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An adaptive demand response framework using price elasticity model in distribution networks

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ABSTRACT

Price elasticity model (PEM) is an appealing and modest model for assessing the potential of flexible demand in demand response (DR). It measures the customer's demand sensitivity through elasticity in relation to price variation. However, application of PEM is partially apprehensible on attributing the adaptability and adjustability along with intertemporal constraints in DR. Thus, this article presents an adaptive economic DR framework with its attributes via a dynamic elasticity approach to model customer's demand sensitivity. This dynamic elasticity is modeled through the deterministic and stochastic approaches. Both approaches envision the notion of load recovery for shiftable/flexible loads to make the proposed framework adaptive and adjustable relative to price variation. In stochastic approach, a geometric Brownian motion is employed to emulate load recovery in addition to intertemporal constraint of load flexibility. The proposed mathematical model shows what should be the customers elasticity value to achieve the factual DR. The numerical study is carried out on standard IEEE 33 distribution system bus load data to assess its technical and socio-economic impact on customers and is also compared with the existing model.

1. Introduction

The fusion of new energy resources and technological breakthrough has transformed the distribution networks (DN) operation and control. Many thanks to smart grids, which have infused DN with resources that exhibit dual nature (i.e., source and sink). In this, energy storage, electric vehicle to grid or vice-versa and prosumers (consumers which produces and consumes power themselves) are newly evolved state-of-the-art technologies in DN [1]. These overhauls have brought many advantages as well as challenges in the system operation and control. In the meanwhile, need of demand response (DR) also has made its presence felt in the system. It is a result due to persistent events such as sudden peak rise, increased penetration of renewable generation, wholesale price fluctuation, reliability and security issues prevailing in the system [2,3]. This led to a pathway for DR that further amplified by increased interaction between utility and customers via advanced metering infrastructure (AMI) [4]. In the past few decades, there have been a lot of research on DR accommodation in the system. It includes mainly two types of approaches viz. price-based demand response (PBDR) and incentive-based demand response (IBDR). PBDR inflicts dynamic price on customers to change their load pattern and IBDR lays out incentive to customers during peak hours for curtailing their loads [5–7]. PBDR is usually modeled as voluntary program such as real time price (RTP), time-of-use (TOU), critical peak pricing (CPP). On the other hand, IBDR programs are categorized into voluntary (direct load control (DLC), interruptible/curtailable services (IC/S)), mandatory (capacity market program (CAP), emergency demand response program (EDRP)), and market clearing (demand bidding, ancillary services(AS)), respectively [6,7].

DR is one of the kinds which is driven by the utilities and executed by the customers. Thus, customers behaviour is essential to understand DR. It is extensively studied in micro-economics, a part of economics [8]. Many of its applications are also being adopted in DR applications. In this, customers preference model, game theory and price elasticity model (PEM) are widely employed approaches in DR modeling as described in literature [9–13]. Where, utility functions are employed to model the customer priority, game theory in rational decision making under the competitive environment and PEM for load modeling of DR in PBDR. This article exclusively covers DR modeling using PEM to observe customer's reaction to price change for analyzing load pattern variation. It aims to model elasticity adaptive for imitating the customer's realistic behavior in DR and is the main motivation of this study.

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Functions and Variables

$\Phi^{-1}(.)$	Inverse cumulative normal distribution.
$\Phi(.)$	Cumulative normal distribution.
$B(.)$	Customer benefit function.
$D_i^{cl}(t)$	Demand after DR of ith customer of class cl at time t.
$L(.)$	Lagrange function.
$N(.)$	Normal distibution.
$S(.)$	Net benefit/Social welfare function.
W	Weiner/Brownian variable.

Indices and Sets

cl, CL	Index and set of customer classes.
i, I	Index and set of customer number.
n, N	Index and set of load bus.
t, τ, T	Index and set of time.
$T_{P/OP/V}$	Set of peak, off-peak and valley hours.

Parameters and Constants

α_i^{cl}	Factor of of load curtailment.
$\Delta D_i^{cl}(t)$	Change in demand of ith customer of class cl at time t.
κ^{cl}	Price coefficient for charging/subsidy over different classes.
λ_i^{cl}	Lagrange multiplier of ith customer of class cl.
μ, σ	Mean and standard deviation.
ρ_t	Utility selling real time price at time t.
ρ_t^{cl}	Real time price offered to class cl at time t.
a_t^{cl}, b_t^{cl}	Lower and upper bound of truncated normal distribution.
$D_{i,o}^{cl}(t)$	Demand before DR of ith customer of class cl at time t.
LF_i^{cl}	Load factor of ith customer of class cl.

Symbols

$\bar{(\cdot)}$	Mean value of (.).
$\underline{(.)}, \overline{(.)}$	Upper and lower bound of (.)

1.1. Literature review

An early approach of PEM application in DR is studied in [9] where authors described the customer's reactions to price change using PEM. The authors explain that self and cross-elasticity of demand can be utilized for setting up price in the centralized electricity market. The authors further extended the work to show the effect of load shifting in the saving of system operating cost in a centralized complex bid-market clearing mechanism with consideration of customer's participation level [14]. A price responsive economic load model is investigated in [5] using PEM under the various PBDR programs. The analysis reveals that in the most of the programs, DR achieve better peak reduction and improved load factor. But from the economic point of view, utility revenues are increased at the cost of customers bills. A comprehensive study is carried out on the different non-linear functions such as logarithmic, exponential and power structure based on the concept of customer benefit and price elasticity for measuring the DR under the dynamic price [15]. The results show that under small elasticity and small price deviation all models give nearly same response. Likewise study is demonstrated in [6] using the various linear and non-linear behavioral models in the electricity market. It concluded that each behavioral model gives distinct load response to the considered market load pattern. This displays dependency of DR behavioral model on different market's load pattern. It is better addressed in [16], where the authors considered a composite weighted DR function amalgamating linear and non-linear functions to assess customer's DR in dynamic price. In contrast to Ref. [5,6], the authors in [17] suggested an approach to determine optimal price using PEM under the various linear and non-linear models. It indicates that different mathematical models exhibit different optimal prices showing dependency upon DR models. In [18,19] authors have utilized the PEM in short and medium-term decision-making models via RTP and TOU model to measure the impact of DR on distribution company's profit. A similar framework is also considered in [20–22], where the authors evaluated elastic/flexible demand under the different PBDR programs. However, in most of these studies [5,9,20–22], DR is evaluated at utility level using an aggregated load approach with the static value of elasticity as also pointed out in [7]. It advises a need to develop an adaptive or flexible elasticity.

Though, a need of disaggregated and discrete elasticity approach is also envisioned in the literature. A disaggregated load profiling approach and distinct elasticity value based on load sector and appliances is considered in [23] to observe the flexible demand in real-time environment with wind power penetration. The author stresses on the development of intelligent demand forecasting algorithm to account time and weather conditions. Further suggested the need of different electricity price patterns to the different customers via wide range of tariff for better participation level in DR. In Ref. [24], PEM is modeled stochastically considering the various scenarios with a predefined range of elasticity value to measure the customers's varying response in DR. In Ref. [25], the authors estimated individual elasticity of customers through sparse construction method using linear regression after the change in the price. A flexible DR program modeling is explored to calculate price elasticity of flexible demand on the basis of price before and after implementation of DR in the competitive electricity markets. It is evaluated on the basis of the assumed price-demand curve and the expected participation level of DR [7]. To better illustrate elasticity, a microscopic approach for residential customers is observed in [26] by considering an appliance-wise elasticity based on survey under IBDR. The results show that this disaggregated approach gives better visualization of flexible load in DR. In similar line, the author in [27] has investigated the various non-linear behavioral models on residential customers to observe the effect of DR on the operating cost and emission of gases in unit commitment problem. It asserted the need of a comprehensive assessment on selecting a suitable DR model for the different customers' load pattern. In Ref. [28], the distinctive elasticity coefficients are assumed for all states, further these coefficients are elaborated distinctively to describe the customers possible reaction in DR. However, there is no meticulous analytical model for assuming these values. Similar coherence is also deliberated in [29], where the authors stated that the price increment/decrement during distinct time periods cannot give same DR due to customer's non-linear behaviour. It assumed low elasticity value for low price periods and high elasticity for high price periods. In [30], the authors have assumed distinct price elasticity matrices for the customer's possible responses in DR to reflect its effect on market-bidding. A better dynamic elasticity extraction approach is suggested in [16], where demand functions are correlated with the price on the basis of fitting of historical data of price and demand. In [31], the authors have postulated an improved price elasticity as a function of customer types and consumption hours in addition with price parameter. Further, elasticity is extracted by fitting historical data into an an exponential function to reflect the customer's non-linear behaviour in DR. This gives the distinct elasticity value at each hour as opposite to fixed value approach considered in the previous works.

The premises of distinct value for each hour put PEM model more closer to customer' pragmatic behaviour in DR. However, PEM model still lacks in incorporating customer's load shifting capability over time horizon, which vary with time elongation. It is characterized by inter-temporal time constraints, which set permissible time limits on load shifting. A few applications of DR with inter-temporal time constraints are reported in [32–34]. In Ref. [32], the authors have applied linear intertemporal constraints to mimic the flexible band of time for shiftable loads. Further, energy balance constraint is applied to ensure no loss of load over the time frame considered. This concept of load recovery is also rendered in [33] using the utility theory and in [34] using flexibility band for different types of loads. In [33], the authors explained that as the customer delay or curtail its demand during high price time period, an exponentially deceasing customer's utility will follow across the other time periods. In PEM, cross-elasticity governs the load recovery for the adjustable/shiftable load. It indicates that cross-elasticity can be means of accommodating intertemporal constraint. Though, modeling of flexibility time constraint is a complex task owing to large number of customers and their different activity pattern. Hence, in this context stochastic approach can be a starting point to model load recovery along with time-flexibility constraints in PEM. It is suggested as a part of the proposed study.

1.2. Motivation and contribution

In the light of above literature review, it can be assessed that PEM is an appealing and an extensively utilized approach to observe the customer's reaction to DR. However, the application of PEM in DR valuation does not fully apprehend its attributes of adaptability and adjustability. These two attributes are exhibited through elasticity from PEM perspective, which measures the relative variation in the demand and price. Though, this relative variation (increment/decrement) does not follow same elasticity pattern due to non-linear behaviour of customers [30]. Further, the application of PEM in DR is generally carried out on the aggregated system demand. This gives overall DR at system level, but may obscure DR reflection occurring at customer class level. Though, the later studies illustrate customer or appliance wise elasticity modeling, showing a paradigm shift. Nonetheless, these applications are limited in literature. More importantly, a less attention has been paid to the valuation of elasticity in the most of the existing studies with few exceptions. It is generally assumed as a fixed value on the different segmented time periods. This diminishes the attribute of elasticity, which requires to be dynamic as demand and price are being time dependent. Further, it lowers the interrelation effect existing between price and demand over the multi-state time framework. Besides, load recovery together with intertemporal constraints is not addressed in PEM to relate the pragmatic DR.

In view of this, the proposed study presents an adaptive economic DR framework using a dynamic elasticity to replicate adaptability and adjustability between peak hours and off-peak/valley hours. The proposed dynamic elasticity is realized through two approaches namely: a deterministic and a stochastic approach. In deterministic approach, elasticity value is being made dynamic to interrelate elasticity of peak and off-peak/valley hours. This dynamic elasticity capsulizes the attributes of self and cross-elasticity for giving lossless DR. In stochastic approach, elasticity is modeled as the stochastic process to exhibit the customers' load recovery stochastically in DR. A geometric Brownian motion (GBM) is utilized to model the cross-elasticity to emulate flexibility evolution over the time. The proposed stochastic model incorporates intertemporal constraints in load recovery for shiftable/flexible loads, which was not explicitly modeled in PEM and DPEM. It presents a realistic load recovery with intertemporal structure of load shift. Besides, a disaggregated load approach for DR valuation at customer level is adopted in contrast to aggregated load as considered in the literature. For this, a bottom-up approach is considered to accommodate different classes and their diversified customers. The main contributions of the paper are summarized as follows:

- An adaptive economic DR framework is developed using a dynamic elasticity in PEM to make customer's behavior in DR adaptive and adjustable on the account of price variations.
- The proposed dynamic elasticity is modeled using a deterministic and stochastic approach. In deterministic, dynamic elasticity establishes a relationship between peak and off-peak/valley hours elasticity to realize complete load recovery for the curtailable/shiftable load.
- A stochastic approach is envisioned to model customers' uncertain behaviour in DR using PEM. For this, a stochastic process, GBM is utilized to imitate load recovery in DR with the adaption of intertemporal time constraint of load flexibility.
- A disaggregated load modeling approach is proposed to assess the effect of DR in class-wise customers. It is illustrated by considering the typical customer load patterns of different classes. Thereafter, diversified customers load pattern are simulated to illustrate heterogeneity existing within the customer classes.

The remaining paper is planned as follows Section 2 describes proposed methodology and the motivation behind it. Section 3 describes the load profiling. Result and comparison analysis is performed in the Section 4. Finally, conclusion of paper is presented in Section 5.

2. Proposed adaptive economic DR framework

The customer response in DR under the ambit of dynamic price is measured as the difference of customer benefit and their payment to the utility. This function is termed as net benefit function or social welfare function. The customers' benefit and its payment vary with the demand capacity, activity usage and societal aspects. Therefore, a disaggregated approach is considered as a better realization to observe the distinct response in DR as suggested in [2]. It is also assumed that all the customers will behave rationally in DR for the ease of exposition. On the basis of this, the net benefit function for each customer of different class can be expressed as follows:

$$S\left(D_i^{cl}(t)\right) = B\left(D_i^{cl}(t)\right) - D_i^{cl}(t)\rho^{cl}(t) \tag{1}$$

where, $S(D_i^{cl}(t))$ and $B(D_i^{cl}(t))$ are net benefit/social welfare and benefit function under demand consumption $D_i^{cl}(t)$ for ith customer of class cl at time t, respectively.$\rho^{cl}(t)$ is offered RTP to the customer of class cl at time t.

When customers participate in DR, they can optimistically expect: i) to get the benefit from participating in DR program and ii) to maintain the overall consumption of energy same. These two expectations can only be coherent, when customers are adaptive and adjustable to DR program. Thus, to make customer's adaptable and adjustable to the DR program, the problem is formulated as a constraint optimization to redistribute the demand optimally. This allows the customer to keep its overall consumption same before DR (BDR) and after DR (ADR). Based on this assumption, the following constraint is imposed on net benefit function and is expressed as follows:

$$\psi_i^{cl} = \sum_{t\in T_P}\Delta D_i^{cl}(t) - \sum_{t\in\{T_{OP}\cup T_V\}}\Delta D_i^{cl}(t) = 0 \tag{2}$$

where, $\Delta D_i^{cl}(t)$ is defined as change in demand of ith customer of class cl at time t. Equation (2) denotes the energy balance constraint BDR and ADR. This constraint transforms the customer' net benefit function into a constraint optimization problem. Such problem formulation can be optimized effectively using a Langrage function, which add the constraint into the objective function and multiply the constraint by a coefficient, called as Lagrange multiplier λ [35]. The Lagrange function $(L(D_i^{cl}(t)))$ of ith customer of class cl at time t for the proposed formulation can be expressed as follows:

$$L\left(D_i^{cl}(t)\right) = S\left(D_i^{cl}(t)\right) + \lambda_i^{cl}\psi_i^{cl} \tag{3}$$

$$L\left(D_i^{cl}(t)\right) = S\left(D_i^{cl}(t)\right) + \lambda_i^{cl}\left(\sum_{t\in T_P}\Delta D_i^{cl}(t) - \sum_{t\in\{T_{OP}\cup T_V\}}\Delta D_i^{cl}(t)\right) \tag{4}$$

The second term in (4) represents the difference of demand exchanged between the peak and valley/off-peak hours. It is to be noted that the set of peak (T_P), valley (T_V) and off-peak (T_{OP}) hours are pre-determined and non-overlapping for the sake of simplicity. Further, substituting (1) and (2) into (3), we get

$$L\left(D_i^{cl}(t)\right) = B\left(D_i^{cl}(t)\right) - D_i^{cl}(t)\rho^{cl}(t) + \lambda_i^{cl}\left(\sum_{t\in T_P}\Delta D_i^{cl}(t) - \sum_{t\in\{T_{OP}\cup T_V\}}\Delta D_i^{cl}(t)\right) \tag{5}$$

where, $D_i^{cl}(t)$ and $D_{i,o}^{cl}(t)$ are defined as BDR and ADR demand of ith customer of class cl at time t, respectively. The optimal condition of the Lagrange function can be defined for the different time periods by taking the first derivative of (6).

$$L\left(D_i^{cl}(t)\right) = B\left(D_i^{cl}(t)\right) - D_i^{cl}(t)\rho^{cl}(t) + \lambda_i^{cl}\left\{\sum_{t\in T_P}\left(D_{i,o}^{cl}(t) - D_i^{cl}(t)\right) - \sum_{t\in\{T_{OP}\cup T_V\}}\left(D_i^{cl}(t) - D_{i,o}^{cl}(t)\right)\right\} \tag{6}$$

$$\frac{\partial L\left(D_i^{cl}(t)\right)}{\partial D_i^{cl}(t)} = \frac{\partial B\left(D_i^{cl}(t)\right)}{\partial D_i^{cl}(t)} - \rho^{cl}(t) - \lambda_i^{cl} \qquad \forall t \tag{7}$$

It can be seen from (7) that the marginal rate of benefit function varies with the price of that hour.

$$\frac{\partial B\left(D_i^{cl}(t)\right)}{\partial D_i^{cl}(t)} = \rho^{cl}(t) + \lambda_i^{cl} \qquad \forall t \tag{8}$$

The second derivative of (8) is defined as following.

$$\frac{\partial B^2\left(D_i^{cl}(t)\right)}{\partial D_i^{cl}(t)^2} = \frac{\partial \rho^{cl}(t)}{\partial D_i^{cl}(t)} = \frac{1}{\varepsilon_i^{cl}(t)}\frac{\rho_o^{cl}(t)}{D_{i,o}^{cl}(t)} \tag{9}$$

It is to be noted that elasticity ($\varepsilon_i^{cl}(t)$) is a result of relative variation in price. Thus, it is denoted as price elasticity. Now, extending the customer benefit function in a quadratic form using Taylor series expansion is expressed as follows: [2].

$$B\left(D_i^{cl}(t)\right) = B\left(D_i^{cl}(t)\right) + B^{'}\left(D_i^{cl}(t)\right)\left(D_i^{cl}(t) - D_{i,o}^{cl}(t)\right) + B^{''}\left(D_i^{cl}(t)\right)\frac{\left(D_i^{cl}(t) - D_{i,o}^{cl}(t)\right)^2}{2} \tag{10}$$

$$B\left(D_i^{cl}(t)\right) = B\left(D_i^{cl}(t)\right) + \left(\rho_o^{cl} + \lambda_i^{cl}\right)(t)\left(D_i^{cl}(t) - D_i^{cl}(t)\right) + \frac{\rho_o^{cl}(t)}{\varepsilon_i^{cl}(t)D_{i,o}^{cl}(t)}\frac{\left(D_i^{cl}(t) - D_{i,o}^{cl}(t)\right)^2}{2} \tag{11}$$

Differentiating (11) and equating it with (8) gives the following expression.

$$\rho^{cl}(t) + \lambda_i^{cl} = \rho_o^{cl}(t)\left\{1 + \frac{\left(D_i^{cl}(t) - D_{i,o}^{cl}(t)\right)}{\varepsilon_i^{cl}(t)D_{i,o}^{cl}(t)}\right\} \tag{12}$$

Rearranging the expression, the customer's demand at any time t ADR will be given by:

$$D_i^{cl}(t) = D_{i,o}^{cl}(t)\left\{1 + \varepsilon_i^{cl}(t)\frac{\left(\rho^{cl}(t) - \rho_o^{cl}(t) + \lambda_i^{cl}\right)}{\rho_o^{cl}(t)}\right\} \qquad \forall t \in T \tag{13}$$

Equation (13) represents the demand ADR using self-elasticity. Similarly, to incorporate the effect of cross-elasticity, PEM can be extended into multi periods as follows [5]:

$$D_i^{cl}(t) = D_{i,o}^{cl}(t)\left\{1 + \varepsilon_i^{cl}(t)\frac{\left(\rho^{cl}(t) - \rho_o^{cl}(t) + \lambda_i^{cl}\right)}{\rho_o^{cl}(t)} + \sum_{\tau\in T}\varepsilon_i^{cl}(t,\tau)\frac{\left(\rho^{cl}(\tau) - \rho_o^{cl}(\tau) + \lambda_i^{cl}\right)}{\rho_o^{cl}(\tau)}\right\} \qquad \forall t\in T,\ \forall\tau\in T \tag{14}$$

Equation (14) represents the combined effect of self and cross-elasticity on the demand. Where, self-elasticity manifests increment/decrement relative to change in the price on the same period, whereas, cross-elasticity evaluates the transverse effect of price on the cross-demand. It is mathematically defined as follows:

$$\varepsilon(t,\tau) = \left(\frac{\Delta D(t,\tau)}{\Delta\rho(t,\tau)}\right)\left(\frac{\rho_o(t,\tau)}{D_o(t,\tau)}\right) \tag{15}$$

$$\begin{cases}\varepsilon(t,\tau) \le 0 & \text{if} t = \tau \\ \varepsilon(t,\tau) > 0 & \text{if} t \ne \tau\end{cases} \tag{16}$$

Equation (16) denotes the notation of self and cross-elasticity, where self-elasticity is attributed to negative value for single time period and positive value is assigned to latter for two different time periods (t, τ).

From the electricity point of view, the magnitudes of both the elasticities vary with the customer's reaction to price change in DR. Thus, assessment of each type of customer requires a rigorous approach, which is cumbersome task due to complex behaviour of customers. Though, with some approximate assumptions (as considered above) an analysis can be carried out to assess the impact of DR using PEM. In this context, this study presents a deterministic and a stochastic approach to model DR using PEM. In deterministic approach, a dynamic elasticity encapsulating the features of self and cross-elasticity is proposed. Likewise, in stochastic PEM, a cross-elasticity is modeled using a geometric Brownian motion (GBM) to imitate customer behaviour under the load recovery.

2.1. Proposed deterministic PEM (DPEM)

In DR, increase or decrease of load in a single time-period is achieved through switching or continuous control and is modeled by self-elasticity. On the other hand, cross-elasticity is defined for the shiftable loads. However, the interrelation between both the elasticities is merely discussed in the literature. Though, in Ref. [9,30] the authors have touched upon the issue of optimal redistribution of demand in DR using PEM. These studies states that the optimal redistribution of load is possible when the following condition is satisfied:

$$\sum_{\tau\in T}\varepsilon(t,\tau)=0 \qquad \forall t \tag{17}$$

Equation (17) describes that the overall change in elasticity (i.e., self and cross-elasticity) over the time horizon should be zero for optimal redistribution of load. It contemplates that change in demand obtained using the elasticity over the time-horizon remains unchanged. This indicates that whatever demand is curtailed using self-elasticity should be adjusted in the cross-time periods (τ) through the cross-elasticity [9]. It defines the loss-less situation (i.e. no change in the consumption BDR and ADR. The aforementioned equation advocates that an interrelation should exist between self and cross-elasticity for an adaptive and adjustable DR framework. In this regard, a dynamic elasticity model attributing the features of both the elasticities is envisioned.

The basis of the proposed dynamic elasticity is to make DR adaptive, which depends upon the customer's load flexibility and time-stretchiness across the cross-periods. Therefore, a dynamic elasticity is proposed for exhibiting the customer's flexibility from peak hours to low-price hours. It establishes an interrelation between peak hours elasticity to off-peak/valley hours for better adaption of DR. As, it is presumed that the shifted demand during off-peak/valley hours is a reflection of curtailed peak demand. In same way, it can be assumed that elasticity during off-peak/valley hours will be reflection of peak hours elasticity. For this assumption to stand, the product of elasticity and demand at any hour is assumed to be equal to product of elasticity and demand at peak hour. This gives elasticity value relative to peak hour elasticity defined as follows:

$$\varepsilon_i^{cl}(t)D_{i,o}^{cl}(t)=\varepsilon_i^{cl}\left(t_{peak}\right)D_{i,o}^{cl}\left(t_{peak}\right) \qquad \forall t\in T\backslash\left\{t_{peak}\right\} \tag{18}$$

Equation (18) defines the relationship of elasticity relative to peak hour's elasticity (t_{peak}) with corresponding demand $D_{i,o}^{cl}(t)$. The assumed relation is defined in such a way that if demand and expected elasticity (i.e. possible peak load curtailment) at peak hour are known, then elasticity of other time states can be evaluated in reference to peak hour. The hypothesized relationship can be explained by considering two states (on and off peak) example for the clarity as depicted in Fig. 1.

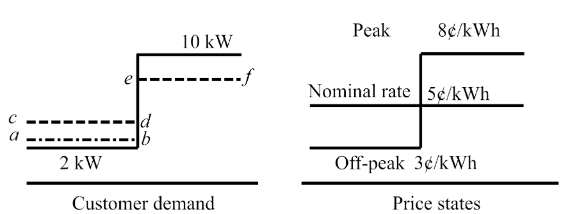

Fig. 1. Schematic diagram of two states under static and dynamic elasticity.

Let the nominal electricity price is 5 ¢/kWh and dynamic price is 3 ¢/kWh and 8 ¢/kWh at off-peak and peak hour, respectively. The corresponding demand before DR is 2 kW and 10 kW at nominal price. Now, if DR in initiated with price variation. The customer will respond to price by varying the demand evaluated using price elasticity. If it is assumed that peak hour elasticity is 0.2. Then, curtailed load demand at peak hour will be 1.2 kW marked by line *ef* calculated using (15). While at off-peak hour the demand is raised by 0.24 kW defined by line *ab* for the same elasticity value. This reflects partial DR. However, through the proposed dynamic elasticity approach, the elasticity value at off-peak will be equal to 1 as evaluated using (18) relative to peak elasticity. In this case the load demand is increased by 1.2 kW denoted by line *cd*, which is equal to curtailed load demand.

The determination of peak hour is another complex task due to different load pattern of customer classes. For this, two approaches are suggested. In first approach, peak hour is that hour, where customers maximum demand occurs. At this hour, elasticity value will be assumed suitably and other hours elasticity will be calculated relatively to peak hour.

$$\varepsilon_i^{cl}(t)=\varepsilon_i^{cl}\left(t_{peak}\right)\left[\frac{\max\left(D_{i,o}^{cl}\right)}{D_{i,o}^{cl}(t)}\right] \qquad \forall t\in T\backslash\left\{t_{peak}\right\} \tag{19}$$

In second approach, peak hour is defined on the basis of peak price. Since, the customer will have highest sensitivity at peak price. Thus, it is chosen as peak hour. Then, at this hour the permissible demand which may be possibly curtailed for DR is assumed to be the fraction of the demand at peak price. Based on this, elasticity value is evaluated at peak hour. The other hour's elasticity is evaluated using (19).

$$\begin{aligned}
&\Delta D_i^{cl}(t)=\alpha_i^{cl}D_{i,o}^{cl}(t)\Big|_{t=t_{peak}\triangleq\rho_{peak}^{cl}=\max(\rho^{cl})}\\
&\varepsilon_i^{cl}\left(t_{peak}\right)=\left(\frac{\Delta D_{i,o}^{cl}(t)}{\Delta\rho^{cl}(t)}\right)\left(\frac{\rho_o^{cl}}{D_{i,o}^{cl}(t)}\right)\Bigg|_{t=t_{peak}\triangleq\rho_{peak}^{c}=\max(\rho^{cl})}\\
&\varepsilon_i^{cl}(t)=\varepsilon_i^{cl}\left(t_{peak}\right)\left[\frac{D_i^{cl}\left(t_{peak}\right)}{D_i^{cl}(t)}\right] \qquad \forall t\in T\backslash\left\{t_{peak}\right\}
\end{aligned} \tag{20}$$

It is to be noted that α_i^{cl} is a proportion of the flexible demand of total available demand of *i*th customer of class *cl*. Moreover, it varies with the customer's activity usage and classes. Once the elasticity at every hour is defined using (19). Then, the final expression of ADR demand of *i*th customer of class *cl* at time *t* is given by the following expression.

$$D_i^{cl}(t)=D_{i,o}^{cl}(t)+\varepsilon_i^{cl}\left(t_{peak}\right)\left[\frac{\max\left(D_{i,o}^{cl}\right)}{D_{i,o}^{cl}(t)}\right]D_{i,o}^{cl}(t)\left(\frac{\rho^{cl}(t)-\rho_o^{cl}(t)+\lambda_i^{cl}}{\rho_o^{cl}(t)}\right) \qquad \forall t\in T,\forall i\in I \tag{21}$$

Equation (21) represents ADR demand using the proposed DPEM, while maintaining the same energy consumption level BDR and ADR. It exhibits the complete load recovery using the proposed dynamic elasticity model. The one of the main feature of the proposed dynamic elasticity model is that, it is a time-variant model, which eliminates the drawbacks of the existing model, where elasticity is treated as static value as considered in the literature [9]. In addition, it amalgamates the attribute of self and cross-elasticity in the dynamic elasticity. It can be understood through the distinct trait of elasticity as discussed earlier. Since, self-elasticity will be relatively low during off-peak hours due to the small relative change in price [29], which makes demand increase small. Thus, the cross-elasticity should cause a significant increase in the demand during off-peak hours. It reflects the customers' adaptiveness to shift the demand on low price hours on the account of high price during peak hours.

2.2. Proposed stochastic PEM (SPEM) modeling using geometric Brownian motion

In this section, elasticity is modeled through the stochastic approach. As, it is discussed above that self and cross-elasticity should be interrelated to make DR adaptive. It can be better elaborated using PEM as shown in (22).

$$\begin{bmatrix} \Delta D(1) \\ \Delta D(2) \\ \vdots \\ \Delta D(T) \end{bmatrix} = \begin{bmatrix} \varepsilon(1,1) & \varepsilon(1,2) & \cdots & \varepsilon(1,T) \\ \varepsilon(2,1) & \varepsilon(2,2) & \cdots & \varepsilon(2,T) \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon(T,1) & \varepsilon(T,2) & \cdots & \varepsilon(T,T) \end{bmatrix} \begin{bmatrix} \Delta\rho(1) \\ \Delta\rho(2) \\ \vdots \\ \Delta\rho(T) \end{bmatrix} \tag{22}$$

Equation (22) evaluates change in demand $(\Delta D(t))$ via corresponding elasticity $(\varepsilon(t,\tau))$ resulted due to change in price $(\Delta\rho(t))$. It is expressed in per unit value. It can be observed from (22) that change in the demand at any time t is the result of change in price of the same time period (t) and other cross time-periods (τ). This correlation between both the elasticities varies with the measure of adaptability and adjustability of DR framework. This adaptability and adjustability of the demand will be complete or lossless, when the overall change in the demand $(\Delta D(t))$ over the time horizon will be zero [36].

$$\sum_{t\in T}\Delta D(t) = \sum_{t\in T}\sum_{\tau\in T}\varepsilon(t,\tau)\Delta\rho(t)D_o(t) = 0 \qquad \forall t, \quad \forall \tau \tag{23}$$

Equation (23) is an extension of (17) suggested by Jiang et al. [36]. It derives optimal distribution of load in terms of change in demand as opposite to (17), where only elasticity is used to obtain optimized load. It defines (23) in a more appropriate way, giving a complete form to observe the effect of price and demand. It clearly states that the summation of change in demand over a time-span is zero, meaning that overall consumption BDR and ADR will be zero. It indicates that price-responsive demand will be redistributed optimally in the cross-time periods. If it does not satisfy that condition, then following relation will hold:

$$\sum_{t\in T}\Delta D(t) < 0 \qquad \forall t \tag{24}$$

Equation (24) indicates the reduction in the demand after DR. This suggests that load recovery of curtailed demand will be partial. From DR perspective, cross-elasticity is a positive constant to balance-out the self-elasticity, which is negative value. This cross-elasticity gives rise to demand in the cross-time periods to compensate for the curtailed demand on high price time period. This indicates that cross-elasticity will reflect the curtailed/shifted demand in the cross-periods as a sort of load recovery. Though, load recovery in the cross-periods governs by the flexibility of intertemporal time constraint to relate the customer's realistic condition as aforementioned. This intertemporal time constraint stipulates that the customer can recover its curtailed demand to an extent time over the cross-periods from the curtailed demand hour [32,37]. In addition, recovery of load will be either decreasing or increasing in the subsequent time periods depending upon the customer' reaction to price. Similar analogy also can be emulated for the cross-elasticity value, which is responsible for the load recovery in PEM. Since, quantification of deterministic value of cross-elasticity over the time is a cumbersome task due to uncertainty associated with the customer activity usage pattern and price. Therefore, a time varying value of cross-elasticity can be perceived as the stochastic process to model the flexible intertemporal constraint of load. In this paper, a geometric Brownian motion (GBM) is employed to estimate the time-varying value of cross-elasticity of PEM over the time [38,39]. It is an extension of Brownian motion and exclusively utilized for predicting the stock price as price being positive [39]. Likewise, cross-elasticity is always positive from DR point of view. Thus, it is suitable for defining the randomness in the cross-elasticity. A stochastic process $\{\varepsilon(t)\}_{t\geq 0}$ is said to be GBM, if it satisfies following stochastic differential equation (SDE) expressed as follows:

$$d\varepsilon(t) = \mu\varepsilon(t)dt + \sigma\varepsilon(t)dW(t) \tag{25}$$

where, ε is defined as the stochastic variable with drift parameter μ as a propagation of the stochastic variable and volatility parameter σ as the variation in the propagation. It is to be noted that ε is assumed to be equivalent to the cross-elasticity here for the clarity. $W(t)$ is called as the Weiner/Brownian variable. The Weiner variable satisfies the following conditions 1) $W(t_0) = 0$ and 2) $W(t_k) = W(t_{k-1}) + N(0, t_k - t_{k-1})$ where, N being a normal distribution with zero-mean and variance equal to $(t_k - t_{k-1})$. If $t_0 < t_1 < .... < t_{k-1} < t_k < T$ are the successive interval over the time horizon, then increment $W(t_k) - W(t_{k-1}) \sim N(0, t_k - t_{k-1})$ are stationary and independent normally distributed random variable. On solving (25) the following expression is obtained.

$$\varepsilon(t) = \varepsilon_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B(t)\right] \tag{26}$$

Where, ε_0 is the initial value at $t = 0$. The generalize expression of GBM is given by (27).

$$\varepsilon(t_{k+1}) = \varepsilon(t_k)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)(t_{k+1} - t_k) + \sigma B(t_{k+1} - t_k)\right] \tag{27}$$

Putting the value of Weiner variable in (27), gives the complete expression as follows:

$$\varepsilon(t_{k+1}) = \varepsilon(t_k)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t_k + \sigma\sqrt{\Delta t_k}N(0,1)\right] \tag{28}$$

The solution of (25) is obtained using Ito's formula [39,40] as expressed in (28). This equation represents the time- based estimation of cross-elasticity in the exponential form. Here, if $t_0 < t_1 < .... < t_{k-1} < t_k < T$ are successive time points, then successive ratios are independent random variable in case of GBM as follows:

$$\frac{\varepsilon(t_1)}{\varepsilon(t_0)}, \frac{\varepsilon(t_2)}{\varepsilon(t_1)}, ..., \frac{\varepsilon(t_k)}{\varepsilon(t_{k-1})} \tag{29}$$

The solution of GBM characterize the stochastic process (i.e. cross-elasticity) using the drift and volatility parameter. Thus, the estimation of cross-elasticity is attributed to these two parameters, which can be evaluated through the historical data of BDR and ADR. It reveals that the value of cross-elasticity varies with the drift parameter $(\mu - \sigma^2/2)$ and variance σ^2. Further, the relationship between drift and volatility defines the movement of the stochastic process. If $(\mu \leq \sigma^2/2)$, then stochastic process decay exponentially in the successive time intervals, which is equivalent to decreasing cross-elasticity with non-linear

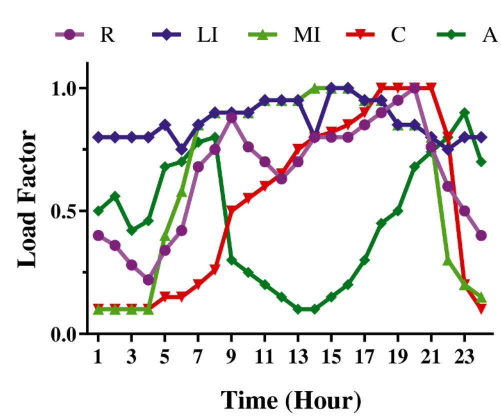

Fig. 2. Load factor pattern of different types of customers classes.

behaviour. This will give partial load recovery with non-linearity. Conversely, for $(\mu > \sigma^2/2)$, stochastic process grow exponentially, which will give increasing value of cross-elasticity over the cross-periods. This in turn will give rise to higher load recovery than the curtailed demand, resulting into a rebound effect in DR (i.e. when a new peak demand higher than base peak is emerged in post DR period) [41]. These two conditions suggest that proposed GBM for cross-elasticity estimation can model the impact of load recovery in DR, effectively. However, this study focuses only on the modeling of load recovery for the former circumstance.

where, $LF_i^{cl}(t)$ is defined as load factor of ith customer of class cl at time t with bound limits of $[a_t^{cl}, b_t^{cl}]$. Equation (31) defines the possible range of variation of load factor for the considered period t. It illustrates the load pattern of customers in the normalized term. Now, to simulate the load patterns of the various customers, a load factor is randomly sampled using a normal distribution. Since, normal distribution generates the variable with the range $(-\infty, \infty)$. Thus, to restrict the variables drawn from a normal distribution within a permissible definite range, a truncated normal distribution is employed. The probability distribution function (PDF) of the truncated normal distribution for each class of customers is expressed as [43]:

$$\psi\left(\mu_t^{cl}, \sigma_t^{cl}, a_t^{cl}, b_t^{cl}; LF_i^{cl}(t)\right) = \begin{cases} 0 & \text{if} LF_i^{cl}(t) < a_t^{cl} \\ \dfrac{\varphi\left(\mu_t^{cl}, \sigma_t^{cl}; LF_i^{cl}(t)\right)}{\Phi\left(\mu_t^{cl}, \sigma_t^{cl}; b_t^{cl}\right) - \Phi\left(\mu_t^{cl}, \sigma_t^{cl}; a_t^{cl}\right)} & \text{if} LF_i^{cl}(t) \le LF_i^{cl}(t) \le b_t^{cl} \\ 0 & \text{if} LF_i^{cl}(t) > b_t^{cl} \end{cases} \tag{32}$$

3. Load profiles

Assessing DR at the customer level requires the knowledge of their electrical-based activity. These activity usages vary within the classes due to the various indigenous and exogenous factors and also across the different classes. This give rise to the diverse load variations within the customer class, which will also reflect on DR. Therefore, diversified load profiles are synthesized to emulate diversified DR for individual customer. For this, an aggregated load pattern using load factor for each class is utilized as shown in Fig. 2 [42]. Then, the variation in each class at each hour is modeled using coefficient of variation (COV). Based on COV, standard deviation at each hour is calculated as follows:

$$\sigma_t^{cl} = \mathrm{COV}_t^{cl} \times \mu_t^{cl} \tag{30}$$

where, μ_t^{cl}, σ_t^{cl} and COV_t^{cl} are mean, standard deviation and COV of customer class cl at time t, respectively. Using mean and standard deviation, the permissible interval of load factor variation within the customer class is defined as:

$$\begin{aligned} LF_i^{cl}(t) &\in \left[a_t^{cl}, b_t^{cl}\right] \in \left[\mu_t^{cl} - \sigma_t^{cl}, \mu_t^{cl} + \sigma_t^{cl}\right] \\ &\in \left[\mu_t^{cl} - \mathrm{COV}_t^{cl} \times \mu_t^{cl}, \mu_t^{cl} + \mathrm{COV}_t^{cl} \times \mu_t^{cl}\right] \end{aligned} \tag{31}$$

To generate a variable from truncated normal distribution the following approach is employed

$$\begin{aligned} &\text{repeat}: \\ &x = rand() \\ &LF_i^{cl}(t) = \Phi^{-1}\left(\mu_t^{cl}, \left(\sigma_t^{cl}\right)^2, x\right) \\ &\text{until}\left(a_t^{cl} \le LF_i^{cl}(t) \le b_t^{cl}\right) \end{aligned} \tag{33}$$

where, μ_t^{cl} and σ_t^{cl} are the values of the original normal distribution. For sampling the load factor from the truncated normal distribution, a

Table 1
Customer class detail.

Classes	Allocated Demand (kW) [45]	No. of Customers	Nodes	S/CF (κ^{cl}) [46]	Demand Range [min, max] (kW)
R	950	121	2-10	-0.2	[2, 15]
C	555	18	11-18	1	[10, 25]
LI	1290	27	19-25	0	[50, 100]
MI	180	6	26-28	0.2	[20, 45]
A	740	63	29-33	-0.5	[8, 18]

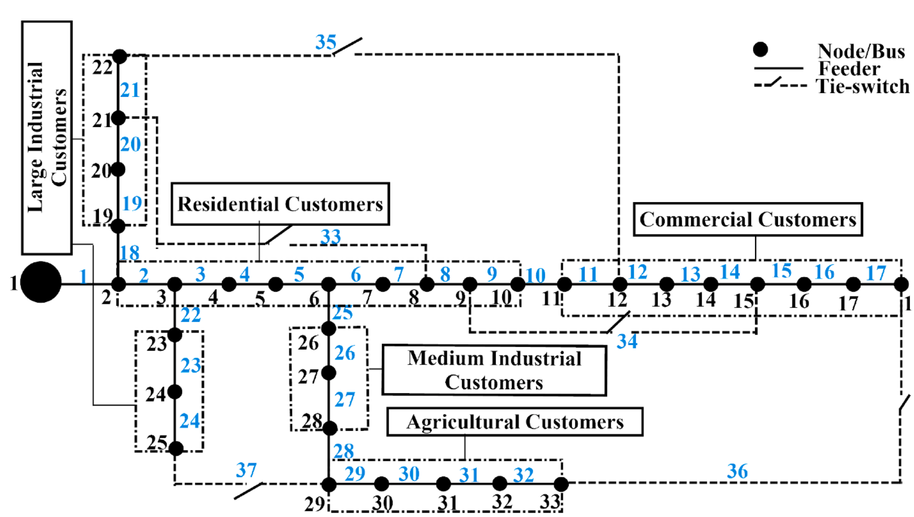

Fig. 3. Single line diagram of IEEE-33 bus distribution test system.

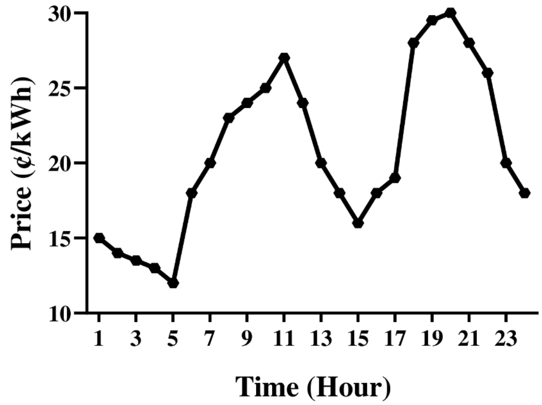

Fig. 4. Dynamic price signal under RTP.

random number (x) is generated. Then the sampled value is evaluated using an inverse cumulative density function (cdf) ($\Phi^{-1}(.)$). The generated value of the sampled load factor is checked against the condition and if it violates then repeat the process. Based on the calculated load factor, the load profile of the customers at each load bus is synthesized using the following expression.

$$D_{i,o}^{cl}(t) = LF_i^{cl}(t) \times \left(\frac{D_{i,o}^{cl}}{\max\left(D_{i,o}^{cl}\right)} \right) \tag{34}$$

where, $D_{i,o}^{cl}(t)$ is BDR demand of ith customer of class cl at time t. The expression (34) defines the customer's load demand in the absolute term.

4. Results

In this section, the effectiveness of the proposed adaptive DR framework using PEM is demonstrated on IEEE-33 distribution system load bus data [44]. The system peak demand is assumed to be 3715 kW. A single line diagram of IEEE-33 bus distribution test system is shown in Fig. 3 [44]. The proposed models are developed in MATLAB® platform on Window 10 based personal laptop Intel(R) Core(TM) i3-8130U CPU @2.20GHz, 4GB RAM. The computation time is 0.8 sec, 0.5 and 1.5 sec for PEM, the proposed DPEM and SPEM, respectively. The load buses of DN are segmented into five different customer classes i.e., residential: (R), commercial: (C), agriculture: (A), large industrial: (LI) and medium industrial (MI) customers to assess the effect of DR on the various load patterns. Each load bus is independently assigned to individual class and the available bus load are stipulated as their demand level for the ease of exposition. Further, the demand proportion of each customer class is assumed to be proportional to the share of energy consumption in Indian energy perspective [45]. The mix has the total 235 number of customers. The customer's demand range is assumed to be faction of actual demand capacity of each customer class. The detail of assigned nodes, total allocated demand, number of customers, subsidy/charging factor (S/CF) and customer demand range for each class is given in Table 1.

Figure 4 depicts RTP offered by the utility and is taken from Ontario Energy Board [47]. The price offered to the various customer classes is calculated using ($\rho^{cl}(t) = \rho(t) \times (1 + \kappa^{cl})$), where κ^{cl} is the subsidy/charging factor (S/CF). It is tailored to mimic the different pricing rates of practical DN, where, each class is being offered the different rates on the account of their demand capacity, activity usage [46]. More importantly, the existence of cross-subsidy among the different customer classes also leads to the different pricing rates, where some customer classes are overcharged for the compensation to the subsidized customer classes. The time span is partitioned into three periods namely: off-peak (0:00-8:00, 23:00-24:00), valley: (12:00-18:00) and peak: (8:00-11:00, 18:00-22:00) for the sake of simplicity. The partitioned time-segments are designed on the basis of utilized tariff rates during winter months in Ontario Energy Board [47]

4.1. Class-wise elasticity

The proposed methods are investigated on class-wise customers and their effectiveness is evaluated in reference to standard PEM. PEM is used as a benchmark model for the comparison analysis. The class-wise elasticity value for R, LI, MI, C and A customers are -0.30, -0.43, -0.54, -0.30 and -0.23, respectively as reported in [48]. It is worth to mention that elasticity values in case of DPEM and SPEM are evaluated using the proposed dynamic and stochastic approach. Though, a peak hour elasticity for DPEM and SPEM is same as the standard PEM, while in case of SPEM, the initial value of cross elasticity is assumed to be equal to the fraction of self-elasticity. It is based on the notion of having low value of cross-elasticity as described in the literature [9]. Here, initial value of cross-elasticity is taken as $(\varepsilon(t,\tau) \times 0.15|_{t=\tau})$ and subsequently evaluated through GBM's descriptive parameters (i.e. μ and σ) using (28). The values of μ and σ are taken as 0.2 and 1.2, respectively for the analysis purpose.

An aggregated class-wise elasticity variation profile using the proposed DPEM is shown in Fig. 5. The figure shows the average elasticity variation over the time horizon for the different classes and aggregated elasticity of all the customer classes. It can be observed from the figure that each customer class exhibits distinct elasticity owing to the different load pattern. It displays nominal variation in elasticity for R and LI

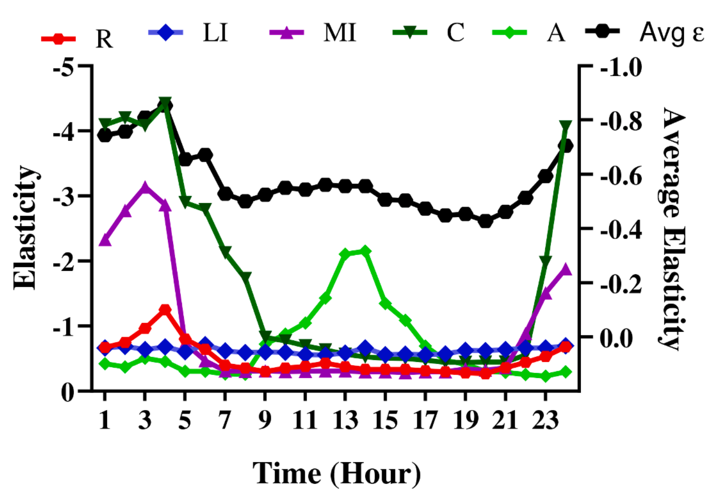

Fig. 5. An aggregated class-wise elasticity variation using the proposed DPEM.

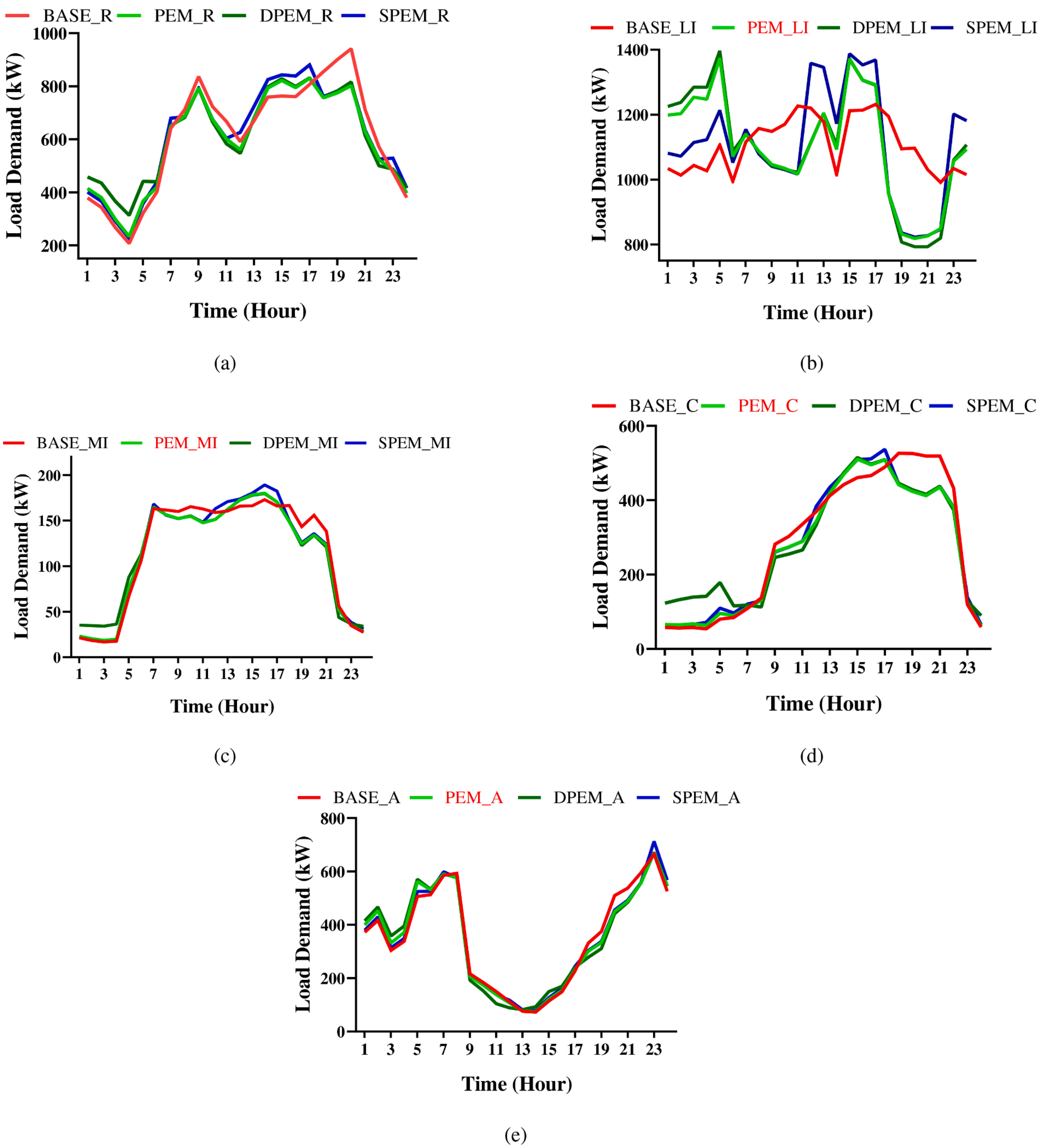

Fig. 6. Aggregated class-wise demand profiles before DR as base case, and after DR using PEM, proposed DPEM and proposed SPEM: (a) Residential Customers, (b) large Industrial Customers, (c) Medium Industrial Customers, (d) Commercial Customers, and (e) Agricultural Customers.

classes. On the other hand, MI, A and C class customers exhibit wide variation in the elasticity. The rationale behind such vast elasticity variation can be explained through the customer aggregated load factor or load/price pattern for more clarity. The figure shows that the customer class which exhibits high relative demand/price ratio between peak to valley or peak to off-peak set forth wide elasticity variation and vice-versa. It can be further substantiated from the elasticity profiles of R and LI class customers, whose relative ratios between the segmented time periods are on nominal range, while wide variation in elasticity are reported for MI, A and C class customers due to the existence of high relative proportions. This elasticity variation indicates that load patterns with high relative ratio will need to exhibit higher load flexibility to achieve true DR, otherwise may face the overall reduction in their energy consumption. On the contrary, customers with nominal proportions among the different time periods can display the load flexibility with ease over the time horizon.

The average elasticity of the overall customer classes is shown in right axis of Fig. 5. It is obtained by taking the weightage average over all customer classes. This demonstrates that a uniform elasticity will perceive at utility level due to the load adjustment induced by DR. The elasticity variation indicates that aggregated assessment of elasticity at utility level may obscure the valuation regarding DR reflection at class-wise customer level. Hence, the suggested disaggregated approach incorporates the class-wise response in DR with better comprehensibility.

4.2. Effect of DR on customer class load pattern

Figure 6 (a)–(e) shows the aggregated class-wise customer demand

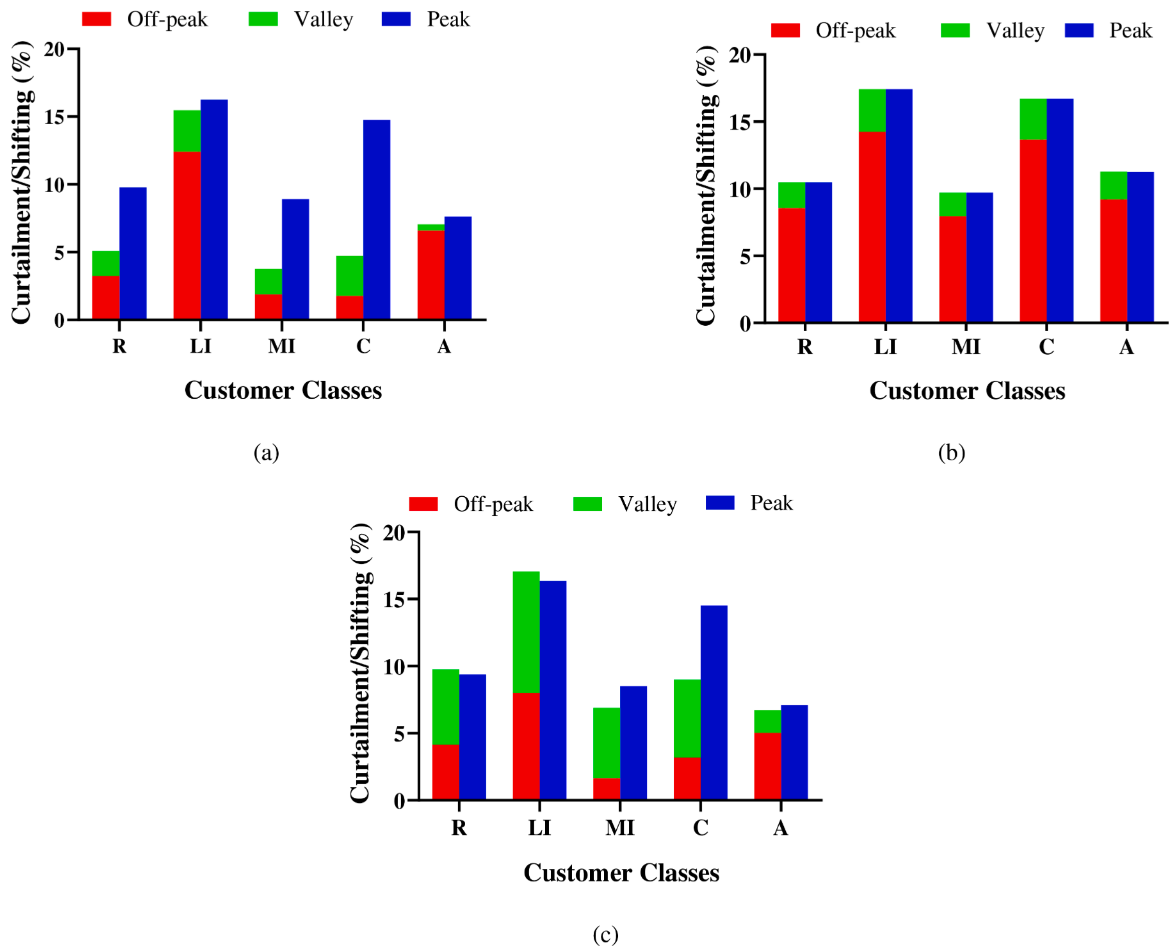

Fig. 7. Curtailed and shifted demand under RTP using (a) PEM, (b) DPEM and (c) SPEM.

for base load BDR and ADR using PEM, the proposed DPEM and the proposed SPEM model. It can be observed from sub-figures that PEM displays DR behaviour persuasively during peak hours. However, its recovery during off-peak/valley hours is not equal to the curtailed demand. It can be shown that R, C and MI customers exhibit partial increase in the demand during off-peak hours. Though, LI and A customers display much better adjustment of load demand during off-peak hours. It can be adjudged through the elasticity variation discussed in the preceding section, where elasticity/demand with nominal ratio can exhibit DR quite easily and contrariwise. On the other hand, the proposed DPEM and SPEM display better curtailment and shifting for all the customer's classes. In case of DPEM, dynamic elasticity makes curtailed load adaptive over the time-frame. It recovers the curtailed load in valley and off-peak price hours, indicating complete load recovery. This satisfies one of assumption of the equivalent energy consumption BDR and ADR. In addition, the proposed DPEM is independent of customers load pattern showing its feasibility to all customer classes load pattern. Likewise, the proposed SPEM also exhibits better load recovery of the curtailed load using the stochastic process accompanied by the inter-temporal constraint. However, this may/may not maintain the energy balance constraint owing to stochastic approach. But, it displays possible uncertainty associated with the customer response in DR, where, some customers will respond to DR actively, while some passively. This gives close to real perception of customer's behaviour in DR.

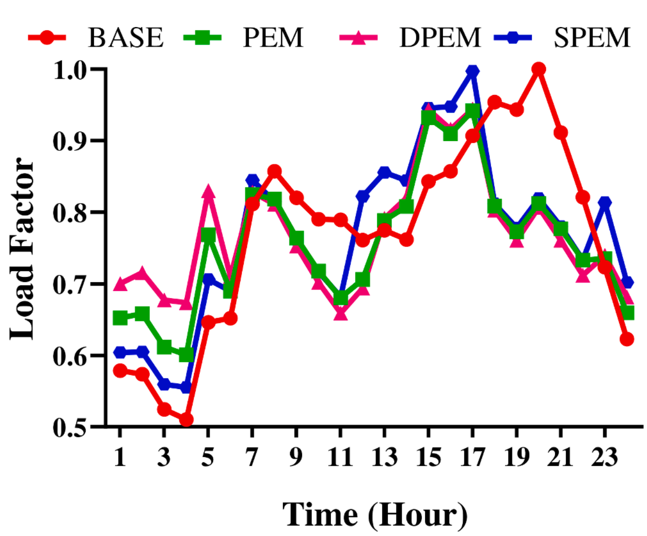

Fig. 8. Aggregated load factor at the utility level.

4.3. Class-wise curtailment and shifting

The aggregated class-wise response in DR can be better illustrated by observing the proportional change in demand in the considered time states. In this order, a term, curtailment and shifting factor (C/SF) is defined. It expresses the relative change in demand BDR and ADR of the segmented time periods to the peak hours demand BDR. It is evaluated using the following expression:

Table 2
Aggregated customer class bills under different price states using the proposed DPEM.

Class	BDR (¢)			ADR (¢)		
	Peak	Valley	Off-peak	Peak	Valley	Off-peak
R	115110.26	72379.75	56868.55	132888.01	67624.62	52412.90
LI	210234.26	147044.53	195118.28	221252.16	140740.60	170027.95
MI	32671.00	24710.61	11800.84	37757.22	23151.34	11615.31
C	148834.80	109778.81	28230.88	163231.89	103621.78	36415.08
A	36303.67	7825.74	43946.21	41393.43	7740.88	37137.38

$$C \Big/ SF = \left| \frac{\left(\sum_{t \in T_{P/V/OP}} D_o^{cl}(t) - \sum_{t \in T_{P/V/OP}} D^{cl}(t) \right)}{\sum_{t \in T_P} D_o^{cl}(t)} \right| \times 100\% \tag{35}$$

where, C/SF is defined in terms of percentage and accounts for the curtailed and shifted demand. The aggregated class-wise outcome under the different methods is depicted in bar graph as shown in Fig. 7. It can be observed from the Fig. 7 (a) that customers of R, C and MI class are partially able to shift the demand, while LI and A customers shift closely to the curtailed load demand. This partial shifting is due to the existence of high relative price/demand ratio between the segmented periods as discussed in Section 4.1. This requires the customers to display higher flexibility to achieve better load adjustment via elasticity. It means that for lossless DR (i.e. complete load recovery), elasticity should be adaptive. This condition can be fulfilled using the proposed DPEM. In DPEM, dynamic elasticity shows better adjustment of demand, where the percentage change in curtailed and shifted demand in each customers class is nearly equal as shown in Fig. 7 (b) as opposite to PEM. Similarly, SPEM gives better load recovery using stochastic process as observed from Fig. 7 (c). Though, it does not maintain energy balance condition BDR and ADR due to the induction of uncertainty by the stochastic model.

4.4. Effect of DR on system load factor

The impact of DR on the system aggregated load factor is illustrated in Fig. 8. The figure indicates that the load factor is lowered considerably during peak hours using all three methods. On the other hand, significant improvement is observed during off-peak/valley hours. The overall increase or decrease balanced out the disproportionate load factor existed between peak and off-peak/valley periods. The figure reveals that the proposed PEM methods give better load factor or equivalent to standard PEM. DPEM approach displays better load factor profile with lossless DR. It gives improved load factor during off-peak period relative to valley period. On the other hand, SPEM model gives preferable load factor during valley period, while inferior load factor during off-peak period. Though, load factor achieved under SPEM is more practicable than DPEM due to the applicability of intertemporal time flexibility constraint in the former approach.

4.5. Effect of DR on economic performance

From the economic point of view, an aggregated class-wise customer electricity bills BDR and ADR for the considered states using the proposed DPEM framework under RTP are shown in Table 2. The objective to illustrate DPEM model based results is due to its deterministic approach with lossless DR. This will give exact results under true DR in contrast to standard PEM. In addition, SPEM model based results may have variable outcome, as uncertainty quantification is not in the scope of the study. Hence, its results are not presented in terms of economic performance.. The results indicate that peak hour bills of customers are increased even after participating in DR. On the other hand, electricity bill during valley and off-peak period are decreased significantly. The results suggest that shifting from flat rate to dynamic rate may not get bill reduction during peak hours. Though, It may balance out to an extent by shifting/delaying the consumption to valleys and off-peak period, where low prices are offered. This relative increment and decrement in the billing of the segmented time period may give overall reduction in billing under desirable condition. Hence, the effectiveness of DR can be better eventuated through the consideration of longer time-span.

The aggregated class-wise bills are summarized in Table 3. It can be observed from the table that R, C and MI customer class bills are increased after participating in DR, with highest increase in C and MI is next to it. This will discourage the customers to participate in DR. The results indicate that participating in DR may or may not be beneficial for each customer class. On the other hand, LI and A customer class electricity bills are decreased. The decrement in the electricity bill of LI and A class customers can be associated with their load patterns. In, both customer classes relative demand ratio between peak to off-peak/valley period are in nominal range. This allows both customer class to adjust their demand adequately. It is also one of the reason that LI class customers are being active DR participant. The overall increase in the electricity bills is around 0.50%.

In terms of utility profit, the revenue gain from the different customer classes is shown in Table 4. The utility revenue is obtained by considering a markup rule to keep the margin of utility as discussed in [49]. The ratio between retail price to wholesale market price is assumed around 1.4-1.5 [49]. It can be observed from the table that utility profit is increased for each customer class except LI and A customers class. Moreover, the overall increase in profit is about 11.84%.

In addition, a comparative economic cost assessment is also performed for PEM, proposed DPEM and SPEM as compiled in Table 5. The results indicate that the total customers bill decreases in the case of standard PEM, whilst increases for DPEM and SPEM. Though, decrease in the bills for PEM is at the expense of the lowered energy consumption ADR, which may cause discomforts to the customers. The bill increase in SPEM is slightly higher than DPEM. Similarly, the total profit under the different DR approaches gives optimistic results as observed from the table. It increases for all three approaches with highest percentage increase in SPEM. The rise in total bills and total revenue using SPEM

Table 3
Aggregated class-wise customer electricity bills using the proposed DPEM.

Class	R	LI	MI	C	A	Overall
BDR (¢)	244358.56	552397.07	69182.46	286844.49	88075.63	1240858.20
ADR (¢)	252925.53	532020.72	72523.87	303268.76	86271.69	1247010.56
Diff (¢)	8566.97	-20376.35	3341.41	16424.27	-1803.94	6152.36
Change (%)	3.51	-3.69	4.83	5.73	-2.05	0.50

Table 4
Aggregated class-wise utility's profit using the proposed DPEM.

Class	R	LI	MI	C	A	Overall
BDR (¢)	64982.38	170037.13	17961.44	67056.01	26847.08	346884.04
ADR (¢)	78758.44	165476.36	22672.80	94222.21	26808.98	387938.80
Diff (¢)	13776.06	-4560.76	4711.36	27166.20	-38.10	41054.76
Change (%)	21.20	-2.68	26.23	40.51	-0.14	11.84

Table 5
Comparative cost analysis under the different DR models.

Method	Total bills (¢)	Change in Bills / (%)	Total profit (¢)	Change in profit / (%)	Total Energy (kWh) / (%)
Base	1240858.20	-	346884.04	-	59402.00 (-)
PEM	1235350.91	-5507.29 (-0.40)	384523.71	37639.67 (10.85)	58553.95 (-1.43)
Proposed DPEM	1247010.56	6152.36 (0.50)	387938.80	41054.76 (11.84)	59402.00 (0)
Proposed SPEM	1254839.626	19488.71 (1.13)	390483.58	43599.54 (12.57)	59265.61 (-0.23)

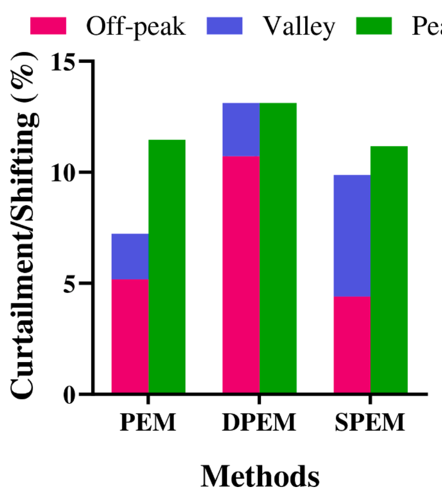

Fig. 9. Curtailed and shifted demand under RTP using PEM, the proposed DPEM and the proposed SPEM.

approach is due to pragmatic load shifting in the valley/off-peak periods with the intertemporal bounds. It is worth to mention that the presented economic analysis gives a snapshot for the considered RTP signal. Thus, the results may vary, when different RTP signal is employed.

4.6. Comparative assessment and discussions

This section investigates the usefulness of the proposed DPEM and SPEM with the existing PEM on an aggregated scale. Therefore, the overall contribution in DR using the existing and the proposed methods is shown in Fig. 9. This figure shows that all the methods demonstrate peak demand curtailment, effectively. However, the absorption of this curtailed demand over the off-peak/valley hours is varying under the different methods. PEM exhibits moderate absorption of the curtailed load due to staticness in elasticity, whereas DPEM displays the complete absorption using the proposed adaptive dynamic elasticity. In addition, both PEM and proposed DPEM adjust its most of curtailed demand to the off-peak period due to high relative demand ratio as shown in figure. This contradicts with the theory of distinct elasticity pattern for the relative increment/decrement in the price. It is further accompanied by the customer's intertemporal constraint of load flexibility, which diminishes non-linearly over the cross-periods (i.e. load recovery decreases as subsequent cross-periods are further away from peak demand hours). Thus, these two conditions make PEM and the proposed DPEM to lack in incorporating intertemporal constraint. On the contrary, the proposed SPEM incorporates both conditions via stochastic process using GBM. This gives realistic load recovery in cross-periods with element of uncertainty. Moreover, it provides an evenly shifted load in the valley and off-peak periods.

The assessment of aforesaid methods indicates that standard PEM model usually employs predetermined static value of elasticity for DR measure. This is better addressed in the proposed DPEM, which makes elasticity adaptive. Besides, it provides a deterministic approach for lossless DR. However, it does not consider time flexibility constraint to reflect the realistic load recovery. This drawback is better illustrated in SPEM model, which provides adaptiveness in elasticity along with time flexibility constraint. However, SPEM approach may/may not provide lossless DR due to uncertainty in the modeling approach of load recovery. This requires an appropriate uncertainty quantification approach to predict DR precisely.

5. Conculsions

This paper presents an adaptive economic DR framework using price elasticity model (PEM) to model demand response (DR) in the distribution networks. The proposed model emulates the key features of DR (adaptability and adjustability) through a dynamic elasticity using deterministic and stochastic approaches, which were partially present in the existing PEM. Both approaches incorporate dynamism in the elasticity to entail load recovery in the cross-periods. In deterministic PEM, the proposed dynamic elasticity inherently manifests the traits of both self and cross-elasticity, and further establishes an interlink between peak, valley and off-peak period to shift load. It demonstrates that curtailed demand during peak hours will be shifted to off-peak hours irrespective of the customers load pattern. Though, the proposed deterministic PEM exhibits true DR, but lacks in incorporating intertemporal constraint of load flexibility. This is effectively overcome in the proposed stochastic PEM, which imitate a feasible load recovery using geometric Brownian motion to model the customers' uncertain behaviour in DR. It can be corroborated from the result analysis that deterministic PEM is more optimistic model, which may provide the overestimated results in the practical situation. On the other hand, stochastic PEM is a progressive framework due to its likeness to realistic condition. Though, its effectiveness depends upon the quantification of uncertainty associated with customers' non-linear behaviour. Besides, deterministic method is proposed under the assumption that customers are rational, whereas stochastic method can be made equally applicable for fully or partially rational DR customers.The one of the outcomes of the economic analysis illustrates that participating in DR programs will not be beneficial for all customers due to the their heterogeneous behaviour. Since, PEM is governed by price and demand pattern. Thus, the present work can be extended in future to observe customer participation level in DR, when the price pattern is designed according to the customer class load pattern.

CRediT authorship contribution statement

Vipin Chandra Pandey: Software, Data curation, Writing – original draft, Conceptualization, Methodology, Writing – original draft, Writing – review & editing, Validation. **Nikhil Gupta:** Visualization, Supervision, Investigation, Formal analysis, Writing – review & editing. **K.R. Niazi:** Visualization, Supervision, Investigation, Formal analysis, Writing – review & editing. **Anil Swarnkar:** Visualization, Supervision, Investigation, Formal analysis, Writing – review & editing. **Rayees Ahmad Thokar:** Visualization, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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