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Effects of Price of Gold on Bombay Stock Exchange Sectoral Indices: New Evidence for Portfolio Risk Management

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ABSTRACT

Using daily data, this paper examines the relationship between the returns of gold and seven sectoral indices in the Bombay Stock Exchange (BSE) for the period from January 2000 to May 2018. Given the importance of gold in India, there are significant issues in a portfolio selection in that country. By addressing the hedged robust portfolio problems, this paper focuses on three vanilla portfolio problems: the maximum return portfolio allocation, the global minimum variance portfolio problem, and the Markowitz portfolio allocation by using various multiple generalized autoregressive conditional heteroskedasticity (GARCH) models. The paper finds that gold returns are significantly independent of the returns of the BSE sectoral indices. Besides, gold returns can help predict the future returns of the Consumer Durables and the Fast-Moving Consumer Goods indices as well as the Oil & Gas equity indices. Finally, the findings also show that gold hedges against the information technology stock index and serves as a robust portfolio diversification tool. With these new results, this paper offers several implications for investors and risk management purposes.

1. Introduction

Gold is an essential financial and physical asset since it performs three noteworthy functions: monetary, non-production (reserve asset), and financial functions. First, due to its features of being a commodity, gold can be used as a medium of exchange; and therefore, it can be used as a currency. This feature is known as the monetary function of gold. It was used as a currency under the first gold standard. According to Lucey et al. (2017), gold can also be used as a hedge against inflation risk due to this monetary function since it is a real asset and can rise with inflation.

Second, gold has the non-production (reserve asset) function; that is, it is used for financial and monetary purposes more than in industrial production processes or purposes. Many countries are still using gold as a part of their foreign reserves to support their exchange rates. These features make gold an important asset not only for investors but also for central bankers and governments since it can be considered an important part of international reserves (Lucey et al., 2017).

Third, gold has been used for hedging and portfolio diversification since it is viewed as a safe haven because it has negative

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correlations with other financial assets like stocks. Investors move to gold as a safe have during crises like the COVID-19 health crisis. This feature is known as the financial function of gold. Put differently; gold is a significant hedge tool. Therefore, it can significantly interact with other financial instruments such as the exchange rate, oil (and other commodities), and stock markets (Baur and McDermott, 2010; Bredin et al., 2015; Choudhry et al., 2015; Ciner et al., 2013; Gil–Alana et al., 2017; Joy, 2011; Lau et al., 2017; Kang et al., 2017; Reboredo, 2013). Note that gold is considered as a safe haven since it can be used as a hedging tool against uncertainty at times of extreme events in the financial markets (Baur and Lucey, 2010; Bialkowski et al., 2015; Li and Lucey, 2017). However, this function of gold has been questioned by Zhen et al. (2018).

In this paper, we focus on the financial function of gold and aim to analyze the interaction between the price of this shiny metal and the stock market returns in India. We should note that India is among the ten countries that hold this yellow metal. There are several investment options in gold in India. These include gold ETFs that invest in physical gold, bullion, E-gold schemes such as gold sovereign bonds (GSBs), issued by the Reserve Bank of India on behalf of the Indian government, and gold mutual funds etc. Gold in India is considered as a status symbol as it symbolizes wealth. It is also believed that it brings luck and happiness during the married life. The rupee plays a vital role in determining the landed cost of the dollar-quoted shiny metal in India. The price of gold in India has also been very volatile, and the Indian government imposed a ban on consignment imports,[1] which disrupted the safe-haven property of this metal that implies the stock markets. Gold is also used to convert black money to white money in the underground economy of India, which constitutes 20-25% of the entire economy of that country.

It is also important to note that gold and gold-based products have traditionally been used as an investment tool in the Indian economy. Given that there has been a solid return performance of gold in the 2000s, the value of private savings of the Indian people has also increased. According to the portfolio choice theory, investors during the times of volatile markets (e.g., during the global financial crisis of 2008-9) move towards the safety of gold instead of investing in risky stock markets (O'Connor et al., 2015). In July 2020, the gold price in India hit an all-time high after the spike in the COVIT-19 pandemic due to its serving as a safe haven asset.

Given that India is still a weak emerging economy and lacks sophisticated investment tools to absorb its 30% saving rate, gold hoarding is even more of an issue in this country since most of the private savings traditionally are evaluated in gold. As it has already been known, this precious metal, like all commodities, is quoted in the USD in global markets (Beckmann et al., 2015). Therefore, during times of uncertainty, the currencies of emerging markets (e.g., the Indian rupee-INR) tend to depreciate (Khalifa et al., 2016) as investors move to safe havens. This issue means that the price of gold can increase in domestic currency even without a change in the international price of gold (Barunik et al., 2016). Therefore, gold is an attractive investment for investors in emerging markets, including India, during times of uncertainty. Specifically, investors will increase the demand for gold if there is an increasing uncertainty due to increases in the INR price of gold. In short, this precious metal can affect the Indian stock markets due to several reasons since gold is a traditional asset in India, has an international price, and has a domestic price that is based on the value of the USD against the INR.

The standard view is that the gold and stock markets are negatively related. That is, when stocks dive, for example, in a downturn, the yellow metal moves up as people find that handling of gold is safer than dealing with cash, and vice versa (British Broadcasting Corporation (British Broadcasting Corporation (BBC, 2012)).[2] There is empirical evidence that confirms this standard view. On the other hand, there are periods like the 2000s when the relationship can generally be considered as a period of co-movements due to the dependence. Thus, the gold-stock relationship may change over time, depending on external conditions, particularly the macroeconomic factors. The risk appetite pinned by opportunity costs is also another factor affecting the relative attractiveness of stocks in comparison to gold. Others include the level of the real interest rates, the value of the USD exchange rate, the pace of economic growth, and the momentum in the gold-stock markets (Kyereboah-Coleman and Agyire-Tette, 2008).

In India, the domestic price of gold has increased continuously due to the high demand for this metal in the Indian economy. Apart from gold being a traditional asset, there are other reasons for demanding this yellow metal in India. The first reason is the "full security" that gold bestows on the economy and financial markets. More specifically, gold provides security since it is a reserve asset of the central banks, and there is no credit risk in investing in gold-based assets. Given that stock markets are labelled as risky assets, gold is a safe asset to invest in (Vigne et al., 2017).

Besides, gold can hedge domestic and global inflation risks, particularly during times of economic uncertainty and geopolitical risks (Bilgin et al., 2018; Gozgor et al., 2019). Global inflation arises from global monetary and supply shocks, which drive prices of goods and services up. However, domestic inflation due to higher domestic aggregate demand or higher costs of factors of production will increase the prices of the goods and services. At this stage, inflation will erode the purchasing power of a domestic currency even within the same amount of money. During such times gold, which is viewed as a "real estate," plays the role of a store of the real value (purchasing power). The decline in purchasing power implies that there will be less money for savings, which can be reflected in the values of the stock market. For example, Batten et al. (2010) focus on the macroeconomic determinants (i.e., business cycles, monetary policy, and financial market sentiments) of price volatility in primary precious metals (gold, silver, platinum, and palladium prices) markets. The authors find that monetary variables, in particular, determine the price volatility of gold. The same authors extend their paper (Batten et al., 2010) and focus on their newer paper (Batten et al., 2014) which deals with the macroeconomic determinants of the gold-inflation relation. Those authors examine the long-term and dynamic relations between global inflation and the gold price and observe that there is no significant co-integration between the price of gold and global inflation if the volatile

[1] India's gold imports in the first five months of 2018 dropped by 39.4% from a year ago to 274.2 tonnes (Reuters report, 2018).

[2] You may also see these references: https://www.usmoneyreserve.com/blog/cash-vs-gold-which-asset-could-prove-better/ https://aheadoftheherd.com/Newsletter/2019/Gold-vs-cash-in-a-crisis.htm https://www.bbc.com/news/business-18644230

period of the early 1980s is neglected. It is important to note that there is a significant time variation in the relationship between those variables, and the co-movement between them has significantly increased during the 2000s and the early 2010s.

There are previous studies that investigate the role of gold in portfolio choices. For instance, O'Connor et al. (2015) present the previous literature on the investment function, and these authors review a wide variety of empirical studies. At this stage, several papers examine the determinants of the price of gold. Specifically, the papers have investigated the driving factors of the price of the yellow metal. Leading factors, which have a drawn special attention from the previous literature, include the price of oil and exchange rates (mainly USD). For example, Ewing and Malik (2013) examine the volatility transmission (spillover) between daily futures returns of gold and oil returns for the period from July 1, 1993, to June 30, 2010. The authors use the univariate and the bivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models with structural breaks and find a significant volatility transmission between gold and oil futures markets. According to their findings, the theory of cross-market hedging is validated, and gold is tagged as a hedging instrument of oil by the futures market participants. Indeed, the previous papers have concluded that investors try to hedge their investments through gold during the time of global crises. For example, Gozgor et al. (2015) indicate that the global financial crisis of 2008–2009 has created a significant change in the portfolio choice, and that there is a significant diversification of four BRIC stock markets (Brazil, Russia, India and China) to traditional assets such as gold. At this stage, the exchange rate and the price of oil are also important drivers of the BRIC stock markets.

According to our empirical results derived from the various causality analysis, gold can help predict the future returns of some diverse equity sectors like consumer durables, oil & gas, and the fast-moving consumer goods (FMCG) stock indices. Concerning the causation of other equity sectors (i.e., capital goods, auto, and information technology (IT)), the results may not be definitive one way or the other. For the Quantile coherence approach, the analysis of the real and the imaginary parts supports the presence of a significant independence between the price of gold and the BSE sectoral equity indices. These findings are also confirmed by the Dynamic Conditional Correlation (DCC) and the ADCC (Asymmetric Dynamic Conditional Correlation) models but not by the Generalized Orthogonal (GO)-GARCH model.

For the portfolio implications, we find that gold is an adequate hedge against the IT stock index and is a useful portfolio diversification tool when considering the Global Minimum Variance Portfolio (GMVP) and the robust version of the vanilla problems. Therefore, we can suggest that changes in the price of gold are not only crucial for the stock market returns in India but also significant for portfolio diversification and risk management.

Our findings contribute to the previous literature on the relationship between the price of gold and the stock market returns. More specifically, we focus on the stock returns of seven sectors in India in regard to changes in the price of gold. Methodologically, we run the nonlinear Granger causality test and the quantile coherence analysis of the relationship between the stock and gold returns. To the best of our knowledge, this is the first paper in the literature that uses the quantile coherence estimations in analyzing the relationship between changes in the price of gold and stock market returns in India. Overall, our paper aims to contribute to the existing empirical literature by using the detailed stock market measures and novel econometric techniques such as the nonlinear Granger causality test and the quantile coherency estimations.

The remainder of the paper is organized as follows. Section 2 provides the data, the model, and the econometric methodology. Section 3 reports the empirical results and discusses the implications of the findings. Section 4 concludes.

2. Data and Econometric Methodology

2.1. Data and Descriptive Statistics

Table 1 reports the descriptive statistics of the data spanning the period between January 3, 2000 and May 25, 2018. This issue is the longest period available to us. The frequency of the data is daily. We focus on seven equity sectors in the BSE, including IT, Metals, Oil & Gas, Auto, Capital Goods, Consumer Durables, and FMCG. Those are the primary sectors that are relevant to our study. The US dollar gold prices are converted to rupees. All the data were obtained from Bloomberg. The preliminary findings in Table 1 illustrate that the means of the seven sectors and gold are positive at a high level for the Auto sector, followed by the Consumer Durables and the IT sectors. Second, the statistics concerning the volatility indicate that the gold market is more stable than the

Table 1
Descriptive Statistics.

	Gold	Auto	Capital goods	Consumer durables	FMCG	It	Metals	Oil and Gas
Mean	0.0401	0.0661	0.0594	0.0616	0.0504	0.0550	0.0521	0.0547
Median	0.0309	0.1081	0.0845	0.1165	0.0590	0.0598	0.0983	0.0550
Maximum	7.1273	10.6265	19.8033	12.4785	11.5338	17.4906	14.9282	17.4844
Minimum	−9.4954	−11.0125	−15.7578	−11.6696	−11.1474	−22.2984	−14.2716	−16.2110
Std. Dev.	1.0955	1.5200	1.8139	1.89167	1.3670	2.2890	2.1600	1.7867
Skewness	−0.0990	−0.2916	−0.0226	−0.26349	−0.0446	−0.3166	−0.2533	−0.3043
Kurtosis	8.9938	6.4868	9.5799	7.24605	8.2242	11.4534	7.1183	10.8693
Jarque-Bera	7256.044	2521.592	8735.408	3693.37	5507.902	14498.26	3473.735	12568.31
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	4842	4842	4842	4842	4842	4842	4842	4842

examined BSE sectoral equity indices. We also observe that the IT sector has a higher risk because of the variability in its return over the sample period (i.e., the returns range from –22% to over 15%).[3] Finally, the results for skewness and kurtosis and the test of the Jarque-Bera confirm that the series are non-normal with a left-skewness. Therefore, the gold market and the BSE equity indices have recorded several adverse shocks rather than positive shocks over the sample period.

The panel "a" of Fig. 1 shows a high oscillation tend (i.e., the instable periods) for the logarithm returns caused by many factors, chief of them is the global financial crisis (hereafter, GFC) during 2008–2009. These volatility periods are shown in panel "b" of Fig. 1, which depicts the squared returns of the series. The daily return volatility, in particular, is much higher for the IT, Oil & Gas, Metal and Capital Goods equity sectors, than for gold over the sample period

To complete the description of the data, we conduct two tests to detect non-normality (using the test of Andrews, 1993) and potential linearity (via the test of Broock et al., 1996) (hereafter the BDS test) in the series. Notably, we regress the BSE stock index returns on daily gold returns. Following Andrews (1993), we test the null hypothesis of constant parameters against the alternative of a one-time structural change at each possible point of time in the entire sample. However, the BDS test considers the null hypothesis of the independently and identically distributed (i.i.d) assumption against an unspecified alternative. All statistics are shown in Tables 2 and 3.

In inspecting these statistics, one can reject the null hypothesis of both tests and not reject the alternative one. This evidence means that the underlying system determined by the BSE and gold returns is nonlinear and non-normal. For this reason, which is different from various previous studies, our paper adopts the adequate methods that fit these time-series characteristics. Next, we discuss the details of the econometric methodology.

2.2. Econometric Methodology

To explore the relationship between gold and stock sector returns of the BSE, we use two distinct measures of connectedness. First, we use the nonlinear Granger causality test to detect the lagged causality between the gold market and the BSE sectoral equity indices. Second, we study the dependence based on a different frequency and quantile levels, using the approach proposed in Baruník and Kley (2019). Note that we need to use nonlinear tests because of the presence of outliers due to shocks and crises. Finally, we complete the analysis by constructing a dynamic portfolio and capturing the weights and hedge benefits.

2.2.1. Linear and nonlinear Granger Causality

Let the BSE sectoral indices and gold returns be two stationary processes defined, respectively, by $X_t = \{x_t\}$ and $Y_t = \{y_t\}$, $t \in \mathbb{Z}$. Moreover, let the information sets of those two prices until time t-1 be $\Psi_{X,t-1}$ and $\Psi_{Y,t-1}$, respectively. In respect to lags l_x and l_y, and $k \geq 1$ (which is the prediction horizon and its value is normally used as $k \geq 1$), $\{x_t\}$ is said to Granger cause $\{y_t\}$ if

$$(y_t, ..., y_{t+k})|\,(\Psi_{Y,t-ly}, \Psi_{X,t-lx}) \sim (Y_t, ..., Y_{t+k})|\,\Psi_{Y,t-ly} \tag{1}$$

where ~ denotes equivalence in distribution · For simplicity, we set $l_x = l_y = 1$. Furthermore, we will not distinguish the latter from the former. We assume $W_t = (X_t,\ Y_t,\ Z_t)$, where $Z_t = Y_{t+1}$; and $W = (X,\ Y,\ Z)$ is used when there is no danger of confusion.

We carry out Hiemstra and Jones (1994, hereafter HJ) and Diks and Panchenko (2005, hereafter DP) tests of general Granger non-causality. According to DP, the nonparametric test of conditional independence between $\{x_t\}$ and $\{y_t\}$ in terms of density functions $f(.)$ is:

$$\text{H0: } \frac{f_{X,Y,Z}(x, y, z)}{f_Y(y)} - \frac{f_{X,Y}(x, y)}{f_Y(y)}\frac{f_{Y,Z}(y, z)}{f_Y(y)} = 0 \tag{2}$$

DP then defined the correlation integral $C_w(\varepsilon)$ as

$$C_w(\varepsilon) = P[\|W_1 - W_2\| \leq \varepsilon] = \int\int I(\|s_1 - s_2\| \leq \varepsilon) f_w(s_1) f_w(s_2) ds_1 ds_2 \tag{3}$$

where W_1, W_2 are self-determining with distributions in the equivalence class of the distribution of W, $I(\|s_1 - s_2\| \leq \varepsilon)$ is the indicator function equalling to 1 if the condition in brackets is satisfied (i.e. $\|s_1 - s_2\| \leq \varepsilon$), and 0 otherwise. $\|x\| = \sup\{|x_i|: i = 1, ..., d_w\}$ describes the supremum norm and $\varepsilon > 0$.

For testing purposes, DP used the residuals of a VAR(p) model. In this case, the time series of the residuals may reflect nonlinear dependencies, whereas the VAR model describes the linear causality. The residuals are then transformed to a standardized scale. For the bandwidth, we set ε equal to 1.5 in all cases.

To test the validity of Eq. (2), HJ claimed that the null hypothesis in the Granger causality test could be expressed for every $\varepsilon > 0$ as:

$$\text{H}_0\text{: } \frac{C_{X,Y,Z}(\varepsilon)}{C_{X,Y}(\varepsilon)} - \frac{C_{Y,Z}(\varepsilon)}{C_Y(\varepsilon)} = 0 \tag{4}$$

or equivalently:

[3] Note that the findings of the standard unit root tests, such as the Phillips–Perron (PP) and the Zivot–Andrews (ZA) tests, confirm that all variables follow a stationary process.

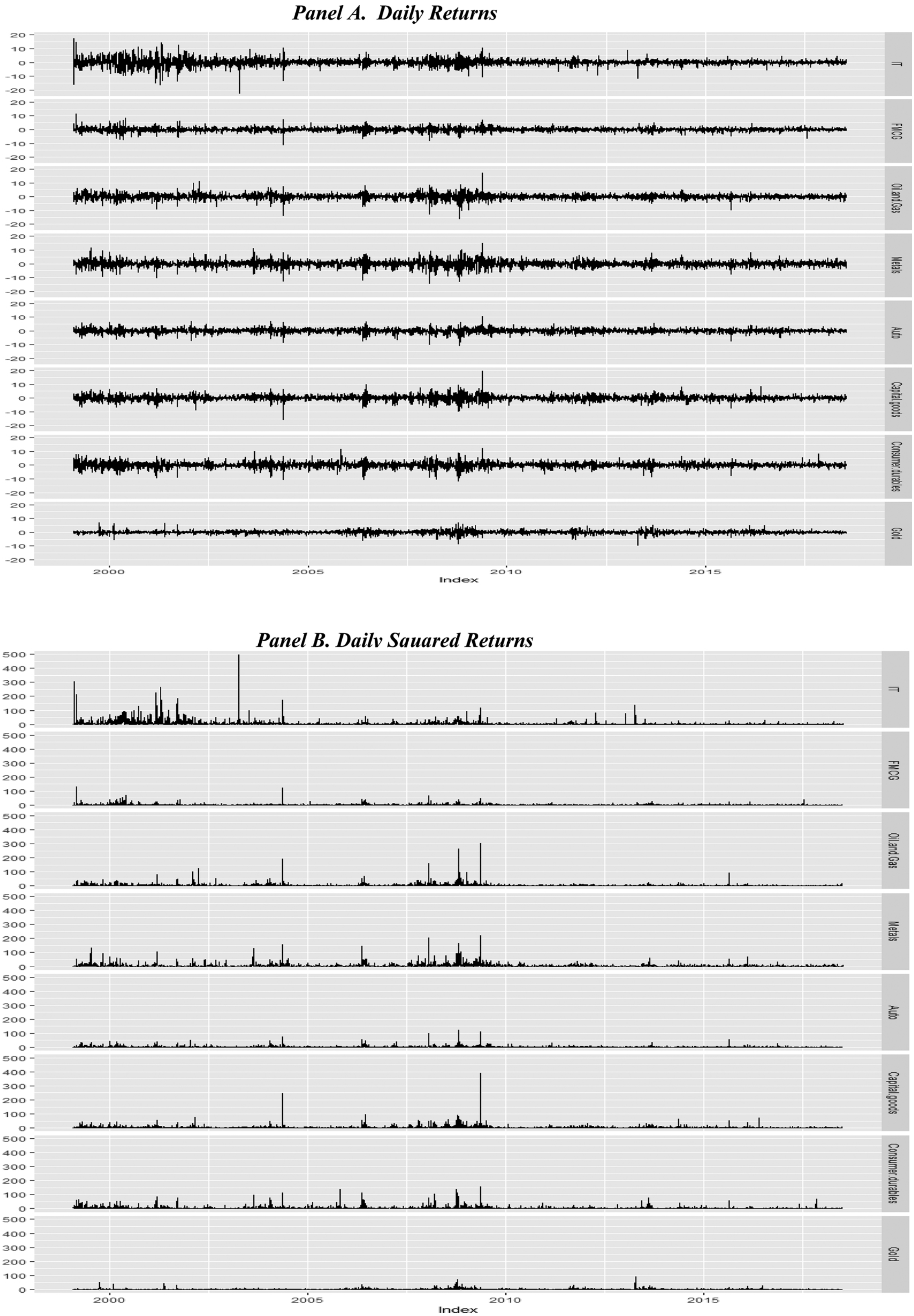


Fig. 1. BSE Sectoral Daily Returns/Squared Returns.

Table 2
Parameter Stability Test of Andrews (1993) (BSE Stock Returns are Regressed on Gold returns).

	Auto	Capital goods	Consumer durables	FMCG	IT	Metals	Oil and Gas
Sup LR	3.0887**	3.8108*	3.3904**	3.4045*	9.1203*	4.0846*	4.6560*
Exp LR	0.8009***	0.9459*	0.8088**	0.9403**	2.2550*	0.9893*	1.3014*
Mean LR	1.4938***	1.7324*	1.4991***	1.7104**	2.7176*	1.8208**	2.3358*
Sup Wald	27.7986**	34.2979*	30.5141**	30.6412*	82.0831*	36.7619*	41.9040*
Exp Wald	9.4566**	13.0165**	11.5806**	12.3044*	35.0816*	13.2765*	16.2274*
Mean Wald	13.4442***	15.5921**	13.4919**	15.3940**	24.4586*	16.3879**	21.0230*

Note: Parameter stability test by Andrews (1993) and Andrews and Ploberger (1994) with the null hypothesis of parameter stability.

Table 3
BDS Test for Nonlinearity (BSE Stock Returns are Regressed on Gold Returns).

	Auto	Capital Goods	Consumer durables	FMCG	IT	Metals	Oil and Gas
$m = 2$	15.18419*	17.59291*	16.37875*	15.70977*	26.46332*	17.21334*	17.41743*
$m = 3$	17.79188*	21.33412*	20.21017*	19.21542*	31.42276*	20.08439*	21.07659*
$m = 4$	19.92905*	24.38120*	22.96433*	21.12797*	34.51946*	22.61259*	23.54845*
$m = 5$	22.07131*	27.09520*	24.97839*	22.66457*	37.68282*	24.79390*	25.87727*
$m = 6$	23.95126*	29.65046*	26.91619*	24.08403*	41.18491*	26.76054*	28.28648*

Note: The entries indicate the z-statistics BDS test based on the residuals of the data series. M denotes the embedding dimension of the BDS test. All hypotheses are rejected at the 1% significance level.

$$\frac{C_{X,Y,Z}(\varepsilon)}{C_Y(\varepsilon)} = \frac{C_{X,Y}(\varepsilon)}{C_Y(\varepsilon)}\frac{C_{Y,Z}(\varepsilon)}{C_Y(\varepsilon)} \tag{5}$$

On the other hand, HJ recommended calculating the correlation integrals for each density and measuring the discrepancy between the two sides of the equation. For that, they propose an alternative estimator of the correlation integral:

$$C_{w,n}(\varepsilon) = \frac{2}{n(n-1)}\sum\sum_{i<j} I_{ij}^{W} \tag{6}$$

where $I_{ij}^{W} = I(\|W_i - W_j\| < \varepsilon)$. Hence, the HJ test can identify the conditions under which the conditional probability holds. However, DP showed that the Granger non-causality does not necessarily imply this condition.

2.2.2. Quantile Coherency Approach

Following Barunik and Kley (2019), the dynamic dependence between $X_t = \{x_t\}$ and $Y_t = \{y_t\}$ can be defined by $\mathfrak{R}^{X,Y}$ $(i.\ e.\ , coherency)$ as following:

$$\mathfrak{R}^{X,Y}(\omega, \tau_1, \tau_2) = \frac{\mathscr{F}^{X,Y}(\omega, \tau_1, \tau_2)}{(\mathscr{F}^{X,X}(\omega, \tau_1, \tau_1).\mathscr{F}^{Y,Y}(\omega, \tau_2, \tau_2))^{1/2}} \tag{7}$$

where $-\pi< \omega< \pi$, $\tau\in [0,1]$, and $\mathscr{F}^{X,Y}$, $\mathscr{F}^{X,X}$ and $\mathscr{F}^{Y,Y}$ are the quantile cross-spectral and the quantile spectral densities of processes $\{x_t\}$ and $\{y_t\}$, respectively.

For the definition of the serial and the cross-section dependence structures of $\{x_t\}$ and $\{y_t\}$, we refer to the kernel matrix of the quantile cross-covariance:

$$\Gamma_k(\tau_1, \tau_2) := (\gamma_k^{\tau_1,\tau_2}(\tau_1, \tau_2))_{XY} \tag{8}$$

where, $\gamma_k^{\tau_1,\tau_2}(\tau_1, \tau_2) := \mathrm{CoV}(I\{X_{t+k} \le q_x(\tau_1)\}, I\{Y_t \le q_y(\tau_2)\})$. For $K \in Z$, and I is a characteristic function. Note that $q_x(\tau_1)$ and $q_y(\tau_1)$ indicate the respective marginal distribution's τ_1 and τ_2 quantiles. In addition to providing information about the cross-section dependence, varying k reveals information about the serial dependence. This yields the following metric for the frequency domain:

$$\mathscr{F}(\omega,\tau_1, \tau_2) := (2\pi)^{-1}\sum_{k=-\infty}^{\infty} \gamma_k^{\tau_1,\tau_2}(\tau_1, \tau_2)e^{-ik\omega} \tag{9}$$

The quantile coherency is estimated via the smoothed quantile cross-periodograms. Among the variety quantile, this paper retains only the coherency for the three quantiles (0.5, 0.05, 0.95) and all their combinations (i.e., the middle, the left tail, and the right tail of the distributions). Examining these quintiles allows us to measure the dependence in different parts of the distributions. Moreover, this study focuses on the three frequencies: the short run (one week), the medium run (one month), and the long run (one year).

It is essential to observe that $\mathscr{F}^{x,y}(\omega,\tau_1, \tau_2)$ in Eq. (9) is complex-valued, and can thus be divided into two parts: the real part which can be attributed to the co-spectrum of the processes $(I\{X_t \leq X(\tau_1)\}_{t\in\mathbb{Z}})$ and $(I\{Y_t \leq q_y(\tau_1)\}_{t\in\mathbb{Z}})$,[4] while the imaginary part aims at removing all sources of the noise coherence that are a consequence of the instantaneous activity.

2.2.3. *Portfolio Risk Analysis*

Gold is used in portfolios as a hedge against risk in the cases of high inflation and crises. Therefore, we also implement the portfolio risk analysis.

(i) *The Hedged Portfolio*. As a test of whether gold displays a hedging potential, we consider the case of an Indian investor who wants to protect the exposure to the BSE sectoral index fluctuations. In terms of returns, this can be denoted as:

$$R_{H,t} = x_t - \Delta_t y_t \tag{10}$$

where $R_{H,t}$ is the return on holding the hedged portfolio between *t-1* and *t* and Δ_t is the hedged ratio, that is, the number of gold contracts that a hedger must sell for each unit of the stock index on which the price is borne. According to Kroner and Sultan (1993) the optimal hedge ratio (Δ_t^*) at time *t* is defined as:

$$\Delta_t^* = \Delta = \rho_t \frac{\sigma_{s,t}}{\sigma_{Gold,t}} \tag{11}$$

where $\sigma_{s,t}$ and $\sigma_{Gold,t}$ denote the conditional variances of the stock and the gold logarithmic returns at time t, respectively. ρ_t is the conditional dynamic correlation estimated from the DCC-AR(1)-GARCH(1,1) model[5] . We suggest that adding the gold futures contracts to the stock holdings will reduce the variance (risk) of the returns of the hedged portfolio. For this reason, we use the hedging effectiveness index to evaluate the hedging effectiveness of gold, obtained as:

$$HE = (Variance_{unhedged} - Variance_{hedged})/Variance_{unhedged} \tag{12}$$

A higher Δ in Eq. (17) indicates higher hedging effectiveness.

(ii) Vanilla Optimum Portfolio Problems. As a test of the gold possible diversification power, we use the solution of three vanilla portfolio problems: The maximum return portfolio allocation, the global minimum variance portfolio (GMVP) allocation, and the Markowitz portfolio allocation.

A The optimization problem of the portfolio that maximizes the return is:

$$\max_w w^T\mu \tag{13}$$

where w are the weights and μ are the logarithm returns.

B The GMVP problem ignores the expected return and focuses on the risks only. The target level is defined by:

$$\min_w w^T\Sigma w \tag{14}$$

C The Markowitz mean-variance framework is:

$$\max_w w^T\mu - \lambda w^T\Sigma w \tag{15}$$

where λ is a parameter that defines the behaviour of a risk-averse investor. Since in practice μ and Σ are unknown, we use here the conditional mean and the conditional covariance matrix with time-varying conditional correlations estimated from the DCC-AR(1)-GARCH(1,1) model. The portfolio problems are subject to a weight constraint.

3. Findings and Discussions

3.1. *Results of Dependence Structures*

The results from the nonlinear Granger-causality are reported in Table 4. One can see that there is a high similarity between the results of the HJ and DP tests. Moreover, there is a strong and a bi-directional causality between gold and the Consumer Durables, Oil & Gas, and the FMCG stock indices. This evidence reflects that gold can help predict the future returns of these sectors, and vice-versa. Concerning the causation of other sectors, the result may not be definitive one way or the other. For instance, there is a significant Granger causality from gold to the auto sector stocks but coming from the second lags. Concerning the causality of the auto equity sector to gold, this causality is not rejected only for the first lag. For the capital goods and IT sectors, a significant causality can be found from the gold market to these sectors but is not valid in the opposite direction.

Fig. 2 shows the results of many cross-dependence tests among the left, the middle, and the right quantiles of the distributions. It can be seen that the plots of the real and imaginary parts are similar and close both to zero. Therefore, there is a significant

[4] The real and the imaginary parts of the Fourier transform of a signal $x(t_0)$ are the Fourier transforms of the signal's even and odd parts, respectively.

[5] Refer to Appendix for more details.

Table 4
Results from Non-linear Granger-Causality.

Gold does not granger cause Auto					Auto does not granger cause Gold				
	HJ	P-value	DP	P-value		HJ	P-value	DP	P-value
lX = lY = 1,	0.672505	0.250631	0.618309	0.268186	lX = lY = 1,	1.811916	0.035	1.732499	0.041592
lX = lY = 2,	2.160551	0.015365	2.012943	0.02206	lX = lY = 2,	0.689559	0.245236	0.374656	0.353958
lX = lY = 3,	2.666854	0.003828	2.255425	0.012053	lX = lY = 3,	0.41844	0.337813	0.258239	0.398111
lX = lY = 4,	2.425823	0.007637	1.691662	0.045355	lX = lY = 4,	0.182566	0.427569	0.065372	0.473939
lX = lY = 5,	2.595266	0.004726	1.906268	0.028308	lX = lY = 5,	−0.78639	0.78418	−0.7259	0.766049
lX = lY = 6,	2.90105	0.00186	2.370624	0.008879	lX = lY = 6,	−0.74753	0.772628	−0.47321	0.681967
lX = lY = 7,	2.697159	0.003497	2.174074	0.01485	lX = lY = 7,	−0.05576	0.522233	0.493712	0.310755
lX = lY = 8,	2.905304	0.001834	2.178447	0.014686	lX = lY = 8,	0.164575	0.434639	0.269831	0.393645
Gold does not granger cause Capital goods					**Capital goods do not granger cause Gold**				
lX = lY = 1,	1.053594	0.146034	1.042276	0.148642	lX = lY = 1,	−0.36835	0.643694	−0.44623	0.672286
lX = lY = 2,	1.953248	0.025395	1.735308	0.041343	lX = lY = 2,	−0.34866	0.636328	−0.57537	0.717479
lX = lY = 3,	2.112303	0.01733	1.758725	0.039312	lX = lY = 3,	−0.53044	0.702097	−0.71873	0.763847
lX = lY = 4,	1.698279	0.044728	1.039026	0.149396	lX = lY = 4,	−0.24239	0.595762	−0.27312	0.607621
lX = lY = 5,	1.502624	0.066468	0.923635	0.177838	lX = lY = 5,	−0.63714	0.737983	−0.38949	0.651541
lX = lY = 6,	1.907923	0.028201	0.827851	0.203878	lX = lY = 6,	−0.61317	0.730118	−0.45404	0.6751
lX = lY = 7,	1.554002	0.060092	0.964711	0.167345	lX = lY = 7,	−0.59437	0.723869	−0.27392	0.607928
lX = lY = 8,	0.791368	0.214365	0.075405	0.469946	lX = lY = 8,	−0.41251	0.660016	−0.00638	0.502543
Gold does not granger cause Consumer durables					**Consumer durables do not granger cause Gold**				
lX = lY = 1,	0.521724	0.300931	0.426294	0.334947	lX = lY = 1,	0.038009	0.48484	0.011543	0.495395
lX = lY = 2,	1.348924	0.088681	1.271469	0.101781	lX = lY = 2,	−0.75261	0.774159	−1.03475	0.849606
lX = lY = 3,	1.614376	0.053223	1.374622	0.084624	lX = lY = 3,	−0.99969	0.841269	−1.32779	0.907876
lX = lY = 4,	0.795736	0.213093	0.094449	0.462376	lX = lY = 4,	−0.96969	0.8339	−1.19003	0.882983
lX = lY = 5,	1.326794	0.092288	0.834134	0.202103	lX = lY = 5,	−1.81995	0.965617	−1.7914	0.963385
lX = lY = 6,	0.504041	0.307116	0.117303	0.45331	lX = lY = 6,	−1.80261	0.964275	−1.81347	0.96512
lX = lY = 7,	−0.05545	0.522114	−0.52404	0.699875	lX = lY = 7,	−1.43048	0.92371	−1.49186	0.932132
lX = lY = 8,	−0.69181	0.755472	−0.43237	0.667264	lX = lY = 8,	−0.24956	0.598536	0.082934	0.466952
Gold does not granger cause FMCG					**FMCG does not granger cause Gold**				
lX = lY = 1,	−0.98876	0.83861	−1.09763	0.863816	lX = lY = 1,	−0.39593	0.63574	−0.3471	0.653922
lX = lY = 2,	0.086697	0.465456	0.116386	0.453673	lX = lY = 2,	0.210442	0.475976	0.060257	0.416661
lX = lY = 3,	0.021587	0.491389	−0.19363	0.576765	lX = lY = 3,	0.50239	0.420991	0.199358	0.307697
lX = lY = 4,	0.25229	0.400408	0.065525	0.473878	lX = lY = 4,	0.309443	0.366645	0.340754	0.378492
lX = lY = 5,	0.721398	0.235332	0.55438	0.289659	lX = lY = 5,	0.0383	0.385077	0.292174	0.484724
lX = lY = 6,	0.897921	0.184614	0.771243	0.220282	lX = lY = 6,	0.200341	0.434511	0.1649	0.420607
lX = lY = 7,	1.518585	0.064433	1.609418	0.053762	lX = lY = 7,	−0.72535	0.848092	−1.02828	0.76588
lX = lY = 8,	1.830295	0.033603	1.615357	0.053117	lX = lY = 8,	0.05575	0.56401	−0.16115	0.47777
Gold does not granger cause IT					**IT does not granger cause Gold**				
lX = lY = 1,	1.17985	0.11903	1.19833	0.115394	lX = lY = 1,	1.760472	0.039164	1.774346	0.038003
lX = lY = 2,	1.975578	0.024101	2.008321	0.022305	lX = lY = 2,	1.38332	0.083283	1.219869	0.111257
lX = lY = 3,	1.049135	0.147058	1.205521	0.114001	lX = lY = 3,	0.97707	0.164267	0.856619	0.195828
lX = lY = 4,	0.755659	0.224927	0.659781	0.254697	lX = lY = 4,	0.329384	0.370933	0.295901	0.383653
lX = lY = 5,	0.838317	0.200926	0.718879	0.236108	lX = lY = 5,	0.517029	0.302568	0.134984	0.446312
lX = lY = 6,	1.461012	0.072006	1.482202	0.069143	lX = lY = 6,	0.829815	0.203322	1.289454	0.09862
lX = lY = 7,	1.329376	0.091862	0.759734	0.223707	lX = lY = 7,	0.525551	0.2996	0.615541	0.269099
lX = lY = 8,	2.391082	0.008399	1.406333	0.079813	lX = lY = 8,	0.394317	0.346673	0.534344	0.296552
Gold does not granger cause Metals					**Metals do not granger cause Gold**				
lX = lY = 1,	0.517837	0.302286	0.367496	0.356625	lX = lY = 1,	−0.0746	0.529734	−0.1034	0.541178
lX = lY = 2,	1.914403	0.027784	1.831705	0.033498	lX = lY = 2,	0.111557	0.455588	−0.27935	0.610013
lX = lY = 3,	2.698298	0.003485	2.529175	0.005717	lX = lY = 3,	−1.02625	0.847613	−1.30449	0.903966
lX = lY = 4,	2.322281	0.010109	1.946226	0.025814	lX = lY = 4,	−0.01056	0.504214	−0.25893	0.602155
lX = lY = 5,	1.231003	0.109161	0.776565	0.218708	lX = lY = 5,	0.049727	0.48017	−0.34572	0.635221
lX = lY = 6,	0.942311	0.173017	0.465372	0.320833	lX = lY = 6,	0.414521	0.339246	0.495698	0.310054
lX = lY = 7,	0.326695	0.371949	−0.14135	0.556203	lX = lY = 7,	0.744947	0.228152	0.884604	0.188185
lX = lY = 8,	0.182323	0.427664	0.18304	0.427383	lX = lY = 8,	1.086269	0.13868	1.190997	0.116827

(*continued on next page*)

Table 4 (*continued*)

Gold does not granger cause Auto					Auto does not granger cause Gold				
Gold does not granger cause Oil & GaS					**Oil & Gas does not granger cause Gold**				
lX = lY = 1,	0.931026	0.17592	1.009235	0.156431	lX = lY = 1,	−0.21824	0.586379	0.032527	0.487026
lX = lY = 2,	1.364219	0.086249	1.157332	0.123568	lX = lY = 2,	−0.1722	0.56836	−0.18435	0.573131
lX = lY = 3,	1.035172	0.150294	0.695056	0.24351	lX = lY = 3,	0.186231	0.426132	0.364838	0.357616
lX = lY = 4,	0.717626	0.236494	0.517972	0.302239	lX = lY = 4,	−0.07197	0.528687	0.35933	0.359674
lX = lY = 5,	0.615506	0.26911	0.440969	0.329618	lX = lY = 5,	−0.59971	0.725649	−0.36944	0.644101
lX = lY = 6,	0.973136	0.165243	0.687811	0.245786	lX = lY = 6,	−0.57674	0.717944	−0.52763	0.701122
lX = lY = 7,	1.138194	0.12752	0.826959	0.20413	lX = lY = 7,	−0.28358	0.611635	0.469539	0.319342
lX = lY = 8,	1.361854	0.086622	1.316914	0.093934	lX = lY = 8,	−0.14541	0.557805	0.312358	0.377384

Note: HJ and DP respectively denote the Hiemstra and Jones (1994) and Diks and Panchenko (2005) causality tests.

independence between the percent change of the gold price and the BSE sectoral stock returns among different frequencies and quantiles. More precisely, this evidence indicates that the bearish and bullish situations in the gold market in India do not affect the BSE composite indices. Hence, gold can be a hedge asset for the Indian investors during tranquil periods. Gold could also allow investors to save their investments during crisis periods (e.g., the GFC period).

3.2. *Results of Vanilla Portfolios*

Having found that there is no significant dependence in the tails (i.e., left or right), and thus the center of distributions of the BSE stock indices and gold is relevant, a natural question arises at this point which is: is the role of gold important in the portfolio choice? Typically, we analyze two strategies: The first one is the hedged strategy. In contrast, the second is the optimal portfolio selection using the mean-variance optimization framework, which selects optimal weights by minimizing (maximizing) risk (mean) under the budget balance restrictions. For this purpose, we use the covariance matrix of the bivariate DCC- GARCH model to estimate the parameters of the optimum portfolios. Based on the Log-likelihood measure, we select the DCC(1,1)-AR(1)-GARCH(1,1) suggested as the best-fitted model of our sample series.

In Table 5, the estimated coefficients for the AR(1) (*ar1*) in the mean equation are positive and statistically significant for all the sectoral equity indices. Still, they are found to be negative and statistically significant in the gold equation. The short-run persistence of a shock (*alpha1*) is evident for each series as the estimated coefficient for *alpha1* is statistically significant. In each case, the short-run persistence of a shock is less than the long-run effect (*beta1*). The estimated coefficient for *beta1* is statistically significant for each variable, underscoring the importance of the long-run effect of a shock. The statistical significance of the *alpha1* and *beta1* terms provides evidence of volatility clustering (i.e., massive changes tend to be followed by large changes, of either sign or small changes tend to be followed by small changes). For the correlation structure, the parameters (i.e., *dcca1*, *dccb1,* and *dccg1*) are positive and statistically significant at the 1% level. These estimated coefficients sum to less than one, indicating that the dynamic conditional correlations are a mean-reverting process.

Fig. 3 also shows the 1000 one-step-ahead dynamic conditional correlations constructed using a rolling-window analysis and a refitting every 20 observations. As can be seen, the correlations fluctuate around zero over time with a negative pattern throughout the GFC period, suggesting that there is a diversification benefit from the gold market for the Indian investors. This evidence seems to be opposite to the findings of Basher and Sadorsky (2016), in which those authors find that gold generally has a positive correlation for emerging markets such as the entire Southeast Asia region.

Fig. 4 depicts the evolution of the hedging ratios of the last thousand in-sample data. We see some similarities in the fluctuations of the different optimal hedge ratios over time, thereby showing a sizeable collective drop in the second half of 2015 (except for the FMCG sector that includes non-durable goods such as packaged foods, beverages, toiletries, over-the-counter drugs, which shows a large drop in the second half of 2017, and the IT sector which indicates a large fall in the first half of 2018)[6] . Moreover, we see also that the optimal hedge values are negative for a long time. This negative value is formed by either buying (going long) or selling (going short) (i.e., between the gold stock and a sectoral equity index). Furthermore, there is a changing trend in the hedge ratios for the gold-BSE stock index pairs, which indicates that investors must adjust their hedging position over time according to the bull- and bear market conditions.

[6] FMCG sector is the 4th largest sector in the Indian economy. Growing awareness and easier access to changing lifestyles have been the key growth drivers for this sector. In addition, the retail market in India is one of the top five retail markets in the world by an economic value, with the modern trade is expected to grow at 20-25 percent per year, which is likely to boost the revenues of the FMCG companies. Concerning the IT sector, India is the leading sourcing destination across the world. Indian IT & ITeS companies (like Infosys, Wipro, TCS and Tech Mahindra, etc.) have set up over 1,000 global delivery centres in about 80 countries across the world. India has also become the digital capabilities hub of the world with around 75 per cent of the global digital talent is present in the country. Finally, India is the topmost offshoring destination for IT companies across the world. Having proven their capabilities in delivering both on-shore and off-shore services to global clients, emerging technologies now offer an entire new gamut of opportunities for the top IT firms in India. (See India brand equity foundation at https://www.ibef.org).

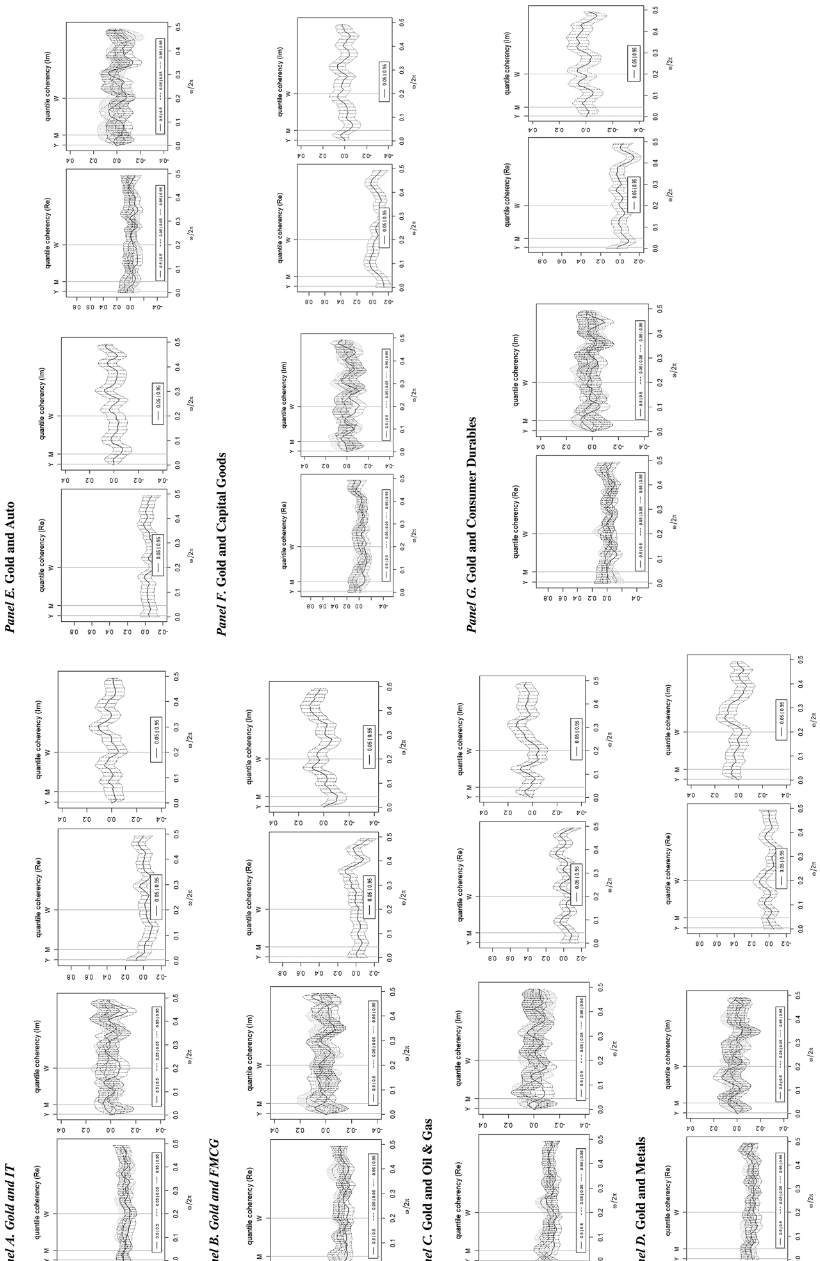


Fig. 2. Quantile Coherency Results.

Table 5
Regression Parameters of the DCC-GARCH Models.

	Estimate	Std. Error	t value	Pr(> |t|)
[IT].mu	0.074481	0.021666	3.4377	0.000587
[IT].ar1	0.043796	0.015226	2.8764	0.004022
[IT].omega	0.036542	0.024201	1.5099	0.131058
[IT].alpha1	0.0863	0.034282	2.5173	0.011825
[IT].beta1	0.910233	0.035765	25.4503	0
[IT].shape	4.810812	0.349819	13.7523	0
[FMCG].mu	0.064576	0.015326	4.2134	0.000025
[FMCG].ar1	0.032564	0.015045	2.1644	0.030433
[FMCG].omega	0.056832	0.018679	3.0426	0.002346
[FMCG].alpha1	0.100699	0.019775	5.0923	0
[FMCG].beta1	0.871304	0.027188	32.0478	0
[FMCG].shape	5.467106	0.42383	12.8993	0
[Oil.and.Gas].mu	0.068086	0.019989	3.4062	0.000659
[Oil.and.Gas].ar1	0.068057	0.015	4.5373	0.000006
[Oil.and.Gas].omega	0.05573	0.014204	3.9235	0.000087
[Oil.and.Gas].alpha1	0.089928	0.013379	6.7216	0
[Oil.and.Gas].beta1	0.892342	0.016039	55.6349	0
[Oil.and.Gas].shape	6.78227	0.640333	10.5918	0
[Metals].mu	0.075405	0.026226	2.8752	0.004037
[Metals].ar1	0.09665	0.014964	6.4589	0
[Metals].omega	0.124288	0.02724	4.5627	0.000005
[Metals].alpha1	0.102987	0.01322	7.7901	0
[Metals].beta1	0.870538	0.016882	51.5658	0
[Metals].shape	6.621708	0.576097	11.4941	0
[Auto].mu	0.103889	0.01989	5.2231	0
[Auto].ar1	0.118596	0.014972	7.9211	0
[Auto].omega	0.064446	0.017372	3.7099	0.000207
[Auto].alpha1	0.100486	0.015328	6.5556	0
[Auto].beta1	0.872044	0.020513	42.5114	0
[Auto].shape	8.481961	0.970083	8.7435	0
[Capital.goods].mu	0.106723	0.022108	4.8273	0.000001
[Capital.goods].ar1	0.118737	0.015115	7.8558	0
[Capital.goods].omega	0.069415	0.018901	3.6725	0.00024
[Capital.goods].alpha1	0.113163	0.017108	6.6147	0
[Capital.goods].beta1	0.868577	0.020288	42.8125	0
[Capital.goods].shape	6.897929	0.657754	10.4871	0
[Consumer.durables].mu	0.109732	0.021634	5.0721	0
[Consumer.durables].ar1	0.082843	0.015004	5.5213	0
[Consumer.durables].omega	0.082373	0.029144	2.8264	0.004707
[Consumer.durables].alpha1	0.107118	0.022023	4.864	0.000001
[Consumer.durables].beta1	0.874222	0.02707	32.295	0
[Consumer.durables].shape	5.452688	0.403329	13.5192	0
[Gold].mu	0.027738	0.011198	2.477	0.013248
[Gold].ar1	−0.02639	0.014102	−1.8715	0.061273
[Gold].omega	0.014886	0.005093	2.9226	0.003471
[Gold].alpha1	0.062277	0.011959	5.2075	0
[Gold].beta1	0.927063	0.014394	64.4064	0
[Gold].shape	4.858405	0.357664	13.5837	0
[Joint]dcca1	0.012509	0.00179	6.9881	0
[Joint]dccb1	0.966855	0.006727	143.7194	0
[Joint]mshape	8.856338	0.321624	27.5363	0
Information Akaike Criteria	25.391			
Log-Likelihood	−61392			
Av.Log-Likelihood	−12.68			
Bayesian Information Criteria	25.496			
Shibata	25.390			
Hannan-Quinn	25.428			

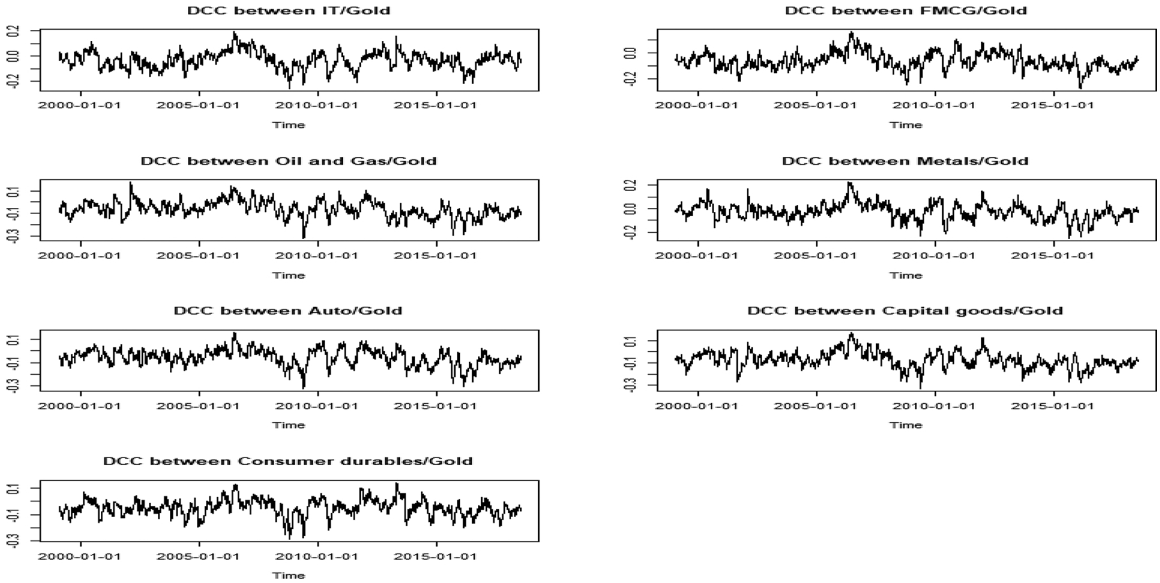


Fig. 3. Plots of DCC between the BSE Sectoral Indices and Gold.

Fig. 4. Optimal Hedge Ratios Computed between the BSE Sectoral Indices and a Position on the Gold.

Table 6
Hedge-Ratio Summary Statistics and Hedging Effectiveness (HE).

	Min.	1 st Qu.	Median	Mean	3rd Qu.	Max.	HE
IT	−0.3178	−0.102	−0.0569	−0.0605	−0.0134	0.1413	0.009036
FMCG	−0.4444	−0.1467	−0.0991	−0.1094	−0.072	0.0299	0.04735
Oil and Gas	−0.7154	−0.2112	−0.1513	−0.1518	−0.0811	0.0833	0.04735
Metals	−0.5786	−0.1818	−0.1032	−0.1162	−0.0437	0.1854	0.02556
Auto	−0.5924	−0.1855	−0.1196	−0.1308	−0.0788	0.091	0.04493
Capital goods	−0.6501	−0.2082	−0.1612	−0.169	−0.116	0.0309	0.03315
Consumer durables	−0.4312	−0.15	−0.1013	−0.1096	−0.0579	0.1135	0.006262

Note. *Refitting* = 20 observations and 1000 path one-step forecasts.

Table 7
Results for Sensitivity Analysis.

	For forecast 500 and re 20				For forecast 1000 and re 10				For forecast 1000 and re 60			
	Mean	Min	Max	HE	Mean	Min	Max	HE	Mean	Min	Max	HE
IT	−0.045	−0.325	0.133	−0.002	−0.060	−0.317	0.144	0.008	−0.060	−0.317	0.141	0.008
FMCG	−0.121	−0.454	−0.005	0.0288	−0.109	−0.444	0.029	0.047	−0.109	−0.435	0.029	0.047
Oil and Gas	−0.150	−0.329	0.007	0.028	−0.151	−0.715	0.086	0.047	−0.151	−0.723	0.079	0.047
Metals	−0.106	−0.415	0.114	0.003	−0.116	−0.578	0.188	0.025	−0.115	−0.581	0.18	0.025
Auto	−0.127	−0.344	0.090	0.012	−0.130	−0.592	0.091	0.044	−0.130	−0.604	0.083	0.044
Capital goods	−0.158	−0.357	−0.021	0.016	−0.169	−0.65	0.030	0.033	−0.168	−0.657	0.030	0.033
Consumer durables	−0.115	−0.343	0.106	0.006	−0.109	−0.431	0.113	0.006	−0.109	−0.439	0.103	0.006

Table 6 also provides some statistical description of the values of the hedge ratio and the hedging effectiveness (HE) of each pair. One can see that the hedge ratios are generally negative, with positive maximum values. This issue has produced an HE mainly closed to zero. This is also the case for which we study the sensitivity of the HE for different forecasting and refitting lengths: 500/20, 1000/10, and 1000/60 (See Table 7). The exception is with the IT sector, which exhibits a negative HE using a path of 500 previously data observations and 20 predict future values (i.e., 20 observations). In sum, these findings highlight that gold is not a useful hedge tool against the BSE sectoral stock indices, except the IT sector index. This result is relatively in line with the findings of Bilgin et al. (2018) and Gozgor et al. (2019) which suggest that gold can be a hedge[7] and a safe haven instrument, particularly during times of economic uncertainty and geopolitical risks.

The previous results have mainly validated the presence of independence between gold and various BSE equity sectors. These results have potentially important implications for investor diversification and risk management. For this reason, we examine the solutions of three problems: the maximum return portfolio allocation, the GMVP allocation, and the Markowitz portfolio allocation. The results of these three problems are presented in Fig. 5.

The results in panel "A" relate to the maximum return portfolio allocation. The optimal solution is to allocate all the budget in the Auto sector without diversification. This evidence seems associated with our preliminary investigation, which shows that this sector has the highest percentage returns. The results related to GMVP allocation are provided in panel "B" of Fig. 5. Here we observe a different solution compared to the above one. In this case, the budget is allocated to most of the sectoral indices and gold to diversify the risk. More precisely, more than 50% of the budget is concentrated in gold, and the rest is diversified among the sectoral indices, with a high percentage in the BSE FMCG sector followed by the Auto sector. Hence, gold can be used as an investment tool to reduce the portfolio risk when investors ignore the return and focus on the risk only.

Finally, the analysis of the mean-variance Markowitz portfolio is illustrated in panel "C." The solution seems to be not very diversified (i.e., 90% BSE auto and 10% BSE consumer durables). This issue is one of the reasons why practitioners do not use it. The other reason has to do with the sensitivity or the lack of robustness of the estimation of the parameters, as we will explore next.

4. Robustness Checks

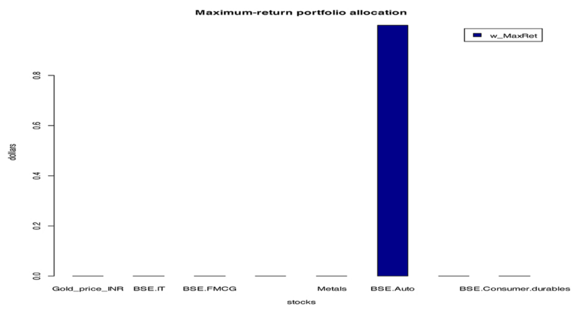
4.1. ADCC Model

For the robustness check, and following Basher and Sadorsky (2016), we propose a regression framework based on the ADCC and GO-GARCH models (the details are reported in Appendix I)[8] . Furthermore, we examine the robustness of the role of gold as a hedge

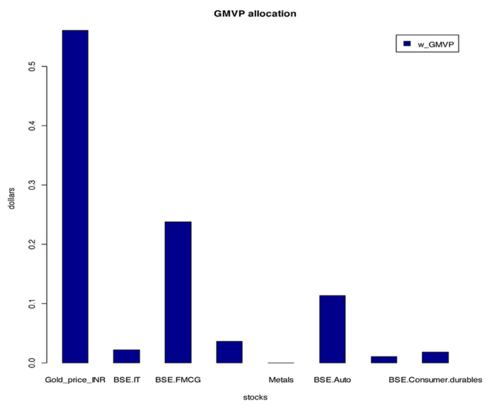
[7] Recall that a hedge is defined by Baur and Lucey (2010) as an asset that is uncorrelated or negatively correlated with another asset or portfolio on average.

[8] These models can be seen as a natural generalization of our methodology, in which ADCC can capture the impact of negative and positive news of gold prices on BSE, while the GO-GARCH model can be used to estimate some possible spill-over effects between time series besides the persistence in volatility and the time-varying correlation.

Panel A. Maximum Return Portfolio Allocation



Panel B. Global Minimum Variance Portfolio Allocation



Panel C. Markowitz Portfolio Allocation

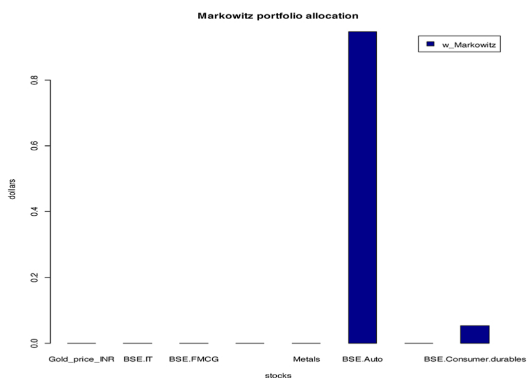


Fig. 5. Vanilla Optimum Portfolio Problems among Gold and BSE Sectoral Indices.

and a diversification with the most robust portfolio optimization (the details are reported in Appendix II). Table 8 reports the estimation results of the ADCC model. The findings from the DCC and ADCC models are very similar.[9] On the other hand, the Gamma1 related to the estimated asymmetric term is positive and statistically significant for all sectors (except the IT sector for which Gamma1 is not significant). This evidence indicates that for the BSE sector index, the negative residuals tend to increase the variance (conditional volatility) more than the positive shocks of the same magnitude. The estimated asymmetrical term is, however, negative

[9] Note that DCC and ADCC are each estimated with multivariate Student t (MVT) distributions. The GO-GARCH is estimated with the multivariate affine normal inverse Gaussian (MANIG) distribution. These distributions are useful for modelling data with heavy tails (Baker and Sadorsky, 2016).

Table 8
Regression Parameters: The ADCC-AR-GARCH Model.

	Estimate	Std. Error	t value	Pr(> |t|)
[IT].mu	0.067749	0.020494	3.3058	0.000947
[IT].ar1	0.044337	0.015216	2.9137	0.003571
[IT].omega	0.049426	0.026514	1.8642	0.062298
[IT].alpha1	0.078183	0.022047	3.5462	0.000391
[IT].beta1	0.893459	0.034463	25.9255	0
[IT].gamma1	0.047553	0.029044	1.6372	0.101579
[IT].shape	4.846449	0.35338	13.7146	0
[FMCG].mu	0.053826	0.015637	3.4423	0.000577
[FMCG].ar1	0.03445	0.015048	2.2893	0.022063
[FMCG].omega	0.064415	0.020336	3.1675	0.001538
[FMCG].alpha1	0.07117	0.014849	4.7929	0.000002
[FMCG].beta1	0.862355	0.028011	30.7867	0
[FMCG].gamma1	0.068742	0.022795	3.0157	0.002564
[FMCG].shape	5.577676	0.436921	12.7659	0
[Oil & Gas].mu	0.059172	0.020571	2.8764	0.004022
[Oil & Gas].ar1	0.069494	0.015037	4.6216	0.000004
[Oil & Gas].omega	0.058559	0.01495	3.917	0.00009
[Oil & Gas].alpha1	0.075746	0.011886	6.3729	0
[Oil & Gas].beta1	0.889698	0.016364	54.3698	0
[Oil & Gas].gamma1	0.031633	0.015446	2.048	0.040557
[Oil & Gas].shape	6.783551	0.642212	10.5628	0
[Metals].mu	0.062255	0.026742	2.328	0.019914
[Metals].ar1	0.099688	0.015119	6.5934	0
[Metals].omega	0.130484	0.029447	4.4312	0.000009
[Metals].alpha1	0.080257	0.012559	6.3906	0
[Metals].beta1	0.868197	0.017889	48.5333	0
[Metals].gamma1	0.046261	0.016913	2.7353	0.006232
[Metals].shape	6.61671	0.57572	11.4929	0
[Auto].mu	0.084129	0.020184	4.1681	0.000031
[Auto].ar1	0.124098	0.015089	8.2242	0
[Auto].omega	0.078645	0.02133	3.6871	0.000227
[Auto].alpha1	0.0629	0.011117	5.6582	0
[Auto].beta1	0.857956	0.023268	36.8722	0
[Auto].gamma1	0.088795	0.023191	3.8289	0.000129
[Auto].shape	8.961126	1.106315	8.1	0
[Capital goods].mu	0.087793	0.022041	3.9832	0.000068
[Capital goods].ar1	0.120562	0.015035	8.0188	0
[Capital goods].omega	0.082757	0.021767	3.802	0.000144
[Capital goods].alpha1	0.074829	0.013034	5.741	0
[Capital goods].beta1	0.858249	0.021997	39.0162	0
[Capital goods].gamma1	0.087355	0.022351	3.9083	0.000093
[Capital goods].shape	7.10896	0.708765	10.0301	0
[Consumer durables].mu	0.100553	0.021833	4.6055	0.000004
[Consumer durables].ar1	0.084859	0.015111	5.6158	0
[Consumer durables].omega	0.096576	0.033014	2.9253	0.003442
[Consumer durables].alpha1	0.090458	0.016798	5.3852	0
[Consumer durables].beta1	0.863876	0.028223	30.6091	0
[Consumer durables].gamma1	0.045072	0.021515	2.095	0.036174
[Consumer durables].shape	5.503579	0.412884	13.3296	0
[Gold].mu	0.033969	0.011187	3.0366	0.002393
[Gold].ar1	−0.02861	0.014109	−2.0281	0.042553
[Gold].omega	0.014984	0.004234	3.5393	0.000401
[Gold].alpha1	0.088475	0.014668	6.0318	0
[Gold].beta1	0.92756	0.011368	81.5945	0
[Gold].gamma1	−0.05365	0.013441	−3.9917	0.000066
[Gold].shape	4.962826	0.371978	13.3417	0
[Joint]dcca1	0.012111	0.001317	9.1938	0
[Joint]dccb1	0.947462	0.01083	87.4889	0
[Joint]dccg1	0.013459	0.004202	3.2026	0.001362
[Joint]mshape	9.165123	0.353414	25.9331	0
Log-Likelihood	−61438			
Av.Log-Likelihood	−12.69			
AIC	25.414			
Bayesian Information Criteria	25.531			
Shibata	25.413			
Hannan-Quinn	25.455			

Table 9
Regression Parameters: The GO-GARCH Model.

	IT	FMCG	Oil & Gas	Metals	Auto	Capital goods	Consumer durables	Gold
Omega	0.01451	0.02646	0.00293	0.03504	0.0151	0.012509	0.0275	0.02871
Alpha1	0.05644	0.07459	0.03164	0.07316	0.05842	0.062208	0.07032	0.07242
Beta1	0.9283	0.89951	0.96392	0.89098	0.92727	0.925519	0.90241	0.89912
Skew	0.11774	−0.0864	−0.064	−0.0785	0.1218	0.00434	−0.0574	−0.0868
Shape	1.7707	1.75704	0.95476	2.02463	1.37729	1.231648	3.07686	1.33093
LL	−61391.5							

and statistically significant for gold, indicating that the negative residuals for this series tend to decrease the variance. Furthermore, the dynamic conditional correlations from the ADCC model are also mean reverting. Finally, the Akaike Information Criteria (AIC) shows that the ADCC model rather than the DCC model is the best fitting model of our sample (see Table 9).

4.2. GO-GARCH Model

For the GO-GARCH model, the parameter estimates are reported in Table 9.[10] Given that this model estimates factors, then there are no standard errors to be estimated. For each factor, the estimated short-run persistence (*Alpha1*) is considerably less than the long-run persistence (*beta1*), which is consistent with the results from the DCC and ADCC models. The signs of the fourth factor are, however, mixed. The correlations estimated from the GO-GARCH model shows similar patterns to those given by the DCC and ADCC models (Fig. 6). Mainly, the correlations from each model estimation have been trending down in the late 2015 and early 2016, with reasonably strong correlations related to those given by the DCC type model. The correlations from the GO-GARCH model, however, show much less variability. Intuitively, the negative relation between gold and the BSE stock returns means that there is a diversification benefit from the gold market for the Indian Investors.

4.3. Model Comparisons

For each pair of correlations, the dynamic conditional correlations produced from the DCC model correlate very highly with those DCCs produced from the ADCC model (See Table 10). The correlations between the correlations of DCC and GO-GARCH (or ADCC and GO-GARCH) are, however, close to zero, which is consistent with Fig. 6. One way to further investigate these differences is to look at the news impact correlation surfaces.

Because the lowest correlation between DCC or ADCC and GO-GARCH occurs in all sectors, we present the results in Fig. 7 that only represent the news impact correlation surface between the IT and gold to save space.[11]

It is found that the DCC news impact correlation surface between IT and gold shows a very similar shape to that of the ADCC (see Fig. 7b). However, the GO-GARCH news impact correlation surface (see Fig. 7c) shows very different patterns. The DCC and ADCC models demonstrate a new shape. Along the z_1 axis (IT), the correlation surfaces between IT and gold trace out a positive to a negative pattern. By comparison, along with the z_2 axis (gold), the correlation surfaces trace out a negative to a positive relationship. In either case, shocks to IT or gold have asymmetric effects on the correlation between these two assets. In the case of the GO-GARCH, the news impact correlation surface is concave, whereas the news impact correlation surface for DCC and ADCC is more convex.

Moreover, the GO-GARCH news impact correlation surface displays more symmetry than either of the DCC or ADCC models. This evidence is expected since the GO-GARCH factors are orthogonalized. Also, one may notice that the correlations between the returns of the IT and gold are all negative in the case of the GO-GARCH. Recall, however, that for the GO-GARCH, the shocks pertain to factors.

4.4. Optimal Hedge Ratios

Fig. 8 shows the optimal hedge ratios computed between a position in the BSE sectoral indices against a position in gold. The DCC and ADCC hedge ratios are very similar since all hedge ratios show a substantial drop in the late 2015 and early 2016.

Other statistics on the hedge ratios are also reported in Table 11. Except for the IT sector, Table 11 shows that the HEs estimated from the ADCC and GO-GARCH models are positive and close to zero, which is in line with the results related to the DCC model. Thus, the risk facing an Indian investor from a position in the IT sector can be hedged by an opposite position in gold. However, gold cannot be an excellent way to hedge for other sectors.

The second robustness check consists of solving robust portfolio selection problems. In these problems, the optimal portfolio considers statistical and modelling errors in the estimates of the relevant market parameters (See Appendix II). Fig. 9 depicts the solution of each robust portfolio problem.

We show that the solutions when we introduce some errors in the returns or/and the covariance matrix appear quite the same

[10] The rotation matrix and the mixing matrix suitable for GO-GARCH estimations are available upon request.
[11] Other plots are also available upon request.

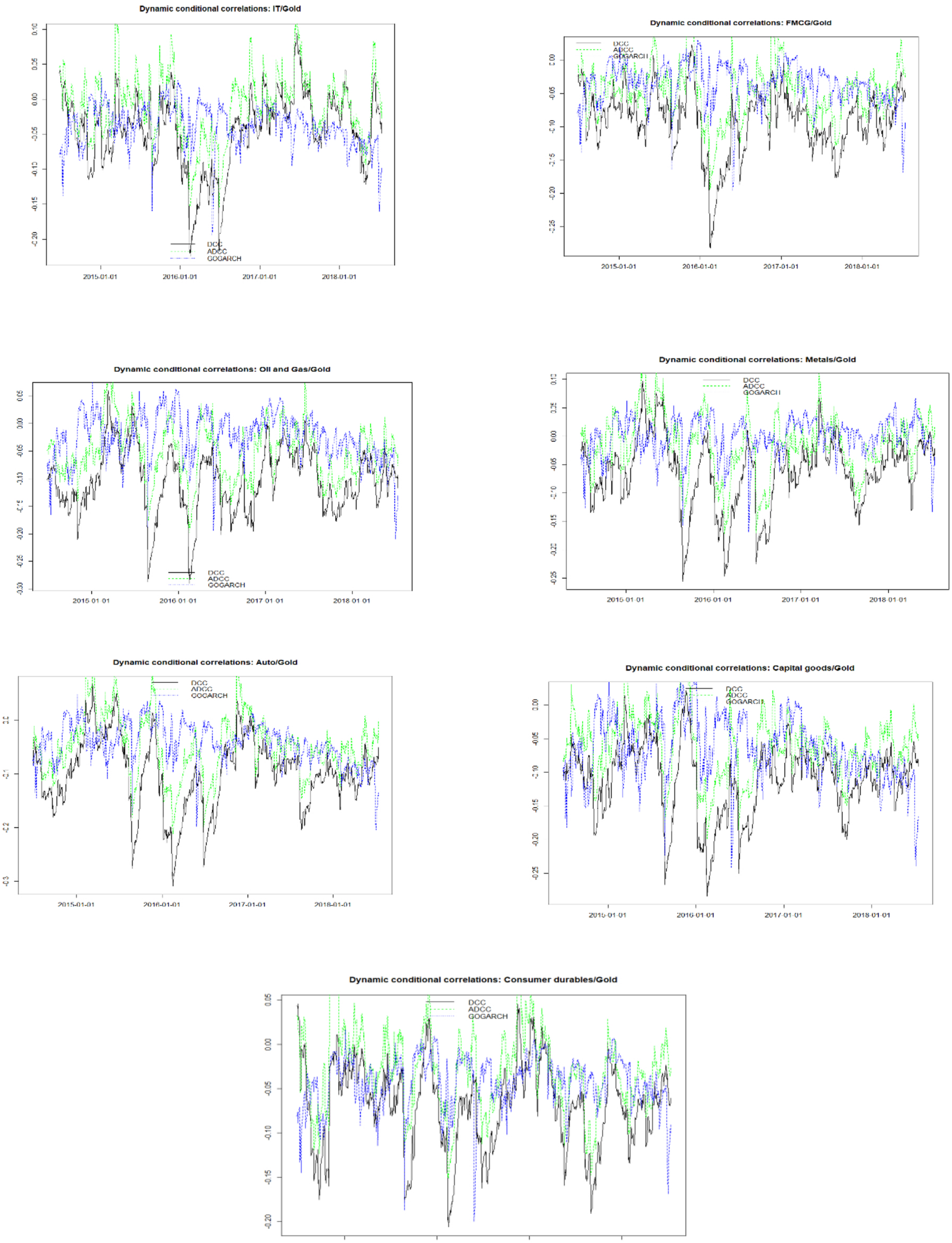
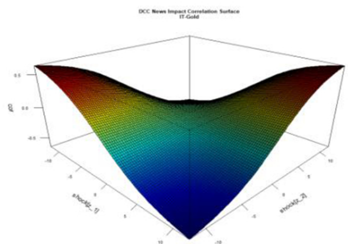


Fig. 6. Comparison between Dynamic Correlations Based on the DCC, ADCC and GO GARCH Models.

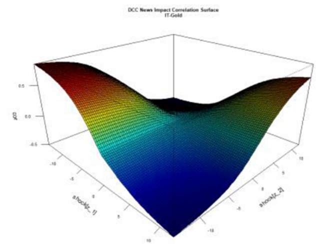
Table 10
Correlations between Correlations.

	IT	FMCG	Oil and Gas	Metals	Auto	Capital goods	Consumer durables
DCC/ADCC	0.9389	0.9439	0.9326	0.9278	0.9359	0.9351	0.9213
DCC/GO-GARCH	0.04806	0.05545	−0.07794	−0.06493	0.007691	−0.1087	0.01232
ADCC/GO-GARCH	0.07588	0.02579	−0.05373	−0.07801	0.001366	−0.138	0.01799
	Rolling results						
DCC/ADCC	0.9413	0.9387	0.9375	0.9374	0.9564	0.9348	0.9223
DCC/GO-GARCH	−0.04295	0.05554	0.1791	0.1273	0.09706	0.04579	0.3291
ADCC/GO-GARCH	0.0211	0.06488	0.1746	0.1216	0.1306	0.05853	0.3018

a) **DCC News Impact Correlation Surface: IT/Gold**



b) **ADCC News Impact Correlation Surface: IT/Gold**



c) **GO-GARCH News Impact Correlation Surface: IT/Gold**

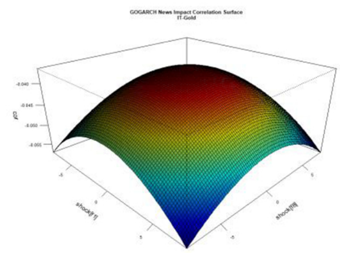


Fig. 7. News Impact Correlation Surface between IT and Gold: The DCC, ADCC and GO-GARCH Models.

from a realization to a realization. Gold is retained in each problem with a percentage of around 50%, but the rest of the budget is well-diversified among the BSE sectors but with a high percentage for the FMCG sector as this sector recorded a high growth rate during the last five years (see Footnote 7). For the optimal weight of gold in the robust portfolios, it seems significantly higher than any estimates found in most previous studies based just on the mean and variance approaches (e.g., Bruno and Chincarini's (2010) suggested a level of 9.5%). This evidence reflects that the optimal weight of gold in the optimal portfolio rises with how much risk investors should have.

Overall, the identification of gold as a highly effective vehicle for diversification is robust when we consider robust portfolio

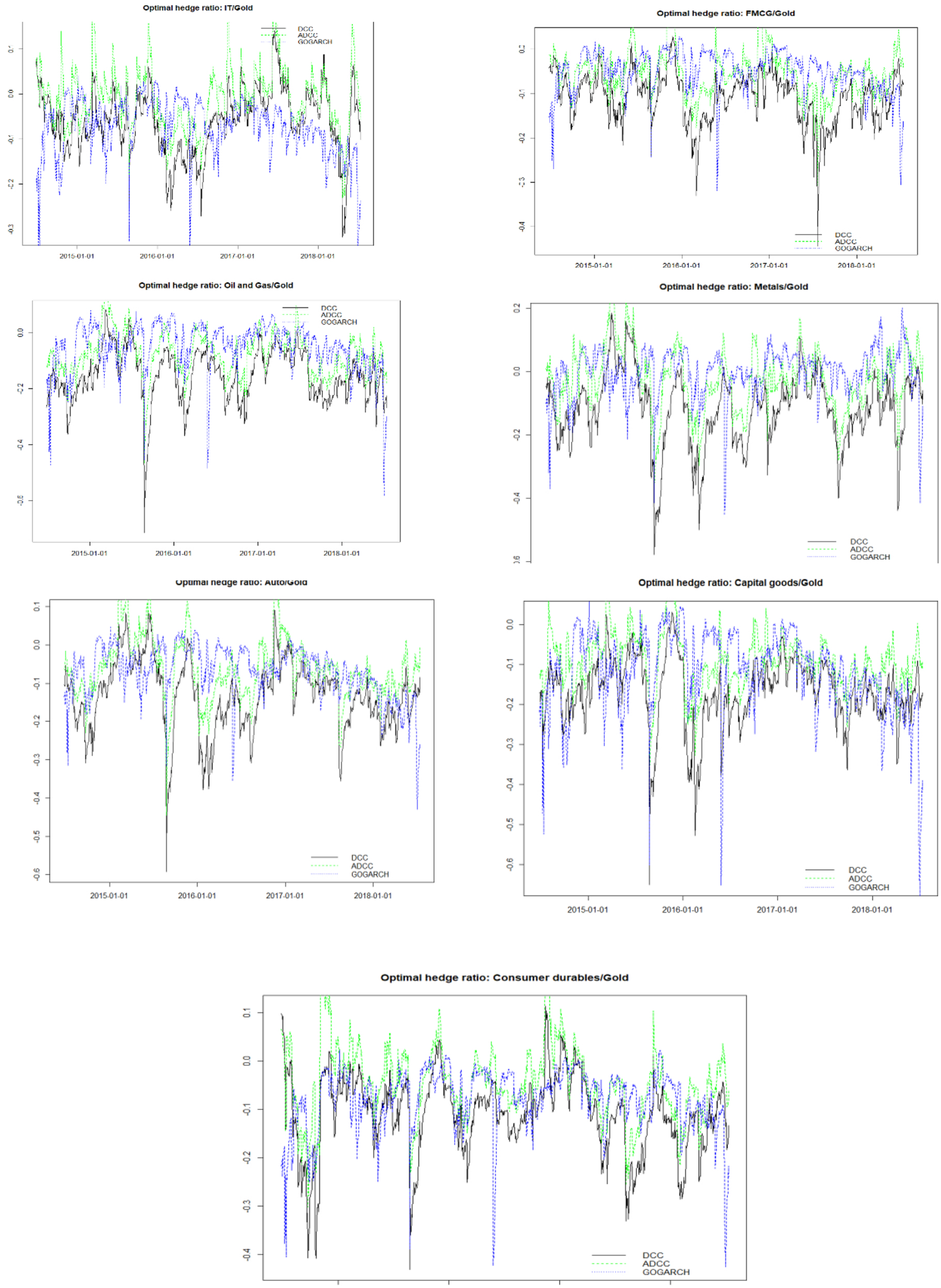


Fig. 8. Optimal Hedge Ratio between the BSE Sectoral Indices and Gold Market.

Table 11
Hedge-Ratio Summary Statistics and Hedging Effectiveness (HE): The ADCC and GO-GARCH Models.

		Min.	1 st Qu.	Median	Mean	3rd Qu.	Max.	HE
ADCC	**IT**	−0.22996	−0.05793	−0.01072	−0.00989	0.03018	0.25852	−0.22996
	FMCG	−0.3072	−0.0873	−0.0521	−0.0545	−0.0298	0.202	0.01325
	Oil and Gas	−0.4669	−0.1275	−0.0744	−0.076	−0.0267	0.2693	0.03314
	Metals	−0.3699	−0.0792	−0.02	−0.0226	0.0377	0.4279	0.0137
	Auto	−0.4449	−0.1106	−0.0664	−0.0607	−0.0201	0.4502	0.03004
	Capital goods	−0.4664	−0.1287	−0.0916	−0.0896	−0.0514	0.2639	0.02094
	Consumer durables	−0.3019	−0.0858	−0.0468	−0.0438	0.0038	0.3845	0.002704
GO-GARCH	**IT**	−0.3882	−0.1038	−0.0651	−0.0735	−0.0284	0.0403	−0.003
	FMCG	−0.3187	−0.0757	−0.0437	−0.0514	−0.0168	0.028	0.00735
	Oil and Gas	−0.5844	−0.0918	−0.0341	−0.0516	0.0097	0.0803	0.01426
	Metals	−0.4537	−0.0386	0.0066	−0.0049	0.0466	0.2015	0.00255
	Auto	−0.4289	−0.1048	−0.0611	−0.0696	−0.0171	0.0469	0.01392
	Capital goods	−0.6862	−0.1769	−0.1075	−0.1208	−0.0428	0.0578	0.01299
	Consumer durables	−0.425	−0.1097	−0.0665	−0.0805	−0.0352	0.0215	0.0102

problems as well as the vanilla GMVP problem. This finding is directly in line with the previous findings of several papers using other markets' data (e.g., Baur and McDermott, 2010; Bredin et al., 2015; Choudhry et al., 2015; Ciner et al., 2013; Gil–Alana et al., 2017; Joy, 2011; Lau et al., 2017; Kang et al., 2017; Reboredo, 2013).

5. Conclusion

In this paper, we have examined the relationship between the returns of gold and changes in seven sectoral indices for the period from January 3, 2000, to May 25, 2018. For this purpose, we have performed the nonlinear Granger Causality test of Diks and Panchenko (2005) and the Quantile Coherency analysis of Baruník and Kley (2019). We have also investigated the role of gold as a hedging and portfolio diversification tool. Specifically, we have focused on three vanilla portfolio problems: The maximum return portfolio allocation, the GMVP problem, and the Markowitz portfolio allocation. The covariance matrix is estimated using the DCC-ARMA-GARCH model, as introduced by Engle (2002). For conducting the robustness checks, we have employed the ADCC and the GO-GARCH models and analyzed the robust portfolio problems.

The novel findings of the paper are as follows. First, the changes in gold prices are significantly independent of the returns of the BSE sectoral indices. This evidence highlights the ability of gold to be a significant diversifying force. Second, gold can predict the future returns of the Consumer Durables, Oil & Gas, and the FMCG stock indices. This issue can help investors predict and react to where the market is headed. Third, gold can be used for hedging purposes against the IT stock index and functioning as a significant portfolio diversification tool when we consider the GMVP problem as well as the robust version of the vanilla problems. This result seems to be in line with extensive literature (See O'Connor et al., 2015), which suggests that gold can have the characteristics of a financial asset and can be incorporated as an investment asset in portfolios.

The paper also discusses potential implications for investors (traders) and risk management purposes. Precisely, the investors must adjust their hedging positions with time according to the bull- and bear market conditions. The paper also finds that gold can be used as an investment tool to reduce the portfolio risk when investors ignore the return and instead focus on the risk only. Finally, the risk facing an Indian investor can be hedged by different strategies. For instance, the IT sector can be hedged by an opposite position in gold since there is a negative relation of the IT sector returns with gold. Therefore, gold can be used for safe haven purposes. For other sectors, gold cannot serve to hedge their risk.

Future works on the subject can focus on other emerging markets (e.g., Brazil, China, and Russia) to analyze the relationship between their domestic stock markets and international financial markets by using the recent econometric techniques.

CRediT authorship contribution statement

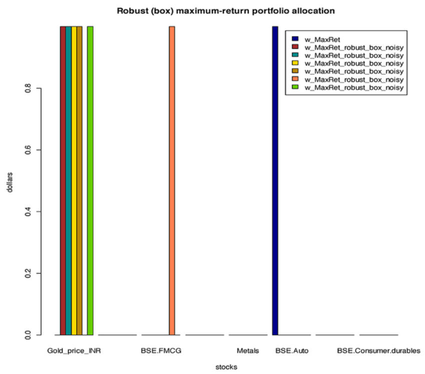
Nader Trabelsi: Writing - original draft. **Giray Gozgor:** Writing - original draft. **Aviral Kumar Tiwari:** Conceptualization, Methodology, Software, Data curation, Supervision. **Shawkat Hammoudeh:** Supervision, Writing - review & editing.

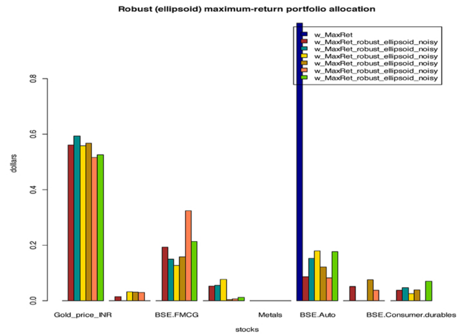
Appendix I. Details of the GARCH Models

DCC Model: One popular approach for estimating the optimal portfolio weights is to use multivariate DCC-GARCH models. Following Engle (2002), this approach considers the time-varying conditional correlation that enables us to detect and incorporate possible changes in the linkage between the BSE stock indices and the price of gold. Based on the likelihood ratio tests of the alternative specifications, the empirical study will be executed by using a bivariate GARCH (1 1) with AR (1) model. This model is defined by a mean equation expressed as:

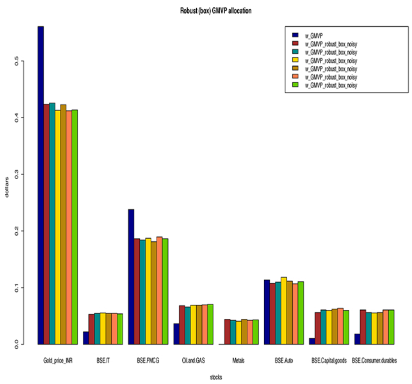
$$r_t = \mu + \phi_{t-1} r_{t-1} + \varepsilon_t \tag{A1}$$

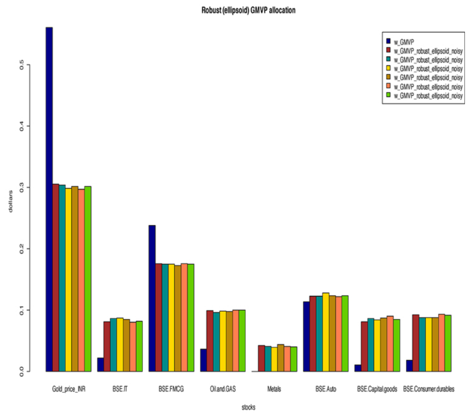
Panel A. Maximum Return Allocation





Panel B. GMVP Allocation





Panel C. Mean-variance Markowitz Allocation

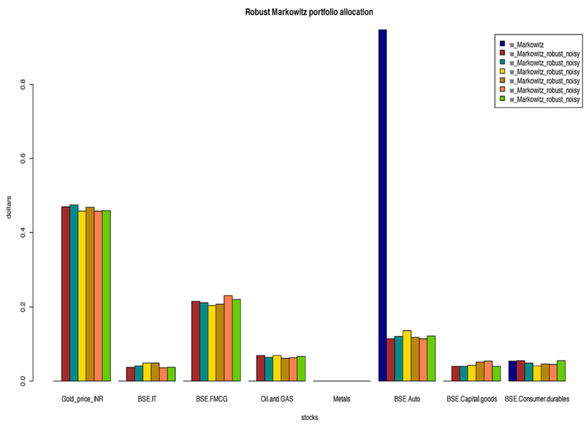


Fig. 9. Results of the Robust Portfolio Allocation.

With

$$\varepsilon_t = H_t^{1/2} z_t \tag{A2}$$

where r_t is (2×1) vector of log-returns of gold and the BSE sectoral indices at time t noted also X_t and Y_t in the tests mentioned above. ε_t is a (2×1) vector of mean-corrected returns of gold and the BSE sectoral indices at time t, i.e., $\mathrm{E}[z_t] = 0$ and $\mathrm{Cov}[z_t] = H_t$. Also, μ_t expresses the time-varying expected value of the conditional r_t following AR(1). The volatility equation is defined by:

$$H_t = D_t \Phi_t D_t \tag{A3}$$

where Φ_t is a (2×2) matrix of the conditional correlations of ε_t at time t. This issue can also be expressed by:

$$\Phi_t = diag(q_{1,t}^{-1/2},\ q_{1,t}^{-1/2}) Q_t (q_{1,t}^{-1/2},\ q_{1,t}^{-1/2}) \tag{A4}$$

with $Q_t = (1 - \theta_1 - \theta_2)\overline{\mathrm{Q}} + \theta_1 z_{t-1} z_{t-1}^{'} + \theta_2 Q_{t-1}$ and $\overline{\mathrm{Q}}$ is the (2×2) unconditional correlation matrix of the standardized residuals z_t. The correlation estimation is:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{q_{i,i} q_{j,j}} \tag{A5a}$$

H_t is (2×2) matrix of conditional variances of ε_t at time t. The element of H_t is:

$$[H_t]_{ij} = \sqrt{h_{i,t} h_{j,t}} \rho_{ij} \tag{5b}$$

D_t is the (2×2) diagonal matrix of conditional standard deviations of ε_t at time t. The elements in the diagonal matrix D_t are standard deviations from univariate GARCH models:

$$D_t = \begin{bmatrix} \sigma_{x,t} = \sqrt{h_{1t}} & 0 \\ 0 & \sigma_{y,t} = \sqrt{h_{2t}} \end{bmatrix} = diag(h_{1,t}^{1/2},\ h_{2,t}^{1/2}), \tag{A6}$$

where $h_{it} = \omega_i + \alpha_{i1} \varepsilon_{i,t-1}^2 + \beta_{i1} h_{i,\,t-1}$.

The estimation of the DCC allows a two-step methodology. In the first step, the parameters of the univariate GARCH model are estimated for each series. The quasi-likelihood is calculated with replacing Φ_t with the identity matrix $\boldsymbol{I}$. In the second stage, the parameters of the correlation structure are estimated using the correctly specified log-likelihood in Eq. (A6) (mentioned above), given the parameters of the correlation structure.

ADCC Model: The second model to be used is the ADCC-GARCH model of Cappiello et al. (2006). This approach contains the following asymmetric term in the variance equation:

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,\,t-1} + d_i \varepsilon_{i,t-1}^2 I(\varepsilon_{i,\,t-1}) \tag{A7}$$

The indicator $I(\varepsilon_{i,\,t-1})$ is equal to one if $\varepsilon_{i,\,t-1}$ is negative and 0 otherwise. The positive value for "d" means that the negative residuals tend to increase the variance more than the positive returns. For the ADCC model, the dynamics of Q are given by:

$$Q_t = (\overline{Q} - A'\overline{Q}A - B'\overline{Q}B - C'\overline{Q}C) + A^{'} z_{t-1} z_{t-1}^{'} A + B^{'} Q_{t-1} B + C' z_t^{-} z_t^{'-} C \tag{A8}$$

where A, B, and C are *two* $\times$ *two* parameter matrices and z_t^{-} are the zero-threshold standardized errors which are equal to z_t when it is less than zero, and zero otherwise. The matrices $\overline{Q}$ and $\overline{Q}'$are the unconditional matrices of $z_t^{-}$and $z_t^{'-}$, respectively.

GO-GARCH Model: Following Van Der Weide (2002), the GO-GARCH model maps a set of asset returns, r_i, onto a set of uncorrelated components, z_i, using a mapping Z.

$$r_t = Z y_t \tag{A9}$$

The unobserved components, y_t, are normalized to have unit variance. A GARCH process can describe each component of yt. In our case, we have:

$$y_t \sim N(0, H_t) \tag{A10}$$

$$H_t = diag(h_{1,t},\ h_{2,t}) \tag{A11}$$

$$H_{i,t} = \omega_i + \alpha_i y_{i.t-1}^2 + \beta_i h_{i,t-1}^2 \tag{A12}$$

The index I runs from 1 to 2. The unconditional covariance matrix of yi is $H_O = I$. the conditional covariance matrix of r_i is:

$$V_t = Z H_t Z' \tag{A13}$$

The matrix Z maps the uncorrelated components y_i to the observed returns r_t. There exists an orthogonal matrix U such that:

$$Z = P \Lambda^{1/2} \psi' \tag{A14}$$

The matrices P and Λ can be obtained from a singular value decomposition on the unconditional variance matrix V.

Appendix II . Robust Portfolio Problems

We introduce "uncertainty structures" for the three vanilla problems indicated above. In particular, we will assume that the data matrix is noisy $\widehat{X}$ and the actual matric can be written as $X = \widehat{X} + \Delta$, where Δ is some error matric bounded in its norm. Thus, we will then model the data matrix as:

$$\mathscr{U}_X = \{X \mid \|X - \widehat{X}\|_F \leq \delta_X\} \tag{A15}$$

For the maximum return portfolio allocation problem, instead of assuming μ is known entirely, we assume it belongs to some convex uncertainty set, denoted by $\mathscr{U}_\mu$. The robust worst-case formulation is:

$$\max_{w} \min_{\mu \varepsilon \mathscr{U}_\mu} w^T \mu \text{ subject to } 1^T w = 1. \tag{A16}$$

We assume an ellipsoid only knows the expected returns:

$$\mathscr{U}_\mu = \{\mu = \hat{\mu} + \kappa S^{1/2}\mathbf{u} \mid \|\mathbf{u}\|_2 \leq 1\} \tag{A17}$$

where one can use the estimated covariance matrix to shape the uncertainty ellipsoid, i.e., $\mathbf{S} = \hat{\mathbf{\Sigma}}$. The robust formulation becomes a SOCP:

$$\max_{w} w^T \hat{\mu} - \kappa \|S^{1/2} w\|_2 \text{ subject to } 1^T w = 1. \tag{A18}$$

For the GMVP, instead of assuming that Σ is known entirely, we now assume it belongs to some convex uncertainty set, denoted by $\mathscr{U}_\Sigma$. The worst robust formulation is:

$$\min_{w} \max_{\Sigma \in \mathscr{U}_\Sigma} w^T \Sigma w \text{ subject to } 1^T w = 1. \tag{A19}$$

The robust problem formulation finally becomes a Second-Order Cone Program as:

$$\min_{w} \|\widehat{X}_w\|_2 + \delta_X \|w\|_2 \text{ subject to } 1^T w = 1. \tag{A20}$$

For the Markowitz problem, the conservative and practical investment approach is to optimize the worst-case objective over the uncertainty sets:

$$\max_{w} \min_{\mu \in \mathscr{U}_\mu} w^T \mu - \lambda \max_{\Sigma \in \mathscr{U}_\Sigma} w^T \Sigma w \text{ subject to } w^T 1 = 1. \tag{A21}$$

Appendix C. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:https://doi.org/10.1016/j.ribaf.2020.101316.

References

Andrews, D.W.K., 1993. Tests for Parameter Instability and Structural Change with Unknown Change Point. Econometrica 61 (4), 821–856.
Andrews, D.W.K., Ploberger, W., 1994. Optimal Tests When a Nuisance Parameter is Present Only under the Alternative. Econometrica 62 (6), 1383–1414.
Baruník, J., Kley, T., 2019. Quantile Coherency: A General Measure for Dependence between Cyclical Economic Variables. The Econometrics Journal 22 (2), 131–152.
Barunik, J., Kočenda, E., Vácha, L., 2016. Gold, Oil, and Stocks: Dynamic Correlations. International Review of Economics and Finance 42, 186–201.
Basher, S., Sadorsky, P., 2016. Hedging Emerging Market Stock Prices with Oil, Gold, VIX, and Bonds: A Comparison between DCC, ADCC and GO-GARCH. Energy Economics 54, 235–247.
Batten, J.A., Ciner, C., Lucey, B.M., 2010. The Macroeconomic Determinants of Volatility in Precious Metals Markets. Resources Policy 35 (2), 65–71.
Batten, J.A., Ciner, C., Lucey, B.M., 2014. On the Economic Determinants of the Gold–Inflation Relation. Resources Policy 41, 101–108.
Baur, D.G., Lucey, B.M., 2010. Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold. Financial Review 45 (2), 217–229.
Baur, D.G., McDermott, T.K., 2010. Is Gold a Safe Haven? International Evidence. Journal of Banking and Finance 34, 1886–1898.
Beckmann, J., Czudaj, R., Pilbeam, K., 2015. Causality and Volatility Patterns between Gold Prices and Exchange Rates. The North American Journal of Economics and Finance 34, 292–300.
Bialkowski, J., Bohl, M.T., Stephan, P.M., Wisniewski, T.P., 2015. The Gold Price in Times of Crisis. International Review of Financial Analysis 41, 329–339.
Bilgin, M.H., Gozgor, G., Lau, C.K.M., Sheng, X., 2018. The Effects of Uncertainty Measures on the Price of Gold. International Review of Financial Analysis 58, 1–7.
Bredin, D., Thomas, C., Potì, V., 2015. Does Gold Glitter in the Long-Run? Gold as a Hedge and Safe Haven across Time and Investment Horizon. International Review of Financial Analysis 41, 320–328.
British Broadcasting Corporation (BBC), 2012. Gold v Paper Money: Which should we Trust More? BBC News Business July 2, 2012.
Broock, W.A., Scheinkman, J.A., Dechert, W.D., Lebaron, B., 1996. A Test for Independence Based On the Correlation Dimension. Econometric Reviews 15 (3), 197–235.
Cappiello, L., Engle, R.F., Sheppard, K., 2006. Asymmetric Dynamics in The Correlations of Global Equity and Bond Returns. Journal of Financial Econometrics 4 (4), 537–572.
Choudhry, T., Hassan, S.S., Shabi, S., 2015. Relationship between Gold and Stock Markets during the Global Financial Crisis: Evidence from Nonlinear Causality Tests. International Review of Financial Analysis 41, 247–256.
Ciner, C., Gurdgiev, C., Lucey, B.M., 2013. Hedges and Safe Havens: An Examination of Stocks, Bonds, Gold, Oil and Exchange Rates. International Review of Financial Analysis 29, 202–211.

Diks, C., Panchenko, V., 2005. A Note On the Hiemstra-Jones Test for Granger Non-Causality. Studies in Nonlinear Dynamics & Econometrics 9 (2), 1–9.
Engle, R., 2002. Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. Journal of Business & Economic Statistics 20 (3), 339–350.
Ewing, B.T., Malik, F., 2013. Volatility Transmission between Gold and Oil Futures under Structural Breaks. International Review of Economics and Finance 25, 113–121.
Gil–Alana, L.A., Yaya, O.S., Awe, O.O., 2017. Time Series Analysis of Co-movements in the Prices of Gold and Oil: Fractional Cointegration Approach. Resources Policy 53, 117–124.
Gozgor, G., Lau, M.C.K., Sheng, X., Yarovaya, L., 2019. The Role of Uncertainty Measures on the Returns of Gold. Economics Letters 185, 108680.
Gozgor, G., Natanelov, V., Van Huylenbroeck, G., 2015. The Impact of the Global Crisis of 2008-09 on the Relationship between Stock Markets and Oil Price in the BRIC Countries. Indian Journal of Finance 9 (4), 7–20.
Hiemstra, C., Jones, J.D., 1994. Testing for Linear and Nonlinear Granger Causality in the Stock Price-Volume Relation. The Journal of Finance 49 (5), 1639–1664.
Joy, M., 2011. Gold and the US Dollar: Hedge or Haven? Finance Research Letters 8, 120–131.
Kang, S.H., Mciver, R., Yoon, S–M., 2017. Dynamic Spillover Effects among Crude Oil, Precious Metal, and Agricultural Commodity Futures Markets. Energy Economics 62, 19–32.
Khalifa, A.A.A., Otranto, E., Hammoudeh, S., Ramchander, S., 2016. Volatility Transmission across Currencies and Commodities with US Uncertainty Measures. North American Journal of Economics and Finance 37, 63–83.
Kroner, K.F., Sultan, J., 1993. Time-varying Distributions and Dynamic Hedging with Foreign Currency Futures. Journal of Financial and Quantitative Analysis 28 (4), 535–551.
Kyereboah-Coleman, A., Agyire-Tettey, K.F., 2008. Impact of Macroeconomic Indicators on Stock Market Performance. Journal of Risk Finance 9, 378.
Lau, M.C.K., Vigne, S.A., Wang, S., Yarovaya, L., 2017. Return Spillovers between White Precious Metal ETFs: The Role of Oil, Gold, and Global Equity. International Review of Financial Analysis 52, 316–332.
Li, S., Lucey, B.M., 2017. Reassessing the Role of Precious Metals as Safe Havens–What Colour is your Haven and Why? Journal of Commodity Markets 7, 1–14.
Lucey, B.M., Sharma, S.S., Vigne, S.A., 2017. Gold and Inflation(s) – A Time-varying Relationship. Economic Modelling 67, 88–101.
O'Connor, F.A., Lucey, B.M., Batten, J.A., Baur, D.G., 2015. The Financial Economics of Gold – A Survey. International Review of Financial Analysis 41, 186–205.
Reboredo, J.C., 2013. Is Gold a Safe Haven or a Hedge for the US Dollar? Implications for Risk Management. Journal of Banking and Finance 37, 2665–2676.
Reuters, 2018. India's May Gold Imports Plummet as Price Rally Dents Demand. Money News Article Reuters, June 6, 2018.
Van der Weide, R., 2002. GO-GARCH: A Multivariate Generalized Orthogonal GARCH Model. Journal of Applied Econometrics 17 (5), 549–564.
Vigne, S.A., Lucey, B.M., O'Connor, F.A., Yarovaya, L., 2017. The Financial Economics of White Precious Metals — A Survey. International Review of Financial Analysis 52, 292–308.
Zhen, H., O'Connor, F., Thijssen, J., 2018. Is Gold a Sometime Safe Haven or an Always Hedge for Equity Investors? A Markov-Switching CAPM Approach for US and UK Stock Indices. International Review of Financial Analysis 60, 30–37.

Further reading

Reuters, 2019. Central Banks Bought More Gold in 2018 than any Year since 1967. WGC. Money News Article Reuters, January 31, 2019.