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A Reactive Power Compensation Method Based on Tracing the Power Flow and Loss Function of Power System

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***Abstract*— this paper proposes a new method for locating and sizing of reactive power compensation of power system. The proposed method is based on the reactive power flow tracing. Defines the power loss distribution coefficient, which has direct response distribution of reactive power sources with clear physical concepts. Utilize this new factor to determine the points of compensation. Calculate the second order loss sensitivity, combined with the principle of most economical operation, to determine the optimal capacity. Application of proposed method is useful for the optimization of reactive power distribution and the reduction of power loss. The simulation result of the IEEE 39-bus system shows the correctness and effectiveness of the proposed method.**

Keywords—Power system;Power flow tracing;Reactive power compensation;Power loss distribution coefficient;The second order loss sensitivity

I. INTRODUCTION

The voltage level of the power system, and the economic operation of the system, has a close relationship with the reactive power levels of the system. By reactive power compensation in several nodes could not only reduce the active power loss and reactive network losses, but also could maintain property voltage levels and improves the stability of power system, to ensure that the power system could operations safely and economy. Therefore, the choice of nodes for reactive compensation is important for stability and economic operation of the system.

Normally, the reactive compensation nodes were determined, in order to improve power system voltage stability or economic operation. For example, reference [1] presents a method based on the concept of pilot node and a realizable algorithm considering random load disturbances to decide the optimal allocation of reactive power, which aims at the reactive power sources allocated most efficiently to minimize the voltage deviation of nodes in point of the whole electric power system. And in [2], a method is implemented in software control of temporal decoupling, based on energy management system, according to the power system soft partition to determine the reactive power source configuration location, in order to optimize the overall voltage. Reference [3] determines the reactive source configuration location by calculation of the sensitivity of the system, aims at minimizing the loss of power system. In recent years, many scholars have tried to solve the reactive power compensation with smart algorithm. For example, taking the costs of reactive power compensation operating into consideration, [4] sets the penalty function, implicating genetic algorithm to determine the compensation position and capacity. In order to minimize operating reactive power compensation costs and raise the voltage stabilization, [5] applicants the particle swarm algorithm and improved particle swarm algorithm to determine reactive power compensation.

Since proposed by J.Bialek in [6], research of application of tracing the flow focuses on determining the cost over the net, share part of using the power grid transmission equipment, and other issues related to electricity market [7-9].

This paper proposes a method of determining the reactive power compensation point, based on power flow tracing, which takes optimization of the distribution of reactive power, and the economic operation into account.

Main work of this article is this: Analyzing the losses in each node of load caused by the reactive power flow by tracing the distribution of reactive load in each branch, and then determining the best position of reactive power compensation and the compensation capacity.

II. APPLICATION OF TRACING THE FLOW IN REACTIVE POWER COMPENSATION

A. DETERMINE THE COMPENSATION NODE USING TRACING THE FLOW

In this section, the system branch reactive power flow will be decomposed to the loads by flow tracing algorithm, to determine the sharing amount of each load for branch reactive power flows.

Assume that the reactive circulating does not exist in the network. Before tracing the reactive flow, adding a virtual node in the middle of all the branches, and equivalent the branch reactive power loss to the reactive load of the virtual node [10]; Line charging power is modeled as reactive power sources in line ends as the π type line equivalent circuit. So the traditional π equivalent model shown in Fig.1 is transformed into the equivalent model for reactive power flow tracing shown in Fig.2.

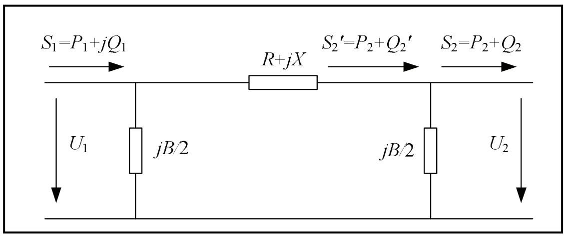

Fig. 1. π shape branch equivalent model

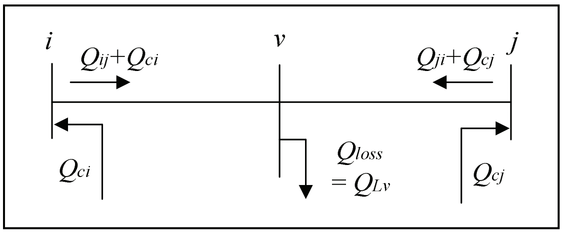

Fig. 2. Equivalent model for reactive power flow tracing

The main principle used to trace the flow is principle of proportional sharing [11]. According to the method in [6], the reactive power contribution of load node for each branch could be easily determined:

$$Q_B = AQ_L \tag{1}$$

Where Q_B is the vector of the reactive flow for each end of the branch, Q_L is the vector for reactive power for load node, Element $A(k,i)$ of A matrix indicates the reactive power contribution of load node i for branch k. Thus, the reactive power contribution of branch k for node i is:

$$Q_B(k,i) = A(k,i) * Q_L(i) \tag{2}$$

Accoding to fig.1, active power loss of the branch is：

$$\begin{aligned} P_{loss} &= I^2 R = \frac{P_2^2 + Q_2^{'2}}{U_2^2} R = \frac{P_2^2}{U_2^2} R + \frac{Q_2^{'2}}{U_2^2} R \\ &= P_{loss}{}^P + P_{loss}{}^Q \end{aligned} \tag{3}$$

Where $P_{loss}{}^Q$ is the active loss caused by the reactive power flow through the branch. Due to the active and reactive power flow of power system can be approximate decoupled, the impact of reactive power compensation for power flow is negligible, so that the primarily impact of reactive power compensation for active loss of the branch is $P_{loss}{}^Q$. According to the "proportional sharing" principle, $P_{loss}{}^Q$ could decomposed to each load node, the factor of load node i for active power loss could be defined as power loss distribution coefficient (γ_i):

$$\gamma_i = \sum_{k=1}^{nl} P_{loss}{}^Q(k) * \frac{Q_B(k,i)}{Q_B(k)} = \sum_{k=1}^{nl} P_{loss}{}^Q(k) * A(k,i) \tag{4}$$

Where nl is the number of branches in the power system.

If there are more reactive source nodes serve reactive power for load node i, and the reactive power transmissions through branch is larger than others, thereby γ_i would be larger than others. So, the active power loss of the system would be reduced effectively, if load node i was compensated properly. Assume the number of node allowed to compensate is k, then γ_i of all load nodes were sorted descending, the largest k_{th} load node would be former elected optimal reactive compensation points.

B. *Determine The Compensation Capacity*

Equal network loss increase ratio is a classical algorithm to determine the capacity for reactive power compensation [12]. Simple derivation process for this algotithm is as follows: The objective function of optimal reactive power compensation capacity distribution is to minimize the active power loss of the system, that is:

$$\min P_{loss}\left(Q_{Gi}\right) \tag{5}$$

Required constraints：

$$\sum_{i=1}^{nb} Q_{Gi} - \sum_{i=1}^{nb} Q_{Li} - Q_{loss} = 0 \tag{6}$$

Using the Lagrange function ：

$$L = P_{loss}\left(Q_{Gi}\right) - \lambda\left(\sum_{i=1}^{nb} Q_{Gi} - \sum_{i=1}^{nb} Q_{Li} - Q_{loss}\right) \tag{7}$$

After derivation calculus to above equation we can get the minimum conditions of the function:

$$\begin{cases} \dfrac{\partial L}{\partial Q_{Gi}} = \dfrac{\partial P_{loss}}{\partial Q_{Gi}} - \lambda\left(1 - \dfrac{\partial Q_{loss}}{\partial Q_{Gi}}\right) = 0 \qquad i = 1、2、\cdots nb \\ \dfrac{\partial L}{\partial \lambda} = \sum_{i=1}^{nb} Q_{Gi} - \sum_{i=1}^{nb} Q_{Li} - Q_{loss} = 0 \end{cases} \tag{8}$$

Equation (8) represents the principle of equal network loss increase ratio.

The steps of using the principle of equal network loss increase ratio to determine the optimum reactive compensation are these:

First, determine the optimal reactive power distribution, select the candidate nodes to compensate, and calculate their network loss increase ratio, compensate the node whose lose increase ratio is least.

After that, recalculated network loss increase ratio of all nodes, then select least network loss increase ratio node to compensate.

And so forth, until the network loss increase rate of all nodes in the system are approximately equal to each other[13], then the system reaches the status of optimal reactive flow.

Thus, solving the above calculation is an iterative process, it’s tedious and time-consuming. This paper tries to find a method which could determine the compensation capacity approach without iterations.

From the solving process above, we could conclude, in determining the compensation capacity, P_{loss} is the function of Q_{Gi}, which is reactive power of node i , and in the stable operating point, P_{loss} can be approximated as a linear function of Q_{Gi}:

$$P_{loss}(Q_{Gi}+\Delta Q_{Gi}) = P_{loss}(Q_{Gi}) + \sum \frac{\partial P_{loss}}{\partial Q_{Gi}} \times \Delta Q_{Gi} \tag{9}$$

Assuming that P_{loss} is a function of Q_{Gi}, and around system operating point, the second derivative of P_{loss} for Q_{Gi} is exist. According to Taylor's theorem,

Near ΔQ_{Gi} equals to 0, $P_{loss} = f(Q_{Gi})$ could expanded to a quadratic function:

$$\begin{aligned} &P_{loss}(Q_{Gi}+\Delta Q_{Gi}) \\ &= P_{loss}(Q_{Gi}) + \frac{\partial P_{loss}}{\partial Q_{Gi}} \times \Delta Q_{Gi} + \frac{1}{2}\frac{\partial^2 P_{loss}}{\partial Q_{Gi}{}^2} \times \Delta Q_{Gi}{}^2 \end{aligned} \tag{10}$$

Active power loss of the power system is sum of active power injected to all nodes:

$$P_{loss} = \sum_{i=1}^{nb} U_i \sum_{j=1}^{nb} U_j \left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij} \right) \tag{11}$$

Where, nb is the number of nodes; G_{ij} and B_{ij} is the real and imaginary part of admittance between node i and node j; δ_{ij} is the difference in the voltage angles between node i and node j; U_i,U_j is the voltage magnitude of node i,j.

Along with the power flow equations, we could conclude:

$$\begin{bmatrix} \dfrac{\partial P_{loss}}{\partial P} \\ \dfrac{\partial P_{loss}}{\partial Q} \end{bmatrix} = \left(J^T\right)^{-1} \begin{bmatrix} \dfrac{\partial P_{loss}}{\partial \delta} \\ \dfrac{\partial P_{loss}}{\partial U} \end{bmatrix} \tag{12}$$

Where J is Jacobi matrix

$$\begin{cases} \dfrac{\partial P_{loss}}{\partial U_k} = 2\displaystyle\sum_{j=1}^{nb} G_{jk} U_j \cos\delta_{kj} \\ \dfrac{\partial P_{loss}}{\partial \delta_k} = 2\displaystyle\sum_{j=1}^{nb} G_{jk} U_k U_j \sin\delta_{jk} \end{cases} \tag{13}$$

The second derivative of P_{loss} for Q_{Gi}, which is node to be compensated, could be calculate by difference method:

$$\frac{\partial^2 P_{loss}}{\partial Q_{Gi}{}^2} = \frac{\dfrac{\partial P_{loss}}{\partial Q_{Gi}}\left(\Delta Q_{Gi} + \Delta\Delta Q_{Gi}\right) - \dfrac{\partial P_{loss}}{\partial Q_{Gi}}\left(\Delta Q_{Gi}\right)}{\Delta\Delta Q_{Gi}} \tag{14}$$

Generally, in a real system, the total reactive power is given, and redistribute the given compensation capacity to each compensation node. Assuming provided compensation capacity is Qc_{sum},

In order to obtain the least system active power loss, conditions below should be satisfied:

$$\frac{\partial P_{Loss}}{\partial Q_{G1}}\Big|_{\Delta Q_{G1}} = \frac{\partial P_{Loss}}{\partial Q_{G2}}\Big|_{\Delta Q_{G2}} = \cdots = \frac{\partial P_{Loss}}{\partial Q_{Gk}}\Big|_{\Delta Q_{Gk}} \tag{15}$$

$$\sum_{i=1}^{k} \Delta Q_{Gi} = Q_{csum} \tag{16}$$

Taking P_{loss} of (10) instead of that in (15), along with (16), optimal compensation capacity of nodes to be compensated is assigned.

III. Case Studies

A. Determine Reactive Power Compensation Scheme

For illustrative purpose, a 39-buses system is used. The topology of the system is shown in Fig. 3.

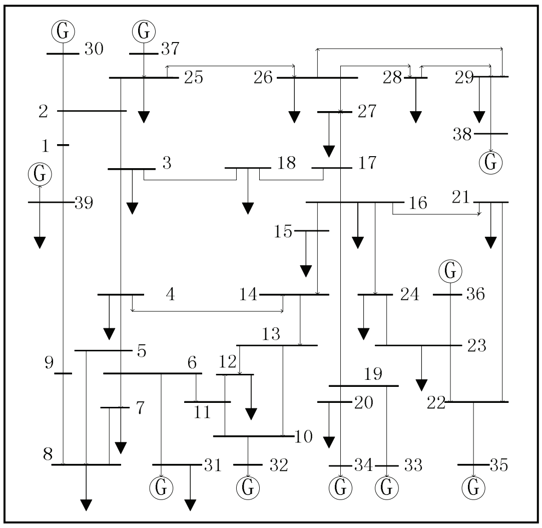

Fig. 3. IEEE 39-bus system topology diagram

According to the result of power flow calculation, reactive power compensation point was determined by the method proposed in section 1.1, and the compensation capacity was determined by the method in section 1.2.

Provided the total reactive power compensation capacity of the system was 1.5, and 4 nodes could be compensated at most.

TABLE I. Information of Compensation Node

nodes	γ**(10⁻³)**	$\partial P_{Loss} / \partial Q_{Gi}$	$\partial^2 P_{Loss} / \partial Q_{Gi}{}^2$	**compensation capacity**
4	2.1636	-0.0046	0.0015	0.3604
8	2.0733	-0.0051	0.0017	0.5897
12	1.5768	-0.0048	0.0030	0.2708
25	1.4321	-0.0050	0.0034	0.2792

According to the descending of power loss distribution coefficient for all nodes, nodes 4,8,12,15 were selected to compensate. The sensitivity of P_{loss} for nodes selected to compensate was calculated in accordance with (12), (13), the second order loss sensitivity was calculated according to (14). Finally, the compensation capacity was determined by (15) and (16). The results are shown in Table 1.

B. *COMPENSATION EFFECT ANALYSIS*

Fig.4 shows the contrast of power loss distribution coefficients of each load nodes before and after compensation, which shows that, power loss distribution coefficients have been effectively reduced. At the same time coefficients of load nodes nearby also decreased, the coefficients of other nodes changed little.

Herein, the results above proof the stability of the coefficient defined and the effective of the method proposed on the other hand.

Since power loss distribution coefficients of the compensated nodes are reduced, so the reactive power coverage of these nodes is reduced, and network loss was reduced and reactive power distribution was improved at the same.

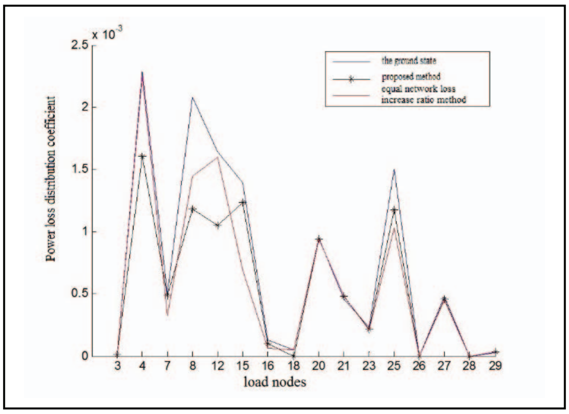

Fig. 4. Comparison of Power loss distribution coefficient between the ground state and compensation

C. *COMPARISON WITH OTHER COMPENSATION METHODS*

To demonstrate the superiority of reactive power compensation method proposed in this paper, for certain reactive power compensation capacity, two reactive power compensation methods were compared:

Scheme Ⅰ: Apply the reactive power compensation method proposed in this paper based on reactive power flow tracing;

Scheme Ⅱ: Using the principle of equal network loss increase ratio to compensate

Results of the two compensation methods were shown in Table 2, in the case of the same total amount of compensation capacity, the two methods are approximate to each other in system power loss reducing. Analyzing of table 2 shows the advantages of the method proposed in this paper:

1) The maximum γ (Power loss distribution coefficient) is least in the compensation state, which ensures that reactive power source of each load nodes absorbed range smaller than other methods, and the distribution of reactive power flow is more reasonable;

2) Computing speeder, proposed method calculate the compensation capacity for each compensation node, based on results of reactive flow tracing. Compare with method of equal network loss increase ratio, which needs multiple iterations, proposed method needs the first and second order loss sensitivity only, so the amount of calculation is less.

TABLE II. RESULTS OF DIFFERENT METHODS OF COMPENSATION

Scheme	**Compensation Information**		**Power loss**	**Maximum γ (10-3)**
	Nodes	***Capacity***		
Ground state			0.4274	2.2864
Scheme I	4	0.3604	0.4205	1.6049
	8	0.5897		
	12	0.2708		
	25	0.2792		
Scheme II	7	0.168	0.4201	2.2455
	8	0.3532		
	15	0.6012		
	25	0.3776		

3) Fig.5 compares the voltage level improving with the two reactive power compensation scheme, which shows that, proposed method has better effect in raising the voltage level.

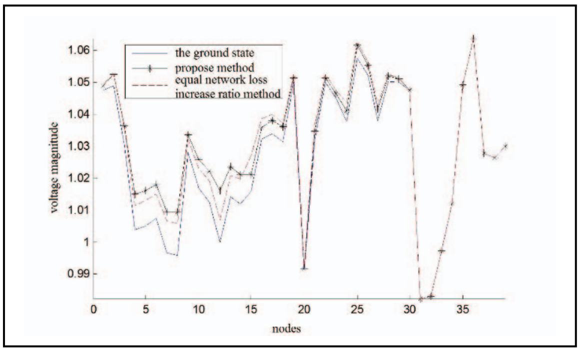

Fig. 5. The improvement of voltage level of different reactive power compensations

IV. CONCLUSION

This paper analyzes the power system reactive power distribution by tracing the flow first, and then proposes a reactive power compensation method based on reactive power flow tracing; Analyzes the distribution of reactive power, defines the power loss distribution coefficient, which is used to determine the compensation point. Compensation capacity was determined by using the second order loss sensitivity.

The method could solve the problem of reactive power compensation effectively, minimize the reactive power transport distance, optimization the distribution of reactive power, and reduce active power loss of the system, which has strong feasibility and practicability in the power system study.

Application in the IEEE 39 buses case shows that, proposed method is effective to solve the reactive power compensation issues.

Proposed power loss distribution coefficient has clear physical concept, responses the effect of reactive power flow contribution to system power loss directly, has a valuable role for continue exploring the power system reactive power flow in physical.

REFERENCES

[1] Xiaoyin Yao, Yuanzhang Sun, Zhifang Wang, "Studies on the optimal allocation of reactive power sources, " Automation of electric power systems, vol. 3, pp. 12-15, 1999

[2] Hongbin Sun, Boming Zhang, Qinglai Guo, et al, "Design for global optimal voltage control system based on soft identification of secondary control zones, " Automation of electric power systems, vol. 8, pp. 16-20, 2003

[3] Yann-Chang Huang, Hong-Tzer Yang, "Solving the capacitor placement problem in a radial distribution system using tabu search approach, " IEEE Trans. on Power Systems, vol. 4, pp. 1868- 1873, 1996

[4] G. Levitin, S. Mazal-Tov, B. Reshef, "Genetic algorithm for optimal planning of transmission system reactive power compensation under security constraints, " Electro- technical Conference, Tel-Aviv, p. 18-20 , May 1998

[5] El-Dib, A. A, "Optimum Var sizing & allocation using particle swarm optimization, " Transmission and Distribution Conference and Exhibition, Dallas, TX, p. 21-24, May 2006

[6] J. Bialek, "Tracing the flow of electricity, " IEEE Proc. Gener. Transm. Distrib, vol. 4, pp. 313-320, 1996

[7] Rao Liu, Tao Tang, Weidong Li, Yun Zou, "Research on power flow tracing method considering the direction of bilateral contract power flows, " Proceedings of the CSEE, vol. 7, pp. 53-58, 2004

[8] Xiaorong Wu, Kaigui Xie, Jiaqi Zhou, "Transmission loss allocation method based on current decomposition and bi-directional trace, " Proceedings of the CSEE, vol 21, pp. 24-29, 2005

[9] Chengmin Wang, Yong Liu, Chuanwen Jiang, "Network losses allocation method based on the market equilibrium principle, " Automation of electric power systems, vol. 6, pp. 35-38, 2005

[10] Ping Wei, Yixin Ni, Fulin Wu, Heng Chen. "Power transfer allocation for open access using graph theory, " Proceedings of the CSEE, vol. 6, pp. 21-25, 2000

[11] J. Bialek, "Topological generation and load distribution factor for supplement charge allocation in transmission open access, " IEEE Trans on Power Systerm, vol. 3, pp. 1185-1190, 1997

[12] Heng Chen. Steady power system analysis, erd ed, Beijing: China Electric Power Press, 2007, pp. 35-46

[13] Longzhang Jin, Yushan Ding. Utility reactive power compensation technology, Beijing: China Water & Power Press, 1997, pp. 77-84