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Cryptocurrency returns and the volatility of liquidity

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ABSTRACT

In this paper I document a positive relation between the volatility of liquidity and expected returns. Specifically, I analyze the relationship between the idiosyncratic volatility of market liquidity and the returns of the five largest cryptocurrencies by market capitalization. I find that the correlation between liquidity volatility and returns is overall significantly positive, but highly time-varying. This implies that investors demand a premium for a high variation in liquidity volatility. I furthermore find that the correlation between returns and the level of liquidity is mostly positive, thus, when liquidity is low, expected returns are high. The results corroborates results from other financial markets.

1. Introduction

In this paper I document a positive relation between the volatility of liquidity and expected returns. Specifically, I analyze the relationship between the volatility of market liquidity and realized returns of the five largest cryptocurrencies[1] The paper documents a significant relationship between the expected returns of a cryptocurrency and it's idiosyncratic volatility of liquidity. The volatility of liquidity is a currency-specific characteristic that measures the uncertainty associated with the level of liquidity of the currency at the time of trade. The positive correlation between the volatility of liquidity and expected returns suggests that risk averse investors require a risk premium for holding currencies with high variation in liquidity.

The level of liquidity of cryptocurrencies are investigated by several authors, see, for example, Brauneis et al. (2020), Yue et al. (2020), Brauneis et al. (2021), and Scharnowski (2020). The second moment of liquidity is less studied. The motivation to study the second moment, or the *volatility*, of liquidity is that investors who need to trade at random points in time might care about the possibility of trading easily in the future. That is, if the variation of the level of liquidity is constant, the investor knows the exact level and how easy it will be to trade the asset in the future. If, however, the volatility is non-constant, there is a chance that the liquidity is completely different when the investor needs to trade in the future. In reality, liquidity does vary over time, though how much varies from asset to asset. Thus, investors are exposed to not only the risk of the level of liquidity, but also the variation in the level of liquidity. This relationship, i.e. between expected return and the volatility of liquidity, has been investigated in the stock market, though with contradictory results, see Chordia et al. (2001), Acharya and Pedersen (2005), Pereira, J.P. and Zhang (2010), and Akbas et al. (2013). In an empirical study of US stock markets, Chordia et al. (2001) finds, surprisingly, evidence of a negative relationship between the volatility of liquidity and returns. Modeling liquidity as a stochastic price-impact process, Pereira, J.P. and Zhang (2010)

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[1] The size is measured by market capitalization as of December 1st, 2020. The currencies includes Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Bitcoin Cash (BCH), and Litecoin (LTC). The market capitalization varies substantially over time, so other currencies might be larger in other periods. Litecoin and Bitcoin Cash has in this period a very similar market capitalization.

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Table 1
This table presents summary statistics for the returns of the cryptocurrencies analyzed in the paper over the period 1.1.2016 through 6.3.2021.

	BTC	ETH	BCH	LTC	XRP
Observations	1891	1891	1346	1891	1891
Minimum	–0.3958	–0.4400	–0.4005	–0.3849	–0.4608
Quartile 1	–0.0118	–0.0218	–0.0256	–0.0220	–0.0198
Median	0.0022	0.0009	–0.0008	0.0000	0.0000
Arithmetic Mean	0.0033	0.0058	0.0041	0.0038	0.0049
Geometric Mean	0.0025	0.0039	0.0011	0.0021	0.0023
Quartile 3	0.0181	0.0309	0.0266	0.0233	0.0186
Maximum	0.2525	0.4667	1.6921	0.8349	1.7945
SE Mean	0.0009	0.0014	0.0023	0.0014	0.0019
LCL Mean (0.95)	0.0015	0.0030	–0.0004	0.0011	0.0013
UCL Mean (0.95)	0.0051	0.0085	0.0087	0.0065	0.0086
Variance	0.0016	0.0037	0.0073	0.0037	0.0066
Stdev	0.0401	0.0607	0.0853	0.0607	0.0811
Skewness	–0.1670	0.4920	6.3482	2.7527	7.4430
Kurtosis	9.1230	6.0086	116.2991	30.0226	138.3058

shows that utility-maximizing, Constant Relative Risk Averse (CRRA) investors adapts trading to the state of liquidity and thereby finds similar results as (Chordia et al., 2001). These results are in contrast to the findings of Akbas et al. (2013), where the authors finds that the relationship is in fact positive. Acharya and Pedersen (2005) finds that systematic liquidity risk is priced in the stock market. The latter two studies are also more in line with intuition: as the volatility of liquidity increases, the risk of the stock increases, and expected returns of the stock should increase as well. Akbas et al. (2013) argues that the reason is that risk averse investors demands a premium for investing in stocks whose liquidity might suddenly disappear. Thus, a risk averse investor might prefer an asset which is more certain to trade a later point to an asset for which it might be hard to trade later. The standard deviation of liquidity captures how much the liquidity varies over time. A bid-ask spread with a low historical standard deviation, combined with a high mean, means an asset with a stable, but low, liquidity. As such, the investor knows that this asset is illiquid, and will most likely stay illiquid in the future. If the standard deviation is high, the investor does not know whether the liquidity will be high or low in the future, which is an additional risk for the investor. Therefore, ceteris paribus, a risk-averse investor may be willing to pay a higher price for an asset that has a lower risk of becoming less liquid at the time of trading, i.e., an asset whose liquidity is less volatile. As such, one can hypothesize that an asset with a high liquidity volatility trades at a discount compared to a similar asset with a less variable liquidity level. The expected returns should then be higher for the asset with the higher liquidity-volatility. In this paper, I document evidence for this hypothesis for cryptocurrencies.

The market for cryptocurrencies has received much attention the last five years, both from regulators, the public, and traders. The trading volume in the largest such currencies has grown exponentially, and with this increase in popularity, the prices increased as well. There are several reasons for this, for example the currencies being traded more in terms of volume, but also that the efficiency of crypto-markets has improved significantly over time, see, for example, Urquhart (2016), Vidal-Tomás and Ibañez (2018), Jiang et al. (2018), Tran and Leirvik (2019), Tran and Leirvik (2020), Aslan and Sensoy (2020), Al-Yahyaee et al. (2020), Kristoufek and Vosvrda (2019), and Naeem et al. (2020). Furthermore, there has been an explosion in research related to many aspects of cryptocurrencies. Corbet et al. (2019) analyzes the cryptocurrency as a financial asset, whereas Chu et al. (2019) investigates the Adaptive Market Hypothesis for the two largest cryptocurrencies, and find evidence that supports the hypothesis of a time-varying market efficiency. Brauneis and Mestel (2019), Choi (2020), Ghabri et al. (2020), and Scharnowski (2020) studies the liquidity of cryptocurrencies, and finds that in general it improves over time. Other studies, such as (Katsiampa et al., 2019) and (Omane-Adjepong et al., 2019), also show that cryptocurrencies are strongly interlinked which reflects volatility spill-over, volatility co-movement, lead-lag effect, market co-movement. For example, Katsiampa et al. (2019) shows that there is an asymmetric volatility relationship in cryptocurrency markets. However, to be able to directly compare the results from cryptocurrencies with results from the stock market, this paper does not take into account a possible asymmetric relationship between the volatility of liquidity and expected stock returns. In this paper the bid-ask spread measure of Corwin and Schultz (2012) is applied, and a rolling window of the spread estimator for the five currencies is computed and analyzed. The wider the spread, the more *illiquid* the asset is. Liquidity, volatility, and market efficiency are closely related, see for example Amihud (2002), Chordia et al. (2008), Leirvik et al. (2017), Wei (2018), and Brauneis and Mestel (2019). First, I find that the liquidity is highly time varying, which is not surprising given the ups and downs in this asset class. However, I also find that the liquidity in general is improving, in terms of narrower spreads, over the period studied. Finally, a positive, and highly significant, relationship between the volatility of liquidity and expected returns is established. These results implies that investors in the cryptocurrency markets accounts for the variation in liquidity and demands a discount for purchasing cryptos with high liquidity-volatility.

2. Data

The markets for cryptocurrencies is relatively new, and the sample used in this analysis covers the period January 1st, 2016, through December 31st, 2020. The reason for cutting the period short is to have a dataset that makes more sense to compare cross-sectionally. Bitcoin, Litecoin and Ripple starts trading in 2013, whereas Ethereum started in 2015 and BCH in 2017. These assest

Table 2
This table presents summary statistics for the bid-ask spread of the cryptocurrencies analyzed in the paper over the period 1.1.2016 through 6.3.2021.

	BTC	BCH	ETH	LTC	XRP
Observations	1891	1346	1891	1891	1891
Minimum	–0.2438	–1.0564	–0.3735	–0.5237	–0.4629
Quartile 1	–0.0120	–0.0170	–0.0149	–0.0097	–0.0163
Median	0.0026	0.0089	0.0102	0.0101	0.0060
Arithmetic Mean	–0.0020	–0.0015	0.0040	0.0050	–0.0011
Geometric Mean	–0.0025	NaN	0.0026	0.0038	–0.0028
Quartile 3	0.0123	0.0239	0.0279	0.0260	0.0194
Maximum	0.1904	0.2138	0.2663	0.2347	0.6021
SE Mean	0.0007	0.0017	0.0012	0.0011	0.0013
LCL Mean (0.95)	–0.0034	–0.0049	0.0017	0.0029	–0.0036
UCL Mean (0.95)	–0.0006	0.0019	0.0063	0.0071	0.0015
Variance	0.0010	0.0040	0.0026	0.0022	0.0032
Stdev	0.0309	0.0632	0.0513	0.0467	0.0567
Skewness	–1.3221	–4.4656	–0.9639	–1.7019	–1.0173
Kurtosis	6.5829	60.4286	6.7915	14.5841	21.2055

has shown to rely on a solid technology compared to other currencies, and has quickly become some of the largest currencies by market capitalization. Note that the sample with observations for BCH starts August 1st, 2017. Freely available data from Coinmarketcap.com is applied, and import all available data via a statistical package named "crypto" in the software R. The data is at daily frequency which contains open, high, low, close prices, volume, and market capitalization[2]Table (1) shows the descriptive statistics for the simple returns of the five currencies I analyze. Simple returns are used because log-returns might give unreliable estimates for assets with extremely high volatility. In fact, for the sample the minimum daily log-return is -130.2%[3]. This is clearly not economically sound. To eliminate the chance of using uneconomic reasonable estimates for returns, I exclusively use simple returns as inputs to the calculations.

In order to control for other variables which might impact cryptocurrency returns, the VIX-index is included, as well as the S&P500 index. In addition, a cryptocurrency index is constructed of twelve different currencies. These currencies are, in addition to the five currencies analyzed in this paper, EOS.IO (EOS), BinanceCoin (BNB), Cardano (ADA), Stellar (XLM), Monero (XMR), Chainlink (LINK), and Tron (TRX). Because Bitcoin is much larger in capitalization than all other currencies, the index is equally weighted so that any sensitivity towards the index is not confused with sensitivity towards Bitcoin. A value-weighted index would be heavily biased towards Bitcoin, as it is several times larger in capitalization than the next largest. If fact, Bitcoin's market capitalization is twice as large as the total capitalization of the next 20 currencies, when ranked by capitalization. As such, an equal-weighted portfolio makes more sense when applied in a regression-analysis. The index is named CRP.

3. Methods

The spread estimator applied is derived by Corwin and Schultz (2012), and is given by

$$S_t = \frac{2 \cdot (\exp(\alpha_t) - 1)}{\exp(\alpha_t) + 1} \tag{1}$$

where the α is given by:

$$\alpha_t = \left(1 + \sqrt{2}\right) \cdot \left(\sqrt{\beta_t} - \sqrt{\gamma_t}\right). \tag{2}$$

Here, the S_t is the estimated size of the bid-ask spread at time t[4]See the Appendix for more details on the bid-ask spread estimator. Summary statistics for the bid-ask spread estimator is given in Table 2 for each of the cryptocurrencies in the dataset applied. Table 2 shows that the median spread is highest for Ethereum, and lowest for Bitcoin. There is substantial variation, where Bitcoin Cash has the highest volatility of the spread, with Bitcoin the lowest.

Furthermore, to analyze the relationship between returns of cryptocurrency i, denoted $r_{i,t}$, and the volatility of liquidity, $\sigma_{LIQ_{i,t}}$, the correlation between these two are computed. A linear regression model is also applied in determining the relationship, using

$$r_{i,t} = \beta_0 + \beta_1 \cdot \sigma_{LIQ_{i,t}} + \beta_2 \cdot R_{CRP,t} + \beta_3 \cdot VIX_t + \beta_4 \cdot R_{SP,t} + \varepsilon_{i,t}. \tag{3}$$

[2] By the end of March 6th, 2021, the market capitalization of the top 5 crypto-currencies in billions USD are: BTC (539.1), ETH (84.1), XRP (10.0), LTC (8.2), BCH (6.4). Tether (USDT) has a market capitalization of 20.9, but with a history of controversies related to its linkage to the US dollar, it is omitted in this paper. Polkadot had a market capitalization of 7.2, but with a very short publicly available price history since only August 20, 2020.

[3] Readers can find the summary statistics using log return in the appendix.

[4] Note that in the rest of the paper, the spread-size, S is referred to as LIQ to avoid confusion with the sample standard deviation.

Table 3
This table presents summary statistics of a 200-day moving window with partial windows requiring a minimum of 25 observations for the volatility of the bid-ask spread of the cryptocurrencies analyzed in the paper over the period 1.1.2016 through 6.3.2021.

	BTC	BCH	ETH	LTC	XRP
Observations	1866	1321	1866	1866	1866
Minimum	0.0124	0.0250	0.0241	0.0161	0.0204
Quartile 1	0.0212	0.0352	0.0375	0.0316	0.0285
Median	0.0266	0.0455	0.0461	0.0387	0.0341
Arithmetic Mean	0.0285	0.0607	0.0509	0.0426	0.0483
Geometric Mean	0.0284	0.0600	0.0507	0.0425	0.0480
Quartile 3	0.0321	0.0895	0.0571	0.0465	0.0643
Maximum	0.0523	0.2290	0.1127	0.0770	0.1066
SE Mean	0.0002	0.0011	0.0004	0.0004	0.0006
LCL Mean (0.95)	0.0280	0.0586	0.0500	0.0419	0.0471
UCL Mean (0.95)	0.0289	0.0627	0.0518	0.0433	0.0495
Variance	0.0001	0.0015	0.0004	0.0002	0.0007
Stdev	0.0096	0.0382	0.0192	0.0156	0.0263
Skewness	0.8990	1.6131	1.2497	0.7420	0.9500
Kurtosis	0.3922	2.2741	1.1219	–0.4476	–0.5430

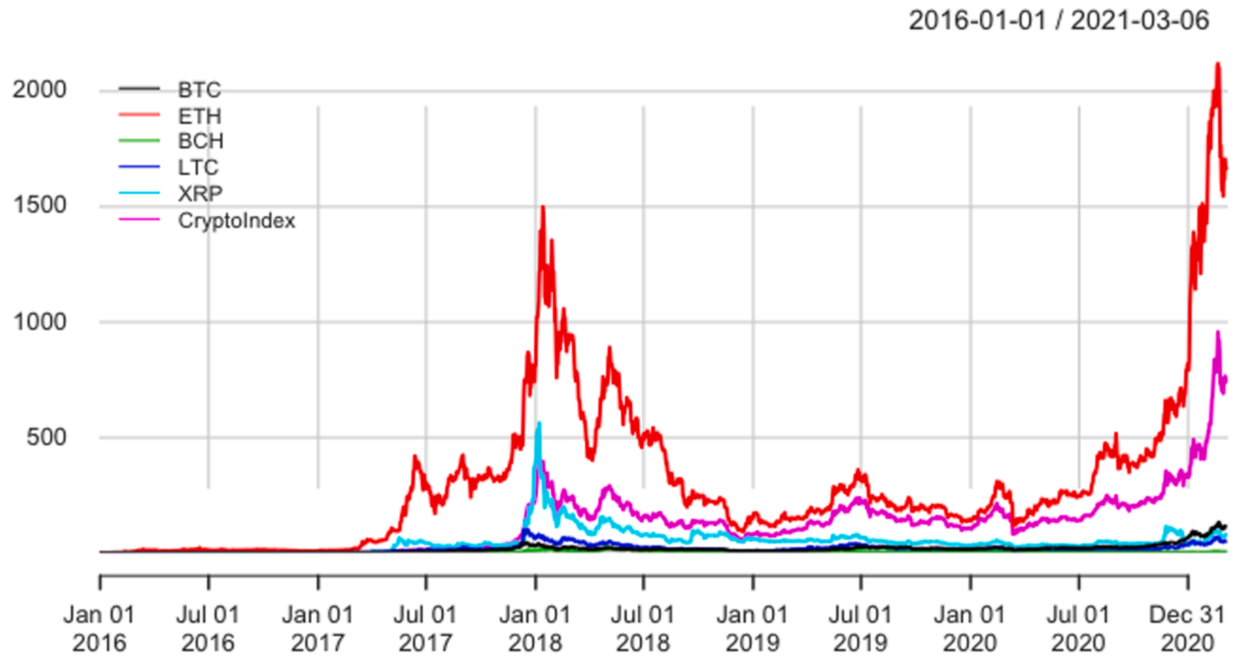

Fig. 1. Time series plot of the prices of the cryptocurrencies analyzed, as well as the constructed cryptocurrency index. All prices are normalized to start with the value 1 for better illustration.

In Eq. 3, the volatility of liquidity is given by σ_{LIQ}, returns on a equal-weighted crytptocurrency-index is given by R_{CRP}, changes in the equity-volatility VIX-index is given in the variable VIX, and the returns on the S&P500 index is given by R_{SP}. Table 3 present summary statistics of the volatility of the bid-ask spread applied in Equation (3). In Table 3 one can see that the mean volatility of the spread is highest for Bitcoin Cash, followed by Ethereum. Bitcoin has the lowest mean volatility of the spread.

4. Empirical Results

In this section the empirical results are presented and discussed. Fig. 1 shows a time series plot of the five cryptocurrencies analyzed in this paper. The time series is normalized so that all prices start at 1 at the beginning of the sample period. Note that for the analysis of bid-ask spread estimates, the time period is January 1st, 2016, except for BCH which starts August 1st, 2017.

Fig. 2 shows a time series plot of a 20-day moving average of the spread-estimator for Ethereum. It is worth noting that the spread estimator is in general positive, with a mean of about 0.4%. However, the spread-estimator produces several negative estimates. For equity markets, the spread estimator by Corwin and Schultz (2012) is known to produce a large share of negative values. After accounting for overnight trading, where the closing price at time t is different from the open price at time $t+1$, the values produced by

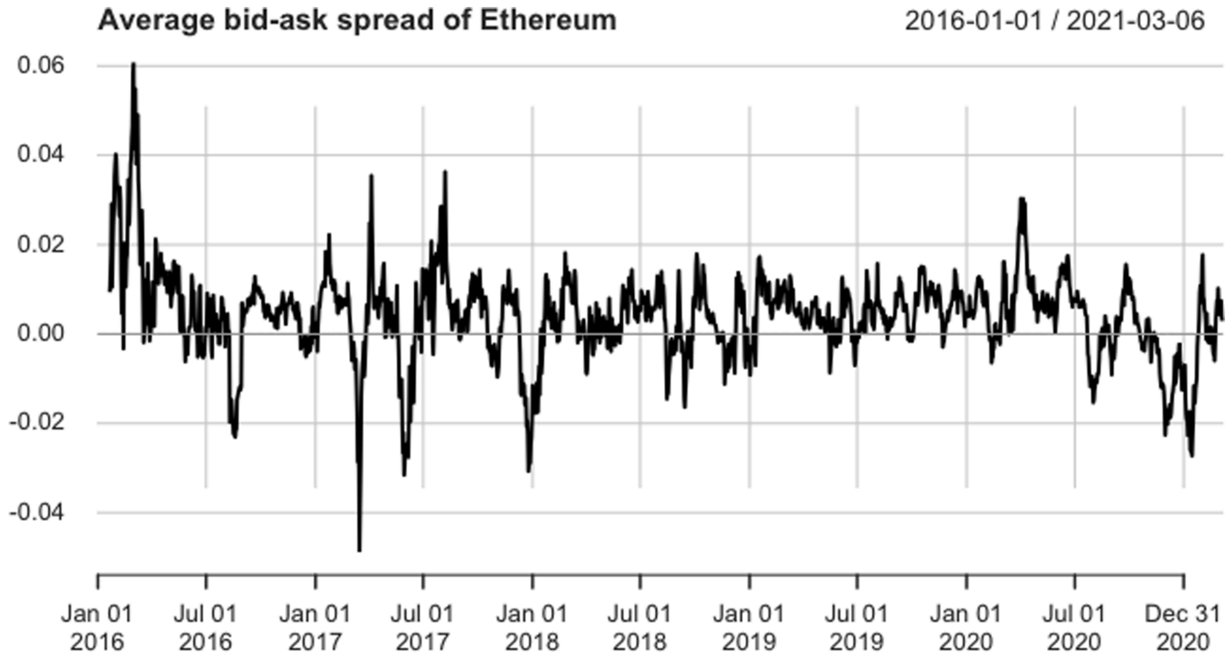

Fig. 2. Time series plot of bid-ask spread estimator for Ethereum.

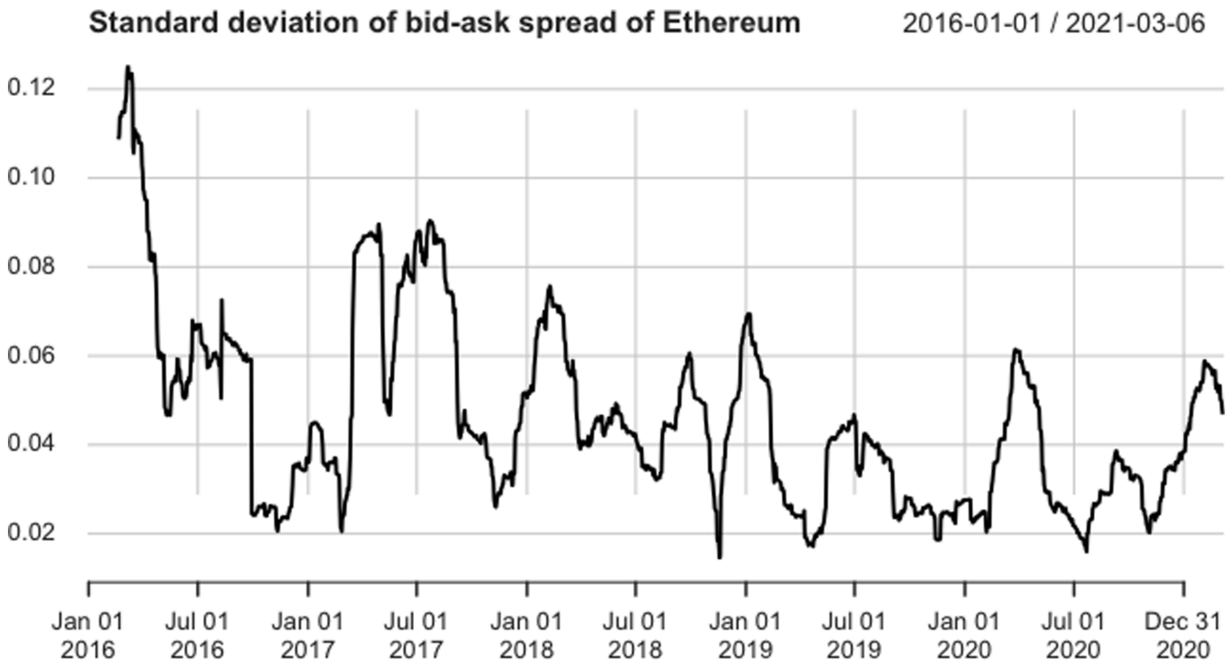

Fig. 3. Rolling window of the standard deviation of the bid-ask spread of Ethereum. It seems the volatility is on average higher in 2016-18 than in 2019-20, but there are some significant jumps early 2019 and early 2020.

the estimator is significantly improved. As cryptocurrencies trades around the clock, there is no large differences between $close_t$ and $open_{t+1}$ prices, which means that the number of negative estimates are reduced, though not eliminated.

Fig. 3 shows a 50-day rolling window of the standard deviation of the spread estimator of ETH. Even though the long-run average seems to be around 5%, the figure illustrates that the volatility of the spread has decreased over time. However, there are a few abrupt changes in the recent period, but the overall level is lower than in the beginning of the period analyzed. Indeed, the man and volatility of the spread in the first half of the period analyzed is $\overline{LIQ}_{ETH} = 0.0047$ and $sd(LIQ_{ETH}) = 0.063$, whereas for the last half these are

Table 4
Regression output. All currencies show a positive and significant relationship between the volatility of liquidity and returns. Not surprisingly, all currencies are also positively related to the crypto-index (CRP). Interestinlgy, the relationship to the stock market level (SP500) and its volatility (VIX) is not homogenous across the assets.

	Dependent variable:				
	R_{BTC}	R_{BCH}	R_{ETH}	R_{LTC}	R_{XRP}
	(1)	(2)	(3)	(4)	(5)
σ_{LIQ}^{BTC}	0.023***				
	(0.006)				
σ_{LIQ}^{BCH}		0.174***			
		(0.005)			
σ_{LIQ}^{ETH}			0.303***		
			(0.010)		
σ_{LIQ}^{LTC}				0.109***	
				(0.007)	
σ_{LIQ}^{XRP}					0.198***
					(0.005)
R_{CRP}	0.524***	0.419***	0.575***	0.766***	0.565***
	(0.009)	(0.035)	(0.029)	(0.021)	(0.026)
R_{SP500}	2.495***	3.858***	– 1.022**	1.104***	- 1.154***
	(0.108)	(0.462)	(0.413)	(0.169)	(0.228)
R_{VIX}	– 0.011	0.364***	– 0.684***	- 0.267***	- 0.500***
	(0.013)	(0.054)	(0.053)	(0.024)	(0.029)
Constant	– 0.002***	– 0.010***	– 0.009***	– 0.004***	- 0.006***
	(0.0004)	(0.0005)	(0.001)	(0.0002)	(0.0002)
Observations	1864	1319	1864	1864	1864
R^2	0.746	0.832	0.769	0.784	0.795
Adjusted R^2	0.746	0.831	0.768	0.783	0.795
Residual Std.Err	0.002	0.004	0.005	0.003	0.004
F Statistic	1,367***	1,624***	1,544***	1,685***	1,806***
Note:				*p<0.1; **p<0.05; ***p<0.01	

$\overline{LIQ}_{ETH} = 0.0034$ and $sd(LIQ_{ETH}) = 0.038$. Qualitatively, the same holds for the other currencies studied: both the mean of the spread and its volatility decreases over time.

Estimating the parameters of Eq. (3) over the whole sample period, I find that the β_1 parameter is positive for all currencies. This indicates that there is a positive relationship between the volatility of liquidity and returns. All parameter estimates for the relationship between liquidity-volatility and returns are highly significant. The parameter estimates for the cryptocurrency index are also all positive and significant. This is not surprising, as the five currencies analyzed in this paper are the five largest by market capitalization, and has a total of about 80% ov the total market capitalization. However, the index is equally weighted so small-cap currencies have a equally large impact on the index returns as the largest currency (Bitcoin). It is interesting to note that the sensitivities against the S&P500 index are positive for Bitcoin, BitcoinCash, and Litecoin, but negative for Ethereum and Ripple. In this setting, Bitcoin is not affected by stock-market volatility, as measured by the VIX index. The parameter estimate is negative (–0.011), but not significant. For the other currencies, Bitcoin Cash has a positive relationship with the VIX-index, whereas the VIX-index has a negative impact on the returns of Ethereum, Litecoin, and Ripple. Table 4 shows the results from the regression.

These results indicate that there is a positive relationship between the volatility of liquidity and returns. This corroborates the results of Akbas et al. (2013), and how financial markets are expected to work; investors consider liquidity risk as something they should be compensated for. However, these results contrasts those of Chordia et al. (2001). The reason might be that (Chordia et al., 2001) applies another liquidity-measure, which is related to turnover. However, Brauneis et al. (2021) shows that spread estimator of Corwin and Schultz outperform the time-series properties of liquidity compared to the measure applied in Chordia al al. (2001). The reason is that the liquidity measure applied in this paper is somewhat more directly related to what investors consider liquidity-risk relevant for trading: the bid-ask spread. It is interesting to note that the R^2 is relatively high. This indicates that the model explains large parts of the variation in returns for these cryptocurrencies. In line with Akbas et al. (2013), the analysis in this paper show that there is in general a positive relationship between the volatility of liquidity and cryptocurrency returns. This implies that when liquidity volatility increases, expected returns increases as well.

5. Conclusion

This paper analyzes the relationship between the volatility of liquidity and returns of five large capitalization cryptocurrencies. The results indicates that there is a positive relationship between the volatility of liquidity and returns in general. This means that investors consider the time-variation of liquidity as a risk which should be compensated with higher returns. For Bitcoin, the largest crypto-currency, this relationship varies over time, and it is found that the relationship between the volatility of liquidity and returns is the

Table 5
This table presents summary statistics for the log-returns of the cryptocurrencies analyzed in the paper over the period 1.1.2016 through 8.12.2020.

	ETH	BTC	XRP	BCH	LTC
Observations	1803	1803	1803	1224	1803
Minimum	–1.3029	–0.4647	–0.6163	-0.4491	-0.5613
Quartile 1	–0.0229	–0.0106	–0.0190	–0.0183	–0.0319
Median	0.0000	0.0020	–0.0023	-0.0004	-0.0031
Arithmetic Mean	0.0027	0.0019	0.0023	0.0017	-0.0002
Geometric Mean	NaN	0.0012	0.0003	0.0002	-0.0033
Quartile 3	0.0277	0.0160	0.0173	0.0204	0.0266
Maximum	0.4103	0.2251	1.0274	0.5114	0.4316
SE Mean	0.0016	0.0009	0.0015	0.0012	0.0018
LCL Mean (0.95)	–0.0003	0.0002	–0.0006	-0.0007	–0.0037
UCL Mean (0.95)	0.0058	0.0037	0.0052	0.0041	0.0033
Variance	0.0047	0.0015	0.0043	0.0029	0.0061
Stdev	0.0685	0.0386	0.0654	0.0540	0.0783
Skewness	–3.4695	–0.9495	3.0050	0.7489	0.4324
Kurtosis	72.0353	14.7795	46.4623	13.5539	8.4200

lowest, yet positive, among the currencies studied. This again indicates that investors in Bitcoin consider liquidity less a risk compared to the other currences, which might be due to the popularity of this particular currency.

CRediT authorship contribution statement

Thomas Leirvik: Conceptualization, Formal analysis, Project administration, Writing - original draft, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests.

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Appendix A. Appendix

In the paper by Corwin and Schultz (2012), the bid-ask spread estimator is given in Eq. 4, but can be significantly simplified to the one given as Eq. 5:

$$S_t = \frac{\sqrt{2\cdot\beta} - \sqrt{\beta}}{3 - 2\cdot\sqrt{2}} - \frac{\sqrt{\gamma}}{\sqrt{3 - 2\sqrt{2}}} \tag{4}$$

$$\begin{aligned} &= \frac{\sqrt{\beta}\left(\sqrt{2}-1\right)}{3-2\cdot\sqrt{2}} - \frac{\sqrt{\gamma}}{\sqrt{3-2\sqrt{2}}} \\ &= \left(1+\sqrt{2}\right)\left(\sqrt{\beta}-\sqrt{\gamma}\right) \end{aligned} \tag{5}$$

Eq. 5 is easier to read, somewhat easier to compute and interpret. The spread estimator is basically the difference between the average and the maximum of the two-day log-range of maximum and minimum prices. The beta is given as the two-day sum of the squared natural logarithm of the ratio highest-to-lowest price each day t:

$$\beta_t = \mathbb{E}\left[\sum_{j=t-1}^{t}\left(\ln\left(\frac{H_j}{L_j}\right)\right)^2\right]$$

This is an approximate measure of the two-day volatility. The gamma is given by the squared natural logarithm of the ratio of the two-day maximum and minimum prices:

$$\gamma_t = \ln\left(\frac{H_{t,t-1}}{L_{t,t-1}}\right)^2$$

where $H_{t,t-1}$ ($L_{t,t-1}$) is the maximum (minimum) price of the days t and $t-1$. The spread estimator S_t is derived based on a high-low volatility measure, and is an approximate measure of the liquidity of an asset. The spread measure is widely used in scientific articles where the liquidity of stocks are investigated, see, for example, Mclean and Pontiff (2016) and Koch et al. (2016), and the references within.

Summary statistics of the log-returns for the cryptocurrencies are given in Table 5. As can be seen, the log-returns produce some extremes that are not necessarily economically sound. For example, the minimum daily return of ETH is -130%, which is not possible, as the price would then be negative.

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