-Markov kernel corresponding to the process $\{(X_n, T_n),\, n = 0, 1, \ldots\}$, where

$$Q_{i,j}(t) = P\{X_1 = j,\, T_1 \le t \mid X_0 = i\},\quad i, j = 1, 2, \ldots N \quad and \quad j \ne i. \tag{1}$$

The interval between the i - thshock and $(i+1)$ - th shock s_{i+1} satisfies an exponential distribution with a parameter λ because components are subject to shocks that follow the Poisson process$\{N(t),\, t \ge 0\}$. The cumulative distribution function (cdf) of the time interval between two consecutive shocks is as follows.

$$F_{s_i}(t) = 1 - e^{-\lambda t},\quad t \ge 0. \tag{2}$$

The random variables $s_1, s_2, \cdots, s_n$ are independent and identically distributed (i.i.d), so $S_n = s_1 + s_2 + \cdots + s_n$ satisfies Gamma distribution with parameters of n and λ. Its cdf is as follows.

$$F_{S_n}(t) = 1 - e^{-\lambda t}\sum_{m=0}^{n-1}\frac{(\lambda t)^m}{m!},\quad t \ge 0. \tag{3}$$

The values of $Q_{i,j}(t)$ under different conditions are discussed in the following.

If the component state transfers from i to j $(i < j < N)$ after M shocks, then

$$\begin{aligned} Q_{i,j}(t) &= P\{X_1 = j,\, T_1 \le t \mid X_0 = i\} \\ &= \textstyle\sum_{n=1}^{\infty} P\{X_1 = j,\, T_1 \le t,\, M = n \mid X_0 = i\} \\ &= \textstyle\sum_{n=1}^{\infty} P\{X_1 = j,\, S_M = T_1 \le T,\, M = n \mid X_0 = i\}. \end{aligned} \tag{4}$$

In this case, after $M = n$ shocks, the state can be transferred, so the magnitudes of the previous $n-1$ shocks are $\alpha(l)d_i < D_{i,i}$ and the magnitude of the n - thshock is$D_{i,j-1} \le \alpha(l)d_n < D_{i,j}$. Substituting the above conditions and Eq. (3) into Eq. (4), we have

$$\begin{aligned} Q_{i,j}(t) &= \textstyle\sum_{n=1}^{\infty} P\{X_1 = j,\, S_M = T_1 \le t,\, M = n \mid X_0 = i\} \\ &= \textstyle\sum_{n=1}^{\infty} P\{0 \le d_1, d_2, \ldots, d_{n-1} < D_{i,i},\, D_{i,i} \le d_n < D_{i,j},\, S_M \\ &\qquad \le t \mid X_0 = i\} \\ &= \textstyle\sum_{n=1}^{\infty} [G(D_{i,i})]^{n-1}[G(D_{i,j}) - G(D_{i,j-1})][1 - e^{-\lambda t}\sum_{m=0}^{n-1}\frac{(\lambda t)^m}{m!}], \end{aligned} \tag{5}$$

where d_i represents $\alpha(l)d_i$, similarly hereinafter.

When $j = N$,

$$Q_{i,N}(t) = \sum_{n=1}^{\infty} [G(D_{i,i})]^{n-1}[1 - G(D_{i,N-1})][1 - e^{-\lambda t}\sum_{m=0}^{n-1}\frac{(\lambda t)^m}{m!}]. \tag{6}$$

In order to calculate it conveniently, the Laplace transform is taken on $Q_{i,j}(t)$, where $1 \le i < N$ and $i < j < N$, we have

$$\begin{aligned} Q_{i,j}^*(s) &= \textstyle\sum_{n=1}^{\infty} [G(D_{i,i})]^{n-1}[G(D_{i,j}) - G(D_{i,j-1})][\frac{1}{s} - \sum_{m=0}^{n-1}\frac{(\lambda)^m}{(\lambda+s)^{m+1}}], \\ Q_{i,N}^*(s) &= \textstyle\sum_{n=1}^{\infty} [G(D_{i,i})]^{n-1}[1 - G(D_{i,N-1})][\frac{1}{s} - \sum_{m=0}^{n-1}\frac{(\lambda)^m}{(\lambda+s)^{m+1}}]. \end{aligned} \tag{7}$$

After obtaining the component transition probability, the steady-state probability can be calculated according to the following formula,

$$\begin{cases} \pi_i = \sum_j \pi_j Q_{i,j}(t) \quad i, j \in E, \\ \sum_i \pi_i = 1 \quad i \in E. \end{cases} \tag{8}$$

3.1. The reliability of the system

In this part, a new semi-Markov process is established, the process satisfies $Z(t) = X_n$ at $t \in [T_n, T_{n+1})$. Based on assumption (c), the system is considered to be failed when it enters state N. Then the life of the system, T_L, can be expressed as $T_L = \inf\{t \ge 0,\, Z(t) = N\}$. It is assumed that the component is in state k $(k \in W)$ at time 0. The reliability of the component can be expressed as follows.

$$\begin{aligned} R(t) &= P\{T_L > t \mid X_0 = k\} \\ &= P\{T_L > t,\, T_1 > t \mid X_0 = k\} + P\{T_L > t,\, T_1 \le t \mid X_0 = k\}. \end{aligned} \tag{9}$$

Considering that the component is in working state at time T_0 and $T_L \ge T_1$, Eq. (9) is

$$\begin{aligned} R(t) &= P\{T_L > t,\ T_1 > t \mid X_0 = k\} + P\{T_L > t,\ T_1 \le t \mid X_0 = k\} \\ &= 1 - P\{T_1 \le t \mid X_0 = k\} + P\{T_L > t,\ T_1 \le t \mid X_0 = k\} \\ &= 1 - \textstyle\sum_{i\in E} P\{X_1 = i,\ T_1 \le t \mid X_0 = k\} \\ &\quad + \textstyle\sum_{j\in W} P\{T_L > t,\ X_0 = j,\ T_1 \le t \mid X_0 = k\} \\ &= 1 - \textstyle\sum_{i\in E} Q_{k,i}(t) + \sum_{j\in W} P\{T_L > t,\ X_0 = j,\ T_1 \le t \mid X_0 = k\}, \end{aligned} \tag{10}$$

where

$$\begin{aligned} & P\{T_L > t,\ X_0 = j,\ T_1 \le t \mid X_0 = k\} \\ &= \int_0^t P\{T_L > t \mid X_1 = j,\ T_1 = u,\ X_0 = k\} dP\{X_1 = j,\ T_1 \le u \mid X_0 = k\} \\ &= \int_0^t R_j(t-u) dQ_{k,j}(u) = R_j(t) * Q_{k,j}(t). \end{aligned} \tag{11}$$

Substituting Eq. (11) into Eq. (10), we have

$$R(t) = 1 - \sum_{i\in E} Q_{ki}(t) + \sum_{j\in W} R_j(t) * Q_{kj}(t). \tag{12}$$

The Laplace transform is taken for Eq. (12) and the result is as follows.

$$R^*(s) = \frac{1}{s} - \sum_{i\in E} Q^*_{k,i}(s) + \sum_{j\in W} R^*_j(s)\hat{Q}_{k,j}(s), \tag{13}$$

where

$$R^*(s) = \int_0^\infty e^{-st}R(t)dt, \quad \hat{Q}_{k,j}(s) = \int_0^\infty e^{-st}dQ_{k,j}(t),$$
$$Q^*_{k,i}(s) = \int_0^\infty e^{-st}Q_{k,i}(t)dt.$$

In order to obtain the reliability of the component, Let

$$\mathbf{A} = \begin{bmatrix} R_1^*(s) \\ R_2^*(s) \\ \vdots \\ R^*_{N-1}(s) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{s} - Q^*_{1,j}(s) \\ \frac{1}{s} - Q^*_{2,j}(s) \\ \vdots \\ \frac{1}{s} - Q^*_{N-1,j}(s) \end{bmatrix} \quad \text{and} \quad \mathbf{C}$$
$$= \begin{bmatrix} 1 & -\hat{Q}_{1,2}(s) & \cdots & -\hat{Q}_{1,N-1}(s) \\ 0 & 1 & \cdots & -\hat{Q}_{1,N-1}(s) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}.$$

$R_i^*(s)$ can be obtained by calculating $\mathbf{A} = \mathbf{C}^{-1}\mathbf{B}$. The component reliability can be obtained by the inverse Laplace transform.

Since the system is composed of two identical components in series, the reliability of the system can be obtained from the following formula.

$$R_Z(t) = R_1(t)R_2(t). \tag{14}$$

3.2. The state of the component at time t

In this part, it is assumed that the component enters state i at time 0. The probability $G_{i,j}(t)$ that the component state is j at time t can be calculated, where

$$\begin{aligned} & G_{i,j}(t) = P\{X_t = j \mid X_0 = i\} \\ &= P\{X_t = j,\ T > t \mid X_0 = i\} + P\{X_t = j,\ T \le t \mid X_0 = i\}. \end{aligned} \tag{15}$$

Then, the values of $G_{i,j}(t)$ of the component can be considered under three different cases as follows.

CASE 1: The component state has not been transferred before time t at the case of $X_0 = i$ $(i \in W)$ and we have

$$G_{i,i}(t) = P\{X_t = i,\ T > t \mid X_0 = i\} + P\{X_t = i,\ T \le t \mid X_0 = i\}. \tag{16}$$

Since the state has not been transferred, the second term on the right side of Eq. (16) is 0 and the first term on the right is

$$P\{X_t = i,\ T > t \mid X_0 = i\} = 1 - \sum_{j=i+1}^{N} Q_{i,j}(t). \tag{17}$$

Substituting Eq. (17) into Eq. (16), we have

$$G_{i,i}(t) = 1 - \sum_{j=i+1}^{N} Q_{i,j}(t),\ i \in W. \tag{18}$$

The Laplace transform is performed on Eq. (18), where

$$G^*_{i,i}(s) = \frac{1}{s} - \sum_{j=i+1}^{N} Q^*_{i,j}(s), i \in W. \tag{19}$$

CASE 2: The component state is j $(i < j < N)$ at time t at the case of $X_0 = i$ $(i \in W)$ and we have

$$\begin{aligned} G_{i,j}(t) &= P\{X_t = j,\ T > t \mid X_0 = i\} + P\{X_t = j,\ T \le t \mid X_0 = i\} \\ &= 0 + \textstyle\sum_{k\in W} P\{X_t = j,\ X_1 = k,\ T \le t \mid X_0 = i\} \\ &= \textstyle\sum_{k\in W} \int_0^t P\{X_t = j \mid T = u,\ X_1 = k\} dP\{X_1 = k,\ T \le u \mid X_0 = i\} \\ &= \textstyle\sum_{k\in W} \int_0^t P\{X_t = j \mid T = u,\ X_1 = k\} dQ_{i,k}(u) \\ &= \textstyle\sum_{k\in W} \int_0^t G_{k,j}(t-u) dQ_{i,k}(u) = \sum_{k\in W} G_{k,j}(t) * Q_{i,k}(t). \end{aligned} \tag{20}$$

In Eq. (20), the first term on the right side is 0 because of the transition of component states. After taking the Laplace transform of Eq. (20), the following equation can be obtained.

$$G^*_{i,j}(s) = \sum_{k\in W} \hat{G}_{k,j}(s)Q^*_{i,k}(s). \tag{21}$$

CASE 3: The component state is N at time t at the case of $X_0 = i$ $(i \in W)$ and we have

$$G_{i,N}(t) = P\{X_t = N,\ T > t \mid X_0 = i\} + P\{X_t = N,\ T \le t \mid X_0 = i\}. \tag{22}$$

In Eq. (22), the first term on the right side is 0 and the second term on the right side can be derived according to the following formula.

$$\begin{aligned} & \textstyle\sum_{k\in E} P\{X_t = N,\ X_1 = k,\ T \le t \mid X_0 = i\} \\ &= \textstyle\sum_{k\in W} P\{X_t = N,\ X_1 = k,\ T \le t \mid X_0 = i\} + P\{X_1 = N,\ T \le t \mid X_0 = i\} \\ &= \textstyle\sum_{k\in W} \int_0^t G_{k,N}(t-u) dQ_{i,k}(u) + Q_{i,N}(t) \\ &= \textstyle\sum_{k\in W} G_{k,N}(t) * Q_{i,k}(t) + Q_{i,N}(t). \end{aligned} \tag{23}$$

Substituting Eq. (23) into Eq. (22), we have

$$G_{i,N}(t) = \sum_{k\in W} G_{k,N}(t) * Q_{i,k}(t) + Q_{i,N}(t). \tag{24}$$

Taking Laplace transform on Eq. (24), we have

$$G^*_{i,N}(s) = \sum_{k\in W} G^*_{k,N}(s)\hat{Q}_{i,k}(s) + Q^*_{i,N}(s). \tag{25}$$

In summary, the values of $G_{i,j}(t)$ of the component have been calculated in this section.

4. Maintenance strategy

In this section, a periodic inspection strategy is considered for the proposed series system. It is assumed that the system is inspected at every interval T. The inspection time is assumed to be negligible and the inspection cost is c_i. The state of the component can be known only through inspection, but component failure can be found immediately.

Three maintenance methods are taken into consideration for maintenance decisions. When the component fails, the replacement is executed and the cost of replacement is c_f. When the component state is i $(N_2 \le i < N)$ at the epoch of inspection, preventive maintenance is executed immediately at the cost of c_p. The component is inspected immediately when the other component fails during the inspection interval. If the state of the inspected component is j ($N_1 \leq j < N_2$), opportunistic maintenance is executed on it. If the state of the inspected component is j ($N_2 \leq j < N$), preventive maintenance is executed on it, otherwise no repair action is needed. The cost of opportunistic maintenance is c_o. The replacement, preventive maintenance and opportunistic maintenance of component are all instantaneous and perfect. The relationship among the costs involved in those three maintenance actions is $c_o < c_p < c_f$.

The objective is to minimize the long run expected average cost per unit time by choosing the best combination of N_1^*, N_2^* and T^*. Let π_{i_1} and π_{i_2} represent the steady-state probability of components. $G_{i_1}(t)$ and $G_{i_2}(t)$ represent states of components at time t respectively.

The following three scenarios may occur at the epoch of the inspection.

Scenario 1: If both components are in state $[1, N_2)$, no repair action is needed.
Scenario 2: If one component is in state $[1, N_2)$, no repair action is needed for it. The other component is in state $[N_2, N)$, preventive maintenance is executed on the component immediately.
Scenario 3: If both components are in state $[N_2, N)$, preventive maintenance is executed on the two components immediately.

If one of the components fails during the inspection interval, the failed component shall be replaced and the state of the other component shall be inspected at the same time. The following four scenarios may occur.

Scenario 4: If the state of the other component is in $[1, N_1)$, no repair action is needed for this one.
Scenario 5: If the state of the other component is in $[N_1, N_2)$, opportunistic maintenance is executed immediately for the component.
Scenario 6: If the state of the other component is in $[N_2, N)$, preventive maintenance is executed immediately for the component.
Scenario 7: If the two components happen to be failed at the same time, failure replacement is executed immediately for the two components.

Costs and the corresponding probabilities for the above seven scenarios are listed in Table 1.

In a word, the period length of the system is

$$L = T_L I_{\{X \leq T\}} + T I_{\{X > T\}},$$

where T_L represents the lifetime of the component, $I_{\{\}}$ denotes the indicator function which equals 1 if the argument is true and 0 otherwise.

Therefore, the average period length is

$$\begin{aligned} E(L) &= \sum_{i_1=1}^{N_2} \sum_{i_2=1}^{N_2} \pi_{i_1} \pi_{i_2} \left[\int_0^T t dF_z(t) + \int_T^\infty T dF_z(t)\right] \\ &= \sum_{i_1=1}^{N_2} \sum_{i_2=1}^{N_2} \pi_{i_1} \pi_{i_2} \int_0^T (1 - F_z(t)) dt \\ &= \sum_{i_1=1}^{N_2} \sum_{i_2=1}^{N_2} \pi_{i_1} \pi_{i_2} \int_0^T R_{i_1}(t) R_{i_2}(t) dt, \end{aligned}$$

where $F_z(t)$ represents the lifetime function of the system.

The expected cost of a period is

$$E(C) = \sum_{n=1}^{7} C_n P_n \ .$$

For preventive and opportunistic maintenance thresholds, the objective function $\min C$ can be obtained by solving the following linear equation.

$$\min C = \frac{E(C)}{E(L)}$$

$$= \frac{\sum_{n=1}^{7} C_n P_n}{\sum_{i_1=1}^{N_2} \sum_{i_2=1}^{N_2} \pi_{i_1} \pi_{i_2} \int_0^T (1 - F_z(t)) dt} = \frac{\sum_{n=1}^{7} C_n P_n}{\sum_{i_1=1}^{N_2} \sum_{i_2=1}^{N_2} \pi_{i_1} \pi_{i_2} \int_0^T R_{i_1}(t) R_{i_2}(t) dt}. \quad (26)$$

5. Numerical examples

5.1. Parameter setting

In this part, a realistic example of rolling bearings in Section 1 is used to demonstrate the proposed model. It is assumed that a system is composed of two identical rolling bearings in series and states of the rolling bearing can be divided into different stages according to the degree of damage.

Here a three-stage accelerated damage shock model is considered. Let $L_1 = 2$, $L_2 = 4$ and $L_3 = 6$ which means that states of the first stage are $\{1, 2\}$, states of the second stage are $\{3, 4\}$ and states of the third stage are $\{5, 6\}$respectively, where state 1 represents a totally new state and the rolling bearing is considered to be failed when it is in state 6. It is assumed that $\alpha(l)$ is modeled by the function of $\alpha(l) = \frac{l + L - 2}{L - 1}$ $(1 \leq l \leq L)$. In this model, $L = 3$, $\alpha(1) = 1$, $\alpha(2) = 1.5$ and $\alpha(3) = 2$.

Rolling bearings are subject to shocks that follow the Poisson process$\{N(t), t \geq 0\}$ and $\lambda = 0.5$/month. The magnitude of each shock satisfies exponential distribution with the rate parameter λ_D=0.2/Gpa. The $D_{i,j}$ set in assumption (e) for this model are listed in Table 2.

Table 1
Costs and the corresponding probabilities under different scenarios.

Scenario	Cost	Probability
1	$C_1 = c_i$	$P_1 = \pi_{i_1} \pi_{i_2} G_{i_1,j_1}(T) G_{i_2,j_2}(T)$, $1 \leq i_1, i_2, j_1, j_2 < N_2$.
2	$C_2 = c_i + c_p$	$P_2 = P_{21} + P_{22}$.
		$P_{21} = \pi_{i_1} \pi_{i_2} G_{i_1,j_1}(T) G_{i_2,j_2}(T)$, $1 \leq i_1, i_2, j_1 < N_2$ and $N_2 \leq j_2 < N$.
		$P_{22} = \pi_{i_1} \pi_{i_2} G_{i_1,j_1}(T) G_{i_2,j_2}(T)$, $1 \leq i_1, i_2, j_2 < N_2$ and $N_2 \leq j_1 < N$.
3	$C_5 = c_i + 2c_p$	$P_5 = \pi_{i_1} \pi_{i_2} G_{i_1,j_1}(T) G_{i_2,j_2}(T)$, $1 \leq i_1, i_2 < N_2$ and $N_2 \leq j_1, j_2 < N$.
4	$C_4 = c_i + c_f$	$P_4 = P_{41} + P_{42}$.
		$P_{41} = \pi_{i_1} \pi_{i_2} G_{i_1,N}(t) G_{i_2,j_2}(t)$, $1 \leq i_1, i_2, j_2 < N_1$.
		$P_{42} = \pi_{i_1} \pi_{i_2} G_{i_1,j_1}(t) G_{i_2,N}(t)$, $1 \leq i_1, i_2, j_1 < N_1$.
5	$C_5 = c_i + c_f + c_o$	$P_5 = P_{51} + P_{52}$.
		$P_{51} = \pi_{i_1} \pi_{i_2} G_{i_1,N}(t) G_{i_2,j_2}(t)$, $1 \leq i_1, i_2 < N_2$ and $N_1 \leq j_2 < N_2$.
		$P_{52} = \pi_{i_1} \pi_{i_2} G_{i_1,j_1}(t) G_{i_2,N}(t)$, $1 \leq i_1, i_2 < N_2$ and $N_1 \leq j_1 < N_2$.
6	$C_6 = c_i + c_f + c_p$	$P_6 = P_{61} + P_{62}$.
		$P_{61} = \pi_{i_1} \pi_{i_2} G_{i_1,N}(t) G_{i_2,j_2}(t)$, $1 \leq i_1, i_2 < N_2$ and $N_2 \leq j_2 < N$.
		$P_{62} = \pi_{i_1} \pi_{i_2} G_{i_1,j_1}(t) G_{i_2,N}(t)$, $1 \leq i_1, i_2 < N_2$ and $N_2 \leq j_1 < N$.
7	$C_7 = 2c_f$.	$P_7 = \pi_{i_1} \pi_{i_2} G_{i_1,N}(t) G_{i_2,N}(t)$, $1 \leq i_1, i_2 < N_2$.

Table 2
Values of thresholds $D_{i,j}$ when $i = 1, 2...5$ and $j = i, i + 1, ..., 5$.(Unit: Gpa.)

i/j	1	2	3	4	5
1	2.40	4.80	7.20	9.60	14.40
2		1.60	4.00	6.40	9.60
3	\	\	1.60	4.80	7.20
4	\	\	\	0.80	4.00
5	\	\	\	\	0.80

5.2. System reliability evaluation

The evolution of rolling bearing states can be described by a Markov renewal process $\{(X_n, T_n), n = 0, 1, ...\}$ with kernel $\boldsymbol{Q}(t) = \{Q_{i,j}(t), i, j = 1, 2,...,N\}$ can be calculated by Eqs. (5)–(7), we have

$$Q_{i,j}^*(s) = \begin{cases} \sum_{n=1}^{\infty} H_1^{n-1}[G(D_{i,j}) - G(D_{i,j-1})]H_2, \\ i = 1, 2, 3, 4, 5 \text{ and } i < j < 6, \\ \sum_{n=1}^{\infty} H_1^{n-1}[1 - G(D_{i,5})]H_2, \quad i = 1, 2, 3, 4, 5 \text{ and } j = 6, \\ 1, i = 6 \quad j = 6, \\ 0, \textit{otherwise}, \end{cases} \tag{27}$$

where $H_1 = G(D_{i,i})$ and $H_2 = \frac{1}{s} - \sum_{m=0}^{n-1} \frac{(\lambda)^m}{(\lambda + s)^{m+1}}$.

Then, the rolling bearing reliability $R(t)$ can be derived from Eqs. (12), (13) and the inverse Laplace transform. Fig. 2 shows the reliability of the rolling bearing given that the rolling bearing enters state 1 at time 0.

Then, the system reliability $R(t)$ can be obtained through Eq. (14). The result is shown in Fig. 3.

The rolling bearing state at time t can be obtained according to Eqs. (18), (20) and (24). In Fig. 4, curves are drawn for probabilities of transferring to state 1, 2, 3, 4, 5, 6 respectively at time t when the rolling bearing is in state 1 at time 0. It can be seen that the probability from state 1 to 1 decreases monotonically from 1 to 0. The probability from state 1 to 6 increases monotonically from 0 to 1. Probabilities that go from state 1 to intermediate states go up and then go down to 0 gradually.

5.3. Opportunistic maintenance strategy

After obtaining relevant indexes of the rolling bearing, the maintenance strategy of the system is going to be constructed.

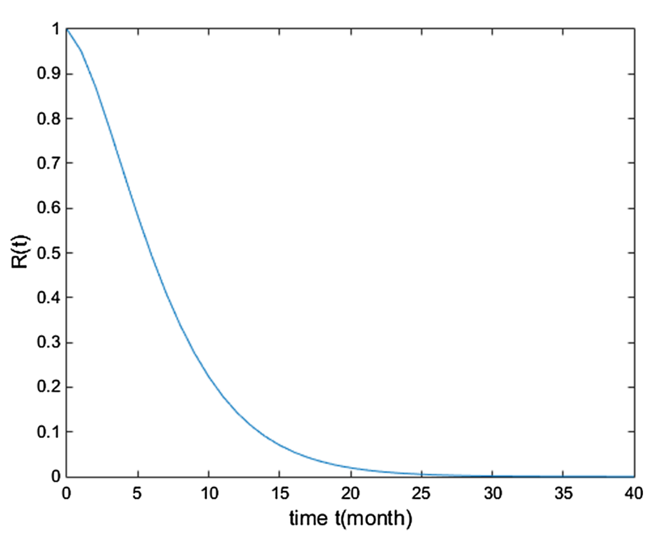

Fig. 2. Reliability of the rolling bearing given that the rolling bearing enters state 1 at time 0.

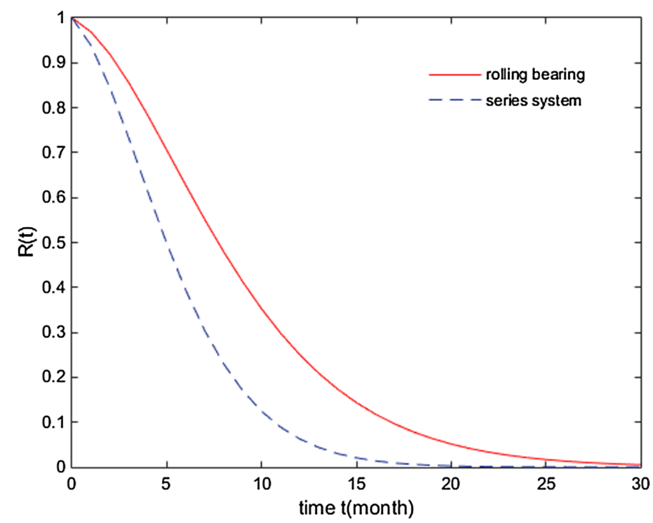

Fig. 3. Reliability of the system given that both rolling bearings enter state 1 at time 0.

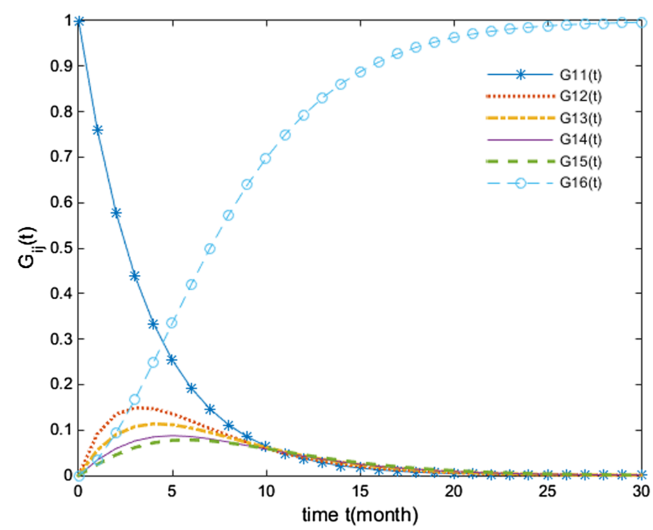

Fig. 4. Probabilities of transferring to state 1, 2, 3, 4, 5, 6 at time t when the rolling bearing is in state 1 at $t = 0$.

It is assumed that the system is inspected at every interval T and the inspection time can be ignored. Replacement, preventive maintenance (PM) and opportunistic maintenance (OM) of bearings are all instantaneous and perfect. Let $c_i = 5,\ c_o = 10,\ c_p = 30$ and $c_f = 60$.N_1 and N_2 represent the thresholds of opportunistic maintenance and preventive maintenance respectively and $1 < N_1 < N_2 < N$.

The optimal inspection interval T^* under different N_1, N_2 combinations is calculated based on Eq. (26) in order to minimize the average cost per unit time. Fig. 5 shows cost curves under different N_1, N_2 combinations. The minimum cost in each curve and the corresponding inspection interval are recorded in Table 3.

As shown in Table 3 and Fig. 5, when the opportunistic maintenance threshold, preventive maintenance threshold and inspection interval are 2, 3 and 2.40 respectively. The average cost per unit time can be minimized to 18.28. The calculation methods and results of the optimal inspection interval, the optimal opportunistic maintenance threshold and preventive maintenance threshold in this paper are valuably managerial suggestions for engineers in order to minimize average cost per unit time in reality.

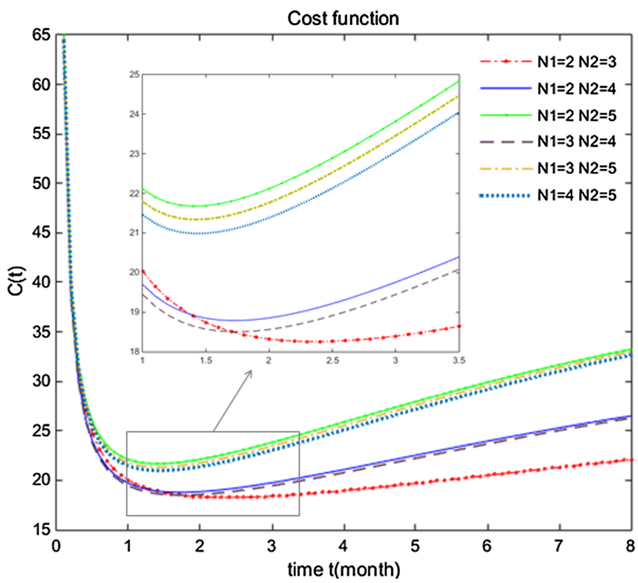

Fig. 5. Cost functions under different threshold combinations.

Table 3
Optimal inspection interval and the corresponding minimum cost under different thresholds combinations.

Case	N_1	N_2	T^*	$C(T^*)$
1	**2**	**3**	**2.40**	**18.28**
2	2	4	1.80	18.79
3	2	5	1.40	21.68
4	3	4	1.50	18.50
5	3	5	1.40	21.34
6	4	5	1.50	20.99

Table 4
The optimal inspection interval, minimum cost and maintenance threshold under different maintenance costs combinations.

$c_o = 10;\ c_p = 30;\ c_f = 60$				$c_i = 5;\ c_0 = 10;\ c_f = 60$			
c_i	T^*	$C^*(T)$	(N_1^*, N_2^*)	c_p	T^*	$C^*(T)$	(N_1^*, N_2^*)
0	0.00	12.65	(3,4)	15	1.90	14.10	(2,3)
5	2.40	18.26	(2,3)	30	2.40	18.26	(2,3)
10	3.30	20.38	(2,3)	45	1.80	21.64	(3,4)
15	5.50	28.19	(2,3)	60	2.50	24.71	(3,4)
$c_i = 5;\ c_p = 30;\ c_f = 60$				$c_i = 5;\ c_o = 10;\ c_p = 30$			
c_o	T^*	$C^*(T)$	(N_1^*, N_2^*)	c_f	T^*	$C^*(T)$	(N_1^*, N_2^*)
10	2.40	18.26	(2,3)	30	6.90	13.63	(2,3)
20	2.30	18.55	(2,3)	45	3.10	16.26	(2,3)
30	2.40	18.84	(2,3)	60	2.40	18.26	(2,3)
60	2.30	19.71	(2,3)	90	1.90	21.67	(2,3)

5.4. Sensitivity analysis of maintenance costs

In this part, the impacts of maintenance and replacement costs involved in this model on the optimal inspection interval, the optimal opportunistic maintenance threshold and preventive maintenance threshold are analyzed in order to improve the credibility of the proposed model. In Table 4, the optimal inspection interval, minimum cost and maintenance threshold under different maintenance costs combinations are recorded.

According to the sensitivity analysis of maintenance costs, the results are recorded in Table 4. Concretely as,

a) The optimal threshold of opportunistic maintenance and preventive maintenance is (3,4) at the case of $c_o = 0$ and $T^* = 0$, which can be considered as the condition that the states of rolling bearings are continuously monitored by a sensor, otherwise the value of optimal threshold is (2,3) and the optimal inspection interval increases with the increase of inspection cost. Therefore, it is necessary to increase the inspection interval for the rolling bearings when the inspection cost increases.
b) With the increase of the cost of preventive maintenance, the value of optimal threshold turns from (2,3) to (3,4). The preventive maintenance cost is positively correlated with the optimal inspection interval. It is a wise choice to take measures of opportunistic maintenance and preventive maintenance for rolling bearings later when the cost of preventive maintenance increases.
c) The result of sensitivity analysis is consistent with the fact that the higher the replacement cost, the shorter the inspection interval. That is, as the cost of rolling bearings increases, rolling bearings should be inspected more frequently in order to minimize maintenance costs.

6. Conclusions

In this paper, a series system which consists of two components with multi-stage accelerated damage is studied. External shocks with the same magnitude may have different effects on the component which is in different stages. Based on these assumptions, relevant reliability indexes of the component and the system are derived. Then, a periodic inspection strategy for the system is constructed. The combination of the optimal opportunistic maintenance threshold, preventive maintenance threshold and the optimal inspection interval is determined by taking the minimum average cost per unit time as the objective function. Finally, a three-stage accelerated damage shock model is presented in the numerical example and sensitivity analysis of maintenance costs is also discussed, explaining the model presented in this paper.

The establishment and realization of this model are more practical and enrich the research content of shock models. In future studies, not only the shock-induced degradation but also internal degradations should be considered for shock models. The situation that the overall system has a parallel or more complicated structure could be considered in future studies. In addition, other popular maintenance strategies could be considered in shock models in future studies, such as time-based maintenance and condition-based maintenance.

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References

Alvarez, E. J., & Ribaric, A. P. (2018). An improved-accuracy method for fatigue load analysis of wind turbine gearbox based on SCADA. *Renewable Energy, 115*, 391–399.
Cavallaro, G., & Nelias, D. (2005). Analysis of high-speed inter-shaft cylindrical roller bearing with flexible rings. *Tribology & Lubrication Technology, 61*(11), 72–82.
Chen, G., & Qu, M. (2019). Modeling and analysis of fit clearance between rolling bearing outer ring and housing. *Journal of Sound and Vibration, 438*, 419–440.
Chen, J. Y., & Li, Z. (2008). An extended extreme shock maintenance model for a deteriorating system. *Reliability Engineering & System Safety, 93*(8), 1123–1129.
Cui, L. R., & Li, H. J. (2006). Opportunistic maintenance for multi-component shock models. *Mathematical Methods of Operations Research, 63*(3), 493–511.
Eryilmaz, S. (2015). Assessment of a multi-state system under a shock model. *Applied Mathematics and Computation, 269*, 1–8.
Eryilmaz, S. (2017a). Computing optimal replacement time and mean residual life in reliability shock models. *Computers & Industrial Engineering, 103*, 40–45.
Eryilmaz, S. (2017b). Delta-shock model based on Polya process and its optimal replacement policy. *European Journal of Operational Research, 263*(2), 690–697.
Hao, S. H., Yang, J., Ma, X. B., & Zhao, Y. (2017). Reliability modeling for mutually dependent competing failure processes due to degradation and random shocks. *Applied Mathematical Modelling, 51*, 232–249.

Li, W. J., & Pham, H. (2005). Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks. *IEEE Transactions on Reliability, 54*(2), 297–303.

Li, X. Q., Jiang, H. K., & Xiong, X. (2019). Rolling bearing health prognosis using a modified health index based hierarchical gated recurrent unit network. *Mechanism and Machine Theory, 133*, 229–249.

Liu, B., Xie, M., Xu, Z. G., & Kuo, W. (2016). An imperfect maintenance policy for mission-oriented systems subject to degradation and external shocks. *Computers & Industrial Engineering, 102*, 21–32.

Mallor, F., & Omey, E. (2001). Shocks, runs and random sums. *Journal of Applied Probability, 38*(2), 438–448.

Mao, Y. Z., Wang, L. Q., & Zhang, C. (2018). Influence of ring deformation on the dynamic characteristics of a roller bearing in clearance fit with housing. *Reliability Engineering & System Safety, 042*, 122–130.

Rafiee, K., Feng, Q. M., & Coit, D. W. (2017). Reliability assessment of competing risks with generalized mixed shock models. *Reliability Engineering & System Safety, 159*, 1–11.

Shen, J. Y., Cui, L. R., & Yi, H. E. (2018). System performance of damage self-healing systems under random shocks by using discrete state method. *Computers & Industrial Engineering, 125*, 124–134.

Shi, X. J., & Wang, L. Q. (2015). TEHL analysis of high-speed and heavy-load roller bearing with quasi-dynamic characteristics. *Chinese Journal of Aeronautics, 28*(4), 1296–1304.

Su, C., & Cheng, L. F. (2018). An availability-based warranty policy considering preventive maintenance and learning effects. *Proceedings of the Institution of Mechanical Engineers Part O-Journal of Risk and Reliability, 232*, 576–586.

Wang, H. Y., Wang, W. B., & Peng, R. (2017). A two-phase inspection model for a single component system with three-stage degradation. *Reliability Engineering & System Safety, 158*, 31–40.

Wang, X. Y., Zhao, X., & Sun, J. L. (2019). A compound negative binomial distribution with mutative termination conditions based on a change point. *Journal of Computational and Applied Mathematics,* 237–249.

Yang, L., Ma, X., & Zhao, Y. (2017a). A condition-based maintenance model for a three-state system subject to degradation and environmental shocks. *Computers & Industrial Engineering, 105*, 210–222.

Yang, L., Ma, X. B., Peng, R., Zhai, Q. Q., & Zhao, Y. (2017b). A preventive maintenance policy based on dependent two-stage deterioration and external shocks. *Reliability Engineering & System Safety, 160*, 201–211.

Zhao, X., Cai, K., Wang, X. Y., & Song, Y. B. (2018a). Optimal replacement policies for a shock model with a change point. *Computers & Industrial Engineering, 118*, 383–393.

Zhao, X., Guo, X. X., & Wang, X. Y. (2018b). Reliability and maintenance policies for a two-stage shock model with self-healing mechanism. *Reliability Engineering & System Safety, 172*, 185–194.

Zhao, X., Wang, S. Q., Wang, X. Y., & Cai, K. (2018c). A multi-state shock model with mutative failure patterns. *Reliability Engineering & System Safety, 178*, 1–11.

Zhao, X., Wang, S. Q., Wang, X. Y., & Fan, Y. (2020). Multi-state balanced systems in a shock environment. *Reliability Engineering & System Safety, 193*. https://doi.org/10.1016/j.ress.2019.106592.

Zhu, W. J., Fouladirad, M., & Berenguer, C. (2015). Condition-based maintenance policies for a combined wear and shock deterioration model with covariates. *Computers & Industrial Engineering, 85*, 268–283.