Admittance-Based Stability Analysis: Bode Plots, Nyquist Diagrams or Eigenvalue Analysis?

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***Abstract*—In the literature, Bode plots and Nyquist diagrams are used extensively in stability analysis for admittance-based models. In this letter, we demonstrate that eigenvalues obtained from the total admittance lead to accurate prediction of stability while Bode plots and Nyquist diagrams show limitations. A voltage source converter with weak grid interconnection is used as an illustrative example to demonstrate the pros and cons of the three methods applied to dq-frame admittance matrices.**

I. Introduction

FREQUENCY-domain methods (Bode plots and Nyquist diagrams) are used extensively in stability analysis for impedance- or admittance-based systems, e.g., [1], [2]. Alternatively, closed-loop system eigenvalues can also be obtained from s-domain admittance matrix [3]. The latter, however, is adopted in few literature. In this letter, we demonstrate that eigenvalues lead to accurate prediction of stability while Bode stability criterion has theoretic limitations and Nyquist criterion has implementation limitations. A voltage source converter (VSC) with weak grid interconnection is used as an illustrative example to demonstrate the two analysis methods.

Section II presents a nutshell of the two types of stability analysis for a system represented by two admittances. Section III presents the example system stability analysis via eigenvalues, Bode plots, and Nyquist diagram. Concluding remarks are also presented in Section III.

II. Admittance Model-based Small-Signal Analysis in A Nutshell

We use a simple system with two impedances to illustrate the stability criterion. The example system in Fig. 1 represents a converter connected to a grid. The converter is modeled in Norton representation as a current source $i_c(s)$ in parallel with an admittance $Y_{\rm conv}(s)$. Its impedance is notated as $Z_{\rm conv}$ and $Z_{\rm conv} = Y_{\rm conv}^{-1}$. The grid interconnection is represented as a Thevenin equivalent, a voltage source $v_g(s)$ behind an impedance $Z_g(s)$. The current flowing into the converter can be derived as follows.

$$i(s) = (I + Y_{\rm conv} Z_g)^{-1} (Y_{\rm conv} v_g(s) - i_c(s)) \tag{1}$$

where I is the identity matrix. For the above system, two assumptions are made. (i) The grid voltage $v_g(s)$ is stable; (ii) the system is stable when the grid impedance Z_g is zero, i.e., $Y_{\rm conv} v_g(s) - i_c(s)$ is stable. The first assumption is valid for the real-world scenarios as long as the grid voltage is within the limits. The second assumption is valid as long as the inverter converter admittance $Y_{\rm conv}$ is stable and current order i_c is stable. For properly designed converters, the second assumption is also true.

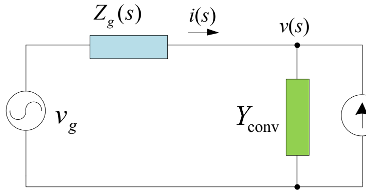

Fig. 1: Impedance model of a converter connected to a grid.

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Therefore, for the current $i(s)$ to be stable, we only need to examine $(I + Y_{\rm conv} Z_g)^{-1}$. This circuit analysis problem may be treated as a feedback system, as shown in Fig. 2, where the input u is $(Y_{\rm conv} v_g(s) - i_c(s))$ and the output y is $i(s)$.

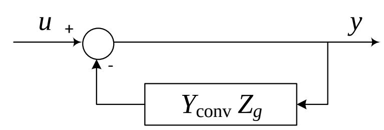

Fig. 2: Circuit analysis problem is converted to a feedback control problem.

Similarly, we may formulate a nodal equation for analysis:

$$v(s) = (Y_g + Y_{\rm conv})^{-1} (Y_g v_g(s) + i_c(s)) \tag{2}$$

Stability Criterion 1: Roots of the characteristic polynomial located in the LHP. The system's stability is guaranteed if the characteristic polynomial has no zero in the right half plane (RHP) [4], or $\det(I + Y_{\rm conv} Z_g) = 0$ should have all roots in the LHP.

Since $\det(I + Y_{\rm conv} Z_g) = \det(Z_g)\det(Y_g + Y_{\rm conv})$, with the assumption that Z_g and Y_g are stable, the stability criterion is equivalent to the following: the roots of $\det(Y)$ should all be located in the LHP, where Y is the total admittance and $Y = Y_g + Y_{\rm conv}$.

Stability Criterion 2: Generalized Nyquist Stability Criterion or Bode Plots More often, open-loop analysis is used for analysis. For scalar admittances or impedances, the Root-Locus method and Nyquist Criterion can be applied. For 2×2 admittances, Generalized Nyquist Criterion developed in 1970 [5] has been popularly used in the field of frequency-domain analysis for machines and converters, e.g., [6], [7]. Stability can be examined by checking the eigen loci or the Nyquist plots of the eigenvalues of open-loop gain $Y_{\rm conv} Z_g$ or $Z_g Y_{\rm conv}$. If the eigen loci do not encircle $(-1, 0)$, then the system is stable. In Bode plots, when phase shift happens, the gain should be less than 1 for a stable system. It has to be noted that

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the Bode stability criterion is a sufficient, but not necessary, condition for instability. In addition, Bode stability criterion applies to stable open-loop systems and it does not apply to those open-loop systems with RHP zeros, or non-minimum phase systems.

III. An Example: VSC in weak grids

In this section, we illustrate the two approaches of stability analysis using an example: a grid-following voltage source converter (VSC) with weak grid interconnection.

The grid-following VSC is assumed to achieve two control functions: regulate the dc-link voltage and regulate the point of common coupling (PCC) voltage. The admittance of the VSC viewed from the PCC bus is desired. To find the admittance, the integration system is constructed to have the PCC bus connected to the grid voltage source through a very small impedance (R_g and L_g). The topology is shown in Fig. 3.

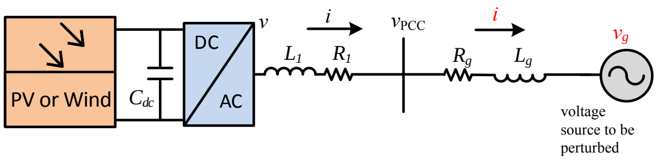

Fig. 3: Grid-connected VSC circuit diagram.

The analytical model of the system is constructed in the dq-frame. This model has been used for stability analysis related to VSC in weak grids [8]–[11]. The full details of the modeling blocks are shown in Fig. 6(a). Using numerical perturbation (e.g., MATLAB command `linmod`), lineraized model at an operating condition can be found. An input/output linearized model is found with the dq-axis voltages as input and the dq-axis currents as output, shown in the following.

$$\begin{bmatrix} i_d(s) \\ i_q(s) \end{bmatrix} = -\underbrace{\begin{bmatrix} Y_{dd}(s) & Y_{dq}(s) \\ Y_{qd}(s) & Y_{qq}(s) \end{bmatrix}}_{Y_{\rm conv}} \begin{bmatrix} v_{gd}(s) \\ v_{gq}(s) \end{bmatrix} \tag{3}$$

Fig. 6(b) presents the frequency-domain responses of the admittance matrix viewed at a bus location close to the PCC bus. The two buses are connected by a small impedance at $0.01 + j0.1$ pu.

The analytical model shown in Fig. 6 has been derived in our prior work [8]–[11] and can be used to demonstrate low-frequency oscillations. Fig. 4 presents the system response subject to a small step change in dc-link voltage order when the grid is weak or transmission line reactance assumes a large value ($X_g = 0.8$ pu). Note that the system is stable when X_g is less than 0.8 pu.

a) Eigenvalue Analysis: We now approach to view the system as a one-node system with two shunt admittances: $Y_{\rm conv}$ and Y_g. The node is the PCC bus. The total admittance viewed at the PCC bus is as follows: $Y = Y_{\rm conv} + Y_g$, where $Y_g = Z_g^{-1}$ and $Z_g = \begin{bmatrix} R_g + sL_g & -\omega_0 L_g \\ \omega_0 L_g & R_g + sL_g \end{bmatrix}$ and ω_0 is the nominal frequency. In this example, ω_0 is 377 rad/s.

The system's eigenvalues are the roots of $\det(Y)$. We use Matlab function `tzero(Y)` to find the eigenvalues. The system has a total 9 eigenvalues. Six right most eigenvalues

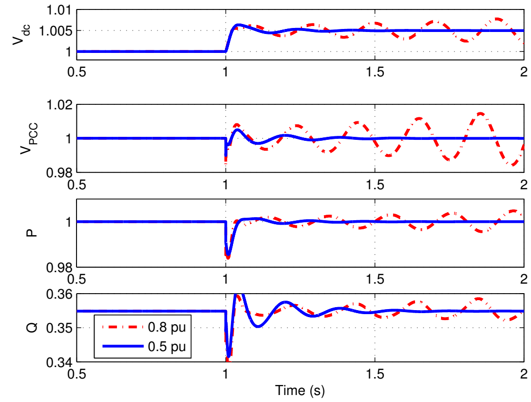

Fig. 4: Dynamic responses of the VSC grid integration system subject to 0.5% change in dc-link voltage order. Dash dotted line: $R_g = 0.08$ pu, $X_g = 0.8$ pu. The system is unstable with growing 5 Hz oscillations. Solid line: $R_g = 0.05$ pu, $X_g = 0.5$ pu. The system is stable.

at two different operating conditions ($X_g = 0.5$ pu versus $X_g = 0.8$ pu) are plotted in Fig. 5. One pair of eigenvalues with frequency of 5 Hz move to the RHP when X_g is 0.8 pu.

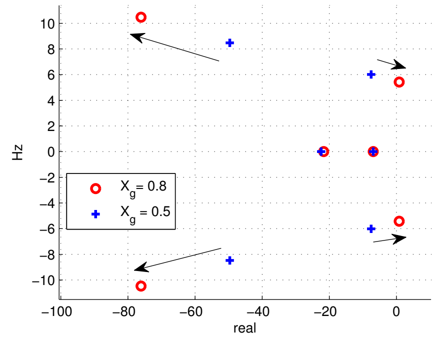

Fig. 5: Eigenvalues obtained using admittance-based approach. The arrows notate eigenvalue loci when X_g increases. The system is stable when X_g is 0.5 pu and unstable when X_g is 0.8 pu.

It can be seen that the eigenvalue analysis results correctly indicate the system is unstable with a pair of eigenvalues located in the RHP when $X_g = 0.8$ pu.

b) Bode plots and Nyquist Diagrams: Bode plots and Nyquist diagrams based on the eigenvalues of $Y_{\rm conv}Z_g$ at every frequency are also plotted. In Bode plots (Fig. 7), two different colors notate two different eigenvalues computed at every frequency. The phase shifting points are examined. It is found that one eigenvalue loci has phase shift at 5.4 Hz for both cases ($X_g = 0.5$ pu and $X_g = 0.8$ pu). The corresponding open-loop gain magnitudes are all greater than 1. This indicates that both systems are unstable. This conclusion is obviously not correct since the simulation results indicate that the system is stable when X_g is 0.5 pu.

This example indicates that Bode plots have limited application scope. It is well-known that Bode plots cannot be used for non-minimum phase system for stability analysis.

It is indicated in [12] that the Bode stability criterion cannot be used to make definitive statement about the system stability.

Fig. 8 gives Nyquist plots of the open-loop gain from -100 Hz to -1 Hz and from 1 Hz to 100 Hz. Stability is indicated by if $(-1, 0)$ is encircled clockwise if the open-loop system is

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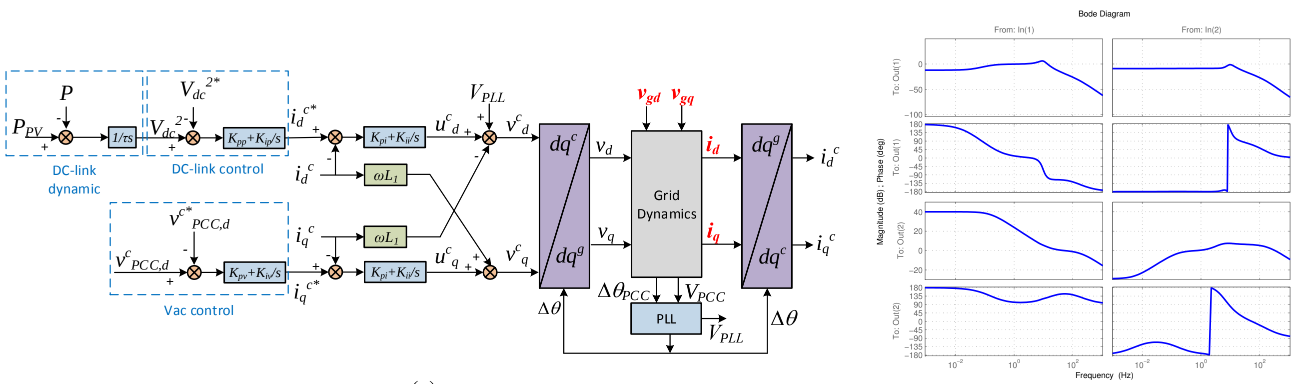
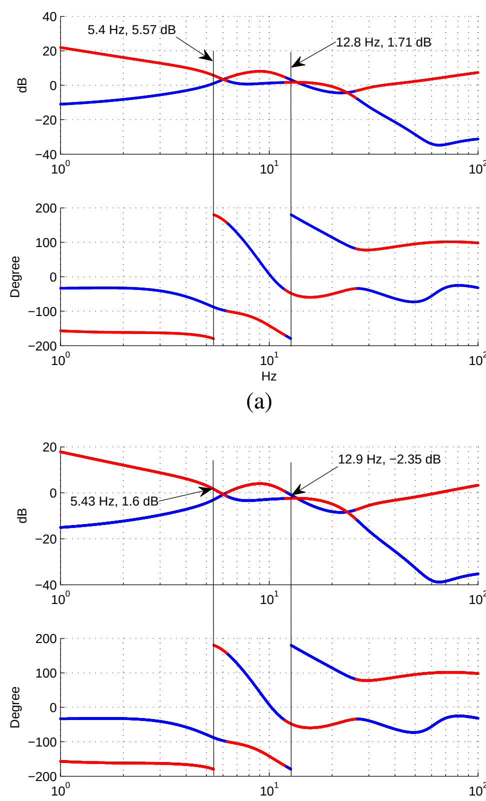


(a) (b)

Fig. 6: Block diagram of the analytical model and its admittance frequency-domain responses. Parameters of the test case in pu: $(R_g, X_g) : (0.001, 0.1)$, $(R_L, X_L) : (0.003, 0.15)$, inner current PI controller: $(K_{pi}, K_{ii}) : (0.3, 5)$, outer loop PI parameters: $(1, 100)$, second-order PLL: $(K_{p,\text{PLL}}, K_{i,\text{PLL}}) = (60, 1400)$. Dc-link capacitor time constant τ: 0.0272 s.



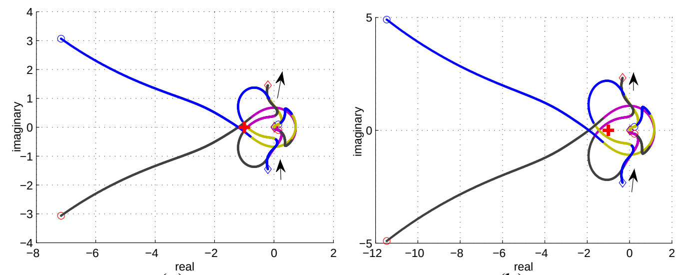
(a)

(b)

Fig. 7: Bode plots of the system at two X_g conditions. (a) $X_g = 0.5$ pu. (b) $X_g = 0.8$ pu. Bode plots indicate the system is unstable under two conditions.

has no zero in the RHP. The number of encirclement should be counted otherwise. This poses difficulty since (i) zeros of the two single-input single-output (SISO) open-loop systems based on the eigenvalues of the 2×2 matrix: $Y_{\text{conv}} Z_g$ are not straightforward to be computed; and (ii) it is difficult to examine encirclement visually. Thus, Nyquist diagram is not straightforward for stability check.

Concluding Remarks: In this letter, we use the VSC with weak grid interconnection example to demonstrate that eigenvalues computed based on a total s-domain admittance can provide accurate stability analysis. We also show the limitations of the frequency-domain approaches: Bode plots cannot



(a) (b)

Fig. 8: Nyquist diagrams of the eigenvalues of $Y_{\text{conv}} Z_g$. (a) $X_g = 0.5$ pu. (b) $X_g = 0.8$ pu.

predict stability analysis accurately while Nyquist diagrams are not straightforward for stability check.

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