Multi-Objective PMU Allocation for Resilient Power System Monitoring

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***Abstract*—Phasor measurement units (PMUs) enable better system monitoring and security enhancement in smart grids. In order to enhance power system resilience against outages and blackouts caused by extreme weather events or man-made attacks, it remains a major challenge to determine the optimal number and location of PMUs. In this paper, a multi-objective resilient PMU placement (MORPP) problem is formulated, and solved by a modified Teaching-Learning-Based Optimization (MO-TLBO) algorithm. Three objectives are considered in the MORPP problem, minimizing the number of PMUs, maximizing the system observability, and minimizing the voltage stability index. The effectiveness of the proposed method is validated through testing on IEEE 14-bus, 30-bus, and 118-bus test systems. The advantage of the MO-TLBO-based MORPP is demonstrated through the comparison with other methods in the literature, in terms of iteration number, optimality and time of convergence.**

***Index Terms*—Multi-Objective Optimization, Phasor Measurement Unit, Power System Resilience, Teaching-Learning Based algorithm, Voltage Stability**

I. INTRODUCTION

The traditional power grids are in the process of transforming into smart grids with inclusion of intelligent devices. However, the integration of these digitalized devices has driven power networks more complex and vulnerable to cyber-physical-human (CPH) threats [1]. Among these devices, phasor measurement units (PMUs) can provide system operators a set of synchronized phasor measurements for better system monitoring [2]. As the installation of PMUs in power grids is still an expensive investment, it is not cost effective to install PMUs at each bus in order to achieve accurate monitoring and reach to the full observability [3]. Therefore, how to allocate PMUs in power systems in order to achieve full system observability in the case of any disruption is still a preeminent challenge.

Several research efforts have been dedicated in the optimal PMU placement (OPP) problem, including different optimization problem formulation and solution algorithms. Authors of [4] implement a minimum spinning tree-genetic algorithm (MST-GA) to solve the OPP problem considering the maximum redundancy. In [5], a binary imperialistic algorithm (BIA) is implemented by considering the impact of zero injection buses (ZIBs) on bulk power systems. Authors in [6] solve the OPP problem with binary particle swarm (BPSO) algorithm by considering the measurement contingencies such as line outage or PMU failure. Authors in [7] solve the OPP problem with the objective of maximizing the observability via binary harmony search algorithm (BHSA).

Moreover, different objectives and indices are considered simultaneously to form the multi-objective PMU placement (MOPP) problem. In [8], a multi-objective OPP problem is investigated based on the cellular learning automata (CLA) algorithm considering the minimum number of PMUs and maximizing the observability. Authors in [9] propose a multi-objective framework for enhancing the reliability and minimizing the cost of PMU deployment in power systems via NSGA-II algorithm. In [10], a multi-objective OPP in power systems is studied by implementing the gravitational search algorithm, considering four cases of PMU loss, line outage, ZIB, and normal PMU placement. Furthermore, PMU placement problem is connected with other functions in power system operation and control. For example, a new integer linear programming method is applied to OPP problem in [2] and [11] for minimizing the cost of installation for robust static state estimation. A voltage stability constrained optimal simultaneous PMU placement with the goal of enhancing the measurement reliability is presented in [12]. Authors in [13] propose a voltage stability-based PMU placement based on the active power and the angle differences.

Considering the above literature, determination of the appropriate number and location of PMUs has raised the concern of fully system monitoring in the case of any disruption, which is a major challenge from the perspective of power system resilience enhancement (future preparedness). Especially, any PMU loss or line outages could result in voltage instability. This paper aims to formulate and solve the multi-objective PMU placement consisting of minimizing the number of PMUs, maximizing the observability redundancy with the impact of phasing installation, and the voltage stability, in order to provide a more resilient PMU placement (RPP). Additionally, a modified teaching-learning-based optimization (MO-TLBO) algorithm is applied to solve the single and multi-objective PMU placement problem. The advantages of this method is to achieve optimal solutions with less iteration numbers and less computational time compared to existing methods, and its validity is tested on different case studies.

The rest of the paper is organized as follows. The mathematical formulation of multi-objective RPP is presented in Section II, including the quantification of observability and voltage stability index, and different impact factors in MORPP. The solution algorithm of MO-TLBO is introduced in Section III. Section IV presents the simulation results and analysis. Finally, section V concludes the paper.

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II. MULTI-OBJECTIVE PROBLEM FORMULATION

A. Minimizing the Number of PMUs

The first objective of MORPP is to determine the minimum number of PMUs with optimal locations to ensure the system observability. The integer linear programming (ILP) formulation is defined as below [14]:

$$\min OF_1 = \sum_{j=1}^{\mathrm{N_{bus}}} x_j \qquad \forall j \subseteq N_{Bus} \tag{1a}$$

$$f_i = \sum_{j=1}^{\mathrm{N_{bus}}} x_j \,.\, a_{ij} \geq 1 \qquad \forall i, j \subseteq N_{Bus} \tag{1b}$$

where x_j is a binary variable, which equals to 1 if bus j is equipped with PMU, otherwise equals to zero. N_{bus} is the set of all network buses. The observability constraint f_i consists of two terms x_j and a_{ij}, which a_{ij} is the bus connectivity parameter and can be defined as below:

$$a_{ij} = \begin{cases} 1, & \text{if } i = \text{j} \\ 1, & \text{if buses } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

The aforementioned objective function can be extended to the cost function, by changing the term x_j to $w_j x_j$. w_j is the installation cost at bus j, and defined as $w_j = (1 + 0.1 * n)C$, where n is the number of measurement channels and C is the cost of each PMU [15].

B. Maximizing the Observability

One major concern in the PMU allocation problem is to achieve the maximum observability of the entire system even in the case of any disruption. Therefore, the second objective function of MORPP is to maximize the observability. The mathematical formulation is shown as below:

$$\max OF_2 = \sum_{j=1}^{\mathrm{N_{bus}}} f_i \qquad \forall i, j \subseteq N_{Bus} \tag{3a}$$

$$f_i = \sum_{j=1}^{\mathrm{N_{bus}}} x_j \,.\, a_{ij} \geq 1 \tag{3b}$$

$$f_i \leq \text{Maximum Connectivity of Bus } i \tag{3c}$$

$$\sum_{j=1}^{\mathrm{N_{bus}}} x_j \leq TN_{PMU} \qquad \forall j \subseteq N_{Bus} \tag{3d}$$

where OF_2 defines the system observability redundancy, TN_{PMU} is the total number of PMUs based on the availability. Equation (3c) demonstrates the constraint of maximum line connectivity of bus i. Additionally, Equation (3d) is to consider that system operators have to install a limited number of PMUs in power system during multiple time stages, due to economic reasons, or communication limitations, which is called phasing installation [16]. In this paper, the impact of phasing installation on observability is also considered.

C. Minimizing the Voltage Stability

1) Objective Function: Voltage stability is one major concern in power system operation. It can be defined as the ability of a power system to keep the voltage of all nodes in the acceptable range under normal condition or in the case of any disruptions. In this paper, the third objective function is defined as minimizing the voltage stability index-based PMU placement, as formulated in the following [17]:

$$\min OF_3 = \sum_{i \; and \; j=1}^{\mathrm{N_{bus}}} VSI_{ij} \,.\, x_j \qquad \forall i, j \subseteq N_{Bus} \tag{4a}$$

$$2 \leq VSOI \leq MNB + 1 \tag{4b}$$

where VSI_{ij} is the voltage stability index with the maximum value of 1, and MNB is the maximum number of branches connected to each bus. If the VSI value is near to the unity, it represents that the line is under stressed condition and needs to be relieved. Otherwise, with a small increase in load, which consequently a change in reactive power, the line may be tripped. On the other hand, if its value is small, it shows that the line works properly. The corresponding line is regarded as a critical line. PMUs should be placed at a bus in order to monitor the critical line that has a higher value of VSI. Therefore, each of the buses that have higher VSI should be monitored by at least 2 PMUs. Accordingly, the voltage stability observability index (VSOI) can be formulated as equation (4b). The whole system observability index (WSOI) will be defined as the summation values of $VSOI$ and f_i. Voltage stability will be considered in PMU placement problems using the index developed as below.

2) Voltage Stability Index: In this paper, based on the π model of transmission lines that connect two buses with one line, the voltage stability index (VSI) can be defined as [18]:

$$VSI_{mn} = \frac{4\left(Z_{mn}\right)^2 Q_n}{(V_m)^2 X_{mn}} \leq 1 \tag{5}$$

where Z_{mn} and X_{mn} are the line impedance and reactance between buses m and n. V_m is voltage at sending bus, and Q_n is reactive power at receiving bus. The calculation procedure of VSI follows three steps:

Step 1: The VSI of each line is calculated based on the results of optimal power flow using equation (5).

Step 2: For any line, increase the reactive power absorption at receiving bus until the VSI value getting close to unity. If the value goes beyond 1, one of the buses connected to the line will experience a voltage drop and cause the system collapse.

Step 3: Obtain the maximum value of VSI at each line, and rank the maximum loadability in the ascending order. The smallest maximum-loadability is ranked the highest, which indicates buses should be monitored more than one PMU.

D. Impact Factors in PMU Allocation Problem

For enhancing power system monitoring in the case of any disruption, and accordingly improve system resilience, different impact factors are considered in this paper. These factors are impact of ZIBs, contingencies such as line and PMU outage.

1) Impact of ZIBs: To consider the impact of zero-injection buses on RPP, an auxiliary binary parameter s is defined, which equals to 1 if it is a zero-injection bus; otherwise 0. In the formulation of the first objective, equation (1b) should be replaced by the following equation:

$$f_i = \sum_{j=1}^{\mathrm{N_{bus}}} x_j \,.\, a_{ij} \;+\; \sum_{j=1}^{\mathrm{N_{bus}}} s_j \,.\, a_{ij} \,.\, y_{ij} \geq 1 \qquad \forall j \subseteq N_{Bus} \tag{6a}$$

$$s_j = \sum_{j=1}^{\mathrm{N_{bus}}} a_{ij} \,.\, y_{ij} \tag{6b}$$

where s_j is a parameter related to ZIB, and y_{ij} is an auxiliary variable of buses i and j. For each non-ZIB, the number of auxiliary variables is equal to the number of ZIBs connected to that bus. And for each ZIB, the number of auxiliary variables is equal to the number of ZIBs connected to that bus plus one.

2) Impact of Measurement Outage: Power system may experience a contingency, due to equipment failure, natural disasters, etc. One of the major contingencies could be the loss of measurement. To achieve a resilient observable system against PMU losses, each bus must be observed by at least two PMUs to ensure the observability of adjacent lines. In the MOPP formulation, constraint (1b) will be modified as below:

$$f_i \;+\; s_j \geq 2 \qquad \forall j \subseteq N_{Bus} \tag{7a}$$

$$s_j = \sum_{j=1}^{\mathrm{N_{bus}}} a_{ij} \,.\, y_{ij} \tag{7b}$$

3) Impact of Line Outage: Compared to the single PMU loss condition, another common contingency is the single line outage. The impact of single line outage can be added to the MOPP formulation by modifying equations (1b) to (8a) and (8b) as below:

$$f_i^k = \sum_{j=1}^{\mathrm{N_{bus}}} x_j \,.\, a_{ij}^k \;+\; \sum_{j=1}^{\mathrm{N_{bus}}} s_j \,.\, a_{ij}^k \,.\, y_{ij}^k \geq 1 \qquad \forall j \subseteq N_{Bus} \tag{8a}$$

$$s_j = \sum_{j=1}^{\mathrm{N_{bus}}} a_{ij}^k \,.\, y_{ij}^k \tag{8b}$$

where k is the line index. In addition, the binary connectivity parameter when line k is out, a_{ij}^k, is defined as a_{ij} if buse and j are connected with line k; otherwise 0.

III. Solution Algorithm

There are many nature inspired optimization algorithn such as genetic algorithm (GA), artificial bee colony (ABC particle swarm optimization (PSO), artificial neural networ etc. These algorithms suffer from issues such as difficulty determining the optimal controlling parameters like populati size, crossover rate and mutation rate, etc. Therefore, a chan in the algorithm parameters greatly impacts the effectivene of the algorithm. However, based on the comparison in [19], the advantage of TLBO algorithm is highly recognized, which obtains global optimal solutions with high consistency and less computational effort, as well as without having any issues mentioned above. In this paper, a modified TLBO algorithm is used for solving single and multi-objective OPP by considering different scenarios.

In this study, considering the MO-TLBO algorithm, the students are divided into various groups of learners. Corresponding to the knowledge level of students, different teachers are assigned to these groups, in which teachers try to enhance their own students' knowledge based on their adaptive teaching factors. In the learning phase, the knowledge of students in each group will be updated using the knowledge of other students in the same group as well as by self-learning. Then the major goal of MO-TLBO is to maximize students' knowledge towards teachers' knowledge based on different teaching and self-learning factors [20].

IV. Simulation Results

The performance and optimality analysis of the MO-TLBO algorithm on the RPP is validated by testing on different IEEE test cases with various impact factors. Finally, the results for multi-objective PMU placement considering all three objective functions is presented.

A. Performance Analysis of Solution Algorithm

In literature, IEEE 14-bus test system without considering impact of ZIBs is a benchmark for testing PMU placement algorithms. The comparison of different solution algorithms, in terms of convergence, is shown in Fig. 1. It can be seen that MO-TLBO has the advantage of achieving the same optimal solution (e.g. 4 PMUs) in much less iteration number. All other algorithms, including GA, PSO, ABC, hybrid GA and spanning tree method (STM) and binary imperialistic competition algorithm (BICA) need more iterations, while MO-TLBO achieves the convergence after 5 iterations.

B. Optimality Analysis through Comparisons

In order to analyze the optimality of MO-TLBO algorithm, Table I summarizes the results of minimizing the number of PMUs considering the ZIBs, using different methods. Additionally, the best locations to equip PMUs considering ZIBs are also shown in Table II. Total nine different solution algorithms are tested against IEEE 14-, 30- and 118-bus systems. It is shown that the MO-TLBO algorithm achieves the optimal solutions with the fastest convergence speed.

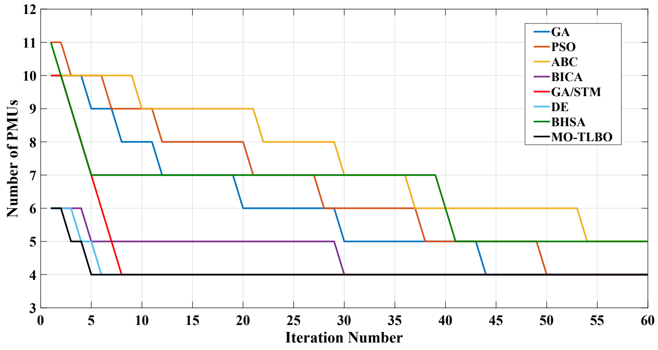

Fig. 1. Convergence comparison of different algorithms on IEEE 14-bus test system.

TABLE I: Comparison of Total Number of PMUs Using Different Algorithms Considering ZIBs

Algorithm	IEEE 14-bus	IEEE 30-bus	IEEE 118-bus
ILP [14]	3	-	29
ABC [21]	3	8	-
PSO [6]	3	7	29
BHSA [7]	3	8	-
BICA [5]	3	7	28
IGA [22]	3	7	29
GA/STM [23]	3	-	29
MO-TLBO	3	7	28

TABLE II: Results of Minimizing PMUs Numbers With ZIBs

Test system	Considering ZIBs
IEEE 14 Bus	2,6,9
IEEE 30 Bus	1,7,10,12,18,23,27
IEEE 118 Bus	2,9,11,13,17,21,25,28,34,37,40,45,49,53,56,62,72,75,77 80,85,86,90,94,102,105,110,114

C. Results of the Impact of Measurement Loss

In the case of any PMU loss, all other lines should be monitored by at least one secondary PMU in the neighborhood. This redundancy requirement will lead to locate as many PMUs as possible to address system resilience from the monitoring perspective. Table III presents the results of minimizing the number of PMUs in the case of any measurement loss. Compared to the base case with ZIBs, more PMUs are installed to monitor the system and maintain the system observable in the case of any measurement loss, including 4, 5, and 33 more PMUs for IEEE 14-, 30- and 118-bus systems, respectively.

D. Results of the Impact of Line Outage

Table IV summarizes the results of minimizing the number of PMUs in the case of line outage. Taking IEEE 30-bus test system as an example, 13 PMUs are needed to observe the system with line outage. Compared to the base case, 6 more PMUs are added to the system to maintain the observability. Moreover, as the impact of line outage is less severe than the impact of measurement loss, it requires less number of extra PMUs for the line outage, as shown in Table III and Table IV.

E. Results of the Impact of Phasing on Observability

Considering the second objective function, the results are presented in Table V and Fig. 2. Take IEEE 118-bus system as an example, as shown in Fig. 2, it
network planner has limited budget to in
network through 3 time stages. At the firs
installed to enable total 72 buses observ
approximately 61% observability of the
second stage, with additional 13 PMUs
buses can be observable to achieve abo
of the system. Finally in the last stage, v
installed, all 118 buses are observable wit
of the system. The phasing installation en
to install PMUs in critical areas requir
Consequently, it assists planners to evalu
dictions of installing PMUs to reach maximum observability at each time stage.

TABLE III: Results of Impact of Measurement Loss

Test System	The Impact of Measurement Loss
IEEE 14 Bus	1,4,5,6,9,10,13
IEEE 30 Bus	1,3,4,7,10,12,15,16,18,19,20,24,27,29,30
IEEE 118 Bus	1,3,6,8,9,11,12,15,17,19,20,21,23,26,27,28,29,32,34,35,40 42,44,45,46,49,51,52,54,56,57,59,62,66,68,70,71,75,76,77 78,80,83,85,86,87,89,91,92,94,96,100,101,105,106,108,110 111,112,115,117

TABLE IV: Results of Impact of Line Outage

Test System	The Impact of Line Outage
IEEE 14 Bus	1,3,6,10,11,13
IEEE 30 Bus	1,3,4,10,12,13,15,16,17,19,23,26,29
IEEE 118 Bus	1,6,10,11,12,15,17,19,21,23,24,25,27,29,32,34,35,40,42,44 46,49,51,53,56,5,59,62,63,70,73,75,76,78,80,83,85,87,89 91,92,94,96,100,102,105,106,109,111,112,115,166,117

F. Results of System Redundancy

The redundancy values of system observability for each of three IEEE test cases are presented in Table VI. Considering the benchmark, the system redundancy value is lower than the second scenario considering ZIBs. The reason is ZIBs improve the system observability, but prevent from any additional redundant measures from PMUs. For the scenario of line outage and PMU loss, the system observability redundancy value is greater than first two scenarios. The reason is that in the case of any disruption, each of the PMUs should take the responsibility of observing the neighboring lines. As a result, each PMU measures more than one time, and consequently the system observability redundancy increases.

G. Results of Multi-Objective PMU Placement

Due to the page limit, the results of voltage stability analysis is only presented for IEEE 14-bus system in Table VII. The

TABLE V: Impact of Phasing in IEEE 14-Bus System

TN_{PMU}	Bus Location	Observability (%)
1	6	0.4286%
2	6,9	0.7143%
3	2,6,9	0.9286%
4	2,6,7,9	100%

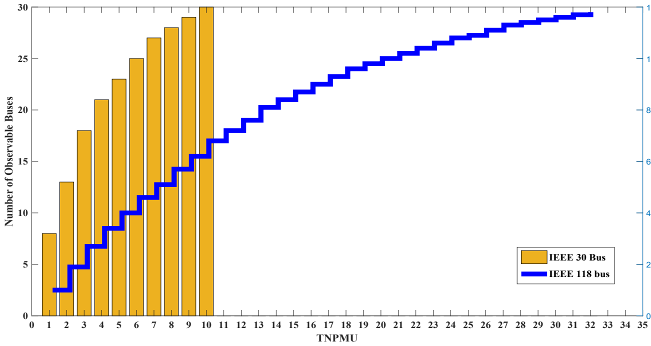

Fig. 2. The effect of Phasing in OPP for different case studies.

TABLE VI: System Redundancy in Different Scenarios

Test System	Ignoring ZIB	Considering ZIB	Line Outage	PMU Loss
IEEE 14 Bus	19	16	34	34
IEEE 30 Bus	50	42	56	59
IEEE 118 Bus	178	159	245	288

TABLE VII: Results of VSI Calculation - IEEE 14-Bus System

Load Bus	Qmax (pu)	VSI Index	Critical Line	Load Bus	Qmax (pu)	VSI Index	Critical Line
9	0.436	0.9874	4-9	6	0.985	0.9902	5-6
13	0.538	0.9966	12-13	4	1.159	0.9924	3-4
14	0.540	0.9941	13-14	5	1.186	0.9970	1-5
12	0.826	0.9998	6-12	3	1.280	0.9795	2-3
11	0.908	0.9903	10-11	10	1.528	0.9892	9-10

TABLE VIII: Results of Multi-objective PMU Placement

Test System Test System	Multi-Objective PMU Locations	WSOI
IEEE 14 Bus	2,6,9,12,14	23
IEEE 30 Bus	3,4,10,12,14,15,18,24,26,29,30	76
IEEE 118 Bus	1,2,9,12,13,18,21,26,28,29,34,39,42,45,47 49,51,54,59,62,69,70,72,77,85,87,90,91,94 99,100,101,106,110,114	279

procedure of calculating VSI is same for other two test systems as discussed in section II, part C. The tables are arranged in the ascending order of maximum reactive power loadability. The results of MOPP considering all objective functions are presented in Table VIII. Taking the IEEE 14-bus system as an example, the most critical buses are the ones with the minimum limit of the load increase. In order to monitor these buses properly in every possible situation, they must be observed by at least 2 PMUs. This means the observability redundancy should be at least 2 for these weak buses and demonstrates the fact that these buses can be fully observable, even in the case of any disruption.

V. CONCLUSION

In this paper, a new modified algorithm of MO-TLBO is implemented on single and multi-objective PMU placement problems with various objective functions. The algorithm is tested on three IEEE test systems and compared with other best algorithms in the literature. In order to achieve a resilient PMU placement, a multi-objective RPP is solved based on minimizing VSI and the number of PMUs, and maximizing observability. Simulation results demonstrate that the proposed MO-TLBO algorithm present better results with less iteration number and less computational time.

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