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Integration of development and advertising strategies for multi-attribute products under competition

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ABSTRACT

In recent years, an increasing number of competing firms have started to integrate their product attribute development and advertising strategies by targeted advertising of the best-developed attribute of their product. With a focus on such integration and its potential effects, we establish an analytical model in which two competing firms sell multi-attribute products. Each firm can choose which attribute to focus on in its development strategy and which attribute to emphasize in its advertising strategy. By optimizing the development and advertising strategies of these firms, we first verify the integration of these two strategies as each firm advertises only its developed attribute. Second, we find that advertising, according to the firms' development choices, plays distinctive roles on profit generation: the difference-alleviating or difference-strengthening effect. Therefore, when determined endogenously, the advertising intensities show qualitatively different changing trends: a monotonous curve in the same-attribute-development case but an inverted-U shaped curve in the different-attribute-development case. Third, we find advertising establishes a connection between firms' attribute development choices, making them depending not only on their own development cost. A smaller difference between the development costs of these firms or a higher advertising efficiency increases the firms' willingness to develop a same attribute. We also extend our analysis to the flexible quality level of the non-developed attribute, asymmetric substitution degrees between attributes and prices, or the endogenous attribute development levels, and study how the interactions between the product development and advertising are affected. This paper highlights the importance of making development and advertising strategies jointly.

1. Introduction

While developing and producing products, an increasing number of production firms have started to launch advertising campaigns to enhance the competitiveness and market demand of their products. For instance, through aggressive advertising, Oppo and Vivo, two early, little-known phone makers, have steadily developed a strong presence in several markets, such as China, India, and Southeast Asia. A new report from IDC Quarterly Mobile Phone Tracker revealed that through advertising, Oppo's annual sales in China increased to 78.4 million, up 122 percent year-on-year. At the same time, Vivo achieved 69.2 million shipments, up 97 percent from the previous year (IDC, 2017; Russel, 2017). These companies account for nearly one-third of the Chinese smartphone market, which is the world's largest based on sales (IDC, 2017). Following this phenomenon, Russel (2017) argues that aggressive advertising campaigns have become one of the most important tools available to companies for stimulating product demand.

In their advertising campaigns, both Oppo and Vivo emphasize only a single key, well-developed product attribute. For example, for its new product, Vivo V5 Plus, Vivo advertises only its advanced camera, which is the most advantageous, well-developed attribute of this smartphone. Accordingly, some media, JIEMIAN (2018), claimed that Vivo's focused advertisement shifts consumers' attention to the photography function of smartphones, which is the most competitive product attribute for Vivo V5 Plus, thereby highlighting the major advantage of this smartphone and using advertisements to maximize the product competitiveness. A production firm's decision to integrate the key attribute of their product development and the focus of their advertising strategy may have a mutual promotion effect. In other words, advertisement increases the advantage of the major attribute of a product relative to others, and in turn, the high competitiveness of a product attribute enhances the role of advertising.

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Related topics are also discussed in academia. Many empirical and experimental studies have shown that advertising can play a key role in influencing consumers' preference formation and product evaluation (Ayanwale, Alimi, & Ayanbimipe, 2005; D'Souza & Rao, 1995; Hoch & Ha, 1986; Tellis, 1988). Carpenter & Nakamoto (1989, 1990) and Zajonc (1968) suggest that advertising changes consumer preference by affecting their tastes for the target product attributes through repeated exposure. These studies verify the existence of a relationship between advertisement and product attributes. That is, an advertisement works by changing the consumers' evaluation of a product attribute. However, the interaction between the product attribute development strategy and the advertising investment strategy has not received much discussion in the literature, even if these strategies can be determined endogenously by firms. How are the profits of competitive firms affected by these potential interactions? Moreover, how do these companies optimize these decisions?

Various attribute-decision outcomes for competing firms can be observed in the market. In many cases, competing firms have an advantage in different aspects of the R&D process that drives them to choose those attributes they specialize in and subsequently produce a different-attribute outcome. For instance, BMW, one of the two dominating competitors in the U.S. premium car market, holds a relatively strong position in the performance dimension, whereas Lexus is positioned strongly in the comfort dimension (Joshi, Reibstein, & Zhang, 2015). Advertising Age reports that since 1975, BMW has strived to advertise its vehicles as "the ultimate driving machine" (Ries, 2005). Additionally, since its U.S. launch in 1989, Lexus has advertised its "relentless pursuit of perfection" with a focus on those attributes at the heart and soul of its brand, namely, comfort, quality, and dependability (Halliday, 1998). Additionally, Edmunds.com, an independent and popular automotive information website, argues that BMW offers "superior levels of driving enjoyment",[1] whereas Lexus offers "utterly refined luxury vehicles".[2] Nevertheless, in some cases, competing firms also choose the same attribute to develop and advertise. For instance, as mentioned above, both Oppo and Vivo chose the photography function, specifically, the selfie cameras, of their smartphones (e.g., "Soft light selfie to brighten your beauty" in their advertising slogans JIEMIAN, 2018), as a common key point in their development and advertising strategies, thereby leading to a same-attribute outcome. These various attribute equilibria highlight the differences in competing firms' attribute decisions, the difficulty in making decisions, and the importance of conducting research.

Based on these considerations, we try to answer several questions. First, how do the competing firms make attribute choices in their development and advertising strategies? Second, how does product attribute development affect the optimal advertising strategies? Third, how does the presence of advertising investment influence attribute choices in new product development? Fourth, what factors will affect the interactions between attribute development and advertising? If so, how?

To answer these questions, we establish a model in which two competing firms determine their product attribute development and advertising strategies for their respective products. Each product has two main attributes. Each firm can choose only one product attribute to focus on for its development strategy and one attribute to emphasize in its advertising strategy. Based on this framework, regarding the development strategy, we further assume that one firm (the low-cost firm) has a lower development cost than the other (the high-cost firm). Regarding the advertising strategy, we establish that both firms have the same efficient advertising cost and that each can endogenously determine its advertising intensity by paying a fixed investment cost. After deriving and analyzing each of these firms' decisions and profits, we generate some novel managerial insights.

First, we verify that each firm chooses to integrate its development and advertising strategies by advertising only its developed product attribute. In this case, integration plays a mutual promoting role in stimulating demand. This integration further qualitatively changes the competing firms' attribute choices in their product development and advertisement strategies.

Second, we find that advertising, according to the firms development choices, plays qualitatively distinctive roles in profit generation. When firms choose different attributes, advertising becomes another source of competition in addition to price. Therefore, a highly efficient advertising investment has a difference-strengthening effect by enlarging the profit difference between low- and high-cost firms. By contrast, when firms choose the same attribute, advertising makes the commonly developed attribute preferable to consumers, thereby exerting a difference-alleviating effect on the profits of these firms. Consequently, when determined endogenously, the advertising intensities demonstrate qualitatively different changing trends. For instance, the high-cost firm's advertising intensity decreases monotonically along with investment costs in the same-attribute case but initially increases and then decreases in the different-attribute case. Such increases merely result from the negative difference-strengthening effect for the high-cost, disadvantaged firm.

Third, regarding the competing firms' attribute development choices, we propose that while choices depend only on the firms' own development cost in the absence of advertising, advertising establishes a connection between two competitors, thereby making their attribute decisions related to not only their own cost but also that of their rival. Especially, we find that the low-cost, advantaged firm prefers to develop the same attribute when the high-cost firm's cost disadvantage is small. This is because that the low-cost firm is concerned with the inevitable free ride in advertising investment and prefers to develop the same attribute only when the other firm's disadvantage is small. Moreover, when establishing a connection, advertising per se affects the firms' attribute choices. A lower advertising efficiency increases the low-cost firm's incentive to develop an attribute different from that of its rival. With inefficient advertising, the firm suffers more from the free ride and prefers to develop a different attribute.

Fourth, we find that the interactions between the attribute development and advertising are influenced by many factors. For instance, we present an interesting insight that the substitution degrees of the attributes and prices have opposite effects on the equilibrium results. While the substitution degree of attributes decreases on the effect of advertising on the firms' attribute choices and the equilibrium results, that of prices increases the effect of advertising. Besides, we also find that the reduced quality difference between the developed and nondeveloped attributes decreases the effectiveness of advertising on firms' attribute choices; specifically, it makes the different-attribute development cases more likely to arise in equilibrium.

The rest of this paper is organized as follows. Section 2 reviews the related literature. In Section 3, we make assumptions and establish our model. Section 4 analyzes the firms' decisions and profits under the combined effects of product development and advertising. Section 5 explores these competing firms' optimal advertising strategies under different attribute-development cases. Section 6 studies these firms' preferences in attribute development under the effects of advertising. Section 7 extends our results by relaxing some assumptions. Section 8 concludes our work.

[1] http://www.edmunds.com/bmw.
[2] http://www.edmunds.com/lexus.

2. Literature review

Our research lies at the intersection among (i) the competing firms' multi-attribute product development, (ii) advertising under competition, and (iii) joint decisions in product development and advertising. We describe in the following sections how our work relates to the literature in these areas.

2.1. Literature on the competitors' multi-attribute product development

Our work contributes to the literature on the competing firms' multi-attribute product development. Previous studies have mainly focused on the multi-attribute product differentiation strategy, which can be classified into vertical-differentiated attribute development (Garella & Lambertini, 2014; Lauga & Ofek, 2011; Vandenbosch & Weinberg, 1995; Zia & Kumar, 2017), horizontal-differentiated attribute development (Barigozzi & Ma, 2018; Irmen & Thisse, 1998), and vertical-and-horizontal-differentiated attribute development (Degryse & Irmen, 2001) according to the types of differentiated attributes. Our study relates to the first stream of literature, which assumes that the attributes developed by competing firms are differentiated vertically, that is, differentiated in the quality dimension. In this stream, Vandenbosch & Weinberg (1995) focus on the product and price competitions in a two-dimensional vertical differentiation model by assuming the marginal costs independent of attribute levels chosen. Lauga & Ofek (2011) discuss how firms manage competition by choosing the development levels for multiple attributes. They assume that marginal costs are increasing in quality but that the marginal costs for the two attributes are the same. Garella & Lambertini (2014) focus on the bidimensional vertical attribute development problem by assuming the same development cost for both firms and attributes. Zia & Kumar (2017) extend the two-attribute product setting to that of a three-attribute product.

We compare our work with the previous literature in terms of model setup and results. First, while focusing on the attribute-level competition in a multi-attribute setting, previous works usually assume no marginal cost (Irmen & Thisse, 1998; Vandenbosch & Weinberg, 1995) or the same cost for all of the attributes (Barigozzi & Ma, 2018; Garella & Lambertini, 2014; Lauga & Ofek, 2011), ignoring the effects of different development costs. In contrast, we establish a model that captures the differentiated costs for two firms in two attributes and analyzes development cost roles. Second, we obtain some interesting findings when we consider another key factor, namely, advertising investment. While previous works show that competing firms often differ in one (Barigozzi & Ma, 2018; Irmen & Thisse, 1998) or several (Garella & Lambertini, 2014; Lauga & Ofek, 2011; Vandenbosch & Weinberg, 1995; Zia & Kumar, 2017) attributes, we find that, with consideration of advertising, competitive firms may choose to develop the same attribute; that is, they do not differ in either attribute. This finding can help explain some realities in the manufacturing and electronics industries. Additionally, in contrast to previous studies, we also show that a firm's attribute development strategy depends on not only its own cost but also that of its rivals in the presence of advertisement.

2.2. Literature on advertising under competition

Another related body of literature is research on advertising, which can be divided into informative view, complementary view, and persuasive view (Bagwell, 2007; Chen, Joshi, Raju, & Zhang, 2009; Shi, Liu, & Petruzzi, 2019) according to its specifical roles. First, the informative view posits that advertising works by increasing consumer awareness and reducing search costs. Second, the complementary view posits that advertising provides an additional utility to consumers. Third, the persuasive view posits that advertising can alter consumers' preferences and valuations to achieve mostly spurious product differentiation. Specifically, our study relates to the persuasive view and examines the question of how persuasive advertising affects competition and interacts with product development. Previous studies in this stream focus mainly on how advertising affects product differentiation (or substitutability) (Fehr & Stevik, 1998; Shaffer & Zettelmeyer, 2004) and competition (Bloch & Manceau, 1999; Chen et al., 2009). In terms of product differentiation, Fehr & Stevik (1998) discuss how advertising affects perceived product difference, changes in ideal product variety, and consumers' willingness to pay. Shaffer & Zettelmeyer (2004) examine the relationship between advertising level and product substitutability by assuming that advertising can influence consumers' transportation cost in the Hotelling model. In terms of product competition, researchers find that persuasive advertisements may reduce or intensify competition. Bloch & Manceau (1999) investigate the effect of advertising when this factor induces a shift between brands without increasing demand. By assuming that advertising can change the distribution of consumer ideal points in the Hotelling model, they find that advertising can increase the advertised product's price when both products are sold by the same firm but reduce the price when these products are sold by different firms. Chen et al. (2009) focus on the effects of advertising on the competition by assuming that advertising acts as a force that pulls consumers closer to a firm. They find that the advertising can lead to an anti-competitive or pro-competitive outcome (even a prisoner's dilemma) depending on consumer response.

Our work addresses the question of how persuasive advertising affects competition. Bloch & Manceau (1999) and Chen et al. (2009) explore the effects of advertising on competitors' optimal price decisions and profit performances. Based on these studies, we establish a bilevel-decision model to elucidate the in-depth effects of advertising on the competitors' upper decisions regarding attribute development. We find that advertising can have a difference-strengthening or difference-alleviating effect depending on whether firms develop the same or different attributes, thereby affecting these firms' attribute development choices. Consequently, advertising intensities show qualitatively different changing trends when determined endogenously. Specifically, advertising intensity shows a monotonic curve in the same-attribute development case but an inverted U-shaped curve in the different-attribute development case.

2.3. Literature on the joint decisions in product and advertising

This paper is also related to the literature on joint decisions regarding product development and advertising. Based on extensive studies that discuss product and advertising separately, some scholars focus on the price-quality, price-advertising, and price-quality-advertising relationships. Erickson (2012) establishes a dynamic model in which a firm is composed by an operations department optimizing the product backlog and a marketing department optimizing the advertising. The author finds that the firm can coordinate these departments to maximize its profit by a committed transfer price. Liu, Zhang, & Tang (2015) extend the work by assuming that the operations department can determine the product quality and propose the firm's optimal transfer pricing strategy between the two departments. Caulkins et al. (2017) explore the optimal time paths for pricing, advertising, and quality for the experience products such as services where the market demand is influenced both by the experience quality and advertising. Wang, Hu, & Liu (2017) study the price and quality strategies in a market that is price-sensitive or quality-sensitive. They

focus on the effects of customer loyalty and the market type on equilibrium channel structure. Chenavaz & Jasimuddin (2017) discuss the relationship between advertising and product quality in an optimal control model. They find that the advertising may increase or decrease with quality depending on the tradeoff between the demand and supply effects. Liu et al. (2018) study the optimal quality and quantity provisions for centralized vs. decentralized distribution under the effects of market size uncertainty. They find that market size uncertainty increases the quantity differential but decreases the quality differential. Chenavaz et al. (2020) study the interplay between price, advertising, and quality in an optimal control model and propose the profitable opportunities of a firm managing a more complex marketing mix. Chenavaz & Eynan (2021) discuss how the monopoly employs advertising to impact the price-demand relationship and find advertising makes Veblen effect more prevalent. Lu and Navas (2021) explore the optimal quality improvement and advertising efforts in a supply chain that faces a potential brand crisis.

We compare our work with these studies in two aspects, namely, competition and the product. First, most previous works establish a monopoly setting from the perspective of dynamic optimization. Caulkins et al. (2017); Chenavaz & Eynan (2021); Chenavaz et al. (2020); Chenavaz & Jasimuddin (2017); Liu et al. (2018) discuss how a monopoly can maximize its profit through the marketing mix. Erickson (2012); Liu et al. (2015) study how a firm coordinates its two departments. In contrast, our work focuses on a duopoly setting in a multi-stage model and the price-advertising-quality effects on the competition. Second, instead of the one-attribute product discussed in all of the works mentioned above, we focus on the competing firms' multi-attribute product development. Relative to the product's quality level, we pay attention to the firms' attribute development choices and show the interactive effects between attribute development and advertising investment.

3. Model

We consider two competing firms that sell multi-attribute products in the market. Each of these firms can choose which attribute to focus on in its development strategy and which attribute to emphasize in its advertising strategy to maximize profit. The following sections introduce the product attribute development and advertising investment strategies of these firms.

3.1. Attribute development

We consider a product having several key attributes. For instance, a mobile phone offers several functions to its users, including communication and photography. A premium car also has various characteristics, such as its performance, comfort, quality, and dependability. For simplicity, we assume two main attributes, A and B. Each firm can choose only one of these attributes to develop, such as BMW positioning itself in the performance dimension and Lexus positioning itself in the comfort dimension (Joshi et al., 2015). This practice is common in real-world contexts because of company philosophies, brand concepts, or product positions. These trends may also result from some limitations, such as budget restrictions or limited expertise of the development team.

If a firm chooses one attribute to develop, then the development level of that attribute, namely, its quality, is assumed to be 1 while that of the other attribute is 0. In Section 7.1, we extend the quality level of the nondeveloped attribute to be $\delta \in [0, 1)$ with a flexible quality level and study its specific impact on equilibrium results. In Section 7.3, we further extend the attribute development level to be endogenous rather than fixed on 1 to analyze the firms' optimal attribute development levels. To incorporate the difference in development cost, we assume that one firm has a lower marginal cost than another firm. Specifically, the former firm's marginal development costs for attributes A and B are c_a and c_b, respectively, whereas the latter firm's marginal development costs for attributes A and B are $c_a + \tau_a$ and $c_b + \tau_b$, respectively, where $\tau_a \geq 0$ and $\tau_b \geq 0$ indicate the differences between these firms. Thus, we refer to the former as the low-cost firm (denoted by L) and the latter as the high-cost firm (denoted by H) throughout the rest of this paper.

On the basis of the literature that considers both price and quality competitions (Jabarzare & Rasti-Barzoki, 2020; Karaer & Erhun, 2015; Matsubayashi & Yamada, 2008; Wang et al., 2017), we initially set that the demand function for the differentiated products, for tractability, follows a linear, downward-sloping function

$$D_i^N = \theta + (a_i - \gamma a_j) + (b_i - \gamma b_j) - (p_i - \gamma p_j),\ i, j \in \{L, H\},\ i \neq j,$$

in which θ indicates the based potential market size, $\gamma \in [0, 1]$ measures the substitution degree of firm j's product relative to that of firm i; that is, the cross effect on a change in the attribute development level (or the price) for a firm i caused by a change in that of firm j, a_i (or b_i) is firm i's development level for attribute A (or B), a_j (or b_j) is the development level of the rival firm for attribute A (or B), and p_i and p_j are the product prices of firm i and its rival. In addition to price competition, the above demand formulation further captures the quality competitions in terms of product attributes A and B; these quality competitions are incorporated in a similar way as the price competition with a common substitution degree γ for simplicity. In Section 7.2, we make the substitution degree of the attributes and that of prices different to examine the effect on the main results.

The demand function D_i^N serves as a base function that considers only the effect of competing firms' attribute development choices. In the following, we incorporate the influence of advertising to obtain a demand function that captures the combined effects of advertising and development.

3.2. Advertising strategy

After product development, we assume that each firm can choose one attribute to emphasize in its advertising strategy. The intensity of advertisement of this attribute is assumed to be χ_i, $i \in \{L, H\}$, and the intensity for the nonadvertised attribute is 0. We assume that each firm can endogenously determine its advertising intensity χ_i with a fixed cost $\frac{k\chi_i^2}{2}$, where k indicates the advertising cost coefficient.

Firms usually emphasize only several key attributes in their advertising strategies. For instance, BMW focuses on their vehicles' performance and emphasizes "the ultimate driving machine" in its advertising campaign (Ries, 2005). In contrast, Lexus has used the tagline "the relentless pursuit of perfection," which highlights attributes of the comfort dimension (Halliday, 1998).

Through repetitive exposure, advertising can enhance consumers' attention to and evaluation of the advertised attribute, as indicated in the empirical and experimental literature (Ayanwale et al., 2005; D'Souza & Rao, 1995; Hoch & Ha, 1986; Tellis, 1988; Zajonc, 1968). For instance, from experimental evidence, Zajonc (1968) finds that mere repeated exposure of an individual to a stimulus object can improve his/her attitude toward the object. Following this notion, the investment of advertisement for one attribute can increase consumers' attention to this attribute and subsequently increase the degree of demand for this attribute. In this way, the demand function under the joint influences of attribute development choices and advertising intensities can be formulated

as

$$D_i = \theta + (1 + \chi_{ia} + \chi_{ja})(a_i - \gamma a_j) + (1 + \chi_{ib} + \chi_{jb})(b_i - \gamma b_j) - (p_i - \gamma p_j),\ i, j \in \{L, H\},\ i \neq j,$$

in which χ_{ia} (χ_{ib}) is the intensity of firm i's advertisement invested into attribute A (B) and χ_{ja} (χ_{jb}) is the intensity of firm j's advertisement invested into attribute A (B). Taking attribute A as an example, $a_i - \gamma a_j$ reveals that a firm i's demand D_i often increases with its development level a_i, but decreases with its rival's development level a_j, due to competition. Based on this, we multiply $a_i - \gamma a_j$ by $1 + \chi_{ia} + \chi_{ja}$ (the intensity of advertisement invested into attribute A) to incorporate the role of advertising in increasing the consumers' attention toward such attribute. A higher intensity of advertisement investment into attribute A can focus the consumers' attention on attribute A and subsequently increase the changing demand degree affected by it.

This demand specification follows from the quadratic consumption utility of a representative consumer,[3] which is given by

$$U(q_i, q_j) = \left[\alpha + (1 + \chi_{ia} + \chi_{ja})\, a_i + (1 + \chi_{ib} + \chi_{jb})\, b_i\right] q_i + \left[\alpha + (1 + \chi_{ia} + \chi_{ja})\, a_j + (1 + \chi_{ib} + \chi_{jb})\, b_j\right] q_j - (\beta q_i^2 + 2\lambda q_i q_j + \beta q_j)/2,$$

in which $\alpha = \frac{\theta}{1-\gamma}$, $\beta = \frac{1}{1-\gamma^2}$, and $\lambda = \frac{\gamma}{1-\gamma^2}$. This is an extended case of the quadratic and strictly concave utility function $U_0(q_i, q_j) = \alpha q_i + \alpha q_j - (\beta q_i^2 + 2\lambda q_i q_j + \beta q_j)/2$, proposed in Shubik and Levitan (1980), which can give rise to a linear demand structure and help to simplify the calculations. Similar quadratic utility functions have been used in Singh and Vives (1984), Raju, Sethuraman, and Dhar (1995), Jerath and Zhang (2010), and Abhishek et al. (2016). Compared with the base utility $U_0(q_i, q_j)$, this utility $U(q_i, q_j)$ further considers the effects of attribute development and advertising. Note that $\partial U/\partial q_i - \partial U_0/\partial q_i = (1 + \chi_{ia} + \chi_{ja})\, a_i + (1 + \chi_{ib} + \chi_{jb})\, b_i$, in which the partials indicate the consumer's additional utility when consuming another product i. The marginal utility $\partial U/\partial q_i$ increases with the attribute development level a_i (or b_i). Moreover, the increment increases with the intensity of advertisement invested into the attribute $1 + \chi_{ia} + \chi_{ja}$ (or $1 + \chi_{ib} + \chi_{jb}$) because consumers become more sensitized to the attribute given the increased advertising.

3.3. Game sequence

A three-stage game proceeds as follows. In the first stage, two firms simultaneously determine the attribute development strategies. In the second stage, two firms simultaneously determine the attribute advertising strategies. In the third stage, two firms simultaneously determine the prices.

On the basis of some literature considering both marketing efforts and price, we assume that the advertising decisions precede the price decisions due to the discrepancy in the periodicity, i.e., the timing and frequency of these decisions. Karray (2013) and Karray and Martín-Herrán (2019) further propose that the decision sequence is affected by the form of advertising and the type of product. The product studied in this paper is in the category that requires research and development and thus mostly belongs to the long-cycle products, such as cars and computers. The advertising campaigns in these industries are more likely to be conducted through traditional media outlets, such as TV, radio, and similar media. Thus, the advertising decisions are usually set for a longer period than prices and should be determined at an earlier stage.

4. Competing firms' profits in different cases

In this section, we show the competing firms' profit functions after they have determined their attribute choices in their product development and advertising strategies. We first classify these firms' attribute choices in their product development strategy and then discuss their attribute choices in their advertising strategy.

Combining the attribute-development choices of firms has four possible outcomes that can be classified into two groups. First, both firms develop the same attribute, including Cases AA and BB; Second, both firms develop different attributes, including Cases AB and BA. We then choose one case in each group as a representative and show the competing firms' profit functions for the selected case. Finally, we derive the optimal decisions in these representative cases.

When both firms develop attribute A (i.e., in Case AA), they first decide their advertising intensities and then their prices. Firms' advertising choices yield four cases: both firms advertise attribute A, and both firms advertise attribute B, the low-cost firm advertises attribute A and the high-cost firm advertises attribute B, and the low-cost firm advertises attribute B and the high-cost firm advertises attribute A. Only the first case emerges in equilibrium (See Appendix for the proof) and, in this case, the profit functions of both firms are as follows:

$$\pi_L = (p_L - c_a)(\theta + (1 + \chi_L + \chi_H)(1 - \gamma) - (p_L - \gamma p_H)) - \frac{k\chi_L^2}{2},$$
$$\pi_H = (p_H - c_a - \tau_a)(\theta + (1 + \chi_L + \chi_H)(1 - \gamma) - (p_H - \gamma p_L)) - \frac{k\chi_H^2}{2}.$$

When the low-cost firm develops attribute A and the high-cost firm develops attribute B (i.e., in Case AB), their advertising choices also yield four cases. Specifically, the low-cost firm advertises attribute A and the high-cost firm advertises attribute B, the low-cost firm advertises attribute B and the high-cost firm advertises attribute A, both firms advertise attribute A, and both firms advertise attribute B. Only the first case emerges in equilibrium (See Appendix for the proof) and, in this case, the profit functions of both firms are as follows:

$$\pi_L = (p_L - c_a)(\theta + (1 + \chi_L) - (1 + \chi_H)\gamma - (p_L - \gamma p_H)) - \frac{k\chi_L^2}{2},$$
$$\pi_H = (p_H - c_b - \tau_b)(\theta - (1 + \chi_L)\gamma + (1 + \chi_H) - (p_H - \gamma p_L)) - \frac{k\chi_H^2}{2}.$$

We apply the backward induction to solve these subgames. To improve readability, we present the specific solving processes in the Appendix and show only the optimal solutions in Proposition 1.

Proposition 1 (Optimal Decisions in Representative Cases)**.** *Each firm advertises only its developed attribute.*

The optimal solutions in Cases AA and AB are listed in Table 1.

Proposition 1 shows that a competing firm has an incentive to advertise only its developed attribute. Otherwise, the firm would rather give up advertising. This finding is in line with the real-world examples of Oppo, Vivo, BMW, and Lexus, among others. By advertising their best-developed, advantaged attribute, firms can

[3] There is a continuum of consumers of the same type in the market. A representative consumer maximizes $U(q_i, q_j) - \Sigma p_i q_i$. From the first partial derivatives with q_i and q_j, we can obtain the above demand function. This utility function $U(q_i, q_j)$ for a representative consumer assumes that a positive noninteger amount of each product may be consumed. As noted in Abhishek, Jerath, and Zhang (2016), this type of a formulation for a representative consumer is consistent with a formulation in which every individual consumer in the population consumes zero or one unit of one product.

Table 1
Optimal decisions in cases.

Cases	Optimal decisions of the prices and advertising intensities
AA	$\chi_L^{AA} = \frac{2(1-\gamma)\left(\left(4-\gamma^2\right)(\theta+(1-\gamma)(1-c_a))k+\left(\gamma(2-\gamma)k-2(1+\gamma)(1-\gamma)^2\right)\tau_a\right)}{\left(4-\gamma^2\right)\left(\left(2-\gamma^2\right)k-4\left(1-\gamma^2\right)\right)k}.$
	$\chi_H^{AA} = \frac{2(1-\gamma)\left(\left(4-\gamma^2\right)(\theta+(1-\gamma)(1-c_a))k-\left((2-\gamma)\left(2-\gamma^2\right)k-2(1+\gamma)(1-\gamma)^2\tau_a\right)\right)}{\left(4-\gamma^2\right)\left(\left(2-\gamma^2\right)k-4\left(1-\gamma^2\right)\right)k}.$
	$p_L^{AA} = \frac{(2+\gamma)\left((2-\gamma)(\theta+(1-\gamma)+c_a)k-4c_a(1-\gamma)^2\right)+\left(\gamma(2-\gamma)k-2(1+\gamma)(1-\gamma)^2\right)\tau_a}{(2+\gamma)\left(\left(2-\gamma^2\right)k-4\left(1-\gamma^2\right)\right)}.$
	$p_H^{AA} = \frac{(2+\gamma)\left((2-\gamma)(\theta+(1-\gamma)+c_a)k-4c_a(1-\gamma)^2\right)+\left(\gamma(2-\gamma)k-2(3+\gamma)(1-\gamma)^2\right)\tau_a}{(2+\gamma)\left(\left(2-\gamma^2\right)k-4\left(1-\gamma^2\right)\right)}.$
AB	$\chi_L^{AB} = \frac{2\left(2-\gamma^2\right)\left(\left(4-\gamma^2\right)\left((2+\gamma)\theta+\left(2-\gamma^2\right)(1-c_a)-\gamma(1-c_b-\tau_b)\right)k-2(1+\gamma)\left(2-\gamma^2\right)(\theta+(1-\gamma)(1-c_a))\right)}{\left((2+\gamma)(2-\gamma)^2k-2(1-\gamma)\left(2-\gamma^2\right)\right)\left((2-\gamma)(2+\gamma)^2k-2(1+\gamma)\left(2-\gamma^2\right)\right)}.$
	$\chi_H^{AB} = \frac{2\left(2-\gamma^2\right)\left(\left(4-\gamma^2\right)\left((2+\gamma)\theta-\gamma(1-c_a)-\left(2-\gamma^2\right)(1-c_b-\tau_b)\right)k-2(1+\gamma)\left(2-\gamma^2\right)(\theta+(1-\gamma)(1-c_b-\tau_b))\right)}{\left((2+\gamma)(2-\gamma)^2k-2(1-\gamma)\left(2-\gamma^2\right)\right)\left((2-\gamma)(2+\gamma)^2k-2(1+\gamma)\left(2-\gamma^2\right)\right)}.$
	$p_L^{AB} = \frac{k\left(4-\gamma^2\right)\left(\left(4-\gamma^2\right)\left((2+\gamma)\theta+\left(2-\gamma^2\right)(1-c_a)-\gamma(1-c_b-\tau_b)\right)k-2(1+\gamma)\left(2-\gamma^2\right)(\theta+(1-\gamma)(1-c_a))\right)}{\left((2+\gamma)(2-\gamma)^2k-2(1-\gamma)\left(2-\gamma^2\right)\right)\left((2-\gamma)(2+\gamma)^2k-2(1+\gamma)\left(2-\gamma^2\right)\right)}+c_a.$
	$p_H^{AB} = \frac{k\left(4-\gamma^2\right)\left(\left(4-\gamma^2\right)\left((2+\gamma)\theta-\gamma(1-c_a)-\left(2-\gamma^2\right)(1-c_b-\tau_b)\right)k-2(1+\gamma)\left(2-\gamma^2\right)(\theta+(1-\gamma)(1-c_b-\tau_b))\right)}{\left((2+\gamma)(2-\gamma)^2k-2(1-\gamma)\left(2-\gamma^2\right)\right)\left((2-\gamma)(2+\gamma)^2k-2(1+\gamma)\left(2-\gamma^2\right)\right)}+c_b+\tau_b.$

enhance their consumers' evaluation of such attribute and highlight their products' competitiveness relative to others, thereby leading to the mutual-promoting role of their development and advertising strategies in stimulating demand. In Section 7.1, we extend our analysis to the nonzero quality level of the nondeveloped attribute and find that firms remain to benefit by advertising their developed attribute if the quality level of the developed attribute is greater than that of the nondeveloped attribute.

In addition, to ensure the existence of the only optimal solution, we impose $k > \frac{4(1-\gamma^2)}{2-\gamma^2}$ as the participation constraint in the same-attribute outcomes (e.g., Case AA) and $k > \frac{2(1+\gamma)(2-\gamma^2)}{(2-\gamma)(2+\gamma^2)}$ in the different-attribute outcomes (e.g., Case AB). We also propose the competing firms' profit functions and solve their optimal advertising and pricing decisions in Case BB by using the same method applied in Case AA (solve Case BA by using the same method applied in Case AB).

Next, we analyze the competing firms' profit performances under combined effects of development and advertising strategies. We present an interesting insight that, combined with different attribute-development outcomes, advertising investment plays distinctive roles in the difference between these competing firms' profits as presented in Proposition 2.

Proposition 2 (Trends of Profits with Advertising Cost Coefficient)**.** *(i) When the competing firms develop the same attribute (e.g., in Case AA), the enhanced efficiency of the advertising investment reduces the profit difference between low- and high-cost firms, i.e.,* $\frac{\pi_L^{AA}-\pi_H^{AA}}{\pi_L^{AA}}$ *increases with k.*

(ii) When the competing firms develop different attributes (e.g., in Case AB), the enhanced efficiency of the advertising investment increases the profit difference between low- and high-cost firms, i.e., $\frac{\pi_L^{AB}-\pi_H^{AB}}{\pi_L^{AB}}$ *decreases with k.*

Proposition 2 indicates that, under different attribute-development equilibriums, the advertising investment exerts completely contrary effects on the difference between these firms' profits. Specifically, given that a lower cost coefficient k indicates a higher efficiency of the advertising investment, we find that increased advertising efficiency decreases the profit difference between low- and high-cost firms in the same-attribute-development case but increases such profit difference in the different-attribute-development case.

These findings can be ascribed to the distinctive roles of advertising in the same- and different-attribute-development outcomes. When firms develop different product attributes (e.g., Case AB), the demand function $D_i = \theta + (1+\chi_i) - (1+\chi_j)\gamma - (p_i - \gamma p_j)$ shows that a firm can enhance only its own demand by increasing its advertising intensity, which means that advertising investment becomes another competition method in addition to prices. Therefore, the higher efficiency of the competition method will increase the profit difference between the advantaged and disadvantaged firms in equilibrium. On the contrary, when firms develop the same product attribute (e.g., Case AA), the demand function $D_i = \theta + (1+\chi_i+\chi_j)(1-\gamma) - (p_i - \gamma p_j)$ shows that the part $(1+\chi_i+\chi_j)(1-\gamma)$ is expanded by both firms' advertising investments. A firm's advertising investment benefits not only itself but also its rival. Thus, along with the increased efficiency, the difference between the advantaged and disadvantaged firms becomes less remarkable.

Proposition 2 presents a key insight that the attribute-development outcome changes the effects of advertising qualitatively. Advertising plays a difference-alleviating role in the same-attribute-development case but a difference-enlarging role in the different-attribute-development case. This proposition initially presents the interaction between attribute development and advertising strategies, which represents the main contribution of this paper. In the following sections, we study these firms' optimal advertising and development strategies to show how they interact with one another.

5. Competing firms' advertising strategies

In this section, we analyze the competing firms' optimal advertising strategies while taking into account their different attribute-development choices. Trends of the competing firms' optimal advertising intensities from the perspectives of advertising cost and development cost are studied, respectively. The former is presented in Proposition 3 and illustrated in Fig. 1 and the latter is summarized in Proposition 4.

Proposition 3 (Trends of Advertising Intensities with Advertisinge-Cost)**.**

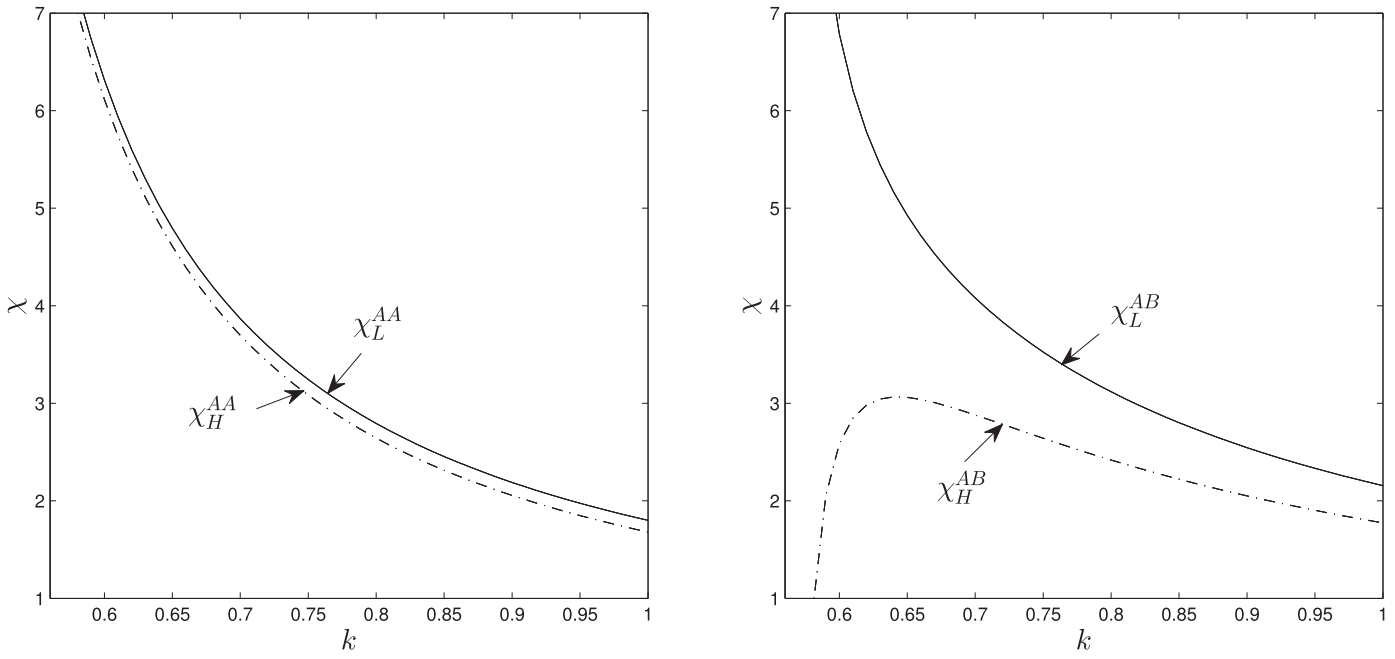

Fig. 1. Trends in the competing firms' advertising intensities with the advertising cost coefficient. *Note:* In the example, $\theta = 2$, $c_a = 0.5$, $c_b = 0.5$, $\tau_a = 0.3$, $\tau_b = 0.3$, and $\gamma = 0.5$.

(i) *When competing firms develop the same attribute (e.g., in Case AA), both of their advertising intensities χ_L^{AA} and χ_H^{AA} consistently decrease along with advertising cost coefficient k.*

(ii) *When competing firms develop different attributes (e.g., in Case AB), the low-cost firm's advertising intensity χ_L^{AB} consistently decreases along with the advertising cost coefficient k, whereas the high-cost firm's advertising intensity χ_H^{AB} initially increases and then decreases.*

Proposition 3 shows a key finding that the competing firms' choices regarding product attribute development qualitatively affect the change trends in their advertising intensities. Result (i) suggests that when they develop the same attribute, the advertising intensities of both firms decrease along with the cost coefficient, which is in line with the expectation that a high cost will reduce the advertising investment. However, result (ii) indicates that, in the different-attribute-development outcome, while the low-cost firm's advertising intensity constantly decreases along with the cost coefficient, the high-cost firm's advertising intensity becomes non-monotonous.

The above proposition can be ascribed to the in-depth influence of the distinctive roles of advertising in the same- and different-attribute-development outcomes. In the same-attribute-development outcome, a firm's advertising investment benefits not only itself but also its rival, thereby giving rise to a free ride between competitors. In this case, each firm wants its rival to invest more in advertising and is less willing to make such investment on its own. Therefore, with the increased advertising cost coefficient k, both firms' incentives to invest in advertising decrease. On the contrary, in a different-attribute-development outcome, these firms compete though their prices and their respective advertising levels. In the case of a fairly low cost coefficient k, the enhanced advertising efficiency significantly intensifies the advertising competition between two firms, thereby driving the low-cost, advantaged firm to increase greatly its advertising level. By contrast, the high-cost firm has a disadvantage in development cost and a lower marginal profit in sales relative to its rival. The ongoing expansion of market demand resulting from advertising investment will amplify such disadvantage more remarkable, thereby further harming the high-cost firm. Consequently, this firm will decrease its advertising level and focus less on the advertising competition, thereby yielding a trend that differs from that recorded in the same-attribute-development outcome.

In sum, the competing firm advantaged in development can enjoy a highly efficient advertising investment, whereas the other disadvantaged firm suffers a limitation and may be harmed by the efficient advertising investment. After deriving the trends in the competing firms' optimal advertising intensities with advertising cost, we examine those trends from another perspective of the development cost as shown in Proposition 4. The trends with development cost are related to advertising efficiency, thereby illustrating the interaction between attribute development and advertising.

Proposition 4 (Trends of Advertising Intensities with Development Cost)**.**

(i) *When competing firms develop the same attribute (e.g., in Case AA), the low-cost firm's advertising intensity χ_L^{AA} decreases along with the development cost difference τ_a for $\frac{4(1-\gamma)^2}{(2-\gamma)^2} < k < \frac{2(1+\gamma)(1-\gamma)^2}{\gamma(2-\gamma)}$ and increases for $k > \frac{2(1+\gamma)(1-\gamma)^2}{\gamma(2-\gamma)}$, whereas the high-cost firm's advertising intensity χ_H^{AA} decreases consistently.*

(ii) *When competing firms develop different attributes (e.g., in Case AB), the low-cost firm's advertising intensity χ_L^{AB} increases along with the development cost difference τ_b consistently, whereas the high-cost firm's advertising intensity χ_H^{AB} decreases consistently.*

Proposition 4 shows the qualitative difference in the trends of the competing firms' advertising willingness with the development cost difference between same- and different-attribute-development outcomes. In particular, when these firms develop the same attribute, the changing directions with development cost difference depend on the advertising cost coefficient k.

Result (ii) shows the trends of the competing firms' advertising investment when they develop different attributes. As the cost difference increases, the high-cost firm's disadvantage in unit development cost increases relative to that of the low-cost firm, thereby decreasing its marginal profit in selling products and its incentive to increase its product demand via advertising investment. Con-

sequently, the high-cost firm's optimal advertising intensity decreases along with the cost difference. By contrast, a higher cost difference increases the low-cost firm's advantage in production and its marginal profit in sales. Therefore, the low-cost firm becomes willing to increase its advertising intensities to increase its production demand and consolidate its production advantage.

Nevertheless, result (i) shows that when competing firms develop the same attribute, the trend of the high-cost firm's advertising intensity remains the same, whereas that of the low-cost firm changes qualitatively. Such trend depends on the advertising cost coefficient. When this coefficient is sufficiently high, the low-cost firm remains willing to increase its advertising investment and subsequently increase its product demand given an increased unit cost difference. However, when the advertising cost coefficient is low, the low-cost firm will reduce its advertising intensity as the cost difference increases. At this time, both firms develop the same attribute, thereby leading to the possibility of the free ride. Especially, a higher value of τ_b enlarges the difference between the low- and high-cost firms, causing the low-cost, advantaged firm suffer greatly from the free ride in advertising investment. When the negative effect of the free ride dominates the positive effect of competition advantage on development, the firm prefers to reduce its investment in advertising, indicating that the firm with an increased advantage does not have a greater willingness to advertise.

6. Competing firms' attribute development strategies

We then explore the competing firms' preferences regarding product attribute development under the effects of advertising. We first present these firms' attribute development strategy when they do not consider the impacts of advertising as a benchmark in Section 6.1. Afterward, we study these firms' attribute development strategies in the presence of advertising in Section 6.2.

6.1. Benchmark in the absence of advertising investment

In this section, we discuss firms' attribute development choices without considering their interactions with advertising. We show the solution process and optimal solutions in Appendix A and describe only the firms' attribute choices by comparing their profits under cases in Lemma 1.

Lemma 1 (Analysis in the Absence of Advertising Investment)**.** *If each firm determines its attribute development strategy without considering an advertising strategy, then each of these firms benefits from developing the attribute with the lower cost, regardless of which attribute its rival chooses.*

Lemma 1 shows that when a firm separates its attribute development and advertising strategies, its attribute development choice is related only to its own attribute development cost. That is, this firm always chooses to develop the low-cost attribute. The optimal attribute development strategy of this firm does not interact with that of its rival, distinctive from those when they develop make attribute development and advertising strategies jointly, which we will discuss in the next section.

6.2. Analysis in the presence of advertising investments

We then study the competing firms' attribute development choices in consideration of their advertising strategies and then derive the equilibrium results. We find that a firm's preference depends not only on its own development cost but also on the development cost of its rival and advertising efficiency.

6.2.1. Firms' respective attribute choices

Proposition 5 and Fig. 2 show the firms' respective attribute strategies. We derive the low-cost firm's choice by comparing π_L^{BA} and π_L^{AA} given that the high-cost firm chooses to develop attribute A. Similarly, we derive the high-cost firm's choice by comparing π_L^{AB} and π_L^{AA} given that the low-cost firm chooses to develop attribute A. A firm' choices when their rival chooses attribute B can be determined by similar methods.

Proposition 5 (Firms' Attribute Preferences)**.**

(i) *Given that its rival chooses attribute A, the low-cost firm benefits from developing attribute A (i.e., $\pi_L^{AA} > \pi_L^{BA}$) for a low development cost difference τ_a.*
(ii) *Given that its rival chooses attribute A, the high-cost firm benefits from developing attribute A (i.e., $\pi_H^{AA} > \pi_H^{BA}$) for a low development cost difference τ_a.*[4]

First, result (ii) is in line with what we expect because the low development cost difference τ_a reduces the cost of developing attribute A for the high-cost firm. The high-cost firm certainly prefers the attribute A. However, result (i) shows that the low-cost firm also prefers to develop attribute A in this situation, which is counterintuitive to some extent. Generally, a higher cost difference τ_a increases the low-cost firm's advantage and competitiveness relative to that of the high-cost firm. One may expect the low-cost firm to develop the same attribute to enjoy such an advantage. However, this is not necessarily the case. Note that firms that choose the same attribute to develop will advertise simultaneously to make their consumers prefer the chosen attribute and then share the demand generated by their total advertising investments. Consequently, the advantaged firm gives a free ride to the disadvantaged firm, and the sufficiently high cost difference τ_a between these firms will intensify the free ride. Given such an increased burden in advertising investment, the advantaged, low-cost firm would choose to develop an attribute different from that of the firm. In contrast, when the cost difference is low, the low-cost firm's concern regarding the free ride decreases. It subsequently increases its willingness to develop the same attribute with its rival.

Fig. 2 illustrates the above proposition and further provides additional insights from the perspectives of advertising cost coefficient k. Specifically, Fig. 2 shows that as the advertising cost coefficient k decreases, both firms' incentives to develop the same attribute as that of their rival increases, indicating a more remarkable effect of advertising on firms' choice of an attribute. This finding is further explored in the following equilibrium analysis.

6.2.2. Equilibrium analysis

Using the solutions of the different subgames studied in the previous section, we now solve the first stage of the game in which the competing firms determine the attributes they will develop. We first provide the game matrix between the e-tailers in strategic form, as illustrated in Table 2. Then, similar to Zhang and Hezarkhani (2021), we give a method to derive the final equilibrium, as shown in Theorem 1.

By analyzing the matrix, it can be observed that the low-cost firm's choice given that the high-cost firm choose attribute A (B) can be derived by comparing π_L^{AA} and π_L^{BA} (π_L^{BB} and π_L^{AB}). The high-cost firm's choice can be derived by the same analogy. By

[4] This proposition is partly proved by numerical examples. We can prove that $\pi_L^{AA} - \pi_L^{BA}$ is a quadratic function with τ_a and two solutions can lead to $\pi_L^{AA} = \pi_L^{BA}$. However, we can find from numerical experimentation that one solution may be omitted as it hardly occurs in the feasible area. We make two numerical experimentations. We examine (i) the situation for k increasing from 0 to 1.6, given that $\theta = 2$, $\gamma = 0.5$, $c_a = 1.5$, $c_b = 1$ and $\tau_b = 1$, and (ii) the situation for c_a increasing from 0 to 3.5, given that $\theta = 2$, $\gamma = 0.5$, $k = 1$, $c_b = 1$ and $\tau_b = 1$. The result (ii) is proved by similar method. See Appendix for details.

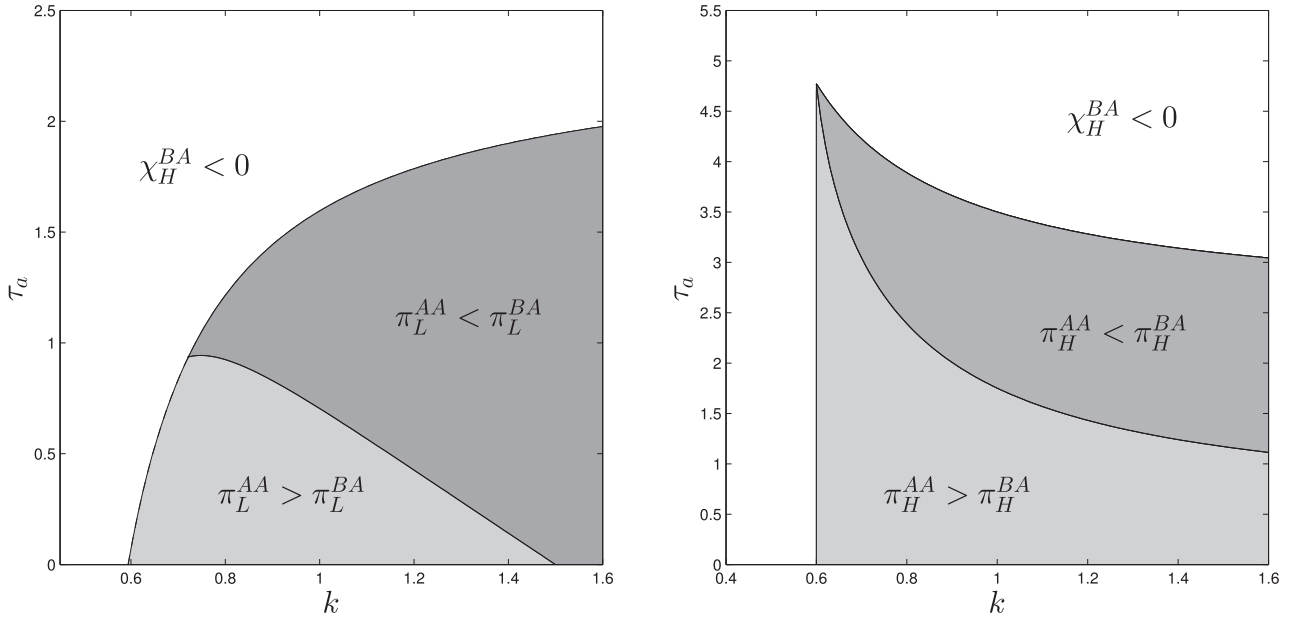

Fig. 2. Firms' respective attribute choices. *Note:* In the example, $\theta = 2$, $\gamma = 0.5$, $c_a = 1.5$, $c_b = 1$ and $\tau_b = 1$.

Table 2
Matrix incorporating the game between the firms
.

		Firm H	
		Attribute A	Attribute B
Firm L	Attribute A	$(\pi_L^{AA},\ \pi_H^{AA})$	$(\pi_L^{AB},\ \pi_H^{AB})$
	Attribute B	$(\pi_L^{BA},\ \pi_H^{BA})$	$(\pi_L^{BB},\ \pi_H^{BB})$

eliminating the dominant strategy, we can obtain the final equilibrium. For example, we consider a possible case in which $\pi_L^{AA} > \pi_L^{BA}$, $\pi_L^{BB} > \pi_L^{AB}$, $\pi_H^{AA} < \pi_H^{AB}$, and $\pi_H^{BB} > \pi_H^{BA}$. We can find from $\pi_L^{AA} > \pi_L^{BA}$ and $\pi_L^{BB} > \pi_L^{AB}$ that the low-cost firm chooses attribute A given that the high-cost firm chooses attribute A and chooses attribute B given that the high-cost firm chooses attribute B. Additionally, regarding the high-cost firm's choice, as $\pi_H^{AA} < \pi_H^{AB}$ and $\pi_H^{BB} > \pi_H^{BA}$, choosing attribute A is always dominated by choosing attribute B. Thus the high-cost firm chooses attribute B regardless of which attribute the low-cost firm chooses. Considering both firms' responses, the equilibrium is BB. By the same analogy, we obtain a method to derive the final equilibrium as below.

Theorem 1 (Method to Derive the Final Equilibrium)**.**

(i) *The equilibrium is AA or BB when the following conditions satisfied.*
- $\pi_L^{AA} > \pi_L^{BA}, \pi_L^{BB} > \pi_L^{AB}, \pi_H^{AA} > \pi_H^{AB}, \pi_H^{BB} > \pi_H^{BA}$.

(ii) *The equilibrium is AB or BA when the following conditions satisfied.*
- $\pi_L^{AA} < \pi_L^{BA}, \pi_L^{BB} < \pi_L^{AB}, \pi_H^{AA} < \pi_H^{AB}, \pi_H^{BB} < \pi_H^{BA}$.

(iii) *The equilibrium is AA when either of the following conditions satisfied.*
- $\pi_L^{AA} > \pi_L^{BA}, \pi_L^{BB} < \pi_L^{AB}, \pi_H^{AA} > \pi_H^{AB}, \pi_H^{BB} > \pi_H^{BA}$.
- $\pi_L^{AA} > \pi_L^{BA}, \pi_L^{BB} > \pi_L^{AB}, \pi_H^{AA} > \pi_H^{AB}, \pi_H^{BB} < \pi_H^{BA}$.
- $\pi_L^{AA} > \pi_L^{BA}, \pi_L^{BB} < \pi_L^{AB}, \pi_H^{AA} > \pi_H^{AB}, \pi_H^{BB} < \pi_H^{BA}$.

(iv) *The equilibrium is BB when either of the following conditions satisfied.*
- $\pi_L^{AA} < \pi_L^{BA}, \pi_L^{BB} > \pi_L^{AB}, \pi_H^{AA} > \pi_H^{AB}, \pi_H^{BB} > \pi_H^{BA}$.
- $\pi_L^{AA} > \pi_L^{BA}, \pi_L^{BB} > \pi_L^{AB}, \pi_H^{AA} < \pi_H^{AB}, \pi_H^{BB} > \pi_H^{BA}$.
- $\pi_L^{AA} < \pi_L^{BA}, \pi_L^{BB} > \pi_L^{AB}, \pi_H^{AA} < \pi_H^{AB}, \pi_H^{BB} > \pi_H^{BA}$.

(v) *The equilibrium is AB when either of the following conditions satisfied.*
- $\pi_L^{AA} > \pi_L^{BA}, \pi_L^{BB} < \pi_L^{AB}, \pi_H^{AA} < \pi_H^{AB}, \pi_H^{BB} > \pi_H^{BA}$.
- $\pi_L^{AA} < \pi_L^{BA}, \pi_L^{BB} < \pi_L^{AB}, \pi_H^{AA} < \pi_H^{AB}, \pi_H^{BB} > \pi_H^{BA}$.
- $\pi_L^{AA} > \pi_L^{BA}, \pi_L^{BB} < \pi_L^{AB}, \pi_H^{AA} < \pi_H^{AB}, \pi_H^{BB} < \pi_H^{BA}$.

(vi) *The equilibrium is BA when either of the following conditions satisfied.*
- $\pi_L^{AA} < \pi_L^{BA}, \pi_L^{BB} > \pi_L^{AB}, \pi_H^{AA} > \pi_H^{AB}, \pi_H^{BB} < \pi_H^{BA}$.
- $\pi_L^{AA} < \pi_L^{BA}, \pi_L^{BB} < \pi_L^{AB}, \pi_H^{AA} > \pi_H^{AB}, \pi_H^{BB} < \pi_H^{BA}$.
- $\pi_L^{AA} < \pi_L^{BA}, \pi_L^{BB} > \pi_L^{AB}, \pi_H^{AA} < \pi_H^{AB}, \pi_H^{BB} < \pi_H^{BA}$.

(vii) *No equilibrium exists when either of the following conditions satisfied.*
- $\pi_L^{AA} < \pi_L^{BA}, \pi_L^{BB} < \pi_L^{AB}, \pi_H^{AA} > \pi_H^{AB}, \pi_H^{BB} > \pi_H^{BA}$.
- $\pi_L^{AA} > \pi_L^{BA}, \pi_L^{BB} > \pi_L^{AB}, \pi_H^{AA} < \pi_H^{AB}, \pi_H^{BB} < \pi_H^{BA}$.

However, considering the variety of cases and the complexity of calculation, it is still hard to depict the equilibrium outcome completely. As mentioned in Theorem 1, we need to make four comparisons: (i) π_L^{AA} and π_L^{BA}, (ii) π_L^{BB} and π_L^{AB}, (iii) π_H^{AA} and π_H^{AB}, and (iv) π_H^{BB} and π_H^{BA}. There are 16 possible combinations in total, which will then lead to 7 types of possible equilibria. Additionally, we consider a three-stage game and optimize firms' multiple decisions. This leads to complex forms of the firms' optimal profits in the subcases. Thus, it is hard to compare these profits and derive the closed-form thresholds.

Considering the difficulty mentioned above, we consider a numerical example, as shown in Table 3, to provide some insights. Similar to the above example, we set $\theta = 2$ and $\gamma = 0.5$ and focus on the combined effect of the advertising coefficient k and the cost parameters c_a, c_b, τ_a, and τ_b. In each numerical case, we respectively derive the low-cost firm's profits $\pi_L^{AA}, \pi_L^{BA}, \pi_L^{BB}$, and π_L^{AB} and obtain the low-cost firm's attribute choice by comparing them. Next, we derive the high-cost firm's profits and attribute choice by the same method. Finally, we provide the equilibrium results considering both firms' responses. Regarding each firm's choice, "A/B" represents that the firm chooses attribute A if the other choose attribute A and chooses attribute B if the other chooses attribute B. "A" ("B") represents that the firm chooses attribute A (B) regardless of which attribute the other chooses. In total, we consider 28 numerical cases divided into seven groups. In each group, the combination of cost parameters c_a, c_b, τ_a, and τ_b remain the same

Table 3
Numerical examples .

No	c_a	c_b	τ_a	τ_b	k	Firm L					Firm H					Equi
						π_L^{AA}	π_L^{BA}	π_L^{BB}	π_L^{AB}	Choi	π_H^{AA}	π_H^{BA}	π_H^{BB}	π_H^{AB}	Choi	
#1	0	0	1	1	0.8	9.75	5.44	9.75	5.44	A/B	6.83	0.96	6.83	0.96	A/B	AA/BB
#2	0	0	1	1	1.2	5.86	4.27	5.86	4.27	A/B	3.53	1.36	3.53	1.36	A/B	AA/BB
#3	0	0	1	1	1.6	4.87	3.93	4.87	3.93	A/B	2.72	1.43	2.72	1.43	A/B	AA/BB
#4	0	0	1	1	2.0	4.41	3.76	4.41	3.76	A/B	2.36	1.45	2.36	1.45	A/B	AA/BB
#5	0	1	0	0	0.8	10.16	0.96	6.50	5.44	A/B	10.16	5.44	6.50	0.96	A/B	AA/BB
#6	0	1	0	0	1.2	5.86	4.27	5.71	3.22	A/B	3.53	1.36	5.71	3.22	A/B	AA/BB
#7	0	1	0	0	1.6	4.59	1.43	2.93	3.93	A	4.59	3.93	2.93	1.43	A	AA
#8	0	1	0	0	2.0	4.08	1.45	2.61	3.77	A	4.08	3.76	2.61	1.45	A	AA
#9	0	0	1	0	0.8	9.75	5.43	10.16	3.39	A/B	6.83	0.96	10.16	3.39	A/B	AA/BB
#10	0	0	1	0	1.2	5.86	4.27	5.71	3.22	A/B	3.53	1.36	5.71	3.22	A/B	AA/BB
#11	0	0	1	0	1.6	4.87	3.93	4.59	3.12	A/B	2.72	1.43	4.59	3.12	B	BB
#12	0	0	1	0	2.0	4.41	3.76	4.08	3.05	A/B	2.36	1.45	4.08	3.05	B	BB
#13	0	1	1	0	0.8	9.75	2.17	6.50	5.44	A/B	6.83	2.17	6.50	0.96	A/B	AA/BB
#14	0	1	1	0	1.2	5.86	2.06	3.65	4.27	A	3.53	2.06	3.65	1.36	A/B	AA
#15	0	1	1	0	1.6	4.87	1.99	2.93	3.93	A	2.72	1.99	2.93	1.43	A/B	AA
#16	0	1	1	0	2.0	4.41	1.95	2.61	3.76	A	2.36	1.95	2.61	1.45	A/B	AA
#17	1	1	1	0	0.8	6.18	3.86	6.50	2.17	A/B	3.90	0.38	6.50	2.17	A/B	AA/BB
#18	1	1	1	0	1.2	3.78	2.91	3.65	2.06	A/B	1.97	0.65	3.65	2.06	B	BB
#19	1	1	1	0	1.6	3.16	2.65	2.93	1.99	A/B	1.49	0.71	2.93	1.99	B	BB
#20	1	1	1	0	2.0	2.88	2.53	2.61	1.95	A/B	1.28	0.73	2.61	1.95	B	BB
#21	0	1	1	1	0.8	9.75	2.17	6.18	7.97	A	6.83	2.17	3.90	0.02	A/B	AA
#22	0	1	1	1	1.2	5.86	2.06	3.78	5.45	A	3.53	2.06	1.96	0.29	A	AA
#23	0	1	1	1	1.6	4.87	1.99	3.16	4.84	A	2.72	1.99	1.49	0.39	A	AA
#24	0	1	1	1	2.0	4.42	1.95	2.88	4.55	A	2.37	1.95	1.28	0.43	A	AA
#25	0	1	2	0	0.8	9.36	3.86	6.50	5.44	A/B	4.16	0.38	6.50	0.96	A/B	AA/BB
#26	0	1	2	0	1.2	6.02	2.91	3.65	4.26	A	1.88	0.65	3.65	1.36	A/B	AA
#27	0	1	2	0	1.6	5.15	2.65	2.93	3.93	A	1.34	0.71	2.93	1.43	B	AB
#28	0	1	2	0	2.0	4.76	2.52	2.61	3.76	A	1.10	0.73	2.61	1.45	B	AB

while the advertising coefficient k changes from 0.8, to 1.2, to 1.6, to 2.0. Therefore, we can analyze the effect of the advertising coefficient k by comparing equilibrium results within each group and the effect of the cost parameters by comparing equilibrium results in different groups.

The results are presented in Observations 1 and 2. The former shows each game participator's response, and the latter shows the equilibrium results combining the two firms' responses.

Observation 1 (Game Participators' Responses)**.**

(i) The high advertising efficiency increases the possibility that the firm will choose the same attribute as the other instead of the attribute with the lower cost.
(ii) The extent of the increase is larger for the high-cost firm than for the low-cost firm.

The first insight can be found from each group. When the advertising coefficient k is sufficiently low, the firms' preferences become A/B regardless of which attribute has the lower cost. In this case, once the firms develop different attributes, the advertising will become a highly efficient competition method in addition to the price. Consequently, the competition will intensify significantly, leading to losses for both firms if they develop different attributes.

The second result can be obtained from the last two groups. In each group, while the cost difference between the two attributes is the same for both firms, the high-cost firm is more likely to adopt A/B than the low-cost firm. This is because the high-cost firm, which holds a production disadvantage relative to the low-cost firm, will be harmed more by the intensified competition under the different-attribute development case.

Observation 2 (Equilibrium Results)**.**

(i) The same-attribute development case is more likely to arise in equilibrium than the different-attribute development case. The latter only arises when the advertising efficiency is sufficiently low.
(ii) In the specially case in which the costs are the same for the two attributes, each of the firms prefers to develop the same attribute as the other all the time. Consequently, the same-attribute development case arises consistently.

First, we can easily observe the second result from the first group. Both firms prefer A/B and then the same-attribute development equilibrium arises in Cases #1–4. This result indicates that each firm is always unwilling to fight with the other when the costs are the same for the two attributes. They forgo developing different attributes to avoid the intensified competition even though they may suffer from the free ride in the same-attribute development case.

The first result can be found for each group. Given the costs of attributes are all the same, the different-attribute development case may arise in the equilibrium only when k is sufficiently high, and the same-attribute development case arises in more cases. This is reasonable according to the two game participators' responses, as mentioned above. As the advertising efficiency increases, each firm is more willing to choose the same attribute. Consequently, firms' attribute choices are more connected, and the same-attribute development case arises with a higher possibility. On the contrary, their decisions interact less. Each firm will choose to develop the attribute with a lower cost, which gives rise to the emergence of the different-attribute development equilibrium.

7. Extensions

In this section, we extend our results by altering some assumptions in the model. The main findings are summarized here, and the details of the analysis and proofs are provided in Appendix B.

7.1. The nonzero quality level of the non-developed attribute

In this section, we extend the quality level of the non-developed attribute to be $\delta \in [0, 1)$ with a flexible quality level. By

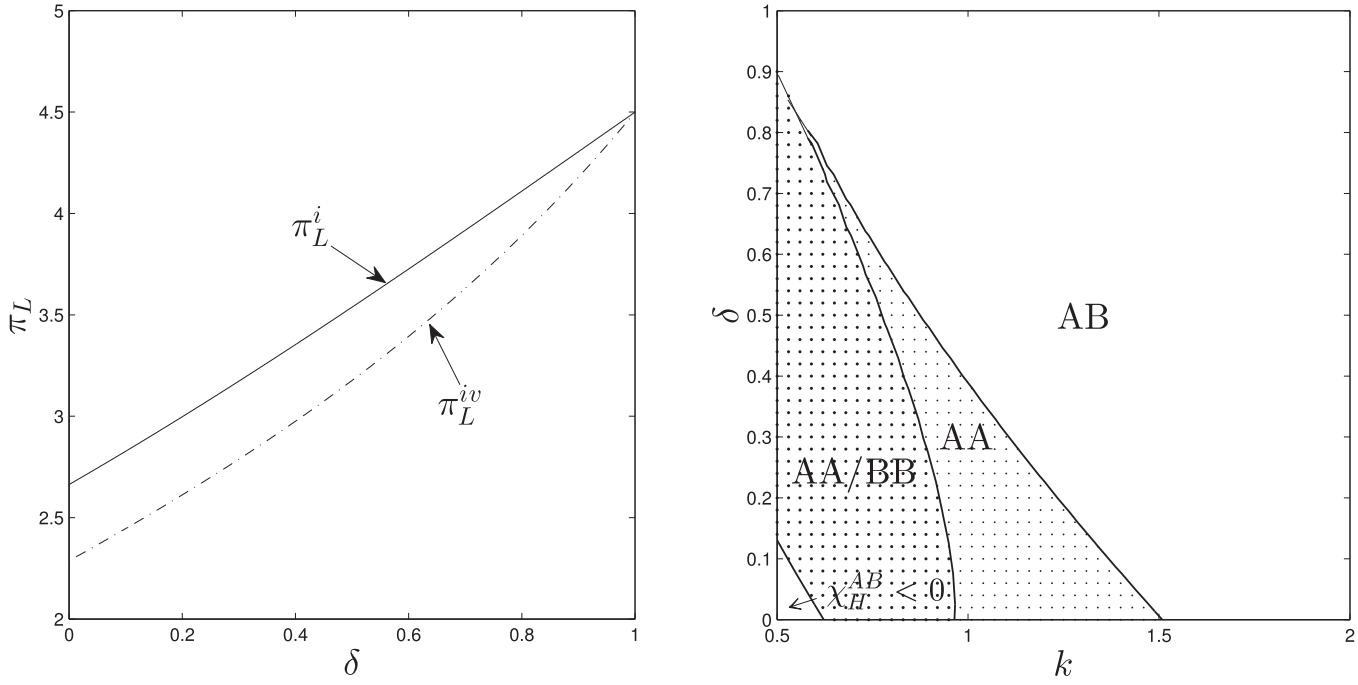

Fig. 3. Impacts of the quality level of the non-developed attribute. *Note:* In this numerical case, $\theta = 2, \gamma = 0.5, c_a = 0, c_b = 1, \tau_a = 2$, and $\tau_b = 0$. Besides, $k = 3$ in the left plot.

solving and comparing the profits (see Appendix B), we find that our main conclusions remain qualitatively unchanged, as shown in Proposition 6. Additionally, we study the specific impact of the quality level of the nondeveloped attribute, as presented in Fig. 3 and summarized in Observation 3. We set $\theta = 2, \gamma = 0.5, c_a = 0, c_b = 1, \tau_a = 2$, and $\tau_b = 0$ in Fig. 3. The values of parameters are same as those in the last group (#25–28) in Table 3, allowing us to compare the results of this extension with those of the main model.

Proposition 6 (Firms' Advertising Choices)**.** *The firms benefit by advertising their developed attribute if the quality level of the developed attribute is larger than that of the nondeveloped attribute.*[5]

Proposition 6 verifies that firms' advertising choices remain the same qualitatively. The left plot in Fig. 3 also shows that the solid line (representing the firm's profit when it advertises the developed attribute) is consistently higher than the dotted line (representing its profit when it advertises the nondeveloped attribute). Additionally, the difference decreases when δ approaches 1. With a reduced quality improvement for the developed attribute relative to the non-developed attribute, the benefits for advertising the developed attribute shrink.

Observation 3 (Impacts of the Quality Level of the Non-developed Attribute)**.**

(i) The reduced quality difference between the developed and nondeveloped attributes decreases the effect of advertising on firms' attribute choices.
(ii) It makes the different-attribute-development cases more likely to arise in equilibrium.

[5] This proposition is partly proved by numerical examples. When firms develop the same attribute, we can prove that they obtain higher profits by advertising the developed attribute. However, when firms develop different attributes, we can find the profit comparison depends on k, γ, and δ but resort to a numerical analysis for the specific results. See Appendix for details.

Observation 3 and the right plot in Fig. 3 show specific impacts of the quality level of the non-developed attribute on the equilibrium. Compared with the last group (#25–28) in Table 3, the equilibrium results remain the same qualitatively, that is, as k increases, the equilibrium changes from AA/BB to AA and then, finally, to AB. In addition, we find that, as δ increases, the range of AB increases. The firms are increasingly willing to develop the attribute with the lower cost rather than the attribute their rival has chosen. The reduced quality difference between the developed and nondeveloped attributes decreases the effect of advertising on firms' attribute choices.

7.2. Asymmetric substitution degrees between attributes and prices

In this section, we extend the substitution degree of the attributes and that of prices, in which the former is changed to be $\lambda \in [0, 1]$ while the latter remains $\gamma \in [0, 1]$. The demand function then follows $D_i = \theta + (1 + \chi_a)(a_i - \lambda a_j) + (1 + \chi_b)(b_i - \lambda b_j) - (p_i - \gamma p_j)$. This setting separately analyzes the effects of substitution degrees regarding the attributes and price.

A numerical example is provided to intuitively show how the attribute developed equilibrium results are affected, as presented in Fig. 4. The left plot presents the effects of λ and k. The right plot presents the effects of γ and k. First, both plots show how the attribute developed equilibrium results changed by the advertising coefficient k remain the same qualitatively. As k increases, the equilibrium changes from AA/BB to AA and, finally, to AB. We summarize the effects of λ and γ on the attribute developed equilibrium separately in the following observation.

Observation 4 (Impacts of the Substitution Degree of the Attributes and that of Prices)**.** The substitution degrees of the attributes and prices have opposite effects on the equilibrium results:

(i) In most cases, the reduced substitution degree λ of the attributes increases the effectiveness of advertising on firms' attribute choices.

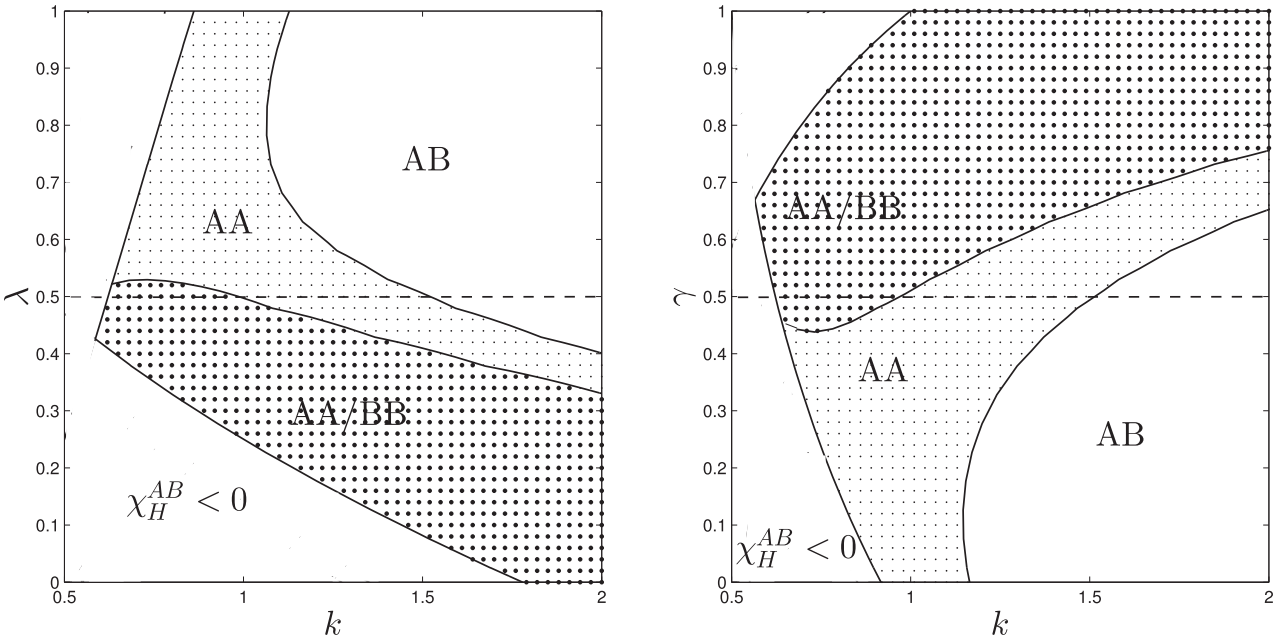

Fig. 4. Substitution degree impacts of attributes and prices. *Note:* In this numerical case, $\theta = 2, \gamma = 0.5, k = 3, c_a = 0, c_b = 1, \tau_a = 2$, and $\tau_b = 0$. In addition, $\gamma = 0.5$ in the left plot and $\lambda = 0.5$ in the right plot.

(ii) In most cases, the enhanced substitution degree γ of the prices increases the effectiveness of advertising on firms' attribute choices.

Observation 4 (i) shows that, when λ approaches 0, the advertising exerts a larger effect, causing firms to develop the same attribute, and the same-attribute development case emerges in a larger range. Given a sufficiently low λ, the cross effect in the attribute development level for a firm caused by a change in that of the other firm is fairly low, thereby decreasing attribute competition. In this case, firms would like to choose the same attribute to enjoy the free ride provided by the rival's advertising investment. Observation 4(ii) shows a contrary effect of γ that, when γ approaches 1, firms prefer to develop the same attribute as the other. In this case, considering the fairly intensified price competition, firms would prefer not to continue to aggressively compete on attributes. Consequently, they choose to develop the same attribute.

7.3. Endogenous attribute development level

In this section, we extend the attribute development level to be endogenous rather than being fixed on 1. The firm's attribute development level is set as $q_i^j, i \in \{L, H\}$ and $j \in \{AA, AB \ldots\}$ with a cost $\frac{1}{2} f q_i^{j^2}$, in which f is the development cost coefficient.

When all of the attribute development levels, advertising intensities, and prices are determined by firms endogenously, the three-stage game becomes highly complicated and finding closed-form solutions is difficult. Thus, we make a numerical study. See six examples as below.[6][-5mm]

Example 1. When $\theta = 2, \gamma = 0.5, c_a = 0.5, c_b = 0.5, \tau_a = 0, \tau_b = 0, k = 3$, and $f = 3$, we obtain

- Case AA/BB: $\chi_L = 0.2198, \chi_H = 0.2198, q_L = 0.6656, q_H = 0.6656, \pi_L = 1.4714, \pi_H = 1.4714$;
- Case AB/BA: $\chi_L = 0.2270, \chi_H = 0.2270, q_L = 0.5278, q_H = 0.5278, \pi_L = 1.4163, \pi_H = 1.4163$.
- Both firm's choice is A/B. The high-cost firm's choice is A/B. The equilibrium is AA/BB.

Example 2. When $\theta = 2, \gamma = 0.5, c_a = 0.5, c_b = 0.5, \tau_a = 0, \tau_b = 0, k = 3$, and $f = 5$, we obtain

- Case AA/BB: $\chi_L = 0.0776, \chi_H = 0.0776, q_L = 0.2743, q_H = 0.2743, \pi_L = 1.4216, \pi_H = 1.4216$;
- Case AB/BA: $\chi_L = 0.1019, \chi_H = 0.1019, q_L = 0.2596, q_H = 0.2596, \pi_L = 1.4086, \pi_H = 1.4086$.
- The low-cost firm's choice is A/B. The high-cost firm's choice is A/B. The equilibrium is AA/BB.

Example 3. When $\theta = 2, \gamma = 0.5, c_a = 0.5, c_b = 0.5, \tau_a = 0, \tau_b = 0, k = 5$, and $f = 5$, we obtain

- Case AA/BB: $\chi_L = 0.0428, \chi_H = 0.0428, q_L = 0.2551, q_H = 0.2551, \pi_L = 1.4177, \pi_H = 1.4177$;
- Case AB/BA: $\chi_L = 0.0580, \chi_H = 0.0580, q_L = 0.2477, q_H = 0.2477, \pi_L = 1.4108, \pi_H = 1.4108$.
- The low-cost firm's choice is A/B. The high-cost firm's choice is A/B. The equilibrium is AA/BB.

Example 4. When $\theta = 2, \gamma = 0.5, c_a = 0.5, c_b = 0.5, \tau_a = 0, \tau_b = 0.5, k = 5$, and $f = 5$, we obtain

- Case AA: $\chi_L = 0.0428, \chi_H = 0.0428, q_L = 0.2551, q_H = 0.2551, \pi_L = 1.4177, \pi_H = 1.4177$;
- Case BA: $\chi_L = 0.0580, \chi_H = 0.0580, q_L = 0.2477, q_H = 0.2477, \pi_L = 1.4108, \pi_H = 1.4108$;
- Case AB: $\chi_L = 0.0667, \chi_H = 0.0352, q_L = 0.2668, q_H = 0.1908, \pi_L = 1.6060, \pi_H = 0.8812$;
- Case BB: $\chi_L = 0.0530, \chi_H = 0.0228, q_L = 0.2691, q_H = 0.1998, \pi_L = 1.6069, \pi_H = 0.8890$;
- The low-cost firm's choice is A/B. The high-cost firm's choice is A. The equilibrium is AA.

Example 5. When $\theta = 2, \gamma = 0.5, c_a = 0, c_b = 0.5, \tau_a = 1, \tau_b = 0.5, k = 5$, and $f = 5$, we obtain

- Case AA: $\chi_L = 0.0830, \chi_H = 0.0159, q_L = 0.3299, q_H = 0.1876, \pi_L = 2.2975, \pi_H = 0.7477$;

[6] Some other numerical examples are put in Appendix to save space.

- Case BA: $\chi_L = 0.0667, \chi_H = 0.0352, q_L = 0.2668, q_H = 0.1908, \pi_L = 1.6060, \pi_H = 0.8812$;
- Case AB: $\chi_L = 0.0997, \chi_H = 0.0291, q_L = 0.3311, q_H = 0.1731, \pi_L = 2.3023, \pi_H = 0.7352$;
- Case BB: $\chi_L = 0.0530, \chi_H = 0.0228, q_L = 0.2691, q_H = 0.1998, \pi_L = 1.6069, \pi_H = 0.8890$;
- The low-cost firm's choice is A. The high-cost firm's choice is A/B. The equilibrium is AA.

Example 6. When $\theta = 2, \gamma = 0.5, c_a = 0.5, c_b = 1, \tau_a = 1, \tau_b = 0, k = 5$, and $f = 5$, we obtain

- Case AA: $\chi_L = 0.0645, \chi_H = 0.0090, q_L = 0.2849, q_H = 0.1468, \pi_L = 1.8079, \pi_H = 0.4829$;
- Case BA: $\chi_L = 0.0489, \chi_H = 0.0233, q_L = 0.2264, q_H = 0.1545, \pi_L = 1.2032, \pi_H = 0.5931$;
- Case AB: $\chi_L = 0.0667, \chi_H = 0.0352, q_L = 0.2668, q_H = 0.1908, \pi_L = 1.6060, \pi_H = 0.8812$;
- Case BB: $\chi_L = 0.0305, \chi_H = 0.0305, q_L = 0.2130, q_H = 0.2130, \pi_L = 1.0406, \pi_H = 1.0406$.
- The low-cost firm's choice is A. The high-cost firm's choice is B. The equilibrium is AB.

By comparing and analyzing these numerical examples, we obtain some observations. Observation 5 shows the firms' profit comparisons under endogenous attribute level and Observation 6 summarizes the firms' decisions on advertising intensities and attribute development levels and the firms' attribute choices.

Observation 5 (When Attribute Levels are Endogenous: Firms' Optimal Profits)**.**

(i) Given the same costs for the two attributes, each firm remains obtaining a higher profit by choosing the same attribute as the other throughout.
(ii) Each firm's profit benefit from the same-attribute case relative to the different-attribute case increases with the advertising-investment efficiency and attribute-development-investment efficiency.

Observation 5 can be obtained from Examples 1–3. Result (i) in Observation 5 further verifies the finding in main model that, when the costs are the same for the two attributes, each firm chooses the same attribute to avoid the intensified competition.

Result (ii) supplements that, as each efficiency (including the advertising investment and attribute-development investment) increases, firms can benefit more from the same-attribute case. The reason is because that, in the different-attribute case, the advertising plays as an additional competition method. Thus the increased advertising efficiency intensifies the competition, leading to greater losses for both firms. On the basis, a higher attribute development level can promote the effect of advertising on demand generation. Considering this promotion role, the highly efficient attribute-development investment will further intensify the competition and hurt the firms more.

Observation 6 (When Attribute Levels are Endogenous: Firms' Optimal Decisions)**.**

(i) Relative to the same-attribute case, both firms will enhance the advertising investment but reduce the attribute-development investment in the different-attribute case.
(ii) When the prices, advertising intensities, and attribute-development levels are all endogenously determined, a firm chooses the attribute with the lower cost if the costs for the two attributes are different and chooses the same attribute as the other if the costs for the two attributes are same.

Observation 6 (i) provides a comparison concerning both the firms' advertising investments and attribute-development investments between the same- and different-attribute cases. We find that firms will invest more on advertising but less on attribute development in the different-attribute case than in the same-attribute case. In the different-attribute setting, the firms need not worry that its increased advertising investment would give a free ride to its rival. Therefore, this firm also has a larger incentive to increase its advertising investment in the different-attribute outcome than in the same-attribute outcome. Observation 6(i) also reveals opposite changing trends between the willingness of advertising investment and that of the attribute-development investment. This is because that, when firms choose different attributes to develop, the product differentiation enhances. This can alleviate the competition on attribute development and then decrease firms' incentive on attribute-development investment. In addition, Observation 6(ii), obtained from Examples 4 and 5, further verifies the findings in the main model. Generally, firms still choose the attribute with a lower cost. However, advertising relates firms' attribute choices to one another, increasing their willingness to choose the same one.

8. Conclusion and discussion

An increasing number of firms have started to integrate attribute development and advertising strategies by choosing one attribute to focus on in their development strategy and to play up in their advertising strategy to stimulate product demand under an intensified competition. This work studies the potential effects of such integration and how these strategies interact with each other.

We establish an analytical model, in which two competing firms sell multi-attribute products. Each firm can choose one attribute to focus on in its development strategy and one attribute to play up in its advertising strategy. After optimizing firms' development and advertising strategies, we obtain some managerial insights. First, we verify the necessity for the integration of these strategies as a firm advertises only its developed attribute; advertising increases the attribute advantage of its product relative to other and, in turn, the high competitiveness of product attribute enhances the role of advertising. Second, we find that, combined with different attribute-development outcomes, the advertising investment plays distinctive roles on firms: a difference-strengthening effect in different-attribute-development outcome but a difference-alleviating effect in same-attribute-development outcome. Third, we find that advertising establishes a connection between the competitors, leading to their attribute-development decisions related to one another.

Despite the encouraging results obtained in this work, some other factors related to the interaction between product development and advertising strategies may be examined in the future. First, we assume that the advertising decisions is usually set for a longer period than prices and should be determined at an earlier stage, but some other periodicity, i.e., the timing and frequency of these decisions, may exist in real contexts, especially for fast moving consumer products. Solving the duopoly model by these different decision sequences may be interesting and practical and generate additional management insights. Besides, we analyze the effect of asymmetric development costs between competing firms. Asymmetric advertising costs also exist in real contexts and may present another interesting direction for future research.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2021.07.053.

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