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Survey on time-domain power theories and their applications for renewable energy integration in smart-grids

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Abstract: The increasing aggregation of renewable-based distributed generating units besides the impressive growing usage of non-linear loads raises unwanted challenges for traditional power terms definition in power engineering. This fact consequently affected the performance of the conventional control frameworks and industrial compensation techniques. In this study, the authors aim to provide an insightful summary over the most recognised time domain-based instantaneous power theories and discuss their advantages and disadvantages within a comprehensive mathematical-conceptual and applicational framework for professionals who are using instantaneous power theories within the smart grid applications. They conclude that there is still a need for a modified power theory which can be validated under non-sinusoidal-unbalanced load/source conditions respecting the physical meaning of different power and current components.

1 Introduction

In the field of power engineering, the power components, such as active, reactive, are the basic and the most important conceptual tools for both system and control design. Under sinusoidal operating conditions, due to the straightforward configuration of a typical power system, there is no ambiguity in the definition of these components for a linear single-phase or linear balanced three-phase system. During the last two decades, microgrid systems containing a variety of nonlinear loads such as power electronic converters and speed drives have been enormously integrated into power systems and started to challenge the traditional power terms concept and their mathematical definitions [1]. An important concern regarding this aggregation of microgrids is their output power oscillations and harmonic injection. The harmonic injection is an unwanted phenomenon which is enforced to the grid by non-linear loads; especially switching devices used in power electronic interfaces. They can considerably reduce the quality of the voltage and current waveforms and causing interference problems in communication, power losses in distribution networks, and operational failures of electronic devices. Power standards (such as IEEE-519) enforce restrictions on the maximum amount of harmonic contents that a certain microgrid can inject to the grid at the point of common coupling (PCC).

On the other hand, three-phase power systems have real-life imbalances, such different voltage drop along with phases making the three-phase voltages imbalanced. Load imbalances in addition to current harmonics will make the system to have additional voltage distortions, plus neutral current flow, affecting the performance of reliability of power-flow usually controlled by droop techniques. Such imbalances might be precursors to more serious contingencies leading to blackouts, excessive losses, insulation degradation, and production interruptions. Also, system imbalance degrades fault location accuracy and state estimation performance. Therefore, an imbalance must be detected and compensated. Symmetrical components transformation converts three phase-measurements into a positive, a negative and a zero sequence. In balanced circuits, it produces a significant value only in the positive sequence, because all phases are rotated to be added in-phase. Monitoring the power system and state estimation methods are very often based on the positive sequence and assume that the network is balanced and can be represented by a single-phase equivalent. Voltage transients and grid voltage unbalanced required added functionality and increased the complexity of the control system, since the performance of conventional control strategies, very often design for ideal and symmetrical conditions, will deteriorate under unbalanced conditions.

Active filters were proposed to compensate for the unwanted parts of voltage and current signals and deal with the associated power quality issues [2]. These filters are three-phase current controlled inverters with a large capacitor on their DC link that is connected in parallel with non-linear loads at PCC. While several methods have been introduced for active filtering implementation within different applications [2–4], the vital step in all the active filtering methods is the reference signal extraction which is done by using power theories [2], where the goal is to extract a reference compensating current or voltage component from the distorted waveform to be used in the final controller design procedure. A lot of research has been conducted during the last few years to address the associated challenges regarding VSC systems design for unbalanced conditions, most of which, are considering the unbalance to arise from the unbalance load condition. However, the frontier studies are arising new concerns regarding situations where the unbalance is originated in the unbalanced voltage source conditions also widely known as the weak-grid. An example of such a condition could be the occurrence of the geomagnetic induced currents caused by the space storms [5]. In this work, we aim to present a survey over the most famous time-domain power theories, in addition to a comprehensive discussion over their applicability under balance, imbalance and weak-grid conditions within a comprehensive mathematical-conceptual and applicational framework.

2 Related work and the proposed approach

The state-of-art literature shows a plethora of studies have been conducted to find a generalised power theory that is applicable under non-sinusoidal and unbalanced conditions [6–40]. However, imperfections in each one of them made them criticised by researchers [41–48]. Among these survey-type articles, some have been characterised by analysing the power theories from their physics viewpoint, while others worked through electrical circuits theory and simulation results. Those example-oriented approaches were not fully validated due to their dependency on how accurately each method has been implemented. Recently in [49], the authors did an informal survey on power theories by reviewing a comprehensive body of articles. However, there is no detailed mathematical overview of their work. It is worth noting that although power theories have been realised in both time and frequency domain grounds, due to real-time implementation capability, most of the reliable and industrial configurations and control schemes have been implemented using time-domain-based theories.

Motivated by the lack of a necessity and sufficient survey in this area, the goal of this paper is to present a comprehensive conceptual-mathematical introduction over most famous power theories, with focus on five widely recognised time-domain-based approaches, which are (i) instantaneous power theory (*pq* theory), (ii) Fryze–Buchholz–Depenbrock (FBD) theory, (iii) Synchronous reference frame (*dq*0) method, (iv) generalised instantaneous power theory (GIPT), and (v) conservative power theory (CPT). Moreover, we discuss a couple of state-of-the-art approaches in addition to a practical comparative case study between one of them and the CPT.

Our paper is organised as follows: first, in Section 3, we introduce different classes of power theories. The most recognised time-domain power theories are then explained in Section 4 with details. Section 5 introduces a couple of state-of-the-art approaches in this field. Section 6 presents a comparative study among both conventional and frontier approaches, including practical simulations and examples. Finally, in Section 7, we have conclusions, relevant in helping the reader, in how to use this paper for modern and future applications of instantaneous power theories for signal decomposition in renewable energy integration in smart-grids.

3 Universe of power theories

Power theories can be categorised into three main classes: (i) time domain, (ii) frequency domain, and (iii) combined time–frequency domain (with wavelet or similar transformation). Fig. 1 summarises main power theories along with their corresponding categories during the last almost 100 years of history.

3.1 Frequency domain

Budeanu in [6] introduced the solution to solve the problem of defining power components under non-sinusoidal conditions by using a frequency-domain based approach. In the frequency domain, the voltage and current signals are expressed as functions of multiples of the fundamental frequency. Some of the frequency domain approaches are Fourier series, Fourier transforms and fast Fourier transform (FFT), which is the most notable method among frequency domain techniques. FFT can decompose current and voltage signals into different frequencies. The main problem of frequency domain approaches is that in the case of many harmonics, especially when the signal contains inter-harmonics cannot works properly and produces a large amount of error [49].

3.2 Time domain

After Budeanu's work, Fryze introduced his theory in the time-domain [7]. In the time-domain approach, voltage and current signals are expressed as a continuous and instantaneous function. The advantage of a time-domain approach is the instantaneous calculation of current and power terms. Of course, the drawback is not to have the capacity to decompose signals in different frequencies for harmonic analysis.

3.3 Time–frequency domain

Time–frequency can be used as an appropriate domain to define power and current terms. On the one hand, incorporating time representation-based approaches such as multiresolution filter banks, it can be implemented within a fast manner to calculate the associated power terms, while on the other hand by using frequency domain alternatives it can visualise the current signals with more details in different frequencies. Yoon and Devaney used time–frequency approach in [28, 29] exploiting the idea of discrete wavelet transform (DWT) for single-phase system. Although this method works in case of single-phase systems, its expansion to multiphase systems is still under discussion [36]. Thus, this method will be used after finding an appropriate time-based approach because in this domain, the base frame is one of the time-based approaches [49].

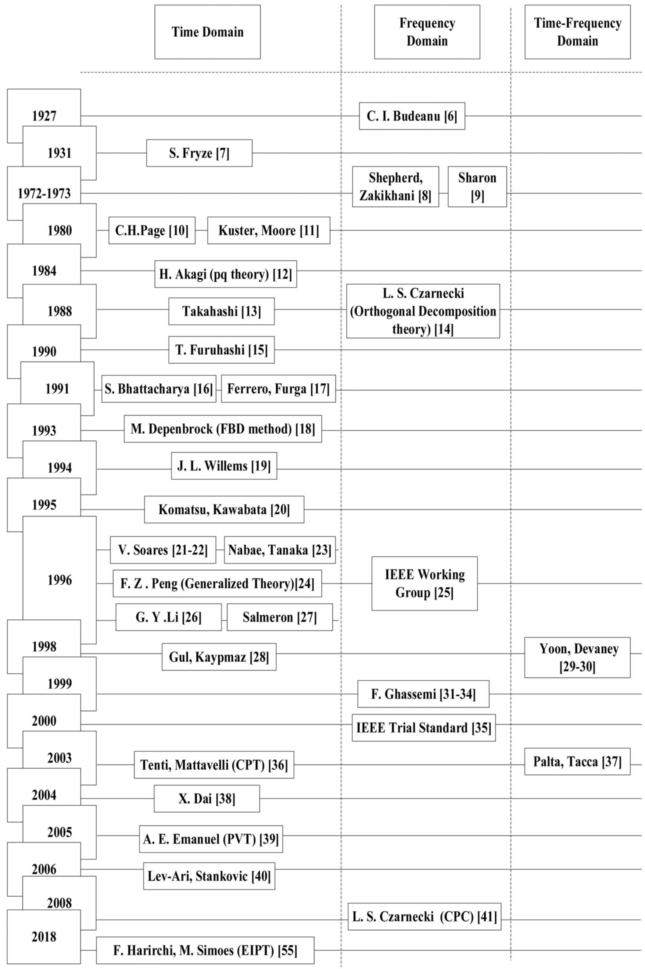

Fig. 1 *Plethora of power theories*

4 Definition of power components under unbalanced and non-linear conditions

In this section, we explain the most relevant time-domain-based power theories (widely known as instantaneous power theories) and discuss their advantages and disadvantages in addition to an overview on their most notable applications among the literature.

4.1 Instantaneous power theory (p - q theory)

In 1984, the instantaneous power theory was introduced by Akagi. Nowadays, it is the most well-known and accepted approach by the power electronics community.

4.1.1 Mathematical analysis formulation: Some authors consider the *p - q* theory as a theoretical tool for active filters control in addition to energy properties' definitions [50, 51]. The *p - q* theory uses Clarke transformation, which transfers three-phase voltages and currents in *a–b–c* coordinates to the *α–β–*0 coordinates (Fig. 2)

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{\sqrt{2}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} \tag{2}$$

Then it is defined

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \tag{3}$$

where p is an instantaneous real power that corresponds to the energy which is transferred from the source to the load and q is the instantaneous imaginary power and it is responsible for the existence of currents which circulate between the phases of the system. Using inverse transformation, real and imaginary current components in α–β–0 frame are obtained as follows:

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{v_{\alpha\beta0}^2} \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} \tag{4}$$

where

$$v_{\alpha\beta0}^2 = v_\alpha^2 + v_\beta^2 + v_0^2 \tag{5}$$

$$i_{\alpha p} = \frac{v_\alpha p}{v_{\alpha\beta0}^2}, \quad i_{\alpha q} = \frac{v_\beta q}{v_{\alpha\beta0}^2} \tag{6}$$

$$i_{\beta q} = \frac{v_\beta p}{v_{\alpha\beta0}^2}, \quad i_{\beta q} = \frac{-v_\alpha q}{v_{\alpha\beta0}^2} \tag{7}$$

Then power terms (p and q) are decomposed to their mean and oscillating parts as follows:

$$p = \tilde{p} + \bar{p} \tag{8}$$

$$q = \tilde{q} + \bar{q} \tag{9}$$

where $\bar{p}$ and $\tilde{p}$ are called the fixed and alternated value of the instantaneous real power, respectively. Moreover, $\bar{q}$ and $\tilde{q}$ are the mean and alternated value of the instantaneous imaginary power, respectively,

$$i_{\alpha p} = i_{\alpha\bar{p}} + i_{\alpha\tilde{p}} = \frac{v_\alpha \bar{p}}{v_{\alpha\beta0}^2} + \frac{v_\alpha \tilde{p}}{v_{\alpha\beta0}^2} \tag{10}$$

$$i_{\alpha q} = i_{\alpha\bar{q}} + i_{\alpha\tilde{q}} = \frac{v_\beta \bar{q}}{v_{\alpha\beta0}^2} + \frac{v_\beta \tilde{q}}{v_{\alpha\beta0}^2} \tag{11}$$

$$i_{\beta p} = i_{\beta\bar{p}} + i_{\beta\tilde{p}} = \frac{v_\beta \bar{p}}{v_{\alpha\beta0}^2} + \frac{v_\beta \tilde{p}}{v_{\alpha\beta0}^2} \tag{12}$$

$$i_{\beta q} = i_{\beta\bar{q}} + i_{\beta\tilde{q}} = \frac{-v_\alpha \bar{q}}{v_{\alpha\beta0}^2} + \frac{-v_\alpha \tilde{q}}{v_{\alpha\beta0}^2} \tag{13}$$

Next, by using $\bar{p}$, $\tilde{p}$, $\bar{q}$, $\tilde{q}$ and inverse Clarke transformation, which is shown in Fig. 3, currents in each phase are decomposed to different components as follows:

$$\begin{bmatrix} i_{a\bar{p}} \\ i_{b\bar{p}} \\ i_{c\bar{p}} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{\alpha\bar{p}} \\ i_{\beta\bar{p}} \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} i_{a\tilde{p}} \\ i_{b\tilde{p}} \\ i_{c\tilde{p}} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{\alpha\tilde{p}} \\ i_{\beta\tilde{p}} \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} i_{a\bar{q}} \\ i_{b\bar{q}} \\ i_{c\bar{q}} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{\alpha\bar{q}} \\ i_{\beta\bar{q}} \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} i_{a\tilde{q}} \\ i_{b\tilde{q}} \\ i_{c\tilde{q}} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{\alpha\tilde{q}} \\ i_{\beta\tilde{q}} \end{bmatrix} \tag{17}$$

4.1.2 Instantaneous power theory applications: In [52, 53] a summary of the basic formulation of the p - q theory is presented and some examples of non-periodic currents and their compensation by active filters are discussed. The authors claim that it is not possible to have perfect compensation at the same time when the source currents are purely sinusoidal and the power flowing in the circuit is constant. With the development of the smart grid, the power quality problems such as voltage and current harmonic distortion and voltage sag/swell are becoming a big concern. In [54], the unified power quality conditioner (UPQC) has been used to improve power quality problems. In the proposed UPQC, a harmonic comprehensive detection algorithm based on improved p - q theory is used.

An instantaneous reactive power compensator consists of switching devices is proposed in [2] which is no practical need to energy storage components. To compensate for voltage and current harmonics of the microgrid, a modified p - q theory (which is suitable for application in control of single-phase active filters and

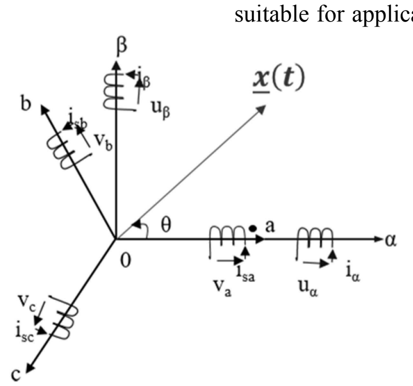

Fig. 2 *Clarke transformation*

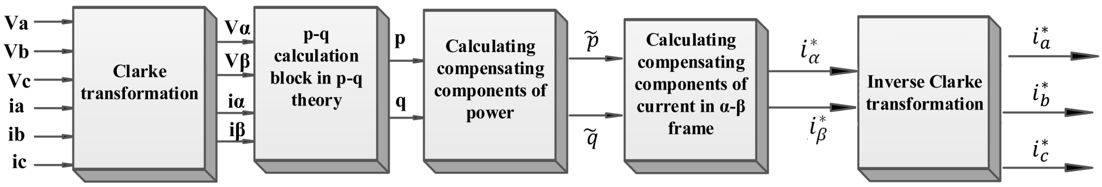

Fig. 3 *Theory-based distortion compensation schematic*

UQPC) is used in [55]. The currents flowing from or into the power system from the coupled photovoltaic system must have no distortion, so the system must have the function of an active power filter to compensate any distortions. In [56] the authors applied the extension of the p - q theory to the coupled photovoltaic system. Another application of using p - q theory is to design a robust synchronising PLL circuit, which can maintain the synchronism in the presence of sub-harmonics, harmonics, and negative-sequence unbalance [57].

4.1.3 Pros and cons: This theory is based on instantaneous voltages and currents in three-phase power systems with or without neutral wire, and it is valid for steady-state or transient. Since the p–q theory works in the time domain, it is used to design real-time and fast controllers for active filters. The averaged powers ($\bar{p}$ and $\bar{q}$) are active and reactive powers and oscillating powers ($\tilde{p}$ and $\tilde{q}$) represent the undesirable powers due to harmonic and unbalanced components in the load current. This theory presents some interesting features, namely

(i) It can be applied to a three-phase system, balanced/unbalanced, with/without harmonics.
(ii) It is based on instantaneous values and allows an excellent dynamic response.
(iii) Its calculations are relatively simple.
(iv) Directly separated zero-sequence components because of using Clarke transformation.

In [41, 42], Willems and Czarnecki have verified that the *pq*-theory faces some problems. Some of the main drawbacks of this theory that are mentioned in the paper are listed here

(i) This theory works just in three-phase systems.
(ii) For ease of calculation, the effect of zero-sequence components is not considered in the imaginary power.
(iii) It does not properly work for distorted voltages.
(iv) It is not able to decompose the unbalanced and harmonic parts of currents separately.

4.2 FBD theory

In 1993, and a few years after the initial introduction of the p - q theory, Depenbrock proposed his theory with the name of FBD (Fryze–Buchholz–Depenbrock) by this reputation that his definition was an extension of Fryze's and Buchholz's power theories which were originally proposed in [17].

4.2.1 Mathematical analysis formulation: FBD theorem is based on the Fryze's current decomposition and Buchholz's instantaneous and RMS values. Decomposed currents were applied to controllers for application of active filtering. This method works in a multiphase power system. The voltages and current vectors are shown with v and i, respectively,

$$v = \begin{bmatrix} v_a \\ v_b \\ v_c \\ \vdots \\ v_n \end{bmatrix}, \quad i = \begin{bmatrix} i_a \\ i_b \\ i_c \\ \vdots \\ i_n \end{bmatrix} \tag{18}$$

Their instantaneous collective values (v_Σ, i_Σ), are defined as

$$i_\Sigma = \sqrt{\sum_{k=1}^{n} i_k^2}, \quad v_\Sigma = \sqrt{\sum_{k=1}^{n} v_k^2} \tag{19}$$

where 'n' indicates the number of phases. Instantaneous power calculated from the inner product of voltage and current vectors

$$p = \boldsymbol{v} \cdot \boldsymbol{i}^T \tag{20}$$

where $\boldsymbol{i}^T$ is the transpose of the current vector. Under periodic conditions, the collective RMS value of currents and voltages can be calculated as

$$I_{\rm rms} = \sqrt{\frac{1}{T}\int_0^T i_\Sigma^2 \mathrm{d}t}, \quad V_{\rm rms} = \sqrt{\frac{1}{T}\int_0^T v_\Sigma^2 \mathrm{d}t} \tag{21}$$

Then collective 'active' power is calculated from

$$P = \frac{1}{T}\int_0^T p\mathrm{d}t \tag{22}$$

Depenbrock decomposed the instantaneous current in each phase of the system (i_k) to active (i_{ak}) and non-active (i_{nk}) currents. Active current contributes to the energy that is transferred from the source to the load

$$i_{ak} = \frac{P}{V_{\rm rms}^2} v_k \tag{23}$$

and non-active currents (i_{nk}) contributes to the amount of energy that used in the system by disturbances and oscillations but i, is not transferred from source to the load

$$i_{nk} = i_k - i_{ak} \tag{24}$$

Moreover, he decomposed i_k to power currents (i_{pk}) which is responsible for the instantaneous power and powerless currents (i_{zk}) which is not contributing to the current that is transferred between source and load and can be easily compensated without energy storage element in the system

$$i_{pk} = \frac{p}{v_{\rm rms}^2} v_k \tag{25}$$

$$i_{zk} = i_k - i_{pk} \tag{26}$$

Finally, he defined variation currents (i_{vk}) which is responsible for the oscillation of the instantaneous power current around its average value

$$i_{vk} = i_{pk} - i_{ak} = i_{nk} - i_{zk} \tag{27}$$

Such a definition of power components is valid and is similar to lots of famous power theories. The orthogonal current decomposition results that

$$i_k^2 = i_{ak}^2 + i_{nk}^2 = i_{ak}^2 + i_{vk}^2 + i_{zk}^2 \tag{28}$$

4.2.2 FBD theory applications: One of the applications of FBD theory is related to the idea of designing a measurement protocol to

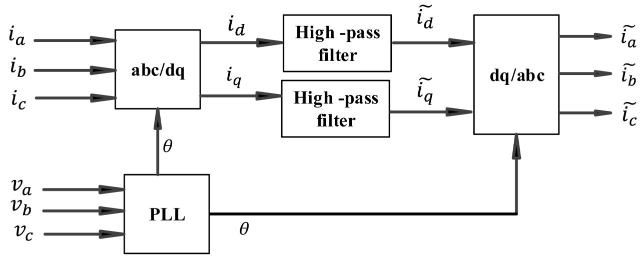

Fig. 4 *Method for current decomposition*

develop a traction substation power quality compensator [58]. In [59–61], a real-time current detection framework based on FBD method is presented to detect the harmonic, negative-sequent and zero-sequent currents in the three-phase four-wire system. The fundamental positive-sequent active and reactive currents, unsymmetrical current and harmonic current can be detected in case of voltage distortion and it is an advantage of using the FBD method. Moreover, the detection method does not need Park and Clarke transformations. As another novel application for this theorem, determining the impact of highly efficient lighting devices such as compact fluorescent lamps (CFLs) and light-emitting diodes (LEDs) on electric grid power quality is proposed using FBD-power theory to the currents absorbed by these lighting devices [62]. In [63], FBD theory has been incorporated for compensating and controlling for the harmonic and reactive power currents and direct and indirect current detection methods for the three-phase power system. An inverter reference current generation using position sampled FBD method is proposed in [64] for renewable energy source interfaced three-phase inverters in the generalised microgrid.

4.2.3 Pros and Cons: There are several concerns with this theory:

(i) The power terms are not defined as conventional terms such as active, reactive and apparent. So here we tried to compare his terms with conventional terms and explained their meanings. He defined i_{ak} as an active current and it is similar in all power theories and i_{pk} is the instantaneous active current and contains active current in addition to harmonics and unbalanced parts which transferred from source to load. Powerless current (i_{zk}) is the instantaneous reactive current in addition to harmonics and unbalanced parts of current that exchanged between phases of the system.
(ii) His theory does not work in case of distorted or unbalanced source voltage because his current terms are proportional to the source voltage. Thus, if the voltage contains distortion, current terms contain distortion as well.
(iii) This theory is not able to decompose current to more detailed parts such as reactive, unbalanced, harmonics, zero-sequence.
(iv) The FBD method is less computationally intensive than the conventional *p–q* theory because it does not require a phase lock loop and Park's transformation. Moreover, it is independent of grid voltage fundamental frequency and it can reject the grid voltage harmonics.

4.3 Synchronous reference frame ($dq0$) *method*

Synchronous reference frame also is widely known as *dq* decomposition is a famous instantaneous power theory developed in 1991 by Bhattacharya *et al*. in [15].

4.3.1 Mathematical analysis formulation: In this theorem, a synchronous reference frame approach is used to transform load currents from the *abc* frame to an alternative mathematical domain, namely, *dq*0 frame

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{29}$$

where θ is the synchronisation angle which is time variant and represents the angular position of the dq frame and it is detected by a phase locked loop (PLL). When θ is calculated by PLL, the phase current transferred to the *dq*0 frame by using (29). After that, the load currents in *dq* frame pass through a high-pass filter to extract their oscillating parts representing the harmonic and unbalanced components of load currents in *dq* frame. The remaining part of the current is the fundamental part. Finally, by using (30) which is the inverse transformation and transfers the currents from the *dq*0 frame to the *abc* frame, those components are transferred to the *abc* frame.

Fig. 4 shows the block diagram of *dq* method for current decomposition, where $\widetilde{i_a}$, $\widetilde{i_b}$, $\widetilde{i_c}$ are oscillating parts of currents which contain unbalanced and harmonic parts.

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \frac{\sqrt{2}}{2} \\ \cos\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) & \frac{\sqrt{2}}{2} \\ \cos\left(\theta + \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \tag{30}$$

4.3.2 Synchronous d–q theory applications: Fault diagnosis is one of the applications of the Synchronous Reference Frame method that has been introduced in [65, 66]. Voltage mode control in boost inverters is another application of *dq* method that is developed in different work such as [67]. This method is also widely used to control smart inverters and multifunctional inverters among the literature [68, 69]. The *dq* method is mature transformation which has also been used for many applications such as modelling and analysing the complex systems considering the models of passive components, complete passive networks, synchronous machines, wind turbine systems, photovoltaic inverters, and others [70], and references therein.

4.3.3 Pros and cons:

(i) This method uses a PLL. In [20, 21] the authors introduced an instantaneous *dq* current decomposition method without using PLL, but it is criticised in [47] not to have proper performance in the case of distorted currents.
(ii) Although this method can be used in active filtering systems for compensation purpose, it is not a power theory because it is not able to define current components according to conventional power terms.
(iii) It is not *able* to separate unbalanced and harmonic parts of currents.

4.4 Generalised instantaneous power theory

In 1996, Peng introduced a GIPT which he claims to work for a three-phase system in different conditions such as balanced/unbalanced, sinusoidal/non-sinusoidal, with/without zero-sequence components.

4.4.1 Mathematical analysis formulation: In the mathematical formulation of his theory, the instantaneous active power *p* is defined as an internal product of voltage and current vectors

$$p = \boldsymbol{v} \cdot \boldsymbol{i} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} [i_a \ i_b \ i_c] = v_a i_a + v_b i_b + v_c i_c \tag{31}$$

Then, the instantaneous reactive power vector is defined as cross-product of voltage and current vectors

$$q = \boldsymbol{v} \times \boldsymbol{i} \tag{32}$$

by using these definitions for active and reactive powers, the corresponding active and reactive current vectors are defined accordingly:

$$\boldsymbol{i}_p = \frac{p}{\boldsymbol{v} \cdot \boldsymbol{v}} \boldsymbol{v} \tag{33}$$

$$\boldsymbol{i}_q = \frac{\boldsymbol{q} \times \boldsymbol{v}}{\boldsymbol{v} \cdot \boldsymbol{v}} \tag{34}$$

Then he proves that the three-phase current vector ($\boldsymbol{i}$) is equal to the sum of the active and reactive current vectors

$$\begin{aligned} \boldsymbol{i}_p + \boldsymbol{i}_q &= \frac{p}{\boldsymbol{v} \cdot \boldsymbol{v}} \boldsymbol{v} + \frac{\boldsymbol{q} \times \boldsymbol{v}}{\boldsymbol{v} \cdot \boldsymbol{v}} = \frac{\boldsymbol{v} \cdot \boldsymbol{i}}{\boldsymbol{v} \cdot \boldsymbol{v}} \boldsymbol{v} + \frac{(\boldsymbol{v} \times \boldsymbol{i}) \times \boldsymbol{v}}{\boldsymbol{v} \cdot \boldsymbol{v}} \\ &= \frac{\boldsymbol{v} \cdot \boldsymbol{i}}{\boldsymbol{v} \cdot \boldsymbol{v}} \boldsymbol{v} + \frac{[-(\boldsymbol{v} \cdot \boldsymbol{i})\boldsymbol{v} + (\boldsymbol{v} \cdot \boldsymbol{v})\boldsymbol{i}]}{\boldsymbol{v} \cdot \boldsymbol{v}} = \boldsymbol{i} \end{aligned} \tag{35}$$

4.4.2 GIPT applications: The GIPT is used for harmonic and reactive power compensation in three-phase power systems in [4]. This theory gives a generalised definition of instantaneous reactive power, which authors claim that it is valid for sinusoidal/non-sinusoidal and balanced/unbalanced three-phase power systems with or without zero-sequence currents/voltages. In [71], Peng's power theory is used for harmonics compensation to reduce the size of the DC-link capacitor. This work implemented a DC voltage regulator for estimating the power loss of the converter and feedback that losses into the Peng's control strategy.

4.4.3 Pros and cons:

(i) We can see such a theory is also valid in the case of single-phase systems

$$p = \boldsymbol{v} \cdot \boldsymbol{i} = VI\cos(\theta) \tag{36}$$

$$q = \boldsymbol{v} \times \boldsymbol{i} = VI\sin(\theta) \tag{37}$$

where V and I are the RMS value of the phase voltage and current signals and θ is their phase differences.
(ii) The conventional relationship for apparent, active and reactive powers still holds in this theory:

$$s^2 = p^2 + q^2 = (\boldsymbol{v} \cdot \boldsymbol{i}_{\mathrm{p}})^2 + \boldsymbol{v} \times \boldsymbol{i}_{\mathrm{q}}^2 = v^2(\boldsymbol{i}_p^2 + \boldsymbol{i}_q^2) = v^2 i^2 \tag{38}$$

(iii) Active and reactive current terms are perpendicular to each other.

$$\boldsymbol{i}_q \cdot \boldsymbol{i}_p = 0 \tag{39}$$

(iv) The author in [37] claims that this theory works only for three-phase systems.
(v) It is not possible to decompose current components, especially the zero-sequence and unbalanced parts of currents.

4.5 Conservative Power Theory

The main problem associated with previously discussed power theories is that they are not appropriate for unbalanced conditions. CPT is the last famous and widely recognised instantaneous power theory proposed by Tenti and Mattavelli [35]; to specifically deal with the challenges associated with unbalanced conditions. It has been claimed to be an appropriate alternative for considering non-linear and unbalanced conditions. Recently, this theory has been used in many types of research related to compensation strategies for microgrid applications.

4.5.1 Mathematical analysis formulation: A major difference between CPT and most of the traditional approaches is that CPT defines the power and current terms among stationary frames. Tentis' definition of instantaneous active power is similar to other power theories

$$p(t) = \boldsymbol{v} \cdot \boldsymbol{i} = \sum_{k=1}^{M} v_k i_k \tag{40}$$

where ($\boldsymbol{v}$) and ($\boldsymbol{i}$) are the voltage and current vectors, respectively, and k indicates the number of phases of the system. Like conventional power theories, the average values of $p(t)$ are defined as active power.

$$P = \bar{p} = \boldsymbol{v} \cdot \boldsymbol{i} = \frac{1}{T} \int_0^T \boldsymbol{v} \cdot \boldsymbol{i} \, \mathrm{d}t = \sum_{k=1}^{K} P_k \tag{41}$$

He defined a new term which he called it as instantaneous reactive energy and it is calculated as follows:

$$w(t) = \hat{v} \cdot \boldsymbol{i} = \sum_{k=1}^{M} \hat{v}_k i_k \tag{42}$$

where $\hat{v}$ is the unbiased integral of the voltage vector

$$\hat{v} = v_f(t) - \overline{v_f(t)} \tag{43}$$

where $v_f(t)$ is the time integral of the voltage vector

$$v_f(t) = \int_0^T v(\tau)\mathrm{d}\tau \tag{44}$$

Moreover, he defines reactive energy as an average value of $w(t)$:

$$W = \bar{w} = \hat{v} \cdot \boldsymbol{i} = \frac{1}{T} \int_0^T \hat{v} \cdot \boldsymbol{i} \, \mathrm{d}t = \sum_{k=1}^{K} W_k \tag{45}$$

Based on the above definitions, the phase currents are decomposed into three basic current components.
Active phase currents are defined by

$$i_{ak} = \frac{v_k \cdot i_k}{v_k^2} v_k = \frac{P_k}{v_k^2} v_k \tag{46}$$

Reactive phase currents are given by

$$i_{rk} = \frac{\hat{v}_k \cdot i_k}{\hat{v}_k^2} \hat{v}_k = \frac{W_k}{\hat{v}_k^2} \hat{v}_k \tag{47}$$

where v_k and $\hat{v}_k$ indicate the RMS value of the phase voltage and unbiased integral of the voltage reactivity, respectively.
Void phase currents are the remaining current terms

$$i_{vk} = i_k - i_{ak} - i_{rk} \tag{48}$$

Which is not an active or reactive current. The active and reactive phase currents can be decomposed into balanced and unbalanced terms. The balanced active currents have been defined as

$$i_{ak}^b = \frac{v \cdot i}{v^2} v_k = \frac{P}{v^2} v_k \tag{49}$$

The balanced active currents represent the portion of the phase currents that flow because of a balanced equivalent circuit and is responsible for conveying the total active power (P) in the circuit. The balanced reactive currents have been defined as

$$i_{rk}^{b} = \frac{\hat{v} \cdot i}{\hat{v}^2}\hat{v}_k = \frac{W}{\hat{v}^2}\hat{v}_k \tag{50}$$

and represent the portion of the phase currents that flow in the system because of the balanced equivalent circuit and is responsible for conveying the total reactive energy (W) in the circuit.

The unbalanced active currents are calculated by

$$i_{ak}^{u} = i_{ak} - i_{ak}^{b} \tag{51}$$

and unbalanced reactive currents are calculated as

$$i_{rk}^{u} = i_{rk} - i_{rk}^{b} \tag{52}$$

Thus, the unbalanced three-phase current defined as

$$i_{k}^{u} = i_{ak}^{u} + i_{rk}^{u} \tag{53}$$

The current vector can be decomposed as

$$i = i_a^b + i_r^b + i_a^u + i_r^u + i_v \tag{54}$$

This theory is based on an orthogonal decomposition, so all the current components are orthogonal to each other and their RMS value equals to

$$I = \sqrt{i_a^{b2} + i_r^{b2} + i_a^{u2} + i_r^{u2} + i_v^2} \tag{55}$$

4.5.2 CPT applications: In [72], CPT is applied to generate the reference current of multi-functional grid-tied inverter for power quality improvement. Using CPT, the load current is decomposed into balanced and unbalanced active, reactive, and voided components. A cooperative control scheme based on CPT is proposed in [73], which is sharing imbalances in a three-phase four-wire system with droop-controller. It has been shown that using CPT, different components of current and power in microgrid, including the balanced, unbalanced, and distorted components can be identified accurately. In [74], a three-phase battery storage system (BSS) with active filtering function to compensate power fluctuation from a wind generator (WG) system is presented and the CPT is used to compute the active output power of the BSS and the WG system.

A non-intrusive appliance load monitoring (NILM) dataset for cognitive power meters based on CPT and pattern recognition techniques are being developed in [75]. This novel dataset is capable of classifying and disaggregating residential appliances for the development of smart or cognitive power meters. The calculated power components from CPT with an appropriate pattern recognition algorithm and a power signature state machine provides proper identification for each appliance.

In [76] the authors are described as a strategy to add an active filtering functionality to a doubly-fed induction generator (DFIG) wind power system. The active filtering is applied through the mathematical formulation of CPT, which is applied into the electric current control loop of the grid side converter. Therefore, by using the CPT method, the wind power system mitigated harmonic currents and improved the power quality in electrical power systems in the presence of non-linear loads. The application of the current decomposition using CPT for load power-sharing in distributed energy resources is proposed in [77].

In [78], by using CPT method, an expert system based on decoupled power/current decomposition and the k-nearest neighbour pattern recognition method is used to identify the correct mitigation solution for power quality improvement in three-phase AC microgrids under non-sinusoidal current and voltage operations. The results show that the proposed algorithm is robust and able to select an appropriate compensation solution easily. In another work, different current components which are being defined using the CPT, is used for load analysis and characterisation [79]. These power and current components can describe the effect of different load characteristic on the power factor calculation. In [80] CPT is used to calculate performance factors for load characterisation and revenue metering where each component calculates by CPT is related to a specific load non-ideality (unbalance, reactivity, distortion). These performance factors are used to characterise the load under different operating conditions and will consider the effect of non-negligible line impedances and supply voltage deviation. In [81], a cooperative control approach based on CPT is proposed for power switching interfaces and any other electronic power processors in the grid, to ensure proper network operation even under severely disturbed conditions.

4.5.3 Pros and Cons: In [46] Czarnecki criticised the CPT from different conceptual perspectives. Here we mention some of the drawbacks:

(i) The physical meaning of ‘reactive energy’ is not clear. In [46], the author shows that the ‘reactive energy does not account for inductive and capacitive energy stored in the load circuit.’ Therefore, loads without any capability of storing energy could have reactive energy, and this confirms that the reactive energy defined in this theory does not have physical meaning and is not related to the phenomenon of energy storage.
(ii) The reactive current is proportional to the reactive energy. Since the physical meaning of the reactive energy is unclear, the physical meaning of the reactive current is also unclear.
(iii) Moreover, it is shown in [46] that the voided power (distortion power) is not related to the distortion of the load current concerning the supply voltage.
(iv) It does not work in case of the distorted or unbalanced source voltage. The active and reactive currents in CPT are proportional with the source voltage, if the source voltage is distorted then active and reactive currents contain distortion and unbalanced parts.

5 State-of-the-art

Three relevant papers were published most recently [82–84]. In [82] the authors present a new power theory called ‘right-angled triangle power theory.’ They have rejected the Budeanu's power theory and defined the power components in a two-dimensional diagram without the need for a standard three-dimensional coordination system used before in Budeanu's theory. They also proved the ability to apply the orthogonality law on all power components in non-sinusoidal systems. In this theory, two new electrical power terms defined as effective active and effective reactive power components. By using these two terms, they approved the orthogonality relationship between fundamental and distorted components. At the same time, they proved that these terms are necessary to complete the form of the right-angled power triangle, which is used to calculate the total apparent power in the non-sinusoidal systems. They called this power component as the apparent effective power to avoid the misconception with the old definitions. Moreover, a new power diagram is proposed to represent all power components in a unified scheme consists of six right-angled triangles; termed ‘right-angled triangle diagram.’

Another advanced power theory is proposed in [83], which expands the application of the p–q theory based on $\alpha\beta 0$ transformation. This theory is especially designed to control the instantaneous power in voltage sourced converters operating under unbalanced conditions with positive, negative, and zero sequence content. This power theory proposes a new transformation which is labelled as *mno* transformation and provides a method to calculate the instantaneous power on general three-phase transmission systems and works under unbalanced conditions. The proposed transformation is based on the creation of the *mno* three-dimensional Cartesian reference frame. It is a time-domain base transformation and adjusts itself instantaneously according to a normal vector which tracks the instantaneous three-phase measured

Table 1 Properties of the most famous power theories

Properties	*pq*	FBD	*dq*	Peng	CPT
Domain	time	time	time	time	time
Fryze/Budeanu	*F*	*F*	*F*	*F*	*B*
works with non-linear loads	✓	✓	✓	✓	✓
works with unbalanced loads	✓	✓	✓	✓	✓
works with unbalanced voltage	✓	×	✓	✓	×
works with distorted voltage	×	×	×	✓	×
unbalanced current decomposition	✓	×	×	×	✓
distorted current decomposition	✓	×	×	×	✓
instantaneous	✓	✓	×	✓	×
harmonics and unbalanced decomposition	×	×	×	×	✓

Table 2 Current components compared in power theories

Current components	*pq*	FBD	*dq*	Peng	CPT
active	$i_{k\bar{p}}$	i_{ak}	$i_{\bar{d}}$	×	i_{ak}^{b}
reactive	$i_{k\bar{q}}$	×	$i_{\bar{q}}$	×	i_{rk}^{b}
distorted	×	×	×	×	i_{vk}
unbalanced	×	×	×	×	$i_{ak}^{u}+i_{rk}^{u}$
harmonics + unbalanced	$i_{k\tilde{p}}+i_{k\tilde{q}}$	×		×	
instantaneous active					×
instantaneous reactive					×
zero sequence		×		×	×

voltage. The method controls both the constant and oscillating terms of the instantaneous three-phase power. It is applied to grounded three-wire and four-wire schemes, especially accommodating zero sequence, unlike previous approaches.

A novel formulation of instantaneous power theory, coined as an Enhanced Instantaneous Power Theory (EIPT), is proposed in [84] for unbalanced and non-linear three-phase power systems. EIPT establishes a proper decomposition of current components for cases of balanced, unbalanced and distorted voltage sources. It can independently calculate currents for unbalanced and non-sinusoidal three-phase power systems, which are valid in case of unbalanced distorted voltage source like weak grid conditions. This formulation is a modified version of both Akagi's and Peng's power theories and can decompose the signal into much more detailed components (such as balance, homopolar and heteropolar unbalanced and harmonic for both active and reactive terms for each phase of the system), so it is more reliable while providing more flexibility and selectivity in terms of control and monitoring applications. Moreover, in the proposed EIPT, all current components are calculated independently, and it works in the case of unbalanced and distorted voltage source conditions as well.

6 Discussion

In this section, we try to discuss why after researchers make all efforts along recent years to define a generalised power theory, there is still no generally accepted approach certified by the power system society.

6.1 Comparative study on conventional approaches

In [48], Emanuel divided power theories into two main classes. (i) Fryze class and (ii) Budeanu class. He divided them into these two classes not because of their different domains (time/frequency) but because of their different methodologies. Fryze divided power or current terms just into two main parts, which are active and reactive terms and then the distortion part is defined as a part of active or reactive components. This is what the authors did in FBD, *pq*, EIPT and Peng's power theory. On the other hand, according to Budeanu's definition, the power and current terms are divided by three main terms which are named active, reactive and distortion terms; this is like what Czarnecki and Tenti suggested in their definitions [35, 40], respectively.

Fryze was severely opposed to his definition of power terms based on Fourier series. The rationale was that by considering the Gibbs phenomenon at discontinuity points, the minimisation of the error produced by approximating a given function with Fourier series would be impossible. In 1961 Usatin [85] criticised Budeanu's methodology; he showed the lack of physical interpretation of distortion power and unauthorised summing up of amplitudes of oscillating components of different harmonics. Moreover, in [48], the author suggests abolishing using a distortion power for power term definitions, because distortion power is a harmonic part of active or reactive powers and if it is defined separately, it lacks physical meaning. Based on the Physics of Electrical System, there are two main paths for transferring the energy: (i) the energy may transfer from source to the load (active power) or (ii) it is exchanged in and out with sections of the system (reactive power). As a conclusion to this discussion, it is worth noting to mention that, to define a valid generalised power theory, one should consider the following facts:

(i) It is very important that each term has an appropriate physical meaning.
(ii) It should be able to work under distorted and unbalanced source voltage as well as distorted and unbalanced load currents.
(iii) It must be able to decompose the unbalanced, distorted, active, reactive, negative sequence, zero-sequence parts of current in each phase for different applications such as active filtering.
(iv) It should be able to expand to the multiphase system.
(v) Time-domain approaches are better than the frequency-based approaches because they are instantaneous and much faster.
(vi) Defining power theories based on Fryze class.

Table 1 summarises the properties of the power theories discussed in this paper. The reader can use this table to compare them and validate for the proper use for the appropriate application. Tables 2 and 3 have a comparison of different current and power terms, respectively, in relevant power theories discussed in this paper (Fig. 5).

Finally, it is worth noting that, over observations form a comprehensive review on most of the state-of-the-art in the power theories applications indicates that, while *pq* is getting much interest on the transmission level applications the CPT is the recognised theory on distribution level applications that are

Table 3 Power terms in different power theories

Power theory	Active power	Reactive power	Distortion power
pq	*P*	*Q* (*imaginary* power)	—
FBD	*P*	*non-active* power	—
dq	—	—	—
Peng	*P*	*Q* (*reactive* power)	—
CPT	*P*	*Q* (*reactive energy*)	*D*

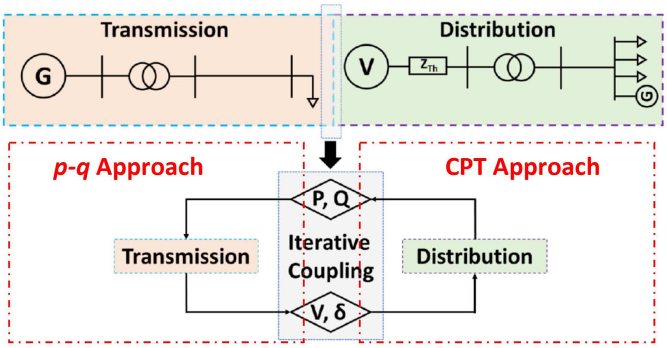

Fig. 5 *pq is the current trend of time-domain power theories in transmission level applications while CPT plays a major role in the distribution level and microgrid systems*

Table 4 Load parameters

500 Ω	1 mH	50 Ω
100 Ω	470 μF	100 μF
300 μH	100 μH	50 Ω

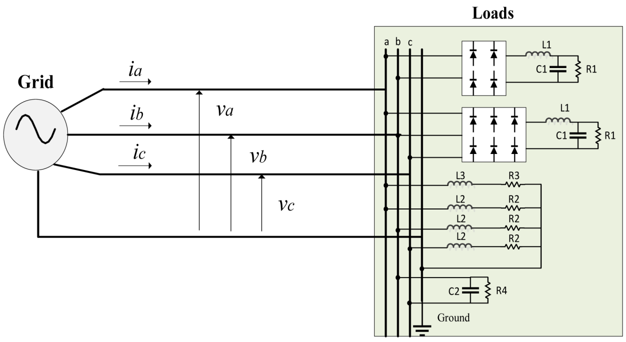

Fig. 6 *Three-phase four-wire system with different types of loads*

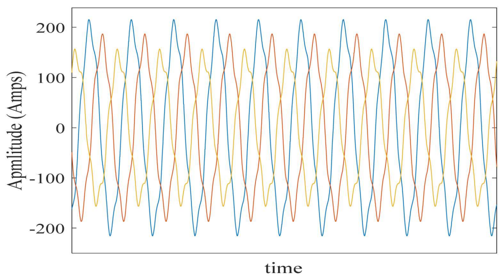

Fig. 7 *Voltage waveforms of the source in phases a, b and c*

trending in the microgrid and multifunctional inverter design and implementation.

6.2 Future trends

As it has been deeply discussed in [86, 87], CPT has demonstrated the best performance among conventional power theories such as *pq* and FBD theories in unbalanced situations. Thus, in this section, a practical case study is developed that compares the performance of CPT versus EIPT as a new trend in power theories under weak-grid conditions.

From another perspective, this case study is a comparison between Emanuel's two classes power theories, where we chose the most recent power theories from each category, EIPT from Fryze class and CPT from Budeanu class in the case of distorted voltage condition. We considered non-linear and unbalanced load (parameters of the grid-connected load are mentioned in Table 4), which is supplied by distorted and unbalanced voltage sources. Figs. 6–8 show the phase voltages and the behaviour of the corresponding load current waveforms in phases *a*, *b* and *c*, respectively.

Fig. 9 illustrates the active and reactive current components calculated by the EIPT and CPT formulations, accordingly. As it is shown, the active and reactive current components extracted by the CPT are suffering from the distorted and unbalanced voltage parts, while EIPT-based approach resulted in pure sinusoidal wave-shapes.

Fig. 10 indicates that the unbalanced and distorted components are well separated through the EIPT approach and Fig. 11 shows the total (heteropolar + homopolar) unbalanced, homopolar and heteropolar current components for the EIPT, respectively.

6.2.1 Application of EIPT-based voltage–current decomposition in an active front end inverter (a power conditioning example): In this part, we considered a typical AFE converter and implemented both the EIPT and CPT approaches as the desired current decomposition module to compensate the harmonic and unbalanced parts of the grid currents. An unbalanced and distorted voltage source is considered, with similar behaviour to the one mentioned in Fig. 7. To this end, the following active filtering scheme has been designed and modelled in the Simulink/Matlab software (Fig. 12).

The main purpose is to achieve close to unit power factor in the grid side in the presence of non-linear unbalanced load/source condition. As we mentioned before, the proposed EIPT-based framework, in addition to CPT, is implemented as a current decomposition method in the control and compensation module. The PI controller is used in the control unit configuration. Fig. 13 shows the output current of the system at PCC, injected into the grid, without any compensation.

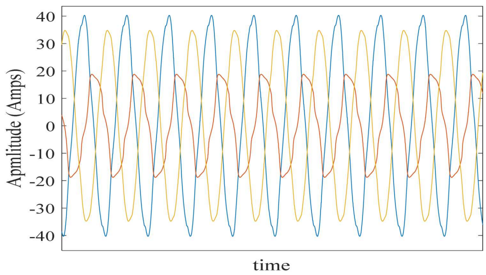

Fig. 8 *Current waveforms of the load in phases a, b and c*

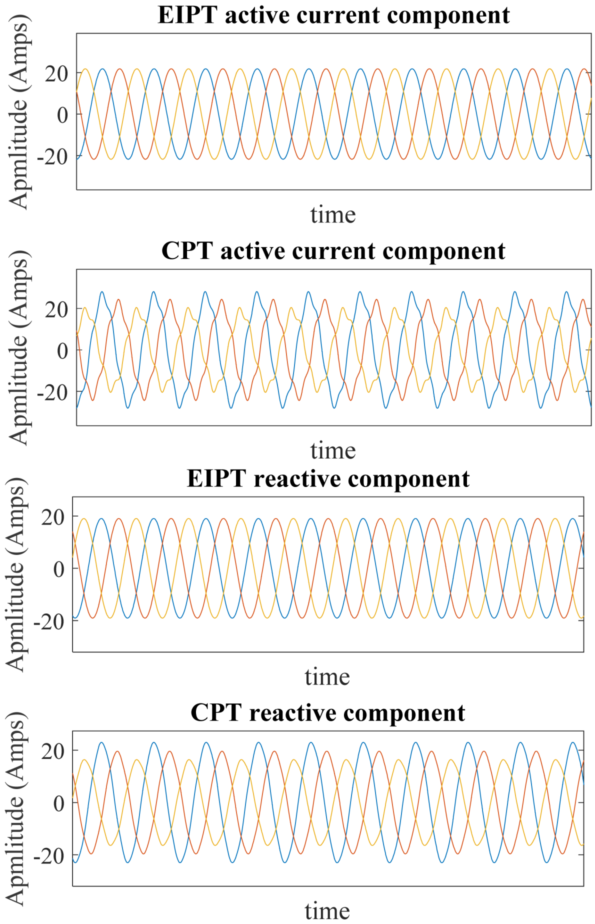

Fig. 9 *Active and reactive current components in EIPT and CPT*

Figs. 14 and 15 illustrate the compensated currents of the grid with a power conditioning system using EIPT and CPT, respectively.

As we can see in Figs. 14 and 15, while CPT does not illustrate a reasonable compensation performance in case of voltage source distortion conditions, exploiting its enhanced mathematical formulation the EIPT approach can compensate both the unbalance and harmonic distortions in case of a weak-grid case study.

7 Conclusion

Aggregation of the new technology of microgrid systems in addition to increasing usage of non-linear loads in power systems results in new challenges. These challenges mainly caused by a variety of uncertain power sources such as renewables, which can have a dynamic impact on voltage regulation, power flow, and system losses. Moreover, upward using of non-linear and unbalanced loads in addition to weak grids, which they do not have regulated voltage and frequency make the traditional power theories invalid and useless.

The improvement of power quality under the current quality approach is very important for distribution systems, and it is well founded through a transient analysis. That is the basis of instantaneous power theory, which can be generalised, or

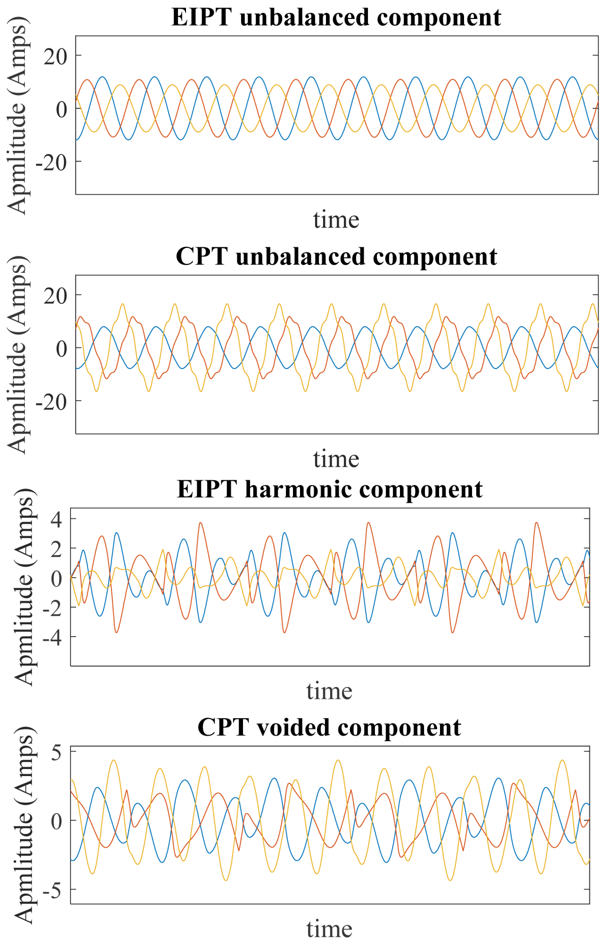

Fig. 10 *Harmonic and unbalance current components in EIPT and CPT*

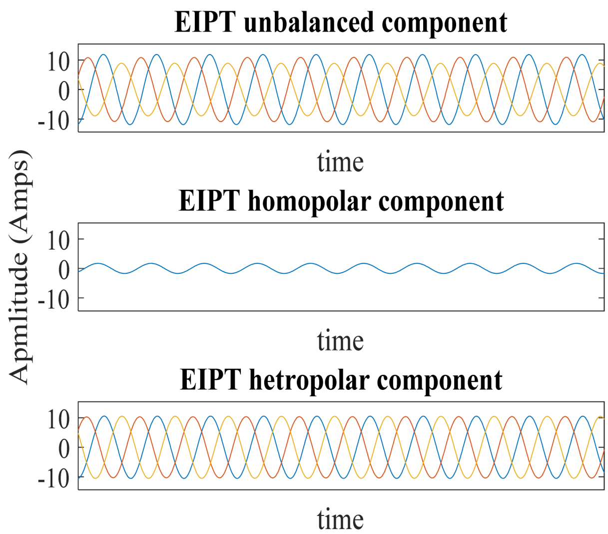

Fig. 11 *Heteropolar and homopolar and total (heteropolar + homopolar) unbalance current components in EIPT*

simplified as the *p*–*q* Akagi theory. This theory is valid for three-phase systems only, and several single-phase *p*–*q* applications mix the transient analysis with the steady-state analysis. It is very typical that in a single-phase system the alpha-axis is associated with the a-phase and a virtual beta-axis is made-up with time-delay, or Hilbert transformation, or gyrators that synchronise a 90° phase shift for a mathematically constructed beta-axis, then the *p*–*q* theory is used. However, all those solutions are not valid during the transient, which is, in fact, the main concern of current quality. Another instantaneous power theory called CPT – that brings other terms to compose the Reactive Energy flow due to reactive energy, imbalance, and void. This theory can perform as well as the

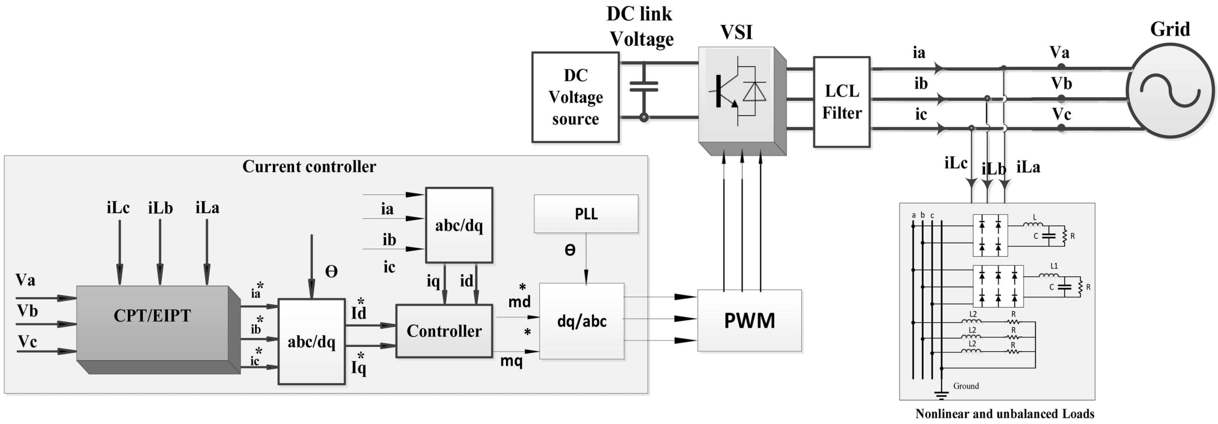

Fig. 12 *Schematic diagram of the power conditioning system with CPT/EIPT*

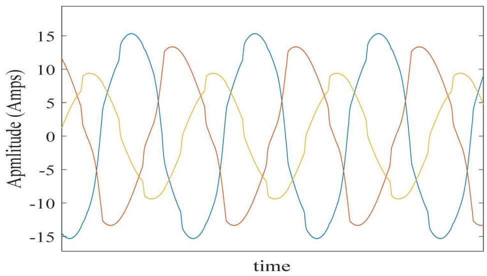

Fig. 13 *Grid currents without compensation*

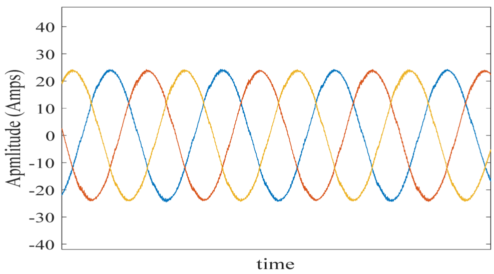

Fig. 14 *Grid currents after compensation by using EIPT*

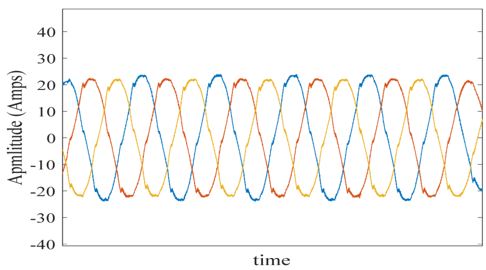

Fig. 15 *Grid currents after compensation by using CPT*

generalised *p–q*, for three-phase, single-phase, multi-phase systems. It has been verified as a powerful alternative technique, and several applications have been developed in the last few years. Instantaneous quantities related to Reactive Energy are presented in the *p–q* (imaginary power, which is not reactive power) and in the CPT terms. This evaluation is validated for active power filters (shunt and series), but they are not valid for maintaining the voltage profile, or reactive support of transmission lines. These instantaneous theories are especially important in the case of distribution lines, with mostly resistive ($R \gg X$) behaviour, i.e. resistance is dominant over reactance.

Various researchers tried to develop a comprehensive power theory which is applicable for different conditions. This paper summarised the most famous power theories and compared them from a mathematical and physical point of view. Moreover, the reasons that why a comprehensive power theory is still required were emphasised and the properties that the new theory must contain was discussed. The EIPT approach has tremendous potential for the applicability of instantaneous power theory and signal decompositions of renewable energy integration for smart-grid systems, because it allows a future definition of power factor valid for a general multiphase, it can support as a generalised theory for a general power factor definition valid for a three-phase under general conditions (imbalance, zero-sequence, and distortions), a so-called 'weak-grid' and it can be formulated with a hybrid CPT approach for single-phase as well as in three-phase systems. Finally, a case study was developed to compare the performance of CPT versus EIPT as a new trend in power theories under weak-grid conditions.

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