Stochastic Security-Constrained Unit Commitment

Lei Wu, Mohammad Shahidehpour, *Fellow, IEEE*, and Tao Li, *Member, IEEE*

***Abstract*—This paper presents a stochastic model for the long-term solution of security-constrained unit commitment (SCUC). The proposed approach could be used by vertically integrated utilities as well as the ISOs in electricity markets. In this model, random disturbances, such as outages of generation units and transmission lines as well as load forecasting inaccuracies, are modeled as scenario trees using the Monte Carlo simulation method. For dual optimization, coupling constraints among scenarios are relaxed and the optimization problem is decomposed into deterministic long-term SCUC subproblems. For each deterministic long-term SCUC, resource constraints represent fuel and emission constraints (in the case of vertically integrated utilities) and energy constraints (in the case of electricity markets). Lagrangian relaxation is used to decompose subproblems with long-term SCUC into tractable short-term MIP-based SCUC subproblems without resource constraints. Accordingly, penalty prices (Lagrangian multipliers) are signals to coordinate the master problem and small-scale subproblems. Computational requirements for solving scenario-based optimization models depend on the number of scenarios in which the objective is to minimize the weighted-average generation cost over the entire scenario tree. In large scale applications, the scenario reduction method is introduced for enhancing a tradeoff between calculation speed and accuracy of long-term SCUC solution. Numerical simulations indicate the effectiveness of the proposed approach for solving the stochastic security-constrained unit commitment.**

***Index Terms*—Lagrangian relaxation, mixed integer program, Monte Carlo simulation, random power outages, scenario aggregation, security-constrained unit commitment, subgradient method, uncertainty.**

NOMENCLATURE:

DR_i	Ramp-down rate limit of unit i.
$E_{e,i}^{ET}(\cdot)$	Emission function of unit i (type ET).
$E_{u,i}^{ET,\max}$	Upper limit of unit i's emission (type ET).
$E_{g,n}^{ET,\max}$	Upper limit of group n's emission (type ET).
$F_{c,itp}(\cdot)$	Production cost function of unit i at time t weekly interval p.
$F_{f,i}(\cdot)$	Fuel consumption function of unit i.
$F_{u,i}^{\min}$	Lower fuel consumption limit of unit i.
$F_{u,i}^{\max}$	Upper fuel consumption limit of unit i.
$F_{g,m}^{\min}$	Lower fuel consumption limit of unit group m.
$F_{g,m}^{\max}$	Upper fuel consumption limit of unit group m.
i	Index of unit.
I_{itp}^{s}	Commitment state of unit i at weekly interval p in scenario s.
l	Index of transmission line.
m	Index of unit fuel group.
n	Index of unit emission group.
NG	Number of units.
NM	Number of fuel groups.
NN	Number of emission groups.
NP	Number of weeks under study (52 weeks).
NT	Number of hours at each weekly interval (168 h).
p	Index of weekly interval.
P_s	Weight of scenario s.
P_{itp}^{s}	Real power generation of unit i at time t at weekly interval p in scenario s.
$P_{D,tp}^{s}$	System demand at time t at weekly interval p in scenario s.
$P_{i,\max}$	Upper limit of real generation of unit i.
$P_{i,\min}$	Lower limit of real generation of unit i.
$PL_{l,\max}$	Maximum capacity of line l.
$PL_{l,tp}^{s}$	Power flow of line l at time t at weekly interval p in scenario s.
S	Number of scenarios.
SU_{itp}^{s}	Startup cost of unit i at time t at weekly interval p in scenario s.
SD_{itp}^{s}	Shutdown cost of unit i at time t at weekly interval p in scenario s.
$SU_{f,itp}^{s}$	Startup fuel consumption of unit i at time t at weekly interval p in scenario s.
$SD_{f,itp}^{s}$	Shutdown fuel consumption of unit i at time t at weekly interval p in scenario s.
$SUET_{e,itp}^{s}$	Startup emission of unit i at time t at weekly interval p in scenario s.
$SDET_{e,itp}^{s}$	Shutdown emission of unit i at time t at weekly interval p in scenario s.
t	Index of time in one week.

Manuscript received June 6, 2006; revised October 1, 2006. Paper no. TPWRS-00345–2006.

The authors are with the Electrical and Computer Engineering Department, Illinois Institute of Technology, Chicago, IL 60616 USA (e-mail: lwu10@iit.edu; ms@iit.edu; litao@iit.edu).

Digital Object Identifier 10.1109/TPWRS.2007.894843

T_i^{off}	Minimum Down time of unit i.
T_i^{on}	Minimum Up time of unit i.
UR_i	Ramp-up rate limit of unit i.
Xon_{itp}^s	On time of unit i at time t at weekly interval p in scenario s.
$Xoff_{itp}^s$	On time of unit i at time t at weekly interval p in scenario s.
$\mathbf{P_{tp}^s}$	Real generation vector at time t at weekly interval p in scenario s.
$\boldsymbol{\gamma}_{\mathbf{tp}}^{\mathbf{s}}$	Phase shifter angle vector at time t at weekly interval p in scenario s.
$\boldsymbol{\gamma}_{\mathbf{max}}$	Upper limit of phase shifter angle vector.
$\boldsymbol{\gamma}_{\mathbf{min}}$	Lower limit of phase shifter angle vector.

I. Introduction

WITH legislative and regulatory mandates for managing electricity restructuring, significant changes are occurring in the electric power industry. Also, the electric power business is rapidly becoming market driven which is based on nodal variations of electricity prices. Meanwhile, the impact of market on energy, environment, and economy requires more efficient, less pollutant, and less costly means of power generation by better managing resource (energy, fuel, and emission) constraints. Accordingly, long-term SCUC could be a proper tool for representing interactions among resource constraints and market objectives [1].

Our proposed stochastic model could be applied to a traditional vertically integrated utility and a centralized energy market (with ISOs). It is quite natural to model long-term fuel and emission constraints for a utility as presented in this paper. In several ISOs such as PJM, MISO, and ISO-NE, SCUC is utilized as a market clearing tool in the day-ahead energy market in which GENCO's energy constraints are modeled. That is, GENCOs submit energy limits which indicate that the sum of daily generation for individual units, or the entire GENCO, would not exceed a prescribed level. The energy constraint is usually a linear function of GENCO's power that is generated by a group of units. In our proposed model, a combination of fuel and emission constraints, which is a quadratic function of power generation by a group of units, represents the energy constraint for a GENCO. In essence, the quadratic model that we are considering is more general than the existing practice of a linear model for representing the energy limit. In this paper, we emphasize the solution of the resource based SCUC model for a vertically integrated utility. However, the same model with an energy constraint could apply to an energy market case.

The solution of long-term SCUC could become cumbersome as we further include system uncertainties such as forced outages of system components and load forecasting inaccuracies. References [2], [3] used a network flow algorithm to solve the fuel dispatch problem and apply a heuristic method for unit commitment. However, as generation constraints become more complicated, such a heuristic method could fail.

Decomposition is a practical method for analyzing unbundled electric power systems as it divides long-term SCUC into a master problem and several tractable subproblems similar to the way the market is set up. A strategy is required to coordinate the master problem with subproblems. Resource allocation targets are used as linking signals to coordinate the mater problem and subproblems for primal approaches and pseudo resource dispatch prices for a dual approach [4]. In the primal method, the master problem allocates long-term fuel consumption, which is directly used by short-term operation scheduling subproblems. The short-term solution would feed back adjustment signals for the reallocation of long-term fuel resource constraints. References [5], [6] used reserve capacity as a coordination indicator between long-term fuel consumption allocation and short-term optimal operation. If the available reserve capacity at any planning interval was less than requested, the long-term solution was updated according to this violation.

Due to operating dynamics of power systems such as the outages of generators and transmission components and uncertainty of system demand, the security of power systems was handled traditionally by providing spinning and operating reserves considering the peak demand case or worst outage case. This is a conservative approach which could lead to a very high cost of operating the power system. Furthermore, outages of multiple components may not be considered in calculating the operating reserve which could also lead to an insecure operation of power systems. Instead of the traditional formulation of system security, we propose a stochastic model in which outages of generators and transmission components as well as uncertainty of system load are simulated as different scenarios. Each scenario would represent a possible system state which would include outages of system components and a possible system demand. The emphasis of this paper is to simulate the impact of uncertainty and the allocation of fuel resources and emission allowance when solving the long term stochastic security- constrained unit commitment problem. The issue of stochastic security will be explored further in our subsequent publications in which probabilistic indexes, such as LOLP (loss of load probability) and ELNS (expected energy not served), will be used for measuring the long-term security of power systems.

[7], [8] used a stochastic model to solve the unit commitment problem without considering transmission security constraints, or fuel and emission limits. [7] used fully and partially random trees to simulate generator failures and [8] created scenarios to simulate generator failures and load forecasting inaccuracies. [8] modeled generator failures by creating scenarios which had demand increases equal to the unavailable generator's capacity; these demand increases were given by approximating the generating capacity loss over a certain period of time. The advantage of the technique was its independence from individual generators, but the approximation neglected the failure in geographical information, and demand increases could even cause the infeasibility of power flows. Furthermore outages of transmission lines should be taken into consideration.

In this paper, we model forced outages of generating units and transmission lines as independent Markov processes, and load forecasting uncertainties as uniform random variables. Different scenarios are generated when stochastic conditions are taken into account. [9] represented an efficient decomposition method for the solution of long-term SCUC. For each scenario, a deterministic long-term SCUC problem is solved and penalty price signals are used to optimize the allocation of fuel consumption and emission allowance based on a hybrid subgradient and Dantzig-Wolfe method. Lagrangian relaxation algorithm is adopted to deal with coupling constraints and to divide the original problem into several tractable subproblems, i.e., short-term SCUC based on mixed-integer program (MIP). Computational requirements for solving scenario-based optimization models depend on the number of scenarios in which the objective is to minimize the weighted-average generation cost over the entire scenario tree. Scenario reduction is adopted in this paper as a tradeoff between computation time and solution accuracy. LMPs calculated from different scenarios reflect the impact of uncertainties. LMPs are strongly influenced by uncertainties which are generally not considered in commercial software packages of SCUC. However, Fig. 9 shows that stochastic LMPs combined with their boundaries provide more accurate economic signals for planning and investment decisions.

The rest of the paper is organized as follows. Section II provides the stochastic formulation of the uncertainty of generation components (units and transmission lines) and the inaccuracy of load forecasting, discusses the scenario aggregation and scenario reduction methods. Section III presents the solution methodology of the stochastic long-term SCUC. Section IV presents and discusses two systems, a 6-bus system and a modified IEEE 118-bus system with 54 units and 91 loads. The conclusion is drawn in Section V.

II. Stochastic Formulation

We consider a set of possible scenarios based on the Monte Carlo simulation method for modeling uncertainties in the long-term SCUC problem. Using the Monte Carlo simulation method we assign a weight P_s to each scenario that reflects the possibility of its occurrence. The Monte Carlo simulation method, scenario aggregation, and scenario reduction technique are presented in Sections II-A–C.

A. Monte Carlo Simulation Method

One of the advantages of the Monte Carlo simulation method is the required number of samples for a given accuracy level is independent of system size which makes it suitable for large scale simulations. We use the Monte Carlo simulation method to simulate the frequency and duration of generator and transmission line outages based on forced outage rates and rates to repair.

Assume the steady state availability of the ith generating unit is p_i and its unavailability is $q_i = 1 - p_i$. Using μ_i and λ_i, we represent the ith component's repair and failure rates in the period respectively, and apply a two-state continuous-time Markov model for representing the ith component. The associated conditional probabilities for component i are defined as follows [6], [11]:

$$p(\varphi_t = 1|\varphi_{t0} = 1) = p_i + q_i * e^{-(\mu_i+\lambda_i)*(t-t0)}$$
$$p(\varphi_t = 0|\varphi_{t0} = 1) = q_i - q_i * e^{-(\mu_i+\lambda_i)*(t-t0)}$$
$$p(\varphi_t = 1|\varphi_{t0} = 0) = p_i - p_i * e^{-(\mu_i+\lambda_i)*(t-t0)}$$
$$p(\varphi_t = 0|\varphi_{t0} = 0) = q_i + p_i * e^{-(\mu_i+\lambda_i)*(t-t0)}.$$

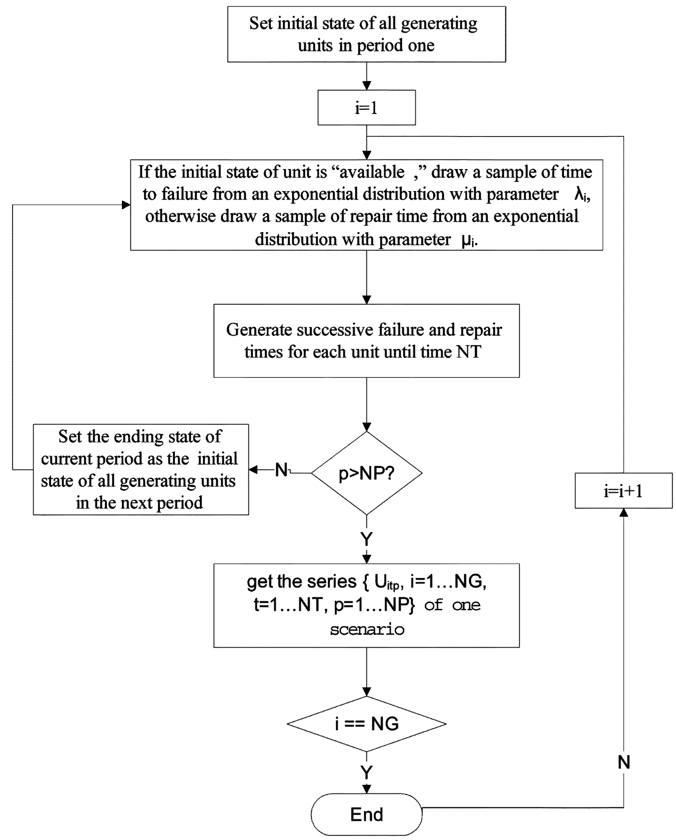

Fig. 1. Flowchart of a unit's state simulation with Monte Carlo method.

We use $\{U_{itp}, i = 1 \ldots NG, t = 1 \ldots NT, p = 1 \ldots NP\}$ in the Monte Carlo simulation method to simulate component availabilities in which $U_{itp} = 1$ indicates that the ith component is available at time t and scheduling period p while $U_{itp} = 0$ indicates otherwise. Fig. 1 depicts the generating unit simulation while a similar procedure can be devised for transmission outages.

A low-discrepancy Monte Carlo simulation method (lattice) is adopted with the possibility of accelerating the convergence rate from $O(1/\sqrt{N})$ associated with ordinary Monte Carlo to nearly $O(1/N)$ where N is the number of sample points generated. Figs. 2 and 3 represent two random number series. Points in Fig. 2 are generated by the ordinary Monte Carlo simulation method and those in Fig. 3 are generated by a rank-1 lattice rule. Accordingly, the random numbers by lattice method would be more evenly distributed. An n-point lattice rule of rank $-r$ in dimension d is defined as follow:

$$\left\{ \sum_{i=1}^{r} \frac{k_i}{n_i} v_i \bmod 1, \quad k_i = 0, 1, \ldots, n_i - 1 \; i = 1, \ldots r \right\}$$

where $v_1, \ldots, v_r$ are linearly independent d-vector of integers. If we draw N independent samples according to the Monte Carlo simulation method, the iteration ends when the relative standard deviation is less than a predefined value (95% relative standard deviation is given as $1.96 * \sigma / sqrt(N)$ where σ is the standard deviation and N is number of simulation). Usually N exceeds several thousands for a relatively small standard deviation. If the low-discrepancy method is used, convergence will

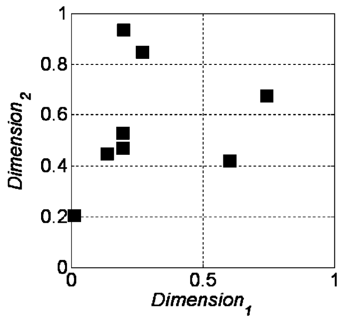

Fig. 2. Random number by Ordinary MC.

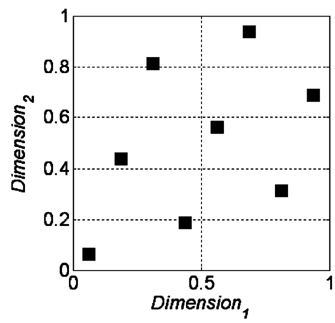

Fig. 3. Random number by Lattice.

be accelerated from $O(1/\sqrt{N})$ to nearly $O(1/N)$ and we can use a relatively smaller number of samples to reach the same convergence [10].

Numerical methods have been adopted for short-term load forecasting. However, forecasting results may be inaccurate due to unusual changes in periodic load demands. Input data for the simulation of outages of generators and transmission lines are forced outage rates and rates to repair, which are independent of representative days/hours in each season. For the load uncertainty simulation, we divide the entire scheduling horizon into several time intervals. For each time interval we create a few scenarios, based on historical data, which are different from the forecasted one. Scenarios in each time interval reflect the representative days/hours chosen for each week/season.

Here again we create multiple scenarios in which probabilities are assigned to occurrences. We divide the entire scheduling period into $N\Gamma$ time units (e.g., four periods based on seasons or several months into several weekly periods). For each time unit we create a few scenarios (say m), based on historical data, that are different from the forecasted one. Then the scenario tree will have $(m^{N\Gamma})$ scenarios, each with a weight of $1/(m^{N\Gamma})$ as shown in Fig. 4. For each scenario we solve a deterministic long-term SCUC problem which is weighted and the final outcome will be an average solution for the long-term SCUC. We further combine simulation samples representing load forecasting inaccuracies with those of component outages in a single Monte Carlo scenario.

B. Scenario Aggregation

The idea of stochastic long-term SCUC is to construct or sample possible options for uncertain circumstances, solve the deterministic long-term SCUC problem for the possible options, and select a good combination of outcomes to represent the stochastic solution.

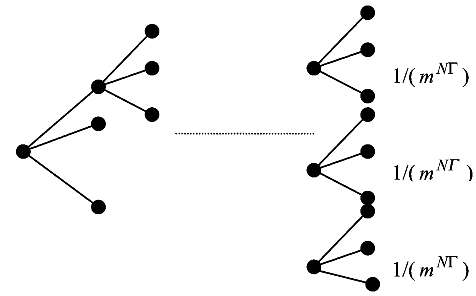

Fig. 4. Scenario tree for the inaccurate load forecasting.

Suppose we have $N\Gamma$ time periods and ξ_h is a vector describing time h, then $\{\xi_0^s, \xi_1^s, \ldots, \xi_{N\Gamma}^s\}$ is a possible scenario. We create a set of scenarios S using the Monte Carlo simulation method where P_s is the possibility of occurrence of s. So the mathematical formulation of the problem for each scenario s is

$$\min P_s \sum_{h=1}^{N\Gamma} f\left(x_h^s, u_h^s, \xi_h^s\right).$$

S.t. system and unit constraints. Here x_h^s, u_h^s, and ξ_h^s represent state variables, decision variables, and scenarios respectively. Accordingly, we have a number of optimal solutions $u^s = (u_0^s, u_1^s, \ldots, u_{N\Gamma}^s)\, s \in S$ where S is the number of scenarios and the possible outcome could be the average solution for each time $h\, \underline{u_h} = \sum_{s\in S} p^s u_h^s$. However, averaging may result in a suboptimal or even infeasible solution for the original stochastic problem [12].

Alternatively, the scenario aggregation method is formulated as follows:

$$\min \sum_{s=1}^{S} P_s \sum_{h=1}^{N\Gamma} f\left(x_h^s, u_h^s, \xi_h^s\right).$$

S.t.

1) Original system and unit constraints for the deterministic problem.
2) Bundle constraint: if two scenarios s and s' are indistinguishable from the beginning to time τ on the basis of information available at time τ, then the decision rendered for the two scenarios must be identical from the beginning to time τ. If $B(s,\tau)$ represents a part of scenario s from the beginning to time τ, then the bundle constraint is given as

$$B(s,\tau) = B(s',\tau) = \Omega_\tau \Rightarrow u_h^s = u_h^{s'} = c_h, h = 1, \ldots, \tau.$$

This problem with coupling constraints for scenarios is solved using the Lagrangian relaxation method, as discussed next, with an implementable decision value defined as

$$c_h = \frac{\sum\limits_{s\in B(s,\tau)=\Omega_\tau} P_s * u_h^s}{\sum\limits_{s\in B(s,\tau)=\Omega_\tau} P_s}.$$

C. Scenario Reduction

Computational requirements for solving scenario-based optimization models depend on the number of scenarios. So an effective scenario reduction method could be very essential for solving large scale systems. The reduction technique is a scenario-based approximation with a smaller number of scenarios and a reasonably good approximation of original system. So we determine a subset of scenarios and a probability measure based on this subset that is the closest to the initial probability distribution in terms of probability metrics. Our scenario reduction technique would control the goodness-of-fit of approximation by measuring a distance of probability distributions as a probability metric. Efficient algorithms based on backward and fast forward methods are developed that determine optimal reduced measures. Simultaneous backward and fast forward reduction methods are briefly introduced here as follows [14], [15]. Let ξ_s $(s = 1, \ldots, N)$ denote N different scenarios, each with a probability of p_s, and $DT_{s,s'}$ be the distance of scenario pair (s, s'). The simultaneous backward reduction includes the following steps:

Step 1: Set S is the initial set of scenarios; DS is the scenarios to be deleted. The initial DS are null. Compute the distances of all scenario pairs: $DT_{s,s'} = DT(\xi_s, \xi_{s'}),\ s, s' = 1, \ldots, N$;

Step 2: For each scenario k, $DT_k(r) = \min DT_{k,s'},\ s', k \in S$ and $s' \neq k$, r is the scenario index that has the minimum distance with scenario k.

Step 3: Compute $PD_k(r) = p_k * DT_k(r),\ k \in S$. Choose d so that $PD_d = \min PD_k,\ k \in S$.

Step 4: $S = S - \{d\},\ DS = DS + \{d\};\ p_r = p_r + p_d$.

Step 5: Repeat steps 2–4 until the number to be deleted meets the request.

The fast forward reduction method is described as follows.

Step 1: Set S is the set of all initial scenarios; DS is the scenarios to be deleted. The initial DS are null. Compute the distances of all scenario pairs: $DT_{s,s'} = DT(\xi_s, \xi_{s'}),\ s, s' = 1, \ldots, N$.

Step 2: Compute $PD_k(r) = \sum_{u \neq k} p_u * DT(\xi_u, \xi_k),\ k = 1, \ldots, N$; Choose d so that $PD_d = \min PD_k,\ k = 1, \ldots, N$.

Step 3: $S = S - \{d\},\ DS = DS + \{d\};\ p_r = p_r + p_d$;

Step 4: Repeat steps 2 to 4 until the number of scenarios to be deleted meets the target.

The General Algebraic Modeling System (GAMS) is used in this study. GAMS provides a tool called SCENRED for scenario reduction and modeling random data processes. The backward reduction, fast forward reduction, and other methods for large scenario reductions are included in the SCENRED library. These scenario reduction algorithms provided by SCENRED determine a scenario subset (of prescribed cardinality or accuracy) and assign optimal probabilities to the preserved scenarios [15]. The computational time for the reduction process from 100 scenarios with 241 dimensions in each scenario to 12 scenarios is about five minutes. The numerical tests are executed on a 3.1 GHz personal computer.

III. Stochastic Long-Term SCUC Problem

A. Stochastic Long-Term SCUC

The formulation of stochastic SCUC problem differs from that of deterministic since spinning and operating reserve constraints are relaxed in the latter case. A specific amount of spinning and operating reserves are considered in the deterministic SCUC problem in order to maintain the security of power system operation when outages occur or actual hourly demands increase unexpectedly. In the stochastic SCUC problem, each possible system state is represented by a scenario in which equipment outages and possible load increases are represented in the SCUC solution and reserve constraints are relaxed accordingly. It was shown in [7] and [8] that the reserve constraint could be relaxed because of the specific consideration of each potential contingency in the stochastic model. That is, a specific commitment schedule is determined in each scenario for each potential contingency. In our stochastic model, we do not enforce the reserve constraint explicitly. However, it does not mean that there is no reserve in the system; reserve is implicitly determined by a specific commitment schedule corresponding to each potential contingency.

The Monte Carlo simulation method is adapted to simulate random characteristics of power systems and scenario aggregation technology is used to solve the stochastic long-term SCUC problem. A major advantage of scenario aggregation technique is that individual scenario problems become simple to interpret, and with aggregation technique the underlying problem structure is preserved.

The objective of stochastic long-term SCUC problem is described as follows:

$$\min \sum_{s=1}^{S} P_s \sum_{p=1}^{NP} \sum_{t=1}^{NT} \sum_{i=1}^{NG} \left[F_{c,itp}\left(P_{itp}^s * I_{itp}^s\right) + SU_{itp}^s + SD_{itp}^s \right]. \quad (1)$$

The objective function (1) is composed of production cost as well as startup and shutdown costs of individual units at all periods. The concept of utilizing scenarios adds another dimension to the SCUC solution that is different from the deterministic long-term SCUC solution. The generation constraints, fuel and emission constraints, network security constraints, and coupling bundle constraints for different scenarios are presented as follows [9], [16], [17].

Generation constraints include the system power balance

$$\sum_{i=1}^{NG} P_{itp}^s * I_{itp}^s = P_{D,tp}^s \quad \forall\, t,\ \forall\, p,\ \forall\, s. \quad (2)$$

Ramping up and down constraints

$$\begin{aligned} P_{itp}^s - P_{i(t-1)p}^s \le& \left[1 - I_{itp}^s\left(1 - I_{i(t-1)p}^s\right)\right] UR_i \\ &+ I_{itp}^s\left(1 - I_{i(t-1)p}^s\right) P_{i,\min} \\ P_{i(t-1)p}^s - P_{itp}^s \le& \left[1 - I_{i(t-1)p}^s\left(1 - I_{itp}^s\right)\right] DR_i \\ &+ I_{i(t-1)p}^s\left(1 - I_{itp}^s\right) P_{i,\min} \\ &\forall\, i,\ \forall\, t,\ \forall\, p,\ \forall\, s. \end{aligned} \quad (3)$$

Minimum up and down time constraints

$$\begin{aligned} &\left[X\mathrm{on}_{i(t-1)p}^s - T_i^{\mathrm{on}}\right] * \left(I_{i(t-1)p}^s - I_{itp}^s\right) \ge 0 \\ &\left[X\mathrm{off}_{i(t-1)p}^s - T_i^{\mathrm{off}}\right] * \left(I_{itp}^s - I_{i(t-1)p}^s\right) \ge 0 \\ &\quad \forall\, i,\ \forall\, t,\ \forall\, p,\ \forall\, s. \end{aligned} \quad (4)$$

Real power generation limits

$$P_{i,\min} I_{itp}^{s} \le P_{itp}^{s} \le P_{i,\max} I_{itp}^{s} \quad \forall\, i,\, \forall\, t,\, \forall\, p,\, \forall\, s. \tag{5}$$

Fuel constraints for individual and groups of units in a GENCO

$$F_{u,i}^{\min} \le \sum_{p=1}^{NP}\sum_{t=1}^{NT}\left[F_{f,i}\left(P_{itp}^{s}\right)*I_{itp}^{s} + SU_{f,itp}^{s} + SD_{f,itp}^{s}\right] \le F_{u,i}^{\max} \quad \forall\, i,\, \forall\, s \tag{6}$$

$$F_{g,i}^{\min} \le \sum_{i\in m}\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left[F_{f,i}\left(P_{itp}^{s}\right)*I_{itp}^{s} + SU_{f,itp}^{s} + SD_{f,itp}^{s}\right] \le F_{g,i}^{\max} \quad \forall\, m,\, \forall\, s. \tag{7}$$

Emission constraints for individual and groups of units in a GENCO

$$\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left[E_{e,i}^{ET}\left(P_{itp}^{s}\right)*I_{itp}^{s} + SUET_{e,itp}^{s} + SDET_{e,itp}^{s}\right] \le E_{u,i}^{ET,\max} \quad \forall\, i,\, \forall\, s, \quad ET = \{SO_2, NO_x\} \tag{8}$$

$$\sum_{i\in n}\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left[E_{e,i}^{ET}\left(P_{itp}^{s}\right)*I_{itp}^{s} + SUET_{e,itp}^{s} + SDET_{e,itp}^{s}\right] \le E_{g,i}^{ET,\max} \quad \forall\, n,\, \forall\, s, \quad ET = \{SO_2, NO_x\}. \tag{9}$$

In a vertically integrated utility, resource (fuel consumption and emission allowance) constraints are formulated as (8) and (9). Since the proposed method can also be applied to energy markets (ISOs), the ISO's energy constraint which is submitted by GENCOs will be a special case of (8) plus (9) as discussed in the introduction.

DC network security constraints (10) and phase shifter angles limits (11)

$$-PL_{l,\max} \le PL_{l,tp}^{s}\left(\mathbf{P_{tp}^{s}}, \boldsymbol{\gamma_{tp}^{s}}\right) \le PL_{l,\max} \quad \forall\, l, \forall\, t, \forall\, p, \forall\, s \tag{10}$$

$$\boldsymbol{\gamma_{\min}} \le \boldsymbol{\gamma_{tp}^{s}} \le \boldsymbol{\gamma_{\max}} \quad \forall\, t, \forall\, p, \forall\, s. \tag{11}$$

Coupling constraints for scenarios

$$B(s,P,\tau) = B(s',P,\tau) = \Omega_{P\tau} \Rightarrow I_{itp}^{s} = I_{itp}^{s'} = c_{itp}, \quad i = 1\ldots NG, (p,t) \in \{P,\tau\}. \tag{12}$$

In (12), $B(s,P,\tau)$ represents a decision part of scenario s from the beginning to time τ in the Pth period. Constraint (12) indicates that if two scenarios s and s' are indistinguishable from the beginning to time τ in the Pth period on the basis of information available at time t, then the decision made for scenario s (here decision includes the units commitment status) must be the same as that of scenario s' from the beginning to time τ in the Pth period.

Accordingly, the objective function (1) combined with a set of constraints (2)–(12) constitutes a stochastic long-term SCUC problem. We find that (2)–(11) are constraints that are related to individual scenarios and only (12) is the coupling constraint connecting different scenarios. We solve the stochastic SCUC problem with Lagrangian relaxation method. A Lagrangian multiplier μ_{itp}^{s} is associated with constraints on each I_{itp}^{s}. The corresponding penalty term $\mu_{itp}^{s}(I_{itp}^{s} - c_{itp})$ is added to the objective function to relax the constraint (12). Then the original problem is separated into S disjoint problems each corresponding to a scenario for deterministic long-term SCUC problem (13) with constraints (2)–(11).

$$\min P_s\left\{\sum_{p=1}^{NP}\sum_{t=1}^{NT}\sum_{i=1}^{NG}\left[F_{c,itp}\left(P_{itp}^{s}*I_{itp}^{s}\right) + SU_{itp}^{s} + SD_{itp}^{s}\right] + \sum_{(s,p,t)\in\{B(S,P,\tau)\}}\sum_{i=1}^{NG}\mu_{itp}^{s}\left(I_{itp}^{s} - c_{itp}\right)\right\}$$
$$s = (1,\ldots,S). \tag{13}$$

The advantages of c_{itp} is that it is implementable, assumed to be the weighted average of the decision made, and represents an optimal solution [12]. The value of c_{itp} which meets $B(s,P,\tau) = B(s',P,\tau)$, is the same for all scenarios and given as

$$c_{itp} = \frac{\sum\limits_{s\in B(s,P,\tau)=\Omega_{P\tau}} P_s * I_{itp}^{s}}{\sum\limits_{s\in B(s,P,\tau)=\Omega_{P\tau}} P_s}.$$

The initial scenario solutions I_{itp}^{s} are expected to be different before Lagrangian multipliers μ_{itp}^{s} are adjusted properly. Finally, scenario solutions I_{itp}^{s} and implementable solutions c_{itp} will converge. However, since the problem is not convex, there is no guarantee that the sequence generated by the scenario aggregation method would entirely converge, i.e., all scenario solutions I_{itp}^{s} and implementable solutions c_{itp} will be equal. Thus approximate scenario solutions are calculated for a fast convergence while maintaining a relatively high accuracy as seen in the case studies.

Starting with initial penalty multiplier μ_{itp}^{s}, S deterministic long-term SCUC problems are solved. If the results satisfy constraint (12), the solution is feasible and we stop. Otherwise, penalty multipliers are updated by the subgradient method and the process is repeated as shown in Fig. 5. As discussed earlier, constraint (12) is not expected to be entirely satisfied. Thus bundle constraint (12) must satisfy the stopping criterion which is stated as the weighted total violation of bundle constraints to be below a certain threshold.

B. Deterministic Long-Term SCUC

The Lagrangian relaxation method is used for the solution of deterministic long-term SCUC (13) with constraints (2)–(11). Coupling constraints (6)–(9) are relaxed and dualized into the objective function (13) with non-negative Lagrangian multipliers.

The relaxed coupling constraints transform the original problem into easier-to-solve subproblems, each of which is a traditional short-term SCUC problem at a certain period without fuel and emission allowance constraints. The Lagrangian relaxation problem is formulated as (14) subject to constraints (2)–(5) and (10)–(11).

For given values of multipliers, the Lagrangian function (14) is decomposed into NP weekly short-term subproblems subject

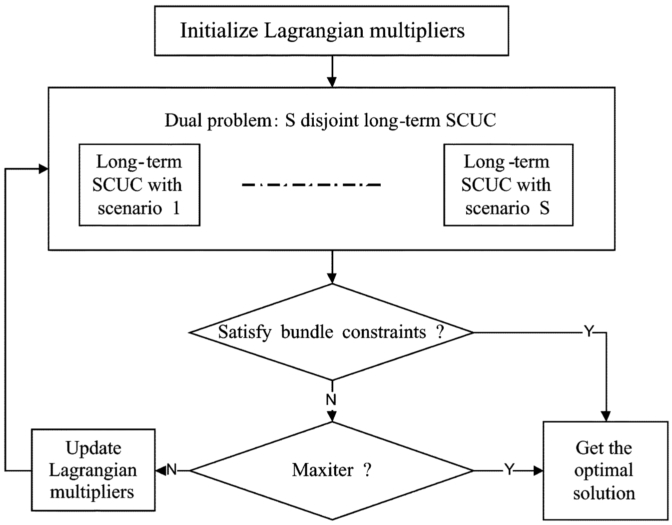

Fig. 5. Flowchart of stochastic long-term SUCU problem.

to constraints (2)–(5) and (10)–(11). Here, we consider the linkage of successive weekly intervals for constraints (3) and (4). Accordingly, the short-term SCUC for the first week is solved by a MIP-based method which provides the initial status for the short-term SCUC in the second week, and so on. See (14) at the bottom of the page.

Various methods could be applied to solve the Lagrangian dual problem. [9] presents an efficient hybrid method based on subgradient and Dantzig-Wolfe decomposition for the solution of deterministic long-term SCUC problem. The hybrid method is adopted to achieve the tradeoff between calculation speed and exact solution. It divides the original problem into a master problem and several subproblems. Long-term fuel and emission constraints are considered in the master problem as coupling constraints over a schedule horizon, pseudo penalty price signals for fuel and emission are calculated and used directly by sub-problems. The sub-problem solutions would feed back adjustment signals for the reallocation of long-term fuel and emission constraints. The numerical examples presented in Section IV will show that an appropriate and near-optimal solution is achieved based on our proposed simulation method for the long-term power system operation.

IV. Numerical Examples

A 6-bus system, the IEEE 118-bus system, and an 1168-bus system are considered in this section to demonstrate the proposed approach for solving the stochastic long-term SCUC with multiple fuel and emission constraints in which the impact of equipment outages and load uncertainties are included.

A. The 6-Bus System

The 6-bus system in Fig. 6 is used to illustrate the proposed method. The system has three units and seven transmission lines as shown in Tables I and II. Unit 1 is coal-fired while unit 2 and 3 are oil-fired and gas-fired, respectively. The last two columns in Table I give the mean up time and mean down time of three units, which show that the larger the generating capacity, the more frequent is the generating unit's outage.

The forced outage rate of a generation unit is calculated as $((\text{mean up time})/(\text{mean up time} + \text{mean down time}))$.

$$\begin{aligned}
\min P_s \Bigg\{ & \sum_{p=1}^{NP}\sum_{t=1}^{NT}\sum_{i=1}^{NG} \left[F_{c,itp}\left(P_{itp}^s * I_{itp}^s\right) + SU_{itp}^s + SD_{itp}^s\right] \\
& + \sum_{i=1}^{NG} \overline{\lambda}_{u,i} \left[\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left(F_{f,itp}\left(P_{itp}^s * I_{itp}^s\right) + SU_{itp}^s + SD_{itp}^s\right) - F_{u,i}^{\max}\right] \\
& - \sum_{i=1}^{NG} \underline{\lambda}_{u,i} \left[\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left(F_{f,itp}\left(P_{itp}^s * I_{itp}^s\right) + SU_{itp}^s + SD_{itp}^s\right) - F_{u,i}^{\min}\right] \\
& + \sum_{m=1}^{NM} \overline{\lambda}_{g,m} \left[\sum_{i\in m}\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left(F_{f,itp}\left(P_{itp}^s * I_{itp}^s\right) + SU_{itp}^s + SD_{itp}^s\right) - F_{g,m}^{\max}\right] \\
& + \sum_{m=1}^{NM} \underline{\lambda}_{g,m} \left[\sum_{i\in m}\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left(F_{f,itp}\left(P_{itp}^s * I_{itp}^s\right) + SU_{itp}^s + SD_{itp}^s\right) - F_{g,m}^{\min}\right] \\
& + \sum_{i=1}^{NG}\sum_{ET} \overline{\mu}_{u,i}^{ET} \left[\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left(E_{e,i}^{ET}\left(P_{itp}^s * I_{itp}^s\right) + SUET_{e,itp}^s + SDET_{e,itp}^s\right) - E_{u,i}^{ET,\max}\right] \\
& + \sum_{n=1}^{NN}\sum_{ET} \overline{\mu}_{g,n}^{ET} \left[\sum_{i\in n}\sum_{p=1}^{NP}\sum_{t=1}^{NT}\left(E_{e,i}^{ET}\left(P_{itp}^s * I_{itp}^s\right) + SUET_{e,itp}^s + SDET_{e,itp}^s\right) - E_{g,n}^{ET,\max}\right] \\
& + \sum_{(s,p,t)\in\{B(S,P,\tau)\}} \sum_{i=1}^{NG} \mu_{itp}^s \left(I_{itp}^s - c_{itp}\right) \Bigg\} \quad s = (1,\ldots,S)
\end{aligned} \tag{14}$$

TABLE I
GENERATOR DATA

U	Bus No.	Unit Cost Coefficients			Pmax (MW)	Pmin (MW)	Ini. St. (h)	Min Dn (h)	Min Up (h)	Ramp (MW/h)	Start Up (MBtu)	Fuel Price ($/MBtu)	Mean Up Time (h)	Mean Down Time (h)
		a (MBtu)	b (MBtu/MWh)	c (MBtu/MW^2h)										
G1	1	176.95	13.51	0.00045	220	100	4	4	4	55	100	1.2469	23.6	0.4
G2	2	129.97	32.63	0.00100	100	10	2	3	2	50	200	1.2461	23.7	0.3
G3	6	137.41	17.69	0.00500	100	10	1	1	1	20	0	1.2462	23.8	0.2

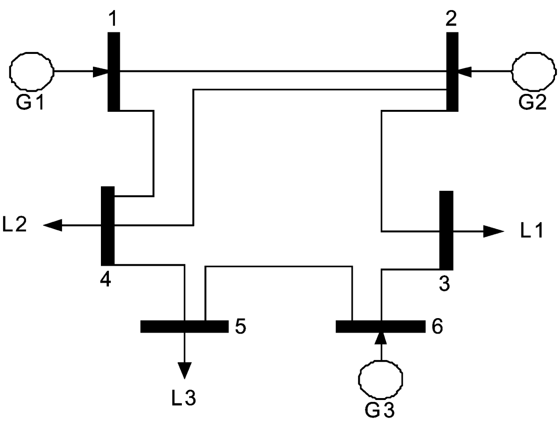

Fig. 6. One-line diagram of six-bus example.

TABLE II
TRANSMISSION LINE DATA

Line No.	From	To	X(pu)	Flow Limit MW)	Mean Up Time (h)	Mean Dn Time (h)
1	1	2	0.170	200	23.5	0.5
2	2	3	0.037	100	23.7	0.3
3	1	4	0.258	100	23.6	0.4
4	2	4	0.197	100	23.6	0.4
5	4	5	0.037	100	23.7	0.3
6	5	6	0.140	100	23.6	0.4
7	3	6	0.018	100	23.8	0.2

TABLE III
WEEKLY PEAK LOAD AS PERCENTAGE OF ANNUAL PEAK LOAD

Week	1	2	3	4
Peak Load	88.0%	90.0%	86.2%	84.5%

TABLE IV
WEIGHT OF EACH SCENARIO AFTER SCENARIO REDUCTION

Scenario	1	2	3	4	5	6
Weight	0.03	0.02	0.03	0.03	0.01	0.01
Scenario	7	8	9	10	11	12
Weight	0.82	0.01	0.01	0.01	0.01	0.01

TABLE V
LONG-TERM GROUP FUEL CONSUMPTION CONSTRAINTS

Fuel	Take-or-Pay Contract(MBtu)	Available(MBtu)
COAL(FGP1)	1,500,000	1,800,000
OIL(FGP2)	-	UNLIMITED
GAS(FGP3)	-	UNLIMITED

TABLE VI
LONG-TERM GROUP EMISSION ALLOWANCE CONSTRAINTS (LBS)

Emission (lbs)	EGP1	EGP2	EGP3
SO_2	400,000	10,000	200,000
NO_X	200,000	5,000	100,000

The mean up time and mean down time of transmission lines are also shown in the last two columns of Table II. The system is tested for a four-week case when the annual peak load is 300 MW and the weekly peak load as a percentage of annual peak loads is listed in Table III.

Three cases are studied to illustrate the impact of component outages and load uncertainty:

Case 1: This is the based case without considering uncertainties. Forced outage rates of system equipment are assumed to be zero. The system load is given in Table III without considering uncertainties.

Case 2: Forced outages of generators and transmission lines are simulated based on the frequency and duration method discussed in Section II. The system load is given in Table III without considering uncertainties. 100 scenarios are created using the low-discrepancy Monte Carlo simulation method with each scenario representing outages of generation units and transmission lines.

Case 3: Component outages and load uncertainties are both taken into account. The simulation of component outages is the same as that in Case 2. Load forecasting inaccuracies are represented by 5% fluctuations in the weekly peak load forecast at the beginning of second week. Consequently, there are $3^3 = 27$ demand scenarios within the four-week horizon (see details in Fig. 4) with a weight of 1/27 for each one. One load curve is selected for each scenario based on the lattice rule.

The computation time for the scenario-based problem depends on the number of scenarios. The scenario reduction method would reduce the total number of scenarios from 100 to 12 as a tradeoff between calculation speed and solution accuracy. Table IV shows the weights of each scenario after reduction. Numerical tests show that after 90% reduction, relative errors in generation cost, fuel consumption and emission level are about 2%. The long-term group fuel consumption and emission allowance constraints, which are the same for scenarios in all cases, are listed in Tables V and VI. Generating unit 1 has fuel supply and emission allowance constraints, while units 2 and 3 have emission allowance constraints.

Fuel consumption, emission level, and cost for scenarios are listed in Tables VII–IX, respectively. In those tables, "EXP" is the expected value of scenarios based on the $\pm 95\%$

TABLE VII
RESULTS OF BASE CASE (CASE 1)

Operation Cost (K$)		2966.51	
Fuel(MBtu)	Coal	Oil	Gas
	1,754,639	2,901	698,537
Emission (lbs)	SO_2(EGp1)	SO_2(EGp2)	SO_2(EGp3)
	350,927	29	139,707
Emission (lbs)	NO_x(EGp1)	NO_x(EGp2)	NO_x(EGp3)
	175,446	15	69,148

TABLE VIII
FUEL CONSUMPTION FOR EACH SCENARIO OF CASE 2

Fuel (MBtu)	Coal	Oil	Gas
Scenario 1	1,682,039	63,271	723,717
Scenario 2	1,636,763	126,814	740,490
Scenario 3	1,646,925	102,979	743,004
Scenario 4	1,630,025	104,166	767,409
Scenario 5	1,642,508	113,815	758,595
Scenario 6	1,601,166	147,724	795,756
Scenario 7	1,626,930	98,554	769,812
Scenario 8	1,622,357	112,240	781,296
Scenario 9	1,643,192	85,151	768,813
Scenario 10	1,624,759	124,201	779,666
Scenario 11	1,602,688	122,210	804,013
Scenario 12	1,697,611	120,398	779,875
EXP	1,629,930 ±27,726	99,720 ±1,965	767,759 ±12,737
Relative Error (%)	1.701	1.971	1.659

TABLE IX
EMISSION LEVEL OF CASE 2

Emission (lbs)	SO_2 (EGp1)	SO_2 (EGp2)	SO_2 (EGp3)
EXP	325,986 ±5,544	997 ±21	153,511 ±2,481
Relative Error (%)	1.701	2.069	1.616
Emission (lbs)	NO_x (EGp1)	NO_x (EGp2)	NO_x (EGp3)
EXP	162,971 ±2,772	510 ±10	75,962 ±1,234
Relative Error (%)	1.701	1.954	1.624

confidence interval. The "Relative Error" is calculated as $((95\% \text{ confidence interval})/\text{expected value}) * 100\%$.

The base case results are presented in Table VII. Without considering outages and load forecasting inaccuracies, large and economical coal-burning unit (generation unit 1) is dispatched as much as possible to supply the system load while the more expensive units 2 and 3 are dispatched to balance the system load. Table VIII shows the fuel consumption in Case 2. The consumptions of coal and gas do not change much from one scenario to another because the generation dispatch of units 1 and 3 is rather fixed for the entire scheduling period. However, the oil consumption of unit 2 changes dramatically from one scenario to another because of changes in unit dispatch. Table IX shows the consumption of emission allowance in Case 2.

When component outages are taken into account, the economical coal-burning unit 1 cannot supply the system load entirely, thus more expensive units 2 and 3 have to be dispatched during certain periods to supply loads with the possibility of load shedding. Accordingly, the expected consumption of oil and gas in Case 2 are higher than those in Case 1. The $\pm$ values in Table IX show the emission consumption in 95% confidence intervals which are calculated based on scenarios. The 95% confidence interval would reflect the uncertainty of power system

TABLE X
FUEL CONSUMPTION OF CASE 3

	Coal	Oil	Gas
EXP of Fuel Consumption (MBtu)	1,664,683 ±2,151	107,696 ±2,088	796,860 ±3,318
Relative Error (%)	0.129	1.937	0.416
EXP of SO_2 Emission (lbs)	SO_2 (EGp1)	SO_2 (EGp2)	SO_2 (EGp3)
	332,937 ±430	1,077 ±21	159,372 ±664
Relative Error (%)	0.129	1.938	0.416
EXP of NO_x Emission (lbs)	NO_x (EGp1)	NO_x (EGp2)	NO_x (EGp3)
	166,452 ±215	551 ±11	78,857 ±327
Relative Error (%)	0.129	1.932	0.415

TABLE XI
TOTAL COST AND LOAD SHEDDING IN CASES 2 AND 3

	Case 2		Case 3	
	Operation Cost ($K)	Load Shedding (MWh)	Operation Cost ($K)	Load Shedding (MWh)
EXP	3038.89 ±49.46	1353.588 ±22.12	3129.59 ±50.95	1498.741 ±49.46
Relative Error (%)	1.627	1.634	1.628	1.046

operation. For instance, the $325{,}986 \pm 5{,}544$ in Table IX shows that 5% of SO_2 emission would be beyond the given interval of $\pm 5{,}544$. The smaller the confidence interval, the more precise the expected value is. In essence, we expect that real time solution to be closer to expected value when the interval is smaller.

Table X shows the expected consumption of fuel and emission allowance in Case 3. Comparison of Cases 2 and 3 shows the inclusion of demand uncertainty results in a more accurate allocation of fuel and emission allowance, measured by relative errors. Table XI shows the expected operating cost and load shedding in Cases 2 and 3. Compared to Case 1, operating cost in Case 2 is relatively higher because the economical unit 1 cannot supply the load entirely due to possible outages. Meanwhile, involuntary load shedding measures are considered. Considering load uncertainties in Case 3, operating cost and load shedding are larger because of upswing in load fluctuations. The fact that relative errors in fuel and emission allowance consumptions and operating cost in Cases 2 and 3 are less than 2% reveals the efficiency of the proposed method. The stochastic approach considering uncertain states of system components and loads could provide a more accurate solution for allocating fuel and emission allowance consumptions.

Fig. 7 depicts the LMP at bus 1 in the 6th day of first week in three cases. In the base case no contingencies are taken into account. Hence bus LMPs (including generation marginal cost, congestion cost and cost of marginal losses) are the same at any instance and equal to the marginal cost of the most expensive dispatched unit (marginal unit). When considering outages of generating units and transmission lines in Case 2, possible transmission congestions could alter LMPs which may be much higher than those of the base case. Furthermore, when adding demand uncertainties in Case 3, LMPs change slightly from those in Case 2.

Compared to Case 1, expected LMPs in Cases 2 and 3 between hours 11 and 16 are much higher because the base unit

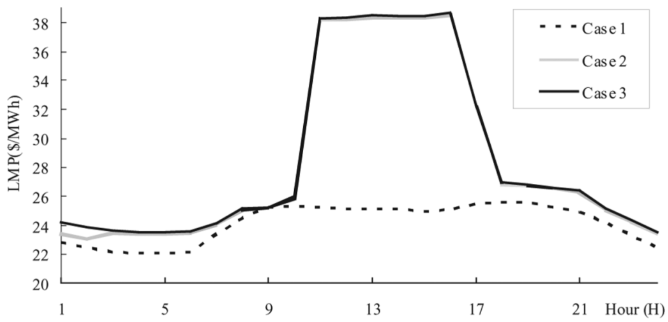

Fig. 7. LMPs of bus 1 in the 6th day of the first week.

(coal-fired unit 1 with a minimum off time of 4 h) is not available at hour 12 and will remain off until hour 16, and transmission lines 1–2 and 2–4 are on outage at hours 15 and 16. The congestion is incurred at line 1–4, so the LMPs are much higher than those of the base case during hour 11 to 16. In spite of high load shedding cost (set at around \$1000 which are much higher than the generating dispatch cost), load shedding will be necessary for managing the security. This example is the tradeoff between security and economics as we take stochastic characteristics of power systems into consideration.

In each scenario, a deterministic long-term SCUC problem is solved in ten iterations for managing long-term fuel and emission allowance constraints. The outer iteration is repeated five times to meet the scenario bundle constraints. The CPU time consumed for the total twelve scenarios is about 3.6 h on a 3.1-HGz personal computer. However, parallel computing used for each scenario, could reduce the CPU time to that of solving a single deterministic long-term problem. If outer iterations for scenario bundle constraints are taken into account, the required CPU time is that of solving one deterministic long-term problem multiplied by the outer iteration number.

B. 118-Bus System

A modified IEEE 118-bus system in Fig. 8 is used in this case. The test data for the 118-bus system are given at http://motor.ece.iit.edu/data/ltscuc. The system is tested in an eight-week case study to demonstrate the stochastic long-term SCUC solution with multiple fuel and emission constraints.

The annual peak load of the system is 6,000 MW and the weekly peak load is listed in Table XII as a percentage of annual peak load. The fuel and emission groups are listed in Table XIII. Generating units, which are burning coal, oil and gas, are represented as fuel group (FGroup) 1, 2, and 3 respectively, and generating units with emission constraints are listed in emission groups (EGroup) 1, 2 and 3. The coal fuel group has upper and lower fuel supply constraints, while the oil fuel group has the upper limit fuel constraint, and the gas fuel group has the lower limit fuel constraint. The entire fuel consumption and emission allowance constraints over eight weeks, which are the same for different scenarios, are listed in Tables XIV and XV.

The low-discrepancy Monte Carlo simulation method is used to create 100 scenarios, each representing possible component outages and load forecasting inaccuracies. The simulation of component outages is based on the frequency and duration

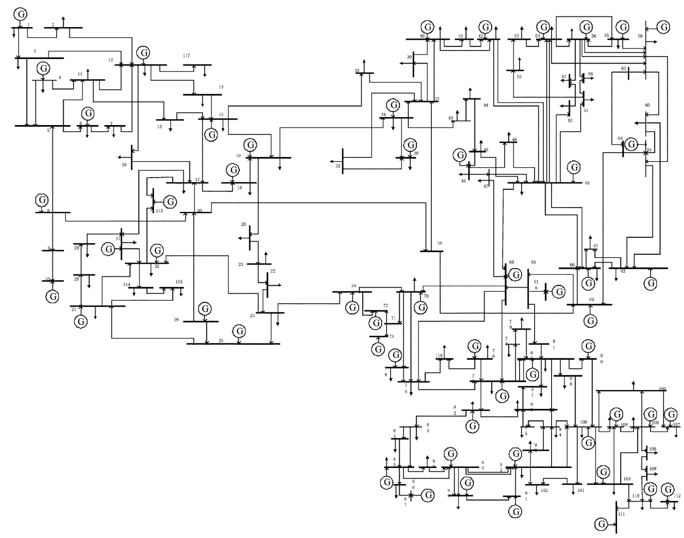

Fig. 8. One-line diagram of IEEE 118-bus system.

TABLE XII
WEEKLY PEAK LOAD AS PERCENTAGE OF ANNUAL PEAK LOAD

Week	1	2	3	4
Peak Load	86.2%	90.0%	90.0%	88.0%
Week	5	6	7	8
Peak Load	88.0%	84.1%	83.2%	80.6%

TABLE XIII
FUEL AND EMISSION GROUPS

Group	**Units**
FGroup1(Coal)	4 5 7 10 11 14 16 19 20 21 22 23 24 25 26 27 28 29 30 34 35 36 37 39 40 43 44 45 47 48 51 52 53
FGroup2(Oil)	31 32 33 38 41 42 46 49 50 54
FGroup3(Gas)	1 2 3 6 8 9 12 13 15 17 18
EGroup1	10 11 16 21 22 23 24 28 29 30 34 35 36
EGroup2	31 32 33
EGroup3	8 9 15 17 18

TABLE XIV
LONG-TERM GROUP FUEL CONSUMPTION CONSTRAINTS

Fuel	**Take-or-Pay Contract(MBtu)**	**Available(MBtu)**
Coal(FGp1)	60,00,000	67,50,000
Oil(FGp2)	-	7,000,000
Gas(FGp3)	1,00,000	UNLIMITED

TABLE XV
LONG-TERM GROUP EMISSION ALLOWANCE CONSTRAINTS

Emission	**Max allowance(lbs)**
SO_2(EGP1)	8,000,000
SO_2(EGP2)	60,000
SO_2(EGP3)	500,000
NO_X(EGP1)	3,500,000
NO_X (EGP2)	20,000
NO_X (EGP3)	200,000

method given in Section II. Load forecasting inaccuracies are represented by 5% fluctuations in weekly peak load forecast at the beginning of second week. Consequently, there are $3^7 = 2187$ demand scenarios within the eight-week horizon (see details in Fig. 4). Each has a weight of 1/2187. For each scenario one load curve is selected based on the lattice rule.

TABLE XVI
WEIGHTS OF EACH SCENARIO AFTER SCENARIO REDUCTION

Scenario	1	2	3	4	5	6
Weights	0.01	0.01	0.28	0.01	0.01	0.39
Scenario	7	8	9	10	11	12
Weights	0.01	0.01	0.01	0.01	0.01	0.24

TABLE XVII
EMISSION LEVEL

Emission (lbs)	**SO_2 (EGp1)**	**SO_2 (EGp2)**	**SO_2 (EGp3)**
EXP	7,384,383 ±3,382	16,611 ±359	144,106 ±4,016
Emission (lbs)	**NO_x (EGp1)**	**NO_x (EGp2)**	**NO_x (EGp3)**
EXP	2,943,753 ±3,645	9,903 ±106	67,614 ±1,378

TABLE XVIII
FUEL CONSUMPTION

Fuel (Btu)	**Coal**	**Oil**	**Gas**
EXP	66,969,534 ±40,225	1,934,656 ±47,980	4,685,651 ±132,257

TABLE XIX
WEEKLY PEAK LOAD

Week	1	2	3	4
Peak Load	83.2%	76.0%	79.3%	87.0%

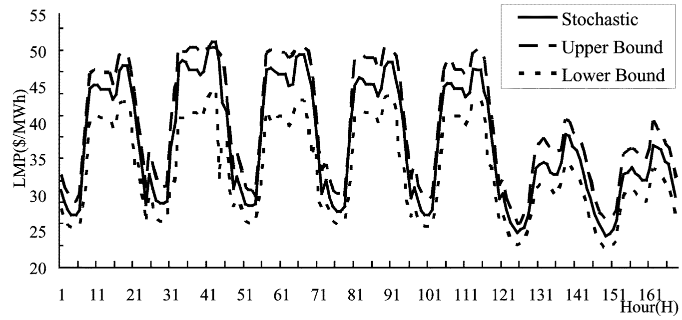

Fig. 9. LMP at bus 1 in the 3rd week.

TABLE XX
FUEL CONSUMPTION FOR EACH SCENARIO (CASE C)

EXP of Fuel Consumption (MBtu)	**Coal**	**Oil**	**Gas**
	7,484,549 ±157,101	1,434,690 ±25,810	4,355,435 ±48,693
EXP of SO_2 Emission (lbs)	**SO_2 (EGp1)**	**SO_2 (EGp2)**	**SO_2 (EGp3)**
	1,347,327 ±28,267	301,167 ±5,418	806,814 ±9,359
EXP of NO_x Emission (lbs)	**NO_x (EGp1)**	**NO_x (EGp2)**	**NO_x (EGp3)**
	765,421 ±16,066	141,457 ±2,543	468,557 ±5,529
Operating Cost ($)	82,405,215 ± 2,127,833		

The computation time for the scenario-based problem depends on the number of scenarios. The scenario reduction technique is then adopted to reduce the total number of scenarios as a tradeoff between calculation speed and solution accuracy. The original scenario tree has 100 scenarios, each with a weight of one percent. After reduction, 12 scenarios are left with weights after reduction shown in Table XVI for each scenario. Numerical tests shows that after the 90% reduction of scenario tree, emission level, fuel consumption, and relative errors of generation cost are about 3%. The eight week emission level, fuel consumption, and total operating cost are listed in Tables XVII–XIX respectively.

When components outages and load forecasting inaccuracies are taken into account, economical coal-burning group units cannot supply system loads over the entire eight weeks and expensive units, such as oil-burning and gas-burning units are committed. When all these random characters are considered, the results for long-term fuel consumption, emission allowance and utilization of generation units are more reasonable. The operating cost is $74{,}251{,}125 \pm 323{,}013$ with less than 3% relative errors in total cost, fuel consumption, and emission level which show the precision of proposed method.

For each scenario, a deterministic long-term SCUC problem is solved in twelve iterations for managing long-term fuel and emission allowance constraints. The outer iteration is repeated five times to meet scenario bundle constraints. For the first iteration, huge penalty factors are used to speed up the convergence and they are reduced proportionally to the change of dual objective function to avoid premature optimization because huge penalty factors could enforce the bundle constraints to be satisfied quickly but with a higher operating cost (a local optimal solution) since the problem is a non-convex optimization problem. By this approach, we could find a better SCUC solution. However, since the problem is not convex, there is no guarantee that all bundle constraints could be satisfied exactly. Thus approximated scenario solutions are calculated for fast convergence while maintaining a relatively high accuracy. Iterations stop when the weighted total bundle constraints violation is below a certain tolerance.

Fig. 9 depicts upper and lower bounds of expected LMPs, which are the expected value of scenarios based on $\pm 95\%$ confidence interval. In practice, when the random events such as outages of generation units and transmission lines as well as load forecasting inaccuracies are taken into account, LMPs are to be calculated for a given interval with a probability of 95%.

C. 1168-Bus System

An 1168-bus power system with 169 units and 1474 branches is used to illustrate the efficiency of our proposed model. The system is tested in a four-week case study to demonstrate the stochastic long-term SCUC solution with multiple fuel and emission constraints. The annual peak load of the system is 22,350 MW and the weekly peak load is listed in Table XIX as a percentage of annual peak loads. The required number of samples for a given accuracy level is independent of the system size when adopting the Monte Carlo simulation method. So we still simulate this large system with 100 scenarios and then reduce the number to 12 after scenario reduction. The total fuel consumption, emission level, and operating cost in four weeks and their relative errors are listed in Table XX.

The outer number of iterations for scenario bundle constraints is four to six for the given accuracy. The total CPU time for this stochastic problem is about 30 h with serial calculation. However, if parallel computation is further adopted in each short-term SCUC calculation process, the total CPU time can be reduced dramatically.

V. Conclusions

In this paper, we proposed a stochastic long-term SCUC formulation for representing uncertainties in the availability of generation units and transmission lines, and inaccuracies in load forecasting. The component outages are simulated by the Monte Carlo simulation. Lagrangian relaxation is applied to decompose the stochastic problem into many deterministic long-term SCUC sub-problems which are solved by a hybrid method. Numerical results show that the efficiency of the proposed solution approach and the impact of outages of system components and demand uncertainties on system operating costs and allocations of energy allocation, fuel consumption, and emission allowance. The merits of proposed temporal model are featured by the simulation of uncertainties in the solution of stochastic long-term SCUC. The stochastic solution provides more reliable decisions on energy allocation, fuel consumption, emission allowance, and long-term utilization of generating units.

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Lei Wu received the B.S. and M.S. degrees in electrical engineering from Xi'an Jiaotong University, China, in 2001 and 2004, respectively. He is pursuing the Ph.D. degree at Illinois Institute of Technology, Chicago.

His research interests include power systems restructuring and reliability.

Mohammad Shahidehpour (F'01) is with the Computer Engineering Department, Illinois Institute of Technology (IIT), Chicago. He is the author of 300 technical papers and four books on electric power systems planning, operation, and control. His books include *Maintenance Scheduling in Restructured Power Systems* (Norwell, MA: Kluwer, 2000), *Restructured Electrical Power Systems* (New York: Marcel Dekker, 2001), *Market Operations in Electric Power Systems* (New York: Wiley, 2002), and *Communication and Control of Electric Power Systems* (New York: Wiley, 2003).

Dr. Shahidehpour is the recipient of the 2004 IEEE Power System Operation Committee's Best Paper Award, 2005 IEEE/PES Best Paper Award, the Edison Electric Institute's Outstanding Faculty Award, HKN's Outstanding Young Electrical Engineering Award, Sigma Xi's Outstanding Researcher Award, IIT's Outstanding Faculty Award, and the University of Michigan's Outstanding Teaching Award.

Tao Li (M'06) received the B.S. and M.S. degrees in electrical engineering from Shanghai Jiaotong University, Shanghai, China, and the Ph.D. degree from the Illinois Institute of Technology, Chicago, in 1999, 2002, and 2006, respectively.

He is currently a Senior Research Associate in the Electric Power and Power Electronics Center, Electrical and Computer Engineering Department, Illinois Institute of Technology. His research interests include power system economics and optimization.