# Voltage Drop and Power Loss Suppression of DCAT System with Dynamic Characteristics

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***Abstract*-DC auto-transformer (DCAT) traction power supply system has been studied for the rail potential and stray current issues in urban rail transit. By transferring rail current to the specific lines with supply voltage doubling, DCAT system helps mitigate the voltage drop and power loss issues additionally. Both voltage drop and power loss analysis of DCAT system with dynamic characteristics are carried out based on its equivalent model. Moreover, the voltage drop and power loss comparisons between DCAT system and the existing system are analyzed in detail. Finally, extensive simulation results verify the theoretical analysis effectively.**

***Keywords*-Urban rail transit, DCAT system, voltage drop, power loss, dynamic characteristics**

## I. INTRODUCTION

Nowadays, urban rail transit generally develops towards high speed and heavy duty, and the train requires higher power, which causes significant voltage drop issue on the overhead line (or third rail) and rail [1]. In practice, the train load has many different conditions, such as traction, coasting, braking, and so on [2],[3]. And the voltage drop corresponds to the positive train current (i.e. traction and coasting conditions), and the over-voltage corresponds to the negative train current (i.e. braking condition). Moreover, the recommended supply voltage fluctuation ranges is 500□900V in DC750V railways, while the value is 1000□1800V in DC1500V railways [4]. Beyond the recommended range may threaten the normal operation of power supply equipment. Meanwhile, the higher power loss also poses challenges in the operation cost and heat dissipation.

Some measures has been proposed for the voltage drop and power loss issues. Here the traditional solutions are mainly classified as three types: reducing the distance between substations, decreasing the rail resistance, and increasing the voltage level of power supply system. But it should be noted that the costs of these solutions are high while the effects are limited. Thus the power electronic based solutions have been proposed, such as medium voltage DC (MVDC) three-wire supply system [5], AC traction power supply system [6], energy storage schemes (i.e. supercapacitor, battery, flywheel, etc.) [7], and energy feedback schemes[8].

However, MVDC three-wire supply system only adopts one DC chopper positioned in the middle of rail, then it leads to the poor dynamic performance of system. And AC traction power supply system adopts the higher voltage supply (such as AC25kV), which leads to the larger insulation distance and construction cost. What's more, the energy storage schemes are generally large and expensive, and more suitable for the lower voltage supply (i.e. DC750V or DC600V) at present. And the energy feedback schemes introduce significant power loss and harmonics, when the energy feedback to high-voltage side. Thus it's necessary to study the more feasible power electronic solutions for the voltage drop and power loss mitigation.

DC auto-transformer (DCAT) traction power supply system has been studied for the rail potential and stray current issues [9],[10], and it seems one of effective solutions for the voltage drop and power loss issues simultaneously. By transferring rail current to specific lines, the supply voltage of DCAT system doubling compared with the existing system, then DCAT system may alleviate the voltage drop and power loss level. And the voltage drop and power loss analysis are discussed detailly in this paper. The rest of this paper is organized as follows. The DCAT system configuration and operation principles are analyzed in Section II. Then the voltage drop and power loss comparisons between DCAT system and the existing system under the constant train load and variable train load with dynamic characteristics are discussed in Section III. Moreover, the simulation results of DCAT system and the existing system are shown in Section IV. Finally, the conclusions are presented in Section V.

## II. DCAT SYSTEM

Fig. 1 shows the configuration of DCAT system, which adds the return line (also named as the negative feeder) and several DCATs compared with the existing system. The typical DCAT configuration is shown in Fig. 2, which consists of two DC-link capacitors $C_1$, $C_2$, four switches $S_1$□$S_4$, one resonant capacitor $C_r$ and one resonant inductor $L_r$. And the DCAT's terminals (i.e. positive terminal, zero terminal and negative terminal) are connected to the overhead line, rail and return line respectively. Obviously, DCAT transfers the traction current from the rail to overhead line and return line, thus the rail current only exists in the train-running section, and the rail current of the no-train sections is theoritically zero. Thus DCAT system solves the stray current and rail potential issues fundamentally.

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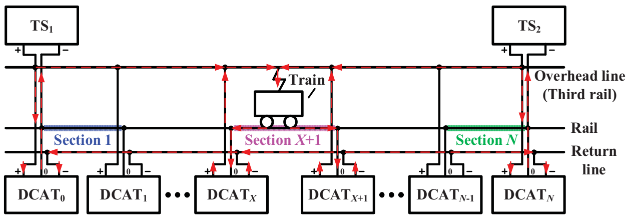



Fig. 1. Configuration of DCAT system

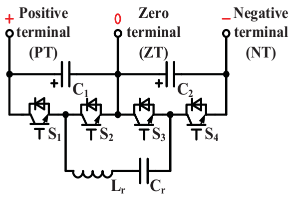
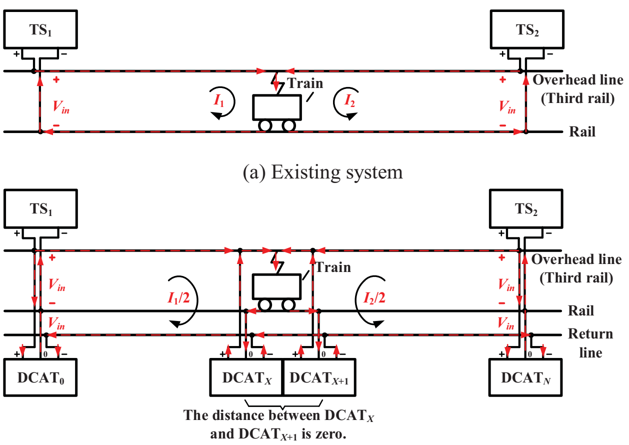



Fig. 2. Typical configuration of DCAT

Meanwhile, for no-train sections, the traction substation (TS) feeds the train through the overhead line and return line. This means that if the section number is high enough, the no-train sections can be approximately regarded as the whole rail. Considering the voltage between overhead line and return line is doubled compared with that between overhead line and rail, the overhead line current reduces to its half as shown in Fig. 3. Supposing the resistances of the overhead line, rail and return line are equal, then the voltage different between TSs and train and the power loss of supply lines are both reduced to 1/4 of the original. And it should be noted that, for DCAT system, the raill voltage difference is zero in DCAT system because there is no current on the rail.




(a) Existing system

(b) DCAT system with infinite sections

Fig. 3. Current distributions of existing system and DCAT system.

However, the section number of DCAT system can not be infinite in practice, and the recommended sections number is 3 to 5 under the different distance between TSs [9]. Thus the train-running section will inevitably introduce more voltage drop and power loss into DCAT system. Therefore, in order to accurately obtain and reduce the voltage drop and power loss of DCAT system with finite sections, the above issues should be studied in detail with the theoretical analysis.

## III. Voltage Drop and Power Loss Analysis

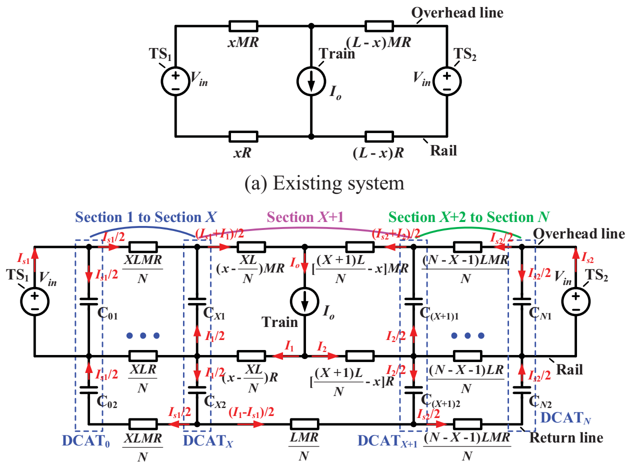
To analyze the voltage drop and power loss in DCAT system, the equivalent models shown as Fig. 4 are built. With the following assumptions:

1) TSs are equivalent to the voltage source $V_{in}$, and the train is equivalent to current source $I_o$;

2) Rail resistance is $R$, while the overhead line and return line resistances are $M$•$R$, here $M$ means the resistance ratio of rail to overhead/return line;

3) The distance between $TS_1$ and $TS_2$ is $L$, and the distance between $TS_1$ and the train is $x$;

4) Rail is divided into $N$ sections with $DCAT_0$ to $DCAT_N$, while the train is running on the section $X$+1 ($X$=0, 1, 2, …, $N$-1).




(a) Existing system

(b) DCAT system

Fig. 4. Configuration of the equivalent model.

### A. Constant Train Load Condition

As shown in Fig. 4(a), the voltage drop of existing system between TSs and train is obtained as

$$\Delta V_{\mathrm{E}} = \frac{(L-x)x}{L}(M+1)RI_o \quad ,0 \le x \le L \tag{1}$$

For the equivalent model of DCAT system as shown in Fig. 4(b), based on the operating principle of DCATs, the current distributions are highlighted in red, then the relationship among load current, rail currents and TS currents is given as

$$I_o = I_1 + I_2 = I_{s1} + I_{s2} \tag{2}$$

Through the further derivation, the voltage drop of DCAT system is shown as

$$\begin{aligned}\Delta V_{\mathrm{DCAT}} = &\left(\frac{M}{2}+1\right)\frac{RI_o}{NL}(Nx-XL)\left[(X+1)L-Nx\right] \\ &+\frac{MRI_o}{2L}x(L-x) \quad ,\frac{XL}{N} \le x < \frac{(X+1)L}{N}\end{aligned} \tag{3}$$

Meanwhile, for the existing system and DCAT system with ignoring the power loss of DCATs, the relationship between the power loss and voltage drop can be derived as

$$P_{loss} = \Delta V I_o \tag{4}$$

Taking $M$=1 as an example, based on (1), (3), and (4), the voltage drop and power loss comparison are shown in Fig. 5. Here, $V_x$=$LRI_o$/2 and $P_x$=$LRI_o^2$/2. Obviously, the voltage drop and power loss of DCAT system are reduced with increasing the section number $N$. Define $\lambda$ as the average voltage drop and power loss ratio of DCAT system to the existing system under constant train load condition, then the average ratio $\lambda$ can be calculated as

$$\lambda = \frac{\Delta V_{\mathrm{DCATav}}}{\Delta V_{\mathrm{Eav}}} = \frac{P_{loss\_\mathrm{DCATav}}}{P_{loss\_\mathrm{Eav}}} = \frac{MN + M + 2}{2N(M+1)} \tag{5}$$

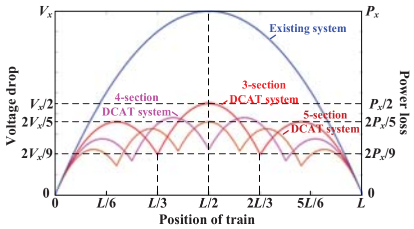


Fig. 5. Voltage drop and power loss comparisons ($M$=1)

Clearly, based on (5), $\lambda$=1/4 when $M$=1 and $N$ is infinite, which proves that the voltage drop and power loss are both reduced to 1/4 under the high section number and equal resistances among the overhead line, rail and return line. Moreover, the voltage drop and power loss ratio curves based on the recommended sections (i.e. $N$=3, 4, 5) are shown as Fig. 6. The average ratio $\lambda$ is about 0.5, and it increased with the resistance ratio $M$. That means that higher overhead/return line resistance leads to higher voltage drop and power loss.

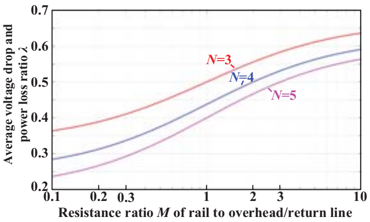


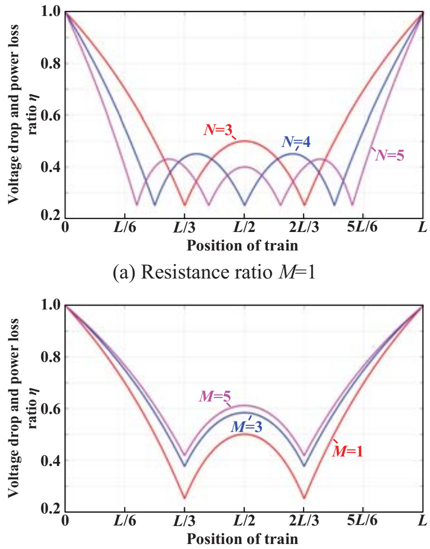
Fig. 6. Voltage drop and power loss ratio curves

*B. Variable Train Load Condition*

Considering the value and direction of the train current changing with the different working conditions, the average ratio $\lambda$ is no longer applicable to the variable train load with dynamic characteristics. Define $\eta$ as the voltage drop and power loss ratio of DCAT system to the existing system under variable train load, and the ratio $\eta$ can be given as

$$\eta = \frac{M}{2(M+1)} + \frac{(M+2)(Nx - XL)[(X+1)L - Nx]}{2N(M+1)(L-x)x}, \quad \frac{XL}{N} \le x < \frac{(X+1)L}{N} \tag{6}$$

Obviously, the ratio $\eta$ changes with the position of train, and it reaches the minimum value $\eta_{\min}$=$M$/[2($M$+1)] at the position $x$=$kL/N$ ($k$=1, 2, …, $N$-1). Fig. 7 shows voltage drop and power loss ratio $\eta$ comparisons under the different resistance ratio $M$ and number of sections $N$. And the ratio $\eta$ is close to 1 near TSs, which means DCAT system has little effect on reducing the voltage drop and power loss at these positions, but these issues near TSs are usually more serious than other positions. Therefore, for DCAT system with the certain $M$ and $N$, it is necessary to reduce the ratio $\eta$ near TSs for the voltage drop and power loss suppression.



(a) Resistance ratio $M$=1

(b) Number of sections $N$=3

Fig. 7. Voltage drop and power loss ratio comparisons

Taking $M$=1, $N$=3 as an example, for the traditional DCAT system, the rail is divided into 3 sections equally by installing $\mathrm{DCAT}_0$ to $\mathrm{DCAT}_3$ (i.e. $x_{\mathrm{DCAT1}}$=$L$/3 and $x_{\mathrm{DCAT2}}$=2$L$/3, here $x_{\mathrm{DCAT}}$ means the distance between $\mathrm{TS}_1$ and DCAT, moreover, $\mathrm{DCAT}_0$ and $\mathrm{DCAT}_3$ are installed in $\mathrm{TS}_1$ and $\mathrm{TS}_2$ respectively). And for the moved DCAT system, $\mathrm{DCAT}_1$ position change from $L$/3 to $L$/6, while $\mathrm{DCAT}_2$ position change from 2$L$/3 to 5$L$/6. Then the voltage drop and power loss ratio comparisons with DCAT movement are shown in Fig. 8 through further derivation. Obviously, the ratio $\eta$ of the moved DCAT system decreases near TSs, at the cost of increasing the ratio $\eta$ in the middle of TSs.

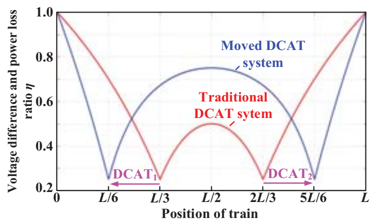


Fig. 8. Voltage drop and power loss ratio comparisons with the movement

In order to further analyze the voltage drop and power loss with dynamic characteristics, Beijing Subway Yizhuang Line (DC750V) from Songjiazhuang Station to Xiaocun Station as shown in Fig. 9 is taken as an example. Moreover, the rail resistance $R$ and resistance ratio $M$ are given as 30mΩ/km and 1, and 3-section DCAT system is adopted for the voltage drop and power loss suppression. Meanwhile, the train load voltage and power loss of DCAT system are derived as

$$V_T(x) = V_{in} - \frac{(L-x)x}{L}(M+1)RI_o(x)\times\eta(x) \tag{7}$$

$$P(x) = \frac{(L-x)x}{L}(M+1)RI_o^2(x)\times\eta(x) \tag{8}$$

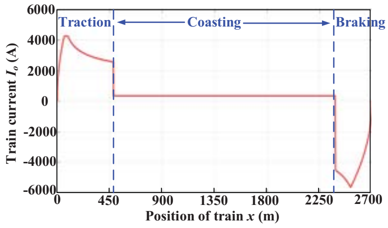
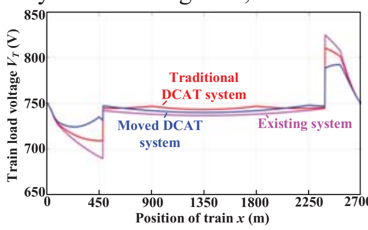
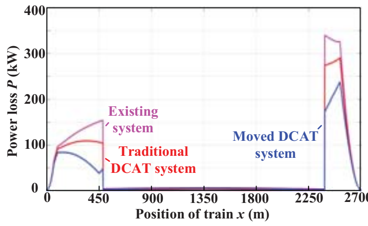



Fig. 9. Dynamic characteristics of Beijing Subway Yizhuang Line

Based on (7) and (8), the train load voltage and power loss comparisons with the dynamic characteristics of Beijing Subway Yizhuang Line are shown in Fig. 10. Obviously, the train load voltage fluctuation and power loss of DCAT system are reduced compared with the existing system, thus DCAT system may solve the voltage drop and power loss issues effectively. Moreover, the voltage fluctuation and power loss can be further reduced with DCAT movement. Thus the moved DCAT system may be more suitable for urban rail transit with dynamic characteristics comparing with the traditional DCAT system. Moreover, it should be noted that the optimal position of DCAT sytem still needs the comprehensive consideration, including the space, cost, rail potential and stray current mitigation, and so on.




(a) Train load voltage




(b) Power loss

Fig. 10. Train load voltage and power loss comparisons with the dynamic characteristics of Beijing Subway Yizhuang Line

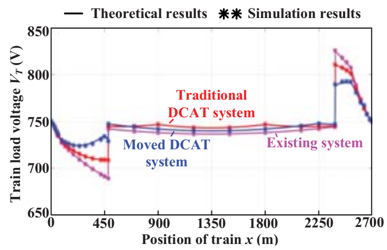
## IV. Simulation Verification

To verify the above-mentioned theoretical analysis, the simulation models with the dynamic characteristics as shown in Fig. 7 are performed in Matlab/Simulink environment, and the simulation parameters are listed in Table 1. What's more, all the components are assumed ideal to simplify the analysis. And each DCAT adopts the phase-shift control to achieve the voltage-balancing between DC-link capacitors [10].

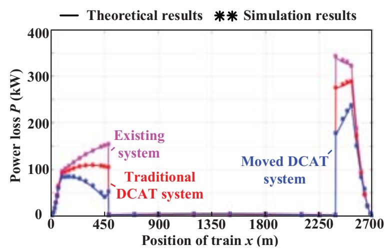
TABLE I
SIMULATION PARAMETERS

| Parameter | Symbol | Value |
|---|---|---|
| TSs voltage | $V_{in}$ | 750V |
| Distance between TSs | $L$ | 2.7km |
| Rail resistance | $R$ | 30mΩ/km |
| Resistance ratio | $M$ | 1 |
| Section number | $N$ | 3 |
| DC-link capacitor of DCATs | $C_1$, $C_2$ | 10mF |
| Resonant capacitor of DCATs | $C_r$ | 2mF |
| Resonant inductor of DCATs | $L_r$ | 2μH |
| Resonant frequency of DCATs | $f_r$ | 2.5kHz |
| Switching frequency of DCATs | $f_s$ | 5.0kHz |

The train load voltage and power loss comparisons between the theoretical results and simulation results are shown in Fig. 11, while the voltage drop and power loss ratio comparisons are given in Fig.12. And the comparisons results are listed in Table 2. Obviously, the simulation results are basically same as the theoretical results, and the mean errors are limited in 1.13%. Comparing with the existing system, DCAT system helps alleviate the load voltage fluctuation and power loss effectively. Here, the maximum voltage drop, over-voltage and power loss of the existing system are 60.6V, 75.7V and 343.5kW, while that of traditional DCAT system is reduced to 41.2V, 60.8V and 288.6kW (i.e. reduce to 67.99%, 80.32% and 84.02%). Meanwhile, the maximums of moved DCAT system reduced to 25.7V, 42.3V and 234.7kW (i.e. reduce to 42.41%, 55.88% and 68.33%) respectively. Therefore, the simulation results prove that DCAT system solves the voltage drop and power loss issues effectively, and the moved DCAT system shows the better performance compared with the traditional DCAT system.




(a) Train load voltage




(b) Power loss

Fig. 11. Train load voltage and power loss comparisons between the theoretical results and simulation results

TABLE II
SIMULATION COMPARISONS RESULTS

| | Theoretical results | | | Simulation results | | |
|---|---|---|---|---|---|---|
| | Maximum train voltage | Minimum train voltage | Maximum power loss | Maximum train voltage | Minimum train voltage | Maximum power loss |
| Existing system | 825.1V | 689.4V | 340.1kW | 825.7V | 689.4V | 343.5kW |
| Traditional DCAT system | 810.3V | 709.1V | 291.0kW | 810.8V | 708.8V | 288.6kW |
| Moved DCAT system | 729.1V | 724.4V | 237.6kW | 792.3V | 724.3V | 234.7kW |

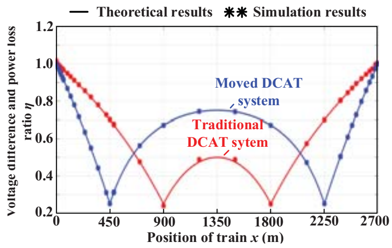


Fig. 12. Voltage drop and power loss ratio comparisons between the theoretical results and simulation results

## V. CONCLUSIONS

This paper presented the voltage drop and power loss mitigation of DCAT system. And the voltage drop and power loss issues of DCAT system with dynamic characteristics are analyzed based on equivalent models. Then the comparisons between DCAT system and the existing system are analyzed detailly in this paper. And define the voltage drop and power loss ratio of DCAT system to the existing system to measure the effect at different positions. However, the traditional DCAT system with equal sections may have the poor effect near TSs. Therefore, the influence of DCAT position has been discussed for the performance improvement. Finally, the correctness and effectiveness of theoretical analysis are validated with the simulation results. And all results show that DCAT system solves the voltage drop and power loss issues effectively, which proves DCAT system is suitable for urban rail transit.

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