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Tax evasion, audits with memory, and portfolio choice

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ABSTRACT

In this study, we consider the memory property of tax audits to investigate the tax evasion problem from the perspective of portfolio choice. We explore the implications of the memory property for tax evasion, consumption, and asset allocation. Assuming that tax audits and jumps in the risky asset both follow self-exciting Hawkes processes, we provide a semi-analytical solution to this problem for an agent with constant relative risk aversion (CRRA) utility. We find that the memory feature does not change the agent's effective holding in the risky asset, and its effects on tax evasion and consumption are determined by the agent's risk aversion. It is suggested that government should treat agents differentially by their risk preferences and set audit-related parameters carefully to avoid unnecessary public expenditure.

1. Introduction

Tax evasion is one of the most intractable and serious issues for government. Rampant tax evasion means insufficient government revenues for debt repayments (Chan et al., 2013). Furthermore, tax evasion may lead to a rise in inequality, since the wealthy can more readily evade their taxes (Alstadsæter et al., 2019). Yet, tax evasion remains an inevitable global problem (Slemrod, 2007). According to data from Murphy (2011), tax evasion in 145 countries cost no less than USD 3.1 trillion in lost revenue in 2010, which exceeds 5% of their GDP and 54.9% of their expenditure on health. Although recent empirical evidence shows a decrease in tax evasion, the average size of the shadow economy, as a proxy for tax evasion, still exceeds 30% of GDP (e.g., Schneider et al., 2010; Buehn & Schneider, 2012; Schneider, 2012; Medina & Schneider, 2018). Consequently, combatting tax evasion efficiently and rapidly has become an urgent task for government and has drawn extensive attention and interest from researchers. In the literature, Allingham and Sandmo (1972) and Srinivasan (1973) originally provided some plausible reasons for and answers to the tax evasion issue. Since then, many others have explored this phenomenon from the perspectives of government (Roubini and Sala-i-Martin, 1995; Varvarigos, 2017; Akhtar et al, 2019), financial markets (Blackburn et al., 2012; Hanlon et al., 2015) and firms (Abdixhiku et al., 2017; Crocker & Slemrod, 2005; Gokalp et al., 2017; López, 2017). Some studies have sought to shed light on the mechanism of tax evasion (e.g., Levaggi & Menoncin, 2012; Lin & Yang, 2001; Richardson, 2006). However, the literature seldom explores the relationship between tax evasion, tax audit, and agents' behavior. Levaggi and Menoncin (2016), as an exception, offer a portfolio approach to study the effects of taxation and fines on agents' investment and consumption behavior in the presence of tax evasion; however, they pay little attention to the interplay between tax audit strategy and agents' behavior.

In this study, we focus on the relationship between agents' investment and consumption decision and tax audit strategy. In terms of tax audit strategy, considering past audit results is a natural idea to improve the efficiency of a tax audit. For instance, the experiment in

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Kleven et al. (2011) indicates that tax evasion responds negatively to prior audits, which implies that people tend to reinforce their beliefs about detection probability following a tax audit. On the other hand, Alm et al. (1993) show that the memory property of tax audits, which means the audit rate depends on the past audit results, could increase taxpayer compliance; it thus has an important influence on taxpayers' decisions. Note that, for simplicity, we hereinafter refer to the memory property of tax audits as *audit memory*. However, to the best of our knowledge, no one has incorporated this property into the theoretical study of tax evasion. We follow the same approach as was used in Levaggi and Menoncin (2016) to establish the linkage between tax audits with memory, tax evasion, and agents' investment and consumption choice. In addition, a clustering of downward jumps of financial asset prices has been observed during some periods, such as the recent 2007–08 financial crisis and the COVID-19 pandemic, which is a type of contagion and has been empirically verified (e.g., see, Lee and Mykland (2007) and Aït-Sahalia et al. (2015)). To coincide with these extreme market circumstances, we adopt a Hawkes jump-diffusion process to model the risky asset prices, in which jumps in a risky asset price are described by a self-exciting Hawkes process.

Considering audit memory, we investigate how an agent makes decisions about consumption, investment, and tax evasion in a financial market allowing for jump contagion. For agents with a CRRA utility function, we obtain the semi-analytical expressions for optimal consumption, investment, and tax evasion by adopting the method initiated by Aït-Sahalia and Hurd (2015). We find that, in the case of logarithm-utility, the level of taxation is positively correlated with tax evasion, and an increase in the penalty for tax evasion and audit intensity can effectively lower tax evasion. In the power utility case, we obtain some interesting findings. First, the effects of audit memory on tax evasion and consumption vary with the agent's risk aversion. More specifically, in the presence of audit memory, a high risk-averse agent whose relative risk aversion coefficient is more than 1 will evade more tax and reduce their consumption expenditure, while a low risk-averse agent whose relative risk aversion coefficient is less than 1 will be more compliant in taxation and promote their consumption.[1] This suggests that the audit fashion should be varied from agent to agent, in that agents with different risk aversion might behave contrarily in the same audit fashion. Second, audit memory will not affect the effective holding in the risky asset. The strategy of tax audits only influences an agent's decision on tax evasion and will not influence the agent's portfolio choice with one risky asset and one riskless asset. Third, tax audits with memory will enhance the positive effect of tax reduction and penalty for evasion on reducing tax evasion if an agent has high risk aversion and mitigate this effect if the agent has low risk aversion. Finally, when audit memory is considered, policymakers ought to be cautious about the choice of audit memory-related parameters since these parameters will determine the effects of the audit memory.

The remainder of this paper is organized as follows. Section 2 sets up the model. In Section 3, we discuss the optimal tax evasion, consumption, and asset allocation problem for the agent with CRRA utility. Section 4 explores the effects of audit memory on tax evasion, consumption, and investment. Section 5 concludes this paper.

2. Model setup

We assume that there are two assets in the economy. One is a risk-free asset paying interest at a constant rate r. The other is a risky asset paying no dividends, with price S_t at time t. In the literature, the jump-diffusion process is extensively applied to the modeling of the risky asset price, where the jump arrivals mostly follow the Lévy process. However, the Lévy process cannot capture the empirically detected jump clustering effect during financial market turmoil. In this study, we adopt the self-exciting Hawkes process to model the jump arrivals, and we assume the risky asset price is driven by the following jump-diffusion process:

$$dS_t = (\mu - \varphi\lambda_{1,t})S_t dt + \sigma S_t dB_t + S_{t-} d\sum_{i=1}^{N_{1,t}} Y_i, \tag{1}$$

where μ is the expected return, σ is the volatility of the risky return, and B_t is a standard Brownian motion. $\{Y_i\}_{i=1}^{\infty}$ are independent and identically distributed jump sizes with mean φ and support on $(-1, \infty)$. $N_{1,t}$ is a Hawkes process with the jump arrival intensity $\lambda_{1,t}$, given by

$$d\lambda_{1,t} = \beta_1\left(\overline{\lambda}_1 - \lambda_{1,t}\right)dt + \alpha_1 dN_{1,t}, \tag{2}$$

where α_1, β_1 and $\overline{\lambda}_1$ are all positive constants. For the Hawkes β_1 process $N_1(t)$, once a jump occurs, the intensity will instantly increase by α_1 and then the increment decays exponentially at a rate over time. Note that the Hawkes process $N_1(t)$ can describe the jump contagion risk of the risky asset, and it will degenerate to a Poisson process when $\alpha_1 = \beta_1 = 0$.

In this study, we consider an agent subject to capital gains taxation. As in Levaggi and Menoncin (2016), we assume that (i) capital gains are taxed continuously and in a symmetric manner, that is, the agent pays tax if the change in asset price is positive and receives a refund if it is negative[2]; and (ii) consumption will not be taxed. Denote by τ_G and τ the tax rates levied on the payoff of the riskless asset and risky asset, respectively. Accordingly, we define $r_G = r(1-\tau_G)$ as the after-tax or tax-adjusted return of the riskless asset. In this economy, the agent might conceal part of their investment in the risky asset in order to evade tax, while evasion is subject to a fine when it is detected, and the fine is a proportion of the total amount of revenue evaded, denoted by $\theta(\tau) \in [0, 1]$. As illustrated in Levaggi and

[1] Hereinafter, we refer to an agent as the high (low) risk-averse agent if he is more (less) risk-averse than the log-utility 1.

[2] The reasoning for this assumption is given in Levaggi and Menoncin (2016), and this assumption is also made in Cai et al. (2017).

Menoncin (2016), this fine function $\theta(\tau)$ includes two important cases: a fine proportional to the value of the evaded asset and a fine proportional to the tax evaded.

In terms of tax audits, in the existing literature, it is generally assumed that audits are conducted with constant intensity (state-independent), and thus the Poisson process is often applied to the modeling of audits. In contrast, it is more practical that, once the evasion is discovered by the audit authority, the agent will be faced with a larger audit intensity; nonetheless, the audit authority will gradually decrease the audit intensity to the usual level until tax evasion is found again. In other words, the audit intensity should be dependent on the past evasion. To describe this feature of tax audits, we use another Hawkes process $N_{2,t}$ to model the arrival of audits. The arrival intensity $\lambda_{2,t}$ of audits is driven by

$$d\lambda_{2,t} = \beta_2\left(\overline{\lambda}_2 - \lambda_{2,t}\right)dt + \alpha_2 1\left(\pi_t^e > 0\right)dN_{2,t}, \tag{3}$$

where $1(\cdot)$ is the indicator function, and α_2, β_2 and $\overline{\lambda}_2$ are all positive constants. It means that, when evasion is discovered, the audit intensity will instantly increase by α_2 and then the increment of intensity decays at an exponential rate β_2. Meanwhile, we assume that B_t, $N_{1,t}, N_{2,t}$ and Y_t are mutually independent.

Let W_t be the agent's wealth at time t. We assume that the agent consumes c_t at time t and there is no endowment stream. Therefore, their wealth follows the dynamics:

$$\begin{aligned} dW_t &= r_G W_t\left(1 - \pi_t - \pi_t^e\right)dt + (1-\tau)\frac{\pi_t W_t}{S_{t-}}dS_t + \frac{\pi_t^e W_t}{S_{t-}}dS_t \\ &-\theta(\tau)\pi_t^e W_t dN_{2,t} - c_t dt. \end{aligned} \tag{4}$$

where π_t and π_t^e denote the investment weights in the non-concealed risky asset and the concealed risky asset, respectively. The agent is unable to short sell the concealed investment account, and hence π_t^e must be nonnegative. Since capital gains are symmetrically taxed, the actual investment return in the risky asset, including the non-concealed account and the concealed account, would be $((1-\tau)\pi_t + \pi_t^e)W_t$ times the return of one unit risky asset. Thus, $(1-\tau)\pi_t + \pi_t^e$ can be regarded as the effective holding weight in the risky asset hereinafter denoted by κ_t. By combining (1) and (2), we obtain

$$\begin{aligned} dW_t &= \left\{\left[r_G + \pi_t[(1-\tau)(\mu - \varphi\lambda_{1,t}) - r_G] + \pi_t^e(\mu - \varphi\lambda_{1,t} - r_G)\right]W_t - c_t\right\}dt \\ &+\sigma\kappa_t W_t dB_t + \kappa_t W_t Y_t dN_{1,t} - \theta(\tau)\pi_t^e W_t dN_{2,t}, \end{aligned} \tag{5}$$

3. Optimal consumption, investment and tax evasion

In this section, we study the infinite-horizon consumption and investment problem for an agent who has utility function $U(x)$. With a positive initial wealth W_0, the agent's objective is to maximize the discounted expected utility of consumption by determining their consumption, and the fractions of their wealth invested in the non-concealed and concealed risky asset. Specifically, the optimal problem for the agent is to achieve

$$\max_{\{\pi_s, \pi_s^e, c_s : s \geq 0\}} \mathbb{E}\left[\int_0^\infty e^{-\rho s} U(c_s) ds\right] \tag{6}$$

where the wealth process W_t is governed by (2).

We now turn to solve the optimal consumption and asset allocation problem by using the stochastic control approach. Following the standard procedure, the indirect utility function or the value function is defined by

$$J(t, w, \boldsymbol{\lambda}) = \max_{\{\pi_s, \pi_s^e, c_s : s \geq t\}} \mathbb{E}_{t,w,\boldsymbol{\lambda}}\left[\int_t^\infty e^{-\rho s} U(c_s) ds\right], \tag{7}$$

where $\mathbb{E}_{t,w,\boldsymbol{\lambda}}[\cdot]$ is the expectation with initial conditions $W_t = w$ and $(\lambda_{1,t}, \lambda_{2,t}) = \boldsymbol{\lambda} = (\lambda_1, \lambda_2)$. According to the principle of optimal stochastic control, the following Hamilton-Jacobi-Bellman (HJB) equation for the indirect utility function J is achieved:

$$\begin{aligned} 0 &= J_t + \max_{\pi, \pi^e, c}\left\{e^{-\rho t} U\left(c\right) + \sum_{i=1} \beta_i\left(\overline{\lambda}_i - \lambda_i\right)J_{\lambda_i} + \left(r_G\left(1 - \pi - \pi^e\right)w\right.\right. \\ &+\kappa\left(\mu - \varphi\lambda_1\right)w - c\Big)J_w + \frac{1}{2}\kappa^2\sigma^2 w^2 J_{ww} \\ &+\lambda_1(\mathbb{E}[J((1+Y\kappa)w, \boldsymbol{\lambda} + \boldsymbol{\alpha}_1)] - J) \\ &+\lambda_2(J((1-\theta(\tau)\pi^e)w, \boldsymbol{\lambda} + \boldsymbol{\alpha}_2 1(\pi^e > 0)) - J)\Big\} \end{aligned} \tag{8}$$

where J_t, J_w, J_{λ_i} denote the partial derivatives of $J(t, w, \boldsymbol{\lambda})$ with respective to t, w, λ_i, and J_{ww} denotes the second-order partial derivative. In addition, $\boldsymbol{\alpha}_1 = (\alpha_1, 0)$ and $\boldsymbol{\alpha}_2 = (0, \alpha_2)$.

Due to the time-homogeneity of the value functions for general infinite-horizon problems, the value function J is of the form

$$J(t,w,\boldsymbol{\lambda})=e^{-\rho t}J(0,w,\boldsymbol{\lambda}):=e^{-\rho t}L(w,\boldsymbol{\lambda}).$$

Subsequently, the HJB equation reduces to

$$\begin{aligned}0=&-\rho L+\max_{\pi,\pi^e,c}\Big\{U(c)+\sum_{i=1}\beta_i\left(\bar{\lambda}_i-\lambda_i\right)L_{\lambda_i}+\Big[r_G(1-\pi-\pi^e)w\\&+\kappa(\mu-\varphi\lambda_1)w-c\Big]L_w+\frac{1}{2}\kappa^2\sigma^2w^2L_{ww}\\&+\lambda_1(\mathbb{E}[L((1+Y\kappa)w,\boldsymbol{\lambda}+\boldsymbol{\alpha}_1)]-L)\\&+\lambda_2(L((1-\theta(\tau)\pi^e)w,\boldsymbol{\lambda}+\boldsymbol{\alpha}_2 1(\pi^e>0))-L)\Big\}\end{aligned} \tag{9}$$

According to (9), the first order conditions for the optimal consumption c^* and investment weights π^*,π^{e*} are

$$U'(c^*)=L_w, \tag{10}$$

$$\sigma^2\kappa^*w=-\frac{L_w}{L_{ww}}\left(\mu-\varphi\lambda_1-\frac{r_G}{1-\tau}\right)-\frac{\lambda_1\mathbb{E}[YL_w((1+Y\kappa^*)w,\boldsymbol{\lambda}+\boldsymbol{\alpha}_1)]}{L_{ww}}, \tag{11}$$

and

$$\begin{aligned}\sigma^2\kappa^*w=&-\frac{L_w}{L_{ww}}(\mu-\varphi\lambda_1-r_G)-\frac{\lambda_1\mathbb{E}[YL_w((1+Y\kappa^*)w,\boldsymbol{\lambda}+\boldsymbol{\alpha}_1)]}{L_{ww}}\\&+\frac{\theta(\tau)\lambda_2L_w((1-\theta(\tau)\pi^{e*})w,\boldsymbol{\lambda}+\boldsymbol{\alpha}_2 1(\pi^e>0))}{L_{ww}},\end{aligned} \tag{12}$$

where $\kappa^*=(1-\tau)\pi^*+\pi^{e*}$. Subtracting (11) from (12), we obtain

$$L_w\frac{r_G\tau}{1-\tau}=\theta(\tau)\lambda_2L_w((1-\theta(\tau)\pi^{e*})w,\boldsymbol{\lambda}+\boldsymbol{\alpha}_2 1(\pi^e>0)). \tag{13}$$

Next, the optimal consumption, tax evasion and asset allocation for the agent with the CRRA utility will be derived.

3.1. Logarithm utility

For the log-utility agent with utility function $U(x)=\log x$, the optimal policy is summarized in the following proposition.

Proposition 3.1. *Let c^*, π^* and π^{e*} denote the optimal consumption, the optimal fractions of wealth invested in the unconcealed risky asset and the concealed risky asset, respectively. Let $\kappa^*=(1-\tau)\pi^*+\pi^{e*}$. Then the optimal policy for the log-utility agent is given by*

$$\begin{cases}c^*=\rho w,\\ \pi^{e*}=\dfrac{1}{\theta(\tau)}-\dfrac{(1-\tau)\lambda_2}{r_G\tau},\\ \kappa^*=\dfrac{1}{\sigma^2}\left\{\mu-\dfrac{r_G}{1-\tau}+\lambda_1\left(\mathbb{E}\left[Y(1+Y\kappa^*)^{-1}\right]-\varphi\right)\right\}.\end{cases} \tag{14}$$

Proof. See Appendix A.

Corollary 3.2. *For the log-utility agent, if short selling is not allowed in the market, the optimal weight in the unconcealed risky asset decreases with the jump intensity and increases with the audit intensity. The optimal weight in the concealed risky asset is independent of the jump intensity, while it decreases with the audit intensity.*

Proof. See Appendix B.

According to Proposition 3.1 and Corollary 3.2, it is intuitive that tax evasion decreases with the fine $\theta(\tau)$, which means penalty for tax evasion is helpful in curtailing tax evasion. If the fine function $\theta(\tau)$ is proportional to the value of the evaded asset, as given in Allingham and Sandmo (1972), clearly tax evasion will be increasing in the tax rate. If the fine function $\theta(\tau)$ is proportional to the tax evaded, say $\theta(\tau)=\alpha\tau$ with $\alpha>1$, the relationship between tax evasion π^{e*} and tax rate τ depends on fiscal parameters $(\theta(\tau),\lambda_2,\tau,\tau_G)$ and the interest rate r, which is consistent with the finding in Levaggi and Menoncin (2016). Specifically, it follows from (14) that $\frac{\partial\pi^{e*}}{\partial\tau}=\frac{\alpha\lambda_2-r_G}{\alpha r_G}\frac{1}{\tau^2}$. Therefore, the effect of tax rate on tax evasion is positive when $\alpha\lambda_2>r_G$. However, the effect is negative when $\alpha\lambda_2<r_G$, i.e., an increase in tax rate leads to a decline in tax

evasion, which is counter-intuitive, as also found by Yitzhaki (1974) and Levaggi and Menoncin (2016).

On the other hand, the log-utility agent will reduce tax evasion when the audit intensity increases. It is intuitive; their holding in the unconcealed risky asset is increasing in audit intensity. These results support the view that increasing the enforcement of tax audits is an effective way to reduce tax evasion and can improve investment activity in formal financial markets. It has also been observed that, for the log-utility agent, jumps in the risky asset merely affect their unconcealed holding, which means that their decision on tax evasion has nothing to do with the market's jump risk.

3.2. Power utility

We now specialize this problem for an agent with power utility

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma \in (0,1) \cap (1,\infty).$$

The optimal policy for the power-utility agent is concluded in the following proposition.

Proposition 3.3. *Let c^*, π^* and π^{e^*} denote the optimal consumption, the optimal fractions of wealth invested in the unconcealed risky asset and the concealed risky asset, respectively. Then the optimal policy for the agent with power utility is*

$$\begin{cases} c^* = w[h(\boldsymbol{\lambda})]^{-1/\gamma}, \\ \pi^{e^*} = \dfrac{1}{\theta(\tau)}\left\{1 - \left(\dfrac{\lambda_2\theta(\tau)(1-\tau)}{r_G\tau}\dfrac{h(\boldsymbol{\lambda}+\boldsymbol{\alpha}_2 1(\pi^{e^*}>0))}{h(\boldsymbol{\lambda})}\right)^{\frac{1}{\gamma}}\right\}, \\ \kappa^* = \dfrac{1}{\gamma\sigma^2}\left\{\mu - \varphi\lambda_1 - \dfrac{r_G}{1-\tau} + \lambda_1\dfrac{h(\boldsymbol{\lambda}+\boldsymbol{\alpha}_1)}{h(\boldsymbol{\lambda})}\mathbb{E}\left[Y(1+Y\kappa^*)^{-\gamma}\right]\right\}, \end{cases} \tag{15}$$

where

$$h(\boldsymbol{\lambda}) = \left\{-\int_0^\infty \mathbb{E}_{0,\boldsymbol{\lambda}}\left[e^{\int_0^s K(\boldsymbol{\lambda}_s)}\right]ds\right\}^\gamma$$

with $K(\cdot)$ given by (C.8).

Proof. See Appendix C.

According to Proposition 3.3, it seems that the optimal consumption and portfolio choice are "analytical". However, they are not.[3] For ease of computation, we consider a special case in which the asset price jumps follow a Poisson process, i.e.,

$$dS_t = \left(\mu - \varphi\bar{\lambda}_1\right)S_t dt + \sigma S_t dB_t + S_{t-} d\sum_{i=1}^{\tilde{N}_{1,t}} Y_i, \tag{16}$$

and the agent's wealth process follows:

$$\begin{aligned} dW_t = &\left\{\left[r_G + \pi_t\left[(1-\tau)\left(\mu - \varphi\bar{\lambda}_1\right) - r_G\right] + \pi_t^e\left(\mu - \varphi\bar{\lambda}_1 - r_G\right)\right]W_t - c_t\right\}dt \\ &+\sigma\kappa_t W_t dB_t + \kappa_t W_t Y_t d\tilde{N}_{1,t} - \theta(\tau)\pi_t^e W_t dN_{2,t}, \end{aligned} \tag{17}$$

where $\tilde{N}_{1,t}$ is a Poisson process with constant intensity $\bar{\lambda}_1$ and the corresponding result for this case is given below.

Corollary 3.4. *Suppose that the jumps of the risky asset are driven by a Poisson process with constant intensity $\lambda_1 \equiv \bar{\lambda}_1$, which is equivalent to the Hawkes process $N_{1,t}$ with $\alpha_1 = \beta_1 = 0$. Then, the optimal policy is*

$$\begin{cases} c^* = w[p(\lambda_2)]^{-1/\gamma}, \\ \pi^{e^*} = \dfrac{1}{\theta(\tau)}\left\{1 - \left(\dfrac{\lambda_2\theta(\tau)(1-\tau)}{r_G\tau}\dfrac{p(\lambda_2+\alpha_2 1(\pi^{e^*}>0))}{p(\lambda_2)}\right)^{\frac{1}{\gamma}}\right\}, \\ \kappa^* = \dfrac{1}{\gamma\sigma^2}\left\{\mu - \varphi\bar{\lambda}_1 - \dfrac{r_G}{1-\tau} + \bar{\lambda}_1\mathbb{E}\left[Y(1+Y\kappa^*)^{-\gamma}\right]\right\}, \end{cases} \tag{18}$$

where $p(x)$ is the solution to the following ordinary differential equation (ODE):

[3] The expressions of some functions related to the optimal rules in Aït-Sahalia and Hurd (2015) are similar to the expression of $h(\boldsymbol{\lambda})$.

$$\beta_2\left(\bar{\lambda}_2 - x\right)(\log p(x))' + \gamma\left(\frac{\theta(\tau)(1-\tau)}{r_G\tau}\right)^{\frac{1}{\gamma}-1}\left(\frac{xp(x+\alpha_2 1(\pi^{e^*}>0))}{p(x)}\right)^{\frac{1}{\gamma}}$$
$$+\gamma\left(p(x)\right)^{-\frac{1}{\gamma}} - x + \left(1-\gamma\right)\left\{r_G\left(1-\frac{1}{1-\tau}\kappa^* + \frac{\tau}{(1-\tau)\theta}\right)\right.$$
$$\left. +\kappa^*\left(\mu - \varphi\bar{\lambda}_1\right) - \frac{1}{2}\kappa^{*2}\sigma^2\gamma\right\} + \bar{\lambda}_1\left[\mathbb{E}(1+Y\kappa^*)^{1-\gamma} - 1\right)\Big] - \rho = 0. \tag{19}$$

Proof. Let $h(\bar{\lambda}_1, \lambda_2) = p(\lambda_2)$. Then, substituting $\lambda_1 = \bar{\lambda}_1$ and $\alpha_1 = \beta_1 = 0$ into (C.6) and (18) easily yields the desired result.

It is found from (18) that κ^* is independent of tax audits and naturally has nothing to do with the memory feature of tax audits. The economic explanation for this result is simple. Recall that κ^* can be regarded as the effective investment weight in the risky asset. Since tax audits do not influence the intrinsic risk and expected return of the risky asset, the agent with a power utility function will not adjust their effective holding in the risky asset. On the other hand, due to $\kappa^* = (1-\tau)\pi^* + \pi^{e^*}$ and $0 < 1-\tau < 1$, the rise in tax evasion, caused by the decline in audit intensity, is associated with a decrease in the total investment weight in the risky asset and an increase in the weight in the safe asset, notwithstanding the unchanged expected return and risk of the risky asset. Thus, when a representative agent has a power utility function, tax evasion could have a negative impact on investment activity in the capital market; this provides an alternative explanation for the inactivity in Brazil's stock market discussed in Kenyon (2008).

In what follows, we provide a method to numerically solve the optimal policy for the power-utility agent in the special case. According to the third equation in (18), the numerical solution for κ^* can easily be obtained given the law of Y. Then we need only to yield $p(x)$ which is determined by the ODE (19). Intuitively, if there is no restriction imposed on investment, π^{e^*} will be a monotonically decreasing continuous function of λ_2 , taking on values from the positive to the negative as λ_2 increases. It implies that there exists a threshold, denoted by λ^{thold} , such that $\pi^{e^*}(\lambda^{\text{thold}}) = 0$ and $\pi^{e^*}(\lambda_2) < 0$ for $\lambda_2 > \lambda^{\text{thold}}$. On the other hand, short selling in the concealed account is impractical, so π^{e^*} must be positive or zero. Accordingly, $\pi^{e^*} = 0$ for all $\lambda_2 \geq \lambda^{\text{thold}}$, i.e., the agent will not conceal any of their investment as the audit intensity passes some threshold. Because tax evasion will vanish when the audit intensity equals the threshold, we have $\lambda^{\text{thold}} = \frac{r_G\tau}{\theta(\tau)(1-\tau)}$, which is deduced from the expression of π^{e^*} in (18). Let $y(x) = \ln p(\lambda^{\text{thold}} - x)$, then the ODE (19) can be transformed into the following delay differential equation (DDE):

$$y'(x) = \frac{B\exp\left(-\frac{1}{\gamma}y(x)\right) + C\left\{\frac{\left(-\left(x-\lambda^{\text{thold}}\right)\right)\exp(y(x-\alpha_2))}{\exp(y(x))}\right\}^{\frac{1}{\gamma}} - D(x-\lambda^{\text{thold}}) + E}{(A_1 + A_2(-(x-\lambda^{\text{thold}})))} \tag{20}$$

where

$$A_1 = \beta_2\bar{\lambda}_2, A_2 = -\beta_2, B = \gamma, C = \gamma\left(\frac{\theta(\tau)(1-\tau)}{r_G\tau}\right)^{\frac{1}{\gamma}-1}, D = -1$$
$$E = \left\{-\rho + (1-\gamma)\left[r_G\left(1 - \frac{1}{1-\tau}\kappa^* + \frac{\tau}{\theta(\tau)(1-\tau)}\right) + \kappa^*\left(\mu - \varphi\bar{\lambda}_1\right) - \frac{1}{2}\kappa^{*2}\sigma^2\gamma\right] + \lambda_1\left[E\left[(1+Y\kappa^*)^{1-\gamma}\right] - 1\right]\right\}$$

Since $\pi^{e^*}(\lambda_2) = 0$ and $p(\lambda_2) = p(\lambda^{\text{thold}})$ for all $\lambda_2 > \lambda^{\text{thold}}$, we have $y(x) = y(0)$ for all $x < 0$. It gives rise to the following boundary condition:

$$y(0) = -\gamma\ln\left\{\frac{1}{\gamma}\left(\left(1-\gamma\right)\left(\frac{1}{2}\kappa^{*2}\sigma^2\gamma - r_G - \kappa^*\left(\mu - \varphi\bar{\lambda}_1 - \frac{r_G}{1-\tau}\right)\right) - \bar{\lambda}_1\left[\mathbb{E}(1+Y\kappa^*)^{1-\gamma} - 1\right] + \rho\right)\right\} \tag{21}$$

where κ^* is the solution to the third equation in (18). Once the DDE (20) with boundary condition (21) is solved,[4] $p(x)$, c^* and π^{e^*} can in turn be derived.

4. Comparative static analysis

In this section, to demonstrate more clearly the implications of audits with memory for the power-utility agent's tax evasion, consumption and investment decisions, we consider the Poisson jump-diffusion model (16) for the risky asset price; accordingly our analysis is based on the results in Corollary 3.4.

For comparison, we take the constant audit intensity (and constant jump intensity) case as the benchmark. In the benchmark model, tax audits are memoryless and the wealth process is given by

$$dW_t = \left\{\left[r_G + \pi_t\left[(1-\tau)\left(\mu - \varphi\bar{\lambda}_1\right) - r_G\right] + \pi_t^e\left(\mu - \varphi\bar{\lambda}_1 - r_G\right)\right]W_t - c_t\right\}dt$$
$$+\sigma\kappa_t W_t dB_t + \kappa_t W_t Y_t d\tilde{N}_{1,t} - \theta(\tau)\pi_t^e W_t d\tilde{N}_{2,t}, \tag{22}$$

[4] For instance, we can solve the DDE using the function *dde 23* in Matlab.

Table 1
The parameter values for the base scenario.

Parameter	Value	Parameter	Value
Expected rate of return	$\mu = 0.1$	Volatility of risky asset	$\sigma = 0.2$
Jump intensity	$\overline{\lambda}_1 = 0.13$	Mean of jump size	$\varphi = 0$
Variance of log jump size	$\sigma_J = 1.04 \times 10^{-4}$	Riskless interest rate	$r = 0.04$
Discount factor	$\rho = 0.04$	Penalty proportion	$\theta = 0.07$
Tax rate on riskless asset	$\tau_G = 0.27$	Tax rate on risky asset	$\tau = 0.235$
Long-term audit intensity	$\overline{\lambda}_2 = 0.095$	Current audit intensity	$\lambda_2 = 0.105$
Increment of audit intensity	$\alpha_2 = 0.1$	Decay rate	$\beta_2 = 0.15$

where $\tilde{N}_{i,t}$ are Poisson processes with constant intensities $\overline{\lambda}_i$, $i = 1, 2$. The optimal policy for the benchmark model can be directly derived by setting $\alpha_2 = \beta_2 = 0$ in Corollary 3.4. In the following analyses, we assume each jump size Y_i is log-normally distributed, say, $\log(Y_i + 1)\tilde{\ }N(\mu_J, \sigma_J)$, and then the mean of Y_i is $\varphi = \exp\left(\mu_J + \frac{1}{2}\sigma_J^2\right) - 1$. Under this assumption, the optimal tax evasion, consumption and asset allocation for the benchmark model can be found in Ma et al. (2019).

Based on Andersen et al. (2002), Wang et al. (2016), and Levaggi and Menoncin (2016), the parameter values for the base scenario are presented in Table 1. Note that, to avoid Yitzhaki's (1974) counterintuitive result, we assume the fine is proportional to the value of the evaded asset, i.e., $\theta(\tau) \equiv \theta$. In the following comparative static analyses, the parameters will take on the values as given in Table 1 unless otherwise specified.[5]

4.1. Effect of audit memory on tax evasion

In this subsection, we mainly examine how agents with different risk aversion make decisions on tax evasion when audit memory is present and the effect of audit memory on tax evasion.

As shown in Fig. 1, in the presence of audit memory, the high risk-averse agent has a lower level of tax evasion than the low risk-averse one. This result is intuitive, for, the risk of being audited and being punished for the concealed account can be regarded as a source of risk; compared to the low risk-averse agent, the high risk-averse one will be less inclined to tax evasion if all other factors are the same. This result is in line with the empirical finding in Alabede et al. (2011), which shows a positive relationship between a taxpayer's risk preference and their tax compliance behavior. In addition, as expected, the negative relationship between audit intensity λ_2 and the concealed investment fraction π^e is confirmed for both the high risk-averse and the low risk-averse agents, regardless of whether tax audits have memory. Intuitively, the agent, regardless of their risk preference and the tax audit fashion, should cut down on their concealed investment when tax audits occur more frequently. This finding is also supported by many other studies in the literature (e.g., Fischer et al., 1992; Levaggi & Menoncin, 2016; Ma et al., 2019).

In what follows, we continue to examine the effect of audit memory on tax evasion. First, compared to the benchmark, the high (low) risk-averse agent tends to conceal more (less) of their risky investment when tax audits have memory property. It means that, although the negative relationship between tax evasion and audit intensity is always true whatever audit fashion is applied, the effect of audit memory on tax evasion is uncertain and depends on the agent's degree of risk aversion. This is consistent with the empirical finding in Mohdali et al. (2014). There is a plausible economic explanation for this result. In the circumstance that tax audits have the memory property, once tax evasion is detected, audit intensity will increase sharply and the rational action for the agent will be to close their concealed account until audit intensity returns to an acceptable level. As a result, the agent will suffer a substantial "loss" in the future because of the absence of tax evasion, which implies that audit memory is an indirect punishment on tax evasion (penalty is a direct one). This punishment will change the agent's investment opportunity set temporally. Thus, he is supposed to take this change into consideration and react appropriately. As Kim and Omberg (1996), Liu et al. (2003), Liu and Pan (2003), and Branger et al. (2009) have explained, the reaction of the agent depends on whether he is more or less risk-averse than the log-utility agent. For the agent who is more risk-averse than the log-utility agent, since their utility is unbounded from below, he is more careful about hedging and, therefore, will conceal more risky investment in the case of audits with memory, when tax evasion is still profitable at present, to pursue extra profit to compensate for the potential "loss" in the future. This tax evasion behavior is a version of "enjoying pleasure in good time". In contrast, for the agent who is less risk averse than the log-utility agent, their utility is bounded from below but unbounded from above, and hence hedging is a less attractive choice for him. Additionally, the indirect punishment renders concealing their investment less attractive. Therefore, the low risk-averse agent is prone to hold less concealed asset when audits have the memory property. From Fig. 1, in the presence of audit memory, the sensitivity of tax evasion to audit intensity for the high risk-averse agent increases as audit intensity decreases, and an opposite result obtains for the low risk-averse agent. The rationale for this is, because the concealed investment decreases with audit intensity, the potential reduction in investment return resulting from indirect punishment tends to increase, and the hedging demand increases as audit intensity decreases.

[5] We have also considered a case where the tax rates for the risky and riskless assets are the same. We find that the results are quite similar with those in the case where the tax rates are different. Due to the limited space, the results in this special case are not presented in this paper; they are available upon request.

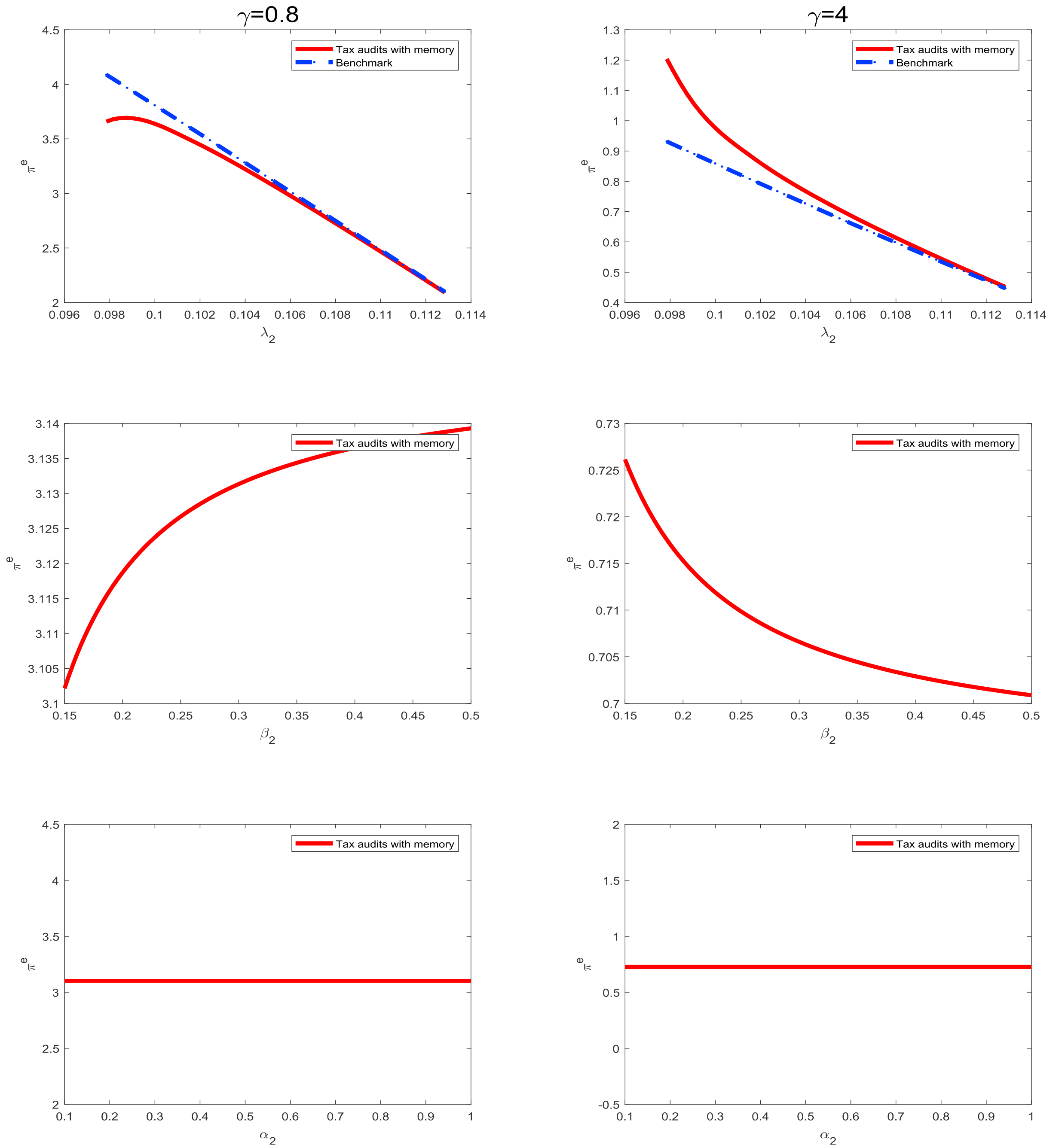

Fig. 1. The sensitivity of tax evasion (π^e) to the audit-related parameters (λ_2, β_2 and α_2). Left graphs: the results for a low risk-averse agent ($\gamma = 0.8$); right graphs: the results for a high risk-averse agent ($\gamma = 4$).

Second, as the decay rate β_2 rises, the agent with low (high) risk aversion will boost (reduce) their concealed position. This result is obvious since the speed of audit intensity reverting to the normal level increases with the decay rate. Therefore, for any specific agent, the effects of the decay rate and audit intensity on tax evasion are opposite. In addition, since the increment of audit intensity as a punishment for tax evasion is set to make the agent refrain from concealing any investment at least for a while, it is likely that the agent will not react to a larger increment of audit intensity.

Our findings in this subsection reveal that the effect of audit memory on tax evasion depends on agent's risk aversion. Therefore, as Mohdali et al. (2014) say, the effect of the threat of punishment on tax evasion varies from group to group, and thus policymakers should consider taxpayers' risk preference when formulating their tax audit strategy. More specifically, policymakers should consider the history of the taxpayer's tax evasion if he is low risk-averse, while they should choose a memoryless tax audit strategy for the taxpayer with a high risk aversion. It is worth noting that, if policymakers adopt a tax audit strategy with the memory property, it is unnecessary for policymakers to set an extremely high punishment in terms of audit intensity. Instead, they should pay more attention to the decay rate of audit intensity since it partly determines the effect of audit memory.

4.2. Effect of tax rate and fine on tax evasion

From Fig. 2, tax evasion can be reduced by increasing the fine or decreasing the tax rate, but the extent depends on agents' risk aversion in the presence of audit memory. Specifically, for the high (low) risk-averse agent, the impact of the tax rate and fine on tax evasion will be strengthened (weakened) by adding memory to tax audits. The potential loss made from being caught and fined is the main explanation for this result. Intuitively, when the tax rate decreases or the fine increases, tax evasion will decrease and the hedge demand for the potential loss will decline, which makes the high risk-averse agent reduce their current investment in the concealed asset. Therefore, for the high risk-averse agent, audit memory enhances the effects of the tax rate and fine on tax evasion. On the other hand, the lower the concealed investment, the lower the potential loss of revenue caused by indirect punishment. Thus, for the low risk-averse agent who is more sensitive to the potential loss of revenue, audits with memory make their concealed investment less sensitive to the tax rate and fine than memoryless audits. These results indicate that audit memory reduces evasion more (less) efficiently by reducing the tax and increasing the fine for the high (low) risk-averse agent. Accordingly, if the authority decides to adopt a tax audit fashion which possesses the memory property, they should consider taxpayers' degree of risk aversion before using taxes and fines as a means reduce tax evasion.

4.3. Optimal consumption

As illustrated in Fig. 3, first, in comparison to the case of memoryless audits, the high (low) risk-averse agent will consume less (more) when audits have memory property. This implies that the optimal consumption also depends on the degree of risk aversion of the agent. In general, this result stems from the fact that the substitution effect caused by an audit's memory property outweighs the corresponding income effect. In the presence of audits with memory, a high risk-averse agent temporarily takes higher audit risk, which

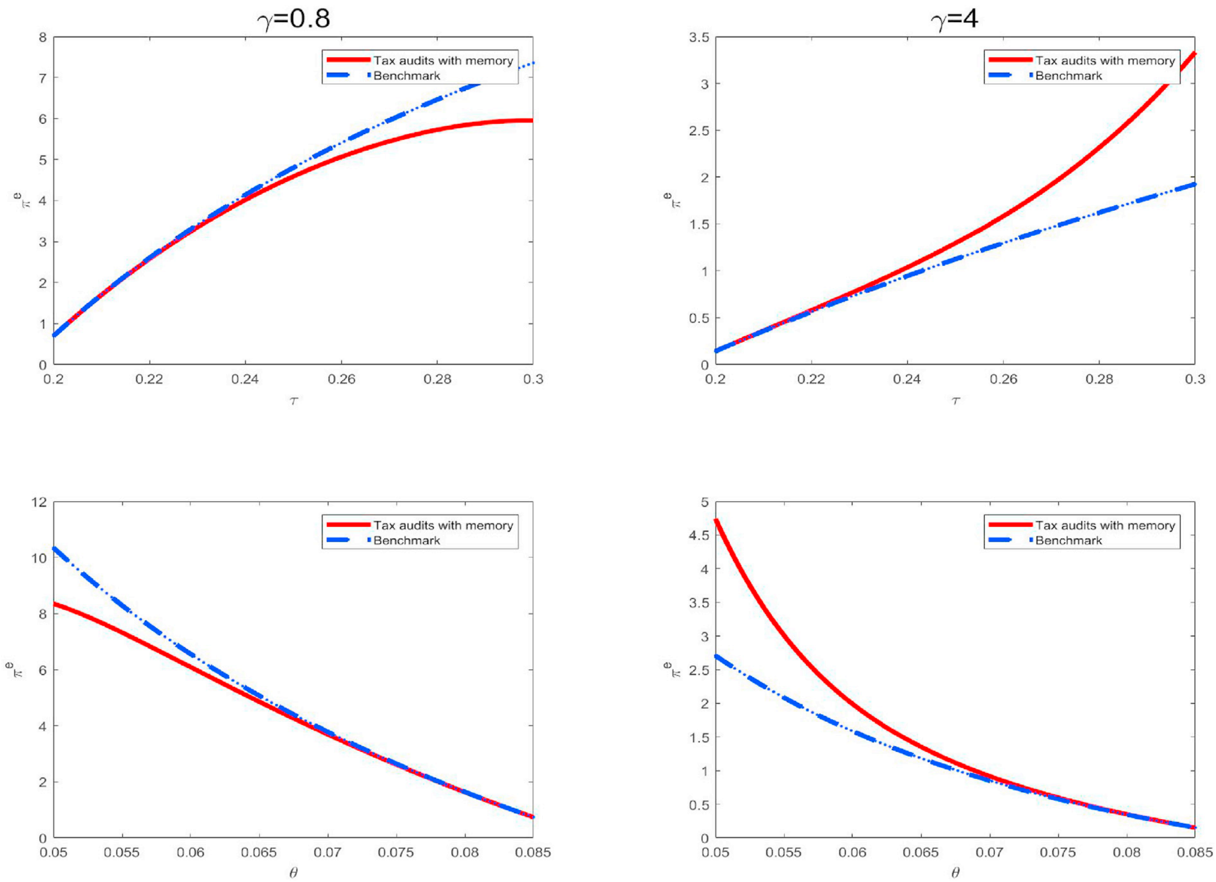

Fig. 2. The sensitivity of optimal tax evasion (π^e) to the tax rate and the fine (τ and θ). Left graphs: the results for a low risk-averse agent ($\gamma = 0.8$); right graphs: the results for a high risk-averse agent ($\gamma = 4$).

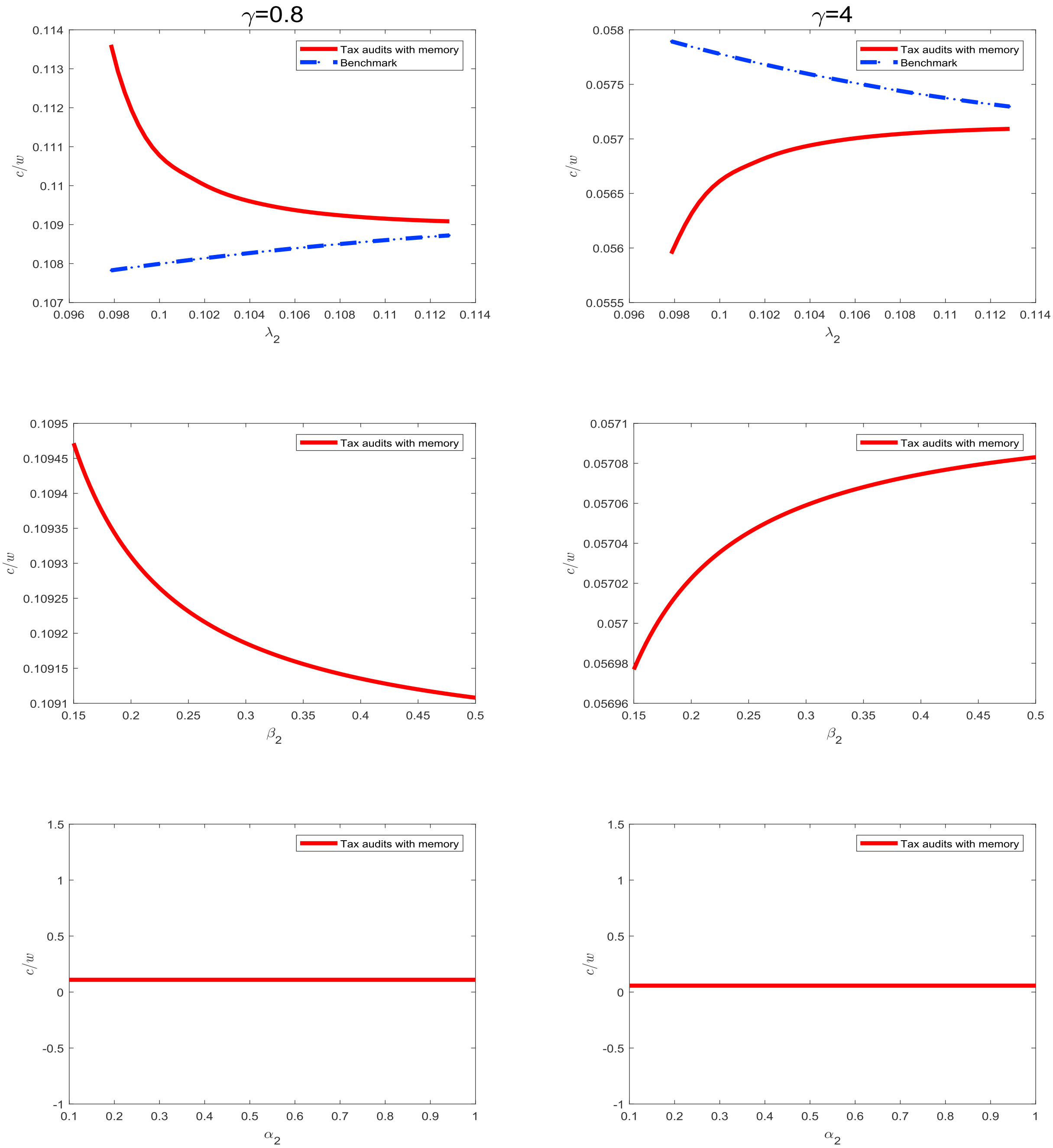

Fig. 3. The sensitivity of optimal consumption-wealth ratio (c/w) to the audit-related parameters (λ_2, β_2 and α_2). Left graphs: the results for a low risk-averse agent ($\gamma = 0.8$); right graphs: the results for a high risk-averse agent ($\gamma = 4$).

increases the relative price of consumption and generates a substitution effect. Although, on the other hand, the higher concealed investment also increases the agent's net expected return and buying power, the high risk-averse agent usually cares more about risk. As a result, the substitution effect is stronger than the income effect. For the low risk-averse agent, compared with the weaker income effect generated by the decreased income, the substitution effect produced by the decreased income and the increased punishment from tax audits will more sharply lower their interest in earning money and will promote their interest in consumption and enjoyment of leisure time. Thus, in the presence of audits with memory, the low risk-averse agent is more willing to enjoy consumption, whereas the high risk-averse agent has less interest therein.

Second, for the high (low) risk-averse agent, their consumption is positively (negatively) related to audit frequency when tax audits have the memory property, which is in total contrast to their consumption behavior when audits are memoryless. When audit intensity rises, so does the likelihood of being caught in evasion. Therefore, the net expected return on concealed investment, for all agents, would decline and thereby weaken the income effect. However, the substitution effect declines more rapidly than the income effect. On the one hand, the relative price of consumption increases with the intensity. On the other hand, the indirect punishment has less impact on the agent's decision about the concealed account and consumption. As a consequence, the substitution effect will diminish steeply, and the high risk-averse agent will consume more goods while the low risk-averse agent moderates their consumption when audit intensity increases.

5. Conclusion

In this study, we develop a theoretical model of tax evasion to investigate the relationship between tax audits with memory and agents' tax evasion, investment, and consumption decisions. We find that tax evasion will curtail the total investment in the risky asset and increase investment in the riskless asset. However, the memory feature of audits does not affect an agent's effective holding in the risky asset. This finding provides theoretical support for the negative relationship between tax evasion and prosperity in the capital markets. We also find that the impact of audit memory on tax evasion is closely related to the degree of agents' risk aversion. Specifically, in comparison to the case of memoryless audits, when audits have the memory property, the low risk-averse agent will reduce tax evasion and the high risk-averse agent behaves contrarily. The negative relationship between audit intensity and tax evasion will be enhanced (reduced) for the low (high) risk-averse agent when the audit intensity rises. Although the audit intensity decay rate plays an important role in the battle against tax evasion, increasing the punishment through audit intensity is only effective up to a certain threshold. Likewise, the effects of a tax reduction and an increase in penalty on tax evasion also vary with an agent's degree of risk aversion. Finally, an agent's consumption behavior in the case of an audit with memory is opposite to that in the absence of the memory property, depending also on their risk aversion as well as audit-related parameters.

From a policy point of view, policymakers should treat agents differentially in terms of tax audits. For the low risk-averse agent, tax audits should be dependent on their history of evasion since this audit fashion will reduce their evasion and increase their consumption expenditure, while for the high risk-averse agent tax audits should be memoryless. In addition, an increase in audit intensity as punishment for tax evasion should be modest since it is costly and its effect neutral at and beyond a certain threshold level. Where policymakers adopt the audit fashion with the memory feature, they should be careful when using tax reduction and increased fines to reduce tax evasion, because the effectiveness of these measures depend on an agent's degree of risk aversion. Lastly, policymakers need to attain a balance between audit cost and policy effect when deciding on an appropriate audit intensity decay rate.

CRediT authorship contribution statement

Yong Ma: Conceptualization, Methodology, Formal analysis, Writing-original draft, Supervision, Funding acquisition.
Hao Jiang: Software, Validation, Data curation.
Weilin Xiao: Methodology, Investigation, Formal analysis.

Declaration of competing interest

The authors declared that they do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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Appendix A

Proof of Proposition 3.1. For the log-utility agent, we conjuncture that

$$L(w,\lambda)=f(\lambda)+\frac{1}{\rho}\log w. \tag{A.1}$$

Then

$$L_w=\frac{1}{\rho w},\quad L_{ww}=-\frac{1}{\rho w^2},\quad L_{\lambda_i}=\frac{\partial f(\lambda)}{\partial\lambda_i},\quad i=1,2. \tag{A.2}$$

It follows from (10) that the optimal consumption process is

$$c^*=\rho w. \tag{A.3}$$

Combining (A.2) and (13), we obtain

$$\pi^{e^*} = \frac{1}{\theta(\tau)} - \frac{\lambda_2(1-\tau)}{r_G \tau}. \tag{A.4}$$

Thus, (11) can be rewritten as

$$\sigma^2 \kappa^* = \mu - \frac{r_G}{1-\tau} + \lambda_1 \left(\mathbb{E} \frac{Y}{1+Y\kappa^*} - \varphi \right). \tag{A.5}$$

Appendix B

Proof of Corollary 3.2. In (A.4), taking the partial derivatives of π^{e^*} with respect to λ_1 and λ_2 yields

$$\frac{\partial \pi^{e^*}}{\partial \lambda_1} = 0 \quad \text{and} \quad \frac{\partial \pi^{e^*}}{\partial \lambda_2} = -\frac{(1-\tau)}{r_G \tau} \le 0.$$

In (A.5), taking the partial derivative of κ^* with λ_1 and λ_2, we obtain

$$\frac{\partial \kappa^*}{\partial \lambda_1} = \frac{\mathbb{E}\left[Y(1+Y\kappa^*)^{-1}\right] - \varphi}{\sigma^2 + \lambda_1 \mathbb{E}\left[Y^2(1+Y\kappa^*)^{-2}\right]} \quad \text{and} \quad \frac{\partial \kappa^*}{\partial \lambda_2} = 0.$$

It implies that the after-tax weighted proportion invested in the risky asset has nothing to do with the audit intensity. If short selling is not allowed in the market, then $0 < \kappa^* \le 1$ and $\mathbb{E}[Y(1+Y\kappa^*)^{-1}] - \varphi < 0$. Thus

$$\frac{\partial \pi^*}{\partial \lambda_1} = \frac{1}{1-\tau}\left(\frac{\partial \kappa^*}{\partial \lambda_1} - \frac{\partial \pi^{e^*}}{\partial \lambda_1} \right) = \frac{1}{1-\tau} \frac{\partial \kappa^*}{\partial \lambda_1} < 0$$

and

$$\frac{\partial \pi^*}{\partial \lambda_2} = \frac{1}{1-\tau}\left(\frac{\partial \kappa^*}{\partial \lambda_2} - \frac{\partial \pi^{e^*}}{\partial \lambda_2} \right) = -\frac{1}{1-\tau} \frac{\partial \pi^{e^*}}{\partial \lambda_2} \ge 0.$$

Appendix C

Proof of Proposition 3.2. For the power-utility agent, a candidate value function would be

$$L(w, \boldsymbol{\lambda}) = \frac{w^{1-\gamma}}{1-\gamma} h(\boldsymbol{\lambda}) \tag{C.1}$$

for some function h. Obviously,

$$L_w = \frac{1-\gamma}{w} L, L_{ww} = -\frac{\gamma(1-\gamma)}{w^2} L, L_{\lambda_i} = \frac{w^{1-\gamma}}{1-\gamma} \frac{\partial h(\boldsymbol{\lambda})}{\partial \lambda_i}, i = 1, 2. \tag{C.2}$$

It follows from (10) and (13) that the optimal consumption is

$$c^* = W[h(\boldsymbol{\lambda})]^{-1/\gamma} \tag{C.3}$$

and the optimal weight in the unconcealed risky asset is

$$\pi^{e^*} = \frac{1}{\theta(\tau)} \left\{ 1 - \left(\frac{\lambda_2 \theta(\tau)(1-\tau)}{r_G \tau} \frac{h(\boldsymbol{\lambda} + \boldsymbol{\alpha}_2 1(\pi^{e^*} > 0))}{h(\boldsymbol{\lambda})} \right)^{\frac{1}{\gamma}} \right\} \tag{C.4}$$

In addition, (11) can be rewritten as

$$\sigma^2 \kappa^* = \frac{1}{\gamma} \left\{ \mu - \varphi \lambda_1 - \frac{r_G}{1-\tau} + \lambda_1 \frac{h(\boldsymbol{\lambda} + \boldsymbol{\alpha}_1)}{h(\boldsymbol{\lambda})} \mathbb{E}[Y(1+Y\kappa^*)^{-\gamma}] \right\}. \tag{C.5}$$

The optimal consumption and investment are given by (C.3), (C.4) and (C.5). However, they are the functionals of $h(\boldsymbol{\lambda})$. Now we proceed to derive the function $h(\boldsymbol{\lambda})$. Substituting the optimal consumption and investment into the HJB equation (3), we have

$$0 = -\rho h(\boldsymbol{\lambda}) + \gamma[h(\boldsymbol{\lambda})]^{1-\frac{1}{\gamma}} + \sum_{i=1} \beta_i\left(\overline{\lambda}_i - \lambda_i\right)\frac{\partial h(\boldsymbol{\lambda})}{\partial \lambda_i} + (1-\gamma)\left\{ r_G(1-\pi^* - \pi^{e*}) + (\mu - \varphi\lambda_1)\kappa^* - \frac{1}{2}\sigma^2\gamma\kappa^{*2}\right\}h(\boldsymbol{\lambda}) + \lambda_1\left[\mathbb{E}(1+Y\kappa^*)^{1-\gamma}h(\boldsymbol{\lambda}+\boldsymbol{\alpha}_1) - h(\boldsymbol{\lambda})\right] + \lambda_2\left[\left(1-\theta(\tau)\pi^{e*}\right)^{1-\gamma}h\left(\boldsymbol{\lambda}+\boldsymbol{\alpha}_2 1\left(\pi^{e*}>0\right)\right) - h(\boldsymbol{\lambda})\right] \tag{C.6}$$

Multiplying $\frac{1}{\gamma}h^{\frac{1}{\gamma}-1}(\lambda)$ on both sides of (C.6), we have

$$-1 = K(\boldsymbol{\lambda})h^{\frac{1}{\gamma}}(\boldsymbol{\lambda}) + \sum_{i=1}\left\{\beta_i\left(\overline{\lambda}_i - \lambda_i\right)\frac{\partial h^{\frac{1}{\gamma}}(\boldsymbol{\lambda})}{\partial \lambda_i} + \lambda_i\left[h^{\frac{1}{\gamma}}(\boldsymbol{\lambda}+\boldsymbol{\alpha_i}) - h^{\frac{1}{\gamma}}(\boldsymbol{\lambda})\right]\right\} \tag{C.7}$$

where

$$K(\boldsymbol{\lambda}) = \frac{1}{\gamma}\left\{-\rho + (1-\gamma)\left[r_G(1-\pi^* - \pi^{e*}) + \kappa^*(\mu - \varphi\lambda_1) - \frac{1}{2}\kappa^{*2}\sigma^2\gamma\right] + \frac{\mathbb{E}(1+Y\kappa^*)^{1-\gamma}}{\mathbb{E}(1+Y\kappa^*)^{-\gamma}}\left(\sigma^2\kappa^*\gamma - \mu + \varphi\lambda_1 + \frac{r_G}{1-r}\right) + \frac{r_G\tau(1-\theta(\tau)\pi^{e*})}{\theta(\tau)(1-\tau)}\right\} - \lambda_1^{1-\frac{1}{\gamma}}\left(\frac{\sigma^2\kappa^*\gamma - \mu + \varphi\lambda_1 + \frac{r_G}{1-\tau}}{\mathbb{E}(1+Y\kappa^*)^{-\gamma}}\right)^{\frac{1}{\gamma}} - \lambda_2^{1-\frac{1}{\gamma}}\left(\frac{r_G\tau(1-\theta(\tau)\pi^{e*})^{\gamma}}{\theta(\tau)(1-\tau)}\right)^{\frac{1}{\gamma}} + \left(1-\frac{1}{\gamma}\right)(\lambda_1+\lambda_2). \tag{C.8}$$

Let $g(\boldsymbol{\lambda}) = h^{\frac{1}{\gamma}}(\boldsymbol{\lambda})$. Then

$$[\mathscr{A}g](\boldsymbol{\lambda}) + K(\boldsymbol{\lambda})g(\boldsymbol{\lambda}) = -1. \tag{C.9}$$

where $\mathscr{A}$ is the Markov generator given by

$$[\mathscr{A}g](\boldsymbol{\lambda}) = \sum_{i=1}\left[\beta_i\left(\overline{\lambda}_i - \lambda_i\right)\frac{\partial g(\boldsymbol{\lambda})}{\partial \lambda_i} + \lambda_i(g(\boldsymbol{\lambda}+\boldsymbol{\alpha}_i) - g(\boldsymbol{\lambda}))\right]. \tag{C.10}$$

According to Feynman-Kac formula,

$$g(\boldsymbol{\lambda}) = -\mathbb{E}_{0,\lambda}\left[\int_0^\infty e^{\int_0^s K(\lambda_s)}ds\right] = -\int_0^\infty \mathbb{E}_{0,\lambda}\left[e^{\int_0^s K(\lambda_s)}\right]ds.$$

References

Abdixhiku, L., Krasniqi, B., Pugh, G., & Hashi, I. (2017). Firm-level determinants of tax evasion in transition economies. *Economic Systems, 41*(3), 354–366.
Aït-Sahalia, Y., Cacho-Diaz, J., & Laeven, R. J. (2015). Modeling financial contagion using mutually exciting jump processes. *Journal of Financial Economics, 117*(3), 585–606.
Aït-Sahalia, Y., & Hurd, T. R. (2015). Portfolio choice in markets with contagion. *Journal of Financial Econometrics, 14*(1), 1–28.
Akhtar, S., Akhtar, F., John, K., & Wong, S.-W. (2019). Multinationals' tax evasion: A financial and governance perspective. *Journal of Corporate Finance, 57*, 35–62.
Alabede, J. O., Ariffin, Z. Z., & Idris, K. M. (2011). Individual taxpayers attitude and compliance behaviour in Nigeria: The moderating role of financial condition and risk preference. *Journal of Accounting and Taxation, 3*(3), 91–104.
Allingham, M. G., & Sandmo, A. (1972). Income tax evasion: A theoretical analysis. *Journal of Public Economics, 1*(3–4), 323–338.
Alm, J., Cronshaw, M. B., & McKee, M. (1993). Tax compliance with endogenous audit selection rules. *Kyklos, 46*(1), 27–45.
Alstadsæter, A., Johannesen, N., & Zucman, G. (2019). Tax evasion and inequality. *The American Economic Review, 109*(6), 2073–2103.
Andersen, T. G., Benzoni, L., & Lund, J. (2002). An empirical investigation of continuous-time equity return models. *The Journal of Finance, 57*(3), 1239–1284.
Blackburn, K., Bose, N., & Capasso, S. (2012). Tax evasion, the underground economy and financial development. *Journal of Economic Behavior & Organization, 83*(2), 243–253.
Branger, N., Kraft, H., & Meinerding, C. (2009). What is the impact of stock market contagion on an investor's portfolio choice? *Insurance: Mathematics and Economics, 45*(1), 94–112.
Buehn, A., & Schneider, F. (2012). Shadow economies around the world: Novel insights, accepted knowledge, and new estimates. *International Tax and Public Finance, 19*(1), 139–171.
Cai, J., Chen, X., & Dai, M. (2017). Portfolio selection with capital gains tax, recursive utility, and regime switching. *Management Science, 64*(5), 2308–2324.
Chan, S., Chau, G., & Leung, P. (2013). Tax audit and investigation in China and Hong Kong. *International Tax Journal, 39*, 31.
Crocker, K. J., & Slemrod, J. (2005). Corporate tax evasion with agency costs. *Journal of Public Economics, 89*(9–10), 1593–1610.
Fischer, C. M., Wartick, M., & Mark, M. M. (1992). Detection probability and taxpayer compliance: A review of the literature. *Journal of Accounting Literature, 11*(1), 1–46.
Gokalp, O. N., Lee, S.-H., & Peng, M. W. (2017). Competition and corporate tax evasion: An institution-based view. *Journal of World Business, 52*(2), 258–269.
Hanlon, M., Maydew, E. L., & Thornock, J. R. (2015). Taking the long way home: Us tax evasion and offshore investments in us equity and debt markets. *The Journal of Finance, 70*(1), 257–287.
Kenyon, T. (2008). Tax evasion, disclosure, and participation in financial markets: Evidence from brazilian firms. *World Development, 36*(11), 2512–2525.
Kim, T. S., & Omberg, E. (1996). Dynamic nonmyopic portfolio behavior. *Review of Financial Studies, 9*(1), 141–161.
Kleven, H. J., Knudsen, M. B., Kreiner, C. T., Pedersen, S., & Saez, E. (2011). Unwilling or unable to cheat? Evidence from a tax audit experiment in Denmark. *Econometrica, 79*(3), 651–692.

Lee, S. S., & Mykland, P. A. (2007). Jumps in financial markets: A new nonparametric test and jump dynamics, 12 *Review of Financial Studies, 21*(6), 2535–2563.

Levaggi, R., & Menoncin, F. (2012). Tax audits, fines and optimal tax evasion in a dynamic context. *Economics Letters, 117*(1), 318–321.

Levaggi, R., & Menoncin, F. (2016). Optimal dynamic tax evasion: A portfolio approach. *Journal of Economic Behavior & Organization, 124*, 115–129.

Lin, W.-Z., & Yang, C.-C. (2001). A dynamic portfolio choice model of tax evasion: Comparative statics of tax rates and its implication for economic growth. *Journal of Economic Dynamics and Control, 25*(11), 1827–1840.

Liu, J., Longstaff, F. A., & Pan, J. (2003). Dynamic asset allocation with event risk. *The Journal of Finance, 58*(1), 231–259.

Liu, J., & Pan, J. (2003). Dynamic derivative strategies. *Journal of Financial Economics, 69*(3), 401–430.

López, J. J. (2017). A quantitative theory of tax evasion. *Journal of Macroeconomics, 53*, 107–126.

Ma, Y., Shan, S., & Xu, W. (2019). Optimal investment and consumption in the market with jump risk and capital gains tax. *Journal of Industrial and Management Optimization, 15*(4), 1937–1953.

Medina, L., & Schneider, F. (2018). *Shadow economies around the world: What did we learn over the last 20 years? IMF working paper No. 18/17*.

Mohdali, R., Isa, K., & Yusoff, S. H. (2014). The impact of threat of punishment on tax compliance and non-compliance attitudes in Malaysia. *Procedia-Social and Behavioral Sciences, 164*, 291–297.

Murphy, R. (2011). *Collect the evaded tax, avoid the cuts* (Vol. 25). The Guardian.

Richardson, G. (2006). Determinants of tax evasion: A cross-country investigation. *Journal of International Accounting, Auditing and Taxation, 15*(2), 150–169.

Roubini, N., & Sala-i Martin, X. (1995). A growth model of inflation, tax evasion, and financial repression. *Journal of Monetary Economics, 35*(2), 275–301.

Schneider, F. (2012). *The shadow economy and work in the shadow: What do we (not) know? IZA discussion paper No. 6423*.

Schneider, F., Buehn, A., & Montenegro, C. E. (2010). New estimates for the shadow economies all over the world. *International Economic Journal, 24*(4), 443–461.

Slemrod, J. (2007). Cheating ourselves: The economics of tax evasion. *The Journal of Economic Perspectives, 21*(1), 25–48.

Srinivasan, T. (1973). Tax evasion: A model. *Journal of Public Economics, 2*(4), 339–346.

Varvarigos, D. (2017). Cultural norms, the persistence of tax evasion, and economic growth. *Economic Theory, 63*(4), 961–995.

Wang, C., Wang, N., & Yang, J. (2016). Optimal consumption and savings with stochastic income and recursive utility. *Journal of Economic Theory, 165*, 292–331.

Yitzhaki, S. (1974). A note on income tax evasion: A theoretical analysis. *Journal of Public Economics, 3*, 201–202.