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Pricing strategy in the product and service market

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ABSTRACT

Existing studies have mainly focused on pricing in either primary markets or aftermarkets. However, in practice, prices in primary markets and aftermarkets are closely correlated. This study examines the joint pricing strategy in both primary markets and aftermarkets based on customer utility and establishes a pricing model for profit-maximization firms. Our results show that overpricing in the aftermarket is caused by customer myopia, while the motivation of the firm to avoid customer myopia depends on its pricing strategy. A quantity−price contract in the aftermarket is designed to raise the firm's profit.

1. Introduction

With intensifying competition in the customer market, revenue in the primary market is shrinking, which is earning the high-profit aftermarket increased attention. More and more companies are trying to extend their businesses to product services in the aftermarket, thus forming a product service system (Bates et al., 2013). Aftermarket services are those required by customers during their subsequent usage of purchased goods, including replacement, upgrade, and maintenance (Carlton and Waldman, 2010). In developed areas such as Europe and the United States, aftermarket services have become an important way to overcome unemployment and provide added value (Bikfalvi et al., 2013). Even in Asia, where the economy is relatively less-developed, the proportion of GDP generated by aftermarket services has exceeded 75% since 2000 (Asian Development Bank, 2017). In China, over 30% of companies have attempted to extend their businesses to product after-market services (Li et al., 2015). In recent years, the product service system has been combined with Big Data and Industry 4.0, resulting in a variety of emerging business models such as the "Power-by-the-Hour" maintenance contract for aircraft engines offered by GE Aviation. This contract is signed right after the aircraft engine sale, and it covers complete maintenance services for the engine for a fixed sum per flying hour.

However, because of the lock-in effect[1] and installed base effect,[2] the complexity of operations in the aftermarket goes far beyond simply manufacturing and selling products in the primary market. For example, while some manufacturers regard

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[1] The lock-in effect refers to a situation in which customers purchase aftermarket services provided by the manufacturer of the original product.

[2] The installed base effect refers to a situation in which the demand for aftermarket services relies heavily on sales of the original product in the primary market.

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2096-2320/

aftermarket services as a tool to enhance sales in the primary market, others provide cheap products and make profits from the services provided in the aftermarket, leading to overpricing (Emch, 2003). Therefore, to succeed in the aftermarket, it is important to create a portfolio of service products carefully (Cohen et al., 2006). Hence, it is worthwhile studying the relationship between prices in the primary market and aftermarket.

This study aims to answer the following three questions: (1) Why does overpricing occur in the aftermarket? (2) What is the optimal pricing strategy for a profit-maximization firm facing complex aftermarket mechanisms? (3) Can a firm improve its profit in the aftermarket by educating customers (e.g., by advertising the service effect)? Based on the characteristics of the product service aftermarket and using a quadratic customer utility function, we establish a game theoretical model that incorporates a monopoly and a customer. We also consider the heterogeneity of customer myopia and the firm's corresponding pricing strategies. This study makes four main contributions. First, when demand in the primary market is endogenously affected by the price in the primary market, customer heterogeneity pertaining to the aftermarket utility no longer leads to overpricing in the aftermarket; by contrast, myopic customer behavior leads to overpricing. Second, a quantity−price contract in either the primary market or the aftermarket is designed to raise the firm's profit. Furthermore, it is more efficient to implement this contract in the aftermarket, especially when customers are more myopic than xxxx or the installed base effect is weak. Third, when the degree of customer myopia is heterogeneous, our numerical study indicates that the performance of the quantity−price contract in the aftermarket is usually optimal in a sophisticated market environment composed of different market sizes, price elasticities, installed base effects, and customer myopia levels. Finally, the firm's motivation to avoid customers' myopic behavior through public education depends on the firm's pricing strategy. Firms that directly conduct price differentiation or adopt a quantity−price contract in the primary market have an incentive to guide customers to avoid myopic behaviors. On the contrary, the decisions made by firms that adopt a quantity−price contract in the aftermarket should take the specific effect of education into account.

The structure of the remainder of this paper is as follows. In Section 2, we review the related literature to highlight our contribution. Section 3 introduces the basic model for firms facing rational customers and discusses the firm's pricing strategy in this context. Section 4 introduces the concept of the "myopic customer" to study the impact of such myopia on the pricing of aftermarket services. In Section 5, a quantity−price contract is designed to raise the firm's profit. The efficiency of the three pricing strategies is compared in Section 6. Section 7 assumes that the degree of customer myopia is heterogeneous and discusses the corresponding pricing strategies of the firm in this environment. Section 8 concludes the paper.

2. Literature review

2.1. Pricing strategy in the aftermarket

There has been growing interest in aftermarket pricing. Owing to the antitrust controversy in the Kodak case in 1994, some studies of the aftermarket have focused on social welfare. For example, Borenstein et al. (1994) claimed that the customer surplus is lowered by aftermarket services because of the lock-in effect. Carlton and Waldman (2010) built a model of a competitive primary market and a monopoly aftermarket. They found that, if customers are reasonable, the monopoly in the aftermarket raises social efficiency since customers will not purchase in the primary market if the price in the aftermarket is unfair. Zēgners & Kretschmer (2017) also studied a monopoly in the aftermarket. They used a Hotelling framework to model the competitive primary market and assumed the aftermarket is monopolized. They further assumed that customers are heterogeneous and found that firms offer a low product price and make profits in the aftermarket since they cannot distinguish customers' types. However, if firms' power in the aftermarket is too strong, the adverse selection effect will reduce their profits. Emch (2003), under the assumption of customer heterogeneity, studied pricing problems in the primary market and aftermarket in both monopoly and oligopoly settings. He stated that overpricing in the aftermarket is caused by customer heterogeneity. Miao (2010) studied the impact of myopic customers on prices in the primary market and aftermarket. These studies all focused on the relationship between prices in the primary market and aftermarket and explored the reasons for overpricing in the latter. In our study, we also examine why overpricing occurs in the aftermarket. However, we focus on the pricing strategy for a profit-maximization firm.

In recent years, aftermarkets, as a high margin business model, have begun to attract more attention as manufacturers' margins have been reduced. Researchers have thus paid attention to firms' operation management in the aftermarket. Cohen et al. (2006) summarized the methods used to maintain competitiveness in the aftermarket. Borchardt et al. (2018) studied aftermarket services in the automobile industry and proposed several key success factors using a case study. Other scholars have studied pricing strategies in the aftermarket. For example, Ferrer et al. (2010) studied bundled pricing with quality differentiation in the aftermarket using a consumer choice model and dynamic programming, while Liang et al. (2017) considered the issue of coordination in the supply chain based on bundled aftermarket services. However, these two studies only considered the installed base effect, ignoring the important lock-in effect prevailing in the aftermarket. Kurata and Nam (2010) used a quantitative model to study the interaction between products and services in the aftermarket. They assumed two types of consumers: one type only purchases products, whereas the other also purchases value-added services. Based on these assumptions, they built a model for different types of games and market structures. The study found that the firm's profit and consumer surplus conflict without coordination mechanisms. Zhang et al. (2019) discussed the impact of information sharing on firms' pricing strategies in the aftermarket. They assumed that aftermarket services can be provided by either a manufacturer that produces products or a retailer that has private market information. They found that sharing information in the aftermarket may cause a prisoner's dilemma, leading the profits of both the manufacturer and the retailer to decline. Therefore, investment in demand forecasting may damage the supply chain. However, again, these studies only considered the installed base effect. In this study, the lock-in effect is also considered to model customers' behavior in the aftermarket.

2.2. Differentiated pricing

Another literature stream to which our work is related concerns differentiated pricing. Shapiro and Varian (1998) suggested that a differentiated pricing strategy can benefit firms. Li et al. (2013) assumed that customers' value function is linear and examined firms' optimal differentiated pricing strategy. The optimal quality levels and prices for multiple versions were obtained. Luo et al. (2017) examined the pricing strategies of differentiated brands in a supply chain, finding that intensified competition lowers supply chain efficiency. Liu et al. (2019) investigated the differentiated pricing problem in the context of a dual channel supply chain. They assumed that consumers have different degrees of network acceptance and showed the optimal pricing strategy of the firm. In this study, we also explore firms' differentiated pricing strategies based on customers' utility. However, we examine the problem in the aftermarket, which has rarely been studied, to the best of our knowledge.

3. Pricing strategies for rational customers

3.1. Customer's utility function

For a utility function to exist, it must satisfy the following five characteristics: completeness, reflexivity, transitivity, continuity, and strong monotonicity (Mas-Colell et al., 2005). In particular, for risk-averse customers, the utility function is concave. A quadratic-form utility function, which corresponds to the linear demand function,[3] has been frequently used in extant work (e.g., Mas-Colell et al., 2005). In this study, we assume that customer utility in the primary market $U(q_1)$ is a quadratic function of product consumption as follows:

$$U_1(q_1) = Aq_1 - \gamma {q_1}^2 - p_1 q_1$$

where p_1 refers to the product price in the primary market, q_1 refers to the product consumption of the customer, γ is the sensitivity coefficient for product consumption in the primary market, and A is a coefficient related to potential demand in the primary market. Without loss of generality, assume $A > 0$ and $\gamma > 0$. For the monotonicity of the utility function, $q_1 \le \frac{A}{2\gamma}$.

Owing to the installed base effect, services purchased by customers in the aftermarket relate to the product quantity purchased in the primary market. To model this phenomenon, we assume that the potential demand of customers in the aftermarket is a proportional function of product consumption in the primary market. The firm provides a uniform service level and charges according to the frequency of service provision. This business model is popular when the service level provided is difficult to measure. For example, when Goldwind, a wind turbine manufacturer, provides standardized wind turbine maintenance, it collects service fees based on the number of turbines maintained. In summary, the utility of the customer in the aftermarket $U(q_2)$ can be expressed as

$$U_2(q_2) = \lambda q_1 q_2 - \theta {q_2}^2 - p_2 q_2$$

where p_2 refers to the service price in the aftermarket, q_2 refers to the service consumption of the customer, θ is the sensitivity coefficient for service consumption in the aftermarket, and λ measures the installed base effect. Without loss of generality, assume $\lambda > 0$ and $\theta > 0$. For the monotonicity of the utility function, $q_2 \le \frac{\lambda q_1}{2\theta}$.

Before purchasing products in the primary market, a rational customer will synthetically consider his or her utility in both the primary market and the aftermarket. Therefore, the total utility of a rational customer is

$$U = Aq_1 - \gamma {q_1}^2 + \lambda q_1 q_2 - \theta {q_2}^2 - p_1 q_1 - p_2 q_2$$

3.2. Firm's pricing decisions

Emch (2003) assumed that demand in the primary market is irrelevant for the market price since customers only choose to buy or not. If they choose to buy, they pay the same price and become potential customers of the aftermarket. In this study, we relax this assumption and model a more realistic case. The problem for the profit-maximization firm can be expressed as

[3] For a customer whose utility function is $U(q) = Aq - \gamma q^2$, the optimal quantity purchased is $q^* = \frac{A}{2\gamma} - \frac{p}{2\gamma}$.

$$\max_{p_1,p_2}\pi = p_1q_1(p_1,p_2) + p_2q_2(p_1,p_2) \tag{1}$$

$$\text{s.t.}\max_{q_1,q_2} U = \mathrm{A}\mathrm{q}_1 - \gamma \mathrm{q}_1^2 + \lambda \mathrm{q}_1 q_2 - \theta q_2^2 - p_1q_1 - p_2q_2 \tag{2}$$

Using backward induction, we first obtain Proposition 1 from equation (2).

Proposition 1. *There exists an optimal solution to the utility-maximization problem of a rational customer in the product service system if and only if* $\lambda<2\sqrt{\theta\gamma}$. *The optimal solution is*

$$\begin{cases} q_1^* = \dfrac{2\theta A - 2\theta p_1 - \lambda p_2}{4\theta\gamma - \lambda^2} \\ q_2^* = \dfrac{\lambda A - \lambda p_1 - 2\gamma p_2}{4\theta\gamma - \lambda^2} \end{cases}$$

The proof is given in Appendix 1.

If we substitute the results of Proposition 1 into equation (1), we have

$$\max_{p_1,p_2}\pi = \frac{1}{4\theta\gamma - \lambda^2}\left(- 2\theta p_1^2 - 2\lambda p_1p_2 - 2\gamma p_2^2 + 2\theta A p_1 + \lambda A p_2\right) \tag{3}$$

Calculating the first- and second-order partial derivatives of equation (3), we have Proposition 2.

Proposition 2. *There exists an optimal solution to the profit-maximization problem of the firm, which is*

$$\begin{cases} p_1^* = \dfrac{A}{2} \\ p_2^* = 0 \end{cases}$$

The proof is given in Appendix 2.

Proposition 2 shows that, even if the demand of a rational customer in the primary market is affected by the price set by the firm, the firm will still provide services in the aftermarket at a price equal to the marginal cost and the appropriate customer surplus through the high price in the primary market, which is consistent with Emch (2003). However, in this situation, the firm can no longer seize the entire customer surplus by charging in the primary market. Therefore, customer heterogeneity cannot influence the pricing strategies of the firm in the aftermarket, which goes against the conclusion derived by Emch (2003).

Indeed, for customers with different parameters (γ, θ, λ), the firm's pricing strategies in the primary market and aftermarket are not affected. However, for customers who differ in parameter A, the firm's pricing strategy in the primary market should be adjusted to $p_1{}^* = \frac{EA}{2}$, where EA is the expectation of parameter A. Therefore, in practice, firms should not change their pricing strategy when customers' sensitivity toward prices in the primary market and aftermarket changes. However, they should adjust their prices when the market size changes.

4. Pricing strategies for myopic customers

Proposition 2 indicates that, even if customer demand in the primary market is influenced by the firm's pricing strategy, the firm has no incentive to overprice in the aftermarket. Furthermore, the existence of heterogeneous customers does not affect the firm's behavior in the aftermarket. However, in practice, overpricing in the aftermarket is common. For example, in the famous case of Kodak in 1992, the company monopolized its service aftermarket for photocopiers and other imaging equipment by refusing to provide spare parts to third-party service providers, and thus profited by charging high prices in the aftermarket (Goldfine and Vorrasi, 2004). To explain this behavior, Miao (2010) introduced the concept of myopic customers and assumed that a proportion of myopic customers exist in the market who only pay attention to the primary market utility, regardless of the future utility in the aftermarket. Below, we discuss the impact of customer myopia on our model.

4.1. Description of customer myopia

Myopic customers focus more on immediate utility in the primary market than on future utility in the aftermarket. Calzada and Valletti (2012) introduced the discount rate, δ, to model customer myopia. In this study, we follow their method and assume that the customer's utility gained in the aftermarket will be discounted by δ:

$$U_2(q_2) = \delta\left(\lambda q_1 q_2 - \theta {q_2}^2 - p_2 q_2\right)$$

where $0 \le \delta \le 1$, which means that customers are more willing to gain immediate utility from the primary market than future utility from the aftermarket. When $\delta = 0$, the customer does not care about the utility obtained in the aftermarket, which corresponds to the completely myopic customer in Miao's (2010) model. When $\delta = 1$, the customer takes full consideration of the utility received in both the primary market and the aftermarket, which corresponds to the rational customer outlined in Section 3. In reality, customers usually possess bounded rationality and are neither completely myopic nor fully foresighted. The model below applies to this scenario.

4.2. Firm's pricing decisions

With myopic customers, the firm's profit function can be expressed as

$$\max_{p_1,p_2} \pi = p_1 q_1(p_1, p_2) + p_2 q_2(p_1, p_2) \tag{4}$$

$$s.t. \max_{q_1,q_2} U = A q_1 - \gamma q_1^2 - p_1 q_1 + \delta\left(\lambda q_1 q_2 - \theta q_2^2 - p_2 q_2\right) \tag{5}$$

Using backward induction, we first obtain Proposition 3 from equation (5).

Proposition 3. *There exists an optimal solution to the utility-maximization problem of the myopic customer if and only if* $\lambda < 2\sqrt{\theta\gamma/\delta}$. *The optimal solution is*

$$\begin{cases} q_1^* = \dfrac{2\theta A - 2\theta p_1 - \delta\lambda p_2}{4\theta\gamma - \delta\lambda^2} \\ q_2^* = \dfrac{\lambda A - \lambda p_1 - 2\gamma p_2}{4\theta\gamma - \delta\lambda^2} \end{cases}$$

The proof is given in Appendix 3.

The impact of customer myopia can be summarized as follows.

Corollary 1. *An increase in the degree of customer myopia reduces demand in the aftermarket. However, the effect of customer myopia on demand in the primary market depends on the prices set by the firm. The increase in the degree of customer myopia will raise demand in the primary market if and only if*

$$p_1 < A - \frac{2\lambda^2 p_2}{\gamma^2}$$

and vice versa.

The proof is given in Appendix 4.

The existence of myopic customers reduces demand in the aftermarket because the utility obtained by customers in the aftermarket decreases. As a result, customers will reduce their budget in the aftermarket. However, customers do not necessarily consume more in the primary market in this case. If primary market prices are exorbitant, customers will reduce their consumption in both the primary market and the aftermarket, leading to a decrease in demand in both markets.

If we substitute the result of Proposition 3 into equation (4), we obtain the following:

$$\pi = \frac{1}{4\theta\gamma - \delta\lambda^2}\left(- 2\theta p_1^2 - \lambda p_1 p_2 - \delta\lambda p_1 p_2 - 2\gamma p_2^2 + 2\theta A p_1 + \lambda A p_2\right) \tag{6}$$

This leads to Proposition 4.

Proposition 4. *There exists an optimal solution to the firm's problem with myopic customers, which is*

$$\begin{cases} p_1^* = \dfrac{\left(8\theta\gamma - (1+\delta)^2\lambda^2\right)A}{16\theta\gamma - (1+\delta)^2\lambda^2} \\ p_2^* = \dfrac{2(1-\delta)\theta\lambda A}{16\theta\gamma - (1+\delta)^2\lambda^2} \end{cases}$$

The proof is given in Appendix 5.

When customers are myopic, the product price in the primary market decreases, and the service price in the aftermarket is higher than its marginal cost. Miao (2010) suggested that the emergence of myopic customers in an oligopoly market leads to overpricing in the aftermarket. However, Proposition 4 demonstrates that, even in monopoly markets, the myopic behavior of customers will still provide the firm with the motivation to offer services at a price above the marginal cost, thus resulting in aftermarket overpricing. Furthermore, combined with the conclusion of Proposition 2, firms should decrease the price in the primary market and make profits in the aftermarket if their customers are heterogeneous, which is a critical feature in emerging markets such as games, videos, and other online services.

5. Quantity–price contract

From Proposition 4, the optimal profit of the firm with myopic customers and the customer's utility can be derived as

$$\pi_{mf} = \frac{2\theta A^2}{16\theta\gamma - (1+\delta)^2\lambda^2} \tag{7}$$

$$U_{mf} = \frac{4\left(16\theta\gamma + (\delta^3 + 2\delta^2 - 3\delta)\lambda^2\right)\theta^2\gamma A^2}{(4\theta\gamma - \delta\lambda^2)\left(16\theta\gamma - (1+\delta)^2\lambda^2\right)^2} \tag{8}$$

It is clear that $U_{mf} > 0$, which means that positive utility can be achieved by the customer. In other words, the firm cannot capture the entire customer surplus through the pricing mechanism in Section 4. To increase its profit, the firm can use a quantity–price contract to sell in bulk in either the primary market or the aftermarket, thereby stimulating customer consumption and appropriating the entire customer surplus.

5.1. Quantity–price contract in the primary market

In practice, quantity–price contracts are often seen in the primary market for large-scale equipment such as railway facilities and elevators in high-rise buildings. The firm determines a quantity–price combination in the primary market and sets a uniform service price in the aftermarket. The problem for the firm in this setting can be expressed as

$$\max_{p_1, q_1, p_2} \pi = p_1 q_1 + p_2 q_2$$

$$\text{s.t.} \begin{cases} q_2 = \underset{q_2}{\operatorname{argmax}} A q_1 - \gamma q_1^2 - p_1 q_1 + \delta\left(\lambda q_1 q_2 - \theta q_2^2 - p_2 q_2\right) \\ U = \max_{q_2} A q_1 - \gamma q_1^2 - p_1 q_1 + \delta\left(\lambda q_1 q_2 - \theta q_2^2 - p_2 q_2\right) \geq 0 \end{cases}$$

The firm and customers conduct a Stackelberg game. After observing the firm's pricing strategy, the customers decide the quantity of the product purchased in the primary market and the amount of services consumed in the aftermarket. The second constraint represents the rationality of customers, which ensures that those purchasing the product obtain non-zero utility and are willing to purchase.

Proposition 5. *There exists an optimal solution for firms adopting a quantity–price contract in the primary market if and only if $\lambda < 2\sqrt{\theta\gamma/\delta}$. The optimal solution is*

$$\begin{cases} {p_1}^* = \dfrac{\left(4(2-\delta)\theta\gamma + \delta\lambda^2\right)A}{8\left(2-\delta^2\right)\theta\gamma} \\ {p_2}^* = \dfrac{(1-\delta)\lambda A}{2(2-\delta)\gamma} \\ {q_1}^* = \dfrac{A}{2\gamma} \end{cases}$$

The proof is given in Appendix 6.

Comparing the results of Proposition 5 with Proposition 3 and Proposition 4, we can see that, if the firm adopts a quantity–price contract in the primary market, it will increase product consumption in the primary market to expand its customer base in the aftermarket, thereby fully utilizing the installed base effect. In this scenario, the firm obtains the entire customer surplus through the primary market price.

5.2. Quantity–price contract in the aftermarket

The firm can also use a quantity–price contract in the aftermarket. The most common examples of firms that use such contracts are mobile carriers such as T-Mobile and AT&T. Mobile carriers often sell mobile phones at a low price (sometimes even free), but require customers to sign a contract that says they will use the provider's services for a certain period. Such contracts usually stipulate a minimum consumption requirement every month. The problem for the firm using such strategy can be expressed as

$$\max_{p_1,p_2,q_2} \pi = p_1 q_1 + p_2 q_2$$

$$s.t. \begin{cases} q_1 = \underset{q_1}{\operatorname{argmax}} A q_1 - \gamma q_1^2 - p_1 q_1 + \delta\left(\lambda q_1 q_2 - \theta q_2^2 - p_2 q_2\right) \\ U = \max_{q_1} A q_1 - \gamma q_1^2 - p_1 q_1 + \delta\left(\lambda q_1 q_2 - \theta q_2^2 - p_2 q_2\right) \geq 0 \end{cases}$$

This leads to Proposition 6.

Proposition 6. *There exists an optimal solution for firms using a quantity–price contract in the aftermarket*:

$$\begin{cases} {p_1}^* = \dfrac{\delta^2\lambda^2 A}{4\theta\gamma} \\ {p_2}^* = \dfrac{4\gamma\theta - \delta^3\lambda^2 A}{4\delta^2\gamma\lambda} \\ {q_2}^* = \dfrac{\delta\lambda A}{4\theta\gamma} \end{cases}$$

The proof is given in Appendix 7.

From Proposition 7, we can also calculate the consumption of the primary market product in this situation: ${q_1}^* = \frac{A}{2\gamma}$. Compared with Proposition 5, we can see that, although the firm cannot directly determine the customer's product consumption in the primary market, it will sell products at a low price to fully exploit the installed base effect in the aftermarket. The difference is that, under this strategy, the firm seizes the entire customer surplus through the aftermarket service price.

6. Firm's strategy selection

Using Proposition 5 and Proposition 6, we can calculate the firm's maximal profit under each selling strategy:

$$\pi_{ff} = \frac{(4(2-\delta)\theta\gamma + \lambda^2)A^2}{(2-\delta)16\delta\theta\gamma^2} \tag{8}$$

$$\pi_{af} = \frac{(\delta^3\lambda^2 + 4\theta\gamma)A^2}{16\delta\theta\gamma^2} \tag{9}$$

Corollary 2. *The quantity–price contract is better adopted in the aftermarket than in the primary market when* $\delta < 1$. *In particular, if* $\delta = 1$, *which means that customers are fully foresighted, the profits of using either strategy are equal.*

The proof is given in Appendix 8.

Corollary 2 reveals the aftermarket's potential to help firms generate more profits (Cohen et al., 2006). Although a firm using either pricing strategy will try (directly or indirectly) to make customers purchase as many products as possible in the primary market, customers behave differently in the aftermarket. When the firm adopts a quantity–price contract in the primary market, customers choose the extent of services independently in the aftermarket, based only on their utility. Since customers do not care about the efficiency of the entire system, their choices are not systematically optimal. Carlton and Waldman (2010), in observing this phenomenon, stated that, because of the zero-profit conditions of the competitive market, the monopoly firm in the aftermarket can actually increase market efficiency. Similarly, in our model, since the firm can seize the entire customer surplus, if it can control the customer's behavior in the aftermarket, it will intentionally set service consumption to maximize the efficiency of the entire system (as well as its profit).

If we compare the firm's profit under a wholesale contract (Section 4) with that under a quantity–price contract, we obtain the following.

Corollary 3. *The profit of the firm adopting a quantity–price contract in the aftermarket increases if and only if*

$$32\theta^2\gamma^2 - \left(\delta^3\lambda^2 + 4\theta\gamma\right)\left(16\theta\gamma - (1+\delta)^2\lambda^2\right) < 0$$

The proof is given in Appendix 9.

Fig. 1 shows the area in which the firm can increase its profit by taking advantage of the quantity–price contract in the aftermarket.

Fig. 1 depicts the isoprofit lines of the two pricing strategies with different levels of customer myopia. If the customer's sensitivity coefficient in either the primary market or the aftermarket is relatively large (i.e., when customer demand is rigid), it is more efficient for the firm to use a quantity–price contract in the aftermarket. This is because customers with inelastic

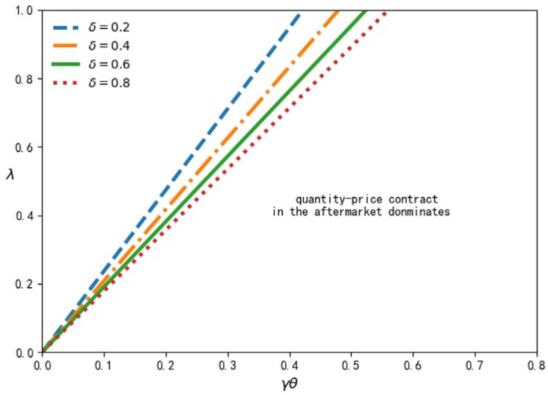

Fig. 1. Isoprofit lines of the firm using a quantity–price contract in the aftermarket and using a wholesale contract.

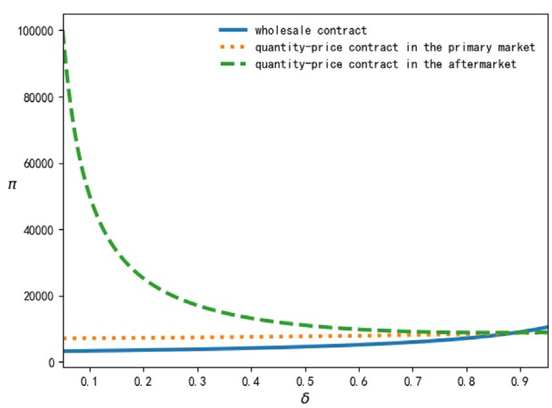

Fig. 2. The impact of customer myopia on the firm's profit under three strategies.

demand are not sensitive to prices, so it is less efficient for the firm to adjust the number of purchases based only on pricing. On the contrary, firms that adopt a quantity–price contract in the aftermarket can directly control the quantity customers purchase, thereby guaranteeing greater efficiency.

Fig. 1 also shows that, when the installed base effect in the aftermarket is weak, it is more efficient for the firm to use a quantity–price contract in the aftermarket because it cannot stimulate customer demand efficiently using a simple pricing mechanism. Therefore, the advantage of the quantity–price contract in the aftermarket is manifested.

In addition, Fig. 1 reveals that if the degree of customer myopia is high, the firm is more likely to adopt a quantity–price contract in the aftermarket. This is because myopic customers do not care about the utility in the aftermarket and hence reduce their service consumption. On the contrary, since the quantity–price contract in the aftermarket allows the firm to directly control customers' service consumption, it can eliminate the adverse effects of customer myopia.

Corollary 3 shows that when the degree of customer myopia increases, the advantage of the quantity–price contract in the aftermarket increases as well. To illustrate the impact of customer myopia on the firm's profit more clearly, Fig. 2 plots the impact under three strategies. In this example, we assume that $\mu = 0.5$, $A = 100$, $\theta = 0.4$, $\lambda = 0.8$, and $\gamma = 0.5$.

Fig. 2 shows that the firm should adopt a quantity–price contract if customers are myopic. When the degree of customer myopia is low, the firm's profit under each strategy is nearly identical. As the degree of customer myopia increases, the profits of the firm adopting a wholesale contract and a quantity–price contract in the primary market decrease, while the profits of the firm adopting a quantity–price contract in the aftermarket increase. This is because, when the degree of customer myopia drops, customers gain more utility and are willing to buy more products. The firm with a wholesale contract sells more products in both markets, thus achieving higher profit. Meanwhile, for the firm with a quantity–price contract in the primary market, the decline in the degree of customer myopia makes customers more willing to consume in the aftermarket; therefore, the profit of the firm increases. However, for the firm with a quantity–price contract in the aftermarket, although it can control customers' service consumption in the aftermarket, it can do nothing when the degree of customer myopia decreases and customers' consumption in the primary market declines. Therefore, the profit of the firm decreases.

7. Discussion on the heterogeneity of customer myopia

In practice, customer types are not unified and customer heterogeneity prevails. To earn more profits, firms pursue differentiated pricing strategies to distinguish customers' types. Considering the relationship between products and services, firms can adopt different types of contracts. For example, different razor blades are sold in packages consisting of several blades, which is a differentiated quantity–price contract. Furthermore, firms can also differentiate the wholesale price. For example, Lenovo provides different services to customers purchasing different products (e.g., customers who purchase a ThinkPad can enjoy customized services called "Think Service"). This section examines the impact of customers with heterogeneous myopic inclinations on the firm's pricing design. Assume two types of customers whose myopia levels are δ_L and δ_H, where $\delta_L < \delta_H$. The proportion of the two types is μ and $1-\mu$, respectively. The firm knows the distribution of customer types but not the specific type of each individual. The profit-maximization firm implements a differentiated pricing strategy and offers customers two contracts. Customers know their types and choose product consumption and service consumption based on a desire to maximize their utility. According to the revelation principle, this section examines the situation in which the price provided by the firm is incentive-compatible.

7.1. *Differentiated wholesale contract*

We first examine a strategy under which the firm provides different wholesale contracts for either customers of type H or customers of type L, which corresponds to the pricing strategy proposed in Section 4. For a customer of type x ($x = L$ or H), the utility-maximization problem can be expressed as

$$\max_{q_{1x},q_{2x}} U = Aq_{1x} - \gamma {q_{1x}}^2 - p_{1x}q_{1x} + \delta_x\left(\lambda q_{1x}q_{2x} - \theta {q_{2x}}^2 - p_{2x}q_{2x}\right)$$

The solution is

$$\begin{cases} q_{1x}^* = \dfrac{2\theta A - 2\theta p_{1x} - \delta\lambda p_{2x}}{4\theta\gamma - \delta_x\lambda^2} \\ q_{2x}^* = \dfrac{\lambda A - \lambda p_{1x} - 2\gamma p_{2x}}{4\theta\gamma - \delta_x\lambda^2} \end{cases} \tag{10}$$

The indirect utility function of the customer is

$$V_x(p_{1x},p_{2x}) = \frac{1}{4\theta\gamma-\delta_x\lambda^2}\left(\theta A^2 - 2\theta A p_{1x} + \theta p_{1x}^2 + \delta_x\lambda p_{1x}p_{2x} - \delta_x\lambda A p_{2x} + \delta_x\gamma p_{2x}^2\right) \quad (11)$$

The firm anticipates the customer's choice and provides different wholesale contracts (p_{1L}, p_{2L}) and (p_{1H}, p_{2H}) for customers of type L and type H, respectively. The profit-maximization problem of the firm can be expressed as

$$\max_{p_{1L},p_{1H},p_{2L},p_{2H}} \pi = \frac{\mu}{4\theta\gamma-\delta_L\lambda^2}\left(2A\theta p_{1L} - 2\theta p_{1L}^2 - \delta_L\lambda p_{2L}p_{1L} + A\lambda p_{2L} - \lambda p_{1L}p_{2L} - 2\gamma p_{2L}^2\right) + \frac{1-\mu}{4\theta\gamma-\delta_H\lambda^2}\left(2A\theta p_{1H} - 2\theta p_{1H}^2 - \delta_H\lambda p_{2H}p_{1H} + A\lambda p_{2H} - \lambda p_{1H}p_{2H} - 2\gamma p_{2H}^2\right)$$

$$s.t.\begin{cases} \frac{1}{4\theta\gamma-\delta_L\lambda^2}\left(\theta A^2 - 2\theta A p_{1L} + \theta p_{1L}^2 + \delta_L\lambda p_{1L}p_{2L} - \delta_L\lambda A p_{2L} + \delta_L\gamma p_{2L}^2\right) \ge 0 \quad (12) \\ \frac{1}{4\theta\gamma-\delta_H\lambda^2}\left(\theta A^2 - 2\theta A p_{1H} + \theta p_{1H}^2 + \delta_H\lambda p_{1H}p_{2H} - \delta_H\lambda A p_{2H} + \delta_H\gamma p_{2H}^2\right) \ge 0 \quad (13) \\ \frac{1}{4\theta\gamma-\delta_L\lambda^2}\left(\theta A^2 - 2\theta A p_{1L} + \theta p_{1L}^2 + \delta_L\lambda p_{1L}p_{2L} - \delta_L\lambda A p_{2L} + \delta_L\gamma p_{2L}^2\right) \ge \\ \quad \frac{1}{4\theta\gamma-\delta_L\lambda^2}\left(\theta A^2 - 2\theta A p_{1H} + \theta p_{1H}^2 + \delta_L\lambda p_{1H}p_{2H} - \delta_L\lambda A p_{2H} + \delta_L\gamma p_{2H}^2\right) \quad (14) \\ \frac{1}{4\theta\gamma-\delta_H\lambda^2}\left(\theta A^2 - 2\theta A p_{1H} + \theta p_{1H}^2 + \delta_H\lambda p_{1H}p_{2H} - \delta_H\lambda A p_{2H} + \delta_H\gamma p_{2H}^2\right) \ge \\ \quad \frac{1}{4\theta\gamma-\delta_H\lambda^2}\left(\theta A^2 - 2\theta A p_{1L} + \theta p_{1L}^2 + \delta_H\lambda p_{1L}p_{2L} - \delta_H\lambda A p_{2L} + \delta_H\gamma p_{2L}^2\right) \quad (15) \end{cases}$$

where equation (12) and equation (13) are individual-rationality constraints, while equation (14) and equation (15) are incentive-compatibility constraints.

This leads to Proposition 7.

Proposition 7. *If the firm implements a differentiated wholesale contract, customers of type L gain greater utility.*

The proof is given in Appendix 10.

7.2. Differentiated quantity–price contract in the primary market

The firm may also design different quantity–price contracts in the primary market for customers to choose from, which corresponds to the pricing strategy proposed in Section 5.1. Specifically, the firm sets a uniform aftermarket service price, p_2, and provides different quantity–price contracts in the primary market, (p_{1L}, q_{1L}) and (p_{1H}, q_{1H}), for customers of type L and type H. For a customer of type x ($x = L$ or H), the utility-maximization problem can be expressed as

$$\max_{q_{2x}} U = Aq_{1x} - \gamma {q_{1x}}^2 - p_{1x}q_{1x} + \delta_x\left(\lambda q_{1x}q_{2x} - \theta {q_{2x}}^2 - p_2 q_{2x}\right)$$

The solution is

$$q_{2x} = \frac{\lambda q_{1x} - p_{2x}}{2\theta}$$

The indirect utility function of the customer is

$$V_x(p_{1x},q_{1x},p_2) = Aq_{1x} - \gamma q_{1x}^2 - p_{1x}q_{1x} + \frac{\delta_x}{4\theta}(\lambda q_{1x} - p_2)^2 \quad (16)$$

Therefore, the firm's profit-maximization problem can be described as

$$\max_{p_{1L},p_{1H},q_{1L},q_{1H},p_2} \pi = \mu\left(p_{1L}q_{1L} + \frac{p_2\lambda q_{1L} - p_2^2}{2\theta}\right) + (1-\mu)\left(p_{1H}q_{1H} + \frac{p_2\lambda q_{1H} - p_2^2}{2\theta}\right)$$

$$s.t.\begin{cases} Aq_{1L} - \gamma q_{1L}^2 - p_{1L}q_{1L} + \frac{\delta_L}{4\theta}(\lambda q_{1L} - p_2)^2 \geq 0 & (17) \\ Aq_{1H} - \gamma q_{1H}^2 - p_{1H}q_{1H} + \frac{\delta_H}{4\theta}(\lambda q_{1H} - p_2)^2 \geq 0 & (18) \\ Aq_{1L} - \gamma q_{1L}^2 - p_{1L}q_{1L} + \frac{\delta_L}{4\theta}(\lambda q_{1L} - p_2)^2 \geq Aq_{1H} - \gamma q_{1H}^2 - p_{1H}q_{1H} + \frac{\delta_L}{4\theta}(\lambda q_{1H} - p_2)^2 & (19) \\ Aq_{1H} - \gamma q_{1H}^2 - p_{1H}q_{1H} + \frac{\delta_H}{4\theta}(\lambda q_{1H} - p_2)^2 \geq Aq_{1L} - \gamma q_{1L}^2 - p_{1L}q_{1L} + \frac{\delta_H}{4\theta}(\lambda q_{1L} - p_2)^2 & (20) \end{cases}$$

where equation (17) and equation (18) are individual-rationality constraints, while equation (19) and equation (20) are incentive-compatibility constraints.

Proposition 8. *If the firm adopts a differentiated quantity–price contract in the primary market*:

(1) *The firm's problem can be simplified as a quadratic programming problem*:

$$\max_{q_{1L},q_{1H},p_2} \pi = \mu\left(Aq_{1L} - \gamma q_{1L}{}^2 + \frac{\delta_L}{4\theta}(\lambda q_{1L} - p_2)^2 + \frac{p_2\lambda q_{1L} - p_2^2}{2\theta}\right) + (1-\mu)\left(q_{1H} - \gamma q_{1H}{}^2 + \frac{\delta_H}{4\theta}(\lambda q_{1H} - p_2)^2 + \frac{p_2\lambda q_{1H} - p_2^2}{2\theta} - \frac{\delta_H - \delta_L}{4\theta}(\lambda q_{1L} - p_2)^2\right)$$

(2) *Customers of type H gain non-negative utility, while customers of type L receive zero utility.*

The proof is given in Appendix 11.

7.3. Differentiated quantity–price contract in the aftermarket

Another alternative pricing strategy of the firm corresponds to Section 5.2, which provides different quantity–price contracts in the aftermarket. The firm specifies a uniform product price, p_1, in the primary market and provides different quantity–price contracts in the aftermarket, (p_{2L}, q_{2L}) and (p_{2H}, q_{2H}), for customers of type L and type H, respectively. For a customer of type x ($x = L$ or H), the utility-maximization problem can be expressed as

$$\max_{q_{1x}} U = Aq_{1x} - \gamma q_{1x}{}^2 - p_{1x}q_{1x} + \delta_x\left(\lambda q_{1x}q_{2x} - \theta q_{2x}{}^2 - p_2 q_{2x}\right)$$

The solution is

$$q_{1x} = \frac{A - p_1 + \delta_x\lambda q_2}{2\gamma}$$

The indirect utility function of the customer is

$$V_x(p_{1x}, q_{1x}, p_2) = \frac{(\delta_x\lambda q_{2x} + A - p_1)^2}{4\gamma} - \delta_x\theta q_{2x}^2 - \delta_x p_{2x}q_{2x} \quad (21)$$

Thus, the firm's problem can be described as

$$\max_{p_{2L},p_{2H},q_{2L},q_{2H},p_1} \pi = \mu\left(\frac{Ap_1 - p_1^2 + \delta_L\lambda p_1 q_{2L}}{2\gamma} + p_{2L}q_{2L}\right) + (1-\mu)\left(\frac{Ap_1 - p_1^2 + \delta_H\lambda p_1 q_{2H}}{2\gamma} + p_{2H}q_{2H}\right)$$

$$s.t.\begin{cases} \dfrac{(\delta_L\lambda q_{2L} + A - p_1)^2}{4\gamma} - \delta_L\theta q_{2L}^2 - \delta_L p_{2L}q_{2L} \geq 0 \quad (22) \\ \dfrac{(\delta_H\lambda q_{2H} + A - p_1)^2}{4\gamma} - \delta_H\theta q_{2H}^2 - \delta_H p_{2H}q_{2H} \geq 0 \quad (23) \\ \dfrac{(\delta_L\lambda q_{2L} + A - p_1)^2}{4\gamma} - \delta_L\theta q_{2L}^2 - \delta_L p_{2L}q_{2L} \geq \dfrac{(\delta_L\lambda q_{2H} + A - p_1)^2}{4\gamma} - \delta_L\theta q_{2H}^2 - \delta_L p_{2H}q_{2H} \quad (24) \\ \dfrac{(\delta_H\lambda q_{2H} + A - p_1)^2}{4\gamma} - \delta_H\theta q_{2H}^2 - \delta_H p_{2H}q_{2H} \geq \dfrac{(\delta_H\lambda q_{2L} + A - p_1)^2}{4\gamma} - \delta_H\theta q_{2L}^2 - \delta_H p_{2L}q_{2L} \quad (25) \end{cases}$$

where equation (22) and equation (23) are individual-rationality constraints, while equation (24) and equation (25) are incentive-compatibility constraints.

Lemma 1. *The optimal strategy for firms using a differentiated quantity–price contract in the aftermarket is one of the optimal solutions to the following two optimization problems*:

(1) Strategy 1:

$$\max_{q_{2L},q_{2H},p_1} \pi = \mu\left(\frac{Ap_1 - p_1^2 + \delta_L\lambda p_1 q_{2L}}{2\gamma} + \frac{(\delta_L\lambda q_{2L} + A - p_1)^2}{4\delta_L\gamma} - \theta q_{2L}^2\right) + (1-\mu) \times \left(\begin{array}{c} \dfrac{Ap_1 - p_1^2 + \delta_H\lambda p_1 q_{2H}}{2\gamma} + \dfrac{(\delta_H\lambda q_{2H} + A - p_1)^2}{4\delta_H\gamma} - \theta q_{2H}^2 \\ -\dfrac{\delta_L(\delta_H\lambda q_{2L} + A - p_1)^2 - \delta_H(\delta_L\lambda q_{2L} + A - p_1)^2}{4\gamma\delta_L\delta_H} \end{array}\right)$$

In this strategy, customers of type H obtain non-negative utility, while customers of type L obtain zero utility.

(2) Strategy 2:

$$\max_{q_{2L},q_{2H},p_1} \pi = \mu\left(\begin{array}{c} \dfrac{Ap_1 - p_1^2 + \delta_L\lambda p_1 q_{2L}}{2\gamma} + \dfrac{(\delta_L\lambda q_{2L} + A - p_1)^2}{4\delta_L\gamma} - \theta q_{2L}^2 \\ -\dfrac{\delta_H(\delta_L\lambda q_{2H} + A - p_1)^2 - \delta_L(\delta_H\lambda q_{2H} + A - p_1)^2}{4\gamma\delta_L\delta_H} \end{array}\right) + (1-\mu)\left(\frac{Ap_1 - p_1^2 + \delta_H\lambda p_1 q_{2H}}{2\gamma} + \frac{(\delta_H\lambda q_{2H} + A - p_1)^2}{4\delta_H\gamma} - \theta q_{2H}^2\right)$$

$$s.t.\begin{cases} p_1 \leq A - \sqrt{\delta_L\delta_H}\lambda q_{2H} \\ q_{2L} \leq q_{2H} \end{cases}$$

In this strategy, customers of type L obtain non-negative utility, while customers of type H obtain zero utility.

The proof is given in Appendix 12.

Lemma 1 reveals two specific strategies for firms using a differentiated quantity–price contract in the aftermarket. The first is that the firm sets a high price in the primary market, in which case myopic customers gain zero utility and rational customers obtain non-negative utility. The main sources of profit are myopic customers. The second is that the firm sets a low price in the primary market, in which case myopic customers gain non-negative utility and rational customers obtain zero utility. The main sources of profit are rational customers.

Lemma 2. *Strategy* 1 *is weakly dominated by Strategy* 2 *in* Lemma 1.

The proof is given in Appendix 13.
From Lemma 1 and Lemma 2, we can easily derive Proposition 9.

Proposition 9. *When the firm uses a differentiated quantity–price contract in the aftermarket*:

(1) The firm's problem can be simplified as a quadratic programming problem:

$$\max_{q_{2L},q_{2H},p_1} \pi = \mu \left(\frac{Ap_1 - p_1^2 + \delta_L \lambda p_1 q_{2L}}{2\gamma} + \frac{(\delta_L \lambda q_{2L} + A - p_1)^2}{4\delta_L \gamma} - \theta q_{2L}^2 - \frac{\delta_H(\delta_L \lambda q_{2H} + A - p_1)^2 - \delta_L(\delta_H \lambda q_{2H} + A - p_1)^2}{4\gamma\delta_L\delta_H} \right)$$
$$+(1-\mu)\left(\frac{Ap_1 - p_1^2 + \delta_H \lambda p_1 q_{2H}}{2\gamma} + \frac{(\delta_H \lambda q_{2H} + A - p_1)^2}{4\delta_H \gamma} - \theta q_{2H}^2 \right)$$
$$s.t. \begin{cases} p_1 \le A - \sqrt{\delta_L \delta_H}\lambda q_{2H} \\ p_1 \le A - \sqrt{\delta_L \delta_H}\lambda q_{2L} \end{cases}$$

(2) Customers of type L gain non-negative utility, while customers of type H receive zero utility.

7.4. Comparison of the three differentiated pricing strategies

Since analytical solutions to the three models in Sections 7.1 to 7.3 do not exist or are too complicated, we compare the three strategies using numerical analysis. To reflect the intricate market environment faced by the firm, in the numerical examples presented in this subsection, we assume that $A = 100$; Table 1 shows the values of the other parameters. Altogether, 204,714 sets of parameter combinations satisfy the constraints of Proposition 3. Since a differentiated quantity–price contract in the aftermarket typically generates the highest profit, we use it as a benchmark for comparison purposes. The box-and-whisker plots in Fig. 3 show the key results of the numerical analysis. The left plot in the figure is the relative profit of the differentiated wholesale contract, while the right plot is the relative profit of the differentiated quantity–price contract in the primary market.

Fig. 3 shows that the profit earned by the firm using a differentiated quantity–price contract in the primary market is lower than that of the firm applying a differentiated quantity–price contract in the aftermarket. In most cases, the profit of the firm using the former strategy is about 25%–70% that of the firm using the latter approach. Although, in some cases, a differentiated wholesale contract generates higher profits than does a differentiated quantity–price contract used in the aftermarket, in most cases the profit gained via the differentiated wholesale contract is only 15%–35% of that generated from the differentiated quantity–price contract in the aftermarket. The expected efficiency of the former contract is even worse than that of the latter.

This explains why, in real life, firms are generally more inclined to agree a differentiated quantity–price contract in the aftermarket. In addition to the mobile carriers mentioned in Section 5, the in-app purchase approach, which has come to be widely adopted by mobile software operators, is another example of such a contract. The in-app purchase approach first emerged in mobile games, wherein the game software itself is free but certain additions or functions incur a cost. The differentiated quantity–price contract is common in the in-app purchase model. For example, game companies often provide different recharge combos for players such as cost-effective recharge combos for casual players and high-charge/high-user-experience combos for loyal players. Reports show that in the United States, 76% of iOS App Store profits come from in-app purchases rather than downloads, while, in Asia, the proportion exceeds 90%.

Table 1
Parameter values in the numerical examples.

Parameter	Value
μ	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
θ	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
γ	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
λ	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
δ_L	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
δ_H	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9

7.5. Impact of customer myopia on the firm's profit

Fig. 4 plots the impact of customer myopia on the profits of the firm using different strategies when the degree of myopia is heterogeneous. In this subsection, we assume that the myopia level of customers of type H is $\delta_H = 0.9$. The values of the other parameters are $\mu = 0.5$, $A = 100$, $\theta = 0.3$, $\lambda = 0.9$, and $\gamma = 0.7$.

Fig. 4 shows that the differentiated quantity–price contract in the aftermarket benefits the firm if customer heterogeneity is strong. Therefore, firms should investigate their target market carefully when designing pricing strategies. If the heterogeneity of their customers is strong, they should use a differentiated quantity–price contract in the aftermarket. Otherwise,

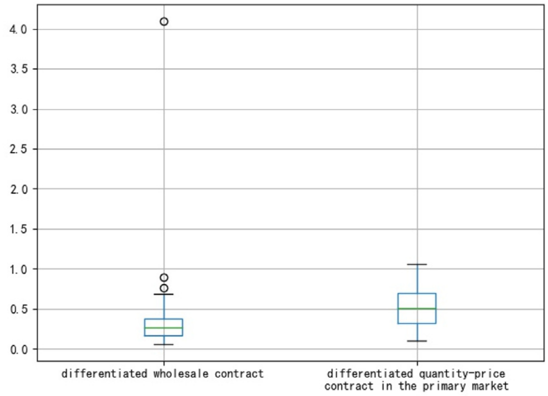

Fig. 3. Comparison of the firm's profit under three pricing strategies.

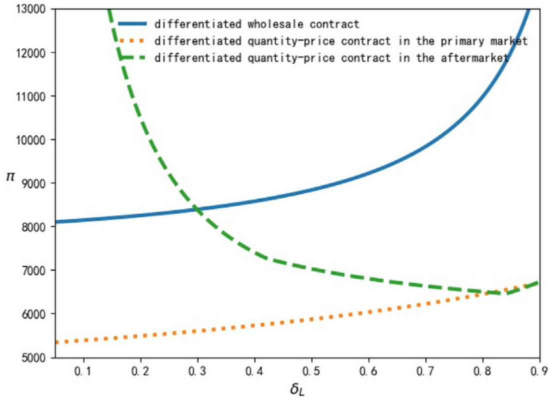

Fig. 4. The effect of myopia level of customers of type L on the profits of the firm using three strategies when customer myopia is heterogeneous.

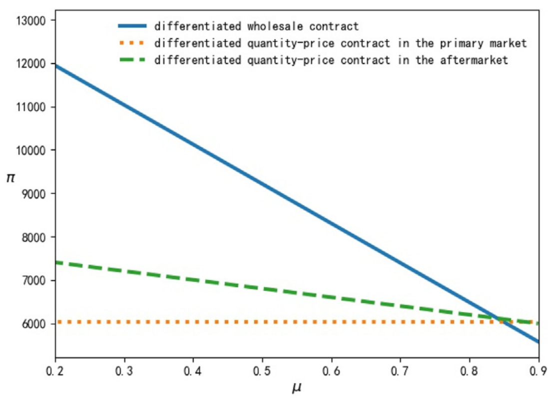

Fig. 5. The effect of the proportion of type L customers on the firm's profits using three strategies when customer myopia is heterogeneous.

they should choose a differentiated wholesale contract. The change in the firm's profit in Fig. 4 with myopic customers of type H is consistent with Fig. 2. This is because, in Fig. 4, we assume a constant proportion of customers of type L. Therefore, the increase in the myopia level of customers of type L pulls the average level of customer myopia in the market down, and the change in the firm's profit is the same as in the case in which customers are homogeneous.

Fig. 5 plots the effect of the percentage of customers of type L on the profits of the firm using different strategies. For comparison purposes, we use the same parameter values as in Fig. 4, namely, $\delta_H = 0.9$, $\mu = 0.5$, $A = 100$, $\theta = 0.3$, $\lambda = 0.9$, and $\gamma = 0.7$. In addition, we assume $\delta_L = 0.6$.

Fig. 5 shows that when the proportion of customers of type L increases, the firm's profits generated by any of the three strategies decreases. Among them, the profits of the firm using a differentiated wholesale contract in both markets decline the fastest, while those of the firm with a differentiated quantity–price contract in the primary market remain almost unchanged. This is because the increase in the proportion of type L customers has two effects on the firm. First, the average degree of customer myopia in the market declines, which is consistent with Figs. 2 and 4, lowering the profits of both the firm using a differentiated wholesale contract and the firm adopting a differentiated quantity–price contract in the primary market. At the same time, the profits of the firm adopting a differentiated quantity–price contract in the aftermarket increase. Second, for the firm using a differentiated wholesale contract, the rise in the proportion of type L customers in the market grants them non-zero utility, which results in a decrease in the firm's profit. For the firm using a differentiated quantity–price contract in the primary market, type H customers gain non-zero utility and thus the increase in the proportion of type L customers leads to a rise in the firm's profit. Therefore, the combined effect results in the synthetic effect depicted in Fig. 5. Indeed, when uncertainty in the market is strong, differentiated quantity–price contracts help the firm maintain a stable cash flow, thereby reducing its operating risks.

Gabaix and Laibson (2006) stated that if the firm's profit in the aftermarket is less than the distortion of social welfare, the firm has an incentive to educate customers to reduce their degree of myopia. However, Miao (2010) suggested that firms are not motivated to reduce the degree of customer myopia. The analysis in this study shows that whether a firm has an incentive to educate customers depends on the strategy it uses and the effect of education. If the firm adopts a differentiated wholesale contract or differentiated quantity–price contract in the primary market, it has an incentive to educate customers to attenuate their myopia level. However, if the firm uses a differentiated quantity–price contract in the aftermarket, it has an incentive to educate customers only if the effect of reducing the proportion of myopic customers through education exceeds the effect of reducing the expectation of the degree of customer myopia in the market.

Combining Proposition 7, Proposition 8, and Proposition 9, we can see that, when a firm uses a differentiated quantity–price contract in the primary market, type H customers will obtain greater utility, while, if the firm adopts either a differentiated wholesale contract or a differentiated quantity–price contract in the aftermarket, type L customers will gain greater utility. Therefore, even if the firm provides education in both cases, customers may be unwilling to receive education, which implies that education is ineffective in some cases.

8. Conclusion

This study models the relationship between product prices in the primary market and service prices in the aftermarket, revealing the root cause for overpricing in the aftermarket: customer myopia. We then examine the efficiency of different pricing strategies in both the primary market and the aftermarket for a profit-maximization firm. Three firm strategies when the degree of customer myopia is either homogeneous or heterogeneous are considered: a wholesale contract, a quantity–price contract in the primary market, and a quantity–price contract in the aftermarket. Our results show that a firm adopting a quantity–price contract in the aftermarket is more efficient than one using a quantity–price contract in the primary market. When the degree of customer myopia is homogeneous, it is better to agree a quantity–price contract in the aftermarket than a differentiated wholesale contract if the customer myopia level is high or the installed base effect is weak. When the degree of customer myopia is heterogeneous, the numerical analysis shows that, in most cases, it is most efficient for the firm to agree the quantity–price contract in the aftermarket.

Another focus of this study is on the firm's strategic decision to reduce the degree of customer myopia through education (e.g., publicizing the effectiveness of aftermarket services through advertising). Our results show that the firm's decision on whether to provide education depends on the strategy it uses and the impact of education. When the firm uses a differentiated wholesale contract or a quantity–price contract in the aftermarket, customers have no incentive to receive education. In this case, the firm should be more circumspect in making decisions.

Declaration of competing interest

The authors declare no conflict of interest.

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Appendix 1

Solve the first- and second-order derivatives of equation (2) with respect to q_1 and q_2:

$$\begin{cases} \dfrac{\partial U}{\partial q_1} = A - 2\gamma q_1 + \lambda q_2 - p_1 \\ \dfrac{\partial U}{\partial q_2} = \lambda q_1 - 2\theta q_2 - p_2 \\ \dfrac{\partial^2 U}{\partial {q_1}^2} = -2\gamma \\ \dfrac{\partial^2 U}{\partial {q_2}^2} = -2\theta \\ \dfrac{\partial^2 U}{\partial q_1 q_2} = \lambda \end{cases}$$

The Hessian matrix is

$$H_c = \begin{bmatrix} -2\gamma & \lambda \\ \lambda & -2\theta \end{bmatrix}$$

There exists an optimal solution to the problem of equation (2) if and only if the objective function is concave or the above Hessian matrix is negative semidefinite. We know the sufficient and necessary condition of a negative semidefinite Hessian matrix is that the second-order principal minor determinant is above zero, while the first-order principal minor determinant is below zero, which is

$$4\theta\gamma - \lambda^2 > 0$$

The optimal solution can be derived from the first-order condition:

$$\begin{cases} q_1^* = \dfrac{2\theta A - 2\theta p_1 - \lambda p_2}{4\theta\gamma - \lambda^2} \\ q_2^* = \dfrac{\lambda A - \lambda p_1 - 2\gamma p_2}{4\theta\gamma - \lambda^2} \end{cases}$$

Appendix 2

Solve the first- and second-order derivatives of equation (3) with respect to p_1 and p_2:

$$\begin{cases} \dfrac{\partial \pi}{\partial p_1} = \dfrac{1}{4\theta\gamma - \lambda^2}(2\theta A - 2\lambda p_2 - 4\theta p_1) \\ \dfrac{\partial \pi}{\partial p_2} = \dfrac{1}{4\theta\gamma - \lambda^2}(-2\lambda p_1 + \lambda A - 4\gamma p_2) \\ \dfrac{\partial^2 \pi}{\partial {p_1}^2} = \dfrac{-4\theta}{4\theta\gamma - \lambda^2} \\ \dfrac{\partial^2 \pi}{\partial {p_2}^2} = \dfrac{-4\gamma}{4\theta\gamma - \lambda^2} \\ \dfrac{\partial^2 \pi}{\partial p_1 p_2} = \dfrac{-2\lambda}{4\theta\gamma - \lambda^2} \end{cases}$$

The Hessian matrix is

$$H_f = \begin{bmatrix} \frac{-4\theta}{4\theta\gamma - \lambda^2} & \frac{-2\lambda}{4\theta\gamma - \lambda^2} \\ \frac{-2\lambda}{4\theta\gamma - \lambda^2} & \frac{-4\gamma}{4\theta\gamma - \lambda^2} \end{bmatrix}$$

H_f is always semidefinite if $4\theta\gamma - \lambda^2 > 0$. Therefore, there exists an optimal solution to the optimization problem of equation (3), which can be derived from the first-order condition:

$$\begin{cases} p_1^* = \frac{A}{2} \\ p_2^* = 0 \end{cases}$$

Appendix 3

Solve the first- and second-order derivatives of equation (5) with respect to q_1 and q_2:

$$\begin{cases} \frac{\partial U}{\partial q_1} = A - 2\gamma q_1 + \delta\lambda q_2 - p_1 \\ \frac{\partial U}{\partial q_2} = \delta\lambda q_1 - 2\delta\theta q_2 - \delta p_2 \\ \frac{\partial^2 U}{\partial {q_1}^2} = -2\gamma \\ \frac{\partial^2 U}{\partial {q_2}^2} = -2\delta\theta \\ \frac{\partial^2 U}{\partial q_1 q_2} = \delta\lambda \end{cases}$$

The Hessian matrix is

$$H_m = \begin{bmatrix} -2\gamma & \delta\lambda \\ \delta\lambda & -2\delta\theta \end{bmatrix}$$

There exists an optimal solution to the problem of equation (5) if and only if the objective function is concave or the above Hessian matrix is negative semidefinite. We know the sufficient and necessary condition of a negative semidefinite Hessian matrix is that the second-order principal minor determinant is above zero, while the first-order principal minor determinant is below zero, which is

$$4\theta\gamma - \delta\lambda^2 > 0$$

The optimal solution can be derived from the first-order condition:

$$\begin{cases} q_1^* = \frac{2\theta A - 2\theta p_1 - \delta\lambda p_2}{4\theta\gamma - \delta\lambda^2} \\ q_2^* = \frac{\lambda A - \lambda p_1 - 2\gamma p_2}{4\theta\gamma - \delta\lambda^2} \end{cases}$$

Appendix 4

The increase in the degree of customer myopia reduces q_2*. To investigate the impact of the degree of customer myopia on product consumption, solve the derivative of q_1* with respect to δ:

$$\frac{\partial {q_1}^*}{\partial \delta}=\frac{2\theta\left((A-p_1)\gamma^2-2\lambda^2 p_2\right)}{\left(4\theta\gamma-\delta\lambda^2\right)^2}$$

Letting the right-hand side of the equation equal zero, we can derive Proposition 4.

Appendix 5

Solve the first- and second-order derivatives of equation (6) with respect to q_1 and q_2:

$$\begin{cases} \dfrac{\partial \pi}{\partial p_1}=\dfrac{1}{4\theta\gamma-\delta\lambda^2}(2\theta A-\lambda p_2-\delta\lambda p_2-4\theta p_1) \\ \dfrac{\partial \pi}{\partial p_2}=\dfrac{1}{4\theta\gamma-\delta\lambda^2}(-\delta\lambda p_1-\lambda p_1+\lambda A-4\gamma p_2) \\ \dfrac{\partial^2 \pi}{\partial {p_1}^2}=\dfrac{-4\theta}{4\theta\gamma-\delta\lambda^2} \\ \dfrac{\partial^2 \pi}{\partial {p_2}^2}=\dfrac{-4\gamma}{4\theta\gamma-\delta\lambda^2} \\ \dfrac{\partial^2 \pi}{\partial p_1 p_2}=\dfrac{-\delta\lambda-\lambda}{4\theta\gamma-\delta\lambda^2} \end{cases}$$

The Hessian matrix is

$$H_{mf}=\begin{bmatrix} \dfrac{-4\theta}{4\theta\gamma-\lambda^2} & \dfrac{-\delta\lambda-\lambda}{4\theta\gamma-\delta\lambda^2} \\ \dfrac{-\delta\lambda-\lambda}{4\theta\gamma-\delta\lambda^2} & \dfrac{-4\gamma}{4\theta\gamma-\lambda^2} \end{bmatrix}$$

Since $\theta > 0$, if $4\theta\gamma - \lambda^2 > 0$, the first-order principal minor determinant is below zero. The second-order principal minor determinant is $\frac{4\theta\gamma-\frac{(1+\delta)^2}{4}\lambda^2}{(4\theta\gamma-\delta\lambda^2)^2}$. Since $\delta < 1$, the above expression is always above zero. Therefore, Hm_f is semidefinite. There exists an optimal solution to the optimization problem of equation (6), which can be derived from the first-order condition:

$$\begin{cases} p_1^*=\dfrac{\left(8\theta\gamma-(1+\delta)^2\lambda^2\right)A}{16\theta\gamma-(1+\delta)^2\lambda^2} \\ p_2^*=\dfrac{2(1-\delta)\theta\lambda A}{16\theta\gamma-(1+\delta)^2\lambda^2} \end{cases}$$

Appendix 6

Customers choose service consumption in the aftermarket to maximize their utility. Solve the first- and second-order partial derivatives of the customer's utility with respect to service consumption q_2:

$$\begin{cases} \dfrac{\partial U}{\partial q_2}=\delta\lambda q_1-2\delta\theta q_2-\delta p_2 \\ \dfrac{\partial^2 U}{\partial {q_2}^2}=-2\delta\theta \end{cases}$$

It is easy to prove that the customer's utility is a concave function of service consumption. Therefore, the best response function of the customer is determined by the first-order condition:

$$q_2{}^* = \frac{\lambda q_1 - p_2}{2\theta}$$

Substituting the above equation into the original problem, the firm's problem can be expressed as

$$\max_{p_1,q_1,p_2} \pi = p_1 q_1 + \frac{\lambda p_2 q_1 - p_2^2}{2\theta} \tag{A1}$$

$$s.t.\begin{cases} Aq_1 - \gamma q_1^2 - p_1 q_1 + \dfrac{\delta}{4\theta}\left(\lambda q_1 - p_2^2\right)^2 \geq 0 \quad (A-2) \\ q_1 \leq \dfrac{A}{2\gamma} \quad (A-3) \end{cases}$$

Equation (A-1) is an increasing function of p_1. Therefore, the constraint (A-2) must be binding. The objective function can be rewritten as

$$\max_{q_1,p_2} \pi = q_1\left(Aq_1 - \gamma q_1{}^2 + \frac{\delta}{4\theta}\left(\lambda q_1 - p_2^2\right)^2\right) + \frac{\lambda p_2 q_1 - p_2^2}{2\theta}$$

Solve the first- and second-order derivatives of the above expression with respect to q_1 and p_2:

$$\begin{cases} \dfrac{\partial \pi}{\partial q_1} = A - 2\gamma q_1 + \dfrac{\delta\lambda^2}{2\theta} q_1 - \dfrac{\delta\lambda}{2\theta} p_2 + \dfrac{\lambda}{2\theta} p_2 \\ \dfrac{\partial \pi}{\partial p_2} = -\dfrac{\delta\lambda}{2\theta} q_1 + \dfrac{\delta}{2\theta} p_2 + \dfrac{\lambda}{2\theta} q_1 - \dfrac{1}{\theta} p_2 \\ \dfrac{\partial^2 \pi}{\partial q_1{}^2} = -2\gamma + \dfrac{\delta\lambda^2}{2\theta} \\ \dfrac{\partial^2 \pi}{\partial p_2{}^2} = \dfrac{\delta}{2\theta} - \dfrac{1}{\theta} \\ \dfrac{\partial^2 \pi}{\partial q_1 p_2} = -\dfrac{\delta\lambda}{2\theta} + \dfrac{\lambda}{2\theta} \end{cases}$$

The Hessian matrix is

$$H_{ff} = \begin{bmatrix} -2\gamma + \dfrac{\delta\lambda^2}{2\theta} & -\dfrac{\delta\lambda}{2\theta} + \dfrac{\lambda}{2\theta} \\ -\dfrac{\delta\lambda}{2\theta} + \dfrac{\lambda}{2\theta} & \dfrac{\delta}{2\theta} - \dfrac{1}{\theta} \end{bmatrix}$$

There exists an optimal solution to the optimization problem if and only if H_{ff} is semidefinite or

$$\begin{cases} -2\gamma + \dfrac{\delta\lambda^2}{2\theta} < 0 \\ \left(-2\gamma + \dfrac{\delta\lambda^2}{2\theta}\right)\left(\dfrac{\delta}{2\theta} - \dfrac{1}{\theta}\right) - \left(-\dfrac{\delta\lambda}{2\theta} + \dfrac{\lambda}{2\theta}\right)^2 > 0 \end{cases}$$

Solve the above equation group:

$$4(2-\delta)\theta\gamma - \delta\lambda^2 > 0$$

The local optimal solution can be derived from the first-order condition:

$$q_1 = \frac{2(2-\delta)\theta A}{4(2-\delta)\theta\gamma - \lambda^2}$$

However, the above local optimal solution does not satisfy constraint (A-3). Therefore, constraint (A-3) must be binding. The original problem can be rewritten as

$$\max_{p_2} \pi = \frac{A}{2\gamma}\left(\frac{A^2}{2\gamma} - \gamma\left(\frac{A}{2\gamma}\right)^2 + \frac{\delta}{4\theta}\left(\frac{\lambda A}{2\gamma} - p_2^2\right)^2\right) + \frac{\frac{\lambda A p_2}{2\gamma} - p_2^2}{2\theta}$$

Solve the above problem:

$$\begin{cases} {p_1}^* = \dfrac{\left(4(2-\delta)\theta\gamma + \delta\lambda^2\right)A}{8\left(2-\delta^2\right)\theta\gamma} \\ {p_2}^* = \dfrac{(1-\delta)\lambda A}{2(2-\delta)\gamma} \\ {q_1}^* = \dfrac{A}{2\gamma} \end{cases}$$

Appendix 7

Customers choose product consumption in the aftermarket to maximize their utility. Solve the first- and second-order partial derivatives of the customer's utility with respect to product consumption q_1:

$$\begin{cases} \dfrac{\partial U}{\partial q_1} = A - 2\gamma q_1 - p_1 + \delta\lambda q_2 \\ \dfrac{\partial^2 U}{\partial {q_1}^2} = -2\gamma \end{cases}$$

It is easy to prove that the customer's utility is a concave function of product consumption. Therefore, the best response function of the customer is determined by the first-order condition:

$$q_1^* = \frac{A - p_1 + \delta\lambda q_2}{2\gamma} \tag{A4}$$

Further, to ensure the monotonicity of the customer's utility function, the solution must satisfy

$$p_1 \geq \delta\lambda q_2 \tag{A5}$$

Substituting equation (A-4) into the original problem, the firm's problem can be expressed as

$$\max_{p_1, p_2, q_2} \pi = p_1 \frac{A - p_1 + \delta\lambda q_2}{2\gamma} + p_2 q_2 \tag{A6}$$

$$s.t.\ \frac{1}{4\gamma}(\delta\lambda q_2 + A - p_1)^2 - \delta\theta q_2^2 - \delta p_2 q_2 \geq 0 \tag{A7}$$

Equation (A-6) is an increasing function of p_2. Therefore, the constraint (A-7) must be binding. The objective function can be rewritten as

$$\max_{p_1, q_2} \pi = \frac{A p_1 - p_1^2 + \delta\lambda q_2 p_1}{2\gamma} + \frac{1}{4\delta\gamma}(\delta\lambda q_2 + A - p_1)^2 - \theta q_2^2$$

Solve the partial derivative of the above expression with respect to p_1:

$$\frac{\partial \pi}{\partial p_1} = -\frac{\lambda}{2\gamma}(1-\delta)q_2 + \frac{1}{2\gamma\delta}\left(-(1-\delta)A - \delta p_1\right)$$

Since the above expression is always below zero, constraint (A-5) must be binding. Substitute constraint (A-5) into the original problem:

$$\max_{q_2} \pi = \frac{A\delta\lambda q_2}{2\gamma} + \frac{A^2}{4\delta\gamma} - \theta q_2^2$$

Solve the above problem:

$$\begin{cases} p_1{}^* = \dfrac{\delta^2\lambda^2 A}{4\theta\gamma} \\ p_2{}^* = \dfrac{4\gamma\theta - \delta^3\lambda^2 A}{4\delta^2\gamma\lambda} \\ q_2{}^* = \dfrac{\delta\lambda A}{4\theta\gamma} \end{cases}$$

Appendix 8

From equation (8) and equation (9):

$$\frac{\pi_{af}}{\pi_{ff}} = \frac{(\delta^3\lambda^2 + 4\theta\gamma)(2-\delta)}{(\lambda^2 + 4(2-\delta)\theta\gamma)\delta}$$

Let the above expression be greater than one:

$$\frac{(\delta^3\lambda^2 + 4\theta\gamma)(2-\delta)}{(\lambda^2 + 4(2-\delta)\theta\gamma)\delta} > 1$$

Simplify the above expression:

$$4\theta\gamma - \frac{1-\delta-\delta^2}{2-\delta}\delta\lambda^2 > 0$$

From Proposition 5, $4\theta\gamma - \delta\lambda^2 > 0$. When $\delta < 1$, the above expression always holds. That is, $\frac{\pi_{af}}{\pi_{ff}} > 1$. When $\delta=1$, the left-hand side of the above expression equals zero. That is, $\frac{\pi_{af}}{\pi_{ff}} = 1$.

Appendix 9

From equation (5) and equation (7):

$$\pi_{mf} - \pi_{af} = \frac{2\theta A^2}{16\theta\gamma - (1+\delta)^2\lambda^2} - \frac{(\delta^3\lambda^2 + 4\theta\gamma)A^2}{16\delta\theta\gamma^2}$$

Let the above expression be than zero:

$$\frac{2\theta A^2}{16\theta\gamma - (1+\delta)^2\lambda^2} - \frac{(\delta^3\lambda^2 + 4\theta\gamma)A^2}{16\delta\theta\gamma^2} < 0$$

Simplify the above expression:

$$32\theta^2\gamma^2 - \left(\delta^3\lambda^2 + 4\theta\gamma\right)\left(16\theta\gamma - (1+\delta)^2\lambda^2\right) < 0$$

Appendix 10

Substitute equation (11) into equation (14):

$$v_L \geq v_H + (\delta_H - \delta_L)(\lambda A - \lambda p_{1H} - \gamma p_{2H})p_{2H}$$

In equation (10), $q_{2x} \geq 0$ or

$$\frac{\lambda A - \lambda p_{1x} - 2\gamma p_{2x}}{4\theta\gamma - \delta_x \lambda^2} \geq 0$$

Therefore,

$$\lambda A - \lambda p_{1x} - \gamma p_{2x} \geq \lambda A - \lambda p_{1x} - 2\gamma p_{2x} \geq 0$$

That is,

$$v_L \geq v_H$$

Appendix 11

Substitute equation (16) into equation (20):

$$v_H \geq v_L + \frac{(\delta_H - \delta_L)(\lambda q_{1L} - p_2)^2}{4\theta} \tag{A8}$$

Since the second term on the right-hand side of the expression is always above or equal to zero:

$$v_H \geq v_L$$

Substitute equation (16) into the original problem:

$$\begin{aligned} \max_{q_{1L}, q_{1H}, p_2, v_L, v_H} \pi = \mu \left(A q_{1L} - \gamma {q_{1L}}^2 + \frac{\delta_L}{4\theta}(\lambda q_{1L} - p_2)^2 + \frac{p_2 \lambda q_{1L} - p_2^2}{2\theta} - v_L \right) \\ + (1-\mu)\left(q_{1H} - \gamma {q_{1H}}^2 + \frac{\delta_H}{4\theta}(\lambda q_{1H} - p_2)^2 + \frac{p_2 \lambda q_{1H} - p_2^2}{2\theta} - v_H \right) \end{aligned} \tag{A9}$$

The objective function is a decreasing function of v_H and v_L. Therefore, constraint (15) and constraint (A-8) must be binding. That is,

$$\begin{cases} v_L = 0 \\ v_H = \dfrac{(\delta_H - \delta_L)(\lambda q_{1L} - p_2)^2}{4\theta} \end{cases}$$

Substituting the above equation group into equation (A-9), the original problem can be expressed as

$$\begin{aligned} \max_{q_{1L}, q_{1H}, p_2} \pi = \mu \left(A q_{1L} - \gamma {q_{1L}}^2 + \frac{\delta_L}{4\theta}(\lambda q_{1L} - p_2)^2 + \frac{p_2 \lambda q_{1L} - p_2^2}{2\theta} \right) + (1-\mu)\left(q_{1H} - \gamma {q_{1H}}^2 + \frac{\delta_H}{4\theta}(\lambda q_{1H} - p_2)^2 + \frac{p_2 \lambda q_{1H} - p_2^2}{2\theta} \right. \\ \left. - \frac{\delta_H - \delta_L}{4\theta}(\lambda q_{1L} - p_2)^2 \right) \end{aligned}$$

Appendix 12

Substitute equation (21) into equation (24) and equation (25):

$$\begin{cases} v_L \geq \dfrac{(\delta_L\lambda q_{2H}+A-p_1)^2}{4\gamma} - \delta_L\theta q_{2H}^2 + \dfrac{\delta_L}{\delta_H}\left(u_H - \dfrac{(\delta_H\lambda q_{2H}+A-p_1)^2}{4\gamma} + \delta_H\theta q_{2H}^2\right) \\ v_H \geq \dfrac{(\delta_H\lambda q_{2L}+A-p_1)^2}{4\gamma} - \delta_H\theta q_{2L}^2 + \dfrac{\delta_H}{\delta_L}\left(u_L - \dfrac{(\delta_L\lambda q_{2L}+A-p_1)^2}{4\gamma} + \delta_L\theta q_{2L}^2\right) \end{cases}$$

Simplify the above equation group:

$$\begin{cases} \delta_H v_L - \delta_L v_H \geq \dfrac{1}{4\gamma}(\delta_H-\delta_L)\left((A-p_1)^2 - \delta_H\delta_L\lambda^2 q_{2H}^2\right) \quad (A-10) \\ \delta_L v_H - \delta_H v_L \geq -\dfrac{1}{4\gamma}(\delta_H-\delta_L)\left((A-p_1)^2 - \delta_H\delta_L\lambda^2 q_{2L}^2\right) \quad (A-11) \end{cases}$$

Substitute equation (21) into the original problem:

$$\max_{q_{2L},q_{2H},p_1} \pi = \mu\left(\frac{Ap_1 - p_1^2 + \delta_L\lambda p_1 q_{2L}}{2\gamma} + \frac{(\delta_L\lambda q_{2L}+A-p_1)^2}{4\delta_L\gamma} - \theta q_{2L}^2 - \frac{v_L}{\delta_L}\right) + (1-\mu)\left(\frac{Ap_1 - p_1^2 + \delta_H\lambda p_1 q_{2H}}{2\gamma} + \frac{(\delta_H\lambda q_{2H}+A-p_1)^2}{4\delta_H\gamma} - \theta q_{2H}^2 - \frac{v_H}{\delta_H}\right)$$

The objective function is a decreasing function of v_H and v_L. Therefore, constraint (22) and/or constraint (23) must be binding. However, since constraint (A-10) and constraint (A-11) cannot be satisfied at the same time (unless q_{2L}=q_{2H}), there are two possibilities for the optimal solution:

$$\begin{cases} \delta_H v_L = \dfrac{1}{4\gamma}(\delta_H-\delta_L)\left((A-p_1)^2 - \delta_H\delta_L\lambda^2 q_{2H}^2\right) \\ v_H = 0 \end{cases}$$

That is,

$$\begin{cases} A - p_1 - \sqrt{\delta_L\delta_H}\lambda q_{2H} \geq 0 \\ q_{2L} \leq q_{2H} \end{cases}$$

Or

$$\begin{cases} \delta_L v_H = \dfrac{1}{4\gamma}(\delta_H-\delta_L)\left((A-p_1)^2 - \delta_H\delta_L\lambda^2 q_{2L}^2\right) \\ v_L = 0 \end{cases}$$

That is,

$$\begin{cases} A - p_1 - \sqrt{\delta_L\delta_H}\lambda q_{2L} \leq 0 \\ q_{2L} \leq q_{2H} \end{cases}$$

The extreme situation when constraint (A-10) and constraint (A-11) are satisfied at the same time (q_{2L}=q_{2H}) is the boundary solution to the two optimization problems.

Appendix 13

For Strategy 1 in Lemma 1, the partial derivative of the firm's profit with respect to product consumption is

$$\frac{\partial\pi}{\partial p_1} = \frac{1}{2\gamma\delta_L}\left(-\delta_L(p_1 - \delta_L\lambda\mu q_{2L}) - ((1-\delta_H)(1-\mu)q_{2H} - \mu q_{2L})\lambda\delta_L - (1-\delta_L)(A-p_1)\right)$$

Since each term in the brackets is negative, the optimal solution of p_1 must be on the boundary:

$$p_1 = A - \sqrt{\delta_L \delta_H}\lambda q_{2L}$$

Therefore, the optimization problem can be expressed as

$$\max_{q_{2L},q_{2H}} \pi = \mu\left(\frac{Ap_1 - p_1^2 + \delta_L \lambda p_1 q_{2L}}{2\gamma} + \frac{(\delta_L \lambda q_{2L} + A - p_1)^2}{4\delta_L \gamma} - \theta q_{2L}^2\right) + (1-\mu)\left(\frac{Ap_1 - p_1^2 + \delta_H \lambda p_1 q_{2H}}{2\gamma} + \frac{(\delta_H \lambda q_{2H} + A - p_1)^2}{4\delta_H \gamma} - \theta q_{2H}^2\right)$$

$$s.t. \begin{cases} p_1 = A - \sqrt{\delta_L \delta_H}\lambda q_{2L} \\ q_{2L} \le q_{2H} \end{cases}$$

The boundary solution ($p_1 = A - \sqrt{\delta_L \delta_H}\lambda q_{2H}$) of Strategy 2 in Lemma 1 can be expressed as

$$\max_{q_{2L},q_{2H}} \pi = \mu\left(\frac{Ap_1 - p_1^2 + \delta_L \lambda p_1 q_{2L}}{2\gamma} + \frac{(\delta_L \lambda q_{2L} + A - p_1)^2}{4\delta_L \gamma} - \theta q_{2L}^2\right) + (1-\mu)\left(\frac{Ap_1 - p_1^2 + \delta_H \lambda p_1 q_{2H}}{2\gamma} + \frac{(\delta_H \lambda q_{2H} + A - p_1)^2}{4\delta_H \gamma} - \theta q_{2H}^2\right)$$

$$s.t. \begin{cases} p_1 = A - \sqrt{\delta_L \delta_H}\lambda q_{2H} \\ q_{2L} \le q_{2H} \end{cases}$$

Since the objective function is non-increasing in p_1, the profit of the boundary solution of Strategy 2 is above or equal to the profit of Strategy 1. In other words, Strategy 1 is weakly dominated by Strategy 2.

References

Asian Development Bank. (2017). *Key Indicators for Asia and the Pacific 2014* (p. 146). Asian Development Bank. Available at: https://www.adb.org/publications/key-indicators-asia-and-pacific-2017.
Bates, K., Bates, H., & Johnston, R. (2013). Linking service to profit: the business case for service excellence. *Int. J. Serv. Ind. Manag., 14*(14), 173–183.
Bikfalvi, A., Lay, G., Maloca, S., et al. (2013). Servitization and networking: large-scale survey findings on product-related services. *Service Business, 7*(1), 61–82.
Borchardt, M., Souza, M., Pereira, G. M., et al. (2018). Achieving better revenue and customers' satisfaction with after-sales services: how do the best branded car dealerships get it? *Int. J. Qual. Reliab. Manag., 35*(3), 1686–1708.
Borenstein, S., MacKie-Mason, J. K., & Netz, J. S. (1994). Antitrust policy in aftermarkets. *Antitrust Law J., 60*(3), 455–482.
Calzada, J., & Valletti, T. M. (2012). *Intertemporal Movie Distribution: Versioning when Customers Can Buy Both versions*. INFORMS.
Carlton, D. W., & Waldman, M. (2010). Competition, monopoly, and aftermarkets. *J. Law Econ. Organ., 26*(1), 54–91.
Cohen, M. A., Agrawal, N., & Agrawal, V. (2006). Winning in the aftermarket. *Harv. Bus. Rev., 84*(5), 129–138.
Emch, E. R. (2003). Price discrimination via proprietary aftermarkets. *Contrib. Econ. Anal. Pol., 2*(1), 1063–1063.
Ferrer, J. C., Mora, H., & Olivares, F. (2010). On pricing of multiple bundles of products and services. *Eur. J. Oper. Res., 206*(1), 197–208.
Gabaix, X., & Laibson, D. (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *Q. J. Econ., 121*(2), 505–540.
Goldfine, D. A. J., & Vorrasi, K. M. (2004). The fall of the *Kodak* aftermarket doctrine: dying a slow death in the lower courts. *Antitrust Law J., 72*(1), 209–231.
Kurata, H., & Nam, S. H. (2010). After-sales service competition in a supply chain: optimization of customer satisfaction level or profit or both? *Int. J. Prod. Econ., 127*(1), 136–146.
Li, J. H., Lin, L., Chen, D. P., et al. (2015). An empirical study of servitization paradox in China. *J. High Technol. Manag. Res., 26*(1), 66–76.
Li, M., Feng, H., Chen, F., et al. (2013). Optimal versioning strategy for information products with behavior-based utility function of heterogeneous customers. *Comput. Oper. Res., 40*(10), 2374–2386.
Liang, L., Xie, J., Liu, L., et al. (2017). Revenue sharing contract coordination of wind turbine order policy and aftermarket service based on joint effort. *Ind. Manag. Data Syst., 117*(2), 320–345.
Liu, Y., Li, J., Ren, W., et al. (2019). Differentiated products pricing with consumer network acceptance in a dual-channel supply chain. *Electron. Commer. Res. Appl., 39*, Article 100915.
Luo, Z., Chen, X., Chen, J., et al. (2017). Optimal pricing policies for differentiated brands under different supply chain power structures. *Eur. J. Oper. Res., 259*, 437–451.
Mas-Colell, A., Whinston, M. D., & Green, J. R. (2005). *Microeconomic theory*. Shanghai University of Finance & Economics Press.
Miao, C. H. (2010). Consumer myopia, standardization and aftermarket monopolization. *Eur. Econ. Rev., 54*(7), 931–946.
Shapiro, C., & Varian, H. R. (1998). Versioning: the smart way to sell information. *Harv. Bus. Rev., 76*(6), 106–114.
Zhang, S., Dan, B., & Zhou, M. (2019). After-sale service deployment and information sharing in a supply chain under demand uncertainty. *Eur. J. Oper. Res., 279*(2), 351–363.
Zēgners, D., & Kretschmer, T. (2017). Competition with aftermarket power when consumers are heterogeneous. *J. Econ. Manag. Strat., 26*(1), 96–122.