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International risk sharing with heterogeneous firms ☆

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ABSTRACT

This paper explores international consumption risk sharing in an open economy macro model with firm heterogeneity and shows that firm entry and the self-selection of more efficient firms into exporting account for better international risk sharing. I show analytically that the conventional unconditional correlation between relative consumption and the real exchange rate is not a good metric for measuring international consumption risk sharing. World trade data covering more than two decades indicate that the extent of international risk sharing is underestimated.

1. Introduction

How do countries hedge against consumption risk? When people can trade securities corresponding to every future state, i.e., Arrow-Debreu security, perfect consumption risk sharing can be ensured, and the marginal utility stemming from one additional unit of nominal wealth is equalized across countries. Specifically, under such complete financial markets, we have

$$U_{C,t} = U^*_{C^*,t} Q_t^{-1},$$

where $U_{C,t}$ and $U^*_{C^*,t}$ represent the marginal utility of home consumption and foreign consumption at time t, respectively. Q_t stands for the real exchange rate at time t defined as the real price of foreign consumption in terms of home consumption. It should be noted that when $Q_t = 1$, the above relation is the case of the Purchasing Power Parity (PPP), which, however, fails to hold in practice. With the constant relative risk aversion utility function and separability between leisure and consumption, the above condition can be specified as

$$C - C^* = \frac{1}{\gamma} Q, \tag{1}$$

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where C and C^* denote home and foreign consumption, respectively. Sans serif font denotes first-order deviations from the trend, and time indices are dropped. In Eq. (1), γ determines the extent of the relative risk aversion. Since we typically have $\gamma \geqslant 1$, the correlation between the relative consumption across countries and the real exchange rate is *positive*: a higher growth in home consumption relative to foreign consumption must be associated with a real depreciation for the home country. Note also that in the case of the PPP ($\mathsf{Q} = 0$), home consumption and foreign consumption comove perfectly ($\mathsf{C} = \mathsf{C}^*$).

Contrary to this prediction under complete financial markets, actual data tells the opposite: Table 1 documents the correlations between the growth rate of per capita consumption and the real exchange rate for each OECD country with respect to the United States. Correlations between relative consumption and the real exchange rate are *close to zero or even negative* for a large number of countries. This "lack" of international risk sharing is known as the Kollmann-Backus-Smith puzzle (Backus and Smith, 1993; Kollmann, 1995, henceforth, KBS), which is featured as a major puzzle in the open economy (Obstfeld and Rogoff, 2000).

In this paper, I discuss international consumption risk sharing in an open economy model with firm heterogeneity. What has been missing in the literature on international consumption risk sharing is an acknowledgment of the changes in product varieties and their qualities, which arise due to the firm entry and the selection of more efficient firms into exporting. The welfare implications as a result of these changes have been discussed in detail in the trade literature.[1] Little is known, however, about its consequences for international risk sharing. This paper aims to fill this gap. To this end, I revisit international consumption risk sharing in a two-country model with firm heterogeneity.

In the theoretical model, firms are assumed to be heterogeneous with respect to their idiosyncratic productivities. Furthermore, the quality of a product variety is set by the firm that produces it. Due to fixed costs for exporting, only a subset of firms export and, hence, "tradedness" (Ghironi and Melitz, 2005) in the economy arises endogenously. With the help of the theoretical model, under the assumption of incomplete financial markets, I analytically show that the KBS correlation is conditional on changes in the number of product varieties and their qualities. The result thus indicates a potential difference between the unconditional KBS correlation and the conditional, i.e., partial KBS correlation. The key to generating a realistic negative unconditional KBS correlation is the wealth effect that makes the real exchange rate appreciate when relative consumption is higher. I detail how the deviation from the PPP arises in the theoretical model with firm heterogeneity. In short, the real exchange rate fluctuates in the present theoretical model because of 1) home bias in consumption, 2) endogenous tradedness in the economy arising from the selection for exporting and, 3) changes in the quality of exports and imports. The mechanism of the real exchange rate appreciation thus hinges on an elaborated Harrod-Balassa-Samuelson effect based on heterogeneous firms. Furthermore, the result is shown to be robust to different extents of measurement error with respect to fluctuations in the number of product varieties and their qualities. In the calibrated model, I also show quantitatively that a realistic unconditional KBS correlation is obtained through a high-quality ladder that drives the wealth effect, among other complementing mechanisms.

I next turn to test the predictions of the theoretical model with actual data. For that purpose, I combine world trade data with the estimates of the quality of exports and imports at the product level. The quality of a product is inferred from supply-side information and its observed unit price and market share (Feenstra and Romalis, 2014). With the regression analysis, I confirm a puzzling close to zero or even negative unconditional KBS relation. By incorporating changes in the number of traded product varieties and their estimated qualities as additional explanatory variables in the KBS regression, however, consumption growth across countries tends to exhibit more covariation with the bilateral real exchange rate growth. Thus the unconditional KBS regression gives a systematic negative bias in assessing the KBS puzzle. Similarly, the unconditional KBS correlation underestimates international consumption risk sharing. This fact is revealed only by considering the KBS puzzle in an open economy model with firm heterogeneity.

Other than the role played by international financial markets in consumption risk sharing, international relative prices, namely, welfare improving transmissions through the terms of trade fluctuations, have been explored in the literature (Cole and Obstfeld, 1991; Acemoglu and Ventura, 2002). However, Corsetti et al. (2008) find that the removal of complete financial markets per se cannot establish a close to zero or even negative unconditional KBS correlation in the theoretical model. To replicate a realistic KBS correlation, Corsetti et al. (2008) emphasize the wealth effect due to a low value of elasticity of substitution between locally produced goods and imported goods and/or a high persistence of productivity shocks. In a model with endogenous firm entry, Hamano (2013) shows that a realistic KBS correlation is obtained through the wealth effect driven by the preference for variety. In the theoretical model here, the wealth effect is further amplified by a higher quality of products. Ghironi and Melitz (2005) discuss a similar mechanism for the real exchange rate appreciation based on heterogeneous firms with constant product quality.[2] From an empirical standpoint, relying on the gravity equation, Fitzgerald (2012) finds biased estimates that emerge from the imperfectly measured real exchange rates in assessing international consumption risk sharing for OECD countries.

[1] See, for instance, Arkolakis et al. (2012), Melitz and Redding (2015), Arkolakis et al. (2019), Feenstra (2018), Krugman (1979), Melitz (2003), Broda and Weinstein (2004), Broda and Weinstein (2006).

[2] Without considering firm heterogeneity and resulting endogenous nontradedness, Benigno and Thoenissen (2008) consider a typical model in an open economy with traded and nontraded sectors. They show that the real exchange rate appreciation may arise due to the Harrod-Balassa-Samuelson effect (Balassa, 1964; Harrod, 1933; Samuelson, 1964) driven by differentiated productivity growth across countries in traded and/or nontraded sector. See also Hamano (2014), where the Harrod-Balassa-Samuelson effect is analyzed with endogenous firm entry in traded and nontraded sectors.

Table 1
The KBS correlation in data.

Australia (AUS)	0.15	Finland (FIN)	−0.05	Luxembourg (LUX)	0.15
Austria (AUT)	0.19	France (FRA)	0.54	Netherlands (NLD)	0.38
Belgium (BEL)	0.70	United Kingdom (GBR)	0.09	Norway (NOR)	0.15
Canada (CAN)	0.21	Greece (GRC)	−0.01	New Zealand (NZL)	−0.13
Switzerland (CHE)	0.23	Hungary (HUN)	−0.13	Poland (POL)	−0.33
Chile (CHL)	−0.36	Ireland (IRL)	0.22	Portugal (PRT)	−0.12
Czech Republic (CZE)	0.36	Iceland (ISL)	−0.56	Slovakia (SVK)	−0.04
Germany (DEU)	0.09	Israel (ISR)	−0.30	Slovenia (SVN)	−0.13
Denmark (DNK)	0.26	Italy (ITA)	0.21	Sweden (SWE)	0.26
Spain (ESP)	0.18	Japan (JPN)	0.05	Mean	0.06
Estonia (EST)	−0.16	Korea (KOR)	−0.35	Median	0.09

Note: The table shows the correlation between relative consumption and the real exchange rate for individual OECD countries with respect to the United States from 1984 to 2011. Data on per capita consumption and the real exchange rate are taken from Penn World Table, version 9.0.

The remainder of this paper is organized as follows. In the next section, I present the benchmark model with financial autarky. In Section 3, the extended model with international bonds is presented. In Section 4, I analytically investigate the nature of international risk sharing under incomplete financial markets. I next calibrate the model and document its quantitative implications. An empirical investigation is conducted in Section 6. In the last section, a conclusion is provided.

2. The model

The theoretical model embeds endogenous firm entry and the selection of more efficient firms into exporting. The world consists of two countries, home and foreign. Each country is populated by one unit mass of atomic households. Throughout the paper, foreign variables are denoted with an asterisk (*). Here, I present the case of financial autarky. In a later section, I relax this assumption by allowing international borrowing and lending.

2.1. Household preference and intratemporal choices

The utility of the representative household in the home country at time t depends on consumption and the labor supply as follows:

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}.$$

In the above expression, γ ($\geqslant 1$) denotes risk aversion as previously defined. χ (> 0) represents the degree of unsatisfaction from supplying labor L_t, and φ ($\geqslant 0$) denotes the Frisch elasticity of the labor supply.

Consumption basket C_t is defined as

$$C_t = \left[C_{H,t}^{1-\frac{1}{\omega}} + C_{F,t}^{1-\frac{1}{\omega}}\right]^{\frac{1}{1-\frac{1}{\omega}}},$$

where ω (> 0) denotes the elasticity of substitution between a local basket of goods ($C_{H,t}$) and an imported basket of goods ($C_{F,t}$). $C_{H,t}$ and $C_{F,t}$ are defined over a set of goods Ω as

$$C_{H,t} = V_{H,t}\left(\int_{\zeta\in\Omega} (q_D(\zeta) c_{D,t}(\zeta))^{1-\frac{1}{\sigma}} d\zeta\right)^{\frac{1}{1-\frac{1}{\sigma}}},\ C_{F,t} = V_{F,t}^*\left(\int_{\vartheta\in\Omega} (q_X^*(\vartheta) c_{X,t}(\vartheta))^{1-\frac{1}{\sigma}} d\vartheta\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

where $V_{H,t} \equiv N_{D,t}^{\psi-\frac{1}{\sigma-1}}$ and $V_{F,t}^* \equiv N_{X,t}^{*\psi-\frac{1}{\sigma-1}}$ in which $N_{D,t}$ and $N_{X,t}^*$ represent the number of domestic and imported product varieties. ψ ($\geqslant 0$) stands for the marginal utility resulting from a one unit increase in the number of varieties in each local and imported basket (Benassy, 1996). Note that the preference becomes the one discussed in Dixit and Stiglitz (1977) when $\psi = \frac{1}{\sigma-1}$. At any given time t, only a subset of goods $\Omega_t \in \Omega$ is available. $c_{D,t}(\zeta)$ and $c_{X,t}(\vartheta)$ represent demand for individual product varieties indexed by ζ and ϑ, which are produced domestically and imported, respectively. $q_D(\zeta)$ and $q_X^*(\vartheta)$ denote the quality of the local product variety ζ and the imported product variety ϑ. σ (> 1) is the elasticity of substitution among varieties. It is assumed that $\sigma \geqslant \omega$.

Demand for domestically produced basket $C_{H,t}$, imported basket $C_{F,t}$, local product variety $c_{D,t}(\zeta)$ and imported variety $c_{X,t}(\vartheta)$ are

$$C_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} C_t,\ C_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\omega} C_t,$$

$$c_{D,t}(\zeta) = (V_{H,t}q_D(\zeta))^{\sigma-1}\left(\frac{p_{D,t}(\zeta)}{P_{H,t}}\right)^{-\sigma}C_{H,t},\ \ c_{X,t}(\vartheta) = \left(V^*_{F,t}q^*_X(\vartheta)\right)^{\sigma-1}\left(\frac{p^*_{X,t}(\vartheta)}{P_{F,t}}\right)^{-\sigma}C_{F,t},$$

where $p_{D,t}(\zeta)$ and $p^*_{X,t}(\vartheta)$ denote the prices of the local product variety ζ and the imported product variety ϑ, respectively. As we can see, local product variety $c_{D,t}(\zeta)$ and imported variety $c_{X,t}(\vartheta)$ depend on relative prices, product varieties ($V_{H,t}$ and $V^*_{F,t}$) and product qualities ($q_D(\zeta)$ and $q^*_X(\vartheta)$) that enter as a demand shifter. The prices of aggregate consumption basket P_t, domestically produced basket $P_{H,t}$, and imported basket $P_{F,t}$ are given by

$$P_t = \left[P_{H,t}^{1-\omega} + P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$

$$P_{H,t} = \frac{1}{V_{H,t}}\left(\int_{\zeta\in\Omega_t}\left(\frac{p_{D,t}(\zeta)}{q_D(\zeta)}\right)^{1-\sigma}d\zeta\right)^{\frac{1}{1-\sigma}},\ \ P_{F,t} = \frac{1}{V^*_{F,t}}\left(\int_{\vartheta\in\Omega_t}\left(\frac{p^*_{X,t}(\vartheta)}{q^*_X(\vartheta)}\right)^{1-\sigma}d\vartheta\right)^{\frac{1}{1-\sigma}}.$$

All prices are denominated in home currency units. In what follows, I choose P_t as the numeraire and define real prices as $\rho_{H,t} \equiv \frac{P_{H,t}}{P_t}$, $\rho_{F,t} \equiv \frac{P_{F,t}}{P_t}$, $\rho_{D,t}(\zeta) \equiv \frac{p_{D,t}(\zeta)}{P_t}$, and $\rho^*_{X,t}(\vartheta) \equiv \frac{p^*_{X,t}(\vartheta)}{P_t}$.

Similar expressions hold in the foreign country.

2.2. Production, pricing, and the export decision

Firms are heterogeneous with respect to their specific productivity levels z, which are distributed over $G(z)$ with support on $[z_{\min}, \infty)$, where $z_{\min}$ stands for a lower bound productivity level of the distribution. Each firm produces a differentiated product variety and thus faces a residual demand curve with constant elasticity σ. Profit maximization of a firm with productivity z yields the following optimal real price produced in the home country:

$$\rho_{D,t}(z) = \frac{\sigma}{\sigma-1}mc_t(z),$$

where $mc_t(z)$ stands for the real marginal cost of production. I assume that producing a product with a higher quality requires a higher marginal cost such that

$$mc_t(z) = \left(1 + \frac{q(z)^{\frac{1}{\phi}}}{z}\right)\frac{w_t}{Z_t z}, \tag{2}$$

where w_t denotes the real wage, ϕ ($0 \leqslant \phi < 1$) is a parameter that determines the quality ladder and Z_t stands for the aggregate productivity level which is common for all firms. Provided a firm-specific productivity level z, the firm chooses its specific quality level $q(z)$ by minimizing the quality-adjusted marginal cost $mc_t(z)/q(z)$. Thus, the firm solves the following minimization problem:

$$\min_{q(z)} \quad mc_t(z)/q(z)$$

subject to (2). The optimal quality level for a firm with productivity level z is therefore given by

$$q(z) = \left(\frac{\phi}{1-\phi}z\right)^{\phi}.$$

Provided $\phi > 0$, a firm with a higher productivity z produces a higher quality of product variety $q(z)$.[3] Observe that when there is no quality ladder ($\phi = 0$), all firms produce a similar quality of goods, irrespective of their specific productivity levels as $q(z) = q = 1$.

In every period, exporting requires fixed operational costs defined as

$$f_X = Z_t l_{f_X,t},$$

where $l_{f_X,t}$ is the amount of labor required to produce f_X amount of fixed costs. Only a subset of firms with productivity level z above a cutoff level $z_{X,t}$ exports by charging sufficiently lower quality-adjusted prices and thus earning positive profits despite the fixed export costs f_X. If a firm exports, its export price is given by $\rho_{X,t}(z) = \tau_t\rho_{D,t}(z)Q_t^{-1}$, where τ_t stands for iceberg trade costs and Q_t is the real exchange rate defined as the price of the foreign consumption basket in terms of the home consumption basket as $Q_t \equiv P^*_t/P_t$.

The profits of the firm with productivity level z that we denote with $d_t(z)$ are decomposed into those from domestic sales $d_{D,t}(z)$ and those from exporting sales $d_{X,t}(z)$ if the firm exports such that $d_t(z) = d_{D,t}(z) + d_{X,t}(z)$. On the basis of the demand functions found previously, the profits in each market are expressed as

[3] We eventually obtain a mapping between firm-specific productivity and its specific quality with alternative modeling. See, for instance, Verhoogen (2008).

$$d_{D,t}(z)=\frac{1}{\sigma}N_{D,t}^{\psi(\omega-1)-1}\left(\frac{\rho_{D,t}(z)}{q(z)}\right)^{1-\omega}C_t,$$

$$d_{X,t}(z)=\frac{Q_t}{\sigma}N_{X,t}^{\psi(\omega-1)-1}\left(\frac{\rho_{X,t}(z)}{q(z)}\right)^{1-\omega}C_t^*-\frac{w_t f_X}{Z_t}\ \text{, when } z>z_{X,t}.$$

$$d_{X,t}(z)=0\ \text{, when } z\leqslant z_{X,t}.$$

Similar expressions hold in the foreign country.

2.3. Firm averages

Among a mass $N_{D,t}$ of domestically producing firms that are distributed over $G(z)$, there is a mass $N_{X,t}=[1-G(z_{X,t})]N_{D,t}$ of exporters in the home country. Following Melitz (2003), we define two average productivity levels, $\widetilde{z}_D$ for domestically producing firms and $\widetilde{z}_{X,t}$ for exporters, as follows:

$$\widetilde{z}_D\equiv\left[\int_{z_{\min}}^{\infty}z^{\sigma-1}dG(z)\right]^{\frac{1}{\sigma-1}},\quad \widetilde{z}_{X,t}\equiv\left[\frac{1}{1-G(z_{X,t})}\int_{z_{X,t}}^{\infty}z^{\sigma-1}dG(z)\right]^{\frac{1}{\sigma-1}}. \tag{3}$$

These average productivity levels summarize all the information about the distribution of productivities.[4]Provided these averages, I define the average real price of domestically produced goods and exported goods as $\widetilde{\rho}_{D,t}\equiv\rho_{D,t}(\widetilde{z}_D)$ and $\widetilde{\rho}_{X,t}\equiv\rho_{X,t}(\widetilde{z}_{X,t})$, respectively. Similarly, the average qualities of domestically produced goods and exported goods are denoted as $\widetilde{q}_D\equiv q_D(\widetilde{z}_D)$ and $\widetilde{q}_{X,t}\equiv q_{X,t}(\widetilde{z}_{X,t})$. Additionally, the average real profits from domestic sales and export sales are defined as $\widetilde{d}_{D,t}\equiv d_{D,t}(\widetilde{z}_D)$ and $\widetilde{d}_{X,t}\equiv d_{X,t}(\widetilde{z}_{X,t})$. Finally, average real profits among all firms in the home country are given by $\widetilde{d}_t=\widetilde{d}_{D,t}+(N_{X,t}/N_{D,t})\widetilde{d}_{X,t}$. Similar expressions hold in the foreign country.

2.4. Firm entry and exit

In every period, there is a mass of $N_{E,t}$ entrants. Prior to entry, these new entrants are identical and face sunk entry costs defined as follows:

$$f_E=Z_t l_{E,t},$$

where $l_{E,t}$ is the amount of labor required to produce f_E amount of fixed costs in the firm setup. Upon entry, each new entrant draws a productivity level z from a distribution $G(z)$. Entrants at time t are assumed to start producing only from time $t+1$. Firms produce unless they are hit by an exogenous depreciation shock, which occurs with probability $\delta\in(0,1)$. This exit-inducing shock is independent of the firm-specific productivity level and assumed to occur at the very end of every period.

Firm entry takes place until post-entry value $\widetilde{v}_t$ (which is defined in Section 2.6) is equalized with entry costs as

$$\widetilde{v}_t=\frac{w_t}{Z_t}f_E.$$

The timing of entry and production implies that the number of firms in the home country evolves according to the following motion: $N_{D,t}=(1-\delta)(N_{D,t-1}+N_{E,t-1})$.

Similar expressions hold in the foreign country.

2.5. Parameterization of productivity draws

$G(z)$ is assumed to be the following Pareto distribution:

$$G(z)=1-\left(\frac{z_{\min}}{z}\right)^{\kappa},$$

where $\kappa\ (>\sigma-1)$ is the shape parameter of the distribution. With the above distribution and the definition of the average productivities in (3), we have

[4] As shown in Melitz (2003), these productivities are defined as the weighted harmonic mean such that

$$\widetilde{z}_D^{-1}\equiv\int_{z_{\min}}^{\infty}z^{-1}\frac{y(z)}{\widetilde{y}_{D,t}}dG(z),\quad \widetilde{z}_{X,t}^{-1}\equiv\frac{1}{1-G(z_{X,t})}\int_{z_{X,t}}^{\infty}z^{-1}\frac{y(z)}{\widetilde{y}_{X,t}}dG(z).$$

As one can see, the weight is the relative production scale with respect to the average production scale in each of the domestic and exporting market.

$$\tilde{z}_D = z_{\min}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}, \quad \tilde{z}_{X,t} = z_{X,t}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}.$$

Additionally, with the Pareto distribution, the share of exporters in the total number of domestic firms is given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^{\kappa}\left(\tilde{z}_{X,t}\right)^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}.$$

A firm with cutoff level productivity $z_{X,t}$ earns zero profits from exporting such that $d_{X,t}(z_{X,t}) = 0$. Combined with the above Pareto distribution and the expression of total average profits, we have the following zero-profit cutoff condition:

$$\tilde{d}_{X,t} = \frac{w_t f_X}{Z_t}\frac{\sigma-1}{\kappa-(\sigma-1)}.$$

Similar expressions hold in the foreign country.

2.6. Household budget constraints and intertemporal choices

Under financial autarky, the budget constraint of the representative household in the home country is given by[5]

$$C_t + \tilde{v}_t(N_{D,t}+N_{E,t})s_{h,t+1} + b_{h,t+1} = w_t L_t + R^s_{h,t}\tilde{v}_{t-1}(N_{D,t-1}+N_{E,t-1})s_{h,t} + R^b_{h,t}b_{h,t}. \quad (4)$$

The home representative household purchases a share $s_{h,t+1}$ of equities as well as $b_{h,t+1}$ amount of bonds both issued at home. The gross returns of equities and bonds in units of home consumption are defined, respectively, as

$$R^s_{h,t+1} \equiv (1-\delta)\frac{\tilde{v}_{t+1}+\tilde{d}_{t+1}}{\tilde{v}_t}, \quad R^b_{h,t+1} \equiv 1 + r_{t+1}.$$

Net returns of bond holdings are denoted by r_{t+1}.[6] The household maximizes the expected intertemporal utility $E_t\sum_{s=t}^{\infty}\beta^{s-t}U_s$ with respect to $s_{h,t+1}, b_{h,t+1}, L_t$, and C_t subject to (4) for all time periods. The optimal equity holdings, bond holdings, and labor supply are given by

$$1 = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^s_{h,t+1}\right], \qquad 1 = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^b_{h,t+1}\right].$$

$$\chi(L_t)^{\frac{1}{\varphi}} = w_t C_t^{-\gamma}.$$

Note that, iterating forward, the first order condition with respect to equity holdings can be rewritten as

$$\tilde{v}_t = E_t\sum_{i=t+1}^{\infty}\beta(1-\delta)^{i-t}\left(\frac{C_i}{C_t}\right)^{-\gamma}\tilde{d}_i.$$

The above gives the post-entry value of firm $\tilde{v}_t$ which is defined as the sum of expected discounted profits using the stochastic discount factor of domestic households.

Similar expressions hold in the foreign country.

2.7. General equilibrium and balanced trade

L_t units of supplied labor are employed for fixed costs of exporting, firm creation, and production of local goods and exported goods. Therefore, the labor market in the home country clears such that

$$L_t = \frac{N_{E,t}\tilde{v}_t}{w_t} + \frac{(\sigma-1)N_{D,t}\tilde{d}_t}{w_t} + \frac{\sigma N_{X,t}f_X}{Z_t}.$$

Under financial autarky, trade is balanced as

$$\int_0^{N^*_{X,t}} p^*_{X,t}(\vartheta^*)c_{X,t}(\vartheta^*)d\vartheta^* = \int_0^{N_{X,t}} p_{X,t}(\vartheta)c^*_{X,t}(\vartheta)d\vartheta.$$

[5] The budget constraint of the representative household in the foreign country is

$$C^*_t + \tilde{v}^*_t\left(N^*_{D,t}+N^*_{E,t}\right)s^*_{f,t+1} + b^*_{f,t+1} = w^*_t L^*_t + \frac{Q_t}{Q_{t+1}}R^s_{f,t}\tilde{v}^*_{t-1}\left(N^*_{D,t-1}+N^*_{E,t-1}\right)s^*_{f,t} + \frac{Q_t}{Q_{t+1}}R^b_{f,t}b^*_{f,t}$$

[6] Equity returns and bond returns issued in the foreign country are $R^s_{f,t+1} \equiv (1-\delta)\frac{\tilde{v}^*_{t+1}+\tilde{d}^*_{t+1}}{\tilde{v}^*_t}\frac{Q_{t+1}}{Q_t}$ and $R^b_{f,t+1} \equiv (1+r^*_{t+1})\frac{Q_{t+1}}{Q_t}$, respectively. Both returns are denominated in the home consumption basket.

On the basis of the previously identified optimal demands, the above expression is rewritten as

$$N_{X,t}^{\psi(\sigma-1)}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\sigma}\rho_{H,t}^{*\sigma-\omega}Q_tC_t^{*}=N_{X,t}^{*\psi(\sigma-1)}\left(\frac{\widetilde{\rho}_{X,t}^{*}}{\widetilde{q}_{X,t}^{*}}\right)^{1-\sigma}\rho_{F,t}^{\sigma-\omega}C_t. \tag{5}$$

The whole system of equations is summarized in Table 2. The benchmark model includes fifty-eight equations and two exogenous shocks. The steady state of the model is detailed in Appendix A.

3. International trade in bonds

I discuss here an alternative financial market structure by introducing internationally traded state non-contingent bonds. Since the model is almost identical to the benchmark model, only modified points are discussed below.

3.1. International bond holdings

With internationally traded bonds, the budget constraint of the home representative household is

$$\begin{aligned}&C_t+\widetilde{v}_t(N_{D,t}+N_{E,t})s_{h,t+1}+b_{h,t+1}+Q_tb_{f,t+1}+\frac{\eta}{2}b_{h,t+1}^2+\frac{\eta}{2}Q_tb_{f,t+1}^2\\&\quad=w_tL_t+R_{h,t}^{s}\widetilde{v}_{t-1}(N_{D,t-1}+N_{E,t-1})s_{h,t}+R_{h,t}^{b}b_{h,t}+R_{f,t}^{b}Q_{t-1}b_{f,t}+T_t^{f}.\end{aligned} \tag{6}$$

The household maximizes the expected intertemporal utility with respect to foreign bonds, $b_{f,t+1}$, and home bonds, $b_{h,t+1}$. To pin down international bond positions in the steady state and ensure the stationarity of the model, quadratic adjusting costs of bond holdings η are introduced. T_t^f is the free rebate of adjusting costs. The first-order conditions for home and foreign bond holdings are given by

$$1+\eta b_{h,t+1}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{h,t+1}^{b}\right],\qquad 1+\eta b_{f,t+1}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{f,t+1}^{b}\right].$$

Other first-order conditions are identical to the benchmark model.

3.2. General equilibrium and net foreign asset dynamics

With internationally traded bonds, the balanced trade condition (5) is replaced by the dynamics of net foreign assets. Net foreign assets (denominated in home consumption units) at the end of period t are defined as

$$NFA_{t+1}\equiv b_{f,t+1}Q_t-b_{h,t+1}^{*}. \tag{7}$$

Since there are no cross-border equity holdings by assumption, only cross-border bond holdings appear in the definition of net foreign assets. Using (7), budget constraint (6) can be rewritten as

$$NFA_{t+1}=NX_t+NFA_tR_{h,t}^{b}+\xi_{h,t},$$

where NX_t denotes net exports and $\xi_{h,t}$ stands for "excess returns" between $t-1$ and t relative to returns on home bonds $R_{h,t}^{b}$. Precisely, NX_t and $\xi_{h,t}$ are given by

$$NX_t=\frac{1}{2}\left[w_tL_t+N_{D,t}\widetilde{d}_t-Q_t\left(w_t^{*}L_t^{*}+N_{D,t}^{*}\widetilde{d}_t^{*}\right)\right]-\frac{1}{2}\left[(C_t-N_{E,t}\widetilde{v}_t)-Q_t\left(C_t^{*}-N_{E,t}^{*}\widetilde{v}_t^{*}\right)\right],$$

and

$$\xi_{h,t}\equiv b_{f,t}Q_t\left(R_{f,t}^{b}-R_{h,t}^{b}\right).$$

Note that excess returns are zero in the first-order dynamics because of the zero bond holdings at the steady state. Finally, asset markets clear for all time periods as

$$b_{h,t+1}+b_{h,t+1}^{*}=b_{f,t+1}+b_{f,t+1}^{*}=0.$$

Table 3 summarizes the set of equations replaced or added. The extended model includes sixty-four equations and two exogenous shocks. The steady state of the extended model remains the same as that in the benchmark model.

4. International risk sharing with heterogeneous firms

In this section, I detail international risk sharing with heterogeneous firms. First, the solution of the puzzle is shown to be possible with the combination of financial market incompleteness and firm heterogeneity. Specifically, the KBS relation is shown to be conditional on changes in the number of product varieties and their qualities, driving a wedge between relative

Table 2
The model.

Price indices	$\rho_{H,t}^{1-\omega}+\rho_{F,t}^{1-\omega}=1,\ \rho_{H,t}=N_{D,t}^{-\psi}\frac{\widetilde{\rho}_{D,t}}{\widetilde{q}_D},\ \rho_{F,t}=N_{X,t}^{*-\psi}\frac{\widetilde{\rho}_{X,t}^*}{\widetilde{q}_{X,t}^*}$
	$\rho_{F,t}^{*1-\omega}+\rho_{H,t}^{*1-\omega}=1,\ \rho_{F,t}^*=N_{D,t}^{*-\psi}\frac{\widetilde{\rho}_{D,t}^*}{\widetilde{q}_D^*},\ \rho_{H,t}^*=N_{X,t}^{-\psi}\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}$
Pricing	$\widetilde{\rho}_{D,t}=\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t\widetilde{z}_D},\ \widetilde{\rho}_{X,t}=\tau_t\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t\widetilde{z}_{X,t}}Q_t^{-1},$
	$\widetilde{\rho}_{D,t}^*=\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t^*\widetilde{z}_D^*},\ \ \widetilde{\rho}_{X,t}^*=\tau_t\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t^*}{Z_t^*\widetilde{z}_{X,t}^*}Q_t$
Profits	$\widetilde{d}_t=\widetilde{d}_{D,t}+\frac{N_{X,t}}{N_{D,t}}\widetilde{d}_{X,t},\quad \widetilde{d}_{D,t}=\frac{1}{\sigma}N_{D,t}^{\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{D,t}}{\widetilde{q}_D}\right)^{1-\omega}C_t$
	$\widetilde{d}_{X,t}=\frac{Q_t}{\sigma}N_{X,t}^{\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\omega}C_t^*-\frac{w_tf_X}{Z_t}$
	$\widetilde{d}_t^*=\widetilde{d}_{D,t}^*+\frac{N_{X,t}^*}{N_{D,t}^*}\widetilde{d}_{X,t}^*,\quad \widetilde{d}_{D,t}^*=\frac{1}{\sigma}N_{D,t}^{*\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{D,t}^*}{\widetilde{q}_D^*}\right)^{1-\omega}C_t^*$
	$\widetilde{d}_{X,t}^*=\frac{Q_t^{-1}}{\sigma}N_{X,t}^{*\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{X,t}^*}{\widetilde{q}_{X,t}^*}\right)^{1-\omega}C_t-\frac{w_t^*f_X^*}{Z_t^*}$
Free entry	$\widetilde{v}_t=\frac{w_t}{Z_t}f_E,\qquad \widetilde{v}_t^*=\frac{w_t^*}{Z_t^*}f_E^*$
Labor market clearing	$w_tL_t=N_{E,t}\widetilde{v}_t+(\sigma-1)N_{D,t}\widetilde{d}_t+\sigma N_{X,t}\frac{w_tf_X}{Z_t}$
	$w_t^*L_t^*=N_{E,t}^*\widetilde{v}_t^*+(\sigma-1)N_{D,t}^*\widetilde{d}_t^*+\sigma N_{X,t}^*\frac{w_t^*f_X^*}{Z_t^*}$
Export share	$\frac{N_{X,t}}{N_{D,t}}=z_{\min}^{\kappa}\left(\widetilde{z}_{X,t}\right)^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}},\quad \frac{N_{X,t}^*}{N_{D,t}^*}=z_{\min}^{\kappa}\left(\widetilde{z}_{X,t}^*\right)^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}$
Zero-profit cutoffs	$\widetilde{d}_{X,t}=\frac{w_tf_X}{Z_t}\frac{\sigma-1}{\kappa-(\sigma-1)},\quad \widetilde{d}_{X,t}^*=\frac{w_t^*f_X^*}{Z_t^*}\frac{\sigma-1}{\kappa-(\sigma-1)}$
Export quality	$\widetilde{q}_{X,t}=\left(\frac{\phi}{1-\phi}\widetilde{z}_{X,t}\right)^{\phi},\ \ \widetilde{q}_{X,t}^*=\left(\frac{\phi}{1-\phi}\widetilde{z}_{X,t}^*\right)^{\phi}$
Number of firms	$N_{D,t+1}=(1-\delta)(N_{D,t}+N_{E,t}),\quad N_{D,t+1}^*=(1-\delta)\left(N_{D,t}^*+N_{E,t}^*\right)$
Euler shares	$1=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{h,t+1}^s\right]$
	$1=\beta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}R_{f,t+1}^s\frac{Q_t}{Q_{t+1}}\right]$
Euler bonds	$1=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{h,t+1}^b\right]$
	$1=\beta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}R_{f,t+1}^b\frac{Q_t}{Q_{t+1}}\right]$
Labor supply	$\chi(L_t)^{\frac{1}{\varphi}}=w_tC_t^{-\gamma},\ \ \chi(L_t^*)^{\frac{1}{\varphi}}=w_t^*C_t^{*-\gamma}$
Balanced trade	$N_{X,t}^{\psi(\sigma-1)}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\sigma}\rho_{H,t}^{*\sigma-\omega}Q_tC_t^*=N_{X,t}^{*\psi(\sigma-1)}\left(\frac{\widetilde{\rho}_{X,t}^*}{\widetilde{q}_{X,t}^*}\right)^{1-\sigma}\rho_{F,t}^{\sigma-\omega}C_t$

Table 3
The model with international bonds.

Euler bonds	$1+\eta b_{h,t+1}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{h,t+1}^b\right]$
	$1+\eta b_{f,t+1}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{f,t+1}^b\right]$
	$1+\eta b_{f,t+1}^*=\beta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}R_{f,t+1}^b\frac{Q_t}{Q_{t+1}}\right]$
	$1+\eta b_{h,t+1}^*=\beta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}R_{h,t+1}^b\frac{Q_t}{Q_{t+1}}\right]$
Bond market clearing	$b_{h,t+1}+b_{h,t+1}^*=0,\quad b_{f,t+1}+b_{f,t+1}^*=0.$
Net foreign assets	$NFA_{t+1}=NX_t+NFA_t(1+r_{t+1})+\xi_t$
Net exports	$NX_t=\frac{1}{2}\left[w_tL_t+N_{D,t}\widetilde{d}_t-Q_t\left(w_t^*L_t^*+N_{D,t}^*\widetilde{d}_t^*\right)\right]$
	$-\frac{1}{2}\left[(C_t+N_{E,t}\widetilde{v}_t)-Q_t\left(C_t^*+N_{E,t}^*\widetilde{v}_t^*\right)\right]$
Excess returns	$\xi_{h,t}\equiv b_{f,t}Q_t\left(R_{f,t}^b-R_{h,t}^b\right)$

consumption growth and fluctuations in the real exchange rate. Second, I show that the measurement error with respect to precise fluctuations in the number of product varieties and their qualities can also contribute to the solution to the puzzle. Finally, I note that the measurement error has the potential to solve the puzzle independent of financial market incompleteness.

4.1. The KBS relation with heterogeneous firms and incomplete financial markets

Here I closely follow the discussion presented in Corsetti et al. (2008), in which they discuss the role played by financial market incompleteness and wealth effects for international risk sharing in a two-country international real business cycle model. Similar to the discussion in Corsetti et al. (2008), I show that financial market incompleteness per se cannot solve the puzzle. However, the solution to the puzzle is shown to be possible with the combination of firm heterogeneity and incomplete financial markets. To demonstrate this point, I discuss the relationship between relative consumption growth and the real exchange rate under financial autarky. I begin by decomposing fluctuations in the real exchange rate in the theoretical model considered here as

$$\mathsf{Q} = (2S_{ED} - 1)\mathsf{TOL} + \psi S_{ED}\mathsf{N}_D^R - (1 - S_{ED})\left[\psi \mathsf{N}_X^R + \left(1 + \frac{1}{\phi}\right)\tilde{\mathsf{q}}_X^R\right], \quad (8)$$

where $\mathsf{TOL} \equiv -(\mathsf{w}^R - \mathsf{Z}^R)$ represents fluctuations in *theterms of labor* (Ghironi and Melitz, 2005) in which $\mathsf{w}^R \equiv \mathsf{w} - \mathsf{w}^*$ and $\mathsf{Z}^R \equiv \mathsf{Z} - \mathsf{Z}^*$ represent fluctuations in relative real wages and productivities across countries, respectively. Similarly, $\mathsf{N}_D^R \equiv \mathsf{N}_D - \mathsf{N}_D^*$, $\mathsf{N}_X^R \equiv \mathsf{N}_X - \mathsf{N}_X^*$ and $\tilde{\mathsf{q}}_X^R \equiv \tilde{\mathsf{q}}_X - \tilde{\mathsf{q}}_X^*$ are the relative changes in the number of domestically available varieties, the number of exported varieties, and the average quality of exports, respectively. S_{ED} represents the steady-state expenditure share of domestically produced goods. Note that a depreciation in the terms of labor ($\mathsf{TOL} > 0$) makes the real exchange rate depreciate ($\mathsf{Q} > 0$) when $S_{ED} > 1/2$. Furthermore, all other things being equal, a higher number of domestic varieties ($\mathsf{N}_D^R > 0$) also makes the real exchange rate depreciate ($\mathsf{Q} > 0$). The same is true for a higher number of imported varieties ($\mathsf{N}_X^R < 0$) or a higher average quality of imported goods ($\tilde{\mathsf{q}}_X^R < 0$).Given the above expression (8), the relationship between relative consumption growth and the real exchange rate fluctuations under financial autarky is expressed as[7]

$$\mathsf{C} - \mathsf{C}^* = \frac{2S_{ED}\omega - 1}{2S_{ED} - 1}\mathsf{Q} + \frac{(\omega - 1)S_{ED}}{2S_{ED} - 1}\left[\psi(\mathsf{N}_X^R - \mathsf{N}_D^R) + \left(1 + \frac{1}{\phi}\right)\tilde{\mathsf{q}}_X^R\right]. \quad (9)$$

This relationship is found to be *conditional* on changes in the relative number of product varieties and the relative quality of traded goods. This observation leads us to the following proposition.

Proposition 1. *With firm entry, the self-selection of more efficient firms into exporting, and incomplete financial markets, there exists a wedge between the unconditional KBS relation and the one conditioned on fluctuations in the number of product varieties and their qualities.*

The *unconditional*KBS relationship between relative consumption and the real exchange rate can differ from the conditional one, which is $\frac{2S_{ED}\omega-1}{2S_{ED}-1}$. With the expenditure share on domestic goods, which is typically higher than one-half ($S_{ED} > 1/2$), and the elasticity of substitution, which is typically higher than unity ($\omega > 1$), the conditional relation is unambiguously positive as $\frac{2S_{ED}\omega-1}{2S_{ED}-1} > 0$, whereas the unconditional KBS relation can be negative. Put differently, the real exchange rate can appreciate ($\mathsf{Q} < 0$) and relative consumption can be higher ($\mathsf{C} - \mathsf{C}^* > 0$) due to a strong wealth effect driven by appreciation in the terms of labor in the unconditional relationship. Simultaneously, a higher number of exported varieties relative to domestic varieties ($\mathsf{N}_X^R - \mathsf{N}_D^R > 0$) and/or a higher quality of exported goods ($\tilde{\mathsf{q}}_X^R > 0$) can be realized such that the appreciation in the real exchange rate hinges on an elaborated Harrod-Balassa-Samuelson mechanism with the endogenous tradedness of exporters as detailed in Ghironi and Melitz (2005).[8] However, note that, by shutting down the impact arising from firm heterogeneity, the discrepancy between the conditional and unconditional relationships disappears. In such a case, we find the same expression as found in Hamano (2013) in a model with firm entry and *homogeneous* firms such that $\mathsf{C} - \mathsf{C}^* = \frac{2S_{ED}\omega-1}{2S_{ED}-1}\mathsf{Q}$.[9] Note that it is firm heterogeneity that creates the wedge between conditional and unconditional relations. Therefore, the financial market incompleteness per se cannot solve the puzzle although it is possible when it is combined with firm heterogeneity.

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[7] In the model, using the demand systems found previously, the balanced trade condition (5) is expressed as

$$\omega\mathsf{Q} - (\mathsf{C} - \mathsf{C}^*) + \psi(\omega - 1)\mathsf{N}_X^R - (\omega - 1)\left[\mathsf{w}^R - \mathsf{Z}^R - \left(1 + \frac{1}{\phi}\right)\tilde{\mathsf{q}}_X^R\right] = 0.$$

Plugging the decomposition of the real exchange rate (8) into the above balanced trade condition, we obtain (9).

[8] The wealth effect in the theoretical model is discussed and explored quantitatively in the following section.

[9] Compared to the expression found in Hamano (2013), which has only the first term on the right-hand side of Eq. (9), the second term in the square brackets arises due to the presence of fixed exporting costs. By setting $f_X = f_X^* = 0$, all firms export independent of their specific productivities. As a result, we do not see any changes in the cutoff level and, thus, quality: $\tilde{z}_X^R = \tilde{\mathsf{q}}_X^R = 0$ and the numbers of exporters and domestic producers coincide as $\mathsf{N}_D^R = \mathsf{N}_X^R$. In the homogeneous firm setting, as in Hamano (2013), the wedge between the unconditional KBS relation and the conditional KBS relation disappear.

How far then is the allocation under financial autarky from that implied under complete asset markets? Even under financial autarky, the mechanism of international risk sharing is at work thanks to welfare improving fluctuations in relative prices (Cole and Obstfeld, 1991; Acemoglu and Ventura, 2002). Importantly, the following corollary is derived by comparing (9) and the allocation under complete financial markets (1):

Corollary 1. *With firm entry and the self-selection of more efficient firms into exporting, the equilibrium allocation under financial autarky mimics perfectly that obtained with complete asset markets when* $\omega = \gamma = 1$.

Corollary 1 is a generalization of the result discussed in Cole and Obstfeld (1991). With firm heterogeneity, international transmission depends on not only the fluctuations in the terms of labor (relative prices) but also those in the relative number of varieties and their qualities, through which a welfare improving transmission can be established.

4.2. The KBS relation with heterogeneous firms, incomplete financial markets, and measurement error

The structural KBS relationship between relative consumption and the real exchange rate in Eq. (9) takes into account complete fluctuations in the number of product varieties and their qualities. In investigating the KBS puzzle with actual data, one should note that these fluctuations are observed or measured imperfectly (Broda and Weinstein, 2004; Broda and Weinstein, 2006). In this subsection, I show that such measurement error can also contribute to solving the puzzle. Following Ghironi and Melitz (2005), I start by defining the theoretical counterpart to the empirical measures such as CPI. I denote the fluctuations in the real exchange rate in the empirical measures, which I denote with $\widehat{\mathsf{Q}}$, in the following way:[10]

$$\widehat{\mathsf{Q}} \equiv \mathsf{Q} - \psi\lambda_1 \mathsf{N}_D^R + \psi\lambda_2 \mathsf{N}_X^R + \lambda_3 \widetilde{\mathsf{q}}_X^R, \tag{10}$$

where the parameters λ_1, λ_2 and λ_3 capture the extent of (in) efficiency of statistical agencies in measuring fluctuations in the relative number of domestic varieties N_D^R, exported (imported) varieties N_X^R, and the average quality of exports (imports) $\widetilde{\mathsf{q}}_X^R$.[11] Depending on the values of these parameters, the definition of $\widehat{\mathsf{Q}}$ differs. On the one hand, when $\lambda_1 = \lambda_2 = \lambda_3 = 0$, there is no discrepancy between the true fluctuations and those in the empirical measures as $\widehat{\mathsf{Q}} = \mathsf{Q}$. On the other hand, when $\lambda_1 = S_{ED}$ and $\lambda_2 = \lambda_3 = 1 - S_{ED}$, statistical agencies completely ignore fluctuations in the number of varieties and their qualities.[12] In general, when $\lambda_1 > 0$ ($\lambda_1 < 0$), there is an underestimation (overestimation) of the impact of domestic varieties, while when $\lambda_2 > 0$ ($\lambda_2 < 0$) and $\lambda_3 > 0$ ($\lambda_3 < 0$), there is an underestimation (overestimation) of the impact of product varieties and the average quality of exports and imports. In a similar way, the fluctuations in consumption in the empirical measures are defined as

$$\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* \equiv \mathsf{C} - \mathsf{C}^* - \psi\lambda_1 \mathsf{N}_D^R + \psi\lambda_2 \mathsf{N}_X^R + \lambda_3 \widetilde{\mathsf{q}}_X^R. \tag{11}$$

On the basis of both (10) and (11), the relation (9) is transformed to the one in the empirical measures as

$$\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* = \frac{2S_{ED}\omega - 1}{2S_{ED} - 1}\widehat{\mathsf{Q}} + \frac{(\omega - 1)S_{ED}}{2S_{ED} - 1}\left\{\psi\left[(1 - 2\lambda_2)\mathsf{N}_X^R - (1 - 2\lambda_1)\mathsf{N}_D^R\right] + \left[(1 - 2\lambda_3) + \frac{1}{\phi}\right]\widetilde{\mathsf{q}}_X^R\right\}. \tag{12}$$

Here again a wedge between the *unconditional KBS relation* and the conditional KBS relation remains. The first term on the right-hand side of Eq. (12) is the same one argued in Corsetti et al. (2008) in a classical two-country model. Note that the first term on the right-hand side of Eq. (12) has exactly the same coefficient as in (9). However, the coefficients on the number of domestic varieties N_D^R, the number of traded varieties N_X^R and the average product quality $\widetilde{\mathsf{q}}_X^R$ are different from the true relationship depending on λ_1, λ_2 and λ_3. As expected, when $\lambda_1 = \lambda_2 = \lambda_3 = 0$ and thus there is no measurement error, the expression in the empirical measures (12) coincides with the true one (9). In Section 6, I explore the above structural relation with regression analysis with real data.

The above point is also true in a model with firm entry but without firm heterogeneity. Specifically, by removing the fixed costs for exporting, all firms export; hence, no changes in quality occur. In such a case, (12) collapses to

$$\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* = \frac{2S_{ED}\omega - 1}{2S_{ED} - 1}\widehat{\mathsf{Q}} + \frac{2\psi(\omega - 1)(\lambda_1 - \lambda_2)S_{ED}}{2S_{ED} - 1}\mathsf{N}_D^R. \tag{13}$$

These observations lead us to the following proposition:

Proposition 2. *With or without the self-selection of more efficient firms into exporting, with firm entry and incomplete financial markets, the measurement error creates a wedge between the unconditional KBS relation and the one conditioned on fluctuations in the number of product varieties and their qualities.*

I thus confirm the point discussed in Hamano (2013): the solution to the KBS puzzle is possible with a combination of measurement error, firm entry, and financial market incompleteness.[13] This point is true with or without firm heterogeneity.

[10] Ghironi and Melitz (2005) and Hamano (2015) provide a similar decomposition but without quality.

[11] One can imagine the possibility of a time-variant measurement error over the business cycles. Aghion et al. (2019) explore how systematically these coefficients are endogenously determined, while I leave the issue aside for the sake of simplicity.

[12] This is the case in Corsetti et al. (2007) when they discuss the empirically relevant measures.

[13] Note that by setting $\lambda_1 = S_{ED}$ and $\lambda_2 = 1 - S_{ED}$, expression (12) becomes similar to the one found in Hamano (2013) as $\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* = \frac{2S_{ED}\omega - 1}{2S_{ED} - 1}\widehat{\mathsf{Q}} + 2\psi(\omega - 1)S_{ED}\mathsf{N}_D^R$.

4.3. Financial market completeness and measurement error

The last point to make is that the measurement error with respect to changes in the number of product varieties and their qualities per se contributes to solving the puzzle regardless of the extent of financial market incompleteness. Indeed, in a model with firm entry (with or without firm heterogeneity), the measurement error can be an important source of the wedge, even under complete asset markets. The point is demonstrated clearly in the derivation of the KBS relationship in the empirical measures under complete asset markets. By plugging (10) and (11) into (1), we thus obtain

$$\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* = \frac{1}{\gamma}\widehat{\mathsf{Q}} - \left(1 - \frac{1}{\gamma}\right)\left\{\psi\left[S_{ED}\lambda_1 \mathsf{N}_D^R - (1 - S_{ED})\lambda_2 \mathsf{N}_X^R\right)\right] - (1 - S_{ED})\lambda_3 \tilde{\mathsf{q}}_X^R\right\}. \tag{14}$$

In the above Eq. (14), the strong positive association between relative consumption and the real exchange rate in the original welfare-based relation (1) is broken.[14] Note that the wedge under complete markets is also possible with the entry of homogeneous firms but without the selection of efficient firms for exporting. In this case, the KBS relation becomes as

$$\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* = \frac{1}{\gamma}\widehat{\mathsf{Q}} - \left(1 - \frac{1}{\gamma}\right)\psi[S_{ED}\lambda_1 - (1 - S_{ED})\lambda_2)]\mathsf{N}_D^R.$$

These observations lead us to the following proposition:

Proposition 3. *Independent of the degree of financial market incompleteness, the measurement error creates a wedge between the unconditional KBS relation and the one conditioned on fluctuations in the number of product varieties and their qualities.*

To summarize this section, firm heterogeneity, measurement error, and financial market incompleteness all contribute to driving a wedge between the conditional KBS relation and the unconditional KBS relation. In the following section, before assessing the prediction of the model with data, I explore how the observed close-to-zero or even negative unconditional KBS correlation can be achieved in the theoretical model. Namely, I focus on the wealth effects that could potentially drive a wedge and thus a realistic KBS correlation.

5. The KBS correlation in the theoretical model

In the previous section, the KBS relation is found to be conditional on changes in product varieties and their qualities. In this section, I explore the mechanism with which a realistic unconditional KBS correlation is obtained. As argued previously, the key to replicating a realistic correlation is the wealth effect that brings the real exchange rate into appreciation and simultaneously realizes higher consumption, i.e., the Harrod-Balassa-Samuelson mechanism based on heterogeneous firms. Among the complementing mechanisms, I emphasize the one that is driven by the quality ladder and quantitatively explore its implications that hold independently of financial market incompleteness.

5.1. Calibration

The theoretical models are calibrated with the parameter values in Table 4. The calibration is conducted on a quarterly basis. The values of constant risk aversion γ, discount factor β, Frisch elasticity of labor supply φ, and the elasticity of substitution between local goods and imported goods ω are in line with the literature. The values of the elasticity of substitution among product varieties σ, preference for variety ψ, death shock δ, trade cost τ, shape of Pareto distribution κ, fixed entry costs f_E, and fixed export costs f_X are set according to Ghironi and Melitz (2005). Specifically, the calibration of f_X and κ is based on the empirical findings of Bernard et al. (2003), according to which the share of exporters is 21%. The parameter value that determines the quality ladder ϕ comes from Feenstra and Romalis (2014), who estimate the elasticity of firm-specific quality with respect to firm-specific productivity.

The productivity process is selected from Backus et al. (1992) such that

$$\begin{bmatrix} \mathsf{Z}_{t+1} \\ \mathsf{Z}^*_{t+1} \end{bmatrix} = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}\begin{bmatrix} \mathsf{Z}_t \\ \mathsf{Z}^*_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon^*_t \end{bmatrix},$$

where ε_t and ε_t^* are assumed to be zero mean i.i.d. The standard deviation of the productivity innovations is set to 0.00852 and the correlation to 0.258.

5.2. Wealth effects

A close-to-zero or even negative *unconditional* KBS correlation in the model can be achieved through the wealth effect with which the movement of the terms of trade (the terms of labor) and, hence, the real exchange rate end up increasing a wedge in consumption across countries under incomplete financial markets. Corsetti et al. (2008) note a lower elasticity

[14] Indeed, product quality and variety work here as a preference shock that breaks the tight relationship between bilateral consumption growth and the real exchange rate fluctuations implied by complete asset markets; see Stockman and Tesar (1995), Raffo (2010) and Mandelman et al. (2011).

Table 4
Baseline parameter values.

γ	constant risk aversion	2
β	discount factor	0.99
φ	Frisch elasticity of labor supply	2
ω	elasticity of substitution between home and foreign goods	2
σ	elasticity of substitution among product varieties	3.8
ψ	preference for variety	0.3571
δ	death shock	0.025
τ	trade cost	1.3
κ	Pareto distribution	3.34
f_E	fixed entry costs	1
f_X	fixed costs for export	0.0158
ϕ	quality ladder	0.61

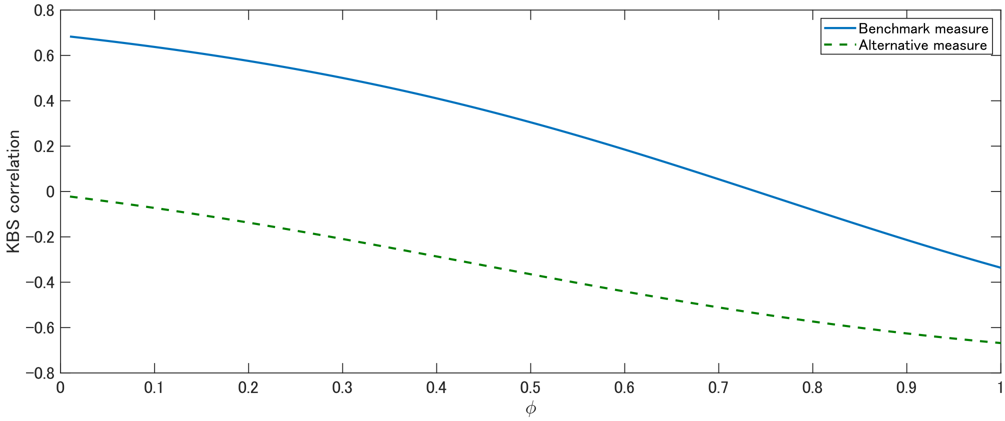

Fig. 1. The KBS Correlation and Quality Ladder (Balanced Trade). Note: The figure reports the sensitivity results of the unconditional KBS correlation in the theoretical model against a quality ladder, ϕ, with the benchmark measurement error ($\lambda_1 = \lambda_2 = \lambda_3 = 1$) and the alternative measurement error ($\lambda_1 = S_{ED}$ and $\lambda_2 = \lambda_3 = 1 - S_{ED}$) obtained under balanced trade.

of substitution between domestically produced goods and imported goods and/or a high shock persistence as a driver of such a wealth effect. Hamano (2013) argues that the wealth effect is driven by a higher number of product varieties relative to that of abroad based on a conditional KBS relationship as in Eq. (13).

The theoretical model considered here embeds all the abovementioned wealth effects discussed in the literature. Following a positive productivity shock, the terms of labor and, thus the real exchange rate appreciates vigorously with endogenous entry and the selection of more efficient firms into exporting. The transmission of the wealth effect thus materializes through the Harrod-Balassa-Samuelson mechanism with endogenous tradedness based on heterogeneous firms. I focus specifically on the role played by product quality as a driver of such a strong wealth effect. In the model, as indicated in Eq. (2), when the quality ladder increases with a higher value of ϕ, marginal costs and thus the terms of labor appreciate for the country that produces goods with higher quality. As seen in Eq. (8) and (10), the appreciation in the terms of labor results in an appreciation in the real exchange rate $\widehat{\mathrm{Q}}$ in the empirical measures, together with higher consumption in the country compared to consumption abroad.

Fig. 1 plots the unconditional KBS correlation in the empirical measures in the benchmark theoretical model, namely, $Corr\left(\widehat{\mathrm{Q}}, \widehat{\mathrm{C}} - \widehat{\mathrm{C}}^*\right)$, against the different values of the quality ladder, ϕ. In specifying the (in) efficiency of empirical measures, I follow Feenstra (1994) and Ghironi and Melitz (2005) and set $\lambda_1 = \lambda_2 = \lambda_3 = 1$ as the benchmark case. I also use an alternative measurement error such that $\lambda_1 = S_{ED}$ and $\lambda_2 = \lambda_3 = 1 - S_{ED}$ to assess the robustness of the result. Fig. 1 shows that as ϕ increases from zero, the KBS correlation changes from positive to negative in the benchmark calibration (solid line). A similar pattern is observed for the alternative measurement error (dashed line). Thus, a higher value of quality ladder amplifies the wealth effect, generating further appreciation in the observed real exchange rate.[15]

[15] Fig. 3 in Appendix B provides the KBS correlation with respect to different values of the quality ladder ϕ with the bond economy. A similar pattern is observed under balanced trade. Additionally, Fig. 4 shows the result of a sensitivity analysis with respect to the persistence of a common productivity shock. In the theoretical model here, I confirm the wealth effect due to a high shock persistence, as argued in Corsetti et al. (2008).

Table 5
The KBS correlation and financial market incompleteness.

	$Corr\left(\widehat{Q},\widehat{C}-\widehat{C}^{*}\right)$
World Average (Median)	−0.025 (-0.028)
OECD Average (Median)	−0.017 (-0.039)
OECD Average with US (Median)	0.06 (0.09)
$\lambda_1=\lambda_2=\lambda_3=1.$	
Financial Autarky	0.16
Bond Economy	0.18
Complete Markets	1.00
$\lambda_1=S_{ED},\lambda_2=\lambda_3=1-S_{ED}$	
Financial Autarky	−0.3
Bond Economy	−0.14
Complete Markets	1.00

Note: The table reports the unconditional KBS correlations in the theoretical model under financial autarky, the bond economy, and complete financial markets with two different types of measurement error. Data on per capita consumption and real effective exchange rates are taken from Penn World Table, version 9.0, for the period 1984 to 2011.

5.3. Financial market incompleteness

As stated in Proposition 3, there exists a wedge between the unconditional KBS correlation and the conditional KBS correlation regardless of the extent of financial market incompleteness in a model with firm entry. Table 5 reports the unconditional KBS correlation in the empirical measures under different financial market imperfections, together with those obtained with actual data. The KBS correlation is 0.16 in the benchmark theoretical model. The bond economy provides a very similar correlation of 0.18. With the alternative empirical measures, the KBS correlation is negative: −0.3 under financial autarky and −0.14 with the bond economy. These correlations are similar to those in the actual data: the average KBS correlation among each pair of countries in the world, among each pair of OECD countries, and that between each OECD country and the United States are −0.025, −0.017 and 0.06, respectively. However, with complete financial markets, the KBS correlations are close to unity for either degree of measurement error in the theoretical model. The results indicate that a tight positive relationship under complete markets, as we see in Eq. (1), is quantitatively difficult to break by just introducing a measurement error.

6. The KBS puzzle in data

As argued in the previous section, the structural relationship between bilateral consumption growth and fluctuation in the real exchange rate is conditional on changes in the number of product varieties and their qualities. Therefore, we should be cautious in interpreting the observed unconditional KBS correlations because of their potential *bias*. In this section, I test the prediction of the theoretical model highlighted in Propositions 1–3 with a regression analysis. I find that the KBS correlation is more positive and thus closer to those implied under complete asset markets once fluctuations in the number of traded varieties and their traded qualities are controlled for. The result indicates underestimation of the extent of international consumption risk sharing.

6.1. Data

For the analysis, a panel data set of 178 countries from 1984 to 2011 is used. Product quality is inferred with the observed unit price and market share for a particular product. For instance, we can say that a product of the same price has a better quality if it captures a higher market share. Feenstra and Romalis (2014) assume that firms determine their product qualities as the theoretical model in this paper describes. Based on the demand (market share) and the supply side (production function of firms) information, they provide estimates of the quality of exports and imports for each product (defined by four-digit SITC codes) for each country in the world for the period from 1984 to 2011. Their estimates of product qualities are defined with respect to the world average, which is normalized to unity.[16] Based on their estimates, I *aggregate* the quality of exports and imports at each country-level in each year in the sample. Specifically, based on the estimated quality of a particular good k of export (X) or import (M) of country i for year t, q^i_{kst}, the aggregate quality of that country's exports or imports is defined as

$$q^i_{st}=\sum_{k=1}^{N^i_{st}} ts^i_{kst}q^i_{kst},$$

[16] Their estimates of product qualities are found at http://cid.econ.ucdavis.edu/Html/Quality_Data_Page.html.

where $s = X$ or M, N^i_{st} is the number of exported or imported varieties (or precisely, the number of categories of goods defined with four-digit SITC codes) with the ROW, and ts^i_{kst} is the share of exports or imports of that particular good k in the total value of exports or imports of country i.

In Appendix C, I present descriptive statistics and the evolution of aggregate qualities as well as the number of varieties of exports and imports for a number of selected countries.[17] The average number of export varieties (categories of goods) is 299.6, while that of import varieties is 487.6 for each year. The number of import varieties tends to be much higher than the number of export varieties for emerging and less developed countries such as Egypt, Mexico, Malaysia, Thailand and Zimbabwe, especially at the beginning of the sample years. We see a large drop in the numbers of both export and import varieties beginning in 2009, the time of the "great trade collapse" following the financial crisis. Compared to the number of traded varieties, the aggregate measures of quality of trade has much lower standard deviations, as reported in the table. The quality of exports tends to be higher than the quality of imports for advanced economies (Canada, Germany, France, the United Kingdom and the United States), while we observe some catching up and an upward trend over time for emerging countries such as India, Malaysia and Thailand. China starts to export a large number of varieties, while its quality remains at a lower level over time. Overall, the aggregate measures I use show similar patterns with respect to the evolution of product varieties and qualities of trade as observed in the literature. These are consistent with those in Feenstra and Romalis (2014).

The data on real per capita consumption, price level of consumption goods, and per capita income are taken from the Penn World Table (pwt90).

6.2. Empirical analysis

Provided the abovementioned data, bilateral consumption growth, bilateral fluctuation in the real exchange rate, bilateral growth in the number of traded varieties and their qualities are computed for each country pair in the world. The consumption growth rate of country $i, \Delta C^i_t$, is defined as $\Delta C^i_t \equiv \ln C^i_t - \ln C^i_{t-1}$.[18] The growth rate of the price level is defined as $\Delta P^i_t \equiv -\left(\ln P^i_t - \ln P^i_{t-1}\right)$. The growth rates of the number of exported or imported varieties and those of qualities are defined as $\Delta N^i_t \equiv \ln N^i_{X,t} - \ln N^i_{M,t} - \left(\ln N^i_{X,t-1} - \ln N^i_{M,t-1}\right)$ and $\Delta q^i_t \equiv \ln q^i_{X,t} - \ln q^i_{M,t} - \left(\ln q^i_{X,t-1} - \ln q^i_{M,t-1}\right)$.

I first present the unconditional KBS correlations $Corr\left(\Delta C^i_t - \Delta C^{USA}_t,\ \Delta P^{USA}_t - \Delta P^i_t\right)$ in contrast with the conditional, i.e., partial KBS correlations conditioned on changes in the number of product varieties and their qualities $Corr\left(\Delta C^i_t - \Delta C^{USA}_t,\ \Delta P^{USA}_t - \Delta P^i_t | \Delta N^i_t - \Delta N^{USA}_t,\ \Delta q^i_t - \Delta q^{USA}_t\right)$. These are computed for each OECD country (country i) with respect to the United States during the entire sample period.[19] In Fig. 2, unconditional KBS correlations and conditional KBS correlations are plotted, together with a 45-degree line. The unconditional correlations are close to zero or even negative for some countries. However, once they are conditioned with changes in the number of product varieties and their qualities, the correlations improve for a large number of countries: the conditional correlations are thus above the 45-degree line.

Although useful and intuitive, the above illustration cannot be used for statistical inference because of the limited number of samples. To further investigate the systematic difference, I use the entire sample and perform a panel regression. Based on the analysis in Section 4, the unconditional relation is specified as follows:

$$\Delta C^i_t - \Delta C^j_t = \beta_0 + \beta_1\left(\Delta P^j_t - \Delta P^i_t\right) + \mu^i_j + \nu_t + \epsilon_t, \tag{15}$$

while the conditional relation is

$$\Delta C^i_t - \Delta C^j_t = \beta_0 + \beta_1\left(\Delta P^j_t - \Delta P^i_t\right) + \beta_2\left(\Delta N^i_t - \Delta N^j_t\right) + \beta_3\left(\Delta q^i_t - \Delta q^j_t\right) + \mu^i_j + \nu_t + \epsilon_t, \tag{16}$$

where μ^i_j and ν_t represent country-pair-specific fixed effects and time fixed effects, respectively. ϵ_t denotes i.i.d. shocks. The coefficients of Eq. (16) have thus the following interpretation with conditional covariance and conditional variance ratio:

$$\beta_1 = \frac{Cov\left(\Delta C^i_t - \Delta C^j_t, \Delta P^j_t - \Delta P^i_t | \Delta N^i_t - \Delta N^j_t, \Delta q^i_t - \Delta q^j_t\right)}{Var\left(\Delta P^j_t - \Delta P^i_t | \Delta N^i_t - \Delta N^j_t, \Delta q^i_t - \Delta q^j_t\right)}.$$

The unconditional KBS relation, $Cov\left(\Delta C^i_t - \Delta C^j_t\right)/Var\left(\Delta P^j_t - \Delta P^i_t\right)$ in Eq. (15), is thus *biased* compared to the true structural conditional KBS relation. Note that Eq. (16) corresponds to the structural relation analyzed in Section 4, specifically, Eq. (12).

[17] These countries are ARG (Argentina), CAN (Canada), CHE (Chile), CHN (China), DEU (Germany), EGY (Egypt), FRA (France), GBR (United Kingdom), IND (India), ITA (Italy), JPN (Japan), MEX (Mexico), MYS (Malaysia), THA (Thailand), USA (United States) and ZWE (Zimbabwe). The full data set of all countries is available upon request.

[18] The income growth rate of country i and the world average are defined in a similar way.

[19] The partial correlation can be explained as the association between two random variables after eliminating the effect of all other random variables. These are computed with R's pcor function. See also the corresponding similar notion in regression analysis with multiple explanatory variables.

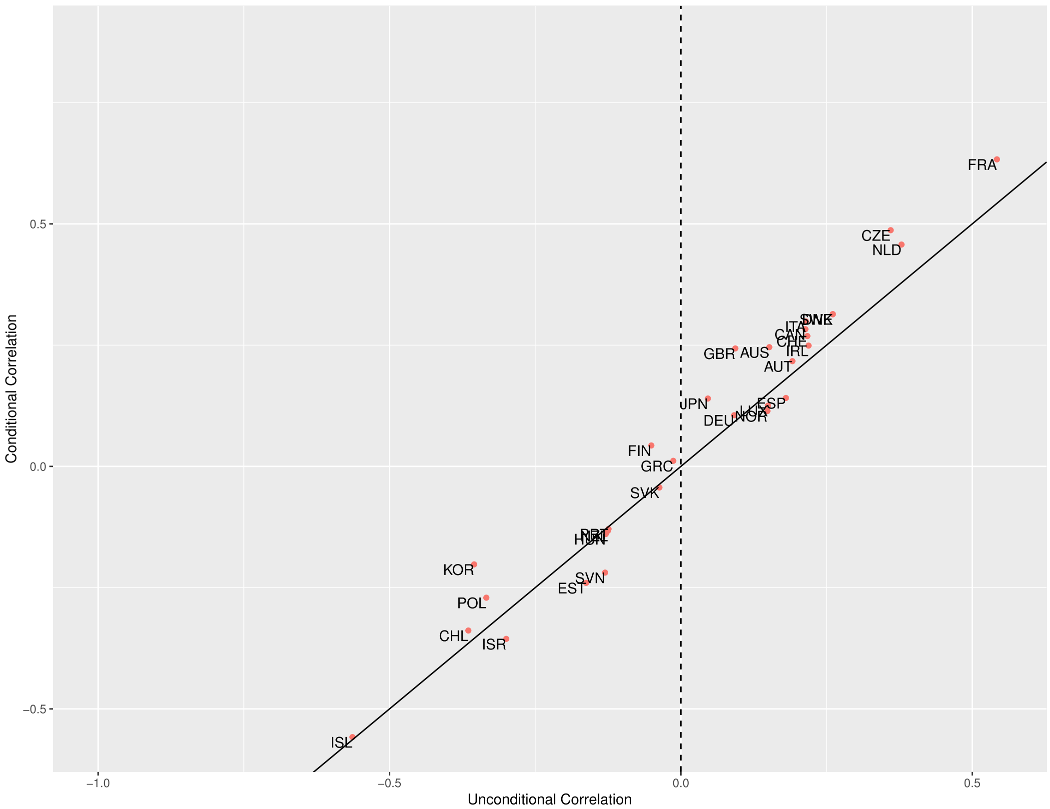

Fig. 2. The Unconditional vs. Conditional KBS Correlations. Note: Unconditional KBS correlations and conditional KBS correlations of each OECD country against the United States are plotted for the period from 1984 to 2011. The solid straight line is a 45-degree line.

Table 6
The KBS regression: full sample.

Dep Var:	$\Delta C_t^i - \Delta C_t^j$							
	Fixed effects				Random effects			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta P_t^j - \Delta P_t^i$	0.005*	0.018***	−0.042***	−0.023***	0.008	0.022	−0.038***	−0.019
	(0.003)	(0.003)	(0.003)	(0.003)	(0.013)	(0.014)	(0.013)	(0.014)
$\Delta N_t^i - \Delta N_t^j$		−0.043***		−0.042***		−0.041***		−0.040***
		(0.002)		(0.002)		(0.008)		(0.008)
$\Delta q_t^i - \Delta q_t^j$		−0.049***		−0.046***		−0.049***		−0.047***
		(0.002)		(0.002)		(0.008)		(0.008)
$\Delta Y_t^i - \Delta Y_t^j$			0.263***	0.225***			0.259***	0.222***
			(0.004)	(0.004)			(0.016)	(0.018)
Observations	405,248	368,227	405,248	368,227	405,248	368,227	405,248	368,227
R^2	0.00001	0.004	0.013	0.013	0.0005	0.004	0.010	0.012
Adjusted R^2	−0.041	−0.039	−0.027	−0.030	0.0005	0.004	0.010	0.012

Note: $\Delta C_t^i - \Delta C_t^j$, $\Delta P_t^j - \Delta P_t^i$, $\Delta N_t^i - \Delta N_t^j$, $\Delta q_t^i - \Delta q_t^j$ and $\Delta Y_t^i - \Delta Y_t^j$ represent the growth rate of consumption, the real exchange rate, the number of traded varieties, the quality of traded products, and the income for country i with respect to country j. Standard errors are reported in parentheses. ***, ** and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 6 shows the results of the estimation of the unconditional KBS relation and the conditional KBS relation. In this full sample case, random effects estimators are preferred to fixed effects estimators according to the Hausman test results.[20] After conditioning on changes in the number of varieties and their qualities, the conditional KBS relation, which corresponds

[20] Random effects estimators rely on the assumption that the variance components are orthogonal to the regressors. I resort to Hausman's specification test for the random effects model, where the null hypothesis is that the variance components are orthogonal to regressors. If the null hypothesis is true, both the random effects and fixed effects estimators are consistent. However, the random effects estimators obtained from the generalized least squares method are efficient while the fixed effects estimators obtained from the least squared dummy variables method are inefficient. Additionally, the random effects estimators could better preserve the degree of freedom (Greene, 2018). The Hausman test fails to reject the null hypothesis, indicating that the random effects model is preferable.

Table 7
The KBS regression: OECD countries.

Dep Var:	$\Delta C_t^i - \Delta C_t^j$ Fixed effects				Random effects			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta P_t^j - \Delta P_t^i$	−0.149***	−0.106***	−0.123***	−0.095***	−0.148***	−0.105***	−0.122***	−0.094***
	(0.007)	(0.007)	(0.006)	(0.007)	(0.007)	(0.007)	(0.006)	(0.006)
$\Delta N_t^i - \Delta N_t^j$		0.009		0.011		0.017		0.020
		(0.018)		(0.017)		(0.017)		(0.017)
$\Delta q_t^i - \Delta q_t^j$		−0.035***		−0.041***		−0.034***		−0.040***
		(0.007)		(0.006)		(0.006)		(0.006)
$\Delta Y_t^i - \Delta Y_t^j$			0.463***	0.332***			0.474***	0.345***
			(0.013)	(0.014)			(0.012)	(0.014)
Observations	12,684	11,372	12,684	11,372	12,684	11,372	12,684	11,372
R^2	0.039	0.024	0.136	0.072	0.039	0.024	0.142	0.076
Adjusted R^2	0.0004	−0.018	0.100	0.032	0.039	0.024	0.142	0.076

Note: $\Delta C_t^i - \Delta C_t^j$, $\Delta P_t^j - \Delta P_t^i$, $\Delta N_t^i - \Delta N_t^j$, $\Delta q_t^i - \Delta q_t^j$ and $\Delta Y_t^i - \Delta Y_t^j$ represent the growth rate of consumption, the real exchange rate, the number of traded varieties, the quality of traded products, and the income for country *i* with respect to country *j*. Standard errors are reported in parentheses. ***, ** and * indicate significance at the 1 %, 5%, and 10% levels, respectively.

to β_1 in Eq. (16), becomes more positive and significant than the unconditional KBS relation, which corresponds to β_1 in Eq. (15). As shown in columns (5) and (6) in the table, the coefficient changes from 0.008 to 0.022. For a further robustness check, I also include the relative income growth rate $\Delta Y_t^i - \Delta Y_t^j$ as a control variable, as in Kose et al. (2009), Hess and Shin (2010) and Baxter (2012). As in Krugman (1989), this value might be considered as a proxy for relative changes in the domestic number of varieties across countries N_D^R in the theoretical relation (12). Interestingly, the unconditional KBS coefficient is negative (−0.038) and statistically significant with respect to relative income growth (column (7) in Table 6) while the conditional KBS coefficient becomes less negative (−0.019) and insignificant when controlling for changes in the number of product varieties and their qualities (column (8) in Table 6). The comparison between the estimated conditional KBS coefficients and the estimated unconditional KBS coefficients for the case of fixed effects estimators shows a similar pattern with or without inclusion of income growth rate.

The above results with the full sample, however, might be noised by giving equal weight to each pair obtained from all possible bilateral combinations among countries irrespective of their income levels. To check the robustness, I present the results among high-income OECD countries in Table 7.[21] Compared to the unconditional KBS coefficients, the conditional KBS coefficients for OECD countries are less negative, indicating a similar bias as that observed for the worldwide data. For further robustness, I restrict the sample by subsetting the country pairs between rich and less developed countries and the use of the same currency unit. The results with the country pairs between "high and upper middle income countries" and "low and lower middle income countries" and those among Eurozone countries are provided in Tables 9 and 10 in Appendix C.[22] The results with these alternative samples show a similar pattern as the case of the full sample and the OECD sample.

Echoing my results, Fitzgerald (2012) estimates the extent of international risk sharing based on the gravity equation in the trade literature. She shows that measured price indices tend to provide puzzling coefficients similar to the original KBS puzzle, indicating less risk sharing across countries. Instead of using such observable real exchange rates as my case, she relies on fixed effects estimators of price indices in the gravity equation as argued in Head and Mayer (2014). In doing so, she finds more favorable evidence for the presence of international risk sharing, especially among OECD countries. Her results thus note the same type of bias that could be provided by unobservable fluctuations in the number of product varieties and their qualities.

To summarize, there exists a systematic bias in the KBS coefficients that is revealed by controlling for cross-country differences in the number of product varieties and their qualities of trade, as the theoretical model predicts. The conditional KBS relations are more positive, indicating better international risk sharing across countries.

7. Conclusion

This paper explores the implications of firm entry and the selection of more efficient firms into exporting for international consumption risk sharing. Specifically, I focus on the correlation between relative consumption growth and fluctuation in the real exchange rate, whose puzzling correlation is known as the KBS puzzle in the literature. In a two-country model with firm heterogeneity, the correlation is analytically shown to be conditional on fluctuations in the number of product varieties and their qualities. The results indicate the existence of systematic bias in the unconditional correlation. The theoretical model is calibrated with different degrees of financial market incompleteness and measurement error with respect to the changes in

[21] For the sample of OECD countries, time fixed effects do not have a significant impact on the dependent variables based on the F test of linear restriction. This is the reason why the results with country pair fixed effects but without time fixed effects are presented in the table.
[22] The country classification due to income level and currency in use follows the one provided by the World Bank in 2018.

the number of product varieties and their qualities. I find that the observed close-to-zero or even negative KBS correlation is obtained through the wealth effect, which hinges on the Harrod-Balassa-Samuelson effect based on heterogeneous firms. Further, I test the implication of the theoretical model using world trade data and find that the KBS correlations become more positive when controlling for fluctuations in the number of product varieties and their qualities. The results show that the extent of international risk sharing is underestimated by just looking at unconditional correlations. Therefore, there are unexplored gains in international trade with respect to international risk sharing that is accounted for by firm entry and the selection of more efficient firms into exporting. In future research, assessing how these gains matter for each country would be very interesting.

CRediT authorship contribution statement

Hamano Masashige: Conceptualization, Methodology, Formal analysis, Software, Data curation, Writing – original draft, Visualization, Investigation, Validation, Writing – review & editing, Project administration, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Steady state

At the symmetric steady state, I assume, without loss of generality, that $Z = Z^* = f_E = f_E^* = z_{\min} = z_{\min}^* = 1$. In this symmetric steady state, I drop the asterisks that denote Foreign variables and the time indices. Note that $NFA = NX = 0$ and $Q = 1$ in the symmetric steady state.

With the above assumptions, first, I solve the value of f_X such that it matches the empirical findings on the share of exporters. The free-entry condition gives $\widetilde{v} = w$. Thus, using the Euler equation for share holdings, we obtain

$$\widetilde{d} = \frac{1-\beta(1-\delta)}{\beta(1-\delta)} w. \tag{17}$$

Therefore, by the definition of $\widetilde{d}$, we obtain

$$\widetilde{d}_D + \frac{N_X}{N_D}\widetilde{d}_X = \frac{1-\beta(1-\delta)}{\beta(1-\delta)} w. \tag{18}$$

Now, we rewrite $\widetilde{d}_D$ and $\widetilde{d}_X$ in the above expression. From the zero-profit export cutoff condition, we have

$$\widetilde{d}_X = wf_X \frac{\sigma-1}{k-(\sigma-1)}. \tag{19}$$

With the above expression and using the steady-state average domestic profits and exporting profits $\widetilde{d}_D$ and $\widetilde{d}_X$, $\widetilde{d}_D$ can be rewritten as

$$\widetilde{d}_D = \frac{1}{\tau^{1-\omega}} \left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} \left[\frac{\sigma-1}{k-(\sigma-1)} + 1\right] wf_X, \tag{20}$$

where we use the fact that $\widetilde{\rho}_D/\widetilde{q}_D = \frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w}{\widetilde{q}_D\widetilde{z}_D}$, $\widetilde{\rho}_X/\widetilde{q}_X = \frac{\sigma}{\sigma-1}\tau\frac{1}{1-\phi}\frac{w}{\widetilde{q}_X\widetilde{z}_X}$ and $\widetilde{q}_D = \left(\frac{\phi}{1-\phi}\widetilde{z}_D\right)^{\phi}$, $\widetilde{q}_X = \left(\frac{\phi}{1-\phi}\widetilde{z}_X\right)^{\phi}$.

Plugging (20) and (19) into (18), we obtain

$$\left[\frac{1}{\tau^{1-\omega}} \left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} \frac{k}{k-(\sigma-1)} + \frac{N_X}{N_D}\frac{\sigma-1}{k-(\sigma-1)}\right] f_X = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}.$$

In the above expression, $\widetilde{z}_D$ is given by the Pareto distribution defined in the main text. $\frac{N_X}{N_D}$ is set to 0.21. Given this value, which is also from the Pareto distribution, $\widetilde{z}_X = 2.9425$ is required with the values of the parameters in the benchmark calibration. By plugging these values into the above equation, f_X can be solved, yielding 0.0158.

Provided this subsidy, the steady-state labor supply is set to unity by controlling χ. Thus, the labor market clearing condition in the steady state yields

$$w = \left[N_E\widetilde{v} + (\sigma-1)N_D\widetilde{d} + \sigma N_X w f_X\right].$$

The equation for the motion of firms in the steady state is $N_E = \frac{\delta}{1-\delta}N_D$. Using (17) and replacing $\widetilde{v}$ as before, the above expression can be rewritten as

$$N_D = \frac{1}{\frac{\delta}{1-\delta} + (\sigma - 1)\frac{1-\beta(1-\delta)}{\beta(1-\delta)} + \sigma \frac{N_X}{N_D} f_X} \tag{21}$$

This equation gives the solution for N_D.

The second equation can be obtained using the steady-state price index as

$$\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} + \tau^{1-\omega}\left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)} = \left(\frac{N_D^{\psi}}{\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w}{\widetilde{q}_X \widetilde{z}_X}}\right)^{1-\omega}$$

By rearranging this equation, we obtain the solution for w:

$$w = \left\{ \left(N_D^{\psi}\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{1}{\widetilde{q}_X \widetilde{z}_X}\right)^{1-\omega} \left[\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} + \tau^{1-\omega}\left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)} \right] \right\}^{\frac{1}{-(1-\omega)}}.$$

Once w is found, N_D can be obtained from (21). The steady-state values of the other variables are relatively easy to determine. Specifically, the value of parameter χ is set to $\chi = wC^{-\gamma}$ such that $L = 1$, which yields 0.2174 given the parameter values of the benchmark calibration.

Finally, we define the following steady-state shares that appear in calibrating the first-order set of equations. The share of domestic and imported goods in total expenditures is

$$S_{ED} \equiv \rho_H^{1-\omega} \text{ and } 1 - S_{ED} \equiv \rho_F^{1-\omega}.$$

The steady-state share of fixed export costs and dividends on domestic, export and total sales relative to C are, respectively, defined as

$$S_{FX} \equiv \frac{N_X w f_X}{C}, S_{DD} \equiv \frac{N_D \widetilde{d}_D}{C}, S_X \equiv \frac{N_X \widetilde{d}_X}{C}, S_D \equiv \frac{N_D \widetilde{d}}{C}.$$

The steady-state share of investments, wage and consumption relative to C are, respectively, defined as

$$S_I \equiv \frac{N_E \widetilde{v}}{C}, \quad S \equiv \frac{w}{C}, \quad S_M \equiv \frac{M}{C}.$$

Appendix B. Sensitivity analysis

See Figs. 3 and 4.

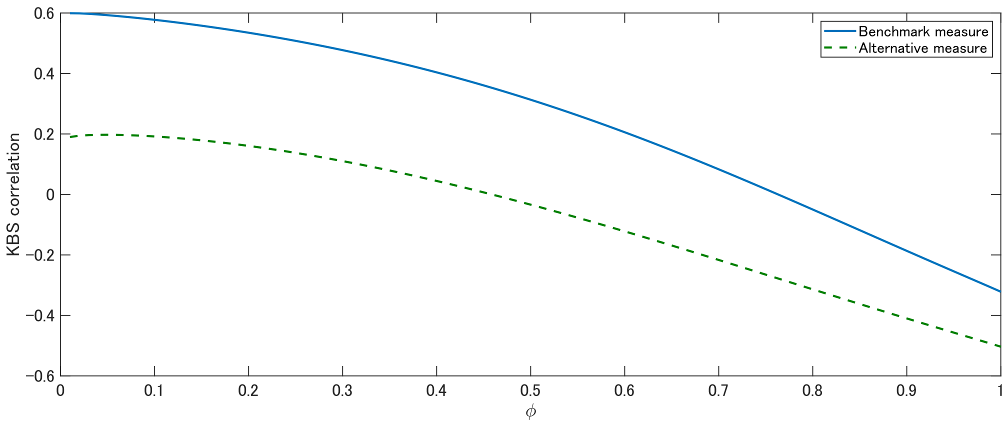

Fig. 3. The KBS Correlation and Quality Ladder (Bond Economy). Note: The figure reports the sensitivity results of the unconditional KBS correlation in the theoretical model against a quality ladder, ϕ, with the benchmark measurement error ($\lambda_1 = \lambda_2 = \lambda_3 = 1$), and the alternative measurement error ($\lambda_1 = S_{ED}$ and $\lambda_2 = \lambda_3 = 1 - S_{ED}$) obtained from the bond economy.

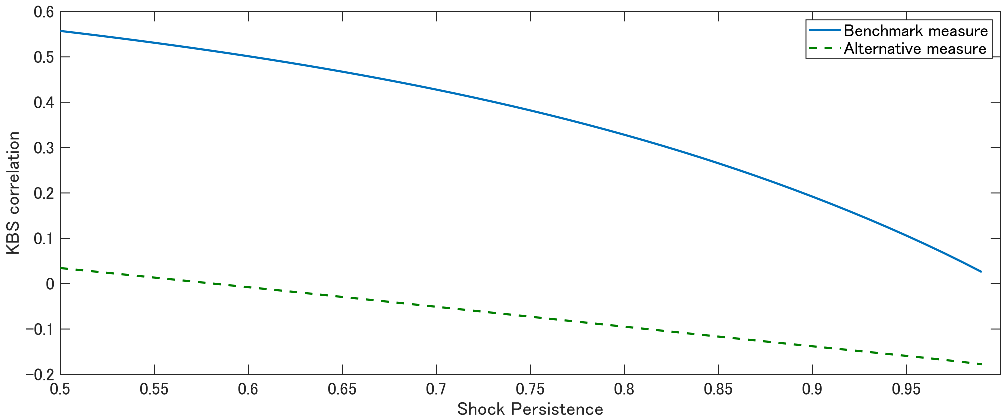

Fig. 4. The KBS Correlation and Shock Persistence (Bond Economy). Note: The figure reports the sensitivity results of the unconditional KBS correlation in the theoretical model against a shock persistence of the productivity process, Z_t, with the benchmark measurement error ($\lambda_1 = \lambda_2 = \lambda_3 = 1$) and the alternative measurement error ($\lambda_1 = S_{ED}$ and $\lambda_2 = \lambda_3 = 1 - S_{ED}$) obtained from the bond economy.

Appendix C. Data and robustness

See Table 8–10 and Figs. 5, 6.

Table 8
Descriptive statistics.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Nb of exported varieties	5,012	299.6	255.5	0	71	550	816
Quality of exported goods	4,673	1.0	0.3	0.2	0.8	1.1	8.0
Nb of imported varieties	5,012	487.6	220.8	0	340	670	845
Quality of imported goods	4,698	1.0	0.1	0.3	0.9	1.0	1.6

Source: Feenstra and Romalis (2014) and the author's calculation.

Table 9
The KBS regression: high income OECD and upper middle income vs. low and lower middle income countries.

Dep Var:	$\Delta C_t^i - \Delta C_t^j$							
	Fixed effects				Random effects			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta P_t^j - \Delta P_t^i$	-0.027^{***}	-0.019^{***}	-0.062^{***}	-0.054^{***}	-0.023^{***}	-0.014^{***}	-0.062^{***}	-0.053^{***}
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
$\Delta N_t^i - \Delta N_t^j$		-0.036^{***}		-0.035^{***}		-0.032^{***}		-0.029^{***}
		(0.003)		(0.003)		(0.003)		(0.003)
$\Delta q_t^i - \Delta q_t^j$		-0.004		-0.002		-0.009^{***}		-0.008^{***}
		(0.003)		(0.003)		(0.003)		(0.003)
$\Delta Y_t^i - \Delta Y_t^j$			0.236^{***}	0.228^{***}			0.254^{***}	0.251^{***}
			(0.006)	(0.006)			(0.006)	(0.006)
Observations	126,635	116,817	126,635	116,817	126,635	116,817	126,635	116,817
R^2	0.0004	0.002	0.014	0.014	0.0003	0.001	0.017	0.017
Adjusted R^2	−0.040	−0.041	−0.026	−0.028	0.0003	0.001	0.017	0.016

Note: $\Delta C_t^i - \Delta C_t^j$, $\Delta P_t^j - \Delta P_t^i$, $\Delta N_t^i - \Delta N_t^j$, $\Delta q_t^i - \Delta q_t^j$ and $\Delta Y_t^i - \Delta Y_t^j$ represent the growth rate of consumption, the real exchange rate, the number of traded varieties, the quality of traded products, and the income for country i with respect to country j. Standard errors are reported in parentheses. ***, ** and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 10
The KBS regression: euro countries.

Dep Var:	$\Delta C_t^i - \Delta C_t^j$ Fixed effects				Random effects			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta P_t^j - \Delta P_t^i$	−0.247***	−0.026	−0.139***	−0.033*	−0.242***	−0.031	−0.136***	−0.039**
	(0.021)	(0.021)	(0.019)	(0.020)	(0.021)	(0.020)	(0.018)	(0.019)
$\Delta N_t^i - \Delta N_t^j$		0.021		0.028		0.036		0.047*
		(0.028)		(0.027)		(0.028)		(0.026)
$\Delta q_t^i - \Delta q_t^j$		0.048***		0.022*		0.050***		0.022**
		(0.012)		(0.011)		(0.011)		(0.011)
$\Delta Y_t^i - \Delta Y_t^j$			0.787***	0.490***			0.783***	0.491***
			(0.021)	(0.027)			(0.021)	(0.026)
Observations	4,137	3,484	4,137	3,484	4,137	3,484	4,137	3,484
R^2	0.032	0.006	0.280	0.098	0.031	0.007	0.281	0.101
Adjusted R^2	−0.009	−0.043	0.249	0.054	0.031	0.006	0.281	0.100

Note: $\Delta C_t^i - \Delta C_t^j$, $\Delta P_t^j - \Delta P_t^i$, $\Delta N_t^i - \Delta N_t^j$, $\Delta q_t^i - \Delta q_t^j$ and $\Delta Y_t^i - \Delta Y_t^j$ represent the growth rate of consumption, the real exchange rate, the number of traded varieties, the quality of traded products, and the income for country *i* with respect to country *j*. Standard errors are reported in parentheses. ***, ** and * indicate significance at the 1%, 5%, and 10% levels, respectively.

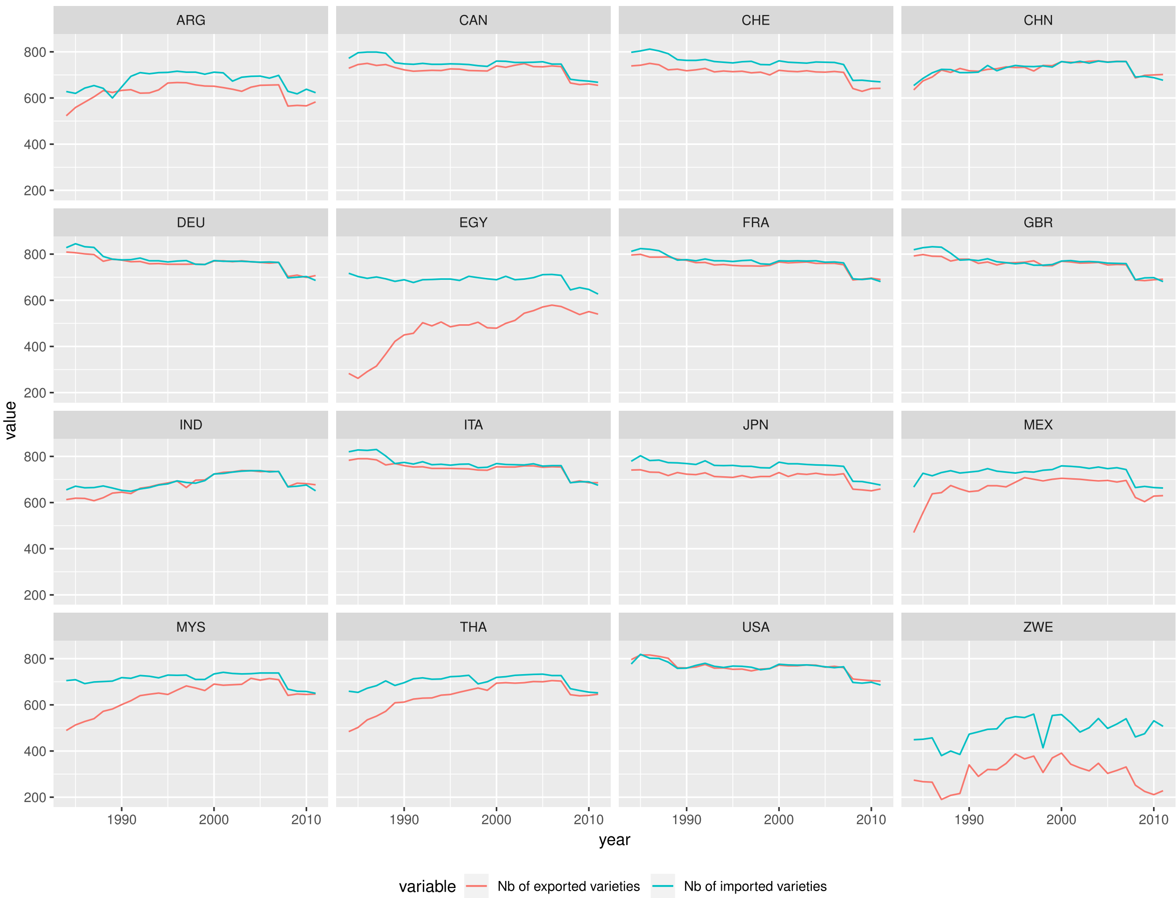

Fig. 5. Evolution of the Number of Exported and Imported Varieties. Note: Evolution of the number of exported and imported varieties of the selected countries from 1984 to 2011. Source: Feenstra and Romalis (2014) and the author's calculation.

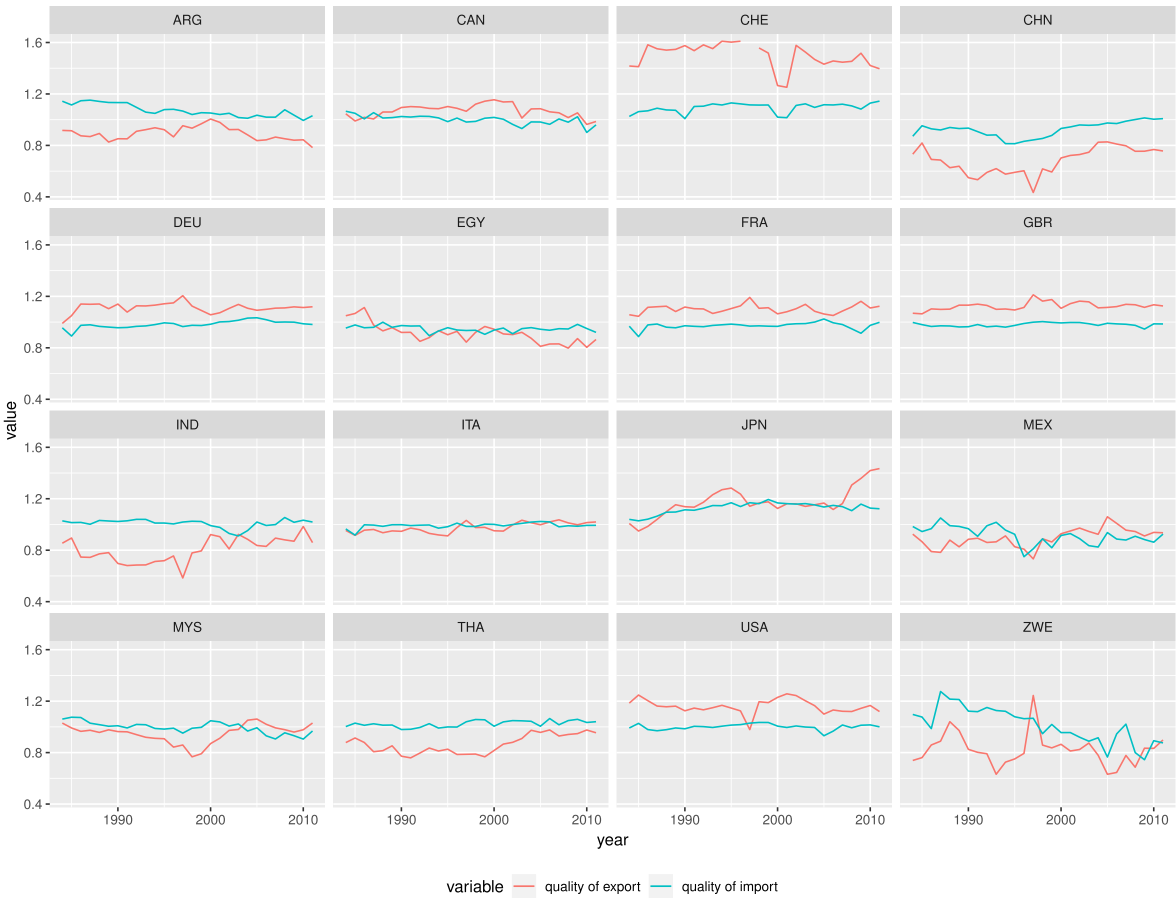

Fig. 6. Evolution of Exported and Imported Quality. Note: Evolution of the number of exported and imported varieties of the selected countries from 1984 to 2011. Source: Feenstra and Romalis (2014) and the author's calculation.

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