Enhanced Instantaneous Power Theory Decomposition for Power Quality Smart Converter Applications

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***Abstract*—Power theories are advancing signal decomposition approaches used for electrical power signal analysis. Currently, they are key tools for designing reliable and efficient power electronic interface controllers. Due to the inherent uncertainty in the power generation within new technology of distributed generation units (such as renewable-based microgrids), most of these theories are not directly applicable in the era of new power engineering systems. Furthermore, the traditional formulations are not appropriate once we face the nonlinearity in the consumption along a weak grid that may not maintain steady voltage or frequency. Therefore, a new signal decomposition method with a higher level of selectivity must be developed to modify the traditional power theories and improve the control strategies from a design viewpoint. This paper describes a novel formulation of an instantaneous power theory, enhanced instantaneous power theory (EIPT), for unbalanced and nonlinear three-phase power systems. EIPT establishes a proper decomposition of current components for cases of balanced, unbalanced, and distorted voltage sources. In addition to mathematical analysis, this research provides comprehensive simulations under different loads and source conditions, which supports the performance of the EIPT in the active filtering methods. Moreover, the performance of the proposed method is compared with the state-of-the-art approach widely known as the conservative power theory. Our results indicate that this new comprehensive approach is helpful in optimizing control strategies for power electronic interfaces and power quality compensators.**

***Index Terms*—Current decomposition, enhanced instantaneous power theory (EIPT), microgrids, power quality, smart-grid.**

Nomenclature and Abbreviations

v_{abc}	Instantaneous voltage vector in the abc frame.
i_{abc}	Instantaneous current vector in the abc frame.
$T_{\alpha\beta 0}$	$\alpha\beta 0$ transformation matrix.
$v_{\alpha\beta 0}$	Instantaneous voltage vector in the $\alpha\beta 0$ frame.
$i_{\alpha\beta 0}$	Instantaneous current vector in the $\alpha\beta 0$ frame.
P	Active power.
Q	Reactive power.
p	Instantaneous active power.
q	Instantaneous reactive power.
p_x	Instantaneous active power in phase x.[1]
q_x	Instantaneou reactive power in phase x.
$p_{\alpha\beta}$	Instantaneous active heteropolar power component.
$\bar{p}_{\alpha\beta}$	Average active balanced heteropolar power component.
$\tilde{p}_{\alpha\beta}$	Oscillating active heteropolar power component.
p_0	Instantaneous active unbalanced homopolar power component.
$q_{\alpha\beta}$	Instantaneous reactive heteropolar power component.
$\bar{q}_{\alpha\beta}$	Average reactive balanced heteropolar power component.
$\tilde{q}_{\alpha\beta}$	Oscillating reactive heteropolar power component.
$q_{\alpha 0}$ and $q_{\beta 0}$	Instantaneous unbalanced homopolar reactive power component.
i_p	Instantaneous active current.
i_q	Instantaneous reactive current.
i_{xp_0}	Active unbalanced homopolar current components in phase x.
$i_{x\bar{p}}$	Active balanced sinusoidal current components in phase x.
$i_{x\tilde{p}}$	Active oscillating current components in phase x.
i_{pux}	Active unbalanced heteropolar current components in phase x.
i_{pHx}	Active harmonic current components in phase x.
i_{xq_0}	Reactive unbalanced homopolar current components in phase x.
$i_{x\bar{q}}$	Reactive balanced sinusoidal current components in phase x.
$i_{x\tilde{q}}$	Reactive oscillating current components in phase x.
i_{qux}	Reactive unbalanced heteropolar current components in phase x.
i_{qHx}	Reactive harmonic current components in phase x.
i_{pux_s}	Active unbalanced part of current components caused by unbalanced part of voltage source in phase x.

[1] Index x stands for the phase name a, b, or c.

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i_{qux_s} Reactive unbalanced part of current components caused by unbalanced part of voltage source in phase x.

i_{pHx_s} Active harmonic part of current components caused by distorted part of voltage source in phase x.

i_{qHx_s} Reactive harmonic part of current components caused by distorted part of voltage source in phase x.

I. Introduction

WITHIN the traditional centralized generation power networks, power quantities, such as active power, reactive power, apparent power, and total harmonic distortion, were well defined under sinusoidal operating conditions for linear and balanced symmetrical multiple-phase or even single-phase systems. However, due to nonlinearity in loads and uncertainty in fast increasing distributed renewable generation sources within the new technology of microgrids and smart-grids, the balancing of active power, the provision of reactive power, and also the compensation of harmonics become new critical issues [1], [2]. Therefore, the lack of proper definitions for power components in the case of distorted current and voltage signals has prompted novel power theories [3]–[12]. Active filtering is a compensating strategy that is widely implemented to stand against all the unwanted terms that appear in the current and voltage components [13]–[16]. The main functionality in all the shunt active filtering methods is the calculation of compensating reference current and used in the controllers to control the current waveform. In nonlinear and unbalanced systems, the traditional definitions of power quantities need to be revised as new equations to use in active filtering applications. This context requires a comprehensive consideration of power theories that must be updated and modified to calculate the compensating currents in the power conditioning or active filtering systems.

Many researchers have redefined power theories under unbalanced, or more generally, distorted current conditions in three-phase systems and a variety of these theories have been introduced [3]–[12]. In general, their research can be divided into two main categories: the first category is the frequency-domain-based approach; and the second category is the time-domain-based approach. The time-domain-based approach usually exploits the instantaneous quantities to define the particular current decompositions and power definitions [4]–[12]. The first attempt to solve the problem of defining power components under nonsinusoidal conditions is historically attributed to Budeanu, who used a frequency domain [3]. Depenbrock used the time-domain approach for the current decomposition (FBD method) [8]. The FBD method is an extension of the Fryze theory [12], and this theory decomposed the current signal to active and nonactive parts.

Akagi *et al.* introduced an instantaneous power definition or pq theory [4]. Conventional pq theory is accepted by the power electronic community [15]–[20] and is an effective method to compensate reactive power and harmonics of nonlinear loads in three-phase systems without energy storage. However, a major drawback with Akagis' approach is that the proposed methodology does not properly take into account the presence of zero sequence [9] and only provides a rough picture of all unwanted (unbalanced plus harmonics) power components in a single power component, oscillating, which limits the alternatives from a control and compensation point of view.

Willems [9] proposed a new interpretation of instantaneous active and reactive currents to generalize the Akagis' framework in terms of m-phase systems, where the direct projection of a current vector over the voltage vector within an m-dimensional subspace is used to directly define the current components. However, Willems' approach only indicates the importance of the consideration of zero sequence components without any mathematical interpretation or justification and fails to generalize the pq theory formulation that is one of the major contributions of this paper.

Peng and Lai [6] proposed a conceptual approach for pq theory for power component definitions, where they initially suggested a definition for instantaneous reactive power (or reactive vector) based on the cross-product operation of the current vector and voltage vector, whereas the instantaneous active power is defined as an inner product of current and voltage vectors similar to Akagi *et al.* [4]. Next, instead of directly defining the current terms (like [9]), he calculates the current components based on the proposed power components definitions. However, this theory is still unable to clearly separate the corresponding active-reactive parts of the current in terms of zero-sequence, nonlinear and unbalanced components within an acceptable resolution.

The definition of power and current terms in a nonlinear and unbalanced system is still under discussion, and there is no standard definition for these quantities under such a condition. Recently, another active filtering method has been introduced by Tenti *et al.* based on conservative power theory (CPT) formulation [10]. The CPT has been quickly named as one of the major power theories of electrical systems with nonsinusoidal and unbalanced currents. The high number of CPT-based papers published recently [16], [21] supports this opinion. However, some terms and concepts suggested by this theory are similar to some known abandoned theories as developed by Budeanu [3] and by Kuster and Moore [22].

In [23], the author concludes that the "reactive energy," as defined in CPT, cannot be regarded as a physical quantity. It was emphasized that the conservative property of the "reactive energy" in this theory is only a mathematical concept and does not have any physical property, and it is demonstrated that the separation of the reactive current can lead to wrong conclusions in a reactive compensator design. Moreover, the active and reactive currents in CPT are proportional to the source voltage, and as a result, if the source voltage is distorted or unbalanced, active and reactive components contain distortion and imbalance as well. Another disadvantage of the CPT is that the voided current definition depends on the other components, and it is not calculated independently (voided currents calculated from the active and reactive currents). It is worth noting that none of the aforementioned power theories are able to decompose current components properly in the case of unbalanced and distorted voltage conditions. Therefore, a new signal decomposition methodology with higher level of selectivity must be developed to modify the traditional power theories.

In this paper an advanced current decomposition method is proposed to independently calculate currents for unbalanced and nonsinusoidal three-phase power systems, which are valid in the case of unbalanced distorted voltage source in weak grid conditions. In this paper, the authors introduce a modified version of both Akagi's and Peng's power theories [4], [6], where our terminology is called an "enhanced instantaneous power theory (EIPT)." The proposed EIPT is able to present a detailed instantaneous current signal decomposition framework[2] where a typical current signal, on each phase of the system, can be decomposed in terms of several components that are clearly describing the contribution of each balance, unbalanced, harmonic, and zero sequence components for both active and reactive terms. This new detailed decomposition approach can be helpful in optimizing the control strategies for power electronic interfaces, especially in renewable-based microgrid systems. In addition, it might be used independently in terms of any signal/vector analytics application.

The most notable advantages of our proposed EIPT are that all current components are calculated independently, and it works in the case of unbalanced and distorted voltage source conditions as well. Moreover, the new proposed approach will decompose the signal into much more detailed components, so it is more reliable while providing more flexibility and selectivity in terms of control and monitoring applications.

The rest of this paper is presented as follows. In Section II, the related mathematical preliminaries and proofs for the proposed power theory under symmetrical balanced, unbalanced, and distorted voltage source conditions will be discussed. Next, in Section III, case studies and comprehensive simulations are presented. The paper ends in Section IV with our final conclusions.

II. EIPT and Current Decomposition Methodology

The basic idea of this paper comes from the pq theory and Peng's power theory [4], [6]. The proposed theory in this paper (EIPT) is developed to decompose the electrical currents of a neutralized three-phase power system (four-wired system) to a set of meaningful subcomponents that are used for active filtering and control of power electronic converters. A three-phase power system is shown in Fig. 1, where instantaneous voltages and currents are defined as instantaneous space vectors

$$\left(v_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \text{ and } i_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \right).$$

In general, our methodology in this paper is divided into three main parts. In the first part, which starts on Section II-B, we considered the voltage source to be a sinusoidal balanced symmetrical voltage, thus, all definitions are developed based on this assumption. Actually, in this situation, the source of nonlinearity and unbalance in the current terms is caused solely by the nonlinear and unbalanced loads. However, in the second part of the methodology Section, Section II-C, we considered an asymmetrical and unbalanced voltage source in addition to the nonlinear and unbalanced loads. Finally, in Section II-D, we extend our EIPT methodology for the distorted voltage source condition. From our mathematical redefinitions, the effects of unbalanced and distorted parts of voltage source has been interpreted along with new definitions and appears within the consequent formulations.

[2]Regarding power definitions in [4] and [6].

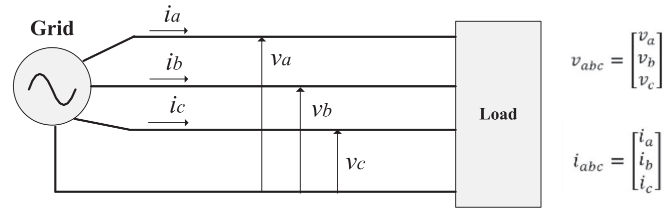


Fig. 1. Three-phase power system.

A. Concordia Transformation ($\alpha\beta 0$ Transformation)

There is an algebraic transformation known as either Clarke or Concordia transformation to map the three-phase instantaneous voltages and currents in the abc coordinates to the $\alpha\beta 0$ coordinates. The Clarke matrix is a constant project matrix widely implemented in electrical drives for control purposes (because the magnitude of voltages and currents are preserved), whereas the Concordia matrix is considered a demodulated signal with power invariance used in faulty conditions and power analysis [24]. The main reason of using this latter transformation is to separate the zero sequence components of the abc phase voltage and current signals. Thus, we used the Concordia transformation to convert instantaneous currents and voltages from abc into $\alpha\beta 0$ frame. The Concordia transformation of three-phase voltages and currents are given by

$$v_{\alpha\beta 0} = \begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = [T_{\alpha\beta 0}] \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \quad i_{\alpha\beta 0} = \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = [T_{\alpha\beta 0}] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{1}$$

where, v_α and v_β are the instantaneous voltages, i_α and i_β are the instantaneous currents, in α and β axis, respectively, and v_0 and i_0 are zero sequence (also termed as homopolar) components of voltages and currents. $T_{\alpha\beta 0}$ is known as $\alpha\beta 0$ transformation matrix defined as follows:

$$[T_{\alpha\beta 0}] = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}. \tag{2}$$

(The reconfiguration of the axes under Concordia transformation is shown in Appendix C.) Similarly, the inverse of Concordia transformation maps the voltages and currents in $\alpha\beta 0$ frame to the instantaneous three-phase voltages and currents in

abc frame:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = [T_{\alpha\beta 0}]^{-1} \begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = [T_{\alpha\beta 0}]^{-1} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix}. \tag{4}$$

Considering the Concordia transformation matrix ($T_{\alpha\beta 0}$), one may easily conclude that

$$T_{\alpha\beta 0} T_{\alpha\beta 0}^T = [I]_{3\times 3}. \tag{5}$$

B. Mathematical Methodology for Current Decomposition in Three-Phase Nonsinusoidal Unbalanced System with Symmetrical Balanced Voltage Source

In this part, we will decompose the current components in the presence of strong grid conditions. Consequently, a balanced voltage source with steady voltage and frequency is used to model the power grid (please find the full list of all quantities and components in the nomenclature).

1) Fundamental Definitions: The definition of instantaneous active power p is almost the same in most of the power theories. This electrical quantity is defined as a scalar product of instantaneous current and voltage vectors. Within the $\alpha\beta 0$ frame notation, p is calculated as

$$\begin{aligned} p = v_{\alpha\beta 0}.i_{\alpha\beta 0} = v_{\alpha\beta 0} {i_{\alpha\beta 0}}^T = \begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} \begin{bmatrix} i_\alpha & i_\beta & i_0 \end{bmatrix} \\ = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 \end{aligned} \tag{6}$$

where "." denotes the dot (internal or scalar) product of two vectors. By definition, p is the instantaneous active power and corresponds to the energy per time unity, which is transferred from the power supply to the load. We separate (6) into the following two arbitrary terms:

$$p_{\alpha\beta} = v_\alpha i_\alpha + v_\beta i_\beta \tag{7}$$

$$p_0 = v_0 i_0 \tag{8}$$

where $p_{\alpha\beta}$ is termed instantaneous active heteropolar power, and p_0 is termed instantaneous active unbalanced homopolar power.

Property 1: The following equality can be obtained for instantaneous active power in $\alpha\beta 0$ frame versus abc frame:

$$p = v_{abc}.i_{abc} = v_{\alpha\beta 0}.i_{\alpha\beta 0}. \tag{9}$$

After a quick search beyond the related articles, one may understand that similar to the active power definition, the corresponding definition of the instantaneous active current i_p is also the same within different power theories:

$$i_p = \frac{p.v_{abc}}{v_{abc}.v_{abc}} \xrightarrow{\text{in } \alpha\beta 0 \text{ frame}} i_p = \frac{p.v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} \tag{10}$$

where the denominators can be expanded as follows:

$$\begin{aligned} v_{abc}.v_{abc} = v_{\alpha\beta 0}.v_{\alpha\beta 0} = {v_a}^2 + {v_b}^2 + {v_c}^2 \\ = {v_\alpha}^2 + {v_\beta}^2 + {v_0}^2. \end{aligned} \tag{11}$$

Now, following a similar approach as Peng (which has been developed in the abc frame), using the cross product of the instantaneous voltage and the instantaneous current vectors, we define the following instantaneous space vector q as the reactive power vector in the $\alpha\beta 0$ frame (alternatively $q_{\alpha\beta 0}$):

$$q = v_{\alpha\beta 0} \times i_{\alpha\beta 0} \tag{12}$$

where "$\times$" denotes the cross product, and $v_{\alpha\beta 0} \times i_{\alpha\beta 0}$ is a vector that is perpendicular to $v_{\alpha\beta 0}$ and $i_{\alpha\beta 0}$. Consider the following definition for the reactive current:

$$i_q = \frac{q \times v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}}. \tag{13}$$

In order to validate our definitions for active and reactive currents in (12) and (13), we need to show that the following statement is valid:

$$i_p + i_q = i \tag{14}$$

where q is the instantaneous reactive or nonactive power, and i_q is the instantaneous reactive current. Applying (10) and (13) in (14), we have

$$\begin{aligned} i_p + i_q &= \frac{p.v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} + \frac{q \times v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} \\ &= \frac{(v_{\alpha\beta 0}.i_{\alpha\beta 0}).v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} + \frac{(v_{\alpha\beta 0} \times i_{\alpha\beta 0}) \times v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} \end{aligned} \tag{15}$$

utilizing the following vector product property:

$$(x \times y) \times z = -(y.z)\,x + (x.z)\,y \tag{16}$$

we end up with

$$\begin{aligned} &\rightarrow i_p + i_q = \frac{(v_{\alpha\beta 0}.i_{\alpha\beta 0}).v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} + \frac{(v_{\alpha\beta 0} \times i_{\alpha\beta 0}) \times v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} \\ &= \frac{(v_{\alpha\beta 0}.i_{\alpha\beta 0}).v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} + \frac{[-(i_{\alpha\beta 0}.v_{\alpha\beta 0})\,v_{\alpha\beta 0} + (v_{\alpha\beta 0}.v_{\alpha\beta 0})\,i_{\alpha\beta 0}]}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} \\ &= i_{\alpha\beta 0} = i \end{aligned} \tag{17}$$

and as a result, our assumption for the definitions of q and i_q are validated accordingly. Now, we expand (12) as follows:

$$q_{\alpha\beta 0} = v_{\alpha\beta 0} \times i_{\alpha\beta 0} = \begin{bmatrix} \begin{vmatrix} v_\beta & v_0 \\ i_\beta & i_0 \end{vmatrix} \\ \begin{vmatrix} v_0 & v_\alpha \\ i_0 & i_\alpha \end{vmatrix} \\ \begin{vmatrix} v_\alpha & v_\beta \\ i_\alpha & i_\beta \end{vmatrix} \end{bmatrix} = \begin{bmatrix} q_{\beta 0} \\ q_{\alpha 0} \\ q_{\alpha\beta} \end{bmatrix} \tag{18}$$

where $q_{\beta 0}$, $q_{\alpha 0}$, and $q_{\alpha\beta}$ are calculated as:

$$q_{\beta 0} = -v_0 i_\beta + v_\beta i_0 \tag{19}$$

$$q_{\alpha 0} = v_0 i_\alpha - v_\alpha i_0 \tag{20}$$

$$q_{\alpha\beta} = -v_\beta i_\alpha + v_\alpha i_\beta \tag{21}$$

$q_{\alpha 0}$ and $q_{\beta 0}$ are the instantaneous unbalanced homopolar reactive power components, and $q_{\alpha\beta}$ is the instantaneous reactive heteropolar power component.

Property 2: The following relation can be found between Peng's reactive power vector q_{abc} in abc frame versus ours in $\alpha\beta 0$ frame

$$\begin{aligned} q_{\alpha\beta 0} &= v_{\alpha\beta 0} \times i_{\alpha\beta 0} = ([T_{\alpha\beta 0}]\, v_{abc}) \times ([T_{\alpha\beta 0}]\, i_{abc}) \\ &= [T_{\alpha\beta 0}]\,(v_{abc} \times i_{abc}) = [T_{\alpha\beta 0}]\, q_{abc}. \end{aligned} \tag{22}$$

The amount of instantaneous power that is transferred between phases (reactive power) is defined as the magnitude of $q_{\alpha\beta 0}$, which is shown with Q:

$$Q = |q_{\alpha\beta 0}|. \tag{23}$$

In a system without zero sequence components: $v_0 = i_0 = 0$, $q_{\beta 0} = q_{\alpha 0} = 0$, accordingly, as a result:

$$q_{\alpha\beta 0} = q_{\alpha\beta} = v_\alpha i_\beta - v_\beta i_\alpha \tag{24}$$

where $q_{\alpha\beta}$ is equal to the imaginary power in conventional pq theory [4].

Property 3: It can be shown that the voltage vector $v_{\alpha\beta 0}$ is perpendicular to the reactive currents, while it is parallel with active current (mathematical proof can be found in Appendix A):

$$i_p \times v_{\alpha\beta 0} = 0 \tag{25}$$

$$i_q . v_{\alpha\beta 0} = 0. \tag{26}$$

In our framework, the instantaneous apparent power and the power factor are defined as follows:

$$3-\Phi : \begin{cases} P = |v_{\alpha\beta 0} . i_{\alpha\beta 0}| \\ Q = |v_{\alpha\beta 0} \times i_{\alpha\beta 0}| \end{cases} \xrightarrow{\text{yields}} S^2 = P^2 + Q^2$$

$$\xrightarrow{\text{yields}} \quad S = IV = |i_{\alpha\beta 0}|\,|v_{\alpha\beta 0}| \tag{27}$$

where P is the real power, and Q is the reactive power (the proof of (27) can be found in the Appendix B). V and I are the rms values of voltage and current vectors and are defined as follows:

$$V = |v_{\alpha\beta 0}| = \sqrt{{v_0}^2 + {v_\alpha}^2 + {v_\beta}^2} \tag{28}$$

$$I = |i_{\alpha\beta 0}| = \sqrt{{i_0}^2 + {i_\alpha}^2 + {i_\beta}^2}. \tag{29}$$

Finally, power factor is defined as

$$PF = \frac{P}{S} \tag{30}$$

which is the same as conventional definitions.

2) EIPT Current Decomposition Approach. Concepts and Components: In what comes next, (31) shows the relation between different power components according to different voltage and current terms and (32) represents its inverse form:

$$\begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\beta 0} \\ q_{\alpha 0} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & v_0 \\ v_\alpha & v_\beta & 0 \\ 0 & -v_0 & v_\beta \\ v_0 & 0 & -v_\alpha \\ -v_\beta & v_\alpha & 0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \tag{31}$$

$$\begin{aligned} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} &= \frac{1}{{v_{\alpha\beta 0}}^2} \begin{bmatrix} 0 & v_\alpha & 0 & v_0 & -v_\beta \\ 0 & v_\beta & -v_0 & 0 & v_\alpha \\ v_0 & 0 & v_\beta & -v_\alpha & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ &+ \frac{1}{{v_{\alpha\beta 0}}^2} \begin{bmatrix} 0 & v_\alpha & 0 & v_0 & -v_\beta \\ 0 & v_\beta & -v_0 & 0 & v_\alpha \\ v_0 & 0 & v_\beta & -v_\alpha & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ q_{\beta 0} \\ q_{\alpha 0} \\ q_{\alpha\beta} \end{bmatrix}. \end{aligned} \tag{32}$$

These two equations are the basis of the formulation that we use in order to decompose the current components in the $\alpha\beta 0$ frame according to different power components. The corresponding coefficients of the instantaneous active unbalanced homopolar power (p_0) in (31) are utilized to find the active unbalanced homopolar current terms as follows:

$$i_{\alpha p_0} = 0,\; i_{\beta p_0} = 0, \quad i_{0 p_0} = \frac{v_0 p_0}{{v_0}^2}. \tag{33}$$

Applying the inverse Concordia transformation, we calculate

$$i_{a p_0} = i_{b p_0} = i_{c p_0} = \frac{1}{\sqrt{3}} i_{0 p_0} = \frac{1}{\sqrt{3}} \frac{v_0 p_0}{{v_0}^2} \tag{34}$$

where, $i_{a p_0}$, $i_{b p_0}$, and $i_{c p_0}$ are the active unbalanced homopolar current components in phases a, b, and c, respectively. To find the active heteropolar current components, we used the corresponding coefficients of $p_{\alpha\beta}$ in (32):

$$\begin{cases} i_{\alpha p_{\alpha\beta}} = \frac{v_\alpha p_{\alpha\beta}}{{v_{\alpha\beta 0}}^2} \\ i_{\beta p_{\alpha\beta}} = \frac{v_\beta p_{\alpha\beta}}{{v_{\alpha\beta 0}}^2} \\ i_{0 p_{\alpha\beta}} = 0 \end{cases}. \tag{35}$$

The instantaneous active power in $\alpha\beta$ axis $(p_{\alpha\beta})$ can be decomposed into its dc and oscillating parts as follows:

$$p_{\alpha\beta} = \bar{p}_{\alpha\beta} + \tilde{p}_{\alpha\beta} \tag{36}$$

where $\bar{p}_{\alpha\beta}$ is termed the average active balanced heteropolar power, and $\tilde{p}_{\alpha\beta}$ is termed an oscillating heteropolar power, which is caused by nonlinear and unbalanced components between phases in the system. To find the sinusoidal balanced active currents, we use the average active heteropolar power $(\bar{p}_{\alpha\beta})$:

$$\begin{cases} i_{\alpha \bar{p}_{\alpha\beta}} = \frac{v_\alpha \bar{p}_{\alpha\beta}}{{v_{\alpha\beta 0}}^2} \\ i_{\beta \bar{p}_{\alpha\beta}} = \frac{v_\beta \bar{p}_{\alpha\beta}}{{v_{\alpha\beta 0}}^2} \\ i_{0 \bar{p}_{\alpha\beta}} = 0 \end{cases} \xrightarrow[\text{frame}]{\text{in } abc} \begin{bmatrix} i_{a\bar{p}} \\ i_{b\bar{p}} \\ i_{c\bar{p}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{\alpha \bar{p}_{\alpha\beta}} \\ i_{\beta \bar{p}_{\alpha\beta}} \end{bmatrix} \tag{37}$$

where, $i_{a\bar{p}}$, $i_{b\bar{p}}$, and $i_{c\bar{p}}$ are the active balanced sinusoidal current components in phases a, b, and c, respectively. To find distorted

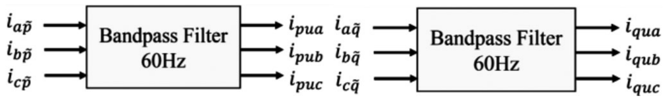


Fig. 2. Bandpass filter used for extraction of unbalanced active and reactive currents.

and unbalanced active current components, we need to use the oscillating active heteropolar power ($\tilde{p}_{\alpha\beta}$) in our calculation:

$$\begin{cases} i_{\alpha\tilde{p}_{\alpha\beta}} = \frac{v_\alpha \tilde{p}_{\alpha\beta}}{{v_{\alpha\beta0}}^2} \\ i_{\beta\tilde{p}_{\alpha\beta}} = \frac{v_\beta \tilde{p}_{\alpha\beta}}{{v_{\alpha\beta0}}^2} \\ i_{0\tilde{p}_{\alpha\beta}} = 0 \end{cases} \xrightarrow{\text{in } abc \text{ frame}} \begin{bmatrix} i_{a\tilde{p}} \\ i_{b\tilde{p}} \\ i_{c\tilde{p}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{\alpha\tilde{p}_{\alpha\beta}} \\ i_{\beta\tilde{p}_{\alpha\beta}} \end{bmatrix} \tag{38}$$

by implementing inverse Concordia transformation, we got $i_{a\tilde{p}}$, $i_{b\tilde{p}}$, and $i_{c\tilde{p}}$, which are the active oscillating current components in phases a, b, and c, respectively.

These oscillating components contain both the distorted and unbalanced parts of the current. These two parts can be easily separated by implementing a bandpass filter with a notch frequency as tightly tuned over 60 Hz (see Fig. 2). i_{pua}, i_{pub}, and i_{puc} are the active heteropolar unbalanced currents in phases a, b, and c, respectively. Moreover, we have

$$\begin{cases} i_{a\tilde{p}} - i_{pua} = i_{pHa} \\ i_{b\tilde{p}} - i_{pub} = i_{pHb} \\ i_{c\tilde{p}} - i_{puc} = i_{pHc} \end{cases} \tag{39}$$

where i_{aHa}, i_{aHb}, and i_{aHc} are the active distorted currents in phases a, b, and c, respectively. The homopolar part of reactive current is calculated from the zero sequence components of reactive vector ($q_{\alpha 0}$ and $q_{\beta 0}$), so from (32), we have

$$\begin{cases} i_{\alpha q_0} = \frac{v_0 q_{\alpha 0}}{{v_{\alpha\beta0}}^2} \\ i_{\beta q_0} = -\frac{v_0 q_{\beta 0}}{{v_{\alpha\beta0}}^2} \\ i_{0 q_0} = \frac{v_\beta q_{\beta 0}}{{v_{\alpha\beta0}}^2} - \frac{v_\alpha q_{\alpha 0}}{{v_{\alpha\beta0}}^2} \end{cases} \xrightarrow{\text{in } abc \text{ frame}} \begin{bmatrix} i_{aq_0} \\ i_{bq_0} \\ i_{cq_0} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} i_{\alpha q_0} \\ i_{\beta q_0} \\ i_{0 q_0} \end{bmatrix} \tag{40}$$

where i_{aq_0}, i_{bq_0}, and i_{cq_0} are termed homopolar reactive current components in phases a, b, and c, respectively. We can also calculate the homopolar reactive current components using $i_{q_0} = i_0 - i_{p_0}$. To find the heteropolar reactive components of the current, we need to use the coefficients of $q_{\alpha\beta}$ in (32):

$$\begin{cases} i_{\alpha q_{\alpha\beta}} = -\frac{v_\beta q_{\alpha\beta}}{{v_{\alpha\beta0}}^2} \\ i_{\beta q_{\alpha\beta}} = \frac{v_\alpha q_{\alpha\beta}}{{v_{\alpha\beta0}}^2} \\ i_{0 q_{\alpha\beta}} = 0 \end{cases} . \tag{41}$$

Similar to (36), one may decompose the instantaneous reactive power in $\alpha\beta$ axis ($q_{\alpha\beta}$) into its average and oscillating components as follows:

$$q_{\alpha\beta} = \bar{q}_{\alpha\beta} + \tilde{q}_{\alpha\beta} \tag{42}$$

where $\bar{q}_{\alpha\beta}$ is termed the average balanced reactive power, and $\tilde{q}_{\alpha\beta}$ as the oscillating reactive power, which is basically caused by nonlinear and unbalanced components between phases in the system. The sinusoidal reactive balanced components of the current are calculated from the $\bar{q}_{\alpha\beta}$ as follows:

$$\begin{cases} i_{\alpha\bar{q}_{\alpha\beta}} = -\frac{v_\beta \bar{q}_{\alpha\beta}}{{v_{\alpha\beta0}}^2} \\ i_{\beta\bar{q}_{\alpha\beta}} = \frac{v_\alpha \bar{q}_{\alpha\beta}}{{v_{\alpha\beta0}}^2} \\ i_{0\bar{q}_{\alpha\beta}} = 0 \end{cases} \xrightarrow{\text{in } abc \text{ frame}} \begin{bmatrix} i_{a\bar{q}} \\ i_{b\bar{q}} \\ i_{c\bar{q}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{\alpha\bar{q}_{\alpha\beta}} \\ i_{\beta\bar{q}_{\alpha\beta}} \end{bmatrix} \tag{43}$$

where, $i_{a\bar{q}}$, $i_{b\bar{q}}$, and $i_{c\bar{q}}$ are sinusoidal balanced reactive current components in phases a, b, and c, respectively. To find distorted and unbalanced reactive current components, the oscillating reactive power component is used in our calculation:

$$\begin{cases} i_{\alpha\tilde{q}_{\alpha\beta}} = -\frac{v_\beta \tilde{q}_{\alpha\beta}}{{v_{\alpha\beta0}}^2} \\ i_{\beta\tilde{q}_{\alpha\beta}} = \frac{v_\alpha \tilde{q}_{\alpha\beta}}{{v_{\alpha\beta0}}^2} \\ i_{0\tilde{q}_{\alpha\beta}} = 0 \end{cases} \xrightarrow{\text{in } abc \text{ frame}} \begin{bmatrix} i_{a\tilde{q}} \\ i_{b\tilde{q}} \\ i_{c\tilde{q}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{\alpha\tilde{q}_{\alpha\beta}} \\ i_{\beta\tilde{q}_{\alpha\beta}} \end{bmatrix} \tag{44}$$

$i_{a\tilde{q}}$, $i_{b\tilde{q}}$, and $i_{c\tilde{q}}$ are distorted and unbalanced reactive current components in phases a, b, and c, respectively. To separate these two parts, we can use bandpass filters (see Fig. 2), where i_{qua}, i_{qub}, and i_{quc} are reactive heteropolar unbalanced currents in phases a, b, and c, respectively. Moreover, we have

$$\begin{cases} i_{a\tilde{q}} - i_{qua} = i_{qHa} \\ i_{b\tilde{q}} - i_{qub} = i_{qHb} \\ i_{c\tilde{q}} - i_{quc} = i_{qHc} \end{cases} \tag{45}$$

where i_{rHa}, i_{rHb}, and i_{rHc} are reactive distorted currents in phases a, b, and c, respectively. Now we can define the instantaneous active and reactive powers for each phase of the system as follows:

$$\begin{cases} p_a = (i_{ap_0} + i_{a\bar{p}} + i_{pHa} + \mathrm{i}_{pua})\, v_a \\ p_b = (i_{bp_0} + i_{b\bar{p}} + i_{pHb} + \mathrm{i}_{pub})\, v_b \\ p_c = (i_{cp_0} + i_{c\bar{p}} + i_{pHc} + \mathrm{i}_{puc})\, v_c \end{cases} \tag{46}$$

$$\begin{cases} q_a = (i_{aq_0} + i_{a\bar{q}} + i_{qHa} + \mathrm{i}_{qua})\, v_a \\ q_b = (i_{bq_0} + i_{b\bar{q}} + i_{qHb} + \mathrm{i}_{qub})\, v_b \\ q_c = (i_{cq_0} + i_{c\bar{q}} + i_{qHc} + \mathrm{i}_{quc})\, v_c \end{cases}, \; q_a(t) + q_b(t) + q_c(t) = 0. \tag{47}$$

The summation of all corresponding reactive power terms over all phases in each sample of time is equal to zero, and it shows that the defined reactive power has physical meaning and it is the amount of power which is transferred between phases of the system and not transferred from source to load.

C. Methodology for Current Decomposition in Three-Phase Nonsinusoidal Unbalanced System With Asymmetrical Unbalanced Voltage Source

In this section, we will decompose the current components in the presence of weak grid conditions, where we do not have balanced voltage in the grid side. As a result, we will have the negative sequence active and reactive current components, as well as zero and positive sequence active and reactive current components.

A note on the source of imbalance: Before going through the details of the mathematical approach, we would like to clarify the following points.

1) In case of a system without neutral wiring, we will only face a single type of imbalance, which is namely the *between phase imbalance*; we term this type as *heteropolar unbalanced phenomenon*.
2) When we are working with a neutral wiring architecture, we will face two of the following types of imbalances in the corresponding current components.
   a) The imbalance between each phase versus neutral wire is modeled in terms of *unbalanced homopolar or zero-sequence components*, which appears in two current terms namely: unbalanced active/reactive homopolar current $i_{\mathrm{x}p_0}, i_{\mathrm{x}q_0}$ (where index x refers to the phase name). These two components are calculated by exploiting homopolar current i_0 and voltage v_0, as detailed in Sections II-B and II-C over each phase.
   b) The imbalance between phases that we arbitrarily term as *unbalanced heteropolar components,* which appears in two current terms namely: unbalanced active/reactive heteropolar current $i_{xup}, i_{\mathrm{x}uq}$. These two components are calculated by exploiting the oscillating parts of active and reactive powers ($\tilde{p}_{\alpha\beta}$ and $\tilde{q}_{\alpha\beta}$) through an auxiliary bandpass filtering step, as detailed and highlighted in Sections II-B and II-C over each phase.

Now, we start the mathematical formulation under unbalanced voltage source condition. Since in this case we are considering the unbalanced voltage source condition, we should first find the zero, positive, and negative sequence components of the voltage on each individual phase using Fortescue transformation as follows:

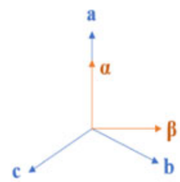
$$\begin{bmatrix} v_0 \\ v_+ \\ v_- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \gamma & \gamma^2 \\ 1 & \gamma^2 & \gamma \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{48}$$

where the phase shift operator is defined as $\gamma = 1\angle 120°$. Inversely the phase voltages are calculated as

$$\begin{aligned} v_a &= v_0 + v_+ + v_- \\ v_b &= v_0 + \gamma^2 v_+ + \gamma v_- \\ v_c &= v_0 + \gamma v_+ + \gamma^2 v_-. \end{aligned} \tag{49}$$

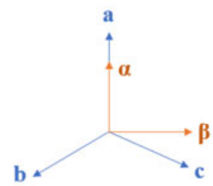
Using the phase shift operator γ, we represent the positive and negative sequences of the voltage in each phase within the following notation:

$$\begin{cases} v_{a+} = v_+ \\ v_{b_+} = \gamma^2 v_+ \\ v_{c_+} = \gamma v_+ \end{cases}, \begin{cases} v_{a-} = v_- \\ v_{b_-} = \gamma v_- \\ v_{c_-} = \gamma^2 v_- \end{cases}. \tag{50}$$



Positive Sequence to αβ $\longrightarrow$

$$\begin{bmatrix} v_{\alpha+} \\ v_{\beta+} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} v_{a+} \\ v_{b_+} \\ v_{c_+} \end{bmatrix}$$



Negative Sequence to αβ $\longrightarrow$

$$\begin{bmatrix} v_{\alpha-} \\ v_{\beta-} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} v_{a-} \\ v_{b_-} \\ v_{c_-} \end{bmatrix} \tag{51}$$

We now exploit this notation to generalize the definition of voltage components in terms of the Concordia transformation to the unbalanced voltage situation as follows:
where $v_{\alpha+}, v_{\alpha-}, v_{\beta+}$, and $v_{\beta-}$ are representing the positive and negative sequence components within the $\alpha-\beta$ transformation domain. Since the zero-sequence component of the voltage will remain the same under Concordia transformation, the square of the voltage magnitude is calculated as

$$|v_{\alpha\beta 0}|^2 = |v_{\alpha+}{}^2 + v_{\alpha-}{}^2 + v_{\beta+}{}^2 + v_{\beta-}{}^2 + v_0|^2. \tag{52}$$

Moreover, within the $\alpha-\beta$ transformation subspace the positive and negative components can be vectorially added to form the total $\alpha-\beta$ voltage subvectors. In other words, from the vector space definitions, each term can be quantized in terms of the following factorized summation:

$$\begin{cases} v_\alpha = v_{\alpha+} + v_{\alpha-} \\ v_\beta = v_{\beta+} + v_{\beta-} \end{cases} \tag{53}$$

as a result, the transformed voltage vector is decomposed as

$$v_{\alpha\beta 0} = \begin{bmatrix} v_{\alpha+} \\ v_{\beta_+} \\ 0 \end{bmatrix} + \begin{bmatrix} v_{\alpha-} \\ v_{\beta_-} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ v_0 \end{bmatrix}. \tag{54}$$

We may now define the corresponding positive and negative power components in terms of voltage and current vectors within the $\alpha-\beta$ transformation domain as follows:

$$\begin{cases} p_{\alpha\beta}{}^+ = v_{\alpha+} i_\alpha + v_{\beta+} i_\beta \\ p_{\alpha\beta}{}^- = v_{\alpha-} i_\alpha + v_{\beta-} i_\beta \end{cases}. \tag{55}$$

The total active power is then calculated as

$$p_{\alpha\beta 0} = p_{\alpha\beta}{}^+ + p_{\alpha\beta}{}^- + p_0. \tag{56}$$

Using a simple mathematical factorization in (55) and considering v_α and v_β from (53), we can easily see

$$p_{\alpha\beta 0} = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 = p_{\alpha\beta} + p_0. \tag{57}$$

This is a similar final notation as in the case of balance voltage source with different components, which are instead calculated by our new definitions in (51). Here, again we decompose the instantaneous active power in terms of average and sinusoidal parts: $p_{\alpha\beta} = \bar{p}_{\alpha\beta} + \tilde{p}_{\alpha\beta}$. In order to find the sinusoidal positive sequence balanced active current components, we need to use the average active power component ($\bar{p}_{\alpha\beta}$) in our calculation. Moreover, the oscillating parts of the positive sequence balanced active current components are defined based on the $\tilde{p}_{\alpha\beta}$ (a similar approach is used to find both the sinusoidal and oscillating current components in terms of reactive power):

$$\begin{cases} {i_{\alpha\bar{p}_{\alpha\beta}}}^{+} = \frac{v_{\alpha+}}{{v_{\alpha\beta 0}}^2}\bar{p}_{\alpha\beta} \\ {i_{\beta\bar{p}_{\alpha\beta}}}^{+} = \frac{v_{\beta+}}{{v_{\alpha\beta 0}}^2}\bar{p}_{\alpha\beta} \\ i_{0\bar{p}_{\alpha\beta}} = 0 \end{cases} \xrightarrow[\text{frame}]{\text{in } abc} \begin{bmatrix} i_{a\bar{p}} \\ i_{a\mathrm{b}} \\ i_{c\bar{p}} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} {i_{\alpha\bar{p}_{\alpha\beta}}}^{+} \\ {i_{\beta\bar{p}_{\alpha\beta}}}^{+} \end{bmatrix}. \tag{58}$$

$i_{a\bar{p}}$, $i_{b\bar{p}}$, and $i_{c\bar{p}}$ are obtained, which are sinusoidal balanced active current components, in phases a, b, and c, respectively. On the other hand, unbalanced active current components caused by heteropolar unbalanced part of the voltage are calculated from

$$\begin{cases} {i_{\alpha\bar{p}_{\alpha\beta}}}^{-} = \frac{v_{\alpha-}}{{v_{\alpha\beta 0}}^2}\bar{p}_{\alpha\beta} \\ {i_{\beta\bar{p}_{\alpha\beta}}}^{-} = \frac{v_{\beta-}}{{v_{\alpha\beta 0}}^2}\bar{p}_{\alpha\beta} \\ i_{0p_{\alpha\beta}} = 0 \end{cases} \xrightarrow[\text{frame}]{\text{in } abc} \begin{bmatrix} i_{pua_s} \\ i_{pub_s} \\ i_{puc_s} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} {i_{\alpha\bar{p}_{\alpha\beta}}}^{-} \\ {i_{\beta\bar{p}_{\alpha\beta}}}^{-} \end{bmatrix} \tag{59}$$

where i_{pua_s}, i_{pub_s} and i_{puc_s} are unbalanced active current components caused by heteropolar unbalanced part of voltage source in phases a, b, and c, respectively. Moreover, the oscillating parts of the active current components are defined as follows:

$$\begin{cases} i_{\alpha\tilde{p}_{\alpha\beta}} = \frac{v_{\alpha}\tilde{p}_{\alpha\beta}}{{v_{\alpha\beta 0}}^2} \\ i_{\beta\tilde{p}_{\alpha\beta}} = \frac{v_{\beta}\tilde{p}_{\alpha\beta}}{{v_{\alpha\beta 0}}^2} \\ i_{0\bar{p}_{\alpha\beta}} = 0 \end{cases} \xrightarrow[\text{frame}]{\text{in } abc} \begin{bmatrix} i_{a\tilde{p}} \\ i_{b\tilde{p}} \\ i_{c\tilde{p}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{\alpha\tilde{p}_{\alpha\beta}} \\ i_{\beta\tilde{p}_{\alpha\beta}} \end{bmatrix} \tag{60}$$

where, $i_{a\tilde{p}}$, $i_{b\tilde{p}}$, and $i_{c\tilde{p}}$ are the distorted and unbalanced active current components in phases a, b, and c, respectively. To separate these two parts, we can use bandpass filters (see Fig. 2). i_{pua}, i_{pub}, and i_{puc} are active unbalanced currents in phases a, b, and c, respectively. Moreover, we have

$$\begin{cases} i_{a\tilde{p}} - i_{pua} = i_{pHa} \\ i_{b\tilde{p}} - i_{pub} = i_{pHb} \\ i_{c\tilde{p}} - i_{puc} = i_{pHc} \end{cases} \tag{61}$$

where i_{pHa}, i_{pHb}, and i_{pHc} are active distorted currents in phases a, b, and c, respectively. We will follow the same procedure for calculating the positive and negative sequences of reactive current. For the positive sequence reactive current we have

$$\begin{cases} {i_{\alpha\bar{q}_{\alpha\beta}}}^{+} = -\frac{v_{\beta+}}{{v_{\alpha\beta 0}}^2}\bar{q}_{\alpha\beta} \\ {i_{\beta\bar{q}_{\alpha\beta}}}^{+} = \frac{v_{\alpha+}}{{v_{\alpha\beta 0}}^2}\bar{q}_{\alpha\beta} \\ i_{0\bar{q}_{\alpha\beta}} = 0 \end{cases} \xrightarrow[\text{frame}]{\text{in } abc} \begin{bmatrix} i_{a\bar{q}} \\ i_{b\bar{q}} \\ i_{c\bar{q}} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} {i_{\alpha\bar{q}_{\alpha\beta}}}^{+} \\ {i_{\beta\bar{q}_{\alpha\beta}}}^{+} \end{bmatrix}. \tag{62}$$

$i_{a\bar{q}}$, $i_{b\bar{q}}$, and $i_{c\bar{q}}$ are balanced sinusoidal reactive current components in phases a, b, and c respectively. On the other hand, unbalanced reactive current components caused by heteropolar unbalanced part of the voltage are obtained from

$$\begin{cases} {i_{\alpha\bar{q}_{\alpha\beta}}}^{-} = -\frac{v_{\beta-}}{{v_{\alpha\beta 0}}^2}\bar{q}_{\alpha\beta} \\ {i_{\beta\bar{q}_{\alpha\beta}}}^{-} = \frac{v_{\alpha-}}{{v_{\alpha\beta 0}}^2}\bar{q}_{\alpha\beta} \\ i_{0q_{\alpha\beta}} = 0 \end{cases} \xrightarrow[\text{frame}]{\text{in } abc} \begin{bmatrix} i_{qua_s} \\ i_{qub_s} \\ i_{quc_s} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} {i_{\alpha\bar{q}_{\alpha\beta}}}^{-} \\ {i_{\beta\bar{q}_{\alpha\beta}}}^{-} \end{bmatrix} \tag{63}$$

where, i_{qua_s}, i_{qub_s}, and i_{quc_s} are the unbalanced reactive current components caused by heteropolar unbalanced part of the voltage source in phase a, b, and c, respectively. Finally, to find the unbalanced and harmonic parts of the reactive current components, we need to use the oscillating reactive power component in our calculation:

$$\begin{cases} i_{\alpha\tilde{q}_{\alpha\beta}} = -\frac{v_{\beta}}{{v_{\alpha\beta 0}}^2}\tilde{q}_{\alpha\beta} \\ i_{\beta\tilde{q}_{\alpha\beta}} = \frac{v_{\alpha}}{{v_{\alpha\beta 0}}^2}\tilde{q}_{\alpha\beta} \\ i_{0\bar{p}_{\alpha\beta}} = 0 \end{cases} \xrightarrow[\text{frame}]{\text{in } abc} \begin{bmatrix} i_{a\tilde{q}} \\ i_{b\tilde{q}} \\ i_{c\tilde{q}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{\alpha\tilde{q}_{\alpha\beta}} \\ i_{\beta\tilde{q}_{\alpha\beta}} \end{bmatrix}. \tag{64}$$

Then $i_{a\tilde{q}}$, $i_{b\tilde{q}}$, and $i_{c\tilde{q}}$ are distorted and unbalanced reactive current components in phases a, b, and c, respectively. To separate these two parts, we can use bandpass filters (see Fig. 2). where i_{qua}, i_{qub}, and i_{quc} are reactive unbalanced currents in phases a, b, and c, respectively. Moreover, we have

$$\begin{cases} i_{a\tilde{q}} - i_{qua} = i_{qHa} \\ i_{b\tilde{q}} - i_{qub} = i_{qHb} \\ i_{c\tilde{q}} - i_{quc} = i_{qHc} \end{cases} \tag{65}$$

where i_{qHa}, i_{qHb}, and i_{qHc} are active distorted currents in phases a, b, and c, respectively. So we can add all heteropolar unbalanced terms together as follows:

$$\begin{cases} i_{pua_s} + i_{qua_s} + i_{pua} + i_{qua} = i_{ua} \\ i_{pub_s} + i_{qub_s} + i_{pub} + i_{qub} = i_{ub} \\ i_{puc_s} + i_{quc_s} + i_{puc} + i_{quc} = i_{uc} \end{cases}.$$

The practical effectiveness of this idea is examined through an active filtering application in Section II-B.

D. Alternative Approach for Current Decomposition in Three-Phase Nonsinusoidal Unbalanced System With Unbalanced and Distorted Voltage Source

Although the proposed approach in Section II-C can address the corresponding challenges in the case of asymmetrical unbalanced voltage source condition, we propose the following EIPT-based decomposition approach to deal with a more challenging situation where one may be faced with a nonsinusoidal and unbalanced voltage source condition. Without loss of generality, the EIPT formulation is first interpreted as a general signal decomposition algorithm in order to decompose the three-phase grid voltage into its detailed components. Next, these components are incorporated in the EIPT formulation to decompose the load currents (see Algorithm 1).

Consider the distorted three-phase grid voltage vector $v_{abc} = [v_a, v_b, v_c]$ at the PCC and the load current vector to be $i_{abc} = [i_a, i_b, i_c]$. Using a symmetrical large resistive load on the grid side, the corresponding grid current behavior is recorded as $i_g = [i_{a\,\mathrm{ref}} i_{b\mathrm{ref}}, i_{c\mathrm{ref}}]$. First, using the sinusoidal nominal grid voltage ($\tilde{V}_{abc}$), we apply the mathematical formulation of the EIPT approach to the i_g and perform this current decomposition over these reference currents, which results in an initial set of (namely) voltage-based current components as follows: $i_{x\bar{p}_{\mathrm{ref}}}, i_{0p_{\mathrm{ref}}}, i_{pHx_{\mathrm{ref}}}, i_{pux_{\mathrm{ref}}}, i_{x\bar{q}_{\mathrm{ref}}}, i_{0q_{\mathrm{ref}}}, i_{qHx_{\mathrm{ref}}}$, and $i_{qux_{\mathrm{ref}}}$.

Since a set of purely resistive loads are used, all the corresponding reference reactive terms (indexed by "q") will be equal to zero. Consider a resistance $R = 1\,\mathrm{k}\Omega$ to be used, multiplying the extracted set of three-phase active current components $i_{x\bar{p}_{\mathrm{ref}}}$, $i_{0p_{\mathrm{ref}}}$, $i_{pHx_{\mathrm{ref}}}$, and $i_{pUx_{\mathrm{ref}}}$ with R, the following set of voltage components are calculated, respectively: $\tilde{v}_{abc} = [\tilde{v}_a,\ \tilde{v}_b, \tilde{v}_c]$, v_{0abc}, $v_{H_{abc}} = [v_{aH}, v_{bH}, v_{cH}]$, and $v_{U_{abc}} = [v_{au}, v_{bu}, v_{cu}]$. Next, we modify the definition of the EIPT-based load current components, as directed in the Algorithm 1. The homopolar current components are still calculated, as directed in Section II-B, whereas the balanced active/reactive components are calculated in step 3. Roughly speaking, in this case, we have two individual sources of imbalance, one related to the load itself and the other one to the voltage source (see steps 4–6); the same concept is valid for the harmonic effect. What we do within Algorithm 1 is that we calculate the contributions of these effects by decomposing the distorted voltage into its subcomponents as well (see step 6), whereas the final (total) unbalanced or harmonic terms in the load current is calculated from the sum of each of these terms, as directed in step 7.

III. Case Studies and Discussions

In order to examine the performance of the proposed EIPT, a set of case studies have been implemented. We start with Case A, where an unbalanced nonlinear load has been fed by a sinusoidal balanced three-phase voltage source to generate desirable distortion patterns in the currents. In the second case study (see Case B), we implemented an unbalanced voltage as a voltage source in the presence of unbalanced and nonlinear load. Within the third case study (Case C), we considered distorted voltage condition, and for the sake of comparison, the decomposition resolution and validity of the decomposed components have been compared versus CPT. The CPT method decomposes the current signals into four components, namely active, reactive, unbalanced, and voided (please refer to [10] for more details). To have a fair comparison, and while EIPT generates finer signal components, the hetropolar and homopolar components are added to generate a total unbalanced term. These three case studies do not include any control unit and purely tries to

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Algorithm 1: Generalized EIPT for Non-Sinusoidal Voltage Source.

Inputs: $i_{abc} = [i_a,\ i_b,\ i_c]$, $v_{abc} = [v_a,\ v_b,\ v_c]$, $\tilde{v}_{abc} = [\tilde{v}_a,\ \tilde{v}_b, \tilde{v}_c]$, $v_{H_{abc}} = [v_{aH}, v_{bH}, v_{cH}]$, and $v_{U_{abc}} = [v_{au}, v_{bu}, v_{cu}]$.

1: The Concordia transform is applied: $i_{abc} \rightarrow i_{\alpha\beta}$, $v_{abc} \rightarrow v_{\alpha\beta}$, $\tilde{v}_{abc} \rightarrow \tilde{v}_{\alpha\beta}$

2: $i_{\alpha\beta}$ & $v_{\alpha\beta}$ $\rightarrow$ *calculating* $\bar{p},\ \bar{q}$. (average part of the calculated powers),
$i_{\alpha\beta}$ & $\tilde{v}_{\alpha\beta}$ $\rightarrow$ calculating $\tilde{p},\ \tilde{q}$, (oscillating part of the calculated powers), and calculate $v^2 = v^2_{a_{\mathrm{rms}}} + v^2_{b_{\mathrm{rms}}} + v^2_{c_{\mathrm{rms}}}$

3. Calculate balanced active/reactive components $i_{x\bar{p}} = \frac{\bar{p}}{v^2}\tilde{v}_x$, $i_{x\bar{q}} = \frac{\bar{q}}{v^2}\tilde{v}_x$ *for* $x \in \{a, b, c\}$

4.1. The homopolar current components are still calculated as directed in Section III-B.

4.2. Calculate oscillating active/reactive components $i_{x\tilde{p}} = \frac{\tilde{p}}{v^2}\tilde{v}_x$, $i_{x\tilde{q}} = \frac{\tilde{q}}{v^2}\tilde{v}_x$ *for* $x \in \{a, b, c\}$

5.1. Calculate load oriented active/reactive heteropolar components by apply a 60 Hz band-pass filter on $i_{x\tilde{p}}, i_{x\tilde{q}}$ and extract i_{pux} & i_{qux} *for* $x \in \{a, b, c\}$.

5.2. Calculate load oriented active/reactive harmonic components such that

$$\begin{cases} i_{x\tilde{q}} - i_{qux} = i_{qHx} \\ i_{x\tilde{p}} - i_{pux} = i_{pHx} \end{cases} for\ x \in \{a, b, c\}$$

6.1. Calculate voltage source oriented active/reactive heteropolar components

$$i_{pux_s} = \frac{\bar{p}}{v^2} v_{xu}\ \&\ i_{qux_s} = \frac{\bar{q}}{v^2} v_{xu}\ for\ x \in \{a, b, c\}$$

6.2. Calculate voltage source oriented active/reactive harmonic components

$$i_{pHx_s} = i_{x\bar{p}} = \frac{\bar{p}}{v^2} v_{xH}\ \&\ i_{qHx_s} = \frac{\bar{q}}{v^2} v_{xH}\ for\ x \in \{a, b, c\}$$

7. Calculate total unbalanced and harmonic components as follows:

$$i_{pux_{\mathrm{total}}} = i_{pux} + i_{pux_s}\ \&\ i_{qux_{\mathrm{total}}} = i_{qux} + i_{qux_s}$$
$$i_{pHx_{\mathrm{total}}} = i_{pHx} + i_{pHx_s}\ \&\ i_{qHx_{\mathrm{total}}} = i_{qHx} + i_{qHx_s}$$
$$\text{for } x \in \{a, b, c\}$$

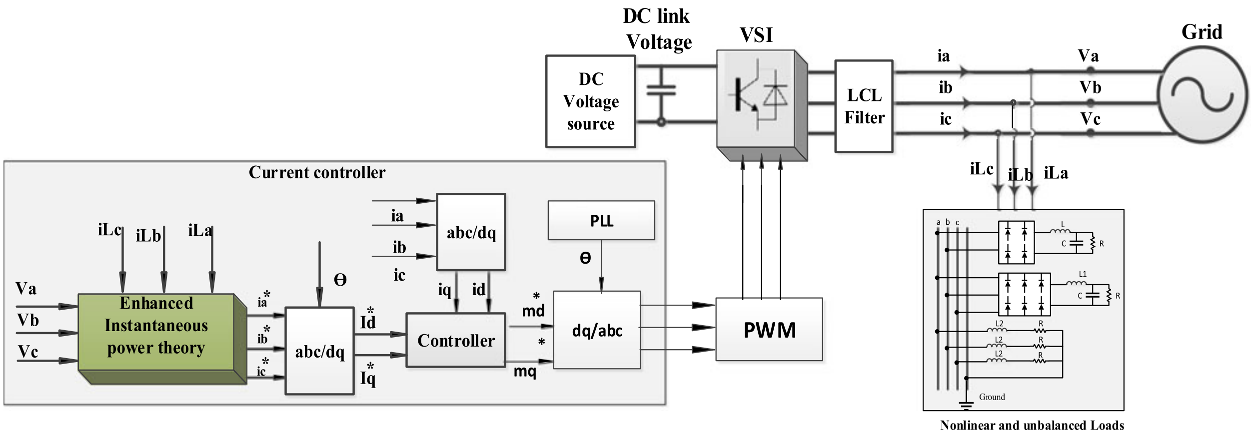

Fig. 3. Schematic of the AFE with the proposed EIPT.

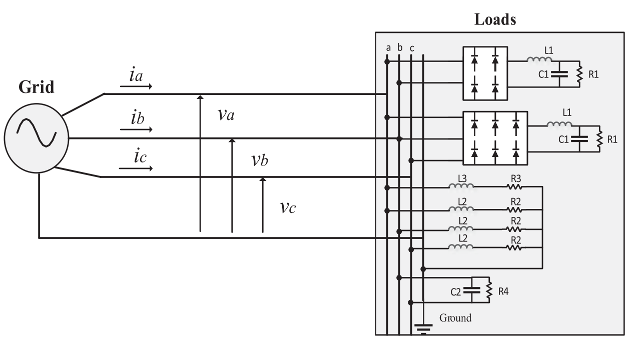

Fig. 4. Three-phase four wire system with different loads.

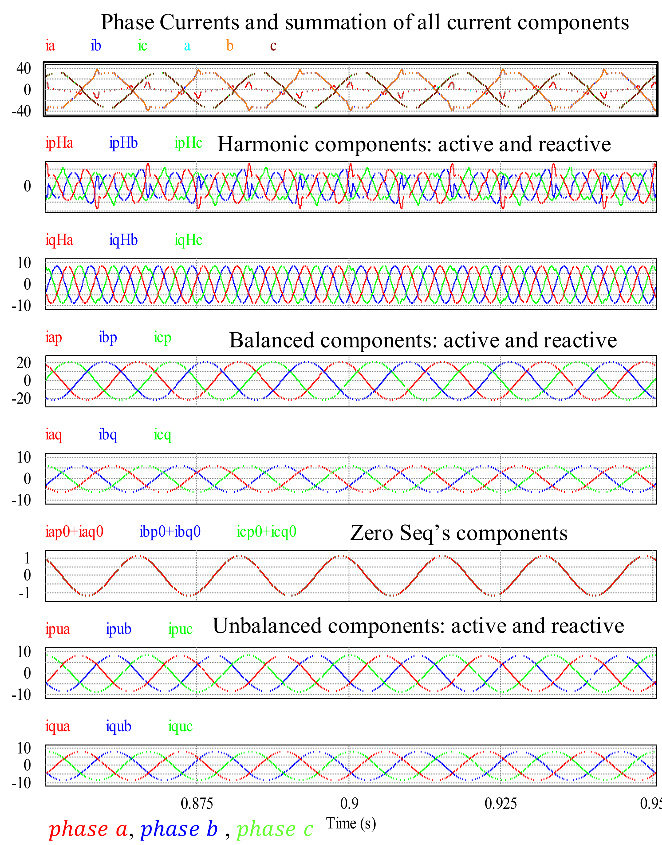

Fig. 5. Decomposed current components in case of unbalanced nonlinear load.

define and extract distorted currents following the mathematical methodology described in Sections II-B, II-C, and II-D, respectively. Finally, Case D explores the examination of the proposed EIPT in terms of an active filtering application (see Fig. 3) using Simulink/MATLAB. This practical control example includes a distorted unbalanced source feeding a near ideal AFE converter. In this case study both EIPT (see Algorithm 1) and CPT were implemented as the signal decomposition module and their performance compared with each other.

A. Case Study 1: Current Decomposition Under Different Load Conditions

This initial case study will illustrate the resolution of the proposed algorithm in addition to justifying the irredundant component-wise selectivity of the proposed signal decomposition framework. Once the distorted current is generated, EIPT formulation is used to decompose currents to different components in each phase. Comprehensive simulations were done and the performance of the proposed theory was examined in different load conditions. Fig. 4 shows the implemented system, which is a three-phase four wire system with variety of loads. Parameters of the system are provided in Appendix D.

To further investigate the performance of the proposed method, a combinational unbalanced and nonlinear load is designed by adding single-phase and three-phase rectifiers. In this case, we expected to simultaneously have the harmonics and unbalanced components in active and reactive currents, and simulation results are verifying this fact (see Fig. 5). As expected, our results in Fig. 5 indicate that, under such a condition, both unbalanced and harmonic components are nonzero. Moreover, different current components are shown in the case of nonlinear unbalanced loads in all of the three phases of the system. Besides, we can see the aggregation of different current components exactly matches the current of each phase.

Reactive powers in different phases and their aggregations are shown in Fig. 6. The aggregation of reactive powers (in all phases) equals zero every time, and this shows that the reactive

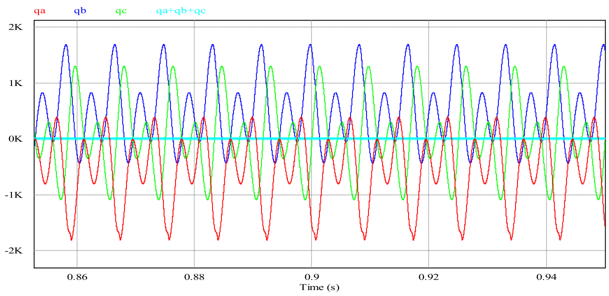

Fig. 6. Reactive powers in different phases and their aggregation in case of unbalanced nonlinear load.

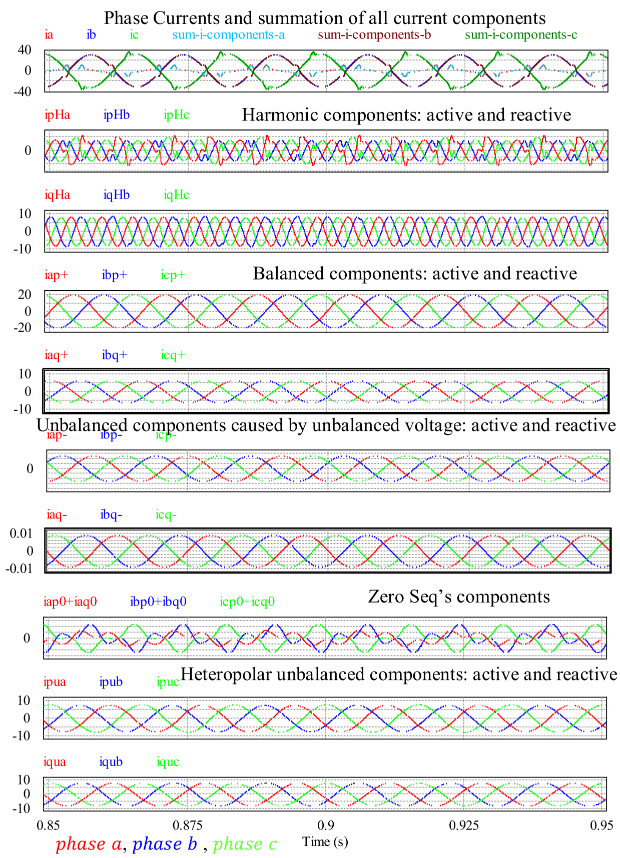

Fig. 7. Decomposed current components in case of unbalanced nonlinear load and unbalanced asymmetrical voltage source.

powers definition in this theory has a physical meaning, and it is equal to the amount of power that is exchanged between the phases and not the one which is transferred from the source to the load.

B. Case Study 2: Current Decomposition Under Asymmetrical Unbalanced Voltage With Unbalanced and Nonlinear Load

In order to present the resolution of the corresponding signal decomposition approach, which has been developed in Section II-C, we implemented an asymmetrical unbalanced voltage as a voltage source in the presence of unbalanced and nonlinear loads and decomposed currents in different phases to their variety of components. Simulation results in Fig. 7 show a good performance of the proposed current decomposition method in this case. Moreover, the aggregation of different current components in each phase of the system exactly matches phasecurrents. Reactive powers in different phases and their aggregations under unbalanced voltage source condition are shown in Fig. 8.

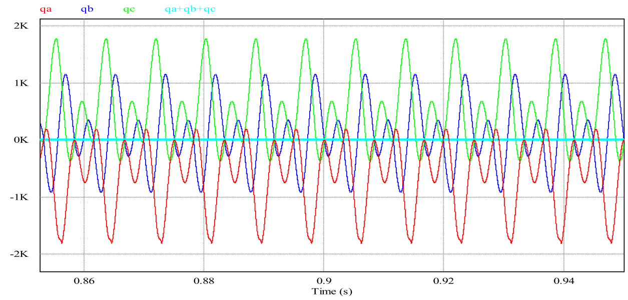

Fig. 8. Reactive powers in different phases and their aggregation in case of unbalanced nonlinear load and unbalanced asymmetrical voltage source.

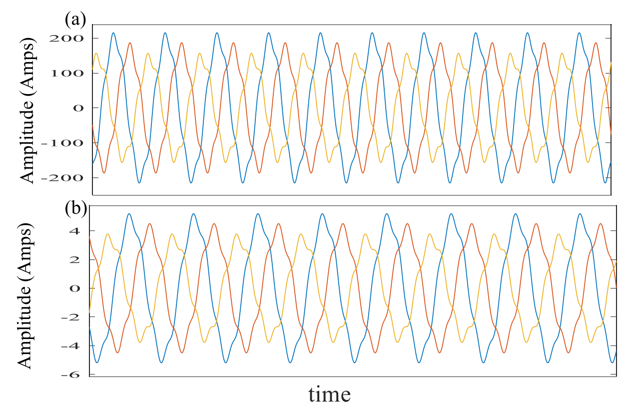

Fig. 9. Voltage waveforms of the source and current waveforms of the load in phases a, b, and c.

C. Case Study 3: Current Decomposition Under Distorted and Unbalanced Voltage Source Conditions and Comparison With CPT Method

Using the idea that was comprehensively discussed in Section II-D, we expanded our methodology for the case of distorted voltage source as well. Here, we show the performance of the proposed algorithm (see Algorithm 1) within the following two examples, as well as its comparison versus the CPT approach.

In the first case study, we considered a set of resistive loads with distorted (harmonic of the order 5 and amplitude of 10% of the sinusoidal voltage) and unbalanced voltage (zero sequence components 20% of positive sequence component). Since the current of resistive load is proportional to the source voltage we expect and observe the distorted-unbalance in the load currents behavior. Fig. 9 is representing the three-phase source voltage and the corresponding load current waveforms in phases a, b, and c. As a result of the voltage source conditions, we would expect to have harmonic parts by the order of 5 and unbalanced zero sequence components in the corresponding decomposed current signals. However, Figs. 10 and 11 are indicating that the

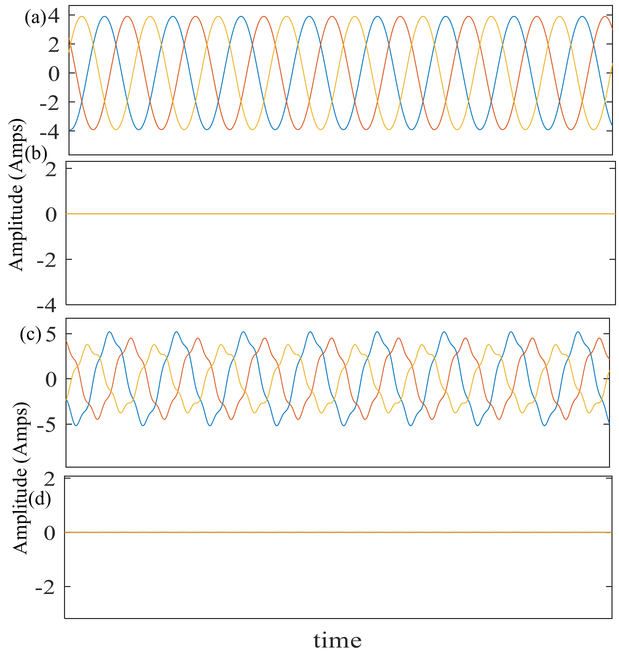

Fig. 10. Temporal behavior of EIPT: a) active and b) reactive current components. CPT: c) active, d) reactive current components.

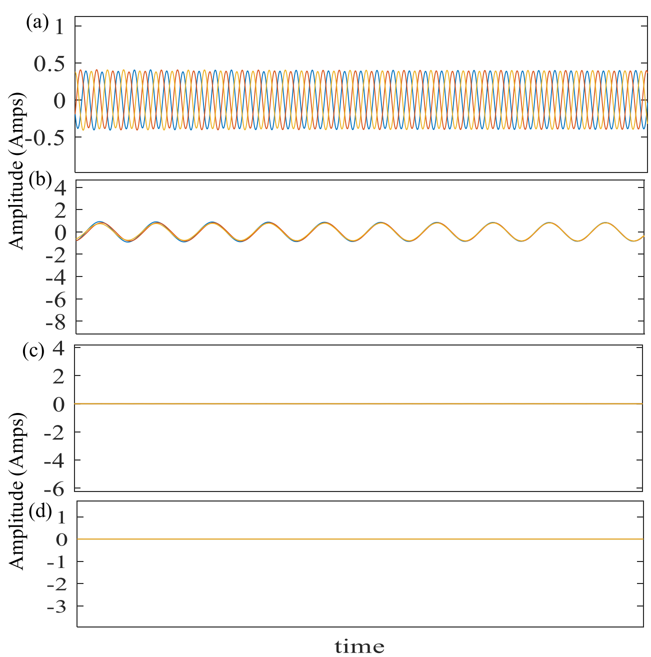

Fig. 11. Temporal behavior of EIPT: a) harmonic, and b) unbalance current components. CPT: c) harmonic, and d) unbalance current components.

CPT approach is not able to decompose the current components properly in this case.

As a matter of fact, the active current component contains some harmonics and unbalanced parts while the voided and unbalanced components are zero. Considering the source-voltage-based mathematical formulation of the CPT, this was an expected phenomenon (also one may refer to the Prof. Tenti's complementary notes, on the CPT limitations in terms

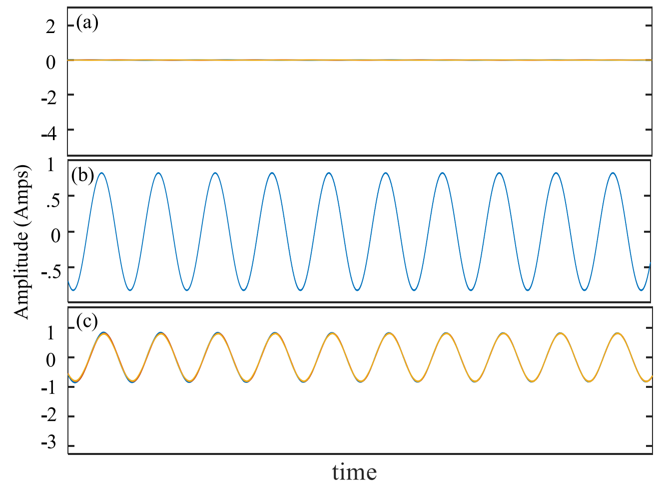

Fig. 12. a) Heteropolar, b) homopolar, and c) the total unbalance current components in the EIPT.

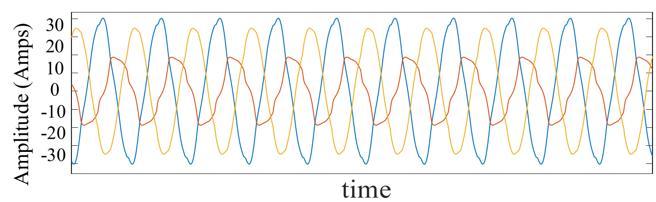

Fig. 13. Current waveforms of the load in phases a, b, and c,.

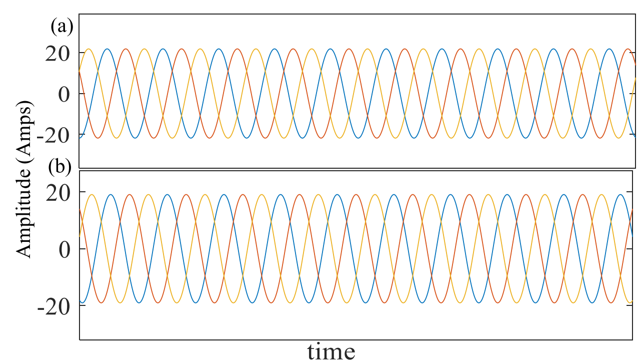

Fig. 14. Temporal behavior of EIPT: a) reactive and b) active current.

of any source of voltage distortions). In contrast, our EIPT-based framework is able to extract harmonics and unbalanced parts with a higher level of accuracy. Fig. 12 shows the total (heteropolar+homopolar) heteropolar, homopolar, and unbalance current components in EIPT, respectively.

In the second case study, we considered nonlinear and unbalanced load (parameters of load are mentioned in Table I) supplied by distorted (including harmonics of the order of 5 and amplitude of 10% of the sinusoidal voltage) and unbalanced voltage source (zero sequence components 20% of positive sequence component). Fig. 13 shows the behavior of the corresponding load current waveforms in phases a, b, and c.

Figs. 14 and 15 illustrate the active and reactive current components calculated by the EIPT and CPT formulations, accordingly. As it is shown, the active and reactive current components

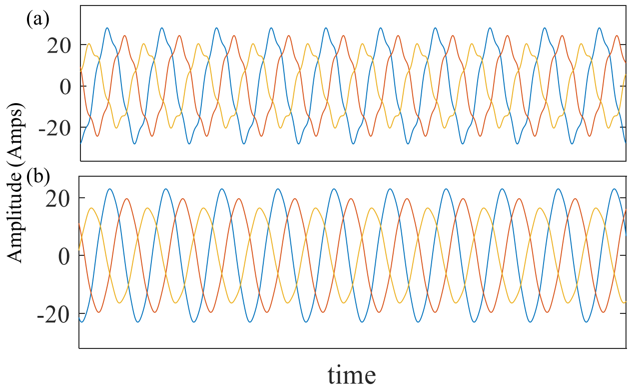

Fig. 15. Temporal behavior CPT: a) active, b) reactive current components.

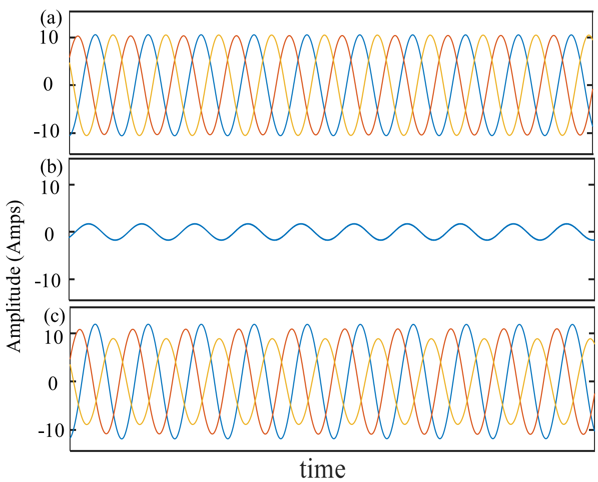

Fig. 16. a) Heteropolar, b) homopolar, and c) total (heteropolar, homopolar) unbalance current components in EIPT.

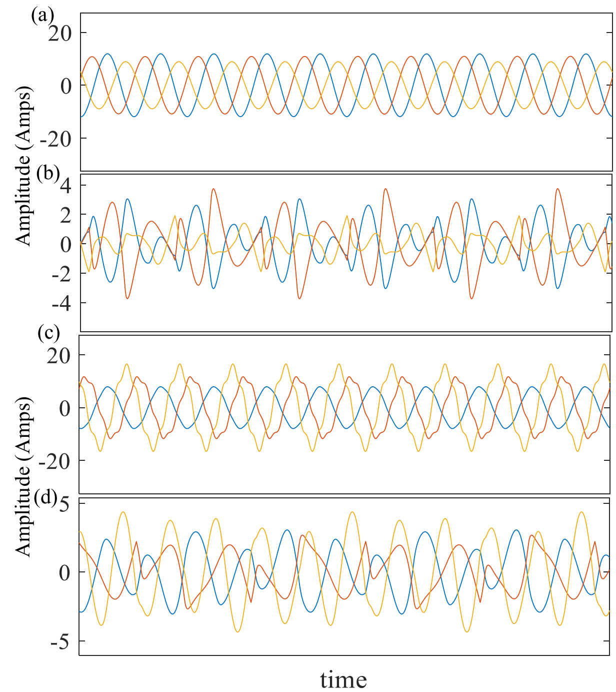

Fig. 17. Temporal behavior of EIPT: a) unbalance, and b) harmonic current components. CPT: c) unbalance, and d) voided current components.

extracted by the CPT are suffering from the distorted and unbalanced voltage parts, whereas EIPT-based approach resulted in pure sinusoidal wave-shapes.

Fig. 16 shows the total (heteropolar+homopolar) unbalanced, homopolar and heteropolar current components for the EIPT, respectively. Finally, Fig. 17 indicates that the unbalanced and distorted components are well separated through the EIPT approach.

D. Application of EIPT-Based Voltage-Current Decomposition in an Active Front End Inverter (A Power Conditioning Example)

In this part, we considered a typical AFE converter and implemented both the EIPT (see Algorithm 1) and CPT approaches as the desired current decomposition module to compensate the harmonic and unbalanced parts of the grid currents. An unbalanced and distorted voltage source is considered with similar behavior to the one mentioned in Fig. 9. To this end, the following active filtering scheme has been designed and modeled in the Simulink/MATLAB software. This active filtering is guarded with the following functionalities:

1) reactive power injection;
2) harmonic compensation;
3) unbalanced compensation.

The main purpose is to achieve close to unit power factor in the grid side in the presence of the nonlinear unbalanced load/source condition.

As we mentioned before, the proposed EIPT-based framework in addition to CPT is implemented as the selected current decomposition methods in the control and compensation module. The PI controller is used in the control unit configuration. It is worth noting that, considering the better performance of the PI controller in the case of dc variables, once the EIPT generates the reference current components, they are converted into the $d-q$ frame subspace using Park transformation as follows:

$$\begin{bmatrix} i_\alpha^* \\ i_\beta^* \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a^* \\ i_b^* \\ i_c^* \end{bmatrix}, \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix}$$
$$= \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \times \begin{bmatrix} i_\alpha^* \\ i_\beta^* \end{bmatrix}.$$

Fig. 18 shows the output current of the system at PCC, injected to the grid, without any compensation.

Fig. 19(a) and (b) is illustrating the compensated currents of the grid with power conditioning system using CPT and EIPT, respectively. To better demonstrate the in-phase characteristic of the current versus voltage signals, Fig. 19(c) represents the grid voltage versus the compensated injected current to the grid at the PCC using EIPT.

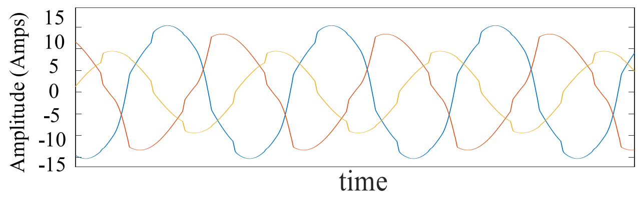

Fig. 18. Grid currents without compensation.

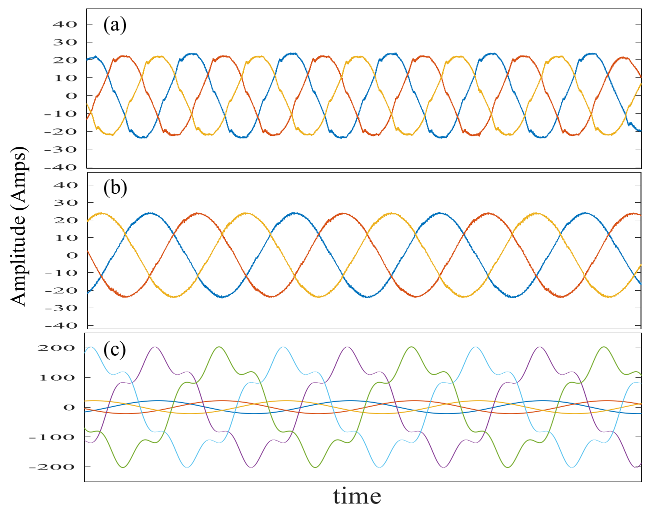

Fig. 19. a) Grid currents after compensation by using CPT, b) grid currents after compensation by using EIPT, c) grid currents versus grid voltages after compensation by using EIPT.

IV. Conclusion

In the last few years, several articles and papers have discussed or presented solutions and methodologies for power quality and weak grid conditions; however, still a more solid approach must be taken for nonsinusoidal and unbalanced waveforms conditions in electrical circuits. It is important to understand how to cope with the ever-increasing use of power electronic equipment with the effects of nonlinearity. This paper proposes an advanced mathematical formulation for instantaneous power signal decomposition (also widely known as power theories) under unbalanced and nonlinear three-phase power systems. The proposed framework is valid for both distorted and unbalanced voltage source conditions. In addition to develop a mathematical analysis, the authors also provided a comprehensive set of case studies over a variety of operational conditions with different load/source distortions. Our advanced instantaneous current decomposition method is proposed for smart grid applications and considers unbalanced and nonlinear three-phase power systems that can decompose the electrical signals into meaningful components. Our terminology is "EIPT." The EIPT is able to decompose the current signal instantaneously into several components (balanced sinusoidal active, balanced sinusoidal reactive, harmonic active, harmonic reactive, homopolar and heteropolar unbalanced active and reactive) for each phase of the system. Additionally, EIPT is able to appropriately decompose current components in the case of asymmetrical, unbalanced, and distorted voltage sources. The effectiveness of the proposed methodology has been examined using extensive case studies developed with Simulink software. Results are indicating good performance of the proposed theory within different load and source conditions. We hope that this new detailed decomposition approach would be helpful in optimizing the control strategies for power electronic interfaces and power quality compensators in smart grids with higher flexibility. Moreover, it might be used independently in any relevant signal processing applications.

Appendix

A. *Proof of Property 2*

1)

$$i_q.v_{\alpha\beta 0} = \frac{q \times v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}}.v_{\alpha\beta 0} = \frac{(v_{\alpha\beta 0} \times i_{\alpha\beta 0}) \times v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}}.v_{\alpha\beta 0}$$
$$= \frac{\left[-\left(i_{\alpha\beta 0}.v_{\alpha\beta 0}\right) v_{\alpha\beta 0} + \left(v_{\alpha\beta 0}.v_{\alpha\beta 0}\right) i_{\alpha\beta 0}\right]}{v_{\alpha\beta 0}.v_{\alpha\beta 0}}.v_{\alpha\beta 0} = 0. \tag{A.1}$$

2)

$$i_p \times v_{\alpha\beta 0} = \frac{p.v_{\alpha\beta 0}}{v_{\alpha\beta 0}.v_{\alpha\beta 0}} \times v_{\alpha\beta 0} = 0. \tag{A.2}$$

B. *Proof of Property 3*

Proof: The cross product is defined as

$$I \times V = \begin{vmatrix} i & j & k \\ i_1 & i_2 & i_3 \\ v_1 & v_2 & v_3 \end{vmatrix} =$$
$$\langle i_2 v_3 - i_3 v_2, i_3 v_1 - i_1 v_3, i_1 v_2 - i_2 v_1 \rangle$$

thus, the magnitude squared of the cross product will be calculated as

$$|I \times V|^2 = (I \times V).(I \times V)$$
$$= i_2^2 v_3^2 - 2 i_2 i_3 v_2 v_3 + i_3^2 v_2^2 + i_3^2 v_1^2 - 2 i_1 i_3 v_1 v_3$$
$$+ i_1^2 v_2^2 + i_1^2 v_2^2 - 2 i_1 i_2 v_1 v_2 + i_2^2 v_1^2. \tag{B.1}$$

On the other hand

$$(|I|\,|V|)^2 = \sqrt{i_1^2 + i_2^2 + i_3^2}\sqrt{v_1^2 + v_2^2 + v_3^2}$$
$$= i_1^2 v_1^2 + i_2^2 v_2^2 + i_3^2 v_3^2 + i_2^2 v_1^2 + i_2^2 v_2^2 + i_2^2 v_3^2 + i_3^2 v_1^2$$
$$+ i_3^2 v_2^2 + i_3^2 v_3^2. \tag{B.2}$$

Moreover,

$$(I.V)^2 = i_1^2 v_1^2 + i_2^2 v_2^2 + i_3^2 v_3^2 + 2 i_1 i_2 v_1 v_2 + 2 i_1 i_3 v_1 v_3$$
$$+ 2 i_2 i_3 v_2 v_3. \tag{B.3}$$

Have a close look at these star-signed equations, we can easily observe that (B.1) = (B.2) – (B.3). In mathematic literature, this is widely known as the Lagrange's Identity. As a result, one may alternatively write

$$|I \times V|^2 = (|I|\,|V|)^2 - (I.V)^2. \qquad (\dagger)$$

Using (†) in (27), we have

$$S^2 = P^2 + Q^2 = |I \times V|^2 + (I.V)^2$$
$$= (|I|\,|V|)^2 - (I.V)^2 + (I.V)^2$$
$$= (|I|\,|V|)^2 = |I|^2|V|^2 \xrightarrow{\text{yields}} S = |I|\,|V|$$

■

C. Schematic of Axis Formation in Concordia Transformation

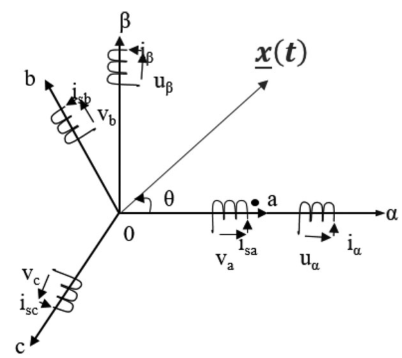

Fig. C.1. Axes for transformation of a three-phase into a two-phase system.

D. Parameters-Related Case Study A (see Fig. 4)

TABLE D.I
GRID AND LOAD PARAMETERS

Grid Voltage	$V_{a\_\mathrm{rms}} = 120$	L_1	300 uH
Grid Frequency(*f*)	60 Hz	L_2	100 uH
R_1	500 Ω	L_3	1 mH
R_2	100 Ω	C_1	470 μF
R_3	50 Ω	C_2	100 μF
R_4	50 Ω		

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REFERENCES

[1] D. Kumar and F. Zare, “Harmonic analysis of grid connected power electronic systems in low voltage distribution networks,” *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 4, no. 1, pp. 70–79, Mar. 2016.
[2] J. Bonaldo, H. Morales Paredes, and J. Antenor Pomilio, “Control of single-phase power converters connected to low voltage distorted power systems with variable compensation objectives,” *IEEE Trans. Power Electron.*, vol. 31, no. 3, pp. 2039–2052, Mar. 2016.
[3] G. Budeanu, “The different opinions and conceptions regarding active power in non-sinusoidal systems,” Rumanian Nat. Inst., Bucharest, Romania, publ. no. 2, 1927.
[4] H. Akagi, Y. Kanazawa, and A. Nabae, “Instantaneous reactive power compensators comprising switching devices without energy storage components,” *IEEE Trans. Ind. Appl.*, vol. IA-20, no. 3, pp. 625–630, May 1984.
[5] H. Akagi, E. H. Watanabe, and M. Aredes, *Instantaneous Power Theory and Applications to Power Conditioning*. Hoboken, NJ, USA: Wiley, 2007.
[6] F. Z. Peng and J.-S. Lai, “Generalized instantaneous reactive power theory for three-phase power systems,” *IEEE Trans. Instrum. Meas.*, vol. 45, no. 1, pp. 293–297, Feb. 1996.
[7] L. S. Czarnecki, “Currents’ physical components (CPC) concept: A fundamental of power theory,” in *Proc. Int. School Nonsinusoidal Currents Compensation*, Jun. 2008, pp. 1–11.
[8] M. Depenbrock, “The FBD method, a generally applicable tool for analyzing power relations,” *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 381–387, May 1993.
[9] J. L. Willems, “A new interpretation of the Akagi-Nabae power components of nonsinusoidal three-phase situations,” *IEEE Trans. Instrum. Meas.*, vol. 41, no. 4, pp. 523–527, Aug. 1992.
[10] P. Tenti, H. K. M. Paredes, and P. Mattavelli, “Conservative power theory, a framework to approach control and accountability issues in smart microgrids,” *IEEE Trans. Power Electron.*, vol. 26, no. 3, pp. 664–673, Mar. 2011.
[11] N. L. Kusters and W. J. M. Moore, “On the definition of reactive power under non-sinusoidal conditions,” *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 5, pp. 1845–1854, Sep. 1980.
[12] S. Fryze, “Active, reactive, and apparent power in nonsinusoidal systems,” *Prz. Elektrot.*, no. 7, pp. 193–203, 1931, (in Polish).
[13] B. Singh, K. Al-Haddad, and A. Chandra, “A review of active filters for power quality improvement,” *IEEE Trans. Ind. Electron.*, vol. 46, no. 6, pp. 960–971, Dec. 1999.
[14] F. Harirchi, M. G. Simoes, M. Babakmehr, A. Al-Durra, and S. M. Muyeen, “Multi-functional double mode inverter for power quality enhancement in smart-grid applications,” in *Proc. 51th IEEE Ind. Appl. Soc. Gen. Meeting*, 2016, pp. 1–8.
[15] M. Ortúzar, R. Carmi, J. Dixon, and L. Morán, “Voltage-source active power filter, based on multi-level converter and ultracapacitor DC link,” *IEEE Trans. Ind. Electron.*, vol. 53, no. 2, pp. 614–623, Apr. 2006.
[16] M. G. Simões, T. D. C. Busarello, A. S. Bubshait, F. Harirchi, J. A. Pomilio, and F. Blaabjerg, “Interactive smart battery storage for a PV and wind hybrid energy management control based on conservative power theory,” *Int. J. Control*, vol. 89, pp. 850–870, 2016.
[17] E. H. Watanabe, M. Aredes, and H. Akagi, “The P-Q theory for active filter control: Some problems and solutions,” *SBA Controle Automação*, vol. 15, no. 1, pp. 78–84, 2004.
[18] H. Kim, F. Blaabjerg, B. Bak-Jensen, and J. Choi, “Instantaneous power compensation in three-phase systems by using p-q-r theory,” *IEEE Trans. Power Electron.*, vol. 17, no. 5, pp. 701–710, Sep. 2002.
[19] E. H. Watanabe, R. M. Stephan, and M. Aredes, “New concept of instantaneous active and reactive powers in electrical systems with generic loads,” *IEEE Trans. Power Del.*, vol. 8, no. 2, pp. 697–703, Apr. 1993.
[20] H. Kim, F. Blaabjerg, and B. Bak-Jensen, “Spectral analysis of instantaneous powers in single-phase and three-phase systems with use of p-q-r theory,” *IEEE Trans. Power Electron.*, vol. 17, no. 5, pp. 711–720, Sep. 2002.
[21] H. K. M. Paredes, F. P. Marafão, P. Mattavelli, and P. Tenti, “Application of conservative power theory to load and line characterization and revenue metering,” in *Proc. IEEE Int. Workshop Appl. Meas. Power Syst.*, Sep. 2012, pp. 1–6.
[22] N. L. Kuster and W. J. Moore, “On the definition of reactive power under non-sinusoidal condition,” *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 5, pp. 1845–1854, Sep. 1980.
[23] L. S. Czarnecki “Critical comments on the conservative power theory (CPT),” in *Proc. Int. Sch. Nonsinusoidal Currents Compensation*, 2015, pp. 1–7.
[24] N. Dharma Rao and H. N. Ramachandra Rao, “Study of symmetrical and related components through the theory of linear vector space,” *Proc. Inst. Electr. Eng.*, vol. 113, no. 6, pp. 1057–1062, 1966.
[25] P. Tenti *et al.*, “Generalized symmetrical components for periodic non-sinusoidal three-phase signals,” *Electr. Power Qual. Utilisation J.*, vol. 13, no. 1, pp. 9–15, 2007.
[26] M. Depenbrock, V. Staudt, and H. Wrede, “Concerning instantaneous power compensation in three-phase systems by using p–q–r theory,” *IEEE Trans. Power Electron.* vol. 19, no. 4, pp. 1151–1152, Jul. 2004.

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