Peer-to-peer trading under subscribed capacity tariffs - an equilibrium approach

Sigurd Bjarghov[a]∗, Magnus Askeland[a,c], Stian Backe[b]
[a]Department of Electric Power Engineering, Norwegian University of Science and Technology – Trondheim, Norway
[b]Department of Industrial Economics, Norwegian University of Science and Technology – Trondheim, Norway
[c]Energy Systems, SINTEF Energy Research – Trondheim, Norway
Email: sigurd.bjarghov@ntnu.no, magnus.askeland@sintef.no, stian.backe@ntnu.no

***Abstract*—Local peer-to-peer (P2P) markets are envisioned as a promising market design to integrate the increasing number of agents in the distribution grid. To incentivize grid-friendly consumption profiles, we suggest a subscribed capacity tariff where end-users pay for a capacity level with a high excess energy term. The P2P market functions as a capacity market where end-users buy capacity from other agents when needed. We demonstrate the concept by formulating the local P2P market equilibrium problem as a mixed complementarity problem (MCP). Analysis of a neighborhood case study shows that both aggregated peak load and agent costs decreases.**

***Index Terms*—Peer-to-peer, capacity tariffs, local markets, battery flexibility**

NOMENCLATURE

Indices and Sets

p	Set of prosumers p
q	Set of prosumers q
t	Time index

Parameters

A_p^{ch}, A_p^{dis}	Battery ch./disch. efficiency [%]
C^a	P2P trading adm. cost $[\frac{\text{€ct}}{kWh}]$
C^h	Grid tariff excess energy cost $[\frac{\text{€ct}}{kWh}]$
C^l	Grid tariff energy cost $[\frac{\text{€ct}}{kWh}]$
C^{sub}	Capacity cost per kW $[\frac{\text{€}}{kW \cdot year}]$
C_t^{DA}	Day-ahead spot price $[\frac{\text{€ct}}{kWh}]$
E_p^{max}	Max. battery SOC [kWh]
G_{pt}^{PV}	PV production [kWh/h]
L_{pt}	Inflexible load [kWh/h]
Q_p^{ch}	Max. battery charging power [kW]
Q_p^{dis}	Max. battery discharging power [kW]

Variables

λ_{pqt}^{P2P}	P2P market clear price between p and q $[\frac{\text{€ct}}{kWh}]$
e_{pt}	Battery state of charge [kWh]
q_{pt}^{ch}	Battery charging [kWh]
q_{pt}^{dis}	Battery discharging [kWh]
x_p^{sub}	Subscribed capacity [kW]
x_{pqt}^{P2P}	P2P electricity bought by p from q. Negative is sold from p to q [kWh]
x_{pt}^{buy}	Total bought electricity [kWh/h]
x_{pt}^{h}	Bought electricity above sub. cap. [kWh/h]
x_{pt}^{l}	Bought electricity below sub. cap. [kWh/h]
x_{pt}^{sell}	Sold electricity [kWh/h]

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I. INTRODUCTION

As part of solving the climate challenge, the EU has emphasised that the consumer's importance changes when forming new incentives and market design[1]. With an increasing worldwide share of variable renewable energy production, the difficulty of balancing supply and demand increases. With the described development, flexibility is expected to be covered by the demand side to a greater extent. In order to unlock flexibility from thermal storage, batteries, and electric vehicles from the end-user, a market design that incentivizes and promotes demand response is needed.

Simultaneously, distribution system operators (DSO) are seeing peak trends in the distribution grid due to increasing demand and more power-intensive assets such as electric vehicles [2]. Today, most grid tariff structures are energy, and not capacity-based, meaning there is a lack of incentive to avoid high consumption peaks. By pricing the scarce resource (capacity), end-users will have better incentives to reduce peak loads and flatten their load profile. Capacity based tariffs were first described in 2005 [3], but have recently gained renewed attention in Norway as the Norwegian regulator has suggested capacity based tariffs to deal with the mentioned challenges [4]. Previous work on the impact of storage when finding optimal subscribed capacity has been done [5], but without coordination with other end-users.

As technologies like smart meters, ICT systems, and distributed energy resources (DER) such as batteries and photovoltaic (PV) have decreased in price, end-users are transforming from consumers to active agents with local production and flexibility, referred to as prosumers. P2P markets have widely been suggested in the literature as a market design that fully empowers the conscious energy citizen. Multiple market designs spanning from community-based to full P2P markets have been described in [6]. Full peer-to-peer markets represent complete democratization of electricity trade, where preferences such as origin, emission-factor, locality, and production type could be embedded into the electricity trade. However, such systems are futuristic due to the drastic need for robust ICT systems, a potentially slow convergence towards trading consensus, and unclarity in regulation [7], [8].

In a neighbourhood, electricity trading is more manageable, and significant cost savings have been shown when imposing a local P2P market in a neighbourhood with storage assets and local production under a centralized control scheme [9]. Also, [10] and [11] showed that the subscribed capacity tariffs provide strong price signals to reduce peak loads in neighborhoods, especially under centralized metering and billing. One of the shortcomings in the mentioned studies is the assumption of centralized control. In energy markets with many agents, complementarity models are more powerful when analyzing the impact of price signals and market designs, as the rational economic behaviour (best response) of each agent is taken into account. Approaches based on non-cooperative game theoretic models with Nash equilibrium (NE) have been considered in multiple studies, often based on Karush-Kuhn-Tucker conditions. A formulation based on alternating direction method of multipliers (ADMM) is shown in [12]. Alternatively, agent-based models based on complementarity constraints can be formulated directly as a mixed complementarity problem (MCP) or as a Stackelberg game that can be used to model agent behaviour under different market designs [13]. Stackelberg games for design of grid tariffs was demonstrated in [14], [15], where the DSO is modelled as the tariff-setting leader under cost-recovery conditions. Although these papers formulate a realistic interaction between the DSO and costumers through grid tariffs, a local market mechanism is not included.

With the presented context, we extend the study presented in [10] by solving the problem using an equilibrium model for decentralized decisions in a local P2P market under subscribed capacity tariffs. The main contribution of this paper is that we show how subscribed capacity tariffs together with local P2P trading can coordinate end-users to reduce peak loads in neighborhoods. Further, we show how a local P2P market can function as an alternative to centralized tariffs.

The rest of the paper is organized as follows: Section II discusses the market- and grid tariff design. The model is the presented in Section III, followed by the case study description in Section IV. Results and discussions are then presented in Section V before concluding remarks are done in Section VI.

II. MARKET DESIGN

A. *Subscribed capacity tariffs*

Norway is currently changing to a capacity-based grid tariff structure to better reflect the upstream costs of the distribution grid. The clear drawback of a volumetric tariff structure is that costs are unevenly distributed as grid investments are mostly related to capacity, not energy. Thus, two end-users with equal annual consumption would have an similar bill, although the end-users trending towards higher peaks in hours with grid scarcity causes a higher cost for the system.

In this paper, we investigate the impact of subscribed capacity tariffs where agents subscribe to a capacity annually and pay for that capacity. The tariff has three cost components, a cost for subscribed capacity C^{sub}, an energy term for consumption below the subscribed capacity C^{l} and an excess energy term C^{h}. The energy term reflects the marginal grid losses, whereas the excess energy term functions as a penalty for excess consumption. This tariff is beneficial compared to a purely volumetric tariff because it reflects the scarce grid capacity.

B. *Local P2P markets*

A local market is essentially a nano-market where end-users can trade with each other as an alternative to buying from the retailer. The advantages of a local market platform are the creation of incentives for local production and possible coordination of flexibility.

Local P2P markets are similar, but have bilateral trades instead of a pool market for trading. The result is discriminatory prices instead of uniform pricing. An interesting advantage of P2P trades is the possibility of treating electricity as a heterogeneous product both concerning where and how it is produced, but also when and for what it is consumed. In this paper, however, we will only consider risk-neutral and rational agents. Discriminatory pricing still benefits from the fact that different agents have different willingness to pay due to the individual tariffs, export of local production, and opportunity costs from batteries.

C. *Synergies of capacity tariffs and local P2P markets*

The analysis in [10] and [11], showed that subscribed capacity tariffs work better on an aggregated level (e.g., a neighborhood) because of the coincidence factor, meaning that not every end-user has peak loads at the same time. However, both studies rely on centralized control to ensure optimal coordination of flexibility. In this paper, tariffs and decisions are decentralized (per agent) instead of centralized. Furthermore, rather than centralized and direct load control, the P2P market handles the coordination of flexibility under decentralized decision-making.

With this tariff structure combined with a P2P market, we introduce a market that serves two purposes: (1) trading of flexibility from battery storage, and (2) a quota market for the right to use capacity. The first concept is widely agreed upon in both real-life projects and research, simply that local markets are useful for sales of excess PV production for local consumption. Besides, batteries can be used for electricity arbitrage based on spot prices. However, arbitrage-based trade is not necessarily beneficial for the power system as new demand peaks can be created. The second purpose (2) answers this challenge by adding capacity to the list of tradeable products. Because each end-user has paid for a capacity limit, excess capacity can be sold in the P2P market. Agents with available capacity either due to coincidence or flexibility assets can sell a capacity quota when needed by other agents who are about to exceed their subscribed capacity. Indirectly, the aggregated consumption of the P2P market will have an incentive to stay below the aggregated subscribed capacity limit.

In fig. 1, a conceptual trading example is visualized. The bottom left agent is consuming precisely the amount he has

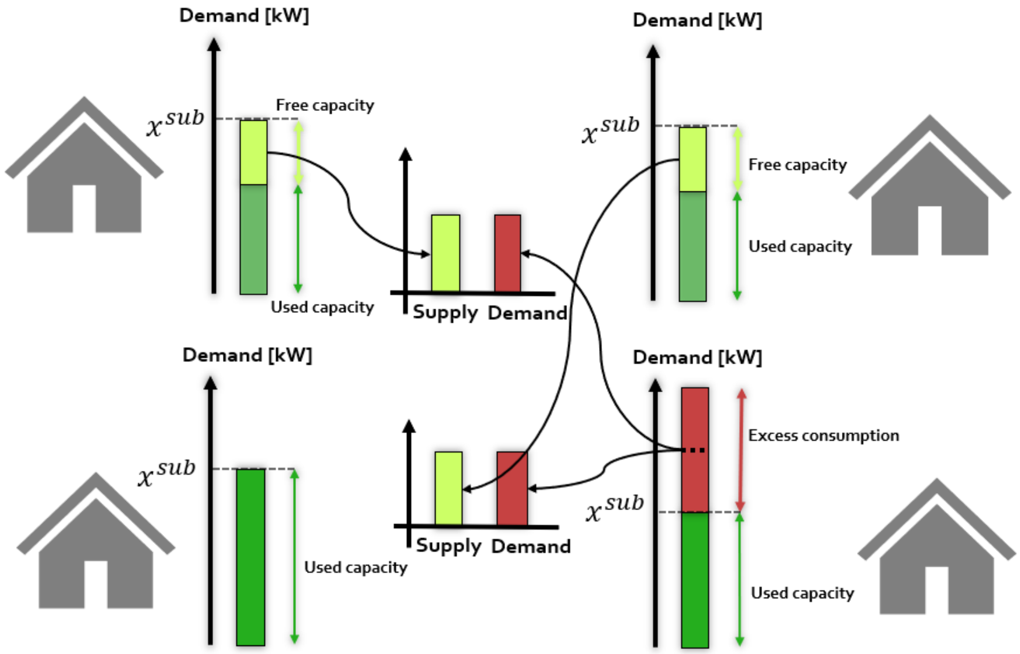

Fig. 1. Capacity peer-to-peer trading example.

subscribed to, whereas the top left and top right agent has some free capacity. As the agents on the bottom right side has excess consumption, he/she is interested in buying the capacity available from the market rather than paying the overcharge fee.

III. MODEL

Modeling decentralized decisions is essential when analyzing the impact of a specific grid tariff or other market design features. In this paper, we show how the DSO can use subscribed capacity tariffs to reduce peak loads in neighborhoods using local markets. The DSO is not modeled explicitly, but we use the grid tariff rates suggested by the Norwegian regulator as a set of exogenous price signals meant to incentivize grid friendly operation of DER. The local market is the enabler, which allows for capacity trading between the agents in the system.

The model is formulated to illuminate the impact of local markets under subscribed capacity tariffs modeled with decentralized decision making. We demonstrate this by formulating the prosumer problem as an electricity bill cost minimization problem, or in essence, maximizing the prosumer's surplus. The local P2P market facilitates capacity trading with discriminatory prices. The prosumers interact with the market through their trades with the retailer and the other agents in the local market.

A. Prosumer problem

The prosumer problem is a cost minimization, where the goal is to minimize the costs of importing electricity to cover the demand. Costs are related to buying electricity on the day-ahead spot market, grid tariff costs, and P2P trading costs. Locally produced electricity can be sold to the day-ahead market or to other peers without grid tariff costs. The objective function is given by (1). The model finds optimal import/export both with the retailer and in the local P2P market. In addition, the subscribed capacity level x_p^{sub} is optimized at each prosumer.

Dual values associated with the constraints are provided and based on the KKT-conditions of this problem, the optimality conditions are formulated as MCP in the Appendix. The MCP formulation allows us to simultaneously solve the prosumer problems with P2P market interaction and derive the Nash equilibrium[1].

$$\begin{aligned}\forall p \quad \min x_p^{sub} C^{sub} + \sum_t [(x_{pt}^{buy} - x_{pt}^{sell}) C_t^{DA} \\ + x_{pt}^{l} \cdot P^{l} + x_{pt}^{h} \cdot P^{h} + \sum_q (\lambda_{pqt}^{P2P} + P^{a}) x_{pqt}^{P2P}] \quad (1)\end{aligned}$$

Import from the grid are split into import below x_{pt}^l and above x_{pt}^h the subscribed capacity x_p^{sub} in (2) and (3).

$$\forall pt \quad x_{pt}^{l} + x_{pt}^{h} - x_{pt}^{buy} = 0 \quad (\nu_{pt}^{tot}) \quad (2)$$

$$\forall pt \quad x_{pt}^{l} - x_p^{sub} \leq 0 \quad (\nu_{pt}^{sub}) \quad (3)$$

The energy balance is given by (4).

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[1] The problem is implemented in GAMS and solved by the PATH solver.

$$\forall pt \quad x^{buy}_{pt} - x^{sell}_{pt} + \sum_{q} x^{P2P}_{pqt} - L_{pt} + G^{PV}_{pt} - q^{ch}_{pt} + q^{dis}_{pt} = 0 \quad (\nu^{eb}_{pt}) \tag{4}$$

Furthermore, the battery state of charge (SOC) balance is given by (5a) and (5b), where (5b) ensures that the SOC in the first and last time period are the same. The bounds on maximum state of charge and max (dis)charging power are given by (5c)-(5e).

$$\forall p(t < t_{end}) \quad e_{p(t+1)} - e_{pt} - q^{ch}_{pt} A^{ch}_{p} + \frac{q^{dis}_{pt}}{A^{dis}_{p}} = 0 \quad (\beta^{soc}_{pt}) \tag{5a}$$

$$\forall p(t = t_{end}) \quad e_{pt_0} - e_{pt_{end}} - q^{ch}_{pt_{end}} A^{ch}_{p} + \frac{q^{dis}_{pt_{end}}}{A^{dis}_{p}} = 0 \quad (\beta^{soc}_{pt_{end}}) \tag{5b}$$

$$\forall pt \quad q^{ch}_{pt} - Q^{ch}_{p} \leq 0 \quad (\beta^{ch}_{pt}) \tag{5c}$$

$$\forall pt \quad q^{dis}_{pt} - Q^{dis}_{p} \leq 0 \quad (\beta^{dis}_{pt}) \tag{5d}$$

$$\forall pt \quad e_{pt} - E^{max}_{p} \leq 0 \quad (\beta^{max}_{pt}) \tag{5e}$$

B. Peer-to-peer market clearing conditions

The market operator ensures balance in all trades between peer p and q, where the dual λ^{P2P}_{pqt} is the discriminatory price between agent p and q as shown in (6). Because we have bilateral trades, prices depend on the objective function of each prosumer.

$$\forall pqt \quad x^{P2P}_{pqt} + x^{P2P}_{qpt} = 0 \quad (\lambda^{P2P}_{pqt}) \tag{6}$$

IV. CASE STUDY

We simulate the problem with four agents for one week with hourly time resolution. Prosumer P1 and P2 have batteries of 10 and 5 kWh, respectively.

- Agent #1: 10 kWh battery, 95 % one-way eff.
- Agent #2: 2 kWp PV, 5 kWh battery, 96 % one-way eff.
- Agent #3: 2 kWp PV
- Agent #4: -

The model determines the optimal subscribed capacity of each agent, as well as the operation of assets and trades with the retailer and the local peer-to-peer market. This is done by simulating with load and PV data from Norway.

We perform the following two case studies:

- Without local P2P markets. End-users optimize their own assets in order to minimize costs.
- With local P2P market. Similar to above, but end-users can interact through P2P trading.

V. RESULTS AND DISCUSSION

By simulating 1 week, we gain insight in optimal operation of flexible assets, subscribed capacity and the share of trades with the retailer and the local P2P market. The results in table I show that by adding a P2P market, a reduction in optimal subscribed capacity for prosumers P3 and P4 is achieved, where as P1 and P2 have relatively similar optimal limits. This reduction is driven by the ability to trade with the other prosumers who have access to battery storage. P1 and P2 can use their batteries actively to sell capacity to P3 and P4 when needed, whereas when no market is available, P3 and P4 must subscribe to higher capacities to lower their bills. The results underline that with the right incentives, local markets facilitate grid friendly consumption patterns due to the locational properties of the market.

TABLE I

OPTIMAL SUBSCRIBED CAPACITY IN KW.

	P1	P2	P3	P4
P2P	1.963	1.905	1.914	1.929
No P2P	1.912	1.917	2.470	2.520

This is further confirmed by looking at fig. 2, where we see a lowering of the highest imports with the P2P market compared to the case without. By using the batteries from P1 and P2, the local P2P market is utilized to provide capacity to agents P3 and P4, allowing them to stay below their reduced subscription limits. As shown in the graph, the imports never exceed their aggregated subscribed capacity, whereas the import is higher in the case with no market. This clearly implies that the market works as a coordination tool and that centralized metering and control is not required to reduce peak loads in a neighborhood.

Battery storage plays a vital role in keeping the import levels below the the subscribed capacity limits. In the No-P2P case, only the agents with battery storage can reduce their import level below the subscription limit. Battery SOC never reaches its maximum in the No-P2P as a consequence, because the agent has no incentive to use the battery. This stands in contrast with the P2P case where both batteries are used to their max. SOC as shown in fig. 4

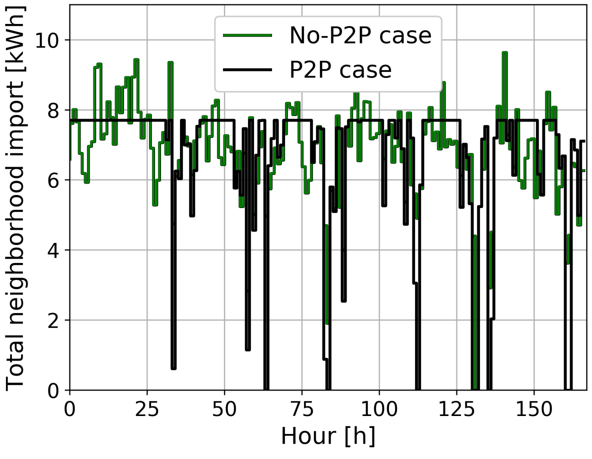

Fig. 2. Total end-user import over 1 week.

The aggregated subscribed capacity can be considered as the ”neighborhood” optimal subscribed capacity, as it allows for zero excess energy consumption as shown in fig. 2. Because

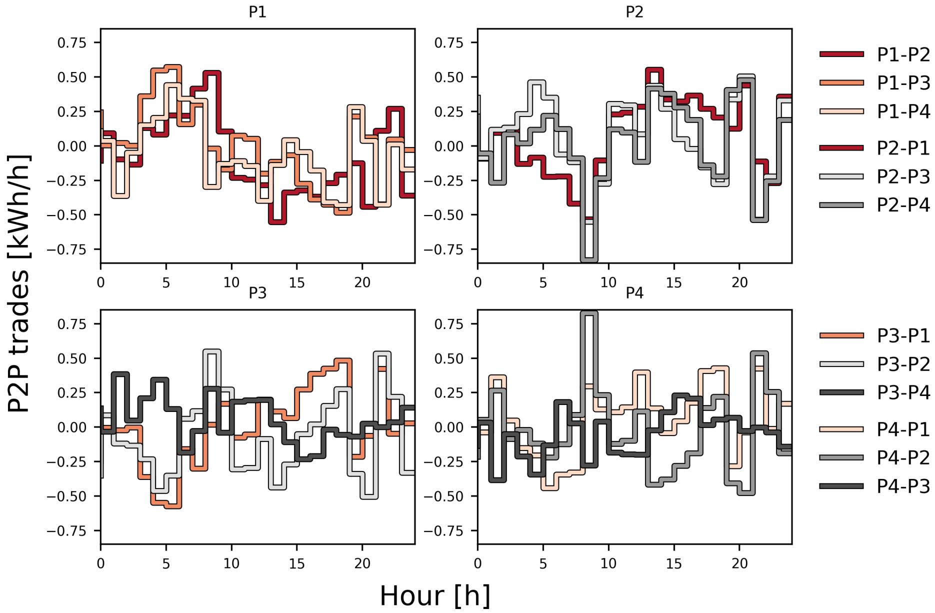

Fig. 3. P2P trading in the first 24 hours of the week.

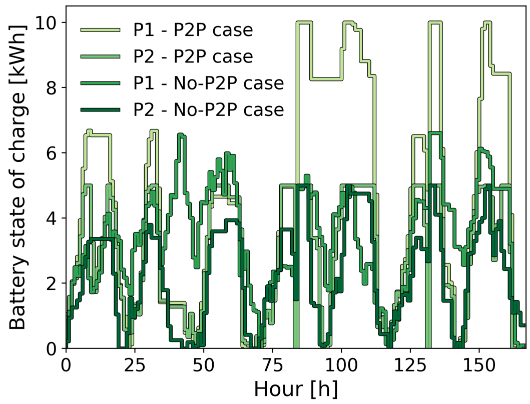

Fig. 4. Battery state of charge in the P2P and no-P2P case.

TABLE II
COSTS PER AGENT IN THE P2P AND NO-P2P CASE IN EURO.

Weekly cost	**P1**	**P2**	**P3**	**P4**	**Total**
No-P2P	€ 13.2	€ 12.1	€ 14.7	€ 15.3	€ 55.3
P2P	€ 13.1	€ 12.0	€ 12.4	€ 13.2	€ 50.7

the P2P market functions as an alternative to centralized coordination, trade happen frequently as a consequence fig. 3. This is the case because the aggregated subscribed capacity is pushed to its minimum, forcing every agent to utilize their limit to the fullest. This strategy results in battery-discharge covered peak loads when the aggregated load surpasses the aggregated subscription limit. In essence, the neighborhood minimizes the possible subscription limit and then uses it to its maximum in the P2P market. This also explains why the aggregated load very often lies on the exact aggregated subscription limit.

Finally, the total electricity costs of the total time horizon for all agents are shown in table II. The reduced costs of €4.6 or 8 % is relatively small. However, it is achieved while still reducing neighborhood peak load by 20 % from 9.64 to 7.71 kWh/h, meaning that these are savings achieved while still saving costs for the DSO. The lost income of the DSO is recovered due to decreased costs, assuming that the tariff is cost reflecting and assures DSO cost recovery. An interesting take is that the agents without batteries are the ones who are reducing their costs the most. This implies that there is a surplus of storage in the case study, which is also confirmed in fig. 4 where agents P1 and P2 most of the time are not using their storage to the fullest, implying a surplus of supply compared to demand in terms of flexibility. In other words, the storage owners compete, resulting in P2P prices close to their alternative opportunity cost of flexibility.

VI. CONCLUSION

We conclude by stating that the local P2P market reduces neighborhood peak loads in combination with capacity tariffs, and works as a useful trading scheme where all agent's preferences are satisfied due to the equilibrium in the market clearing. Peak loads as well as agent costs are decreased, implying synergy between the tariff structure and a local P2P market.

Further work includes cost analysis for each agent, as well as a more complex analysis of how the heterogenous bilateral market price between agent-pairs reflect their opportunity and penalty costs. Futhermore, case studies including investment analysis as well as market efficiency analysis could be performed.

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APPENDIX

As both the market clearing and the prosumer problem are linear, the KKT-conditions are necessary and sufficient for optimality. The final MCP formulation consists of the KKT-conditions of each peer, as well as the P2P market clearing.

First, the market clearing (7):

$$\forall pqt \quad x^{P2P}_{pqt} + x^{P2P}_{qpt} = 0 \quad \perp \lambda^{P2P}_{pqt} \tag{7}$$

followed by the prosumer problem (8a)-(13e).

$$\forall p \quad x^{sub} - \sum_t \nu^{sub}_{pt} \geq 0 \perp x^{sub}_p \geq 0 \tag{8a}$$

$$\forall pt \quad C^l + \nu^{tot}_{pt} + \nu^{sub}_{pt} \geq 0 \perp x^l_{pt} \geq 0 \tag{8b}$$

$$\forall pt \quad C^h + \nu^{tot}_{pt} \geq 0 \perp x^h_{pt} \geq 0 \tag{8c}$$

$$\forall pt \quad C^{DA}_t - \nu^{tot}_{pt} + \nu^{eb}_{pt} \geq 0 \perp x^{buy}_{pt} \geq 0 \tag{8d}$$

$$\forall pt \quad -C^{DA}_t - \nu^{eb}_{pt} \geq 0 \perp x^{sell}_{pt} \geq 0 \tag{8e}$$

$$\forall pqt \quad \lambda^{P2P}_{pqt} + \nu^{eb}_{pt} + P^a \geq 0 \perp x^{P2P}_{pqt} \tag{9}$$

$$\forall pt \quad -\nu^{eb}_{pt} - \beta^{soc}_{pt} A^{ch}_p + \beta^{ch}_{pt} \geq 0 \perp q^{ch}_{pt} \geq 0 \tag{10a}$$

$$\forall pt \quad \nu^{eb}_{pt} + \frac{\beta^{soc}_{pt}}{A^{dis}_p} + \beta^{dis}_{pt} \geq 0 \perp q^{dis}_{pt} \geq 0 \tag{10b}$$

$$\forall p(t > t_0) \quad \beta^{soc}_{p(t-1)} - \beta^{soc}_{pt} + \beta^{max}_{pt} \geq 0 \perp e_{pt} \geq 0 \tag{10c}$$

$$\forall p(t = t_0) \quad \beta^{soc}_{pt_{end}} - \beta^{soc}_{pt_0} + \beta^{max}_{pt_0} \geq 0 \perp e_{pt} \geq 0 \tag{10d}$$

$$\forall pt \quad x^l_{pt} + x^h_{pt} - x^{buy}_{pt} = 0 \quad \perp \nu^{tot}_{pt} \tag{11a}$$

$$\forall pt \quad x^l_{pt} - x^{sub}_p \leq 0 \perp \nu^{sub}_{pt} \geq 0 \tag{11b}$$

$$\begin{aligned} \forall pt \quad & x^{buy}_{pt} - x^{sell}_{pt} + \sum_q x^{P2P}_{pqt} \\ & - L_{pt} + G^{PV}_{pt} - q^{ch}_{pt} + q^{dis}_{pt} = 0 \perp \nu^{eb}_{pt} \end{aligned} \tag{12}$$

$$\forall pt \quad q^{ch}_{pt} - Q^{ch}_p \leq 0 \perp \beta^{ch}_{pt} \geq 0 \tag{13a}$$

$$\forall pt \quad q^{dis}_{pt} - Q^{dis}_p \leq 0 \perp \beta^{dis}_{pt} \geq 0 \tag{13b}$$

$$\forall pt \quad e_{pt} - E^{max}_p \leq 0 \perp \beta^{max}_{pt} \geq 0 \tag{13c}$$

$$\begin{aligned} & \forall p(t < t_{end}) \quad e_{p(t+1)} - e_{pt} \\ & - q^{ch}_{pt}\eta^{ch}_p + \frac{q^{dis}_{pt}}{\eta^{dis}_p} = 0 \quad \perp \beta^{soc}_{pt} \end{aligned} \tag{13d}$$

$$\begin{aligned} & \forall p(t = t_{end}) \quad e_{pt_0} - e^{soc}_{pt_{end}} \\ & - q^{ch}_{pt_{end}} A^{ch}_p + \frac{q^{dis}_{pt_{end}}}{A^{dis}_p} = 0 \perp \beta^{soc}_{pt} \end{aligned} \tag{13e}$$