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Corporate finance, monetary policy, and aggregate demand[☆]

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ABSTRACT

I study how heterogeneity of financial frictions and monopolistic competition influence the pass through of the nominal interest rate to the real lending rate, its transmission into investment, and corporate cash holdings. Firms finance stochastic investment opportunities with either bank-issued credit or money. The market structure generates an aggregate demand externality which doubles transmission at the a policy rate of 4.8% and magnifies the effects of financial frictions on investment. In line with empirical evidence, the cash-to-sales ratio increases with the extent of financial constraints, and rises with the intensity of competition for financially constrained firms. Financial constraints raise firms' sensitivity to monetary policy; and a mean-preserving spread of financial frictions reduces investment and output, strengthens transmission, and reduces the external share of finance.

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1. Introduction

There is strong empirical evidence that the effects of monetary policy differ markedly by industries and are influenced by financial constraints of firms (i.e., Dedola and Lippi, 2005; Ehrmann et al., 2003; Gertler and Gilchrist, 1994). One reason firms differ in their ability to obtain credit is because of underlying heterogeneity in the pledgeability of assets. Such heterogeneity reflects physical properties of assets (portability, tangibility), legal institutions (i.e., bankruptcy law), and also frictions in the market for second-hand sales. Moreover, in a monopolistically competitive environment, an aggregate demand externality arises which causes bottlenecks on constrained firms to reduce investment demand generally. Additionally, firm market power affects the incentive to hold cash. Intuitively, as competition rises, firms wish to sell more output. If financially constrained, however, they end up depending disproportionately on internal finance. Here, I examine how heterogeneous financial frictions and monopolistic competition propagate monetary policy and influence firms' cash holdings.

The setup elaborates on Rocheteau et al. (2017). Entrepreneurs can finance random investment opportunities using retained earnings or external bank credit. However, firms cannot always obtain a loan, and, even if successful, can only borrow

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a limited amount due to limited enforcement, as in Holmström and Tirole (1998) or Kiyotaki and Moore (1997).[1] Banks specialize in monitoring and enforcement and provide liquid short-term liabilities to firms in exchange for illiquid loans. They lend in an over-the-counter market and negotiate with firms over size, interest rate, and down payment. The lending rate is an intermediation premium, which arises from frictions characteristic of over-the-counter financial markets, as discussed by Lagos and Rocheteau (2009). Firms retain earnings both to insure themselves in case they cannot secure a bank loan and to strategically reduce the real lending rate when negotiating with the bank. The precautionary motive for holding cash is consistent with the survey on corporate liquidity management by Sánchez and Yurdagul (2013). A lower policy rate induces firms to hold more cash and enables them to negotiate a lower lending rate.[2]

Credit in the model features both an intensive margin (loan size) and extensive margin (acceptability of loan applications), which accords well with actual corporate credit markets. The presence of both margins is consistent with the evidence in the Joint Small Business Credit Survey (2014). Of survey participants applying for loans, 33% received what they requested, 21% receive less, and 44% were denied. The intensive margin is important in gauging the distance between the financially constrained investment level of a firm and its target, and the extensive margin captures the insurance motive of holding money.

Researchers have tried to quantify the pledgeability of assets in several ways. Berger et al. (1996) sample proceeds from discontinued operations over a sample of Compustat firms over 1984–1993 and impute liquidation values of 72 cents on the dollar in exit value for total receivables, 55 cents for inventory, and 54 cents for fixed assets. Almeida and Campello (2007) apply these weights on manufacturing Compustat firms and find a mean expected liquidation value of 0.526 and standard deviation of 0.169. Williamson (1988) argues that the liquidation value of assets is closely tied to their redeployability; Kessides (1990) and Worthington (1995) measure asset redeployability using the ratio of used to total fixed depreciable capital expenditures in an industry. The latter estimates an elasticity of investment with respect to cash flow of 0.542 on durable manufacturers compared to 0.359 non-durable manufacturers between 1964 and 1989. This finding suggests that assets are less pledgeable in durable industries than non-durables and corroborates the empirical studies on heterogeneous monetary policy transmission. Finally, there is also variation in legal enforcement. Though United States bankruptcy law is a federal statute, business may use state insolvency laws, which differ significantly in the protection they offer creditors. Morrison (2009) documents that less than 20% of failing small businesses filed for bankruptcies in 2003. The rest used state insolvency laws, which include friendly foreclosures, bulk sales, and assignments for the benefit of creditors.

Heterogeneous financial frictions are particularly important in the presence of demand linkages from monopolistic competition, in which firms' desired investment depends on overall output. This channel propagates financial constraints and the effects of monetary policy. Even unconstrained firms face a lower demand as a result. Blanchard and Kiyotaki (1987) and Startz (1989) stress the aggregate demand externality of monopolistic competition, which several recent papers examine in other contexts. Schaal and Taschereau-Dumouchel (2016) generate a coordination problem of investment with demand complementarities between imperfect substitutible intermediate goods and variable capacity utilization. Using the global game approach to select among the multiple equilibria, they find that a sufficiently large shock may prevent firms from coordinating on high output. Jones (2011) studies how linkages between sectors through intermediate goods and complementarity of goods amplifies initial income differences and can more plausibly explains large income differences across countries. This demand externality creates a multiplier formally similar to that of capital accumulation in growth theory. Hsieh and Klenow (2009) show that microeconomic distortions to the allocation of capital and labor can lead to differences in aggregate productivity, which are magnified by the capital accumulation multiplier. Here, I focus on how demand linkages between firms magnify the effects of financial frictions.

Incorporating heterogeneity of financial frictions across entrepreneurs serves three purposes beyond its interplay with demand complementarities. First, it enables us derive cross-sectional implications and link them to empirical work. One such implication is the effect of firm type on pass through and transmission discussed in the opening. The theory here supports size as a proxy for capital market access and implies asymmetric transmission across firm size. A second implication is that heterogeneity in access to capital markets generates differentials in cash holdings. Firms with tighter financing constraints have a higher cash/sales ratio, consistent with empirical studies.

Second, heterogeneity incorporates distributional effects of investment and output. Mean-preserving spreads of financial frictions hurt investment and output because of both the concavity of firms' revenue and the fact that, for unconstrained firms, further increases in pledgeability have no effect. The aggregate demand externality strengthens this effect by reducing the demand for unconstrained firms as well. Moreover, mean-preserving spreads increase monetary policy transmission. Third, heterogeneity eliminates kinks in the effects of policy from transitioning to a region in which financial frictions either bind on all firms or on none.

The financial economics literature has examined the relevance of frictional capital markets and product market competition for firms' cash holdings (normalized by assets or sales). Opler et al. (1999) and Almeida et al. (2004) find that more financially constrained firms hold more cash. Morellec et al. (2014) develop a model in which ex-ante identical firms com-

[1] Bernanke et al. (1999) and Holmström and Tirole (1998) endogenize financial frictions in terms of a principal agent problem with costly state verification, and Rocheteau et al. (2017) provides an alternative rationalization in terms of limited commitment. Other New Monetarist work emphasizing pledgeability includes Lagos (2010), Venkateswaran and Wright (2014), and Williamson (2012).

[2] Fig. 1 in Rocheteau et al. (2017) illustrates the close relationship between the real lending rate and the 3-month Treasury bill rate and the nominal Fed Funds rate. Fig. 2 shows the negative comovement between cash/sales and the 3-month T-bill rate and banks' net interest margin.

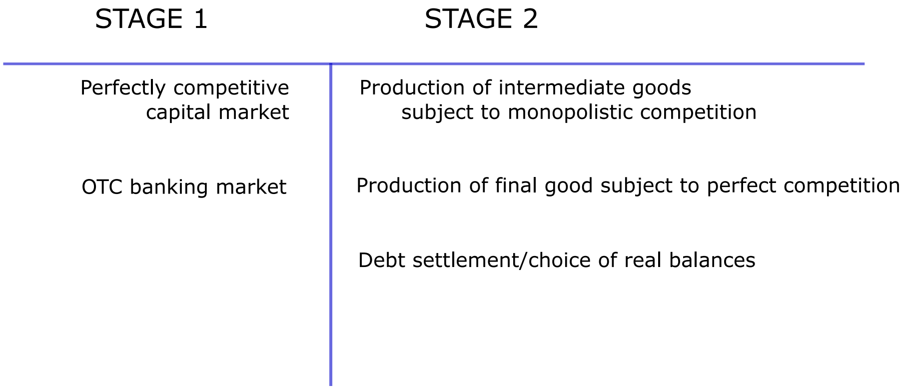

Fig. 1. Timing.

pete à la Cournot and face shocks to their cash flow. They can raise cash to prevent bankruptcy by issuing new equity, which has both a fixed cost and a proportional cost. The main financial friction thus involves underwriting fees. Indeed, Altınkılıç and Hansen (2000) find that 85% of underwriter fees are variable costs, with a rising marginal cost. Morellec et al. (2014) show that, for financially constrained firms, cash holdings rise increase with competition. Moreover, smaller (and more constrained firms) hold more cash. They verify these results on Compustat firms between 1980 and 2007. I find that both results hold in this model. However, whereas Morellec et al. (2014) analyze ex-post heterogeneity of firms, I focus on ex-ante heterogeneous enforcement constraints between creditors and lenders and examine how they interact with monetary policy and monopolistic competition. This framework also stresses the role of over-the-counter market between entrepreneurs and banks; greater bank bargaining power raises pass through to the real lending rate but reduces transmission.

Ottonello and Winberry (2018) also study how the investment channel of monetary policy depends on firm heterogeneity. However, they concentrate on the role of monetary policy shocks (innovations to a Taylor rule) in a business cycle setting. I instead focus on long-run (anticipated) effects of monetary policy. Additionally, they consider heterogeneity in idiosyncratic shocks to firm productivity and the valuation of capital, whereas I study variability in the pledgeability of assets. Finally, the motive for holding cash in this setting depends on the uncertainty of investment opportunities and bank finance.

To link market power and cash holdings, this paper constructs markups from Compustat data according to the production method developed by Hall (1988) and modified subsequently to control for unobserved firm-specific productivity shocks, most recently by Ackerberg et al. (2006) and De Loecker and Eeckhout (2017). This approach only requires data on revenue, total expenditure, the capital stock, and output, where the latter can be obtained from revenue via a price deflator. I construct the capital stock via the perpetual inventory method.

Sections 2 and 3 present the environment and equilibrium. The general model features internal and external finance and heterogeneity of entrepreneurs in terms of financial constraints.[3] Section 4 discusses the empirical analysis and calibration strategy; the corresponding parameter values are used for the theoretical results that follow. This section constructs markups by applying the production approach to Compustat data and provides evidence from panel data regressions that firms with greater market power accumulate more cash, and that cash holdings fall with asset pledgeability. Section 5 presents the main quantitative results of the model. The appendix analyzes special versions of the model which can be fully characterized analytically: pure external finance and perfect enforcement. Section 6 concludes; and the appendix includes proofs, additional figures, and details on estimating markups.

2. Environment and preliminaries

Time is denoted by $t \in \mathbb{N}_0$. Each period is divided into two stages. In the first stage, there is a Walrasian market for capital goods and an over-the-counter market for banking services (loan provisions and means of payment) with search and bargaining. In the second stage, there is a frictionless centralized market where agents settle debts and trade final goods and assets. The capital good k is storable across stages but not across periods. A numeraire consumption good c is produced and traded in the CM. Good c is not storable. Fig. 1 illustrates the timing.

There are four types of agents labeled by $\{e, s, b, f\}$. These consist of entrepreneurs (e), who require capital for investment projects; suppliers (s), who produce the capital; banks (b), who finance the acquisition of capital by entrepreneurs as explained below, and final goods producers (f). The measure of entrepreneurs is normalized to one. All agents have linear preferences $U(c, h) = c - h$, where c is the consumption of the numeraire and h is hours of work. Whereas suppliers, banks, and final goods producers are homogeneous, entrepreneurs are indexed by the pledgeability coefficient $\chi_j \in [0, 1]$.

[3] In general, output depends on the distribution of internal finance and investment, but internal finance and investment are choices that depend on output. Therefore, I solve for a fixed point of output, similar to the way (Aiyagari, 1994) does for capital.

Entrepreneurs can produce intermediate goods by converting the supplier's primary good one-for-one into capital: $y = k$. Entrepreneurs acquire k in Stage 1 and bring it to Stage 2 for production. Second, they can also produce c using their own labor according to a linear technology, $c = h$. Suppliers produce k in Stage 1 with a linear technology, $k = h$. Banks cannot produce c or k.

Fig. 2 details the production chain in which entrepreneurs obtain primary input from suppliers to produce intermediate goods, which are then purchased by perfectly competitive final good firms.

Entrepreneurs face two types of idiosyncratic uncertainty: with probability λ, they receive an investment opportunity, as in Kiyotaki and Moore (1997); with probability α, they can access financing from banks, as in Wasmer and Weil (2004). Thus, the probability that an entrepreneur has an investment opportunity and is matched with a bank is $\alpha\lambda$. With probability $\lambda(1-\alpha)$, an entrepreneur has an investment opportunity but no access to a bank.

Next we discuss the enforcement technology in Stage 2 on debts incurred by entrepreneurs in the DM. Consider an entrepreneur j with k units of capital goods and liability $l > 0$ toward banks. Following production, the entrepreneur can default on his debt and walk away with part of the returns of his capital. I assume no trade credit is possible, and that banks can enforce payment up to χ_j of the value of the investment project. As we discussed, The parameter χ_j reflects both portability and tangibility of capital and legal enforcement.

Limited enforcement can generate a demand for liquid assets. Here, we focus on outside fiat money. Fiat money is storable and evolves according to $A_{m,t+1} = (1+\pi)A_{m,t}$. The price of money in terms of the numeraire is $q_{m,t}$. In a stationary equilibrium, $q_{m,t} = (1+\pi)q_{m,t+1}$, so π is the inflation rate. We assume $\pi > \beta - 1$.

2.1. Problem of the entrepreneur

Consider an entrepreneur at the beginning of Stage 2 with k units of capital goods from Stage 1 and financial wealth ω denominated in the numeraire. Financial wealth is composed of real balances a_m net of obligations. The entrepreneur's lifetime expected utility solves

$$W^e(k,\omega) = \max_{c,h,\tilde{a}_m}\{c - h + \beta V^e(\tilde{a}_m)\} \tag{1}$$

$$\text{s.t.} \quad c = f(k,Y) + h + \omega + T - (1+\pi)\tilde{a}_m \tag{2}$$

in which $f(k, Y)$ is the revenue from k units of capital and aggregate demand Y, which we derive in Section 2.2. Substituting $c - h$ into W^e yields

$$W^e(k,\omega) = f(k,Y) + \omega + T + \max_{\tilde{a}_m}\{-(1+\pi)\tilde{a}_m + \beta V^e(\tilde{a}_m)\} \tag{3}$$

W^e is linear in total wealth, and the choice of real balances $\tilde{a}_m$ is independent of (k, ω).

Similarly, the CM lifetime expected utility of a supplier or bank, $j \in \{b, s\}$ with wealth ω is

$$W^j(\omega) = \omega + \max_{\tilde{a}_m}\{-(1+\pi)\tilde{a}_m + \beta V^j(\tilde{a}_m)\} \tag{4}$$

We next consider the problem of a supplier at the beginning of Stage 1:

$$V^s(\tilde{a}_m) = \max_{k\geq 0}\{-k + W^s(\tilde{a}_m + q_k k)\} \tag{5}$$

A supplier produces k at a linear cost in exchange for a payment $q_k k$. If the market for capital goods is active, $q_k = 1$ and $V^s(\tilde{a}_m) = W^s(\tilde{a}_m)$.

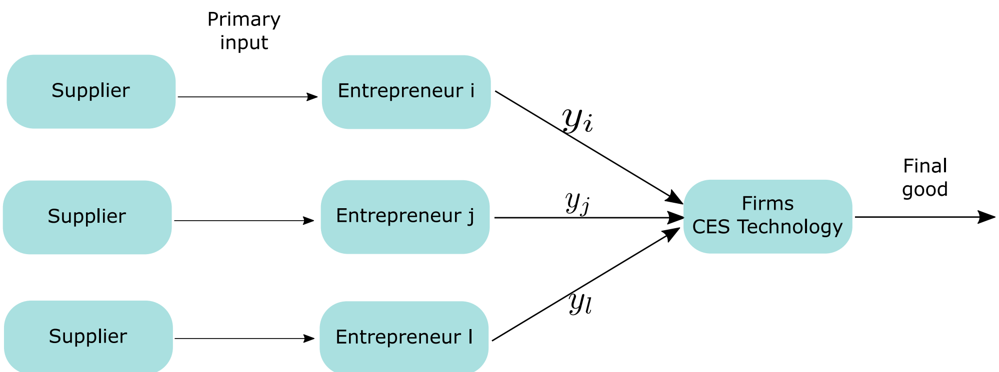

Fig. 2. Production chain.

The portfolio of suppliers solves

$$\max_{\tilde{a}_m}\{-(1+\pi)\tilde{a}_m + \beta\tilde{a}_m\} \tag{6}$$

Provided $\pi > \beta - 1, \tilde{a}_m = 0$. Suppliers hold no real balances as they have no liquidity needs.

An entrepreneurs' lifetime expected utility at the beginning of the DM is

$$V^e(\tilde{a}_m) = \mathbb{E}W^e(k, \tilde{a}_m - \psi - \phi) \tag{7}$$

The entrepreneur purchases k at total cost $\psi = q_k k$ and compensates the bank for its intermediation services with payment ϕ. The total payment $\psi + \phi$ is subtracted from the entrepreneur's financial wealth in Stage 2. If the entrepreneur does not receive an investment opportunity, then $k = 0$. If the entrepreneur is not matched with a bank, then $\phi = 0$.

The choice of real balances solves

$$\max_{\tilde{a}_m}\{-(1+\pi)\tilde{a}_m + \beta\mathbb{E}[f(k,Y) + \tilde{a}_m - \psi - \phi + T]\} \tag{8}$$

Multiplying by $(1+\rho)$, and using the Fisher equation $1 + i = (1+\rho)(1+\pi)$, the portfolio problem reduces to

$$\max_{\tilde{a}_m \geq 0}\{-i\tilde{a}_m + \mathbb{E}[f(k,Y) - k - \phi]\} \tag{9}$$

where (k, ϕ) is a function of $\tilde{a}_m$. The nominal rate i is the opportunity cost of holding money.

2.2. Probem of the final goods firm and revenue function of the entrepreneur

We solve the profit maximization of the final goods firm and use it to derive the revenue function $f(k, Y)$ of entrepreneurs, which determines the return on the investment project.

The quantity index is

$$Y = \left[\int_0^1 y_i^\sigma di\right]^{\frac{\gamma}{\sigma}} \quad \sigma < \gamma, \quad \gamma < 1 \tag{10}$$

where $i \in [0, 1]$ denotes the identity of an individual entrepreneur, y_i denotes the input produced by the entrepreneur in a match with a supplier. The parameter σ measures the substitutability of the various goods and $\gamma \in [\sigma, 1]$ is the returns to scale parameter: preferences range from additively-separable CES ($\gamma = \sigma$) to Dixit–Stiglitz CES ($\gamma = 1$). The more general case allows us to quantify the role of the returns to scale γ and calibrate it to match estimates of the semi-elasticity of aggregate demand with respect to the nominal rate.

The final goods firm takes the output price P and input prices P_i of the intermediate goods firms as given to maximize profits:

$$\max_{y_i \geq 0}\left(PY - \int_0^1 P_i y_i d_i\right), \tag{11}$$

The first order condition yields the individual demand curve

$$y_i = \left(\frac{P\gamma}{P_i}\right)^{\frac{1}{1-\sigma}} Y^{\frac{\gamma-\sigma}{\gamma(1-\sigma)}} \tag{12}$$

I normalize the final output price $P = 1$.[4] Dividing the individual demands of two entrepreneurs yields $y_i/y_j = (P_j/P_i)^{1/(1-\sigma)}$, so that the elasticity of substitution of variety i with respect to j is $1/(1-\sigma)$. Provided that $\sigma < \gamma$, the demand of input i, y_i, depends positively on the output of other entrepreneurs, Y. This positive dependence of y_i on Y is an aggregate demand externality, which amplifies credit frictions.

Rearranging (12) provides the inverse demand curve $P(k, Y)$. An entrepreneur can transform k units of primary input into y units of secondary input, so the total revenue from acquiring k units of primary input from a supplier is

$$f(k,Y) \equiv P(k,Y)k = \gamma Y^{\frac{\gamma-\sigma}{\gamma}} k^\sigma. \tag{13}$$

The revenue function in (13) is Cobb–Douglas with endogenous total factor productivity $\gamma Y^{(\gamma-\sigma)/\gamma}$. The function f satisfies $f(0, Y) = 0, \partial f/\partial k\big|_{k=0} = \infty$, and $\partial f/\partial k > 0 > \partial^2 f/\partial k^2$. From (13), it follows that the optimal level of investment is

$$k^* = (\sigma\gamma Y^{(\gamma-\sigma)/\gamma})^{1/(1-\sigma)} \tag{14}$$

[4] Under perfect competition of the final goods firm, $PY = \int_0^1 P_i y_i d_i$, and using (12), one can show that $P = (\int_0^1 P_i^{-\sigma/(1-\sigma)} d_i)^{-(1-\sigma)/\sigma} \gamma^{-1/\sigma} Y^{(1-\gamma)/\gamma}$, which approaches the standard Dixit-Stiglitz price aggregator as $\gamma \to 1$.

3. Equilibrium with internal and external finance

We now let entrepreneurs accumulate cash in stage 2 to finance investments in stage 1 of the following period. Internal finance serves as an immediate funding source, has no explicit interest payments, and can be used regardless of whether entrepreneurs are matched with a bank. Monetary policy now plays a role via its effect on the cost of holding real balances and thus the lending rate and investment. We refer to the effect of the policy rate on investment as *transmission* and to the effect of the policy rate on the lending rate as *pass through*, in line with Rocheteau et al. (2017).[5]

I first characterize the outside option of an entrepreneur. Given an investment opportunity but no access to a bank, feasibility requires $k \le a_m$ and the surplus from investing is

$$\Delta_m(a_m) = f(k_m, Y) - k_m \quad \text{where} \quad k_m = \min\{a_m, k^*\} \tag{15}$$

The function $\Delta_m(a_m)$ is increasing and strictly concave for all $a_m < k^*$ with $\Delta_{m'}(a_m) = \frac{\partial f}{\partial k} - 1 > 0$.

Suppose next that the entrepreneur is in contact with a bank. Then the terms of the contract specify (1) the investment level k, (2) the down payment d, and (3) the bank's fee ϕ. The surplus from a bank loan is thus $f(k,Y) - k - \phi - \Delta_m(a_m)$. Accordingly, (k, d, ϕ) solves

$$\max_{k,d,\phi} [f(k,Y) - k - \phi - \Delta_m(a_m)]^{1-\theta} \phi^{\theta} \quad \text{s.t.} \tag{16}$$

$$k - d + \phi \le \chi f(k,Y) \tag{17}$$

$$d \le a_m \tag{18}$$

Lemma 1 characterizes the solution of the bargaining given a_m and Y in terms of a threshold of retained earnings a^*, and is analogous to Lemma 1 in Rocheteau et al. (2017).

Lemma 1. *There is an a^* characterized by*

$$(\chi^* - \chi) f(k^*, Y) = (1-\theta) a^* + \theta f(a^*, Y)$$

satisfying $a^* < k^*$, in which $a^* > 0$ if and only if $\chi < \chi^*$. Moreover, if $a_m \ge a^*$, then

$$k_c = k^* \tag{19}$$

$$\phi^* = \theta[f(k^*, Y) - k^* - \Delta_m(a_m)] \tag{20}$$

If $a_m \le a^*$, then $(\phi, k) \in \mathbb{R}_+ \times (\tilde{k}, k^*)$ solves

$$\frac{a_m + \chi f(k_c, Y) - k_c}{(1-\chi) f(k_c, Y) - a_m - \Delta_m(a_m)} = \frac{\theta}{1-\theta} \frac{1 - \chi \frac{\partial f}{\partial k_c}}{(1-\chi)\frac{\partial f}{\partial k_c}} \tag{21}$$

$$k_c + \phi = a_m + \chi f(k_c, Y) \tag{22}$$

If $a_m < a^*$, then Eq. (21) uniquely determines k_c given a_m, and Eq. (22) solves for ϕ given k_c and a_m.

Lemma 2. *If the liquidity constraint binds, then $\frac{\partial k_c}{\partial a_m} > 0$, $\frac{\partial [a_m + \chi f(k_c, Y)]}{\partial a_m} > 1$, and $\frac{\partial k_c}{\partial \theta} < 0$.*

Thus, accumulating reserves increases financing capacity by more than one since pledgeable output increases, and investment rises. Furthermore, a higher bank bargaining power reduces investment.

The lending rate is $r \equiv \phi/(k_c - a_m)$ is

$$r = \begin{cases} \theta \frac{f(k^*, Y) - k^* - \Delta_m(a_m)}{k^* - a_m} & \text{if} \quad a_m \in [a^*, k^*] \\ \frac{\chi f(k_c, Y)}{k_c - a_m} - 1 & \text{if} \quad a_m < a^* \end{cases} \tag{23}$$

We have the following relationship between the real lending rate and cash holdings.

Lemma 3. *$\partial r / \partial a_m < 0$ for all $a_m \in [a^*, k^*]$ and $r \to 0$ as $a_m \to k^*$.*

The lemma says that, if liquidity constraint binds, then the real lending rate decreases with cash holdings and converges to zero as cash approaches the target level of investment. If the liquidity constraint binds, however, then there is no guarantee that $\partial r / \partial a_m < 0$. In particular, for $\theta = 1$, then $r = [\chi f(\hat{k}, Y)]/(\hat{k} - a_m)$, so that $\partial r / \partial a_m > 0$. In this case, bringing more

[5] One difference, however, is that Rocheteau et al. (2017) define transmission into investment and output, respectively, in terms of absolute deviations $\partial Y / \partial i$ and $\partial K / \partial i$, whereas we focus on percentage deviations of these variables from the initial state. The reason is that with monopolistic competition k^* is a function of Y and thus depends on the entire set of parameters; thus, absolute comparisons cannot be made because the initial state is different. For Rocheteau et al. (2017), k^* does not depend on α, λ, σ, θ, or χ, so that such comparisons are appropriate.

assets finances the same level of investment with a smaller loan bearing a higher interest rate. Thus, a priori, the pass through $\partial r/\partial i$ need not be positive, and its sign and magnitude depends closely on the bank bargaining power.

The entrepreneur's choice of real balances solves

$$\max_{a_m \geq 0}\{-ia_m + \lambda(1-\alpha)\Delta_m(a_m) + \alpha\lambda\Delta_c(a_m)\} \tag{24}$$

where $\Delta_c(a_m) \equiv f(k_c, Y) - k_c - \phi$ takes the following form:

$$\Delta_c(a_m) = \begin{cases} (1-\theta)[f(k^*, Y) - k^*] + \theta\Delta_m(a_m) & \text{if} \quad a_m \geq a^* \\ (1-\chi)f(k_c, Y) - a_m & \text{otherwise} \end{cases}$$

There are three cases. If $a_m > k^*$, then the entrepreneur finances k^* without resorting to bank credit and he appropriates the full gains from trade. If $a_m \in [a^*, k^*)$, he can still finance k^*, but only by using bank credit. Finally, if $a_m < a^*$, then the liquidity constraint binds and the entrepreneur's surplus equals the non-pledgeable output net of his real balances. However, given $i > 0$, entrepreneurs always choose $a_m < k^*$, since the marginal benefit of an additional unit of real balances is zero at k^* but the marginal cost is positive. Thus, we can just consider the last two cases and use $k_m = a_m$ and hence $\Delta_m(a_m) = f(a_m, Y) - a_m$. For financially unconstrained firms, note that (24) solves

$$i = \lambda[1-\alpha(1-\theta)]\left(\frac{\partial f}{\partial a_m} - 1\right) \tag{25}$$

Using (25) and (14), we can characterize a_m as a fraction of the optimal investment k^*:

$$\begin{aligned} a_m(Y) &= \Upsilon(i)^{1/(1-\sigma)}[\sigma\gamma Y^{(\gamma-\sigma)/\gamma}]^{1/(1-\sigma)} \\ &= k^*\Upsilon(i)^{1/(1-\sigma)} \end{aligned} \tag{26}$$

where

$$\Upsilon(i) = \left[\frac{\lambda(1-\alpha(1-\theta))}{i+\lambda(1-\alpha(1-\theta))}\right] \tag{27}$$

The cash-to-investment ratio $a_m/k^* = \Upsilon(i)^{1/(1-\sigma)}$. Note that as $i \to 0$, $\Upsilon(i) \to 1$, and $a_m \to k^*$. Moreover, $\Upsilon(i)^{1/(1-\sigma)}$ increases with λ and θ, which encourage cash holdings due to greater investment opportunities and the strategic motive, respectively. The cash-to-investment ratio decreases with α, σ, and i because of greater opportunities of external finance, less market power, and a higher cost of internal finance.

Aggregate demand satisfies

$$Y = \left\{\lambda(1-\alpha)\left[\int_0^1 k_{i,m}^{\sigma} di\right] + \lambda\alpha\left[\int_0^1 (k_{i,c})^{\sigma} di\right]\right\}^{\gamma/\sigma} \tag{28}$$

and can be rewritten using (26) for $\chi \geq \chi^{**}$ as

$$Y = \lambda^{\gamma/\sigma}\left\{[1-G(\chi^{**})]k^{*\sigma}[\alpha + (1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}] + \int_0^{\chi^{**}}[\alpha k_c(\chi)^{\sigma} + (1-\alpha)a_m(\chi)^{\sigma}]dG(\chi)\right\}^{\gamma/\sigma} \tag{29}$$

where $G(\chi_b)$ denotes the cumulative distribution of pledgeability coefficients with density $g(\chi_b)$. A monetary equilibrium with internal and external finance is a list (k_c, a_m, r, Y) that solves (16), (23), (24), and (29).

The first order condition for real balances can be written as

$$i = \lambda[1-\alpha(1-\theta)][\partial f/\partial k_m - 1] \quad \text{if} \quad a_m > a^* \tag{30}$$

$$\frac{i}{\lambda} + 1 = (1-\alpha)\frac{\partial f(k_m, Y)}{\partial k_m} + \alpha(1-\chi)\frac{\partial f(k_c, Y)}{\partial k_c}\frac{\partial k_c}{\partial a_m} \quad \text{otherwise} \tag{31}$$

From Eq. (30), if the liquidity constraint does not bind, the marginal benefit of cash is the product of the probability of receiving an investment opportunity, the probability of internal finance plus the probability of external finance multiplied by the bargaining power of the bank, and the extra revenue net of the investment cost. If the liquidity constraint does bind, then bringing more cash to bank meetings increases the investment level. Thus, if $a < a^*$, then $\frac{\partial k_m}{\partial i} < 0$, $\frac{\partial k_m}{\partial \lambda} > 0$, $\frac{\partial k_m}{\partial \alpha} < 0$ and $\frac{\partial k_m}{\partial \theta} > 0$. The expression $\frac{\partial k_c}{\partial a_m} = h(a_m, k_c, \chi, Y)$ is the effect of higher assets on the bargained level of investment k_c of an entrepreneur with coefficient χ and aggregate demand Y. Appendix C.10 provides the full expression. The latter two conditions show how conditions in the credit market affect money demand. As loans are less accessible or more costly, entrepreneurs hold more cash to compensate. Given $\theta > 0$, entrepreneurs hold cash even if $\alpha = 1$ in order to reduce interest payments to bankers. The next proposition shows that coexistence of cash and credit is robust.

Proposition 1 (Coexistence)**.** *For all $i > 0$ and $\chi > 0$, if $\lambda(1-\alpha) > 0$ or $\lambda\theta > 0$, then cash and credit coexist in equilibrium.*

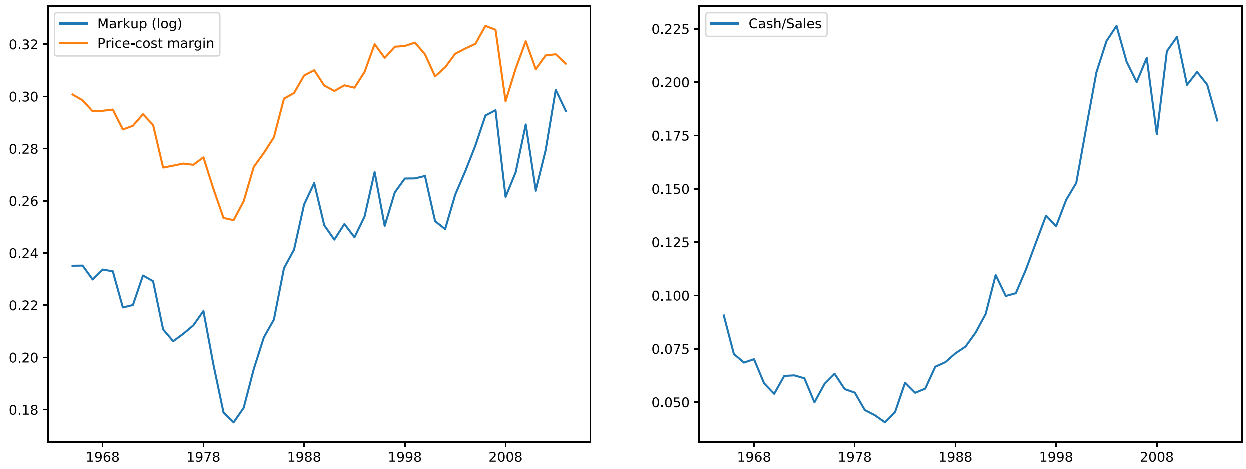

Fig. 3. Estimated markup and the price-to-cost margin in the left-hand panel and the cash-sales ratio in the right-hand panel. All four series have been winsorized with a 1% band.

Money exists in equilibrium provided that the insurance motive, $\lambda(1-\alpha)>0$, or strategic motive, $\lambda\alpha\theta>0$, is active. There is a general motive to use credit if $i>0$, either to finance more investment if the household is liquidity constrained, or to economize on cash holdings.

In Rocheteau et al. (2017), there is a threshold $\bar{i}$ such that for $i<\bar{i}$ policy affects the component of investment financed internally but not that financed through banking. The aggregate demand externality eliminates this dichotomy: smaller internal finance reduces aggregate demand, which reduces entrepreneurs' revenue and thereby decreases investment financed through banking.

If we combine Lemma 1 with (26), we obtain a closed form characterization of the threshold χ^{**} at which the liquidity constraint binds.

Lemma 4.

$$\chi^{**}=\theta+(1-\theta)\sigma-\left\{\theta\Upsilon(i)^{\sigma/(1-\sigma)}+(1-\theta)\sigma\Upsilon(i)^{1/(1-\sigma)}\right\}$$

The first term on the right hand side Lemma 4 coincides with the expression under external finance. The second term on the right hand side is an adjustment for the availability of internal finance. As $i\to\infty$, $\Upsilon\to 0$, and $\chi^{**}\to\chi^{*}=(1-\theta)\sigma+\theta$, which is the threshold under pure external finance (see appendix). As $i\to 0$, $\chi^{**}\to 0$, so that the liquidity constraint never binds. The intuition is that if there is no cost to internal finance, then entrepreneurs can always reach the desired level of investment. Also, note that if $\sigma\to 1$, then entrepreneurs have no incentive to hold cash, and $\chi^{**}=1$, so that all firms are constrained. Note that since the ratio of cash to desired investment level $\Upsilon^{1/(1-\sigma)}$ is independent of the returns to scale γ, the proportion of constrained firms χ^{**} is independent of γ as well. An upward shift in σ raises the target investment level and an increase θ amplifies financial frictions. Therefore, either movement shifts the χ^{**} curve upward. The threshold a^{*} for the bargaining is decreasing in θ, but it does not decrease fast enough so that the first best is attainable.

4. Empirical analysis and calibration

The financial economics literature provides evidence that financial constraints and competition raise firm cash holdings. Opler et al. (1999) examine Compustat firms from 1971–1994 and find that firms with greater access to capital markets hold less cash relative to non-cash assets. Almeida et al. (2004) study manufacturing firms from 1971 to 2000 find that firms facing greater financial constraints save more of their cash flow as cash. Morellec et al. (2014) find evidence that, for financially constrained firms, more competition is associated with greater cash holdings. They measure competition using the price-to-cost margin of the firm, the Herfindahl–Hirschman Index, and the product market fluidity measure developed by Hoberg et al. (2014).

I provide empirical evidence of these results over a longer panel and with a pledgeability measure used to calibrate the model. The data is drawn from the set of publicly traded firms from 1964 to 2017. Publicly traded firms account for about one third of total U.S. employment (Davis et al., 2006) and about 41% of sales (Asker et al., 2014).

The main challenge is estimating markups to test the model predictions. Compustat provides firm-level output and input data but does not provide information on prices or marginal costs. To overcome these limitations, I follow a version of the production approach discussed by Ackerberg et al. (2006) and De Loecker and Eeckhout (2017), which is based on original ideas from Hall (1988). To save space, I relegate all the details of the construction of markups to the data appendix.

Table 1
Cash holdings, competition, and pledgeability. The dependent variable is cash/sales. The independent variable is the constructed markup in the first column and the price-to-cost margin in the second column. The time range is 1964–2017. Standard errors, in parentheses below the estimate, are clustered at the firm level. The symbols ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

	Constructed markup	Price-to-cost margin
Markup (log)	−0.946***	
	(0.041)	
PCM		−0.718***
		(0.024)
Pledgeability	−1.707***	−1.196***
	(0.106)	(0.096)
Size	−0.166***	−0.117***
	0.007	0.006
Market-to-book	0.001	0.000
	(0.001)	(0.001)
Cash flow	0.024***	0.037***
	(0.009)	(0.013)

Fig. 3 shows the time trend of the estimated markup together with the price-to-cost margin in the left-hand panel, as well as the cash-sales ratio in the right-hand panel.[6] Each variable is an average over firms weighted by the market share of sales in that period.[7] The evolution of markups resembles Fig. 1 from De Loecker and Eeckhout (2017), who estimate the production function for each NAICS category separately. Both markups and cash holdings exhibit an upward trend, as discussed by De Loecker and Eeckhout (2017), Morellec et al. (2014), and others. This makes the use of time fixed effects in panel regressions especially important in assessing the relationship between cash holdings and markups. After eliminating missing values, including those which arise from lagged variables, the final dataset has 150,764 firm-year observations and 13,685 firms identified via the gvkey. Given the estimated markups, I regress

$$cash_{i,t} = \beta_1 \mu_{i,t-1} + \beta_2 \chi_{i,t-1} + \beta_3' \Gamma_{i,t-1} + \alpha_j + \lambda_t + _{i,t} \tag{32}$$

where subscripts i, j, t represent, firm, industry, and year, respectively. The outcome variable $cash_{it}$ is cash divided by sales; Fig. 6 shows its relationship with competition and financial frictions in the model. The independent variable $\mu_{i,t}$ is either the log of the constructed markup or the price-to-cost margin, χ_{it} is the measure of asset pledgeability (opposite of financial constraint), $\Gamma_{i,t}$ is a set of firm-level controls, and α_j and λ_t are fixed effects for industry and year, where industry is defined by the 4-digit NAICS code. Following Morellec et al. (2014), the regressors and controls are lagged by one year to ensure they are predetermined relative to the dependent variable. The set of controls $\Gamma_{i,t}$ follow work by Opler et al. (1999) and others and include firm size, the market-to-book ratio, and cash flow. The purpose is to control for other aspects of heterogeneity which affect cash holdings. The market-to-book ratio is a proxy for investment opportunities, firm size measures captures the relevance of scale economies in cash holding, and cash flow tends to lead to more cash, all else equal. The appendix provides more details.

Table 1 indicates that a one-unit change in the log markup or price-to-cost margin, controlling for tangibility, time-and-industry fixed effects, and other firm measures, is associated with a decrease in the cash-sales ratio of -0.946 and -0.718, respectively. The results are statistically significant at the 1% level. Moreover, a one-unit change in the tangibility of assets is associated with an expected decrease of -1.707 and -1.196, respectively. Thus, the estimation supports the model implication that competition and financial frictions increase cash-to-sales. As typically found in the literature, the estimates on cash flow are positive and significant and those on size are negative and statistically significant. It turns out, however, that the market-to-book ratio is economically and statistically insignificant; estimating the regression without the market-to-book ratio yields nearly identical coefficients for pledgeability and the markups.

Fig. 4 is a scatterplot of cash/sale and price-to-cost margin after partialling out each variable for industry-and-year fixed effects, the pledgeability measure, and the set of controls. The dark line is the least-squares regression line. In addition to the generally negative relationship, one striking feature is the extreme dispersion of (demeaned) cash holdings and margins across firms.

[6] In terms of Compustat variables, the price-cost margin is (SALES-COGS)/SALES, and the cash-sales ratio is CHE/SALES.

[7] Let firm i's sales at time t be $S_{it} = P_{it}Q_{it}$. and S_t be total sales. Then $s_{it} = S_{it}/S_t$ is the share of firm i's sales at time t, and the average weighted markup is $M_t = \sum_i s_{it}\mu_{it}$.

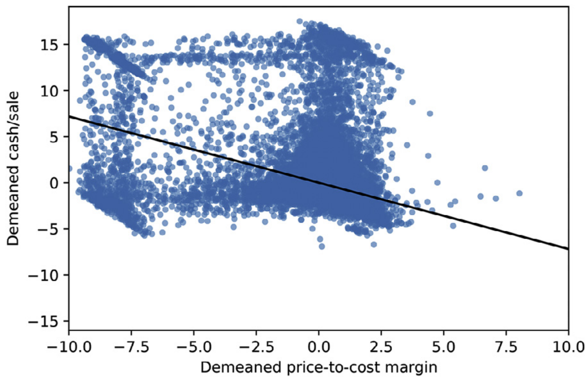

Fig. 4. Cash/sales and the price-to-cost margin. Each series is demeaned by industry-and-year fixed effects, pledgeability, and the set of controls $\Gamma_{i,t}$. The two series are winsorized with a 1% band. The dark line is the lest squares regression line.

The calibration strategy extends the approach by Rocheteau et al. (2017) to include markup and pledgeability information from the Compustat panel, 1964–2017. I interpret *i* as the 3-month Treasury bill secondary market rate, which averages 4.8% from 1964–2017. This value reflects the extraordinarily low rates following the Great Recession. The targets include the semi-elasticity of money and output with respect to the nominal rate. Lucas (2000) estimates a money semi-elasticity of −7 from the data, which is consistent with the estimate using data from Mulligan (1997). Dedola and Lippi (2005) find an output semi-elasticity of −0.7 for the United States: a one percentage point increase in the nominal interest reduces output by 0.7%. Following Rocheteau et al. (2017), I set the real lending rate at 2.4%, which is based on the average difference between the prime lending rate and the 3-month T-bill rate. I solve for the semi-elasticity of money demand (B.11), semi-elasticity of output (B.12), and the real lending rate (B.10) as a system to obtain λ, θ, and γ. Rocheteau et al. (2017) interprets α as the probability that a loan application is accepted and calibrates it to 0.9 according to the level of the Survey of Small Business Finance. For the markup, I target the mean markup of firms weighted by sales. The sales-weighted mean is 34.2%.

An important aspect of the this article concerns the heterogeneity of financial frictions. The approach associates pledgeable output with the expected liquidation value of assets calculated by Almeida and Campello (2007) on manufacturing Compustat firms from 1985–2000. The strategy for measuring pledgeability originates with Berger et al. (1996). They estimate the expected liquidation value of firms' main operating assets (receivables, inventory, and capital stock). Berger et al. (1996) examine discontinued operations between 1984 and 1993 for which it is possible to ascertain the ratio of liquidation value to book value in receivables, inventory, and fixed assets. They then regress this variable on the proportion of book value in receivables, inventory, and fixed assets and estimate that a dollar's book value produces an average of 72 cents of liquidation value for receivables, 55 cents for inventory, and 54 cents for fixed assets. Cash holdings, however, do not lose value in liquidation, and so they implicitly carry a weight of one.[8] Accordingly, Almeida and Campello (2007) construct an empirical measure of pledgeability as follows for firm *i* at time *t*:

$$Pledge\,ability_{it} = \frac{0.715\,Receivables_{it} + 0.547\,Inventory_{it} + 0.535\,Capital_{it} + Cash_{it}}{Total\,Assets_{it}}$$

The numerator in this expression represents the total pledgeable assets. In the model, the pledgeable assets for firm *i* are $\chi_i f(k_c, Y) + a_m^e$. The parameter χ applies to output, not cash, which is fully pledgeable for all firms. The question concerns the proper empirical analogue of $f(k_c, Y)$. Though this quantity represents revenue in the model, it is also the the total non-cash assets of the firm due to full depreciation of the capital stock. Hence, we can construct a firm-year pledgeability component χ_{it} as

$$\chi_{it}(Total\,Assets_{it} - Cash_{It}) = 0.715\,Receivables_{it} + 0.547\,Inventory_{it} + 0.535\,Capital_{it} + Cash_{it}$$

or

$$\chi_{it} = \frac{0.715\,Receivables_{it} + 0.547\,Inventory_{it} + 0.535\,Capital_{it}}{Total\,Assets_{it} - Cash_{it}}$$

In the model, pledgeability is a permanent firm characteristic, so we need to impute a permanent firm-level measure. To do this, we first regress $\chi_{i,t}$ on time dummies and obtain the residuals $\tilde{\chi}_{i,t}$, which eliminates aggregate variability over time. We eliminate the remaining idiosyncratic variability by averaging over firm years:

$$\chi_i = \frac{1}{T_i}\sum_{t=1}^{T_i} \tilde{\chi}_{it}$$

8 In Compustat, Receivables has code RECT, inventory has code INVT, and capital has code PPENT, total assets has code AT, and cash has code CHE.

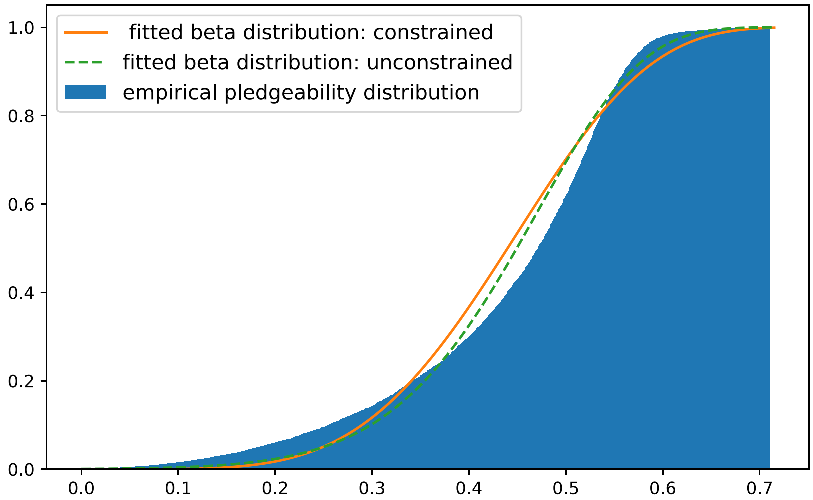

Fig. 5. Cumulative distribution function of pledgeability coefficients on universe of Compustat data, 1964–2007. The dashed line is the cumulative distribution function from a maximum-likelihood estimation of a non-standard beta distribution, whereas the solid line is the corresponding cdf from an MLE estimation of a beta distribution whose endpoints are constrained to the minimum and maximum values found in the data.

Table 2
Calibration strategy.

Parameter	Values	Calibration strategy
γ	0.837	Semi-elasticity of output
α	0.900	Loan application acceptance rate
λ	0.861	Semi-elasticity of money demand
σ	0.745	Sales-weighted gross markup = 1.34
θ	0.583	Real lending rate
i	0.048	3-month T-bill rate (nominal)
μ	0.439	MLE estimation of pledgeability distribution
σ_χ	0.106	MLE estimation of pledgeability distribution
t_0	−46.910	MLE estimation of pledgeability distribution
t_1	0.809	MLE estimation of pledgeability distribution

where T_i number of firm years for firm i.

To discipline the model, I fit a nonstandard beta distribution to the data, with probability density function

$$f(x) = \frac{(x-t_0)^{a-1}(t_1-x)^{b-1}}{B(a,b)(t_1-t_0)^{\alpha+\beta-1}}$$

where a and b are the shape parameters, the support lies on $[t_0, t_1]$, and $B(\cdot)$ is the beta function. Fig. 5 plots the cumulative distribution function of the data together with the distributions implied by maximum likelihood with a non-standard beta distribution. The 'unconstrained' case indicated by the dash line corresponds to full MLE estimation. The 'constrained' case indicated by the solid line sets the location and scale parameters to fit the minimum and maximum values of the data.

It is evident from Fig. 5 that unconstrained maximum-likelihood estimation fits slightly better. This requires setting t_0 highly negative but setting the shape parameters a and b so that there is positive but negligible mass left of 0. The mean pledgeability coefficient is 0.44 and the standard deviation is 0.11. Appendix D shows that the shape of the distribution does not change substantially across each two-digit Standard Industrial Classification group,with the exception of mining.

Table 2 lists the parameter values of the calibrated model.

5. Quantitative results

The appendix outlines how to compute the equilibrium given the calibrated parameter values. There is a fixed point problem for output since the revenue function for entrepreneurs depends on demand overall. Moreover, calculating output requires numerical integration over the production of individual entrepreneurs.

Fig. 6 plots cash divided by average sales $a_m/[\lambda(\alpha f(k_c, Y) + (1-\alpha) f(a_m, Y))]$ with respect to (individual) χ in the left panel and with respect to economy-wide σ in the right panel. In the right-hand panel, we consider a firm with $\chi = \mu - 2\sigma_\chi$. The model reproduces the two major stylized facts between financial constraints, competition, and cash holdings. First, cash-to-sales increase with the extent of financial constraints (fall with pledgeability). Second, they rise with more intense competition (higher σ for all firms) provided that firms are financially constrained. Otherwise, the relationship is nonmonotonic. To be more precise, Proposition 2 analytically characterizes cash-to-sales if the liquidity constraint does not bind.

Proposition 2. *The ratio of cash to expected sales if the liquidity constraint doe snot bind is*

$$= \frac{\sigma}{\lambda}\left[\frac{\Upsilon(i)^{1/(1-\sigma)}}{\alpha + (1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}}\right] \tag{33}$$

Cash-to-sales rise with σ provided that

$$1 + \frac{\sigma}{(1-\sigma)^2}\frac{\alpha \log \Upsilon(i)}{\alpha + (1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}} > 0 \tag{34}$$

which is satisfied for either σ or i sufficiently small.

The intuition underlying Proposition 2 is that, as firms become more competitive, they wish to expand production since the marginal revenue is closer to the price of the good. However, since they obtain lower markups, firms hold less cash relative to investment, i.e., $a_m/k_c = \Upsilon(i)^{1/(1-\sigma)}$ falls. So, there is a tradeoff between higher output (and investment) and lower cash per unit of investment. Provided that σ or the nominal interest rate i are not too high, the first effect dominates, and cash holdings expand.

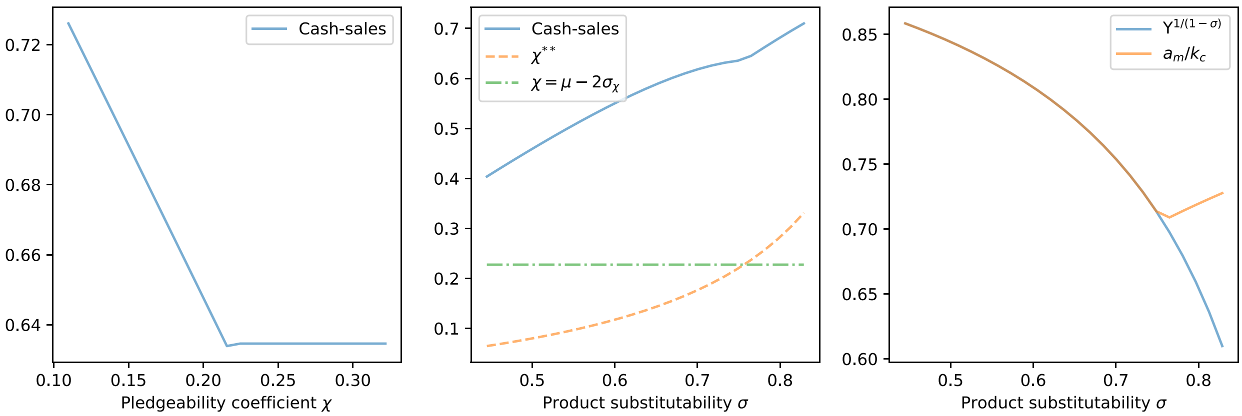

Fig. 6. Cash holdings, product diversity, and financial constraints.

However, the right-hand panel of Fig. 6 notes that the dashed χ^{**} curve rises with σ, eventually intersecting the line $\chi = \mu - 2\sigma_\chi$. This is the point at which the firm becomes constrained. Further competition unambiguously increases the cash-sales ratio. Once the financial constraint binds, entrepreneurs need to hold disproportionately more cash to boost sales. Depending on the point at which the financial constraint binds, cash-sales may rise monotonically with competition. This reasoning also implies that (33) is a lower bound for cash-to-sales in the general model.

Both panels have empirical support. Opler et al. (1999) examine Compustat firms from 1971–1994 and find that firms with greater access to capital markets hold less cash relative to non-cash assets. Almeida et al. (2004) study manufacturing firms from 1971 to 2000 and conclude that firms facing greater financial constraints save more of their cash flow as cash. Morellec et al. (2014) find evidence that, for financially constrained firms, more competition is associated with greater cash holdings. Finally, Thakor and Lo (2015) apply a difference-in-difference approach and find that, as the biopharmaceutical industry became more competitive following the Hatch–Waxman Act, it accumulated more cash relative to other research-and-development intensive industries.

We next run simulations and show how investment, aggregate cash/to aggregate sales, output, the real interest rate, external share of finance, and the proportion of constrained firms vary with respect to the variable of interest. I express aggregate investment and output in percentage deviations from the initial value, which facilitates comparison. The other variables are in levels.[9]

Fig. 7 compares the effects of the policy rate under the calibrated value of $\gamma = 0.84$ and also according to $\gamma = \sigma = 0.75$, which eliminates the aggregate demand externality and corresponds to the use of additively separable preferences. There is a first-order effect of the policy rate on output without any compensating difference in the ability of entrepreneurs to raise external funds. At $i = 4.8\%$, investment falls by over 2% without the aggregate demand externality and 5% with it. Hence, the externality causes investment to fall by over twice as much at this value. The stronger monetary policy transmission of monetary policy is not accompanied by a change in the pass through to the real interest rate. As we noted earlier, the

[9] The ratio of aggregate cash to aggregate sales, the real interest rate, and the share of external finance are already normalized in their respective ways; moreover, the latter two approach zero as $i \to 0$. For reasons of numerical stability, the initial value of the nominal rate is 0.04%.

proportion of constrained firms $G(\chi^{**})$ is independent of γ since the latter does not affect firms' desired cash holdings to target investment.

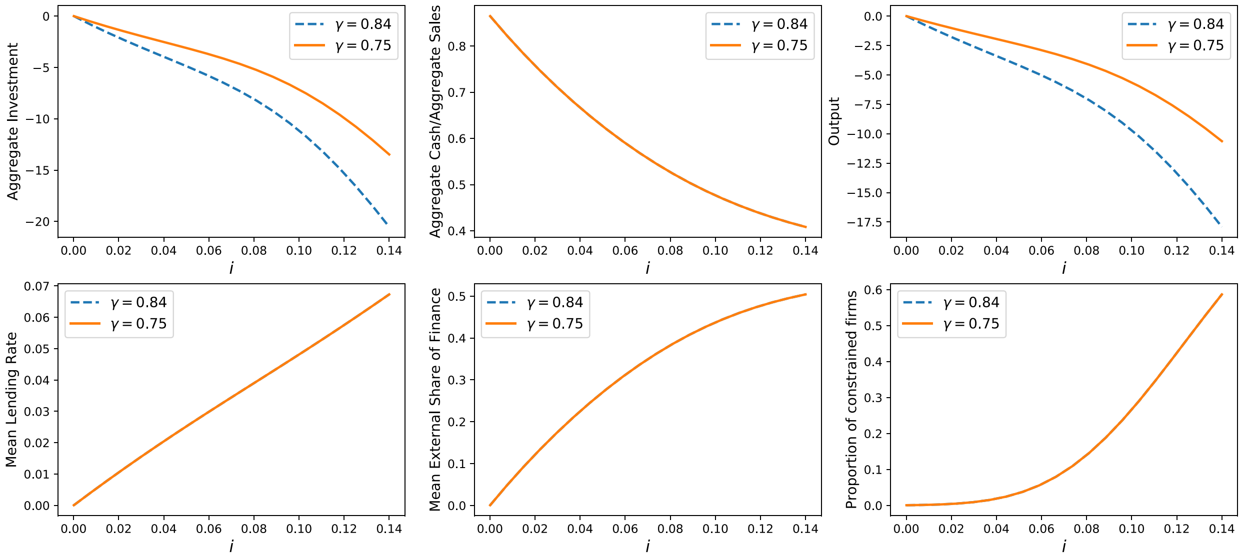

Fig. 7. Pass through of nominal interest rate: different values of γ. The vertical axes in the top panel represent proportional deviations from the initial value. The vertical axes of the bottom panel are in levels.

Fig. 8 illustrates the effects of raising the nominal interest for the calibrated value of mean financial frictions μ together with a standard deviation change in either direction. The diagram showcases the financial multiplier: the effects of monetary policy rise with financial frictions. More financial frictions, unsurprisingly, imply that the mean external share of finance rises less with the nominal interest rate. The effects are nonlinear in μ: the transition from $\mu = 0.44$ to $\mu = 0.33$ induces bigger changes in investment, output, and external share of finance than from $\mu = 0.55$ to $\mu = 0.44$. The nonlinearity relates closely to the concavity of the revenue function; a higher cost of internal financing causes a more precipitous drop in revenue for lower μ, which in turn reduces aggregate demand and further hurts revenue. As the concavity of the revenue function arises from a negatively sloped demand curve, monopolistic competition raises the financial multiplier.

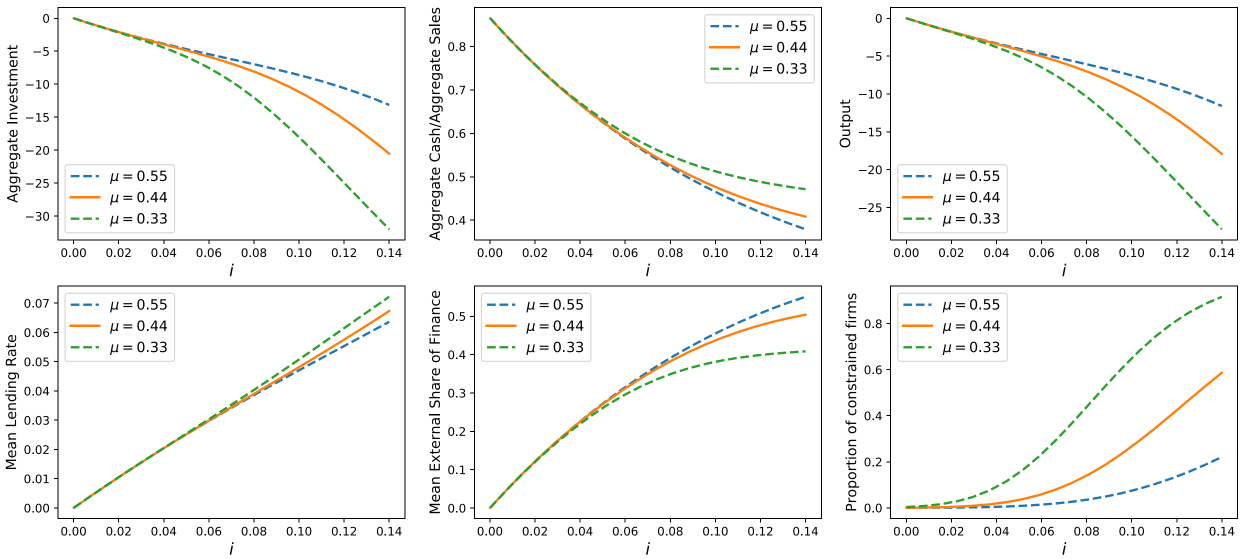

Fig. 8. Pass through of nominal interest rate: different values of μ. The vertical axes in the top panel represent proportional deviations from the initial value. The vertical axes of the bottom panel are in levels.

Fig. 9 illustrates the interaction between financial frictions and the aggregate demand externality. Increasing average pledgeability (decreasing financial frictions) positively affects aggregate investment and output, alongside the external share of finance. A higher γ implies a stronger aggregate demand externality, leading to a stronger demand for the investment of each entrepreneur. At μ increases from 0.33 to 0.44, investment increases by about twice as much in the absence of the aggregate demand externality. As discussed previously, the bottom panels and the ratio of cash to sales do not depend on γ. The mean lending rate barely budges. Slackening financial constraints tend to reduce the real lending rate, but this effect is mitigated by (slightly) declining cash holdings.

Note that there are two competing effects on aggregate internal finance. Higher pledgeability reduces entrepreneurs' need to accumulate cash, but it also raises output and, though the aggregate demand externality, investment opportunities. These greater investment opportunities raise money demand. The calibrated value of γ implies that money demand rises slightly. Either way, the effects on money demand are tiny compared to those of the nominal rate.

The next set of figures examine the relevance of mean-preserving spreads of pledgeability coefficients on macroeconomic outcomes. Fig. 10 illustrates the effects of on aggregate demand and the external share of finance under both the calibrated returns to scale $\gamma = 0.84$ and $\gamma = \sigma = 0.75$. The higher variance affects aggregate demand in three ways. First, a mean-preserving spread increases the proportion of constrained firms $G(\chi^{**})$, which directly reduces output. Second, within the distribution of constrained firms $G(\chi)/G(\chi^{**})$, $0 \leq \chi \leq \chi^{**}$, there is a redistribution toward more heavily constrained firms. Concavity of the revenue function and Jensen's inequality implies that output falls within this set. Furthermore, the aggregate demand externality causes investment demand to decrease even among the financially unconstrained firms. Note that

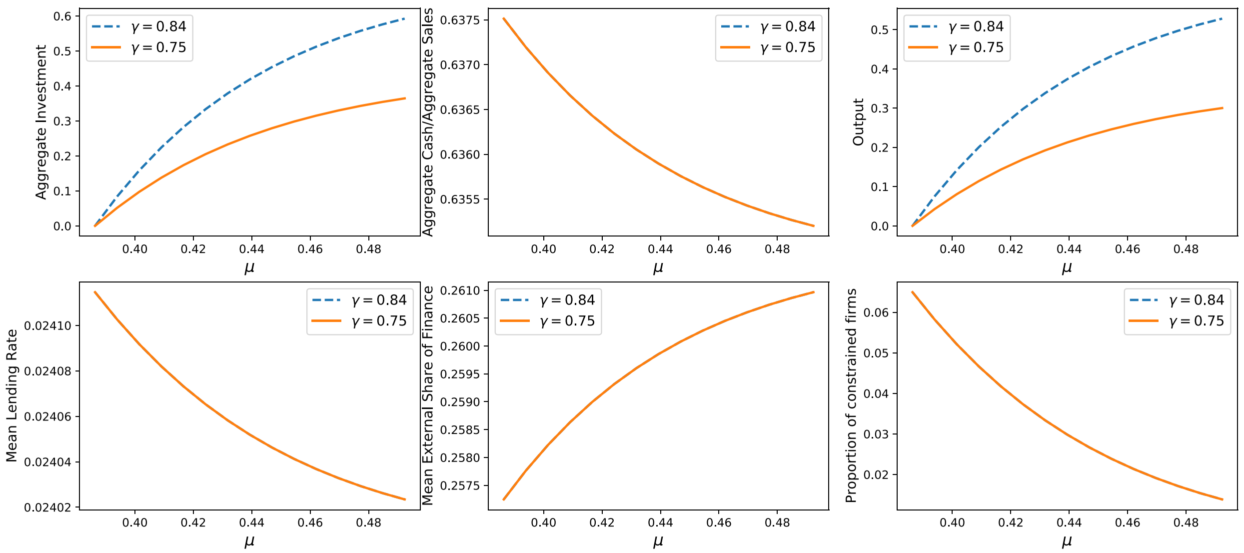

Fig. 9. Effects of higher pledgeability: different values of γ. The vertical axes in the top panel represent proportional deviations from the initial value. The vertical axes of the bottom panel are in levels.

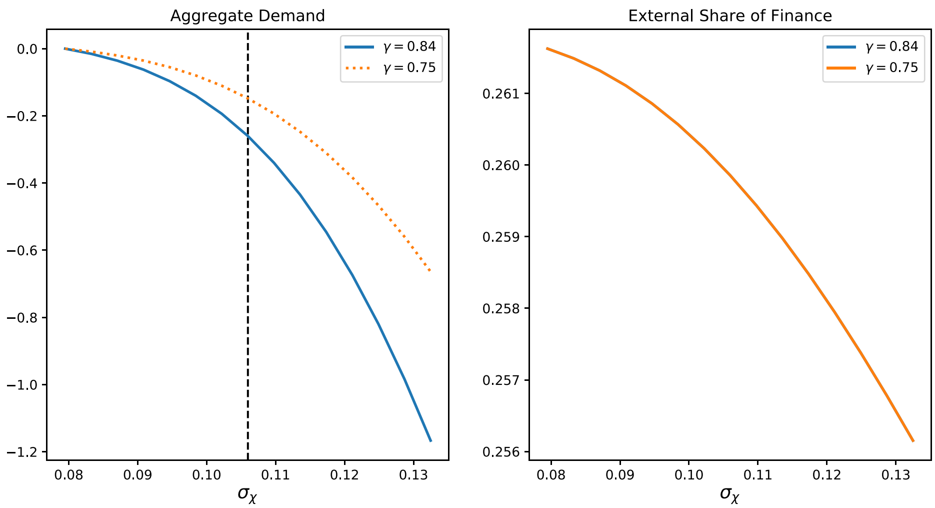

Fig. 10. Effect of a mean-preserving spread on output and the real lending rate. The vertical axis for output is in log deviations multiplied by 100.

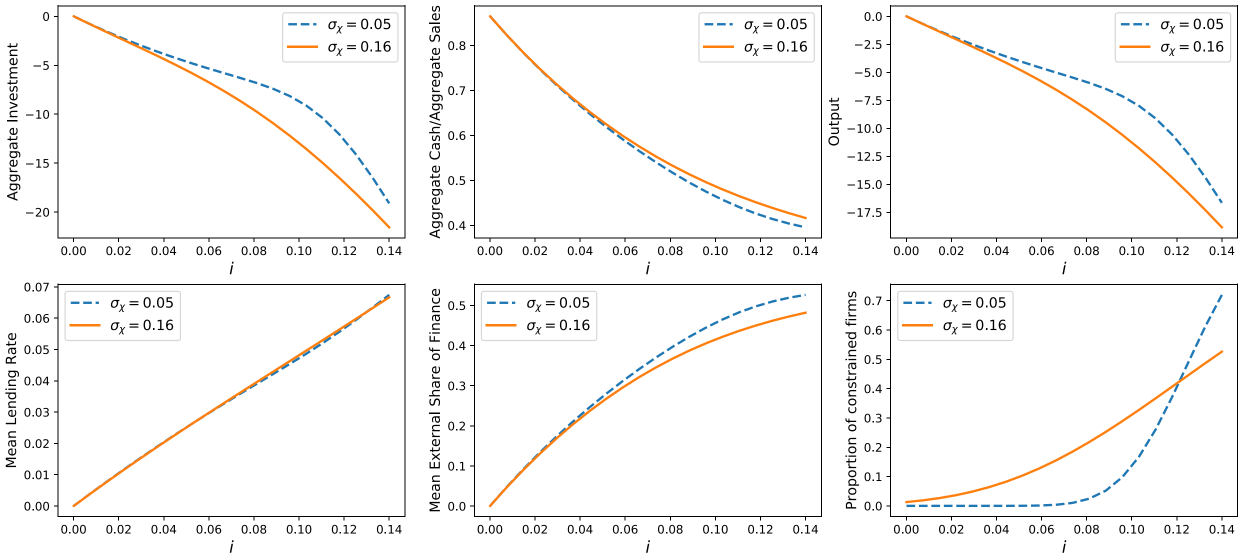

Fig. 11. Effect of nominal interest rate: different values of σ_χ. The vertical axes in the top panel represent proportional deviations from the initial value. The vertical axes of the bottom panel are in levels.

the first two effects do not require the aggregate demand externality, just concavity of the revenue function from product diversity. The external share of finance decreases from the increase in constrained firms. Fig. 10 shows that under the calibration output is about 0.25% lower, and that about half of this effect is due to the aggregate demand externality. Though the loss in output is not too severe at the calibration, further increases in heterogeneity of financial frictions are increasingly costly in terms of output.

Fig. 11 depicts the effects of the policy rate under higher variation of the pledgeability coefficients, with the mean held constant. A mean-preserving of liquidity constraints has almost no effect on the pass through but raises transmission. Moreover, it attenuates the increase in the external share of finance. The reason is that a mean-preserving spread increases the share of constrained firms $G(\chi^{**})$, leading to a bigger reduction of investment at a given i. However, as the policy rate increases, a higher proportion of firms become constrained in the high-spread case but are eventually surpassed by the proportion under the low spread. For this reason, mean-preserving spreads increase transmission the most for values of the policy rate from 7–10%.

Finally, the theory provides a rich cross section of transmission of monetary policy with respect to pledgeability coefficients. Fig. 12 shows the dependence of investment on firm's pledgeability coefficient, for percentiles 5, 10, 25, 50, and 90, together with the external share of finance. Note that, at the calibrated interest rate of 8%, over 95% of firms are uncon-

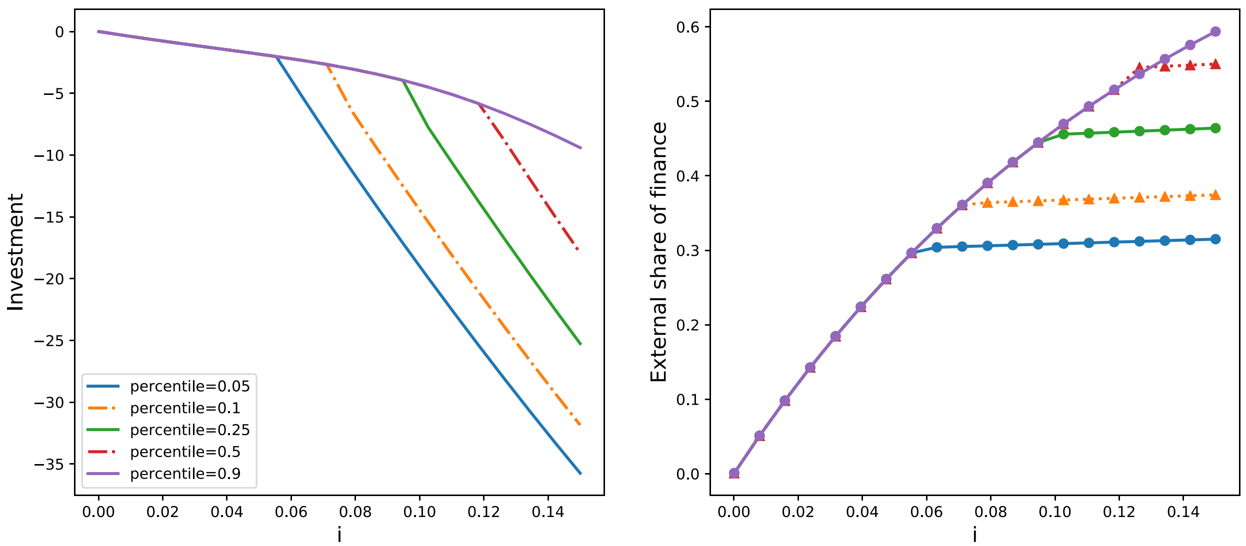

Fig. 12. Cross section of transmission by pledgeability coefficient. Investment is expressed in percentage deviations from the initial value and the external share of finance is in levels.

strained. However, by a rate of 10%, 25% of firms are financially constrained. Compare a firm at the 10th percentile to one at the 25th percentile. In moving from $i = 0\%$ to $i = 10\%$, the former experiences a drop in investment of about 15% compared to about 7% for the latter. From the right panel, we see that, as firms become constrained, they are barely able to increase their external share of finance for further increases in the policy rate. This illustration lends credence to the empirical strategy of identifying firm size with capital market access after controlling for other sources of heterogeneity, as by Gertler and Gilchrist (1994). The main caveat, of course, is that the one-to-one mapping from firm size to pledgeability coefficient only applies to constrained firms.

6. Conclusion

This paper studies the effects of monetary policy and corporate-finance implications of an environment in which monopolistic competition and heterogeneity in firms' ability to borrow interact. Entrepreneurs have both strategic and precautionary motives for holding money. Financial fictions induce firms to hold more cash, and a more competitive environment raises cash holdings for constrained firms. I find empirical support of these predictions by using measures of pledgeability and markups on publicly traded firms from Compustat. The aggregate demand externality from monopolistic competition increases transmission – though not the pass through to the real lending rate–and interacts strongly with financial frictions. Heterogeneous financial frictions implies cross-sectional dispersion of transmission, with monetary policy yielding bigger effects on smaller, more constrained firms, which is consistent with empirical evidence. Finally, greater dispersion of financial frictions reduces output and raises transmission.

There are three limitations to this study that merit discussion. First, the empirical analysis is based on large firms. Many of these firms have relationships with banks. Shocks and crises may sever these relationships and generate persistent effects. Wong et al. (2017) considers a similar environment as Rocheteau et al. (2017) except that it develops a dynamic model of lending relationships and optimal monetary policy. While in a credit relationship, entrepreneurs would have a lower insurance motive to holding cash compared to entrepreneurs outside of a credit relationship. Hence, entrepreneurs matched with creditors would tend to hold less cash. This element should not have much bearing on the interaction between heterogeneous financial frictions and the aggregate demand externality. Lending relationships are more pertinent for major financial shocks in a business cycle rather than the long-run analysis undertaken in this paper.

The second concerns entry margins of firms and banks. Entry of the former is relevant for the extent of product diversity; entry of the latter causes the amount of banks to respond positively to interest rate margins. Bank entry interacts in an interesting way with credit relationships (caveat 1), especially in the context of optimal monetary policy.

Finally, there are no firm dynamics with linear preferences. This limitation does not hinder the objectives of this paper. Senga et al. (2017) and Ottonello and Winberry (2018) offer a rich corporate finance framework with firm dynamics and aggregate shocks. Moreover, Bassetto et al. (2015) distinguish between proprietorships, who who can only grow slowly and rely heavily on retained earnings, and corporations.

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Appendix A. Pure bank credit

The bank can credibly promise a payment to the supplier and enforce payments up to the limit implied by χ. The bank extends a loan to the entrepreneur by crediting a deposit account in his name for the amount l. The deposit claims are liabilities of the bank that can be transferred from the entrepreneur to the supplier in exchange for k. In Stage 2, the supplier redeems the claim on the bank for ψ, and the entrepreneur settles his debt by returning $\psi + \phi$ to the bank. The terms of the loan contract are a pair (ψ, ϕ) with $\psi = k$ determined through bilateral negotiation. The surpluses are

$$S^e \equiv W^e(k, \omega^e - k - \phi) - W^e(0, \omega^e) = f(k, Y) - k - \phi \tag{A.1}$$

$$S^b \equiv W^b(\omega^b + \phi) - W^b(\omega^b) = \phi \tag{A.2}$$

so that the total surplus is $S^e + S^b = f(k, Y) - k$.

The bargaining problem is

$$\max_{k,\phi}[f(k, Y) - k - \phi]^{1-\theta}\phi^{\theta} \quad \text{s.t.} \quad \chi f(k, Y) \geq k + \phi \tag{A.3}$$

Fig. A.13 depicts the frontier in the contract space and utility space, taken with minimal modification from Rocheteau et al. (2017). The quantity $k^*(Y) = \arg\max_k[f(k, Y) - k]$ maximizes the bilateral surplus. The maximum surplus of the bank is $\chi f(\hat{k}, Y) - \hat{k} \leq f(\hat{k}, Y) - \hat{k}$, where $\hat{k}$ solves $\chi \partial f / \partial \hat{k} = 1$. The bargaining solution of $k \in [\hat{k}, k^*]$.[10]

[10] Suppose $k < \hat{k}$. Then a small increase in k increases both the total surplus and the maximal surplus of the bank. Thus, there is some higher intermediation fee that would raise surplus for both parties.

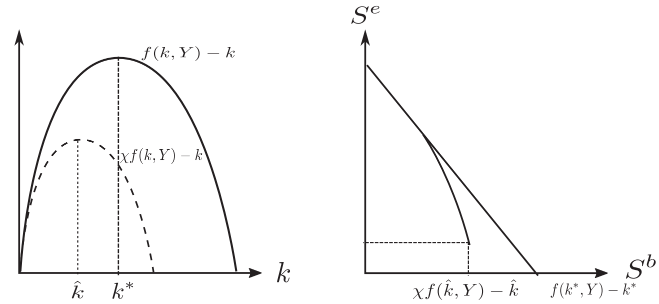

Fig. A.13. Pareto frontier for loan.

The lending rate $r = \phi/k$ is the ratio of interest payment ϕ to the size of the loan k:

Proposition 3. *The liquidity constraint binds if and only if*

$$\chi < \chi^* \equiv \frac{(1-\theta)k^* + \theta f(k^*, Y)}{f(k^*, Y)} = (1-\theta)\sigma + \theta \tag{A.4}$$

The general solution satisfies

$$k = \min\left\{1, \left[\frac{\chi}{(1-\theta)\sigma + \theta}\right]^{\frac{1}{1-\sigma}}\right\}(\sigma\gamma)^{1/(1-\sigma)} Y^{\frac{\gamma-\sigma}{\gamma(1-\sigma)}} \tag{A.5}$$

$$\phi = \min\{\chi f(k, Y) - k, \theta[f(k, Y) - k]\} \tag{A.6}$$

$$r = \theta\left(\frac{1-\sigma}{\sigma}\right) \tag{A.7}$$

According to (A.5), the investment decision of an individual entrepreneur depends on the credit conditions of other entrepreneurs in the economy. If a shock reduces financial access for some mass of firms, then entrepreneur i reduces his own investment provided that $\gamma > \sigma$.

The pledgeability constraint is more likely to bind with a higher σ and θ. First, more substitutability of goods raises the target level of investment, increasing the likelihood that the pledgeability constraint binds. Second, greater bargaining power of banks induces banks to make smaller loans at a higher interest rate without affecting the target level of investment. Given the symmetry of product differentiation and homogeneity of bargaining power, the real lending rate is the same for all firms. Moreover, as f is isoelastic in k, the distribution of financial frictions and the returns to scale do not affect the lending rate, as the interest payment and loan size change proportionately.

The following corollary summarizes comparative statics of investment.

Corollary 1. *If $\chi < \chi^*$, then the constraint binds and the solution k is increasing and continuous in χ, with $k(0) = 0$ and $k(\chi^*) = k^*$. Furthermore, $\frac{\partial \log k}{\partial \theta} = -1/[(1-\theta)\sigma + \theta] < 0$, $\frac{\partial \phi}{\partial \theta} > 0$, and $\frac{\partial \phi}{\partial \chi} > 0$.*

From (10), aggregate output is

$$Y = \left[\int_0^1 \alpha\lambda k_i^\sigma di\right]^{\frac{\gamma}{\sigma}} \tag{A.8}$$

since each entrepreneur i has an opportunity to invest with probability $\alpha\lambda$.

Substituting (A.5) into (29) yields

$$Y = \left[\alpha\lambda(\sigma\gamma)^{\frac{\sigma}{1-\sigma}} Y^{\frac{(\gamma-\sigma)\sigma}{\gamma(1-\sigma)}}\right]^{\gamma/\sigma} F(\sigma, \theta)^{\gamma/\sigma} \tag{A.9}$$

where

$$F(\sigma, \theta) = \left[\int_0^{(1-\theta)\sigma+\theta} \left[\frac{\chi}{(1-\theta)\sigma + \theta}\right]^{\sigma/(1-\sigma)} dG(\chi) + 1 - G[(1-\theta)\sigma + \theta]\right] \tag{A.10}$$

F is a weighting factor that scales output according to the mass of entrepreneurs that are constrained. If $\chi_i \geq (1-\theta)\sigma + \theta \quad \forall i$, then $F = 1$. In general, $F < 1$. F satisfies $\frac{\partial F}{\partial \sigma} < 0$ and $\frac{\partial F}{\partial \theta} < 0$. Using integration by parts, F can be expressed more succinctly as

$$F(\sigma, \theta) = 1 - \frac{\sigma}{1-\sigma}\left[\frac{1}{(1-\theta)\sigma + \theta}\right]^{\sigma/(1-\sigma)} \int_0^{(1-\theta)\sigma+\theta} G(\chi)\chi^{(2\sigma-1)/(1-\sigma)} d\chi \tag{A.11}$$

As $\sigma \to 1$ (allowing $\gamma \to 1$), $F \to 0$, since the first integral approaches zero and $(1-\theta)\sigma + \theta \to 1$. As $\sigma \to 0$, $F \to 1$, which is especially clear from (A.11). Also, $F(\sigma, 1) = \mathbb{E}(\chi^{\sigma/(1-\sigma)})$, so that $F(1/2, 1) = \mathbb{E}(\chi)$. Next, I solve for both Y and aggregate investment $K = \mathbb{E}(k)$ in terms of $F(\cdot)$:

$$Y = (\sigma\gamma)^{\gamma/(1-\gamma)}[\alpha\lambda F(\sigma, \theta)]^{\gamma(1-\sigma)/[\sigma(1-\gamma)]} \tag{A.12}$$

$$K = \alpha\lambda(\sigma\gamma)^{\gamma/(1-\gamma)}(\sigma\gamma)^{1/(1-\sigma)}[\alpha\lambda F(\sigma, \theta)]^{\gamma(1-\sigma)/\sigma(1-\gamma)} \tag{A.13}$$

which depend on the entire distribution of pledgeability coefficients through F.

Definition 1. A bank credit equilibrium is a list (k_i, r, Y) satisfying (A.5), (A.7), and (A.12).

From the uniqueness of Y in (A.12) and the uniqueness of k_i given Y from Proposition 3, it follows that equilibrium exists and is unique.

Proposition 4. *There is a unique bank credit equilibrium.*

From (A.12) and (A.13), it is clear that mean investment and aggregate demand have the same semi-elasticity with respect to a change in $G(\cdot)$ or θ. Furthermore, we can say unambiguously that a distribution of pledgeability coefficients G_1 exhibits more financial frictions compared to G_2 if G_2 first order stochastically dominates G_1, i.e. $G_2(\chi) \leq G_1(\chi)$ for all χ, with the inequality strict at some χ. An increase in financial frictions reduces F, and the effect on investment and output is magnified by the aggregate demand externality. We now derive some more comparative statics, illustrating how the elasticity of output with respect to α, λ, and θ increases with the aggregate demand externality. We can derive the following:

$$\frac{\partial F}{\partial \theta} = \frac{\sigma}{1-\sigma}\left\{\frac{\sigma}{[(1-\theta)\sigma+\theta]^{1/(1-\sigma)}}\int_0^{(1-\theta)\sigma+\theta} G(\chi)\chi^{(2\sigma-1)/(1-\sigma)}d\chi - (1-\sigma)\frac{G[(1-\theta)\sigma+\theta]}{(1-\theta)\sigma+\theta}\right\} \tag{A.14}$$

$$\epsilon_{Y,\alpha} = \epsilon_{Y,\lambda} = \frac{\gamma}{1-\gamma}\frac{1-\sigma}{\sigma} \tag{A.15}$$

$$\epsilon_{Y,\theta} = -\frac{\gamma}{1-\gamma}\theta F(\chi^*)^{-1}\sigma\left\{\frac{\sigma}{[(1-\theta)\sigma+\theta]^{1/(1-\sigma)}}\int_0^{(1-\theta)\sigma+\theta} G(\chi)\chi^{(2\sigma-1)/(1-\sigma)}d\chi - (1-\sigma)\frac{G[(1-\theta)\sigma+\theta]}{(1-\theta)\sigma+\theta}\right\} \tag{A.16}$$

Appendix B. Equilibrium with perfect enforcement

We consider the special case of perfect enforcement $\chi_j = 1$ for each entrepreneur j. Doing so enables us to characterize the interaction between monopolistic competition and monetary policy analytically and derive an upper bound of output that is useful as a benchmark in the more general version. We can obtain output in closed form by combining (29) with (26) and (14):

$$Y = [\lambda(1-\alpha)a_m^{\sigma} + \lambda\alpha k^{*\sigma}]^{\gamma/\sigma} \tag{B.1}$$

$$= [\lambda(1-\alpha)k^{*\sigma}\Upsilon(i)^{\sigma/(1-\sigma)} + \lambda\alpha k^{*\sigma}]^{\gamma/\sigma} \tag{B.2}$$

$$= k^{*\gamma}\lambda^{\gamma/\sigma}[(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)} + \alpha]^{\gamma/\sigma} \tag{B.3}$$

Then we substitute k^*:

$$Y = [\sigma\gamma Y^{(\gamma-\sigma)/\gamma}]^{\gamma/(1-\sigma)}\lambda^{\gamma/\sigma}[(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)} + \alpha]^{\gamma/\sigma} \tag{B.4}$$

$$= (\sigma\gamma)^{\gamma/(1-\sigma)}Y^{(\gamma-\sigma)/(1-\sigma)}\lambda^{\gamma/\sigma}[(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)} + \alpha]^{\gamma/\sigma} \tag{B.5}$$

Hence,

$$Y^{\frac{(1-\gamma)}{(1-\sigma)}} = (\sigma\gamma)^{\gamma/(1-\sigma)}\lambda^{\gamma/\sigma}[(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)} + \alpha]^{\gamma/\sigma} \tag{B.6}$$

which can be rearranged as

$$Y = (\sigma\gamma)^{\gamma/(1-\gamma)}\lambda^{\gamma(1-\sigma)/\sigma(1-\gamma)}\left[(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)} + \alpha\right]^{\gamma(1-\sigma)/\sigma(1-\gamma)} \tag{B.7}$$

The remaining variables are given recursively by

$$k^* = [\sigma\gamma Y^{(\gamma-\sigma)/\gamma}]^{1/(1-\sigma)} \tag{B.8}$$

$$a_m = k^*\Upsilon(i)^{1/(1-\sigma)} \tag{B.9}$$

$$r = \frac{\theta\{[f(k^*, Y) - k^*] - [f(a_m, Y) - a_m]\}}{k^* - a_m} \tag{B.10}$$

Thus, a_m is the product of k^* and a dampening factor $\Upsilon(i)^{1/(1-\sigma)}$, which depends on the availability of investment opportunities, search frictions, bargaining power, the cost of holding real balances, and product differentiation. Higher λ raises investment opportunities and thereby induces cash holdings. Greater α reduces the insurance motive of cash and therefore reduces money demand. Higher bargaining power of banks raises cash holdings through the strategic motive in negotiation.

It is helpful for comparative statics to do a first-order approximation of $\log Y$ at $i = 0$: $\log Y = \log Y|_{i=0} + i\frac{\partial \log Y}{\partial i}|_{i=0}$. Proposition 5 shows the semi-elasticity of aggregate demand, financed capital, internal finance, and revenue with respect to the nominal interest rate.

Proposition 5. *The semi-elasticity of money demand, output, investment, and revenue satisfy*

$$\frac{\partial \log a_m}{\partial i} = -\frac{1}{(1-\sigma)(i+\lambda(1-\alpha(1-\theta)))}\left\{\frac{(\gamma-\sigma)(1-\alpha)}{1-\gamma}\frac{\Upsilon(i)^{\sigma/(1-\sigma)}}{(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}+\alpha}+1\right\} \tag{B.11}$$

$$\frac{\partial \log Y}{\partial i} = -(1-\alpha)\frac{\gamma}{1-\gamma}\frac{\Upsilon(i)^{\sigma/(1-\sigma)}}{[(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}+\alpha](i+\lambda(1-\alpha(1-\theta)))} \tag{B.12}$$

$$\frac{\partial \log k^*}{\partial i} = -(1-\alpha)\frac{(\gamma-\sigma)}{(1-\gamma)(1-\sigma)}\frac{\Upsilon(i)^{\sigma/(1-\sigma)}}{[(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}+\alpha](i+\lambda(1-\alpha(1-\theta)))} \tag{B.13}$$

$$\frac{\partial \log f(k^*, Y)}{\partial i} = \frac{\partial \log k^*}{\partial i} \tag{B.14}$$

Furthermore, the first order approximation of the pass through from the interest rate to output, investment, money, and revenue at $i = 0$ satisfies

$$\frac{\partial \log Y}{\partial i}\Big|_{i=0} = -\frac{\gamma(1-\alpha)}{(1-\gamma)\lambda[1-\alpha(1-\theta)]} \tag{B.15}$$

$$\frac{\partial \log k_c}{\partial i}\Big|_{i=0} = -\frac{\gamma-\sigma}{(1-\sigma)}\frac{1-\alpha}{(1-\gamma)\lambda[1-\alpha(1-\theta)]} \tag{B.16}$$

$$\frac{\partial \log a_m}{\partial i}\Big|_{i=0} = -\frac{1}{(1-\sigma)\lambda[1-\alpha(1-\theta)]}\left[\frac{(\gamma-\sigma)(1-\alpha)}{1-\gamma}+1\right] \tag{B.17}$$

$$\frac{\partial \log f(k_c, Y)}{\partial i}\Big|_{i=0} = \frac{\partial \log k_c}{\partial i}\Big|_{i=0} \tag{B.18}$$

The semi-elasticity of output is increasing in the returns to scale γ, decreasing in access to external finance α, decreasing in the probability of getting an investment opportunity λ, and decreasing in the bargaining power of banks θ. Intuitively, under perfect enforcement, bank-financed investment only changes as a result of the aggregate demand externality, whereas cash-financed investment is directly sensitive to the cost of holding money. More opportunities for investment ($\uparrow\lambda$) or higher bank bargaining power ($\uparrow\theta$) stimulate money demand and consequently reduce transmission. The semi-elasticity of output does not, to a first order, depend on the product diversity σ. As a result, the pass through from the nominal rate to the real rate is also independent of σ, to a first-order approximation.

The semi-elasticity of financed capital k_c is negative provided that $\gamma > \sigma$, and the magnitude falls with σ. Again, the aggregate demand externality induces negative transmission to investment even for i in the neighborhood of $i = 0$, in contrast to Rocheteau et al. (2017).

Define the external share of finance $ext_f = 1 - a_m/[(1-\alpha)a_m + \alpha k_c]$. In characterizing the real lending rate, it is useful to define the composite parameter $\Theta(i)$:

$$\Theta(i) = \frac{1-\Upsilon(i)^{\frac{\sigma}{1-\sigma}}}{1-\Upsilon(i)^{1/(1-\sigma)}}$$

Proposition 6. *The external share of finance satisfies*

$$ext_f = \frac{\alpha(1-\Upsilon^{1/(1-\sigma)})}{\alpha(1-\Upsilon^{1/(1-\sigma)})+\Upsilon^{1/(1-\sigma)}} \tag{B.19}$$

A first order approximation in the neighborhood of $i = 0$ yields

$$ext_f \approx \frac{\alpha i}{(1-\sigma)\lambda[1-\alpha(1-\theta)]} \tag{B.20}$$

Thus, the external share of finance approaches zero as the nominal interest rate approaches zero. In general, the external share of finance is independent of γ but increases with σ. As noted, a rise in σ lowers the ratio of real balances to the target level of investment, making the firm more dependent on external finance.

The following lemma helps us approximate the pass through to a first order.

Lemma 5. $\lim_{i\to 0}\Theta(i) = \sigma$ *and* $\lim_{i\to 0}\Theta'(i) = \frac{\sigma}{2\lambda[1-\alpha(1-\theta)]}$. *As* $i \to \infty$, $\Theta(i) \to 1$.

Proposition 7. *The real interest rate satisfies*

$$r = \theta\left\{\frac{1}{\sigma}\Theta(i) - 1\right\}$$

As $i \to 0$, $r \to 0$, *and as* $i \to \infty$, $r \to \theta(1-\sigma)/\sigma$, *the same value as under external finance alone.*

The first order approximation to r *in the neighborhood of* $i = 0$ *is*

$$r = \frac{\theta i}{2\lambda[1-\alpha(1-\theta)]} \tag{B.21}$$

Eq. (B.21) coincides with the approximation in Rocheteau et al. (2017) without monopolistic competition and aggregate demand externalities. In particular, the real interest rate is independent of the returns to scale and, to a first-order approximation, independent of the elasticity of substitution between goods. Moreover, higher θ has a direct effect raising interest rates for a given holding of real balances a_m, and an indirect effect as entrepreneurs compensate by holding more real balances. However, the first effect predominates. As α increases, entrepreneurs have less need to insure themselves, so they reduce their real balances. The disagreement point in negotiations is worse, so the real lending rate increases. Similarly, higher λ strengthens the outside option and lowers the real lending rate.

Though product diversity has no first-order effect on the real interest rate, in general the latter decreases with σ because, as competition rises, the profitability of investment diminishes and thus the real interest rate that banks can charge falls. The appendix compares the first-order approximation of the lending rate to the global solution and shows that it is generally an upper bound and deteriorates fast for $i > 0.05$.

Appendix C. Proofs

C.1. Proof of Proposition 1

The entrepreneur's problem over real balances is

$$J(a_m, i, Y) \equiv -ia_m + \lambda(1-\alpha)\Delta_m(a_m) + \lambda\alpha\Delta_c(a_m)$$

for $Y > 0$ given. Using $\Delta'_m(0) = \infty$ and $\Delta'_c(0) = \infty$ for all $\theta > 0$, there is a positive solution to $\max_{a_m} J(a_m)$ if $\lambda(1-\alpha) > 0$ or $\lambda\alpha\theta > 0$. We check that $k_c > k_m$ for $i = 0$. If the liquidity constraint does not bind, then $k_c = k^*$ and by (30) $k_m < k^*$. If the liquidity constraint does bind, then the bargaining problem implies

$$\begin{aligned} &(1-\chi)f(k_c, Y) - a_m - \Delta_m(a_m) > 0 \\ &\Leftrightarrow (1-\chi)f(k_c, Y) - f(a_m, Y) > 0 \\ &\Leftrightarrow k_c - a_m > 0 \end{aligned}$$

C.2. Proof of Proposition 2

Write the cash-to-expected sales ratio as

$$\frac{a_m}{\lambda[\alpha f(k_c, Y) + (1-\alpha)f(a_m, Y)]} = \frac{k^*\Upsilon(i)^{1/(1-\sigma)}}{\lambda[\alpha\gamma Y^{(\gamma-\sigma)/\gamma}k^{*\sigma} + (1-\alpha)\gamma Y^{(\gamma-\sigma)/\gamma}a_m^{*\sigma}]} \tag{C.1}$$

$$= \frac{k^*\Upsilon(i)^{1/(1-\sigma)}}{\lambda\gamma Y^{(\gamma-\sigma)/\gamma}(\alpha k^{*\sigma} + (1-\alpha)a_m^{*\sigma})} \tag{C.2}$$

$$= \frac{{k^*}^{1-\sigma}\Upsilon(i)^{1/(1-\sigma)}}{\lambda\gamma Y^{(\gamma-\sigma)/\gamma}[\alpha + (1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}]} \tag{C.3}$$

$$= \frac{\sigma\gamma Y^{(\gamma-\sigma)/\gamma}\Upsilon(i)^{1/(1-\sigma)}}{\lambda\gamma Y^{(\gamma-\sigma)/\gamma}[\alpha + (1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}]} \tag{C.4}$$

$$= \frac{\sigma}{\lambda}\left[\frac{\Upsilon(i)^{1/(1-\sigma)}}{\alpha+(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}}\right] \tag{C.5}$$

proving (33). Differentiating this expression with respect to σ yields

$$\frac{1}{\lambda}\left[\frac{\Upsilon(i)^{1/(1-\sigma)}}{\alpha+(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}}\right]\left\{1+\frac{\sigma}{(1-\sigma)^2}\frac{\alpha\log\Upsilon(i)}{\alpha+(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}}\right\} \tag{C.6}$$

The first expression in brackets is positive and the second is curly braces is ambiguous. Let us restrict attention to the two terms of the curly braces. Since $\Upsilon(i)<1$ for $i>0$, the first term is positive and the second is negative. Thus, the sign of the second term can be made arbitrarily small by letting either σ or i approach 0. Hence, provided i or σ is sufficiently small, the first term dominates and cash-to-sales rise with competition.

C.3. Proof of Proposition 3

The first part of this argument parallels the proof of Proposition 1 in Rocheteau et al. (2017). If $k+\phi\geq\chi f(k,Y)$ does not bind, then the solution to (A.3) maximizes the match surplus $f(k,Y)-k$, while ϕ shares the surplus according to the bargaining powers. The solution is thereby characterized by $k=k^*$ and $\phi=\theta[f(k^*,Y)-k^*]$. The pledgeability threshold χ^* satisfies $k^*+\theta[f(k^*,Y)-k^*]=\chi^* f(k^*,Y)$. Rearranging,

$$\chi^*=\theta\frac{f(k^*,Y)}{f(k^*,Y)}+(1-\theta)\frac{k^*}{f(k^*,Y)}=(1-\theta)\sigma+\theta$$

If the pledgeability constraint binds, then k solves

$$k\in\arg\max[f(k,Y)]^{1-\theta}[\chi f(k,Y)-k]^{\theta}$$

The first order condition yields

$$\phi=\chi f(k,Y)-k \tag{C.7}$$

$$\frac{k}{f(k,Y)}=\frac{\chi\frac{\partial f(k,Y)}{\partial k}-\theta}{(1-\theta)\frac{\partial f(k,Y)}{\partial k}} \tag{C.8}$$

and we recover ϕ from the pledgeability constraint.

As in Rocheteau et al. (2017), the left hand side of (C.7) is increasing in k from 0 to ∞, with limits obtainable by L'Hopital's rule. The right hand side is decreasing for all $k\in[\hat{k},k^*]$. The right hand side evaluated at k^*, $(\chi-\theta)/(1-\theta)$, is smaller than the left hand side given $\chi<\chi^*$. At $k=\hat{k}$, the right hand side is $1/(\partial f/\partial k|_{k=\hat{k}})=1/\chi$, which exceeds the left hand side. Hence, there is a unique solution $k\in[\hat{k},k^*]$ to (C.7).

Combining (A.6) with (C.9) and

$$k=k^* \tag{C.9}$$

$$\phi=\theta[f(k^*,Y)-k^*] \tag{C.10}$$

yields (A.7).

C.4. Proof of Proposition 5

Take the logarithm of (29):

$$\log Y=\frac{\gamma}{1-\gamma}\log(\sigma\gamma)+\frac{\gamma(1-\sigma)}{\sigma(1-\gamma)}\log\lambda+\frac{\gamma(1-\sigma)}{\sigma(1-\gamma)}\log\left\{(1-\alpha)\left[\frac{\lambda(1-\alpha(1-\theta))}{i+\lambda(1-\alpha(1-\theta))}\right]^{\sigma/(1-\sigma)}+\alpha\right\} \tag{C.11}$$

Differentiating (C.11) with respect to i yields:

$$\frac{\frac{\gamma(1-\alpha)}{1-\gamma}\Upsilon(i)^{(2\sigma-1)/(1-\sigma)}\Upsilon'(i)}{(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}+\alpha}$$

Substituting $\Upsilon'(i)=-\Upsilon(i)/(i+\lambda(1-\alpha(1-\theta)))$ and rearranging yields (B.12)

As $i\to0$, $\Upsilon(i)\to1$ and $\Upsilon'(i)\to-1/[\lambda(1-\alpha(1-\theta))]$, and we obtain

$$\frac{\partial\log Y}{\partial i}=-\frac{\gamma(1-\alpha)/(1-\gamma)}{\lambda[1-\alpha(1-\theta)]}$$

Using (B.15) with (B.8) and (B.9) we can derive (B.11): note

$$\frac{\partial\log a_m}{\partial i}=\frac{\partial\log k^*}{\partial i}+\frac{1}{1-\sigma}\frac{\partial\log\Upsilon(i)}{\partial i}$$

$$= \frac{\gamma-\sigma}{\gamma(1-\sigma)}\frac{\partial \log Y}{\partial i} - \frac{1}{1-\sigma}\frac{1}{i+\lambda(1-\alpha(1-\theta))}$$
$$= -\frac{1}{(1-\sigma)(i+\lambda(1-\alpha(1-\theta)))}\left\{\frac{(\gamma-\sigma)(1-\alpha)}{1-\gamma}\frac{\Upsilon(i)^{\sigma/(1-\sigma)}}{(1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}+\alpha}+1\right\}$$

Applying (B.15) yields (B.16) and (B.17). To generate the pass through on revenue, take the logarithm of (13):

$$\log f(k^*, Y) = \log\gamma + \frac{\gamma-\sigma}{\gamma}\log Y + \sigma\log k^*$$

Differentiating with respect to i and applying (B.15) and (B.16) yields (B.18).

C.5. Proof of Proposition 6

Given $ext_f = 1 - 1/[(1-\alpha)a_m + \alpha k_c]$, the substitution of (B.8) yields

$$1 - \frac{1}{(1-\alpha)+\alpha(1/\Upsilon^{1/(1-\sigma)})}$$

which can be algebraically rearranged as

$$\frac{\alpha(1-\Upsilon(i)^{1/(1-\sigma)})}{\alpha(1-\Upsilon(i)^{1/(1-\sigma)})+\Upsilon(i)}$$

Differentiating with respect to i yields

$$-\frac{\alpha\Upsilon'(i)}{1-\sigma}\Upsilon^{\sigma/(1-\sigma)}\left\{\frac{[\alpha(1-\Upsilon^{1/(1-\sigma)})+\Upsilon^{1/(1-\sigma)}]+(1-\Upsilon^{1/(1-\sigma)})(1-\alpha)}{[\alpha(1-\Upsilon^{1/(1-\sigma)})+\Upsilon^{1/(1-\sigma)}]^2}\right\} \tag{C.12}$$

As $i\to 0$, $\Upsilon\to 1$, so the expression in braces converges to 1. Using the limit of $\Upsilon'(i)$ as $i\to 0$, we obtain

$$\frac{\partial ext_f}{\partial i} = \frac{\alpha}{(1-\sigma)\lambda[1-\alpha(1-\theta)]}$$

C.6. Proof of Proposition 7

Using (B.10), (B.9), and (13), we can express the real interest rate as (7). From Lemma 5 and Proposition 7, it immediately follows that as $i\to 0$, $r\to 0$, and, as $i\to\infty$, $r\to\theta[(1-\sigma)/\sigma]$

C.7. Proof of Corollary 1

The only nontrivial part is the proof that $\partial k/\partial\theta = (\sigma-1)/[(1-\theta)\sigma+\theta] < 0$.First, rewrite (C.7) as

$$(1-\theta)\sigma = \chi\frac{\partial f}{\partial k} - \theta$$

and totally differentiate with respect with θ to obtain

$$\frac{\partial k}{\partial\theta} = \frac{1}{\chi\partial^2 f/\partial k^2}$$

Plugging in for $\partial^2 f/\partial k^2$, we obtain

$$\frac{\partial k}{\partial\theta} = \frac{k^{1-\sigma}}{\chi\sigma(\sigma-1)\gamma Y^{(\gamma-\sigma)/\gamma}}$$

Plugging in for k and simplifying, we obtain

$$\frac{\partial k}{\partial\theta} = \frac{\sigma-1}{(1-\theta)\sigma+\theta} < 0$$

C.8. Proof of Lemma 2

Differentiate the right hand side of (21) with respect to θ, obtaining:

$$\frac{1}{(1-\theta)^2}\frac{1-\chi\frac{\partial f}{\partial k_c}}{(1-\chi)\frac{\partial f}{\partial k_c}} - \frac{\theta}{1-\theta}\frac{(1-\chi)\partial^2 f/\partial k_c^2}{[(1-\chi)\frac{\partial f}{\partial k_c}]^2}\frac{\partial k_c}{\partial\theta}$$

Differentiating the left hand side with respect to θ yields

$$-\frac{\partial k_c}{\partial\theta}\left\{\frac{[(1-\chi)f(k_c,Y)-f(a_m,Y)][1-\chi\frac{\partial f}{\partial k_c}]+(a_m+\chi f(k_c,Y)-k_c)(1-\chi)\frac{\partial f}{\partial k_c}}{[(1-\chi)f(k_c,Y)-f(a_m,Y)]^2}\right\}$$

If $\frac{\partial k_c}{\partial\theta}\geq 0$, then the right hand side is positive but the left hand side is negative. Hence, $\frac{\partial k_c}{\partial\theta}<0$. It is clear from inspecting $h(a_m, Y)$ defined in (C.17) that $\frac{\partial k_c}{\partial a_m}>0$. Consequently, $\frac{\partial[a_m+\chi f(k_c,Y)]}{\partial a_m}>1$.

C.9. Proof of Lemma 3

Differentiating the bargaining solution for r (23) with respect to a_m in the case $a > a^*$ yields

$$\frac{-\frac{\partial f}{\partial a_m}(k^*-a_m)+[f(k^*,Y)-k^*-(f(a_m,Y)-a_m)](-1)}{(k^*-a_m)^2} \tag{C.13}$$

$$=\frac{-\frac{\partial f}{\partial a_m}(k^*-a_m)+[f(k^*,Y)-f(a_m,Y)]}{(k^*-a_m)^2} \tag{C.14}$$

$$=\frac{\frac{f(k^*,Y)-f(a_m,Y)}{k^*-a_m}-\frac{\partial f}{\partial a_m}}{(k^*-a_m)} \tag{C.15}$$

$$<0,\quad \text{from concavity of f} \tag{C.16}$$

Furthermore, as $a_m\to k^*$, both the numerator and denominator approach zero, so the form is indeterminate. However, differentiating the numerator and denominator of (23) with respect to a_m yields $\frac{\partial f}{\partial a_m}-1$, which approaches 0 as $a_m\to k^*$.

C.10. Effect of money holdings on investment

The curve $h(a_m, k_c, \chi_i, Y)=\frac{\partial k_c}{\partial a_m}$ specifies the change in investment with respect to real balances for an entrepreneur with coefficient χ_i, economic conditions Y, assets a_m, and investment level k_c.

$$-\frac{\frac{\partial f}{\partial k_c}+\frac{\theta}{(1-\theta)(1-\chi_i)}\frac{\partial f}{\partial a_m}\left[1-\chi_i\frac{\partial f}{\partial k_c}\right]}{\left[\chi_i\frac{\partial f}{\partial k_c}-1\right]\frac{\partial f}{\partial k_c}+[a_m+\chi f(k_c,Y)-k_c]\frac{\partial^2 f}{\partial k_c^2}+\frac{\theta}{(1-\theta)(1-\chi_i)}\left\{[(1-\chi_i)f(k_c,Y)-f(a_m,Y)]\chi_i\frac{\partial^2 f}{\partial k_c^2}-(1-\chi_i)\frac{\partial f}{\partial k_c}\left[1-\chi_i\frac{\partial f}{\partial k_c}\right]\right\}} \tag{C.17}$$

provided that $a_m < a^*$, and is 0 otherwise.

C.11. Proof of Lemma 4

From Lemma 1, we have that the threshold a^* given bargaining power χ is

$$(\chi^*-\chi)=\frac{(1-\theta)a^*+\theta f(a^*,Y)}{f(k^*,Y)}$$

Now, we shift the point of view and find the minimum threshold χ^{**} consistent with the first best. Using $a^*=k^*\Upsilon(i)^{1/(1-\sigma)}$, we have

$$(\chi^*-\chi^{**})=(1-\theta)\frac{k^*\Upsilon(i)^{1/(1-\sigma)}}{f(k^*,Y)}+\theta\Upsilon(i)^{\sigma/(1-\sigma)}$$

Using $\sigma f(k^*,Y)/k^*=1$, we can rewrite this as

$$\chi^{**}=\chi^*-[(1-\theta)\sigma\Upsilon(i)^{1/(1-\sigma)}+\theta\Upsilon(i)^{\sigma/(1-\sigma)}] \tag{C.18}$$

C.12. Proof of Lemma 5

1. The quantity $\Theta(i)$ is an indeterminate form as $i\to 0$, so we differentiate numerator and denominator with respect to i:

   $$-\frac{\frac{\sigma}{1-\sigma}\Upsilon(i)^{(2\sigma-1)/(1-\sigma)}}{-\frac{1}{1-\sigma}\Upsilon(i)^{\sigma/(1-\sigma)}}$$

   As $i\to 0$, $\Upsilon\to 1$, and hence any power of Υ approaches 1. Thus, $\Theta(i)\to\sigma$

2. By the quotient rule,

$$\Theta'(i) = \Upsilon'(i)\frac{(1-\Upsilon(i)^{1/(1-\sigma)})(-\sigma/(1-\sigma)\Upsilon^{(2\sigma-1)/(1-\sigma)} + (1/(1-\sigma))(1-\Upsilon(i)^{\sigma/(1-\sigma)})\Upsilon(i)^{\sigma/(1-\sigma)}}{[1-\Upsilon(i)^{1/(1-\sigma)}]^2}$$

First, calculate $\Upsilon'(i) = -\frac{\lambda(1-\alpha(1-\theta))}{[i+\lambda(1-\alpha(1-\theta))]^2} \rightarrow -\frac{1}{\lambda(1-\alpha(1-\theta))}$ as $i \rightarrow 0$.

The remainder of the expression is an indeterminate form. Differentiating both numerator and denominator yields

$$\frac{-\left(\frac{\sigma}{1-\sigma}\right)\left[\frac{2\sigma-1}{1-\sigma}\Upsilon^{(3\sigma-2)/(1-\sigma)} - \frac{2\sigma}{1-\sigma}\Upsilon(i)^{(3\sigma-1)/(1-\sigma)}\right] + \frac{1}{1-\sigma}\left[\frac{\sigma}{1-\sigma}\Upsilon(i)^{(2\sigma-1)/(1-\sigma)} - \frac{2\sigma}{1-\sigma}\Upsilon(i)^{(3\sigma-1)/(1-\sigma)}\right]}{-\frac{2}{1-\sigma}[1-\Upsilon(i)^{1/(1-\sigma)}]\Upsilon(i)^{\sigma/(1-\sigma)}} \tag{C.19}$$

which is also an indeterminate form. We can simplify as follows:

$$\frac{-\frac{1}{2(1-\sigma)}\left[(1-2\sigma)\sigma\Upsilon(i)^{(3\sigma-2)/(1-\sigma)} + 2\sigma^2\Upsilon(i)^{(3\sigma-1)/(1-\sigma)} + \sigma\Upsilon(i)^{(2\sigma-1)/(1-\sigma)} - 2\sigma\Upsilon(i)^{(3\sigma-1)/(1-\sigma)}\right]}{\Upsilon(i)^{\sigma/(1-\sigma)} - \Upsilon(i)^{(\sigma+1)/(1-\sigma)}} \tag{C.20}$$

Differentiating numerator and denominator again and letting $i \rightarrow 0$ yields the algebraic expression

$$\begin{aligned}
&\frac{1}{2(1-\sigma)}[(1-2\sigma)\sigma(3\sigma-2) + 2\sigma^2(3\sigma-1) + \sigma(2\sigma-1) - 2\sigma(3\sigma-1)]\\
&= \frac{1}{2(1-\sigma)}[\sigma(2\sigma-1)(2-3\sigma+1) + 2\sigma(\sigma-1)(3\sigma-1)]\\
&= \frac{1}{2(1-\sigma)}[3\sigma(2\sigma-1)(1-\sigma) + 2\sigma(\sigma-1)(3\sigma-1)]\\
&= \frac{1}{2}[3\sigma(2\sigma-1) + 2\sigma(1-3\sigma)]\\
&= \frac{1}{2}[6\sigma^2 - 3\sigma - 6\sigma^2 + 2\sigma]\\
&= -\sigma/2
\end{aligned}$$

Consequently, $\lim_{i\rightarrow 0}\Theta'(i) = \frac{\sigma}{2\lambda[1-\alpha(1-\theta)]}$.

Second, the first part of the expression can be rewritten as

$$\frac{\sigma}{1-\sigma}\frac{\Upsilon^{(2\sigma-1)/(1-\sigma)}}{1-\Upsilon(i)^{1/(1-\sigma)}}$$

which approaches zero.

The second part of the expression can be written as the product

$$\frac{\Upsilon(i)^{\sigma/(1-\sigma)}}{1-\sigma}\frac{1-\Upsilon(i)^{\sigma/(1-\sigma)}}{[1-\Upsilon(i)^{1/(1-\sigma)}]^2}$$

The first factor approaches $1/(1-\sigma)$ and the second factor requires L'Hopital's rule:

$$\frac{\frac{-\sigma}{1-\sigma}\Upsilon(i)^{(2\sigma-1)/(1-\sigma)}\Upsilon'(i)}{2[1-\Upsilon(i)^{1/(1-\sigma)}]\left(\frac{-1}{1-\sigma}\right)\Upsilon'(i)} to$$

The first-order approximation of r in the neighborhood of $i = 0$ follows immediately from Lemma 5.

Appendix D. Extra figures

D.1. Effect of bargaining power on pass through and transmission

Fig. D.14 how changes in the bank bargaining bargaining power influences the effects of the policy rate. A higher θ increases the pass through and the external share of finance, but reduces transmission into aggregate investment and output. The intuition is that higher bargaining power induces entrepreneurs to hold more cash, which reduces transmission.

D.2. Distribution of pledgeability coeffients by two-digit SIC codes

I address the issue of whether the distribution of pledgeability coefficients is sensitive to broad industry categories. Accordingly, Fig. D.15 plots the distributions for the two-digit Standard Industrial Classification groups: Agriculture, Forestry and Fishing, Mining, Construction, Manufacturing, Wholesale Trade, Retail Trade, and Services. Most of the distributions are broadly similar. The Services sector has somewhat more mass in the left tail, and Retail Trade and Mining have somewhat less mass in the left tail. Mining is also unusually skewed, with a greater proportion of pledgeable assets.

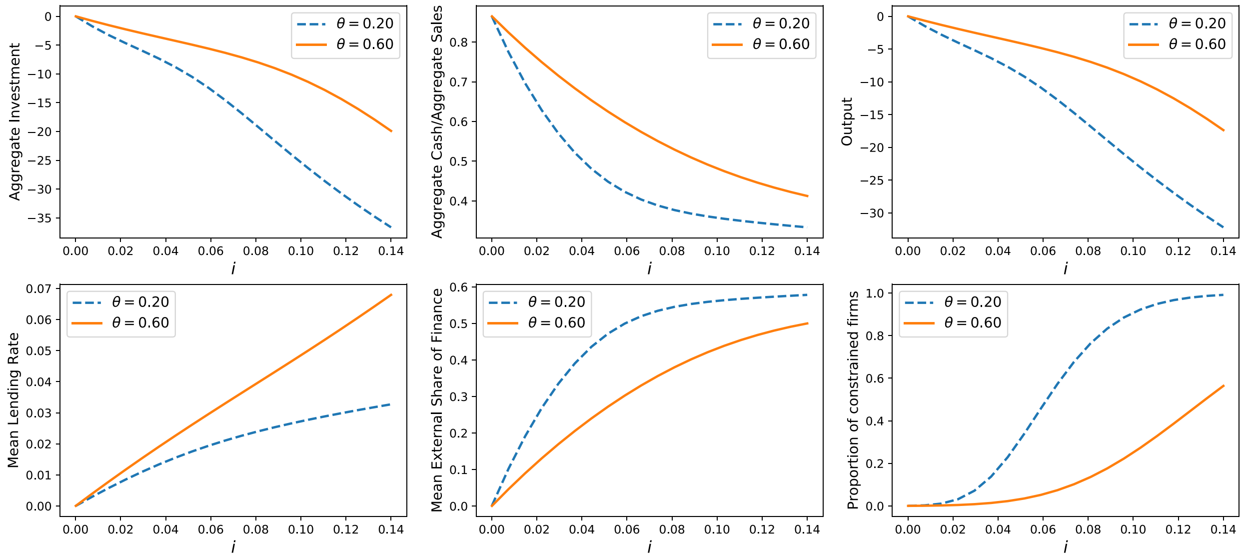

Fig. D.14. Effect of nominal interest rate: different values of θ.

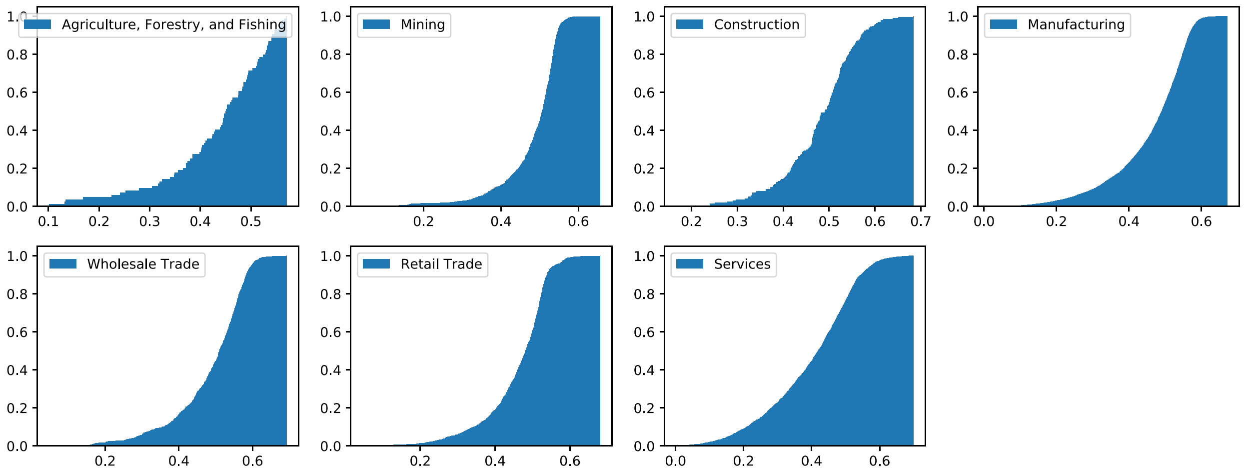

Fig. D.15. Estimated pledgeability distribution by 2-digit SIC group.

Appendix E. Computational strategy under heterogeneous firms with limited enforcement

1. We start with aggregate demand given by (29):

$$Y = \lambda^{\gamma/\sigma}\left\{[1 - G(\chi^{**})]k^{*\sigma}[\alpha + (1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}] + \int_0^{\chi^{**}}[\alpha k_c(\chi)^\sigma + (1-\alpha)a_m(\chi)^\sigma]dG(\chi)\right\}^{\gamma/\sigma} \tag{E.1}$$

$$Y = \lambda^{\gamma/\sigma}\left\{[1 - I_{\chi^{**}}(a,b)]k^{*\sigma}[\alpha + (1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}] + \int_0^{\chi^{**}}[\alpha k_c(\chi)^\sigma + (1-\alpha)a_m(\chi)^\sigma]dG(\chi)\right\}^{\gamma/\sigma} \tag{E.2}$$

2. We define a mapping $L : \mathbb{R}^+ \to \mathbb{R}$ as follows. First goal is to evaluate

$$I_1 = \int_0^{\chi^{**}}[\alpha k_c(\chi)^\sigma + (1-\alpha)a_m(\chi)^\sigma]dG(\chi)$$

3. We approximate I_1 using the composite midpoint method. Divide $[0, \chi^{**}]$ into N evenly spaced subintervals, of length $h = \chi^{**}/N$.
4. Choose sequence of χ_i on midpoints of subintervals: $\chi_i = (i - 1/2)h, i = 1, \dots, N$
5. Compute vector $a_m(\chi)^\sigma$ using (24) and then compute vector $k_c(\chi)^\sigma$

6. Compute

$$Q_N = h\sum_{i=1}^{N}[(1-\alpha)a_m(\chi_i)^\sigma + \alpha k_c(\chi_i)^\sigma]dG(\chi_i)$$

7. Compute quantity $I_2 = [1 - I_{\chi^{**}(a,b)}]k^{*\sigma}[\alpha + (1-\alpha)\Upsilon(i)^{\sigma/(1-\sigma)}]$ and then $Y_{new} = \lambda^{\gamma/\sigma}(Q_N + I_2)^{\gamma/\sigma}$
8. Return $(Y_{new} - Y)/Y$
9. Find zero of function $L(Y)$ in endpoints $(0, \overline{Y})$.
10. Compute vector $a_m(\chi)$ and $k_c(\chi)$
11. Calculate k^* and then aggregate investment again using the composite midpoint rule.
12. Also calculate the following via numerical integration:

$$K_{cons} = \int_0^{\chi^{**}} \lambda[\alpha k_c(\chi) + (1-\alpha)a_m(\chi)]dG(\chi)$$
$$a_{m,cons} = \int_0^{\chi^{**}} a_m(\chi)dG(\chi)$$
$$r_{cons} = \int_0^{\chi^{**}} \left(\frac{a_m(\chi) + \chi f(k_c(\chi), Y) - k_c(\chi)}{k_c(\chi) - a_m(\chi)}\right)dG(\chi)$$

13. We recover mean investment, money holdings, and the real interest rate as follows:

$$K = [1 - G(\chi^{**})]k^*[\alpha + (1-\alpha)\Upsilon(i)^{1/(1-\sigma)}] + K_{cons}$$
$$\mathbb{E}(a_m) = [1 - G(\chi^{**})]k^*\Upsilon(i)^{1/(1-\sigma)} + a^e_{m,cons}$$
$$\mathbb{E}(r) = [1 - G(\chi^{**})]\frac{\theta\{[f(k^*, Y) - k^*] - [f(a_m, Y) - a_m]\}}{k^* - a_m} + r_{cons}$$

The remaining variables are computed similarly.

Appendix F. Empirical analysis

The dataset is drawn from the Compustat North America database from 1964 to 2017. The data excludes utilities (SIC codes 4900–4999), as they are heavily regulated; financial firms (SIC codes 6900–6999); firms not incorporated in the United States; and public administration (SIC codes 9100–9729). We also require require sales, cost of goods sold, cash, and assets to be positive, and for assets to exceed the sum of receivables (RECT), inventory (INVT), and the net value of capital (PPENT). Following Almeida and Campello (2007), observations in which firm sales exceed 100% are excluded, as these indicate mergers or major reorganizations. Moreover, we require firms to operate for at least three years after making the previous exclusions. A separate production function is estimated for each four-digit NAICS group.

Estimating the markup requires data on sales (SALE), total variable costs, (COGS), construction of the capital stock, the GDP Deflator from the Bureau of Economic Analysis (Fred code GDPDEF), and a deflator for the capital stock from Feenstra et al. (2015), Fred code PLKCPPUSA670NRUG. For each firm, the capital stock is initialized using the book value (PPEGT). Future capital is constructed using the law of motion $k_{i,t+1} = k_{i,t} + \text{inv}_{it+1}$, where net investment is calculated as $\text{inv}_{it} = \text{PPENT}_{it} - \text{PPENT}_{it-1}$. The quantity PPENT is the net book value of the capital stock. I linearly interpolate PPENT to reduce missing values. For estimating the effects of markups on cash holdings, we measure the latter as either cash and cash equivalents (CHE) divided by (SALE), which corresponds to the model analogue. We define the price-to-cost ratio as (SALE-COGS)/SALE and winsorize it and the cash series with a 1% band. The estimated markup measure is also winsorized with a 1% band.

I summarize the procedure, which closely follows De Loecker and Eeckhout (2017). Consider an economy with N firms, indexed by $i = 1, \ldots, N$. In each period t, firm i minimizes the cost of producing Q_{it} given production function $Q(\cdot)$:

$$Q(\Omega_{it}, V_{it}, K_{it}) = \Omega_{it}F_t(V_{it}, K_{it})$$

where $V = (V^1, \ldots, V^J)$ are the variable production inputs (labor, energy, intermediate inputs, materials, etc.), K_{it} is the capital stock and Ω_{it} is total factor productivity. The associated Lagrangian is

$$\mathcal{L}(V_{it}, K_{it}, \Lambda_{it}) = P^V_{it}V_{it} + r_{it}K_{it} - \Lambda_{it}(Q(\cdot) - Q_{it})$$

where P^V is the price of the variable input, r is the user cost of capital, and Λ_{it} is the Lagrangian multiplier, which equals the marginal cost. The first order condition with respect to V yields

$$\frac{\partial L_{it}}{\partial V_{it}} = P^V_{it} - \Lambda_{it}\frac{\partial Q_{it}}{\partial V_{it}} = 0$$

Multiplying all terms by V_{it}/Q_{it} and rearranging yields the output elasticity of input V:

$$\theta^V_{it} = \frac{\partial Q}{\partial V_{it}}\frac{V_{it}}{Q_{it}} = \frac{1}{\Lambda_{it}}\frac{P_{it}V_{it}}{Q_{it}}$$

Define the gross markup $\mu = P/\Lambda$ and plug in to obtain

$$\mu_{it} = \theta^V_{it} \frac{P_{it} Q_{it}}{P^{V_j}_{it} V_{it}}$$

To measure the markup, we just need the revenue share of the variable input $P^V_{it} V_{it} / (P_{it} Q_{it})$ and the output elasticity of the variable input θ^V_{it}. From Compustat, we observe sales $S_{it} = P_{it} Q_{it}$ and total variable costs $C_{it} = \sum_j P^{Vj}_{it} V^j_{it}$. Since the data does not decompose the variable costs by input, I follow De Loecker and Eeckhout (2017) and use total variable costs.

For a given industry I use the translog production function:

$$q_{it} = \beta_v v_{it} + \beta_k k_{it} + \beta_{vv} v^2_{it} + \beta_{kk} k^2_{it} + \beta_{vk} vk + \omega_{it} +_{it}$$

where lower cases denote logs and $\omega_{it} = \ln \Omega_{it}$ and q_{it} is log the log of firm sales divided by the GDP deflator. We impute the value of the capital stock for each firm using the perpetual inventory method, discussed above.

The main endogeneity challenge arises from unobserved firm productivity ω_{it}. In general, unobserved productivity can be correlated with both the choice of inputs and output, rendering the estimation inconsistent. An early technique is the use of fixed effects estimation, which requires $\omega_{it} = \omega_{it-1}$ for all t. As this assumption is very strong, I only use it to generate starting values in estimation.

Identification relies on the two-stage approach proposed by Ackerberg et al. (2006). This process relies on the existence of a control function $\omega_{it} = h(v_{it}, k_{it})$ and an AR(1) process for productivity: $\omega_{it} = \rho \omega_{it-1} +_{it}$. The first stage estimates

$$q_{it} = \phi_t(v_{it}, k_{it}) +_{it}$$

where $\phi_t(v_{it}, k_{it}) = \beta_v v_{it} + \beta_k k_{it} + \beta_{vv} v^2_{it} + \beta_{kk} k^2_{it} + \beta_{vk} v_{it} k_{it} + h(v_{it}, k_{it})$. I assume that $h(\cdot)$ can be written in the translog form as well, so that

$$\phi_t(v_{it}, k_{it}) = \gamma_v v_{it} + \gamma_k k_{it} + \gamma_{vv} v^2_{it} + \gamma_{kk} k^2_{it} + \gamma_{vk} v_{it} k_{it} + \gamma_{it} +_{it} \tag{F.1}$$

From (F.1) we obtain the estimate $\widehat{\phi}$. Given a candidate $\beta = (\beta_v, \beta_k, \beta_{vv}, \beta_{kk}, \beta_{vk})$, we obtain $\widehat{\omega}_{it} = \phi_{it} - \beta_v v_{it} - \beta_k k_{it} - \beta_{vv} v^2_{it} - \beta_{kk} k^2_{it} - \beta_{vk} v_{it} k_{it}$. The innovation to productivity $\xi_{it}(\beta)$ is identified as the residual of the regression $\widehat{\omega}_{it}$ on $\widehat{\omega}_{it-1}$. Using the estimate $\widehat{\xi}_{it}(\beta)$ one can form the moment

$$\mathbb{E}\left\{ \xi_{it}(\beta)(k_{it}\ v_{it-1}\ k^2_{it}\ v^2_{it-1}\ v_{it-1} k_{it})' \right\}$$

and estimate β by minimizing the sample analogue. To justify the moment conditions, write $\omega_{it} = \mathbb{E}[\omega_{it} | I_{it-1}] + \xi_{it}$, where I_{it-1} is the information set at time $t-1$. The innovation to productivity is uncorrelated with any last-period information, which includes (predetermined) capital and lagged labor.

The final markup estimate is

$$\mu_{it} = (\beta_v + 2\beta_{vv} + \beta_{vk}) \frac{S_{it}}{C_{it}}$$

Given the estimated markup, we regress

$$cash_{i,t} = \beta_1 \mu_{i,t-1} + \beta_2 \chi_{i,t-1} + \beta'_3 \Gamma_{i,t-1} + \alpha_j + \lambda_t +_{i,t} \tag{F.2}$$

for firm i, industry j, and year t. $Cash_{i,t}$ is cash divided by sales, $\mu_{i,t}$ is the price-to-cost ratio or the log of the constructed markup, $\chi_{i,t}$ is the (time-varying) pledgeability measure, $Y_{i,t}$ is a vector of controls, α_j are industry-fixed effects, and λ_t are time-fixed effects. The regressors and controls are lagged following Morellec et al. (2014) to ensure that they are predetermined relative to cash holdings. The control variables $Y_{i,t}$ consist of firm size (the logarithm of sales divided by the GDP deflator), the market-to-book ratio, and cash flow. The market-to-book ratio is defined as the market value of assets defined as the book value of assets. Specifically, market-to-book is $(AT - CEQ + CSHO * PRCC_C)/AT$, where CEQ is the book value of common equity, $CSHO$ is the number of common shares outstanding, and $PRCC_C$ is the value of the common shares at the closing of the calendar year. Cash flow is defined as $(OIBDP - XINT - TXT - DVC)/AT$, where $OIBDP$ is operating income before depreciation, $XINT$ is interest and related expenses, TXT is income taxes, and DVD are dividends on ordinary stock.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jedc.2019.03.001.

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