Building and Environment 72 (2014) 343–355

Contents lists available at ScienceDirect

Building and Environment

journal homepage: www.elsevier.com/locate/buildenv

Theory and applications of HVAC control systems – A review of model predictive control (MPC)

Abdul Afram*, Farrokh Janabi-Sharifi

Department of Mechanical and Industrial Engineering, Ryerson University, 350 Victoria Street, Toronto, ON M5B 2K3, Canada

ARTICLE INFO

Article history:
Received 5 August 2013
Received in revised form
16 November 2013
Accepted 18 November 2013

Keywords:
Model predictive control (MPC)
HVAC control review
Theory
Applications
Performance
Comparison

ABSTRACT

This work presents a literature review of control methods, with an emphasis on the theory and applications of model predictive control (MPC) for heating, ventilation, and air conditioning (HVAC) systems. Several control methods used for HVAC control are identified from the literature review, and a brief survey of each method is presented. Next, the performance of MPC is compared with that of other control approaches. Factors affecting MPC performance (including control configuration, process type, model, optimization technique, prediction horizon, control horizon, constraints, and cost function) are elaborated using specific examples from the literature. The gaps in MPC research are identified, and future directions are highlighted.

1. Introduction

With the significant increase of energy consumption in buildings, energy saving strategies have become a priority in energy policies in many countries. For instance, building energy consumption in the EU was 37% of the final energy totals in 2004 [1]. In the USA, building energy consumption accounted for 41% of primary energy consumption in 2010 [2]. The categories of building services and heating, ventilation, and air conditioning (HVAC) systems make up the major sources of energy use in buildings (almost 50% [1,2]). Therefore, the development and implementation of effective control techniques for HVAC systems is of primary importance. In particular, with the decreased costs of data processing, storage, and communication over recent years, the design and implementation of more complex control techniques have become feasible.

Despite the similarity of HVAC control to other types of process control, certain features exist that render HVAC system control unique and challenging, including the following:

- Nonlinear dynamics;
- Time-varying system dynamics and set-points;
- Time-varying disturbances;
- Poor data due to low resolution of analog-to-digital converter (ADC) devices, sampling rates, accuracy of sensors, and lack of access to network forecasting and environmental information;
- Interacting and at times conflicting control loops; and
- Lack of supervisory control (in many buildings).

Many control methods have been developed or proposed for HVAC systems. However, because of their simplicity, on/off and PID control are still used in many HVAC systems, resulting in inconsistent performance among such systems. With advances in data storage, computing, and communication devices, it is now feasible to adopt and implement a proper control approach to overcome the inherent issues in HVAC control. The focus of this paper is on a survey of control methods for HVAC systems, and emphasis is placed on the model predictive control (MPC) approach because research on MPC development for HVAC systems has intensified over the last years due to its many inherent advantages, which include

- Use of a system model for anticipatory control actions rather than corrective control;
- Integration of a disturbance model for disturbance rejection;
- Ability to handle constraints and uncertainties;
- Ability to handle time-varying system dynamics and a wide range of operating conditions;
- Ability to cope with slow-moving processes with time delays;

* Corresponding author. Tel.: +1 416 979 5000x7089; fax: +1 416 979 5265.
E-mail addresses: abdul.afram@ryerson.ca, abdulafram@gmail.com (A. Afram), fsharifi@ryerson.ca (F. Janabi-Sharifi).

0360-1323/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.buildenv.2013.11.016

- Integration of energy conservation strategies in the controller formulation;
- Use of a cost function for achievement of multiple objectives;
- Use of advanced optimization algorithms for computation of control vectors;
- Ability to control the system at both the supervisory and local loop levels.

However, a comprehensive survey of MPC approaches for HVAC systems is still lacking. In particular, selected trends and issues related to MPC design must be identified.

The organization of this paper is as follows. First, a review of HVAC systems is presented to outline the spectrum of control tasks in HVAC systems. Section 2 includes a brief review of previous surveys related to HVAC control. Section 3 classifies the approaches to HVAC control according to methodology, scope and implementation to create a framework with which to compare MPC with other methods. Section 4 discusses the comparison of MPC with other methods as well as the factors that affect its performance. Finally, Section 5 includes a summary of important factors that govern MPC design and outlines open design problems for HVAC systems.

2. Previous surveys

A large body of literature has been published on applications of MPC to HVAC systems, but to the best of the authors' knowledge, no recent comprehensive review has been published on the theory and applications of MPC.

Brief reviews of hard and soft control techniques were reported in Refs. [3,4], respectively. The hard control techniques reviewed in Ref. [3] include gain scheduling, optimal control, robust control, MPC, and nonlinear and adaptive control. The soft or intelligent control techniques reviewed in Ref. [4] include controllers based on the artificial neural network (ANN), fuzzy logic (FL), and genetic algorithm (GA). Intelligent control techniques such as neuro- and genetic-fuzzy approaches were also reviewed in Ref. [5]. A review of hybrid controllers resulting from the fusion of hard and soft control techniques was also provided in Ref. [4]. ANN and GA applications for energy conservation in HVAC systems were comprehensively reviewed in Ref. [6]. A review of hybrid and soft techniques (i.e., fuzzy-P, fuzzy-PI, fuzzy-PID, adaptive-fuzzy, fuzzy-neural controllers) and multi-agent control systems (MACs) for energy management was provided in Ref. [7]. A review of fuzzy modeling and control of HVAC systems was published in Ref. [8]. A review of load forecasting in HVAC systems using intelligent control techniques was reported in Refs. [9,10].

An overview of HVAC simulation approaches that covers the modeling of HVAC components, controls, and systems was presented in Ref. [11]. An overview of supervisory and optimal control of HVAC systems was given in Ref. [12]. The optimization techniques used in supervisory control (i.e., least squares, simplex search, gradient-based search, sequential quadratic programming, evolutionary programming and GA) were also reviewed in Ref. [12]. A survey of energy-efficient strategies for HVAC systems (i.e., heat recovery, liquid pressure amplification, and thermal storage) was conducted in Ref. [13]. Automatic controls for HVAC systems (i.e., on/off control, PID control, time control [on/off switch, fixed time boosted start, and optimum start and stop]) were reviewed in Ref. [14]. Additionally, a survey of the theory and applications of adaptive control for HVAC systems was given in Ref. [15].

3. Classification of HVAC control methods

A classification for control methods in HVAC systems is illustrated in Fig. 1. The control methods are divided into classical control, hard control, soft control, hybrid control, and other control techniques. Brief details of each method are provided in the following sections.

3.1. Classical control

Classical controllers consist of the most commonly used control techniques, such as on/off control and P, PI, and PID control. The on/off controller uses an upper and lower threshold to regulate the process within the given bounds. The P, PI, and PID controllers use error dynamics and modulate the controlled variable to achieve accurate control of the process.

Classical controllers are used for the dynamic control of cooling coil units [16,17], room temperature control [18–22], damper gap rate control [17,23], supply air pressure control [17,24], supply air temperature control [25,26], variable air volume (VAV) unit temperature control [27], evaporator supply heat control [27], and heater control [17]. Most of the research is focused on finding optimal tuning and auto-tuning methods for PID controllers.

Although the on/off controller is the most intuitive and easiest to implement, it is unable to control moving processes with time delays. Because of the high thermal inertia of many HVAC processes, a process that is controlled using an on/off controller displays large swings from the set points. The PID controller produces promising results, but tuning the controller parameters is cumbersome, and the performance of the controller degrades if the operating conditions vary from the tuning conditions. Retuning or auto-tuning approaches for the PID controller [28] can be time-consuming. In certain applications, auto-tuning might be unacceptable because of its intrusive nature relative to normal operation [29].

3.2. Hard control

Hard controllers are based on a theory for control systems composed of gain scheduling control, nonlinear control, robust control, optimal control, and MPC.

In gain scheduling control, a nonlinear system is divided into piecewise linear regions. For each of the linear regions, a linear PI or PID controller is designed with a different set of gains. Self-tuning PI or PID controllers are also proposed in the literature to vary the controller gains based on the state of the process. For example, in Ref. [19], two PI controllers are tuned to meet the high and low heat demand conditions in a hydronic-radiator-based HVAC system. In Ref. [24], to control the supply air pressure, a PI controller is used with gains based on the error between the set point and the measured supply air pressure.

For nonlinear controller design, the control law can be derived using Lyapunov's stability theory, feedback linearization and adaptive control techniques. The control law is used to drive the nonlinear system toward a stable state while achieving the control objectives. Nonlinear controllers have been applied to air handling unit (AHU) control [30], cross flow water-to-air heat exchanger control [31], and control of greenhouse environments (ventilation, cooling and moisturizing) control [32].

The purpose of robust control is to design a controller that works well under time-varying disturbances and changes in parameters. Examples of robust control include supply air temperature control [33], supply airflow rate control [33], and zone temperature control [34].

The optimal control algorithm solves an optimization problem to minimize a certain cost function. The objectives of optimization in HVAC systems are generally minimization of energy consumption and control effort and maximization of thermal comfort.

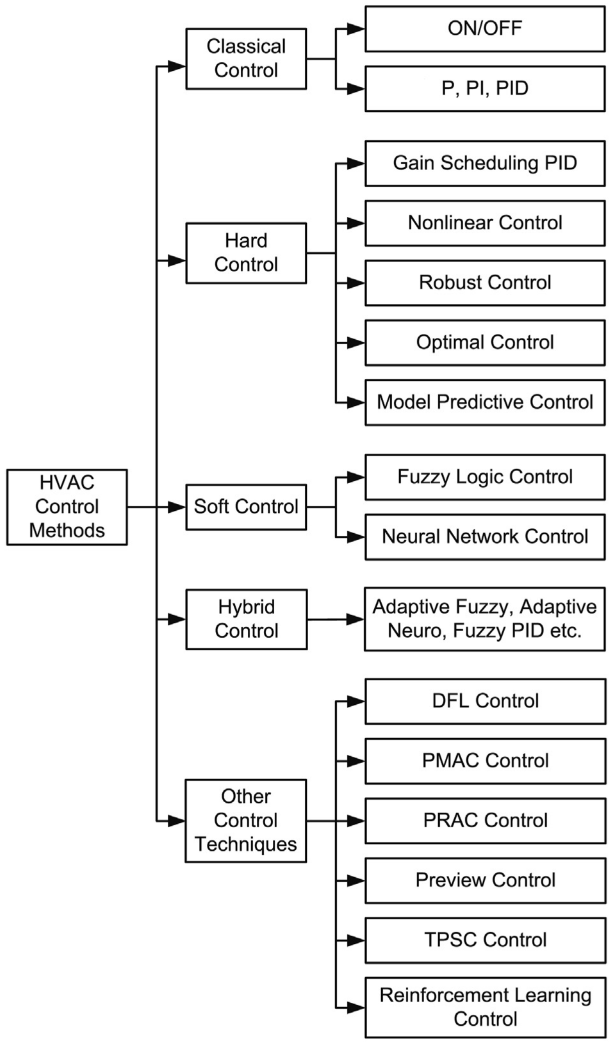

Fig. 1. Classification of control methods in HVAC systems.

Examples of optimal control design include active thermal storage control [35], passive thermal storage control [36], energy optimization of HVAC system [37,38], VAV system control [39], and building heating and cooling control [40,41].

The hard controller techniques are well established in the control system design field. The nonlinear control techniques are effective but require the identification of stable states and complex mathematical analysis for controller design. For gain scheduling control design, the identification of linear regions and design of switching logic between regions is necessary, and the manual tuning of multiple PID controllers in these regions can be quite cumbersome. Optimal control and robust control are promising techniques for HVAC process control because they are capable of rejecting disturbances and time-varying parameters. In general, robustness is difficult to guarantee in HVAC systems, which are subject to varying conditions in buildings. Many of these approaches also require the specification of additional parameters, which could be difficult and impractical for integration in HVAC systems. Among the hard control approaches, MPC is one of the most promising techniques because of its ability to integrate disturbance rejection, constraint handling, and slow-moving dynamic control and energy conservation strategies into controller formulation.

3.3. Soft control

Soft control techniques such as those based on FL [42–45] and ANN [6,26,46–48] are comparatively new techniques made possible by the advent of digital controllers.

In an FL controller [42–45], control actions are implemented in the form of if-then-else statements. The FL also can be incorporated for the auto-tuning of PID controller gains in which PID control represents the local scope of control, and the FL supervisor is often used to optimize the response of the system on the global scale. The fuzzy supervisor also acts as an arbiter and resolves conflicting objectives from the local level controllers by prioritizing certain controllers over others based on the common goals of reduction in energy consumption and maintenance of thermal comfort. Alternatively, the FL can be implemented on both the local and supervisory levels of control. Examples of FL design include predicted mean vote (PMV)-based thermal comfort control [43], which controls temperature, humidity, and air velocity in an AHU. Another example of FL is the design of a three-level hierarchical supervisory-FL controller to generate the operating modes of the water and air subsystems and the set-points for the lower level controllers [49].

The ANN is trained on the performance data of the system and fits a nonlinear mathematical model to the data. The algorithm is a black box modeling technique that does not require an understanding of the underlying physics of the process. The ANN is commonly used in feed-forward control, and ANN can be trained on the controller input–output in an attempt to replace a conventional controller in that application. Examples of ANN design include a PMV-based thermal comfort controller for zone temperature control [50], optimization of air conditioning setback time based on outdoor temperature [51], and fan control of an air cooled chiller [52].

The implementation of FL control requires comprehensive knowledge of the plant operation and its different states, whereas ANN-based control design requires training data on a wide range of operating conditions, which may not be available for many systems. Additionally, industry is usually reluctant to adopt and use a black box approach.

3.4. Hybrid control

Hybrid controllers are produced by the fusion of hard and soft control techniques. Several controllers, including quasi-adaptive fuzzy control [53], adaptive-neuro control [48] and fuzzy-PID control [45], have been proposed in the literature for the control of HVAC systems.

Hybrid controllers are composed of soft control techniques such as ANN at higher levels and hard control techniques such as adaptive controllers at the lower levels of the control structure. In fuzzy-PID systems, controller gains can be auto-tuned using FL. Both hard and soft control techniques complement each other, and a combination of both can solve problems that may not be solved by each technique separately. Examples of hybrid control include a fuzzy self-tuning PI controller for supply air pressure control [24] and a quasi-adaptive fuzzy controller for zone temperature control [53] using convector-radiator power control.

Just as hybrid control benefits from the qualities of both hard and soft control techniques, it also inherits the problems associated with those techniques. For example, the design of a soft control component requires user expertise and large amounts of data for training, and the hard control component may be difficult to design and tune under the wide range of operating conditions often observed in HVAC systems.

3.5. Other control techniques

Other control techniques, such as direct feedback linear (DFL) control [54], pulse modulation adaptive controller (PMAC) [55], pattern recognition adaptive controller (PRAC) [56], preview control [57], two parameter switching control (TPSC) [58], and reinforcement learning control [59,60] have also been proposed for the control of HVAC systems.

The purpose of DFL control is to achieve decoupling between different control loops in HVAC systems and achieve global stability of the system. By applying input–output linearization, the coupled equations of the system are converted to linear uncoupled equations to which conventional linear control techniques can be applied. The DFL has been applied for control of zone temperature in Ref. [54] and demonstrated lower energy consumption, better disturbance rejection, and enhanced transient and steady state performance compared with PID control.

The PMAC is useful only for on/off systems such as fixed capacity compressors. The purpose of PMAC is to reduce the switching frequency of an on/off system to reduce equipment cycling and the associated energy costs and equipment wear. Using a PMAC, on/off systems can be controlled by a closed loop controller such as PID. The PID controller measures the error of the system from its set point and generates an analog signal as its output. The PMAC cascaded with a PID controller generates discrete an on/off pulse-width pulse-frequency modulation (PWPFM) signal corresponding to this analog signal. The PWPFM signal is applied to the discrete input of the system instead of an analog signal. For example, in Ref. [55], PMAC regulated the zone temperature of a direct expansion (DX) system by controlling the single-capacity compressor. The PRAC automatically tunes the gain and integral time of the PI controller based on the closed loop response patterns in self-regulating systems. This method produces near-optimal performance, and according to Ref. [56], it has been applied to HVAC control of many buildings, including offices, high schools, national labs, and hospitals. The process output is measured and fed to a digital PRAC, which estimates the process noise and tunes the PI controller parameters to tightly regulate the process.

The TPSC can be viewed as an improvement to the on/off controller, which uses one sensor or parameter to control the operation. Instead, the TPSC uses two sensors mounted at different points to control the system. For instance, the TPSC has been used to control the flow rate of hot water in radiant floor heating systems based on measurements of slab temperature and air temperature [58]. Compared with the on/off controller, the TPSC reduces oscillations of the air temperature and slab temperature because it increases the control effort by turning the control valve on and off more often.

Reinforcement-learning controller learns from the input and output of the system from past control actions using machine-learning techniques. For example, reinforcement learning control has been applied for thermal energy storage [59,60]. The reinforcement-learning controller savings are comparable to those of conventional control techniques but do not reach the level of MPC cost savings. Reinforcement learning is a model-free method and improves the controller performance based on previous control actions; however, it takes an unacceptably long time to learn and is difficult to implement in practice [12].

3.6. Summary

When considering HVAC control system characteristics, the MPC offers many advantages. Many processes in HVAC systems are slow moving with time delays, and time-varying internal and external disturbances act on the system. The system undergoes a wide range of operating conditions. The actuators exhibit rate and range limit constraints. In many areas, energy has a variable price structure. In the presence of all of these challenges, an ideal controller should be able to handle time-varying disturbances, wide operating conditions, actuator constraints, and variable price structures. Apparently, many control systems display several shortcomings in their application to HVAC control. For instance, the classical controllers require manual tuning and perform sluggishly or too aggressively outside of the tuning band. The hard controllers require rigorous mathematical analysis and the identification of stable equilibrium points for the controller design. Soft control requires massive amounts of data for training and reinforcement, and learning techniques require extensive time, rendering them impractical for industrial implementations. Alternatively, MPC provides a solution to many of the aforementioned problems and therefore constitutes the focus of this survey.

4. Model predictive control (MPC)

Because the focus of this paper is MPC, a comprehensive review of MPC techniques and comparisons with other control techniques are provided in this section.

The MPC uses a system model to predict the future states of the system and generates a control vector that minimizes a certain cost function over the prediction horizon in the presence of disturbances and constraints. The first element of the computed control vector at any sampling instant is applied to the system input, and the remainder is discarded. The entire process is repeated in the next time instant. The cost function can take the form of tracking error, control effort, energy cost, demand cost, power consumption, or a combination of these factors. Constraints can be placed on the rate and range limits of the actuators and the manipulated and controlled variables (e.g., upper and lower limits of the zone temperature, supply airflow rate limits, and range and speed limits for damper positioning). External and internal disturbances acting on the system due to weather, occupant activities, and equipment use are also modeled, and their predicted effects on the system are used during control vector computation. This effort results in a controller that is robust to both time-varying disturbances and system parameters and regulates the process tightly within the given bounds. The MPC is used in both supervisory and local (execution) levels of control in HVAC systems.

4.1. Comparison of MPC with other control approaches

The following comparison metrics are commonly used to compare the performances of various controllers:

- Energy and cost savings [61–66]
- Peak load shifting capability [61]
- Transient response improvement (decrease in rise time, settling time, and peak time) [67–70]
- Steady-state response improvement (decrease in offset error) [62,69]
- Control of variables within bounds [71,72]
- Reduction in fluctuations from a set-point (better regulation) [66]
- Efficiency and coefficient of performance (COP) improvements [68]
- Robustness to disturbances and changes in operating conditions [67,73]
- Indoor air quality and thermal comfort improvement [64,66,72]
- Computation time reduction [73]

Most researchers use one or two of the above performance metrics to evaluate the performance of their proposed controllers against others. In fact, MPC for HVAC systems is shown to outperform most control techniques using the aforementioned performance metrics.

The results obtained for MPC applications can be divided into simulation and experimental categories. To show the significance and advantages of MPC approaches, details of the developed MPC controllers are presented within simulation and experimental platforms in the following subsections.

4.1.1. Simulation results

The zone temperature and damper position in a simulated VAV system were controlled using MPC in Ref. [67]. Compared with a PI controller, the MPC-based supply airflow rate controller displayed better transient response (rise time, settling time, percentage overshoot) and was more robust in the presence of air duct pressure disturbances. During the regulation of a low-flow-rate set point, the PI controller produced a sluggish response that needed additional time to reach the set point. At a high-flow-rate set point, the PI controller responded too aggressively, which resulted in excessive overshoots above the set point. In contrast, the MPC-based technique produced consistent responses in both cases and achieved both faster settling time and lower overshoot. When zone temperature regulation was tested for low cooling load and high cooling load situations, the PI controller was able to regulate the process precisely at the set point, whereas MPC regulated the process within a feasible range close to the set point. However, the control effort put forth by the PI controller was much larger than that of the MPC controller. From inspecting the control signals generated by the PI and MPC controllers, it was observed that the PI controller signal exhibited much more fluctuation under low cooling load and oscillated at a high cooling load, which resulted in the need for re-tuning. In contrast, MPC produced a much smoother control signal under both the high and low cooling load conditions.

Simulations of zone temperature regulation using decentralized, centralized, and distributed MPC were presented in Ref. [66]. The zone temperature was well regulated at the set point using centralized and distributed MPC in the presence of coupling effects between adjacent zones. The PI controllers used a decentralized structure because they do not consider the coupling effects between zones. Each PI controller regulated the zone temperature individually in a multi-zone building without communicating any information to the neighboring controllers. The multi-zone decentralized MPC controllers also behaved in a fashion similar to that of the PI controller. However, the centralized and distributed MPC controllers accounted for the coupling effects of the neighboring zones by making predictions for the coupling effects and communicating the control decisions to the neighboring controllers. Compared with the PI controller, decentralized MPC was able to reduce the energy consumption by approximately 5.5%, whereas centralized MPC and distributed MPC were able to achieve an additional 36.7% increase in thermal comfort and a 13.4% reduction in energy consumption.

When used for the temperature and ventilation control of six zones in Ref. [72], the MPC-based technique was able to regulate the temperature within the limits and provided adequate ventilation levels based on the occupancy of the zones. However, the PI controllers failed to maintain zone temperatures within the desired thermal comfort range at all times and resulted in low ventilation when the occupancy was increased.

To achieve a desired zone temperature, the supply water temperature for a radiant floor heating system was computed using both the numerical Simulink® model (also referred to as the exact solution) and MPC in Ref. [71]. The MPC maintained the room temperature within the desired bounds at all times using weather prediction and accounting for the dead time of the building. However, the exact solution method was unable to maintain the room temperature at the set point at all times because it did not use weather forecasting.

In Ref. [61], MPC was used for zone temperature control. By employing the MPC technique, the peak loads were shifted, and the on-peak power profile was flattened. Compared with the baseline night setup strategy (0%), MPC yielded higher savings (28%) than did the linear-up (17%) and step-up (24%) strategies.

For the charging and discharging control of an ice storage system, MPC outperformed conventional control strategies (i.e., chiller priority control, constant proportion control, and storage priority control), as reported in Ref. [74]. Supervisory MPC was used to generate the optimal zone temperature set point profile, the thermal storage optimal charging and discharging profiles, and the precooling profile in Ref. [75]. Compared with conventional chiller control techniques, which have no thermal storage and chiller priority control, the MPC generated extra energy savings of 27% and 17%, respectively. When a supervisory MPC-based optimal sequence of tank water set points was used in Ref. [65], the energy consumption of heat pump was reduced.

4.1.2. Experimental results

To control the temperature of multiple zones, multi-input/multi-output (MIMO) MPC was used to control the water flow valve (WFV) in Ref. [68]. The MPC was also applied to regulate the evaporator temperature and pressure by controlling the electronic expansion valve (EEV) and compressor speed. For comparison purposes, local level PI controllers were also implemented on the aforementioned processes. It was observed that the MPC outperformed the PI controllers, e.g., improved regulation of superheat temperature and evaporator pressure. Adding supervisory MPC to the system improved the COP of the system by 9.5% and resulted in higher efficiency.

The MPC and PID control simulations for regulating the dry bulb temperature of the off-coil air from the AHU were carried out in Matlab® in Ref. [70]. In the simulations, MPC produced less overshoot and a faster settling time compared with the PID controller. The controller was implemented on a lab-scale pilot HVAC system. The implemented controller also showed improved robustness and superior tracking performance compared with the PID controller.

The supply air temperature of a test room in a factory building was controlled using controllers designed using prescribed error dynamics and MPC techniques in conjunction with feedback linearization [62]. The MPC controller performed remarkably well, demonstrating good trajectory tracking. The MPC could account for process dead time and use future values of the reference signal. Therefore, the MPC controller demonstrated a minimal delay in response, less overshoot, and a shorter settling time compared with a controller designed with prescribed error dynamics.

For zone temperature control in a large university building, the performance of the MPC was compared with that of a finely tuned weather compensated controller that also used weather forecasting in Ref. [64], and the heating curve method in Ref. [63]. The MPC used 29% less energy while maintaining the same thermal comfort level in both applications. Because the building had a time delay of 12 h in its temperature response because of its large thermal capacitance, MPC heated the building in advance to track the reference trajectory more accurately. The weather-compensated controller supplied water to the radiant ceiling heating system at a much higher temperature compared with that of the MPC controller, resulting in higher energy consumption. The heating curve method heated the concrete of the building during the night and turned off the heating in the morning. The MPC also preheated the building during the night, but it did not switch off the heating during day, which resulted in a significant peak energy reduction.

The zone temperature and humidity of a thermal chamber in a university lab were controlled with an MPC and a neural-fuzzy controller in Ref. [69]. Compared with the neural-fuzzy controller, the MPC demonstrated superior performance: it improved the settling time by 25% and the steady-state error for temperature and humidity by 100% and 400%, respectively.

A comparison of on/off control with learning-based MPC (LBMPC) was carried out in Ref. [76] using a single heat pump air conditioning (AC) system installed in a university lab. LBMPC reduced the energy consumption by 30–70% compared with the on/off control. The energy savings were reduced as the occupancy and temperature of outside air increased, resulting in a higher thermal load on the AC.

In summary, both the simulation and experimental results suggest many advantages in the use of MPC for HVAC system control. The remaining sections shed light on the components of the MPC system and its implementation.

4.2. Factors affecting MPC performance

A typical MPC system is composed of a system model, constraints, a disturbance model, a cost function, an optimization method, and a control horizon, which could all affect the performance of MPC. The remainder of this paper examines the effects of these choices on MPC performance.

4.2.1. Control configuration and type

Different MPC configurations can be considered, and such configurations can be categorized into hierarchical, cascaded, centralized, decentralized, and distributed structures.

The controllers can be used in a hierarchical or cascaded design to cater to both fast-moving and slow-moving disturbances in HVAC systems. For instance, MPC was combined with conventional local loop PID controllers in a hierarchical structure in Ref. [63] and cascaded to the PI controller in Ref. [77]. MPC can also be combined with a rule-based control (RBC) in a hierarchical structure to derive control signals using a set of rules [78]. MPC-based controllers can also be used in both the upper (supervisory) and lower (execution) levels of hierarchical control [79] and in both the inner and outer loops of the cascaded configuration [67].

Decentralized, centralized and distributed MPC can be used for a multi-zone building [66]. The decentralized control uses the same local controller separately for each zone without any consideration of thermal coupling between zones. Because zone coupling is not addressed in decentralized implementation, it results in temperature swings in the zone due to heating of neighboring zones at different set points resulting in poor control performance. The centralized controller considers the inputs, outputs, occupancy and thermal coupling for all zones simultaneously. Therefore it is able to track the set point of each zone despite different occupancy periods and zone temperature set points. However, a centralized MPC configuration results in a higher computation time and lower reliability because any problem in the central controller will disable the HVAC system of the entire building. This system is also not scalable to large buildings because implementing the controller would require higher-order MIMO models and a large amount of computing power. The solution is to design a distributed controller similar to the decentralized controller in which each controller communicates with the neighboring controllers to share the zone temperature information and the future course of action. The distributed controller performance [66] is comparable to that of the centralized controller and achieves similar energy savings and temperature regulation and its computational cost is low comparable to decentralized controller.

Robust MPC can be used to provide consistent control performance in the presence of disturbances and over a wide range of operating conditions [67,80].

For instance, a VAV AHU system was controlled using robust MPC, and the results were compared with those of the conventional PI control strategy in Ref. [80]. Compared with the conventional strategy, the robust strategy yielded tighter control of the supply air temperature set point by modulating the cooling coil valve. The robust strategy also showed a faster response compared with that of the conventional strategy in the presence of disturbances. The robust strategy accounted for the uncertainty in the gain and the time delay in the temperature control process and produced a control signal with such actuator constraints as rate and range limits. In contrast, the conventional strategy did not consider uncertainties and constraints, resulting in a sluggish response if the operating conditions deviated from the tuning conditions. In the robust MPC control strategy, control laws were implemented in the form of state feedback control in which the optimum gain was determined by optimizing a cost function based on tracking error.

In another work [67], a robust gain-scheduling MPC was considered with a bi-linear MPC for zone temperature control. The former MPC regulated a damper nonlinear process and managed the fast variations in supply airflow rate due to the change in damper position. The latter MPC controlled the process temperature, which could exhibit time-varying dynamics. The temperature-process MPC produced a reference for the supply airflow rate based on the error between the zone temperature and its set point. The damper-process MPC tracked this supply airflow rate and adjusted the damper position based on the error between the reference and measured supply airflow rate.

4.2.2. Controlled process

An HVAC system is composed of many subsystems that can be controlled independently of one another. The most important controlled variables in the HVAC system are zone temperature, humidity, and ventilation rate. The set points of temperature, pressure, and flow rate in the water and refrigerant loops are also controlled variables that are regulated by fans, pumps, compressor, boiler, and valves. Similarly, the temperature, flow rate, and pressure in the air loop are also controlled variables that are controlled by the heating and cooling water flow-rate valves, fans and dampers. The damper position, valve position, compressor speed, boiler fuel consumption rate, fan speed, and pump speed are all manipulated variables.

MPC was applied to zone temperature control in Refs. [61,65–67,71], damper position control in Ref. [67], HVAC energy consumption control in Ref. [78], hot water supply temperature regulation in Ref. [63], optimal storage water temperature profile generation in Ref. [65], charging and discharging rate control of an ice storage system in Ref. [74], thermal storage of a large-scale cooling system [81], temperature control of multiple-zones in Refs. [66,68,72], evaporator pressure and cooling set point generation in Ref. [68], zone humidity control in Ref. [69], temperature control of a MIMO process in Ref. [73], and ventilation control in Ref. [72].

For instance, an MPC controller was designed for a large university building using a state-space model identified by a subspace state-space identification method [64]. The MPC controlled the room temperature by regulating the heating-water flow rate into a radiant ceiling heating system. The MPC was used to control thermal storage by controlling the condenser water temperature, the chilled water temperature and the chilled water flow rate for a university campus cooling system [81]. In this work, the models of cooling system components and energy consumption were first determined and validated. Next, the MPC controller was designed to produce the set point for the condenser water supply, the chilled water supply, and the chilled water flow rate used to charge the storage tanks during the night. The designed MPC demonstrated an improved COP and a reduction in the electricity costs compared with the baseline case implemented using operator experience. This result was achieved by increasing the set point temperature and the flow rate of chilled water and by reducing the charging time. In the baseline case, operators charged the tanks with a lower temperature for extended periods of time, resulting in overcharging and greater losses that lowered the efficiency.

4.2.3. Building HVAC systems

MPC controllers have been applied to a variety of building HVAC systems. For example, MPC was applied for zone temperature control of a single-story office building with a VAV cooling system without heating or mechanical ventilation [61], zone temperature, and damper process control for a single-zone VAV system [67], and supply-air temperature control of a continuous air volume (CAV) system installed in a factory [62].

HVAC systems serve both single-zone and multi-zone buildings. In single-zone buildings, the set points of thermal comfort and indoor air quality variables are the same in all rooms, whereas in multi-zone buildings, the set points of the different zones can be controlled by the users. It is easier to design a controller for a single-zone building because simplifications can be used for the geometric and thermal properties of the building and because the insulation between the zones is poor. In this case, coupling cannot be neglected and must be modeled properly for the accurate control of zone temperature, humidity, and air quality. This strategy results in more complex MIMO controllers. Several MPC strategies have been applied to both single-zone and multi-zone buildings, i.e., a single-story office building [61], a factory building [62], a small studio apartment [78], a large university building [63,64,82], a test room [66,71,75], a shed [65], and a multi-story office building [74].

4.2.4. Energy conservation strategy

Energy can be conserved by implementing different control strategies, such as thermal storage in the building mass [61] or floor heating mass [65], passive solar gains [65], thermal storage in tank water [74,75], temperature reset during unoccupied hours [3,48], night setbacks, pre-cooling during off-peak periods and set-point changes during peak hours [83,84], optimum start and stop times [85], ventilation control [86,87], and economizer cycle control [3,4,13]. These conservation strategies can be implemented together with MPC to maximize energy savings. The cost function of a predictive controller can be based on energy conservation such that peak loads can be shifted to off-peak hours and energy consumption during peak hours can be minimized. The peak shifting does not always result in lower energy consumptions but may result in lower operating costs in the presence of a variable rate structure.

For example, in Ref. [75], an optimum amount of thermal energy storage in the tank water was used to compare the performance of MPC with that of other conventional energy storage strategies based on chiller priority and storage priority control. It was found in Ref. [75], that thermal energy storage with MPC resulted in a significant operating cost reduction. Even a simple non-predictive strategy such as chiller priority resulted in greater savings than a system without thermal storage. It was shown that passive storage in building mass results in the highest savings for buildings with a large thermal mass [88]. Obviously, passive thermal storage savings are low for buildings with less thermal mass, such as residential buildings.

4.2.5. Prediction basis and disturbances

The MPC algorithm must predict the future state of the system based on an estimate of internal and external disturbances acting on the system. Internal disturbances occur because of occupant activities, equipment use, and lighting. External disturbances primarily occur because of weather variables, e.g., outside temperature, humidity, solar irradiance, wind velocity, and cloud factor. The internal disturbances can be estimated using the known occupancy and lighting and equipment use schedules [61,72,75]. The external disturbances can be estimated using short-term weather forecast models, such as the bin predictor, random walk, and harmonic predictor; linear parametric models such as the auto-regression integrated moving average (ARIMA); and nonlinear models such as ANN [89,90]. The bin predictor models and ANN models can provide near-perfect forecasting.

An MPC that uses a forecast generated by these models can outperform other methods that do not use weather forecasting. For example, the effects of weather forecast uncertainty on HVAC control performance in terms of energy consumption and thermal comfort violations were investigated and reported in Ref. [91]. The room temperature regulation performance of RBC was compared with those of deterministic MPC (DMPC) and stochastic MPC (SMPC). The RBC used expert knowledge in controller design and was used as a benchmark. A theoretical benchmark known as performance bound (PB) was also used to evaluate the theoretical saving potentials among RBC, DMPC, and SMPC. During the computation of PB, it was assumed that the weather forecast was 100% accurate and without any uncertainty. This assumption allows the calculation of the maximum savings potential of DMPC. The DMPC used linear constraints in the MPC formulation and assumed that the weather forecast was accurate, thus remaining at its expected value. Due to this assumption, the uncertainty in weather variables was not considered, and the DMPC did not perform well when the actual weather varied from the forecast. In constraint and cost function formulation for SMCP, the weather uncertainty was assumed to have a Gaussian distribution. This assumption was validated via analysis of the predictions of the weather forecast model and its actual measurements. The performances of RBC, PB, DMPC, and SMPC were compared based on non-renewable primary energy (NRPE) usage and the amount of comfort violation. The PB performed best because it was a theoretical concept and considered no variations in the predicted and actual weather. The RBC outperformed the DMPC in most cases, whereas the SMPC outperformed the RBC in all simulated cases with the lowest NRPE and a minimum amount of comfort violations. The results showed that by incorporating weather uncertainty, the SMPC can serve as a superior controller that consumes less energy and produces a zone temperature within the given bounds most of the time compared with an RBC approach. A good weather prediction model can further enhance the SMPC performance. The amount of comfort violations can be decreased, if desired, if using an SMPC scheme, but doing so results in higher energy usage.

Researchers have also used the future value of the reference signal [62], prediction of tracking error [67], and historical value of the control signal [71] to predict future system states in MPC design.

Apart from the internal and external disturbances discussed above, other disturbances such as coupling between neighboring zones [66], variable air mass flow rates and water inlet temperatures in the AHU [62], and interaction between the evaporators in multi-evaporator systems [68] also act as disturbances in a control system. Certain works used simulated disturbances, e.g., random noise [73] and heating at an unknown rate [68], in their proof of concept.

4.2.6. Model for system dynamics simulation and controller development

The MPC controller can use either physics-based models (also known as analytical first principle or forward models) or data-driven models (also known as black box or inverse models) to predict the system output.

Physics-based models are based on the knowledge of the process, parameters that can be determined from manufacturer documentation and application of parameter estimation techniques on measured process data. Physics-based models have been developed for zones [92–94], mixing boxes [93], AHUs [95,96], compressors [68,97], fans [98], pumps [99], valves [16], dampers [67], and ducts [92]. Physics-based models of thermal processes are analogous to electrical RC networks. For simplicity, these models use lumped thermal capacitance and resistance in place of distributed thermal capacitance and resistance. This strategy results in simple dynamic first-order models that represent the thermal process. Data-driven models fit linear and nonlinear mathematical functions to measured data. Examples of data-driven models include ANN [46,100–102], FL [103,104], support vector machine (SVM) [105], first- and second-order time delay models [106,107], and statistical models (e.g., autoregressive (AR), autoregressive with exogenous (ARX), autoregressive moving average (ARMA), finite impulse response (FIR), autoregressive moving average exogenous (ARMAX), output error (OE), and Box–Jenkins (BJ) models) [108]. The accuracy of data-driven models is high compared with that of physics-based models, but these models suffer from generalization capabilities.

Comprehensive models can be developed using HVAC simulation programs such as EnergyPlus [61,65], TRNSYS [75], and Simulink® [66,71] for HVAC systems and buildings under consideration. Such models produce highly accurate results that are useful for performance analysis and optimization of HVAC systems. However, these models are generally not used for controller development. The controller is generally developed on simpler physics-based and data-driven models that achieve reasonable accuracy and simplicity. Researchers have generally used comprehensive models in conjunction with simpler models, whereas simpler models are used for controller development and comprehensive models are applied to simulate the performance of the controller.

To develop good quality models, the data should have high accuracy, low noise, and appropriate temporal resolution to capture the process dynamics correctly. For fast-moving processes in HVAC systems (i.e., airflow rate and water flow rate measurements), the sampling rate should be higher compared with that of slow-moving processes (i.e., air temperature and water temperature). In HVAC system control, data sampled at one-minute intervals are appropriate for fast-moving processes, and hourly data are appropriate for slow-moving processes. Median and averaging filters can be applied for removing spike noise and quantization noise, respectively [109]. The data should also cover a broad range of operating conditions observed by the HVAC system such as variations in weather parameters and occupancy patterns throughout the year. Due to changes in the building and HVAC parameters over time, the model prediction will deviate from the actual process output. To cope with this situation, the models can be updated online. If performance data are available for multiple years, then it is good practice to train and test models on data sets from different years. The accuracy of models can be increased by clustering the data into different seasons or similar outdoor weather conditions [102]. Multiple models can be trained on these data clusters, and an appropriate model can be selected based on input measurements.

After a model has been developed, model validation is necessary to verify its accuracy. Model validation can be carried out by comparing model outputs with measurements, analytical solutions of a known problem, or with results of other modeling software [110]. Performance metrics are defined to compare prediction results of different models and their deviations from measured data. Models are compared using absolute error (*AE*), maximum absolute error (MAX_{AE}), mean absolute error (*MAE*), mean bias error (*MBE*), mean squared error (*MSE*), absolute percentage error (*APE*), mean absolute percentage error (*MAPE*), standard deviation of absolute error (Std_{AE}), standard deviation of absolute percentage error (Std_{APE}), coefficient of determination (*D*), root mean square error (*RMSE*), coefficient of variation (*CV*), goodness of fit (*G*), relative mean error (*RME*), mean absolute relative error (*MARE*), coefficient of multiple determination (R^2), and correlation coefficient (*CC*) [46,89,101,102,104,111–115].

To obtain further insight into model development and its application in MPC, several examples are taken from the literature. For example, in the application of MPC to the HVAC system of a plant [62], the system included a cooling coil whose outlet air temperature must be controlled by manipulating the position of a chilled water control valve. Physics-based models were developed for the valve gear, hydraulics, cooling coil, and temperature sensor to simulate the plant dynamics. The hydraulics were modeled by measuring the valve actuation signal and the resulting water flow-rate and fitting a third-order polynomial on the data. A cooling coil dynamic model was obtained from a mass and energy balance of the air and water streams, resulting in first-order differential equations for the air and water temperature inside the cooling coil. A model for a temperature sensor was developed using a first-order time delay model. The cooling coil and temperature sensor models were converted into linear discrete state space models.

In another example, physics-based models were developed in the design of a MPC to simulate the control of zone temperature inside a small studio apartment [78]. The focus of this work [78] was primarily on parameter identification in models for zone temperature, HVAC energy consumption and control signals. The estimated parameters included thermal capacitance and conductance of air and structural nodes of the building. Two types of parameter estimation algorithms were presented and applied to measured data to find estimates of the capacitance and conductance of a building structure and the control input and energy consumption. A rule-based MPC controller was subsequently applied to regulate the zone temperature based on models with estimated parameters, and the effect of model mismatches on the controller performance was studied.

Finally, it should be noted that due to the simplicity of linear models in control law development, certain MPC designers often attempt to linearize the obtained models using Jacobian linearization [116] and feedback linearization [62]. Linear models can also be obtained using the prediction error method [68] and system identification techniques [65].

4.2.7. Prediction horizon, control horizon and time step

The prediction horizon refers to the length of time for which system output is computed by the MPC, whereas the control horizon denotes the length of time for which the control signal is computed. The time step (or control sampling time) is the time during which the control signal remains unchanged. Typically, for slow-moving processes in HVAC systems, the prediction horizon is 5–48 h, the control horizon range is 4–5 h, and the time step is between 1 and 3 h [62,63,65]. The control horizon is generally smaller than or equal to the prediction horizon. The selected horizon depends on the controlled process and its dynamics. For instance, in many indoor applications, a time step of 1 h is reasonable because temperature change is a slow-moving process. Using a smaller prediction horizon or a faster sampling time could result in degradation of the controller performance due to delays in the temperature process. Using a longer prediction horizon could lead to increased computation time without any further benefit [75,117]. When applied to fast-moving dynamic processes such as compressor pressure and superheat temperature control, the prediction horizon and control horizon often shrink to a few seconds [68]. In certain applications, a time-variable horizon is also employed. For instance, in optimizing energy consumption over a 24-h period [61], a shrinking horizon scheme is applied in which the prediction horizon reduces as the time progresses towards the end of the day.

4.2.8. Constraints

The MPC is also known as constrained control because of its ability to find a solution that does not violate the constraints placed on the inputs, outputs, and actuators. Types of constraints include equality (e.g., capacity limits of the tank, boiler, and chiller), and inequality (e.g., actuator range and rate limit) constraints. For example, the speed at which a damper moves from a fully open to a fully closed position is finite and can be expressed as a rate limit. As another example, due to either manufacturing imperfections or restrictions on the minimum and maximum ventilation rates, the damper operating range may be limited to positions between fully open and fully closed. This type of constrained damper motion is known as the damper range limit constraint [67]. In addition to placing constraints on actuators, rate and range limit constraints can also be placed on controlled variables. For example, to maintain thermal comfort, the zone temperature may not be allowed to change by more than a specified amount per unit time, and the temperature should be maintained within a certain band.

To further the understanding of the types of constraints in MPC development, selected examples from the literature are described. In temperature process control, the supply air temperature [62] and/or supply airflow rate [67] were constrained to operate in a given range. For room temperature control in Ref. [71], limits were placed on the supply heat flux and indoor temperature. The minimum heat flux was constrained to zero, but its maximum remained unconstrained. For zone temperature and humidity control in Ref. [69], constraints were placed on the supply-air fan speed to remain between 0.1 and 0.75 of the rated value and on the chilled water valve opening to remain between 0.1 and 1. For the generation of an optimal temperature set point profile for tank water storage in Ref. [65], the allowable values of the tank temperature set point were constrained to 30 °C–55 °C. For the control of the charging rate of an ice storage system in Ref. [74], charge and discharge rate constraints and range constraints were placed on the state of charge of the storage tank. The storage tank and chiller capacities were constrained to provide four times and one time the peak cooling load, respectively.

For temperature control of two separate zones in a multi-evaporator system in Ref. [68], the vapor compression cycle (VCC) constraints included the minimum evaporator pressures, the maximum compressor speed and capacity, and the valve maximum opening. The EEVs and WFVs were constrained to operate within 8%–14.5% and 22%–50% of their ranges, respectively. An output constraint was placed on the superheat to remain between 6 °C and 12 °C. In Ref. [66], constraints were also placed on the minimum and maximum of evaporator cooling, evaporator pressure, pressure differential, and max pressure slew rate.

4.2.9. Cost function

The cost function is based on the desired behavior of the system and serves to stabilize the system if the optimal cost can be described by a Lyapunov function [91]. For systems with slow dynamics (i.e., temperature processes), stability is not an issue, and one can choose any form of cost function. The cost function also describes the performance target, such as the minimization of energy consumption and the maximization of thermal comfort in HVAC systems. Maximizing thermal comfort and minimizing energy consumption are two competing objectives, and a trade-off must be found by placing weights on these factors in the cost function. In the quadratic cost function, the weights provide a trade-off between tracking error and control effort. A linear cost function is used in minimizing such economically driven signals as operating cost, terminal cost and energy cost. The following cost functions or combinations of them are widely used in MPC-based HVAC control:

- Weighted sum of tracking error and control effort [62,66–68,70,72,73];
- Quadratic cost function for tracking the error and control effort [63,64,68];
- Sum of the energy cost and demand cost [61];
- Norm of the momentary temperature deviation [71];
- Sum of the tracking error [65];
- Integrated power or energy consumption [61,79];
- Operating cost [74,75];
- Terminal cost [69]; and
- Dissatisfaction cost [79].

Most researchers have attempted to minimize the weighted sum of the tracking error and control effort, and others have only minimized the sum of tracking error or instantaneous error (operating cost will increase as a result). The latter cost function is useful if there is no incentive to save energy and the price of electricity is constant throughout the operating period. However, certain researchers have only minimized power consumption, operating cost and terminal cost and sacrificed thermal comfort. Such a cost function is useful if a significant cost savings exists at the expense of thermal comfort, i.e., in the case of a variable price structure. The dynamic cost function can also be used to place different weights on thermal comfort and energy consumption based on the energy conservation incentive during the day. The following two examples provide further details on the formulation and use of cost functions in an MPC framework.

The proposed MPC for room temperature control in Ref. [76] used a two-term cost function. The first term was the squared sum of the tracking error (i.e., the difference between room temperature and desired temperature over the control horizon). The second term represented the energy conservation over the optimization period as a function of the control signal. Because the occupancy varies widely over the day, as does the weather, these factors together represent the uncertainty faced by the controller.

A hierarchical MPC (H-MPC) was proposed in Ref. [79] for energy consumption reduction in a residential house. The two-level H-MPC consisted of a scheduling MPC (S-MPC) and a piloting MPC (P-MPC). The S-MPC used dissatisfaction and energy consumption cost functions that were minimized over a large horizon of 7 h with a sampling time of 1 h to produce a solution that was partially used by the P-MPC. The S-MPC addressed the slow-moving dynamics and the varying price profile of the electricity. The P-MPC operated on a shorter horizon with a sampling time of 5 min to track the state sequence generated by the S-MPC and manage the disturbances and fast-moving dynamics. Compared with a centralized MPC, the H-MPC showed superior performance in terms of dissatisfaction cost.

4.2.10. Optimization problem

After the formulation of the system model, the disturbance model, the constraints and the cost function, MPC solves a constrained optimization problem to compute the optimum control vector. Because gradient-based techniques are usually designed to work with continuous functions and may not even be able to find global minimum of the function, a variety of optimization methods have been proposed. A classification of linear and nonlinear optimization methods for HVAC control is given in Ref. [12].

Optimization techniques commonly used by HVAC researchers include linear programming (e.g., Simplex method) [61], quadratic programming (QP) [67], dynamic programming (DP) [65], mixed integer programming (MIP) [74], evolutionary algorithm (EA) [118], particle swarm optimization (PSO) [119], and the GA [120]. In addition to EA, PSO, and GA, other meta-heuristic optimization techniques such as simulated annealing [121], differential evolution, ant colony optimization [105], bee algorithms [122], the Tabu search [123], the Harmony search [124], firefly algorithm [125], cuckoo search [126], artificial immune systems [127], the memetic algorithm [128], the cross entropy method [129], and the bacterial foraging method [130] are less common among HVAC researchers and thus present a potential area of research. The following examples illustrate the use of optimization methods in MPC development.

In Ref. [61], the minimization of energy and demand cost was formulated as a linear program solved using a variation of the Simplex method under the Matlab® function 'Linprog'. In Ref. [62], the weighted sum of the tracking error and control effort was minimized using the Matlab® MPC Toolbox. The QP algorithm in the MPC of the temperature process was used to minimize the tracking error and control effort in Ref. [67]. To minimize the quadratic cost function (which penalizes rapid changes in heating water temperature), SciLab's internal quadratic optimization program solver was used in Refs. [63,64]. In Ref. [71] a constrained nonlinear multivariable function was minimized using a variation of sequential QP under the Matlab® 'fmincon' function. The purpose was to minimize the deviations in the indoor temperature and the norm of the momentary temperature. Deviations above and below the comfort range were penalized. The DP algorithm was used to minimize the integrated power consumption rate of a heat pump over a period of interest in Ref. [65]. The operating cost of a cooling plant over a simulation period was minimized in Ref. [74] using DP and MIP. The Wolfe-Dantzig algorithm was applied to solve the QP problem using the QPDANTZ program included in the Matlab® MPC Toolbox in Ref. [68]. Iterative DP (IDP) was applied to solve the convex quadratic optimization problem in Ref. [69] to minimize the terminal cost. The quasi-Newton and DP algorithms were applied for passive and active storage optimization, respectively, to minimize the operating cost for time-of-use-differentiated electricity and fixed-cost natural gas in Ref. [75].

The GA technique was used to compute the control vector for MPC in room temperature control under a variable electricity price structure [120]. Compared with a non-optimized base case, the optimized MPC was able to reduce the operating cost of the HVAC system by 30% by shifting the load to off-peak hours. Discomfort was increased during the optimized control scheme, but temperature was maintained within the upper and lower control limits. The supervisory controller developed using a model-based GA in Ref. [131] resulted in significant energy savings in the winter or mild seasons and a significant indoor air quality (IAQ) improvement in the summer season compared with those of a conventional controller. The GA was used to compute optimal set points for supply air flow rate, chiller temperature, and zone temperature. In Ref. [119], PSO-based MPC was used to control the temperature and ventilation rate of a greenhouse by forced heating and natural ventilation. Compared with a conventional controller, PSO-based MPC was able to reduce the control effort and the heating and ventilation costs, resulting in greater savings and reduced wear of the components. An improved PSO algorithm known as the differential discrete PSO (DDPSO) was proposed in Ref. [132]. The proposed DDPSO achieved a better solution in fewer iterations compared with the standard PSO when applied for building temperature control.

5. Conclusions

Certain important points of MPC development for HVAC control can be summarized as follows:

- Many attractive choices are available for HVAC system control in the form of conventional controllers, hard controllers, soft controllers, and hybrid controllers. These techniques were reviewed, and the advantages and disadvantages of each technique were highlighted. Compared with most of the other control techniques, MPC generally provides superior performance in terms of lower energy consumption, better transient response, robustness to disturbances, and consistent performance under varying conditions.
- The accuracy of the model, weather forecasting and disturbance predictions all affect the energy consumption and performance of MPC. New information such as measured weather variables (wind speed, solar flux, ambient temperature, and humidity) should be incorporated in MPC at each sampling instant to improve controller performance.
- Most of the MPC formulations use discrete linear models of the system obtained by either linearizing the state-space models around a certain equilibrium point or creating linear ARX models from empirical data. Certain MPC formulations use discretized versions of continuous model equations obtained from physics-based models. The system identification techniques are also used to derive simple linear models for MPC formulations from more complicated and comprehensive models developed in EnergyPlus and TRNSYS. The MPC can be interfaced with comprehensive models built in the EnergyPlus, TRNSYS, and Matlab® Simulink® platforms to simulate control performance for a real building and actual weather conditions.
- Selection of the prediction horizon and sampling time affects the accuracy, computational cost, and response time of MPC. H-MPC and cascade MPC are designed to handle both slow- and fast-moving disturbances. The slow dynamics are controlled by a supervisory-level controller, which operates using a longer time horizon of typically 24 h and a slow sampling time of typically 1 h. The fast-moving disturbances are controlled by a lower-level controller that operates on a shorter horizon in the range of 30–60 min and using a fast sampling time of typically 5–10 min.
- Even in its most basic form (such as DMPC), MPC with linear constraints, simple disturbances, and load forecasting models outperforms the conventional control approaches that do not contain any built-in predictive algorithms.
- Energy conservation strategies can be easily integrated into MPC design. Thermal storage presents opportunities for peak shifting and reducing operating costs. The MPC with thermal energy storage outperforms controllers that do not use thermal storage. Buildings with large thermal mass (such as office buildings) could use passive thermal storage by pre-heating or pre-cooling the building during the off-peak period. Buildings with small thermal mass (such as residential buildings) can use tank water for thermal energy storage. The use of thermal storage may result in higher energy consumption but lower costs because of the variable price of electricity throughout the day.

Despite considerable work on MPC development for HVAC systems, possible areas that require further investigation still exist and are summarized as follows:

- Performance comparison of different MPC techniques (i.e., robust MPC, SMPC, D MPC, LBMPC, and H-MPC);
- MPC development for ground source heat pumps;
- Investigation of integrating nonlinear modeling methods (i.e., ANN, FL, and SVM) for use in MPC;
- Study of techniques for comprehensive on-line updates of the model and accurate estimates of disturbances as well as their impact on MPC performance;
- Investigation of integrating meta-heuristic optimization techniques and their impact on MPC performance. Such methods include simulated annealing, differential evolution, ant colony optimization, bee algorithms, the Tabu search, the Harmony search, the firefly algorithm, cuckoo search, artificial immune systems, memetic algorithms, the cross entropy method, and the bacterial foraging method for use in MPC control vector computation;
- Further research on factors that affect MPC performance.

Acknowledgments

This research was financially supported by Ryerson Center for Urban Energy (CUE), Toronto Hydro and Mitacs-Accelerate Program.

References

[1] Perez-Lombard L, Ortiz J, Pout C. A review on buildings energy consumption information. Energy Build 2008;40:394–8.
[2] Annual energy outlook 2012 early release. Energy Information Administration (EIA); 2012.
[3] Naidu DS, Rieger CG. Advanced control strategies for heating, ventilation, air-conditioning, and refrigeration systems – an overview: part I: hard control. HVAC R Res 2011;17:2–21.
[4] Naidu DS, Rieger CG. Advanced control strategies for HVAC&R systems – an overview: part II: soft and fusion control. HVAC R Res 2011;17:144–58.
[5] Mirinejad H, Welch KC, Spicer L. A review of intelligent control techniques in HVAC systems. Cleveland, Ohio, USA: Energytech/IEEE; 2012. pp. 1–5.
[6] Kalogirou SA. Artificial neural networks and genetic algorithms in energy applications in buildings. Adv Build Energy Res 2009;3:83–120.
[7] Dounis AI, Caraiscos C. Advanced control systems engineering for energy and comfort management in a building environment – a review. Renew Sust Energ Rev 2009;13:1246–61.
[8] Singh J, Singh N, Sharma JK. Fuzzy modeling and control of HVAC systems – a review. J of Sci & Ind Res 2006;65:470–6.
[9] Bansal RC, Pandey JC. Load forecasting using artificial intelligence techniques: a literature survey. Int J Comput Appl Technol 2005;22:109–19.
[10] Metaxiotis K, Kagiannas A, Askounis D, Psarras J. Artificial intelligence in short term electric load forecasting: a state-of-the-art survey for the researcher. Energy Convers Manag 2003;44:1525–34.
[11] Trcka M, Hensen JLM. Overview of HVAC system simulation. Autom Constr 2010;19:93–9.
[12] Wang S, Ma Z. Supervisory and optimal control of building HVAC systems: a review. HVAC R Res 2008;14:3–32.
[13] Al-Rabghi OM, Akyurt MM. A survey of energy efficient strategies for effective air conditioning. Energy Convers Manag 2004;45:1643–54.
[14] Harrold MV, Lush DM. Automatic controls in building services. IEE Proc B Electr Power Appl 1988;135:105–33.
[15] Astrom KJ. Theory and applications of adaptive control – a survey. Autom 1983;19:471–86.
[16] Jin G-Y, Tan P-Y, Ding X-D, Koh T-M. Cooling coil unit dynamic control of in HVAC system. In: Conf Ind Electron Appl (ICIEA). Beijing, China: IEEE; 2011. pp. 942–7.
[17] Jette I, Zaheer-uddin M, Fazio P. PI-control of dual duct systems: manual tuning and control loop interaction. Energy Convers Manag 1998;39:1471–82.
[18] Zhang J, Zhang K. A particle swarm optimization approach for optimal design of PID controller for temperature control in HVAC. In: Int Conf Meas Technol Mechatron Autom (ICMTMA). Shanghai, China: IEEE; 2011. pp. 230–3.
[19] Tahersima F, Stoustrup J, Rasmussen H, Nielsen PG. Thermal analysis of an HVAC system with TRV controlled hydronic radiator. In: Conf Autom Sci Eng (CASE). Toronto, Ontario, Canada: IEEE; 2010. pp. 756–61.
[20] Bai J, Wang S, Zhang X. Development of an adaptive Smith predictor-based self-tuning PI controller for an HVAC system in a test room. Energy Build 2008;40:2244–52.
[21] Bai J, Zhang X. A new adaptive PI controller and its application in HVAC systems. Energy Convers Manag 2007;48:1043–54.
[22] Kulkarni MR, Hong F. Energy optimal control of a residential space-conditioning system based on sensible heat transfer modeling. Build Environ 2004;39:31–8.
[23] Soyguder S, Alli H. An expert system for the humidity and temperature control in HVAC systems using ANFIS and optimization with Fuzzy modeling approach. Energy Build 2009;41:814–22.
[24] Pal AK, Mudi RK. Self-tuning fuzzy PI controller and its applications to HVAC systems. Int J of Comput Cogn 2008;6:25–30.
[25] Xu M, Li S, Cai W. Practical receding-horizon optimization control of the air handling unit in HVAC systems. Ind Eng Chem Res 2005;44:2848–55.
[26] Zaheer-uddin M, Tudoroiu N. Neuro-PID tracking control of a discharge air temperature system. Energy Convers Manag 2004;45:2405–15.
[27] Lim D, Rasmussen BP, Swaroop D. Selecting PID control gains for nonlinear HVAC&R systems. HVAC R Res 2009;15:991–1019.
[28] Wang Y-G, Shi Z-G, Cai W-J. PID autotuner and its application in HVAC systems. In: Am control Conf. Arlington, Virginia, USA: IEEE; 2001. pp. 2192–6.
[29] Salsbury TI. A survey of control technologies in the building automation industry. In: Proc IFAC 16th world Congr. Prague, Czech Republic 2005. p. 1396.
[30] Moradi H, Saffar-Avval M, Bakhtiari-Nejad F. Nonlinear multivariable control and performance analysis of an air-handling unit. Energy Build 2011;43:805–13.
[31] Hodgson DA. Investigation of a nonlinear controller that combines steady state predictions with integral action [PhD thesis]. Fort Collins, Colorado, United States: Colorado State University; 2010.
[32] Pasgianos GD, Arvanitis KG, Polycarpou P, Sigrimis N. A nonlinear feedback technique for greenhouse environmental control. Comput Electron Agric 2003;40:153–77.
[33] Anderson M, Buehner M, Young P, Hittle D, Anderson C, Jilin T, et al. MIMO robust control for HVAC systems. IEEE Tran Control Syst Technol 2008;16:475–83.
[34] Al-Assadi SAK, Patel RV, Zaheer-uddin M, Verma MS, Breitinger J. Robust decentralized control of HVAC systems using $H\infty$-performance measures. J Franklin Inst 2004;341:543–67.
[35] Henze GP, Felsmann C, Knabe G. Evaluation of optimal control for active and passive building thermal storage. Int J Therm Sci 2004;43:173–83.
[36] Greensfelder EM, Henze GP, Felsmann C. An investigation of optimal control of passive building thermal storage with real time pricing. J Building Perform Simul 2011;4:91–104.
[37] Dong B. Non-linear optimal controller design for building HVAC systems. In: Int Conf Control Appl (CCA). Yokohama, Japan: IEEE; 2010. pp. 210–5.
[38] Nishiguchi J, Konda T, Dazai R. Data-driven optimal control for building energy conservation. In: Proc SICE Annu Conf. Taipei, Taiwan: IEEE; 2010. pp. 116–20.
[39] Mossolly M, Ghali K, Ghaddar N. Optimal control strategy for a multi-zone air conditioning system using a genetic algorithm. Energy 2009;34:58–66.
[40] Yan Y, Zhou J, Lin Y, Yang W, Wang P, Zhang G. Adaptive optimal control model for building cooling and heating sources. Energy Build 2008;40:1394–401.
[41] Sun J, Reddy A. Optimal control of building HVAC&R systems using complete simulation-based sequential quadratic programming (CSB-SQP). Build Environ 2005;40:657–69.
[42] Gouda MM, Danaher S, Underwood CP. Thermal comfort based fuzzy logic controller. Build Serv Eng Res Technol 2001;22:237–53.
[43] Homod RZ, Sahari KSM, Almurib HAF, Nagi FH. Gradient auto-tuned Takagi-Sugeno Fuzzy Forward control of a HVAC system using predicted mean vote index. Energy Build 2012;49:254–67.
[44] Shepherd AB, Batty WJ. Fuzzy control strategies to provide cost and energy efficient high quality indoor environments in buildings with high occupant densities. Build Serv Eng Res Technol 2003;24:35–45.
[45] Soyguder S, Karakose M, Alli H. Design and simulation of self-tuning PID-type fuzzy adaptive control for an expert HVAC system. Expert Syst Appl 2009;36:4566–73.
[46] Kusiak A, Xu G. Modeling and optimization of HVAC systems using a dynamic neural network. Energy 2012;42:241–50.
[47] Rieger CG. Advanced control strategies for HVAC systems in critical building structures [PhD thesis]. Pocatello, Idaho, United States: Idaho State University; 2008.
[48] Weiss MVG. Adaptive neuro energy management control strategies for HVAC systems in buildings [MASc thesis]. Montreal, Quebec, Canada: Concordia University; 2006.
[49] Yu Z, Dexter A. Hierarchical fuzzy control of low-energy building systems. Solar Energy 2010;84:538–48.
[50] Liang J, Du R. Thermal comfort control based on neural network for HVAC application. In: Conf control Appl. Toronto, Canada: IEEE; 2005. pp. 819–24.
[51] Ben-Nakhi AE, Mahmoud MA. Energy conservation in buildings through efficient A/C control using neural networks. Appl Energy 2002;73:5–23.
[52] Henze GP, Hindman RE. Control of air-cooled chiller condenser fans using clustering neural networks. ASHRAE Trans 2002;108:232–44.

[53] Gouda MM, Danaher S, Underwood CP. Quasi-adaptive fuzzy heating control of solar buildings. Build Environ 2006;41:1881–91.
[54] Li X, Shi Z, Hu S. A novel control method of a variable volume air conditioning system for indoor thermal environment. In: Int Conf Comput Eng Technol (ICCET). Chengdu, China: IEEE; 2010. p. V2-566–V2-570.
[55] Salsbury TI. A new pulse modulation adaptive controller (PMAC) applied to HVAC systems. Control Eng Pract 2002;10:1357–70.
[56] Seem JE. A new pattern recognition adaptive controller with application to HVAC systems. Autom 1998;34:969–82.
[57] Zaheer-uddin M, Al-Assadi SAK, Patel RV. Decentralized preview control for multiple disturbance rejection in HVAC systems. Control Eng Pract 1994;2:989–1000.
[58] Cho SH, Zaheer-uddin M. An experimental study of multiple parameter switching control for radiant floor heating systems. Energy 1999;24:433–44.
[59] Henze GP, Schoenmann J. Evaluation of reinforcement learning control for thermal energy storage systems. HVAC R Res 2003;9:259–75.
[60] Liu S, Henze GP. Experimental analysis of simulated reinforcement learning control for active and passive building thermal storage inventory. Energy Build 2006;38:148–61.
[61] Ma J, Qin J, Salsbury T, Xu P. Demand reduction in building energy systems based on economic model predictive control. Chem Eng Sci 2011;67:92–100.
[62] Rehrl J, Horn M. Temperature control for HVAC systems based on exact linearization and model predictive control. In: Int Conf Control Appl (CCA). Denver, Colorado, USA: IEEE; 2011. pp. 1119–24.
[63] Siroky J, Oldewurtel F, Cigler J, Privara S. Experimental analysis of model predictive control for an energy efficient building heating system. Appl Energy 2011;88:3079–87.
[64] Privara S, Siroky J, Ferkl L, Cigler J. Model predictive control of a building heating system: the first experience. Energy Build 2011;43:564–72.
[65] Candanedo JA, Athienitis AK. Predictive control of radiant floor heating and solar-source heat pump operation in a solar house. HVAC R Res 2011;17:235–56.
[66] Morosan P-D, Bourdais R, Dumur D, Buisson J. Building temperature regulation using a distributed model predictive control. Energy Build 2010;42:1445–52.
[67] Huang G. Model predictive control of VAV zone thermal systems concerning bi-linearity and gain nonlinearity. Control Eng Pract 2011;19:700–10.
[68] Elliott MS. Decentralized model predictive control of a multiple evaporator HVAC system [MSc thesis]. College Station, Texas, United States: Texas A&M University; 2008.
[69] Xi X-C, Poo A-N, Chou S-K. Support vector regression model predictive control on a HVAC plant. Control Eng Pract 2007;15:897–908.
[70] Lü H, Jia L, Kong S, Zhang Z. Predictive functional control based on fuzzy T-S model for HVAC systems temperature control. J of Control Theory Appl 2007;5:94–8.
[71] Karlsson H, Hagentoft C-E. Application of model based predictive control for water-based floor heating in low energy residential buildings. Build Environ 2011;46:556–69.
[72] Yuan S, Perez R. Multiple-zone ventilation and temperature control of a single-duct VAV system using model predictive strategy. Energy Build 2006;38:1248–61.
[73] Xu M, Li S. Practical generalized predictive control with decentralized identification approach to HVAC systems. Energy Convers Manag 2007;48:292–9.
[74] Henze GP, Dodier RH, Krarti M. Development of a predictive optimal controller for thermal energy storage systems. HVAC R Res 1997;3:233–64.
[75] Henze GP, Kalz DE, Liu S, Felsmann C. Experimental analysis of model-based predictive optimal control for active and passive building thermal storage inventory. HVAC R Res 2005;11:189–213.
[76] Aswani A, Master N, Taneja J, Culler D, Tomlin C. Reducing transient and steady state electricity consumption in HVAC using learning-based model-predictive control. Proc of the IEEE 2012;100:240–53.
[77] Wallace M, McBride R, Aumi S, Mhaskar P, House J, Salsbury T. Energy efficient model predictive building temperature control. Chem Eng Sci 2012;69:45–58.
[78] Balan R, Cooper J, Chao K-M, Stan S, Donca R. Parameter identification and model based predictive control of temperature inside a house. Energy Build 2011;43:748–58.
[79] Lefort A, Bourdais R, Ansanay-Alex G, Gueguen H. Hierarchical control method applied to energy management of a residential house. Energy Build 2013;64:53–61.
[80] Huang G, Wang S, Xu X. A robust model predictive control strategy for improving the control performance of air-conditioning systems. Energy Convers Manag 2009;50:2650–8.
[81] Ma Y, Borrelli F, Hencey B, Coffey B, Benga S, Haves P. Model predictive control for the operation of building cooling systems. IEEE Tran Control Syst Technol 2012;20:796–803.
[82] Ferkl L, Siroky J. Ceiling radiant cooling: comparison of ARMAX and subspace identification modelling methods. Build Environ 2010;45:205–12.
[83] Platt G, Ward J, Wall J. Optimal supervisory HVAC control: experiences in Australia. HVAC R Res 2011;17:297–308.
[84] Katipamula S, Lu N. Evaluation of residential HVAC control strategies for demand response programs. ASHRAE Trans 2006;112:535–46.
[85] Canbay CS. Optimization of HVAC control strategies by building management systems case study: Ozdilek shopping center [MSc thesis]. Izmir, Turkey: Izmir Institute of Technology; 2003.
[86] Hart R. Advanced unitary HVAC control sequence. ASHRAE Trans 2012;118:628–35.
[87] Hart R, Callahan JM, Anderson K, Johanning P. Unitary HVAC premium ventilation upgrade. ASHRAE Trans 2011;117:517–24.
[88] Zhou G, Krarti M, Henze GP. Parametric analysis of active and passive building thermal storage utilization. J Sol Energy Eng 2005;127:37–46.
[89] Henze GP, Felsmann C, Kalz DE, Knabe G. Impact of forecasting accuracy on predictive optimal control of active and passive building thermal storage inventory. HVAC R Res 2004;10:153–78.
[90] Florita AR, Henze GP. Comparison of short-term weather forecasting models for model predictive control. HVAC R Res 2009;15:835–53.
[91] Oldewurtel F, Parisio A, Jones CN, Gyalistras D, Gwerder M, Stauch V, et al. Use of model predictive control and weather forecasts for energy efficient building climate control. Energy Build 2012;45:15–27.
[92] Thosar A, Patra A, Bhattacharyya S. Feedback linearization based control of a variable air volume air conditioning system for cooling applications. ISA Trans 2008;47:339–49.
[93] Beghi A, Cecchinato L, Paggiaro F, Rampazzo M. VAVAC systems modeling and simulation for FDD applications. In: Int Conf Control Autom (ICCA). Santiago, Chile: IEEE; 2011. pp. 800–5.
[94] Vasak M, Starcic A, Martincevic A. Model predictive control of heating and cooling in a family house. In: Proc 34th Int convention MIPRO. Opatija, Croatia: IEEE; 2011. pp. 739–43.
[95] Atthajariyakul S, Leephakpreeda T. Real-time determination of optimal indoor-air condition for thermal comfort, air quality and efficient energy usage. Energy Build 2004;36:720–33.
[96] Tashtoush B, Molhim M, Al-Rousan M. Dynamic model of an HVAC system for control analysis. Energy 2005;30:1729–45.
[97] Jin G-Y, Ding X-D, Tan P-Y, Koh T-M. A hybrid water-cooled centrifugal chiller model. In: Conf Ind Electron Appl (ICIEA). Beijing, China: IEEE; 2011. pp. 2298–303.
[98] Wemhoff AP, Frank MV. Predictions of energy savings in HVAC systems by lumped models. Energy Build 2010;42:1807–14.
[99] Xu P, Xu T, Shen P. Advancing evaporative rooftop packaged air conditioning: a new design and performance model development. Appl Therm Eng 2012;40:8–17.
[100] Chen J, Lian Z, Tan L, Zhu W, Zhang W. Modeling and experimental research on ground-source heat pump in operation by neural network. In: Int Conf Comput Distrib Control Intell Environ Monit (CDCIEM). Changsha, China: IEEE; 2011. pp. 459–62.
[101] Kusiak A, Li M, Zhang Z. A data-driven approach for steam load prediction in buildings. Appl Energy 2010;87:925–33.
[102] Tang F. HVAC system modeling and optimization: a data-mining approach [MSc thesis]. Iowa City, Iowa, United States: The University of Iowa; 2010.
[103] Homod RZ, Sahari KSM, Almurib HAF, Nagi FH. RLF and TS fuzzy model identification of indoor thermal comfort based on PMV/PPD. Build Environ 2012;49:141–53.
[104] Soyguder S, Alli H. Predicting of fan speed for energy saving in HVAC system based on adaptive network based fuzzy inference system. Expert Syst Appl 2009;36:8631–8.
[105] Ding L, Lv J, Li X, Li L. Support vector regression and ant colony optimization for HVAC cooling load prediction. In: Int Symp Comput Commun Control Autom (3CA). Tainan, Taiwan: IEEE; 2010. pp. 537–41.
[106] Wang S, Xu X, Huang G. Robust MPC for temperature control of air-conditioning systems concerning on constraints and multitype uncertainties. Build Serv Eng Res Technol 2010;31:39–55.
[107] Bi Q, Cai W-J, Wang Q-G, Hang C-C, Lee E-L, Sun Y, et al. Advanced controller auto-tuning and its application in HVAC systems. Control Eng Pract 2000;8:633–44.
[108] Yiu CMJ. Statistical modelling and forecasting schemes for air-conditioning system [PhD thesis]. Hung Hom, Kowloon, Hong Kong: The Hong Kong Polytechnic University; 2008.
[109] Matlab. Signal smoothing. Signal processing toolbox: MathWorks; 2013.
[110] Energy estimating and modeling methods. ASHRAE handbook fundamentals. ASHRAE; 2009.
[111] Kumar M, Kar IN. Non-linear HVAC computations using least square support vector machines. Energy Convers Manag 2009;50:1411–8.
[112] Mustafaraj G, Chen J, Lowry G. Thermal behaviour prediction utilizing artificial neural networks for an open office. Appl Math Model 2010;34:3216–30.
[113] Li X, Ding L, Deng Y, Li L. Hybrid support vector machine and ARIMA model in building cooling prediction. In: Int Symp Comput Commun Control Autom (3CA). Tainan, Taiwan: IEEE; 2010. pp. 533–6.
[114] Li K, Su H, Chu J. Forecasting building energy consumption using neural networks and hybrid neuro-fuzzy system: a comparative study. Energy Build 2011;43:2893–9.
[115] Wu S, Sun J-Q. A physics-based linear parametric model of room temperature in office buildings. Build Environ 2012;50:1–9.
[116] Maasoumy MH. Modeling and optimal control algorithm design for HVAC systems in energy efficient buildings [MSc thesis]. Berkeley, California, United States: University of California at Berkeley; 2011.
[117] Henze GP, Yuill DP, Coward AH. Development of a model predictive controller for tankless water heaters. HVAC R Res 2009;15:3–23.
[118] Fong KF, Hanby VI, Chow TT. A robust evolutionary algorithm for HVAC engineering optimization. HVAC R Res 2008;14:683–705.

[119] Zou Q, Ji J, Zhang S, Shi M, Luo Y. Model predictive control based on particle swarm optimization of greenhouse climate for saving energy consumption. In: World Autom Congr (WAC). Kobe, Japan: IEEE; 2010. pp. 123–8.
[120] Molina D, Lu C, Sherman V, Harley R. Model predictive and genetic algorithm based optimization of residential temperature control in the presence of time-varying electricity prices. In: Ind Appl Soc Annu meeting (IAS). Orlando, Florida, USA: IEEE; 2011. pp. 1–7.
[121] Aggelogiannaki E, Sarimveis H. A simulated annealing algorithm for prioritized multiobjective optimization – implementation in an adaptive model predictive control configuration. IEEE Trans on Syst Man Cybern Part B (Cybern) 2007;37:902–15.
[122] Zhang X, Fong KF, Yuen SY. A novel artificial bee colony algorithm for HVAC optimization problems. HVAC R Res 2013;19:715–31.
[123] Ha LD, Ploix S, Zamai E, Jacomino M. Tabu search for the optimization of household energy consumption. In: Int Conf Inf Reuse Integr. Waikoloa Village, Hawai, USA: IEEE; 2006. pp. 86–92.
[124] Geem ZW, Kim JH, Loganathan GV. A new heuristic optimization algorithm: harmony search. Simul 2001;76:60–8.
[125] Coelho LDS, Mariani VC. Improved firefly algorithm approach applied to chiller loading for energy conservation. Energy Build 2013;59:273–8.
[126] Yang X-S, Deb S. Cuckoo search via Levy flights. In: Nat Biol Inspired Comput (NaBIC). Coimbatore, India: IEEE; 2009. pp. 210–4.
[127] Wang J, Zhang C, Jing Y. Fuzzy immune self-tuning PID control of HVAC system. In: Int Conf Mechatron Autom (ICMA). Takamatsu, Japan: IEEE; 2008. pp. 678–83.
[128] Hu M, Weir JD, Wu T. Decentralized operation strategies for an integrated building energy system using a memetic algorithm. Eur J Oper Res 2012;217: 185–97.
[129] Rubinstein RY, Kroese DP. The cross-entropy method: a unified approach to combinatorial optimization, Monte-Carlo simulation and machine learning. Springer; 2004.
[130] Passino KM. Biomimicry of bacterial foraging for distributed optimization and control. IEEE Control Syst Mag 2002;22:52–67.
[131] Wang S, Jin X. Model-based optimal control of VAV air-conditioning system using genetic algorithm. Build Environ 2000;35:471–87.
[132] Arpaia P, Donnarumma F, Manfredi S, Manna C. Model predictive control strategy based on differential discrete particle swarm optimization. In: Environ Energy Struct Monit Syst (EESMS). Taranto, Italy: IEEE; 2010. pp. 70–3.