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Dynamic availability and performance deficiency of common bus systems with imperfectly repairable components

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ABSTRACT

Common bus performance sharing (CBPS) abounds in diverse applications such as transportation, power supply and collaborative computing to facilitate efficient utilization of limited system resources. This paper models such a CBPS system with repairable components performing the main system function and repairable bus lines for redistributing surplus performance of some system components to components undergoing performance deficiency. Both time-to-failure and time-to-repair of system units (including components and lines) are random and may follow any arbitrary types of distributions. A probabilistic model based on discrete-state continuous-time stochastic processes is proposed for evaluating instantaneous availability of these repairable system units. The proposed model addresses the general repair policy encompassing minimal repair, perfect repair and imperfect repair. The universal generating function technique is further implemented for evaluating the system level performance metrics, including instantaneous system availability, instantaneous expected performance deficiency, expected system availability and total expected unsupplied demand during a specified mission time. Examples are provided to illustrate the proposed evaluation methodology and its application in prioritizing maintenance improvement actions.

1. Introduction

In the reliability engineering field, the common bus performance sharing (CBPS) system model was firstly suggested in [1], as an extension of the single-directional two-component performance sharing model studied in [2]. Performance sharing is a desired function of many real-world systems with limited system resources. In these systems, each unit has to meet its individual demand. If the performance of a unit exceeds its required demand level, surplus (unconsumed) performance exists, which can be shared with other units experiencing performance deficiency through a common bus redistribution system. For example, in distributed computing environments (e.g., grid computing, cloud computing) [3–5], multiple computing units may collaborate to accomplish a specified large task, which can be divided into many subtasks. Each computing unit involved has its own demand, characterized by the number of subtasks it needs to handle. During the task execution, the surplus processing capacity of a computing unit can be shared with another computing unit with demand exceeding its processing capacity. Performance sharing is also typical in power generating systems. Particularly, the electricity produced in power generating units located at different sites can be shared through common power transmission lines [1,6]. Many studies showed that the performance sharing is valuable to efficient utilization of system resources, reducing system operation cost, and improving system reliability or availability [1,7].

Since the seminal work in [1], the CBPS model has been studied and extended in different directions, particularly, in system structures and optimization problems considered. For example, in [8] the optimal dynamic component connection strategy was investigated for series CBPSs with limitation on the size of the performance sharing group. In [9] the optimal component allocation problem was considered for CBPSs with hierarchical performance sharing groups. In [10] two performance sharing groups with two common bus structures were modeled. In [11], the optimal component allocation problem was solved for CBPSs subject to phased-mission requirements. In [12], the model of [11] was further extended to consider common-cause failures. In [13] reliability analysis was performed for CBPSs with k-out-of-n structures using the universal generating function based technique. In [14] the

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Acronyms

cdf	cumulative distribution function
pdf	probability density function
CBPS	common bus performance sharing
UGF	universal generating function
DSCTP	discrete-state continuous-time process

Nomenclature

τ	time horizon
N	number of components composing the system
M	number of lines composing the common bus
$W_n(t)$	random demand of component n at time t
$A(\tau)$	expected system availability during the time horizon τ
$a(t)$	instantaneous system availability at time t
$UD(\tau)$	total expected unsupplied demand during the time horizon τ
$e(t)$	instantaneous expected performance deficiency of the system at time t
$\langle T_j, X_j\rangle$	event when the jth failure of a component occurs at time T_j and the component spends time X_j in operation mode before the failure
$\Omega_j(t,x)$	function representing joint distribution of random values T_j and X_j
$a_n(t)$ $(\tilde{a}_m(t))$	instantaneous availability of component n (line m)
D_n $(\tilde{D}_m)$	random repair time for component n (line m)
$d_n^{\min}$, $d_n^{\max}$	minimal and maximal possible realizations of D_n
$\tilde{d}_m^{\min}$, $\tilde{d}_m^{\max}$	minimal and maximal possible realizations of $\tilde{D}_m$
K_n	number of possible demand levels of component n
g_n	nominal performance of component n
J_n (J_m)	maximal number of failures of component n (line m) during the mission
c_m	nominal capacity of line m
$f_n(t)$, $F_n(t)$	*pdf, cdf* of time-to-failure distribution of component n
$\tilde{f}_m(t)$, $\tilde{F}_m(t)$	*pdf, cdf* of time-to-failure distribution of line m
$\psi_n(t)$, $\Psi_n(t)$	*pdf, cdf* of repair time for component n
$\tilde{\psi}_m(t)$, $\tilde{\Psi}_m(t)$	*pdf, cdf* of repair time for line m
η_n, β_n	Scale, shape parameters of Weibull time-to-failure distribution of component n
$\rho_{n,j}(t)$	probability that component n is under repair after jth failure at time t
π_n $(\tilde{\pi}_m)$	repair efficiency coefficient of component n (line m)
$G_n(t)$ $(C_m(t))$	random performance of component n (capacity of line m) at time t
$s_n(t)$	random surplus performance of component n at time t
$q_n(t)$	random performance deficiency of component n at time t
$S(t)$	DSCTP of the entire system surplus performance
$Q(t)$	DSCTP of the entire system performance deficiency
$C(t)$	DSCTP of common bus capacity
$\lfloor x \rfloor$	maximum integer value not exceeding x

optimal defense and attack strategies were investigated for CBPSs with identical components subject to intentional attacks. In [15] the optimal component allocation and preventive replacement interval problem was solved for series-parallel CBPSs. In [16] the optimal component loading and protection from the external impacts were studied for series-parallel CBPSs.

While most of literature modeling CBPS systems assume unrepairable components and bus lines, in [15] periodic preventive replacement is considered for each system component and corrective maintenance (repair) is conducted if a component fails between two replacement actions. The model however is limited to fixed repair time under the minimal repair policy where the component is restored to an "as bad as old" condition after each repair. Moreover, only a steady-state achieved availability can be evaluated in [15]. The model in [7] considers a more general repair policy for system components, but the continuous time Markov chain approach is applied to model system components behavior, which limits the model to exponential time-to-failure and time-to-repair distributions. Both works do not explicitly consider failures of the common bus performance redistribution system and thus assume non-repairable bus lines.

This work makes contributions by modeling CBPS systems with repairable components and repairable bus lines. Both time-to-failure and time-to-repair of the system components and bus lines are random and may follow any arbitrary types of distributions. The general repair policy is addressed, which covers perfect repair (the component after repair has an "as good as new" condition), minimal repair (the component after repair has an "as bad as old" condition), and imperfect repair (the component is restored to a condition between the former two) [17,18]. A probabilistic modeling methodology is suggested for evaluating the instantaneous availability of each repairable unit for any given time instant. The universal generating function (UGF) technique is then implemented for assessing instantaneous system availability and instantaneous expected performance deficiency at any given time instant. The expected system availability and total expected unsupplied demand during a specified mission time are further evaluated for the entire system.

The remainder of the paper is organized as follows: Section 2 depicts the CBPS system considered in this work. Section 3 presents the instantaneous availability evaluation method for a repairable component (or bus line) subject to random failure time and repair time. Section 4 presents the system performance metrics considered, including instantaneous system availability, expected system availability, instantaneous expected performance deficiency and total expected unsupplied demand. Section 5 presents the UGF-based technique for evaluating system performance metrics. Section 6 presents illustrative examples and application of the proposed methodology in maintenance

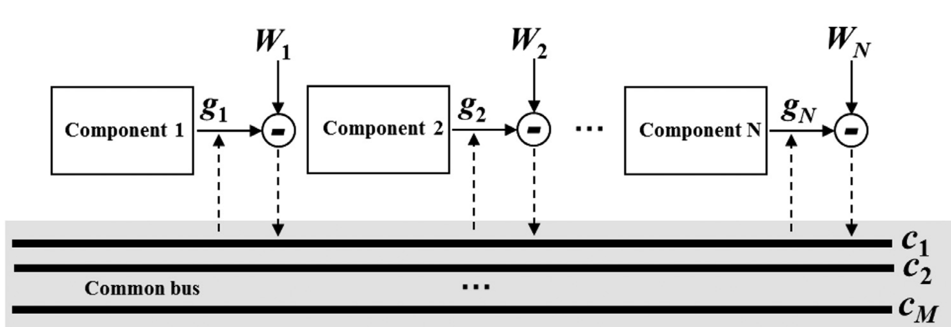

Fig. 1. Structure of CBPS.

improvement actions. Lastly, Section 7 concludes the work and points out future directions of this research.

2. System model

The CBPS considered consists of N repairable components. Each component $n \in \{1, \ldots, N\}$ has its nominal performance g_n and must satisfy its random demand defined as a discrete-state continuous-time process (DSCTP) $W_n(t)$. All the components are connected through a common bus performance redistribution system consisting of M repairable lines, as illustrated in Fig. 1. Line $m \in \{1, \ldots, M\}$ has its nominal transmission capacity c_m. During repairs, the performance of the components and capacity of the transmission lines are zeroed. The total transmission capacity of the common bus at time instant t equals to the sum of capacities of all the lines available at this instant. All of the components that have performance exceeding the demand can send the surplus performance to the components that experience the performance deficiency through the common bus. The system fails if at least one of individual component demands $W_n(t)$ is not satisfied.

The time-to-failure of component n (line m) is characterized by a cumulative distribution function (*cdf*) $F_n(t)$ ($\tilde{F}_m(t)$). When a component (line) fails, the repair/replacement procedure starts immediately. The repair time D for component n (line m) relies on external factors such as availability and efficiency of repair manpower and equipment and is randomly distributed in the interval $[d_n^{\min}, d_n^{\max}]$ ($[\tilde{d}_m^{\min}, \tilde{d}_m^{\max}]$). The *cdf* $\Psi_n(t)$ ($\tilde{\Psi}_m(t)$) of the random repair time of component n (line m) is known and such that $\Psi_n(t) = 0$ for $t < d_n^{\min}$ and $\Psi_n(t) = 1$ for $t > d_n^{\max}$ ($\tilde{\Psi}_m(t) = 0$ for $t < \tilde{d}_m^{\min}$ and $\tilde{\Psi}_m(t) = 1$ for $t > \tilde{d}_m^{\max}$). It is assumed that failures and repairs of all the components and lines are *s*-independent.

A limited time horizon τ is considered for modeling the system behavior. During this time horizon, the maximal number of repairs that can be conducted for component n (line m) is $\tau/d_n^{\min}$ ($\tau/\tilde{d}_m^{\min}$). Thus, the maximal number of failures of component n (line m) is given as $J_n = 1 + \lfloor \tau/d_n^{\min} \rfloor$ ($J_m = 1 + \lfloor \tau/\tilde{d}_m^{\min} \rfloor$).

3. Instantaneous availability of repairable components or lines

In this section, a probabilistic model is presented for evaluating the instantaneous availability $a_n(t)$ of component n at time t. The same algorithm can be applied to evaluate the instantaneous availability $\tilde{a}_m(t)$of line m.

The virtual age-based repair model of [19,20] is adopted. Specifically, let t_0 and t represent the operation time of component n before and after a repair, respectively. If the hazard rate function of component n before a repair is $\zeta_n(t)$, then its hazard rate after the repair is $\zeta_n(\pi_n t_0 + t)$, where the coefficient 0(perfect repair) $\le \pi_n \le$ 1(minimal repair) models repair efficiency of component n [19]. The *pdf* of the time-to-failure of component n after the repair is $f_n^*(t_0,t) = f_n(\pi_n t_0 + t)/[1 - F_n(\pi_n t_0)]$, and the *cdf* is $F_n^*(t_0,t) = [F_n(\pi_n t_0 + t) - F_n(\pi_n t_0)] / [1 - F_n(\pi_n t_0)]$.

Consider a random event $\langle T_j, X_j\rangle$ ($X_j \le T_j$) for component n, representing that its jth failure takes place at time T_j (elapsed from the beginning of the mission) after the component spent time X_j in the operation mode and time $T_j - X_j$ in the repair mode. Each event $\langle T_j, X_j\rangle$ corresponds to an initiation of a repair procedure that takes random time $D \in [d_n^{\min}, d_n^{\max}]$. Thus, for any realization of X_j, T_j can range from $X_j + (j-1)d_n^{\min}$ to $X_j + (j-1)d_n^{\max}$ (when component n spends minimal and maximal time in all of the previous $j-1$ repairs, respectively). Next, we derive the joint distribution function of T_j and X_j denoted by $\Omega_j(t,x)$.

Consider the first failure, i.e., $j = 1$. Because the component has not spent any time in the repair mode before the occurrence of the first failure, $X_1 = T_1$ and

$$\Omega_1(t, x) = \begin{cases} f_n(t) & \text{if } (x = t) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The transition from event $\langle T_j, X_j\rangle$ to event $\langle T_{j+1}, X_{j+1}\rangle$ ($T_{j+1} \ge T_j + d_n^{\min}$, $X_{j+1} \ge X_j$) can take place when the component functions for time $X_{j+1} - X_j$ after a repair procedure, which takes time $D = (T_{j+1} - T_j) - (X_{j+1} - X_j)$. Because $d_n^{\min} \le D \le d_n^{\max}$, the condition $T_{j+1} + X_j - X_{j+1} - d_n^{\max} \le T_j \le T_{j+1} + X_j - X_{j+1} - d_n^{\min}$ should hold to make the event transition $\langle T_j, X_j\rangle \rightarrow \langle T_{j+1}, X_{j+1}\rangle$ possible. Note that $X_j = X_{j+1}$ when component n fails immediately following the jth repair.

Expression (2) shows the recursive evaluation of $\Omega_{j+1}(t,x)$ ($j = 0, \ldots, J_n - 1$) using $\Omega_j(t,x)$, $\psi_n(t)$ and $f_n(t)$. Note that when $t < x + jd_n^{\min}$ or $t > x + jd_n^{\max}$, $\Omega_{j+1}(t,x) = 0$.

$$\begin{aligned} \Omega_{j+1}(t, x) &= \int_0^x \int_{\max(\tilde{x}+(j-1)d_n^{\min}, t+\tilde{x}-x-d_n^{\max})}^{\min(\tilde{x}+(j-1)d_n^{\max}, t+\tilde{x}-x-d_n^{\min})} \Omega_j(\tilde{t}, \tilde{x})\psi_n(t - \tilde{t} - x \\ &\quad + \tilde{x}) f_n^*(\tilde{x}, x - \tilde{x}) d\tilde{t} d\tilde{x} \\ &= \int_0^x \int_{\max(\tilde{x}+(j-1)d_n^{\min}, t+\tilde{x}-x-d_n^{\max})}^{\min(\tilde{x}+(j-1)d_n^{\max}, t+\tilde{x}-x-d_n^{\min})} \Omega_j(\tilde{t}, \tilde{x})\psi_n(t - \tilde{t} - x \\ &\quad + \tilde{x}) \frac{f_n(\pi_n\tilde{x} + x - \tilde{x})}{1 - F_n(\pi_n\tilde{x})} d\tilde{t} d\tilde{x} \end{aligned} \tag{2}$$

If a failure happens at instant t-δ and the repair following the failure takes at least time δ, then the component is in the repair mode at time t. Thus, expression (3) gives the probability that component n is under repair at time t after the jth failure. Note that $\rho_{nj}(t) = 0$ for $t < (j-1)d_n^{\min}$.

$$\rho_{nj}(t) = \int_0^{\min(t, d_n^{\max})} \int_{t-\delta-(j-1)d_n^{\max}}^{t-\delta-(j-1)d_n^{\min}} \Omega_j(t - \delta, x)(1 - \Psi_n(\delta)) dx d\delta \text{ for } t \ge (j-1)d_n^{\min} \tag{3}$$

Because the minimal time when the jth failure may happen is $(j-1)d_n^{\min}$, the number of failures that can happen at time no later than t cannot go beyond $(1 + t/d_n^{\min})$. Thus, the instantaneous availability of component n at time t, i.e., the overall probability that component n is available (not under repair) at time t can be obtained as

$$\begin{aligned} a_n(t) &= 1 - \sum_{j=1}^{\lfloor 1+t/d_n^{\min} \rfloor} \rho_{nj}(t) \\ &= 1 - \sum_{j=1}^{\lfloor 1+t/d_n^{\min} \rfloor} \int_0^{\min(t, d_n^{\max})} \int_{t-\delta-(j-1)d_n^{\max}}^{t-\delta-(j-1)d_n^{\min}} \Omega_j(t - \delta, x)(1 - \Psi_n(\delta)) dx d\delta. \end{aligned} \tag{4}$$

The numerical algorithm for obtaining instantaneous availability of repairable component/line based on the derivations above is presented in [21]. It is shown there that the computational complexity of the algorithm is $O(J_n \delta_\tau^3)$, where δ_τ is the number of equal intervals the time horizon τ is divided into for discretization and J_n is the maximum possible number of component/line failures during the mission. See in [21] also discussion about the choice of δ_τ for providing sufficient accuracy of computations.

4. Instantaneous availability and performance deficiency of the entire system

Having the instantaneous availability of each component n $a_n(t)$ and line m $\tilde{a}_m(t)$ evaluated using the method suggested in Section 3, one can determine DSCTP of random performance $G_n(t)$ of component n and transmission capacity $C_m(t)$ of line m respectively as

$$\Pr(G_n(t) = g_n) = a_n(t), \Pr(G_n(t) = 0) = 1 - a_n(t) \tag{5}$$

$$\Pr(C_m(t) = c_m) = \tilde{a}_m(t), \Pr(C_m(t) = 0) = 1 - \tilde{a}_m(t) \tag{6}$$

Table 1
Parameters of components.

Component n	g_n	η_n	β_n	π_n	$d_n^{\min}$	$d_n^{\max}$	μ_n	σ_n
1	100	50	1.1	0.3	15	20	17	4
2	120	20	2.0	0.7	10	40	15	3
3	50	70	1.5	0.8	18	38	22	6
4	90	80	1.3	0.0	20	30	25	10
5	60	40	2.3	0.2	10	25	15	6

The total transmission capacity of the common bus in any time instant t is determined as the sum of capacities of lines available at this instant. Thus

$$C(t) = \sum_{m=1}^{M} C_m(t). \tag{7}$$

If component n at time t has performance exceeding its demand, the surplus performance can be shared with other components through the common bus. The total surplus performance in the entire system at time t is

$$S(t) = \sum_{n=1}^{N} \max(G_n(t) - W_n(t), 0). \tag{8}$$

The total performance deficiency in the system at time t is

$$Q(t) = \sum_{n=1}^{N} \max(W_n(t) - G_n(t), 0). \tag{9}$$

All components that cannot meet the demand at time t need the total amount of performance $Q(t)$ to compensate their deficiency, whereas the components with surplus performance can provide at this time no more than amount $S(t)$ of performance for this compensation. Therefore, the amount of performance that should be transmitted by the transmission system is min($S(t)$,$Q(t)$). Due to the limited capacity $C(t)$ of the transmission system, the total amount of the performance that can be redistributed at time t is min ($S(t)$,$Q(t)$, $C(t)$), i.e.,

$$\min\left(\sum_{n=1}^{N} \max(G_n(t) - W_n(t), 0), \sum_{n=1}^{N} \max(W_n(t) - G_n(t), 0), \sum_{m=1}^{M} C_m(t)\right). \tag{10}$$

$S(t)$ and $Q(t)$ are statistically dependent DSCTPs, whereas $S(t)$ and $C(t)$ as well as $Q(t)$ and $C(t)$ are statistically independent.

The entire performance deficiency remained after the performance sharing at time t is

$$\begin{aligned}\Delta(t) &= Q(t) - min(S(t), Q(t), C(t)) = max(0, Q(t) \\ &\qquad - min(S(t), C(t))) \\ &= \max\left(0, \sum_{n=1}^{N} \max(W_n(t) - G_n(t), 0) \right.\\ &\qquad \left. - \min\left(\sum_{n=1}^{N} \max(G_n(t) - W_n(t), 0), \sum_{m=1}^{M} C_m(t)\right)\right)\end{aligned} \tag{11}$$

The instantaneous system availability $\alpha(t)$ at time t is defined as the probability that the entire system performance deficiency can be fully compensated by the surplus performance transferred by the common bus at this time. It can be evaluated as:

Table 2
Parameters of common bus lines.

Line m	c_m	$\tilde{\eta}_m$	$\tilde{\beta}_m$	$\tilde{\pi}_m$	$\tilde{d}_m^{\min}$	$\tilde{d}_m^{\max}$	$\tilde{\mu}_m$	$\tilde{\sigma}_m$
1	50	50	1.3	0.4	15	25	20	1
2	70	40	1.0	0.2	10	30	23	6
3	100	70	1.7	0.3	12	25	18	6

Table 3
Demand distributions.

Component n	w_n	y_n
1	60, 40, 10	0.20, 0.55, 0.25
2	80, 70, 50, 30	0.60, 0.15, 0.15, 0.10
3	60, 30	0.70, 0.30
4	50, 30	0.40, 0.60
5	80, 50, 40	0.40, 0.35, 0.25

$$a(t) = \Pr\left\{\max\left(0, \sum_{n=1}^{N} \max(W_n(t) - G_n(t), 0) - \min\left(\sum_{n=1}^{N} \max(G_n(t) - W_n(t), 0), \sum_{m=1}^{M} C_m(t)\right)\right) = 0\right\}. \tag{12}$$

The expected probability of meeting the demand during the time horizon τ (i.e., the expected system availability) is

$$A(\tau) = \frac{1}{\tau}\int_0^{\tau} a(t)dt. \tag{13}$$

The instantaneous expected performance deficiency at time t can be obtained as the expectation of the random variable $e(t) = E(\Delta(t))$. The total expected unsupplied demand during the time horizon τ is $UD(\tau) = \int_0^{\tau} e(t)dt$.

5. Algorithm for determining instantaneous availability and performance deficiency

The UGF technique is implemented to determine $e(t)$, the instantaneous expected performance deficiency at time t. In general, the polynomial in (14) defines the UGF representing the distribution of a DSCTP $Y_j(t)$ that can take k_j possible values [22].

$$u_j(z, t) = \sum_{h=1}^{k_j} x_{j,h}(t) z^{\varepsilon_{j,h}}, \tag{14}$$

where $x_{j,i}(t) = \Pr\{Y_j(t) = \varepsilon_{j,i}\}$. To obtain the UGF representing the DSCTP of a function of h independent DSCTP $\varphi(Y_1(t), \ldots, Y_h(t))$, the composition operator defined in (15) is used.

$$U(z, t) = \underset{\phi}{\otimes}(u_1(z, t), \ldots, u_h(z, t))$$

$$\underset{\phi}{\otimes}\left(\sum_{i_1=1}^{k_1} x_{1,i_1}(t) z^{\varepsilon_{1,i_1}}, \ldots, \sum_{i_h=1}^{k_h} x_{n,i_n}(t) z^{\varepsilon_{n,i_n}}\right) = \sum_{i_1=1}^{k_1}\sum_{i_2=1}^{k_2}\ldots\sum_{i_h=1}^{k_h}\left(\prod_{j=1}^{h} x_{j,i_j}(t)\right) z^{\phi(\varepsilon_{1,i_1}, \ldots, \varepsilon_{h,i_h})} \tag{15}$$

Specifically, Eqs. (16) and (17) give the UGFs representing the DSCTP of random performance of system component n and line m at time instant t.

$$u_n(z, t) = \sum_{i_n=0}^{1} x_{n,i_n}(t) z^{\varepsilon_{n,i_n}}, \tag{16}$$

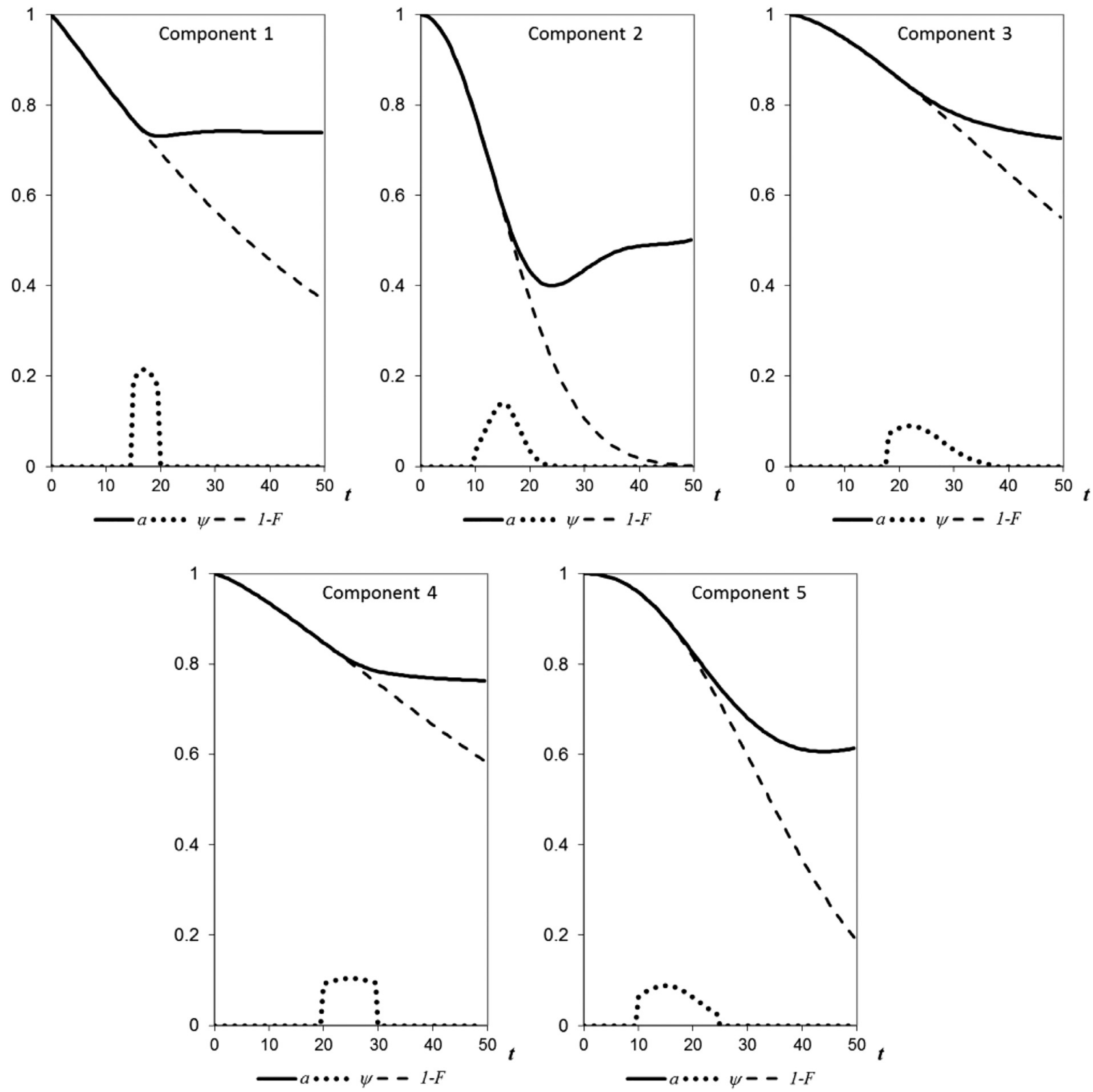

Fig. 2. Instantaneous availability $a(t)$, repair time *pdf* $\psi(t)$ and reliability $1 - F(t)$ of example CBPS components.

$$\tilde{u}_m(z,\, t) = \sum_{j_m=0}^{1} \tilde{x}_{m,j_m}(t) z^{\tilde{\varepsilon}_{m,j_m}}, \tag{17}$$

where $\boldsymbol{x}_{n,1}(t) = a_n(t)$, $\boldsymbol{x}_0(t) = 1 - a_n(t)$, $\varepsilon_{n,1} = g_n$, $\varepsilon_{n,0} = 0$, $\tilde{x}_{m,1}(t) = \tilde{a}_m(t)$, $\tilde{x}_{m,0}(t) = 1 - \tilde{a}_m(t)$, $\tilde{\varepsilon}_{m,1}(t) = c_m$ and $\tilde{\varepsilon}_{m,0}(t) = 0$.

Eq. (18) gives the UGF representing the DSCTP of random demand of component n at time instant t.

$$\hat{u}_n(z,\, t) = \sum_{k=0}^{K_n} y_{n,k}(t) z^{w_{n,k}}, \tag{18}$$

here $\boldsymbol{y}_{n,k}(t) = \Pr(W_n(t) = w_{n,k})$ and K_n is the number of possible demand levels.

Any combination of states of all the N components and their demand levels at any time instant t corresponds to certain realization of surplus performance $S(t)$ and performance deficiency $Q(t)$. As $S(t)$ and $Q(t)$ are dependent DSCTPs, they must be represented by a single UGF that relates probabilities of state combinations to the corresponding realizations of $S(t)$ and $Q(t)$. In order to obtain this UGF one should first obtain the UGF representing the DSCTPs $s_n(t)$ and $q_n(t)$ for each component n.

Having UGFs $u_n(z,\, t)$ and $\hat{u}_n(z,\, t)$ representing DSCTPs $G_n(t)$ and $W_n(t)$ respectively, one can obtain the UGF representing the DSCTPs of $s_n(t)$ and $q_n(t)$ using the composition operator $\underset{\min,\min}{\otimes}$ defined in (19).

$$v_n(z,\, t) = u_n(z,\, t) \underset{\min,\min}{\otimes} \hat{u}_n(z,\, t)$$
$$= \sum_{h=0}^{1} \sum_{k=1}^{K_n} x_{n,h}(t) y_{n,k}(t) z^{\min(0,\varepsilon_{n,h}-w_{n,k}),\min(0,w_{n,k}-\varepsilon_{n,h})}$$
$$= \sum_{j=1}^{I_n} \gamma_{n,j}(t) z^{s_{n,j},q_{n,j}}. \tag{19}$$

The UGF in (19) represents the distribution of probabilities of joint events:

$$\gamma_{n,j}(t) = \Pr\{s_n(t) = s_{n,j} \cap q_n(t) = q_{n,j}\}. \tag{20}$$

Following (8) and (9), one can obtain the UGF $V_{\{1,2,\ldots,N\}}(z,t)$, representing DSCTPs $S(t) = \sum_{n=1}^{N} s_n(t)$ and $Q(t) = \sum_{n=1}^{N} q_n(t)$ using the following recursive procedure:

1. Assign $V_{\{1\}}(z,t) = v_1(z,t)$;
2. For $n = 2, \ldots, N$ obtain $V_{\{1,2,\ldots,n\}}(z,t) = V_{\{1,2,\ldots,n-1\}}(z,t) \underset{+}{\otimes} v_n(z,t)$,

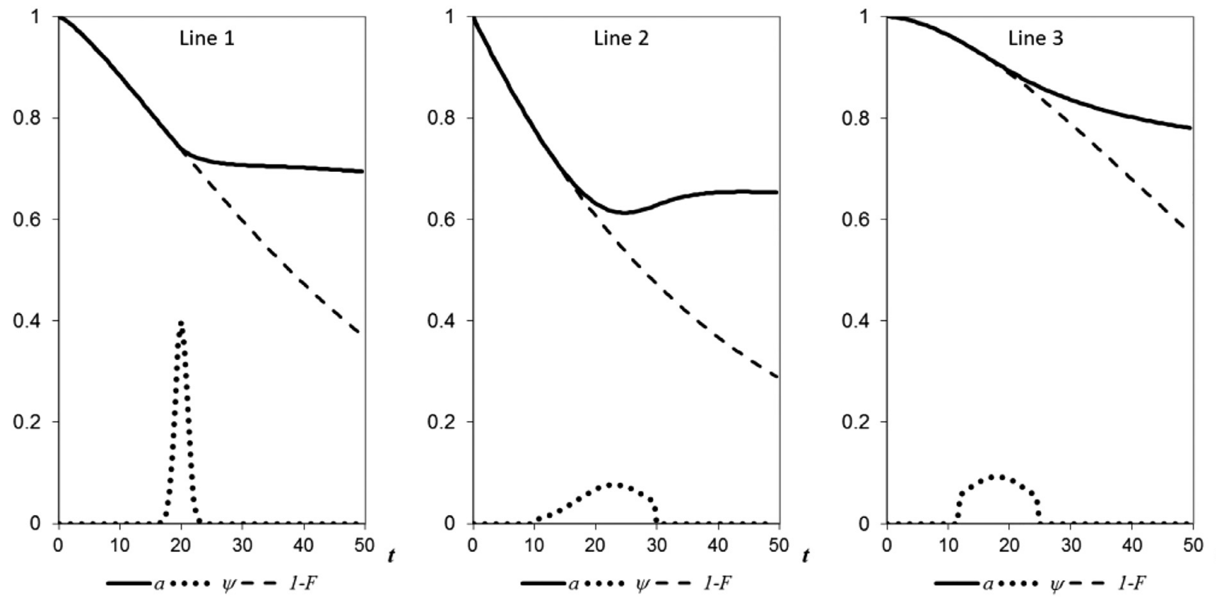

Fig. 3. Instantaneous availability $a(t)$, repair time *pdf* $\psi(t)$ and reliability $1 - F(t)$ of common bus lines.

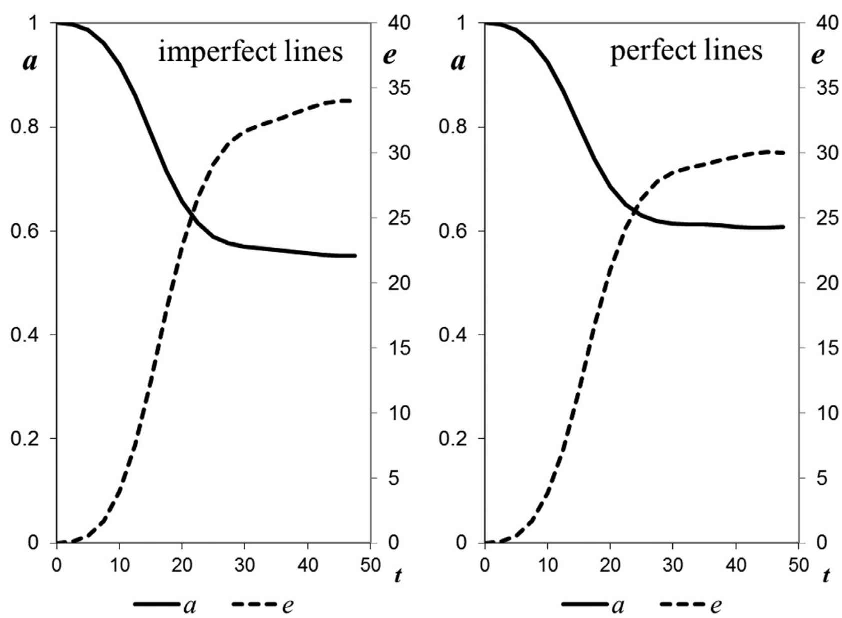

Fig. 4. Instantaneous availability $a(t)$, and expected performance deficiency $e(t)$ of the entire CBPS.

where

$$V_{\{1,\dots,n-1\}}(z,t) \underset{+}{\otimes} v_n(z,t) = \left(\sum_{i=1}^{I} \eta_i(t) z^{a_i,b_i}\right) \underset{+}{\otimes} \left(\sum_{j=1}^{I_n} \gamma_{n,j}(t) z^{s_{n,j},q_{n,j}}\right) = \sum_{i=1}^{I}\sum_{j=1}^{I_n} \eta_i(t)\gamma_{n,j}(t) z^{a_i+s_{n,j},b_i+q_{n,j}}. \tag{21}$$

Following (7), one can obtain the UGF $\Theta_{\{1,2,\dots,M\}}(z,t)$, representing the DSCTP of the entire common bus capacity $C(t) = \sum_{m=1}^{M} C_m(t)$ using the similar recursive procedure:

1. Assign $\Theta_{\{1\}}(z,t) = \tilde{u}_1(z,t)$;
2. For $m = 2, \dots, M$ obtain $\Theta_{\{1,2,\dots,m\}}(z,t) = \Theta_{\{1,2,\dots,m-1\}}(z,t) \underset{+}{\otimes} \tilde{u}_m(z,t)$.

Having the UGF $V_{\{1,2,\dots,n-1\}}(z,t) = \sum_{l=1}^{L} \delta_i(t) z^{\lambda_i,\phi_i}$ representing dependent DSCTPs $S(t)$ and $Q(t)$, and the UGF $\Theta_{\{1,2,\dots,M\}}(z,t) = \sum_{e=1}^{E} \chi_e(t) z^{\vartheta_i}$ representing DSCTP $C(t)$, one can obtain the UGF $U(z,t)$ representing the DSCTPs $\Delta(t)$ of the performance deficiency remained after the performance sharing in accordance with (11). The corresponding composition operator takes the form of

$$U(z,t) = V_{\{1,2,\dots,n-1\}}(z,t) \underset{def}{\otimes} \Theta_{\{1,2,\dots,M\}}(z,t) = \left(\sum_{l=1}^{L} \delta_i(t) z^{\lambda_i,\phi_i}\right) \underset{def}{\otimes} \left(\sum_{e=1}^{E} \chi_e(t) z^{\vartheta_i}\right)$$
$$= \sum_{l=1}^{L}\sum_{e=1}^{E} \delta_i(t)\chi_e(t) z^{\max(0,\phi_i-\min(\lambda_i,\vartheta_i))} = \sum_{h=1}^{H} \xi_h(t) z^{\sigma_h}. \tag{22}$$

The instantaneous system availability (the probability that the performance deficiency does not exist at time t) is equal to the probability that $\Delta(t) = 0$, which corresponds to the coefficient $\xi_h(t)$ of term

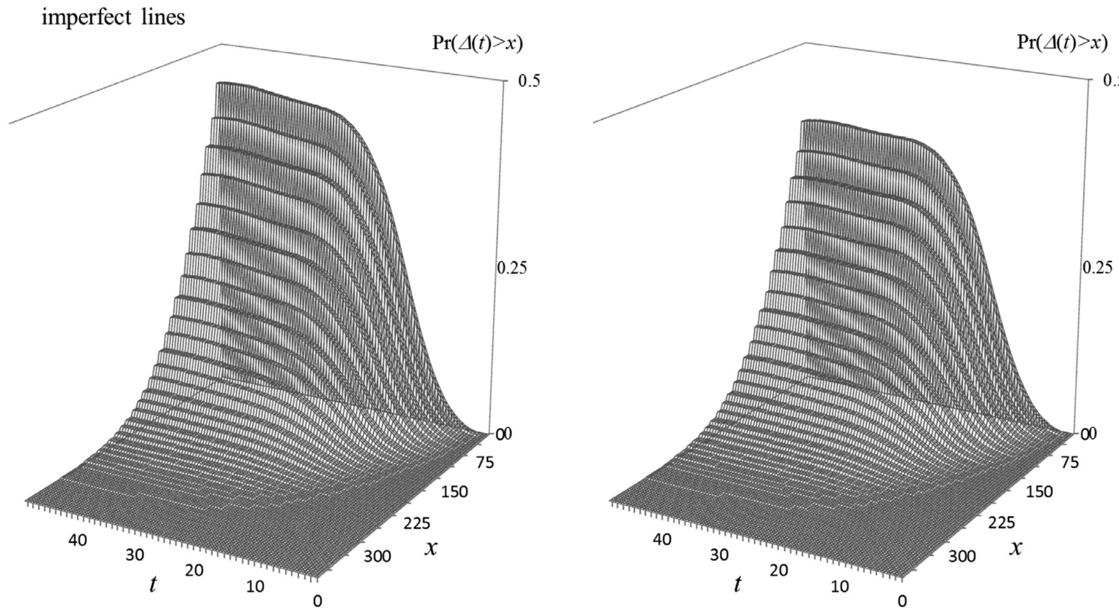

Fig. 5. Dynamic distribution of the entire CBPS performance deficiency $\Pr(\Delta(t) > x)$.

$\xi_h(t)z^0$ in the UGF $U(z,t)$ (22). The expected performance deficiency at time t can be obtained from the UGF (22) as

$$e(t) = \sum_{h=1}^{H} \xi_h(t)\sigma_h. \tag{23}$$

From the description of the procedures presented in this section it can be easily seen that the worst case complexity of the algorithm for determining the expected performance deficiency for each time instance is $O(\prod_{n=1}^{N} 2K_n + 2^M)$,which equals to the maximum number of different state combinations of the components and lines. In practice the total complexity is much lower because different state combinations can produce the same surplus performance and performance deficiency which results in appearance of like terms in the corresponding UGF. Collection of such like terms considerably reduces the algorithm complexity (see [22]).

The verification of the event transition methodology adopted in this work for evaluating component instantaneous availability is presented in [3]. To verify the algorithm designed for the common bus systems, the expected system performance deficiency obtained for extreme case, when the line capacities are zeroed and the system reduces to series multi-state one has been compared with the same index obtained by Monte Carlo simulations. The results produced by the suggested algorithm always fell between the lower and upper bounds corresponding to the 95% confidence intervals obtained in the simulations.

6. Illustrative example

Consider a CBPS with five producing components and three bus lines. It is assumed that the time-to-failure distributions of components and lines are Weibull with scale (η_n) and shape (β_n) parameters [23] presented in Tables 1 and 2. These tables also present the rest of parameters for the system components and lines, including the nominal performance/capacity g_n and c_m, repair efficiency coefficients π_n, parameters of truncated normal distributions of repair time (lower bound, upper bound, mean and standard deviation respectively denoted by $d_n^{\min}$, $d_n^{\max}$, μ_n, σ_n) [24].

Table 3 presents the random demand of each component in the form of vectors of demand values realizations w and corresponding probabilities y. It is assumed that the demand distributions are time independent (probabilities $y_{n,k}(t) = \Pr(W_n(t) = w_{n,k}$ do not depend on time i.e. $y_{n,k}(t) \equiv y_{n,k}$). The considered time horizon is $\tau = 50$ (weeks).

Figs. 2 and 3 present reliability functions $1 - F(t)$, *pdf* of repair times $\psi(t)$ and obtained instantaneous availabilities $a(t)$ of system components and common bus lines. Fig. 4 presents the entire CBPS instantaneous availability and performance deficiency for the cases of lines having parameters presented in Table 2 and perfect lines with unlimited capacity. Fig. 5 presents the dynamic distribution of the CBPS performance deficiency in the form of $\Pr(\Delta(t) > x)$ for the two cases. The expected CBPS availabilities during the time horizon τ for the two cases are $A(\tau) = 0.698$ and $A(\tau) = 0.729$, respectively. The total expected unsupplied demands are $UD(\tau) = 1078.8$ and $UD(\tau) = 972.3$, respectively. It can be seen that the unreliability and limited capacity of the common bus lines affect the performance of the entire system significantly.

Fig. 6 presents the dynamic distribution of the CBPS performance deficiency for several system configurations when the CBPS consists of different groups/subsets of components from Table 1 (with corresponding demands from Table 3). The common bus configuration and parameters remain unchanged for all the examples. It can be seen that the number of the possible realizations of the performance deficiency decreases with a decrease in the number of components. Indeed, the number of different combinations of component performance levels and demand values decreases when fewer components compose the CBPS.

The suggested methodology can be used for comparing effects of possible maintenance improvement actions on the CBPS performance characteristics. For example, Table 4 presents $A(\tau)$ and $UD(\tau)$ indices for cases when the repair time for one of components/lines is improved (by allocating repair facilities nearby) but remains unchanged for the rest of components/lines. It also contains the relative indices improvement $\Delta A(\tau)$ and $\Delta UD(\tau)$ over the case when all the repair times remain unchanged. It is assumed that the improved repair time obeys the truncated normal distribution with parameters $\tilde{d}_m^{\min} = 9$, $\tilde{d}_m^{\max} = 11$, $\tilde{\mu}_m = 10$ and $\tilde{\sigma}_m = 4$. It follows from data in Table 4 that the maintenance improvement of component 2 causes the greatest improvement of the entire CBPS performance. The maintenance improvement of component 1 has greater effects on the CBPS availability than the maintenance improvement of component 4, whereas the maintenance improvement of component 4 has greater effects on the total unsupplied demand than the maintenance improvement of component 1. This can be explained by the fact that component 1 has greater nominal performance, but lower reliability than component 4.

Table 5 presents $A(\tau)$ and $UD(\tau)$ indices for cases when nominal

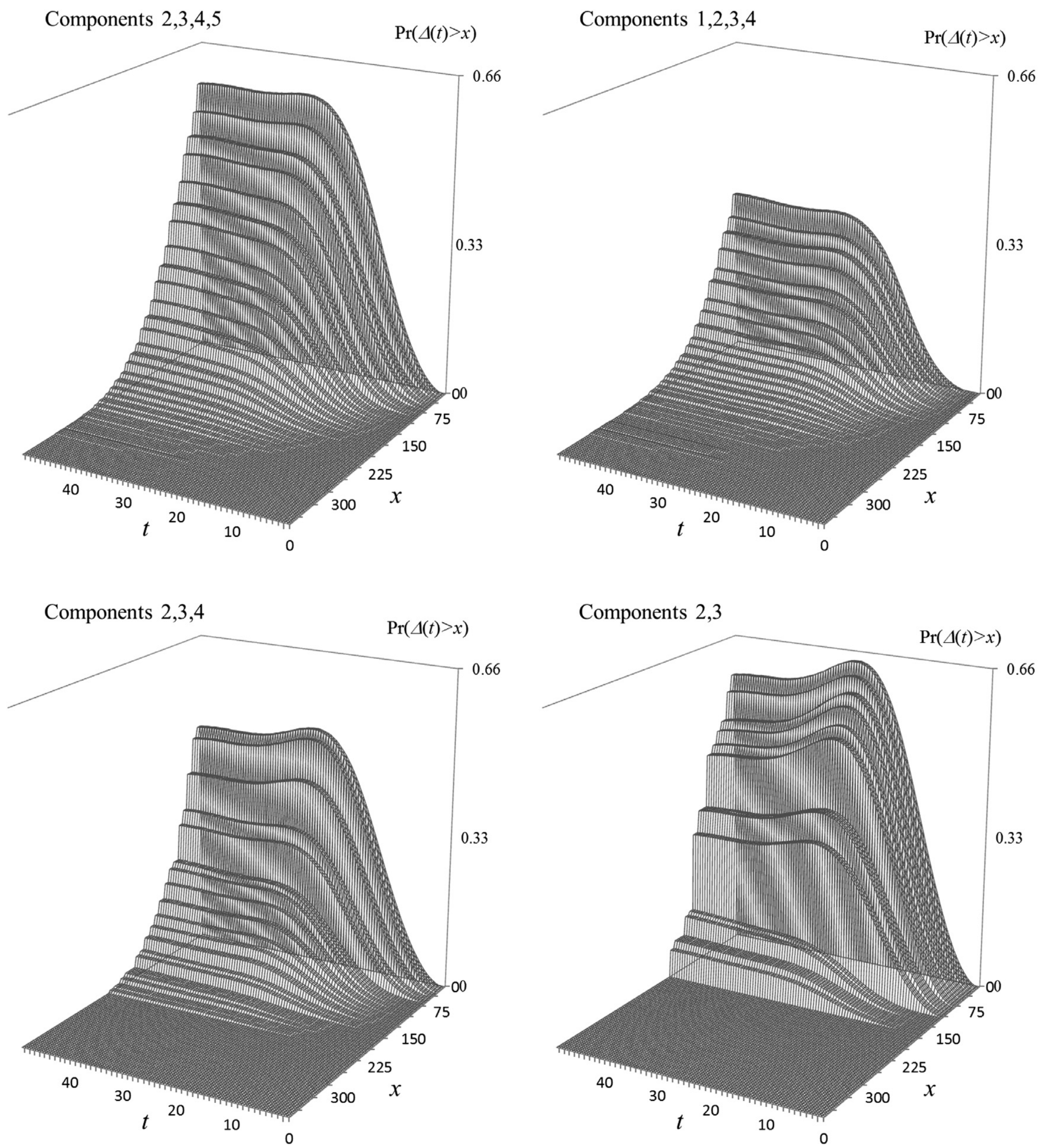

Fig. 6. Dynamic distribution of the entire CBPS performance deficiency $\Pr(\Delta(t) > x)$ for different system configurations (CBPS consisting of four, three and two components).

Table 4
CBPS performance improvement caused by reduction of repair time.

	$A(\tau)$	$UD(\tau)$	$\Delta A(\tau)$ %	$\Delta UD(\tau)$ %
Component 1	0.7244	918.4	3.8	14.9
Component 2	0.7375	906.0	5.7	16.0
Component 3	0.7129	993.2	2.1	7.9
Component 4	0.7236	916.5	3.7	15.0
Component 5	0.7143	991.2	2.3	8.1
Line 1	0.7033	1058.6	0.8	1.9
Line 2	0.7067	1043.2	1.2	3.3
Line 3	0.7070	1044.5	1.3	3.2

Table 5
CBPS performance improvement caused by increase of nominal performance.

	$A(\tau)$	$UD(\tau)$	$\Delta A(\tau)$ %	$\Delta UD(\tau)$ %
Component 1	0.7304	972.4	4.6	9.9
Component 2	0.7093	1048.3	1.6	2.8
Component 3	0.7391	903.2	5.9	16.3
Component 4	0.7334	941.5	5.1	12.7
Component 5	0.7312	935.9	4.8	13.2
Line 1	0.7027	1058.8	0.7	1.9
Line 2	0.7029	1067.9	0.7	1.0
Line 3	0.7015	1071.7	0.5	0.7

performance/transmission capacity of one of components/lines is increased by 20 units, whereas it remains unchanged for the rest of components/lines. It also contains the relative indices improvement $\Delta A(\tau)$ and $\Delta UD(\tau)$ over the case when all the performances/transmission capacities remain unchanged. It can be seen that the increase in the nominal performance of component 3 has the greatest effect on the system performance improvement. The increase in the nominal performance of component 4 affects the system availability greater than the increase for component 5, but affects the unsupplied demand less than component 5 does.

The complex interrelations among demand distributions, performances and reliabilities of components and lines as well as repair time distributions make intuitive ranging the efficiency of maintenance improvement actions impossible. The suggested algorithm can help in making decisions about the priority of maintenance improvement actions.

The running time of the algorithm for evaluating system dynamic performance indices on 3.2 GHz PC is about 100 s.

7. Conclusion and future work

The CBPS model has recently attracted much research attention due to their applications in resource-constrained systems. Most of the existing works assume non-repairable system components. In very few works that consider repairs, either fixed repair time or exponential time-to-repair distribution (or constant repair rate) is assumed for system components and no repair is possible for the common bus performance redistribution system. We contribute to the body of knowledge on CBPS systems by considering both repairable functioning components and repairable common bus performance redistribution system. The proposed methodology is applicable to arbitrary types of time-to-failure and time-to-repair distributions for system components and bus lines. The instantaneous availabilities of system components and common bus lines, instantaneous system availability, instantaneous expected performance deficiency, expected system availability and total expected unsupplied demand during a specified mission time are evaluated for repairable CBPS systems. Examples are provided to demonstrate the application and necessity of the proposed evaluation method in prioritizing maintenance improvement actions for system components and common bus lines.

Based on the proposed evaluation method, one direction of our future work is to formulate and solve the optimal component allocation problem for the considered repairable CBPS system. Another direction is to extend the proposed CBPS model to consider multi-state system components and bus lines that function at several discrete performance levels [25,26]. Empirical studies showed that workloads applied to system components can affect their performance and time-to-failure distributions, and thus the overall system performance [27–29]. Therefore, we are interested in considering component loading for system performance analysis and solving relevant optimization problems. We are also interested in considering common-cause failures with selective effects [30,31] in the analysis and optimization of CBPS systems.

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