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Load pattern-based voltage stability analysis in unbalanced distribution networks considering maximum penetration level of distributed generation

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Abstract: Stability analysis in the power system is becoming more important than ever as more distributed energy resources penetrate in the system. This study presents a novel load pattern voltage stability index (LP_{VSI}) applicable to transmission and distribution systems. By considering the nominal value of voltages, the power network is converted into a two-bus equivalent system. Then, LP_{VSI} is derived by only the real-time measurement of the voltage and deviation of active and reactive power loads. Also, the assessment of distributed generation's penetration level on unbalanced systems, with maximum loadability and power loss reduction constraints, is performed with regard to daily load variations. The accuracy and efficiency of the proposed indicator are tested on an unbalanced 34-node radial distribution system. Obtained results in comparison with some other papers in the literature demonstrate that the proposed voltage stability index is fast and effective in identifying non-trivial instabilities in the power system networks.

Nomenclature

$V_a \measuredangle \delta_a$	complex voltage at *k*th *bus*
$V_{th} \measuredangle \delta_{th}$	complex Thevenin voltage equivalent
R_{th}	Thevenin resistance seen from *k*th bus
X_{th}	Thevenin reactance seen from *k*th bus
I_a	complex current flow to *k*th bus
V_{th_x}	real part of Thevenin voltage equivalent
V_{th_y}	imaginary part of Thevenin voltage equivalent
V_{a_x}	real part of voltage magnitude of *k*th bus
V_{a_y}	imaginary part of voltage magnitude of *k*th bus
P_a	active power injection to *k*th bus
Q_a	reactive power injection to *k*th bus
ΔP	real part of power demand deviation
ΔQ	imaginary part of power demand deviation
W_H	weighting coefficient per hour
$\boldsymbol{V}_{abc}$	voltage matrix of *k*th bus in each phase
$\boldsymbol{Z}_{abc}$	impedance matrix of branches in each phase
R_{abc}	electrical resistance of branches in each phase
$\boldsymbol{I}_{abc}$	current matrix of *k*th bus in each phase
P_{DG}	active power output of DG
P_{Load_i}	active demand loads at *i*th bus
$V_{j,\min}$	lower bounds of the voltage at the bus *j*
$V_{j,\max}$	upper bounds of the voltage at the bus *j*
N	bus numbers
λ	loadability factor
H	24 h
$\boldsymbol{J}$	Jacobian matrix

1 Introduction

The economic growth, abundance of the systems' expected functionalities and environmental constraints have caused the current power systems to be operating in a very intensive and too close to their voltage instability margins. Fast-rising demand of electrical energy on the distribution side, on the other hand, may lead transmission power systems closer to the voltage instability regions, which is deemed to be a major factor in power system blackouts. This is especially more critical when there is not as much enough reserved reactive power to compensate for the voltage drop [1–3]. Planning systems with right reserved required reactive power not only minimise the risk of voltage collapse, but also reduce the transmission power loss with maintaining voltages in its due limits. To this end, it is necessary to identify the weak/critical buses to avoid voltage instability through repair and/or installations of new instruments which supply the appropriate reactive power [4].

In addition, to consider environmental aspects which pave the way to green networks, development of distributed generation units (DGs) has been seen as the most effective solution both to compensate for energy shortage and reduce greenhouse gas emissions. Hence, the spread and deployment of DGs is rapidly growing [5]. This, in turn, will reduce the distance between electricity production and electrical loads, increase energy efficiency and upgrade the postponing investments [6]. Also, DGs, when interconnected, are capable to reduce power losses, improve power quality and enhance related voltage stability issues [7, 8]. However, DGs can have a negative impact on voltage stability if they do not provide a controllable level of reactive power injection to the power network when needed.

The most important challenge one must confront in DGs employment is that related to determining the optimal location and penetration level of the DGs such that they can be easily positioned in a power system without large structural changes, on one hand, and maintain the voltage levels of all the buses within their permissible range, on the other hand. Although very advantageous, DGs may create problems and limitations at high penetration levels, including over-voltage conditions, increased network power losses and oscillatory stability problems [8–10]. Therefore, it seems that the proper penetration level of DGs which are placed at optimal locations, can improve the voltage profile, improve the voltage stability and reduce active and reactive power losses. In [11], the planning of the optimal DG installation is performed by two-stage optimisation method considering the integration of energy storages. They utilise the loss sensitivity factor to maximise

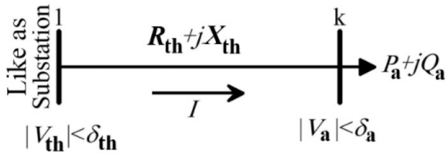

Fig. 1 *Equivalent circuit seen from node k*

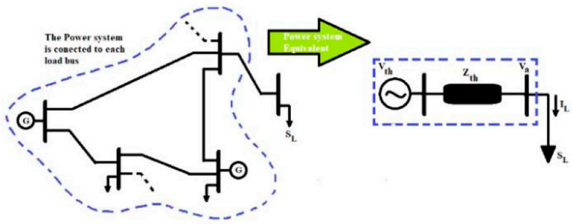

Fig. 2 *Thevenin equivalent circuit seen from a load bus*

investment benefits and reduce the losses to find the best location and optimum size of DGs.

When the power system experience stresses (e.g. due to gradual increase in load), the instability of the voltage can easily occur, especially and often at weak buses [12]. Therefore, planning and operation of a power system in the presence of DGs need to be taken care of both their location selection and penetration level as the most important criteria. In summary, finding nose of the power–voltage (PV) curve and loadability limit (LL) is most important criteria for voltage stability assessment. Voltage instability is mainly a local phenomenon, while voltage collapse affects significant portions of a network. Therefore, studies of voltage stability and voltage collapse in power systems are mostly focused on the detection of weak or critical buses. Using the voltage stability indices, power system operators will be able to monitor the entire network in order to determine locations more prone to the voltage collapse [13–16].

While most studies have focused on voltage stability issues on symmetric and balanced systems, voltage instability in radial distribution systems, which are often configured asymmetric (multi-phase), is mostly associated with asymmetric loading for different phases. The main reason for this is high active and reactive power losses in distribution networks caused by large R/X line ratio [17–20]. In the research context of voltage stability and collapse, many studies have carried out for prediction and detection of voltage collapse and voltage stability: such as the loadability index based on the PV and QV curves analysis [21], optimisation methods [22, 23], sensitivity indicators [24], modal analysis [25], the eigenvalue decomposition method [26], the indicators based on the Thevenin equivalent impedance [2, 27, 28], the line loading indices and the indices extracted from available as well as practical solutions of power flow equations analysis [15, 17, 29–32].

Although in some voltage stability indices, such as FVSI and PSI, the maximum loadability cannot be detected perfectly [9]. This is because in the formulation structure, due to simplifying the equations only active or reactive power is present. Also, in some other voltage stability indices such as the approximation voltage stability index (A-VSI) [14], due to the approximation used in its formulation structure, they are not sufficiently accurate in calculating the loadability of the system. Thus, calculating the optimal values of DG size and location cannot be performed precisely.

Researchers in [2] use a Thevenin equivalent circuit and consider the ***Y***-admittance calculations in the voltage stability index. However, one of the main features of the proposed LP_{VSI} index in the current paper is that it does not require ***Y***-matrix calculations.

Jasmon and Lee [33] introduced a voltage stability indicator (VSI) based on the transmitted power through reduced distribution system lines, without considering the terminal voltage of the line. As a remedy, a VSI has been presented in [17] which includes the terminal voltage and active power equations. In contrast, Zabaiou *et al.* [34] have designed their VSI based on the calculation of the end voltage of the line and the reactive power equations. However, because of oversimplification, their index performs poorly when it comes to the accurate detection of weak buses relative to the voltage instability phenomenon in both transmission and distribution systems [35]. Mahmoud [36] has presented the theory of catastrophe to provide a new indicator in radial distribution networks. Optimal allocation of DG certainly improves the voltage profile and reduces losses in distribution systems, but may affect the issue of voltage stability [9]. Therefore, developing an appropriate voltage stability index in the optimisation algorithm can improve the loadability margin of the networks. In [37], a probabilistic voltage stability index is presented to identify weak buses in the active distribution systems considering uncertainty in the DG output and load demand. However, the voltage stability index provided is similar to the stability index (SI) [29] and catastrophe voltage stability index (CAT-VSI) [36], which are time consuming and at each stage they need to recalculate the power flow for updating. It will be shown that in real-time calculations, the LP_{VSI} index presented in this paper can be more practical in real-time analysis. In addition, a series of methods have been presented for calculating the voltage stability in distribution systems based on the analysis of Jacobian matrix (***J***), network admittance and lines' impedance matrix [38, 39]. However, the associated matrices suffer from singularity issues, making them not suitable for radial distribution networks.

The use of the equivalent circuit, as a common technique in the analysis of voltage stability and in the calculation of the loadability of the system (see Fig. 1), simplifies the analysis and calculation of the voltage and current supplied to the loads [40, 41]. In this regard, Chebbo *et al.* [42] proposed an index for each bus based on calculation of the equivalent impedance amplitude ratio relative to load impedance. They suggested that the voltage instability would occur when the value of the index reached one. In this respect, the impedance matching technique [27, 28] uses local phasor measurement unit to provide an equivalent circuit estimation from the viewpoint of each bus of the transmission network.

Also, there are studies on voltage stability and DGs' penetration levels using repeated load distribution methods, without considering if voltages violate their permissible levels or not [43, 44]. In order to reduce the computational time of the algorithm, a method has been presented in [45] which benefits from the reduced matrix operations for estimating the penetration level of DG. In other studies, DG's penetration levels are discussed in the presence of voltage regulators to improve the voltage profile [46, 47]. So far, few studies have accommodated DG within an unbalanced/multi-phase distribution network in order to improve voltage profile and reduce power loss reduction [48], or similarly, based on unbalanced voltage deviation index [49]. However, neither of these two references has considered the maximum loadability and the voltage limits of the buses in their calculations. In [12], this deficiency has been partly overcome by considering above-mentioned limitations together to examine the effect of DG's penetration level on unbalanced systems. In contrast to these researches, Meng *et al.* [5] consider the stochastic behaviour of DG and the full analysis of the impact of the penetration levels of DGs. It is obvious that the effect of the model selection and/or design of the load distribution systems and daily load variations, as well as the VSIs, are important factors in analysing the system loadability and the penetration level of DGs (Fig. 2).

Also, the maximum loadability margin of the system is discussed in order to maintain voltage stability in the unbalanced distribution systems in [50], considering net-load unbalance of distribution systems connected to transmission systems, the presence of DG and load types, affects the maximum loadability. Therefore, it is important to analyse the voltage stability and determine the maximum loadability of the unbalanced distribution system due to the increasing level of penetration of DGs. In general, it has been shown that the size and location of the DGs can dramatically affect the voltage profile; thus, they must be perfectly designed to keep the bus voltages in their permissible ranges [45].

Hence, the main aim of this paper is to assess the static voltage stability in a distribution electricity network by proposing a novel index to determine the important issue of loadability margin. The proposed load pattern voltage stability index (LP_{VSI}) is able to immediately update its value in the presence of load variations. It is shown that the proposed LP_{VSI} is more accurate than the previous indices by considering the both active and reactive power loads. Also with the aim of the system power loss reduction and the voltage stability index improvement, we propose an iterative method is proposed to determine effectively the penetration level of DGs in this study.

The layout of the paper is as follows. Section 2 includes formulas of the proposed voltage stability index and efficiency analysis in an online estimation of loadability margins. In Section 3, an iterative algorithm for DG allocation is introduced and impact of the penetration level of DG units is analysed in unbalanced networks. Section 4 summarises the main results and demonstrates the efficiency of our approach when applied to the IEEE 34 Node Test Feeder. Finally, the conclusion is given in Section 5.

2 Methodology for load pattern based voltage stability analysis

2.1 Proposed LP_{VSI}

Consideration of a N-node radial distribution system and its conversion to a reduced two-node equivalent circuit for each node (Fig. 1) is presented by Chakravorty and Das in [29].

By considering the Thevenin equivalent circuit for node k of distribution system, following simple equations are used in order to obtain new voltage stability index:

$$\frac{\boldsymbol{V}_{\text{th}} - \boldsymbol{V}_a}{R_{\text{th}} + jX_{\text{th}}} = \boldsymbol{I}_a \tag{1}$$

$$\boldsymbol{I}_a^* \cdot \boldsymbol{V}_a = P_a + jQ_a \tag{2}$$

It can be easily concluded that

$$(\boldsymbol{V}_{\text{th}} - \boldsymbol{V}_a) \cdot \boldsymbol{V}_a = (P_a + jQ_a)(R_{\text{th}} + jX_{\text{th}}) \tag{3a}$$

Then

$$\left((V_{\text{th}_x} - V_{a_x}) - j\left(V_{\text{th}_y} - V_{a_y}\right)\right) \cdot \left(V_{a_x} + jV_{a_y}\right) = (P_a + jQ_a)(R_{\text{th}} - jX_{\text{th}}) \tag{3b}$$

So

$$\begin{aligned}&\left(V_{a_x}\left(V_{\text{th}_x} - V_{a_x}\right) + V_{a_y}\left(V_{\text{th}_y} - V_{a_y}\right)\right)\\ &+ j\left(V_{a_y}\left(V_{\text{th}_x} - V_{a_x}\right) - V_{a_x}\left(V_{\text{th}_y} - V_{a_y}\right)\right) = (P_aR_{\text{th}} + Q_aX_{\text{th}})\\ &+ j(Q_aR_{\text{th}} - P_aX_{\text{th}})\end{aligned} \tag{4}$$

The following two equations can be extracted from (4):

$$\begin{cases} V_{a_x}\left(V_{\text{th}_x} - V_{a_x}\right) + V_{a_y}\left(V_{\text{th}_y} - V_{a_y}\right) = (P_aR_{\text{th}} + Q_aX_{\text{th}}) \\ V_{a_y}\left(V_{\text{th}_x} - V_{a_x}\right) - V_{a_x}\left(V_{\text{th}_y} - V_{a_y}\right) = (Q_aR_{\text{th}} - P_aX_{\text{th}}) \end{cases} \quad (5a, \; b) \tag{5a}$$

Assuming V_{a_x} is known, V_{a_y} is derived by solving the following equation from (5a):

$$V_{a_y}^2 - V_{\text{th}_y}V_{a_y} + (P_aR_{\text{th}} + Q_aX_{\text{th}}) - V_{a_x}\left(V_{\text{th}_x} - V_{a_x}\right) = 0$$

So, V_{a_y} can be calculated if the following inequality is satisfied.

$$V_{\text{th}_y}^2 - 4\left((P_aR_{\text{th}} + Q_aX_{\text{th}}) - V_{a_x}\left(V_{\text{th}_x} - V_{a_x}\right)\right) \geq 0$$

In the worst case, close to the voltage collapse point, it can be observed that

$$\begin{cases} V_{\text{th}_y}^2 - 4\left((P_aR_{\text{th}} + Q_aX_{\text{th}}) - V_{a_x}\left(V_{\text{th}_x} - V_{a_x}\right)\right) = 0 \\ V_{a_y} = V_{\text{th}_y}/2 \end{cases} \tag{6}$$

At this point, it can be obtained that

$$V_{a_x}^2 - V_{\text{th}_x}V_{a_x} + (P_aR_{\text{th}} + Q_aX_{\text{th}}) - V_{\text{th}_y}^2/4 = 0$$

Also

$$\begin{cases} V_{\text{th}_x}^2 - 4\left((P_aR_{\text{th}} + Q_aX_{\text{th}}) - V_{\text{th}_y}^2/4\right) = 0 \\ V_{a_x} = V_{\text{th}_x}/2 \end{cases} \tag{7}$$

Then voltage instability is accrued, if the following inequality is satisfied:

$$V_{\text{th}_x}^2 + V_{\text{th}_y}^2 \leq 4(P_aR_{\text{th}} + Q_aX_{\text{th}}) \tag{8}$$

From the second equation (5b) close to voltage collapse point, the following equation is derived.

$$\frac{R_{th}}{X_{th}} = \frac{P_a}{Q_a} \tag{9}$$

Then, inequality (8) and (9) are combined together in order to define proposed new online voltage stability index

$$\text{LP}_{\text{VSI}} = e^{-\left(\left(\left(V_{\text{th}_x}^2 + V_{\text{th}_y}^2\right)(|R_{\text{th}}| + |X_{\text{th}}|)\right)/\left(4\left(R_{\text{th}}^2 + X_{\text{th}}^2\right)\right)\right)} \tag{10}$$

It should be noted that the maximum stability index identifies the weakest node of the system, which has the highest sensitivity related to the voltage collapse. If the power demands change or the power system is under any gradual increase of demands load, this index is recalculated for each node as follows:

$$\text{LP}_{\text{VSI}} = e^{-\left(\left(\left(\left(V_{\text{th}_x}^2 + V_{\text{th}_y}^2\right)(|R_{\text{th}}| + |X_{\text{th}}|)\right)/4\left(R_{\text{th}}^2 + X_{\text{th}}^2\right)\right) - \Delta P\text{sign}(R_{\text{th}}) - \Delta Q\text{sign}(X_{\text{th}})\right)} \tag{11}$$

where ΔP and ΔQ are the real and imaginary parts of power demand deviation, respectively. Since power demand deviation can be easily calculated from current and voltage of each node, and these two variables are readily available in local measurements, then this index can be used for online estimation to loadability margin.

The proposed indicator has the following advantages:

i. This LP_{VSI} index is easily calculated in each operating point by only two simple experiences.
ii. The proposed index is easily updated by changing the power demands.
iii. It can be used for online estimation of approaching to LL.
iv. As the update of the proposed voltage stability index is only possible by changing active and reactive power of the load demands, so compared to other indices such as CAT-VSI [36] that need to re-analyse the power flow in the system, the proposed method can be updated faster and more effectively. As a result, there is no need to repeat power flow.
v. Due to the simplicity of updating the LP_{VSI} index, it is not necessary to perform again power flow and calculations of Jacobian and network impedance matrices.
vi. Since this proposed voltage stability index depends only on the parameters of equivalent network seen from the each bus of the

system and their active and reactive power deviation, so the proposed index is suitable for both distribution and transmission systems in which the Thevenin equivalent is derivable.

2.2 Load pattern impact

It is known that not only the load model has a significant effect in the determination of system voltage security [35, 51], but also load increase can affect voltage fluctuations in distribution networks [5]. In this paper, the deviation of loads (ΔP and ΔQ) in daily load pattern is considered to the assessment of voltage stability and to find suitable DG penetration level.

The measurements of individual consumer's load curves have been performed in periods of ∼15 days, using electronic equipment. Also, the electric pulses from electronic measurement have been counted and accumulated in given programmed intervals by the user (1, 5 or 15 min) [52]. The consumers' representative curves can be used to obtain daily load curves in any point of the network by aggregation of the consumers' load. Fig. 3 depicts the Iran's peculiar daily demand profile [53] that is used in this paper as real test data. It has been verified that the load forecasting has always been an essential part of an efficient power system planning and operation. Hence, considering the daily load pattern one can infer situations leading to LL in the voltage stability analysis, before the voltage collapse occurs.

The proposed LP_{VSI}, here, is capable of diagnosis of the conditions which leads to voltage instability and provides the operator information of the studied system to take necessary actions in order to avoid instability. The results of predictions can be used to determine the amount of injecting demanded power by DG resources, adding reactive power and even determining of the amount of load shedding required to maintain the system voltage stability.

By normalising the 24 h load curve of Fig. 3, with an average power of almost 30,645 GWh/h and generalising the results to the total active/reactive load demand in the 34-node system studied in this paper, load distribution of the studied system can be obtained in hourly bases and in terms of weighting coefficients W_H, as given in Table 1. In this table, the hourly weight coefficients and the power generation required by the user of the generalised 34-bus system are tabulated, in which $H = 1, 2, \ldots, 24$ represents each hour in a 24 h period.

$$W_H = \frac{P_{H-\text{Iran network}}}{30645},\ H = 1,\ 2, \ldots, 24 \qquad (12)$$

2.3 Thevenin parameter calculation

The following sub-algorithm is used to calculate the equivalent circuit seen from each bus of the radial distribution system:

(a) First, by neglecting all the active and reactive load demands as well as the downstream branches of the bus k, the power flow is performed to obtain the Thevenin voltage of the bus k ($V_{\text{th}}\measuredangle\delta_{\text{th}}$).

(b) By replacing the maximum loadability in bus k, the short circuit current of that bus is calculated $\left(I_{k\text{SC}}\right)$.

(c) Using the equation $Z_{\text{th}} = (V_{\text{th}}\measuredangle\delta_{\text{th}})/I_{k\text{SC}}$, the values of R_{th} and X_{th} for each bus are then obtained.

(d) The above steps are repeated at each bus of distribution system (for all buses).

It is worthwhile to note that for the circumstances when topology changes (such as restructuring), it is necessary to perform the network calculation using the new initial data. Then the new information will be updated by recalculating the power flow in the first part of the proposed algorithm.

3 DG placement and impact of DG penetration level in unbalanced multiphase networks

Proper DG placement and operation will bring benefits for supporting voltage, reducing system power loss and enhancing voltage stability. Therefore, the objective in this paper is to minimise active power losses as well as to maximise system

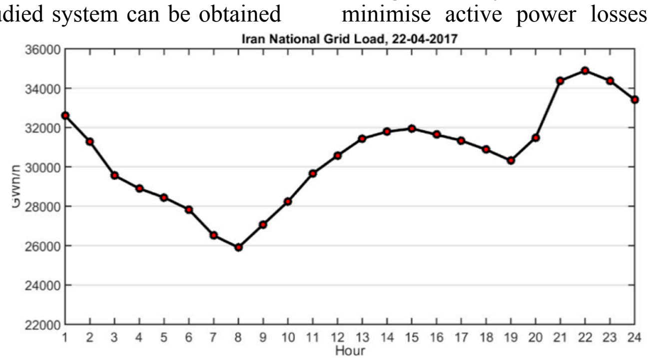

Fig. 3 *Typical daily load curve of Iran Network [53]*

Table 1 Modified daily load curve for case study

Hour	1	2	3	4	5	6	7	8
W_H	1.066	1.023	0.966	0.946	0.929	0.908	0.868	0.847
$P_{34\text{-bus}}$, KW	1886	1810	1674	1674	1643	1606	1536	1498

Hour	9	10	11	12	13	14	15	16
W_H	0.883	0.922	0.969	1.000	1.026	1.039	1.043	1.034
$P_{34\text{-bus}}$, KW	1563	1631	1713	1769	1816	1837	1844	1828

Hour	17	18	19	20	21	22	23	24
W_H	1.022	1.008	0.990	1.028	1.125	1.140	1.125	1.093
$P_{34\text{-bus}}$, KW	1809	1783	1751	1819	1990	2017	1990	1934

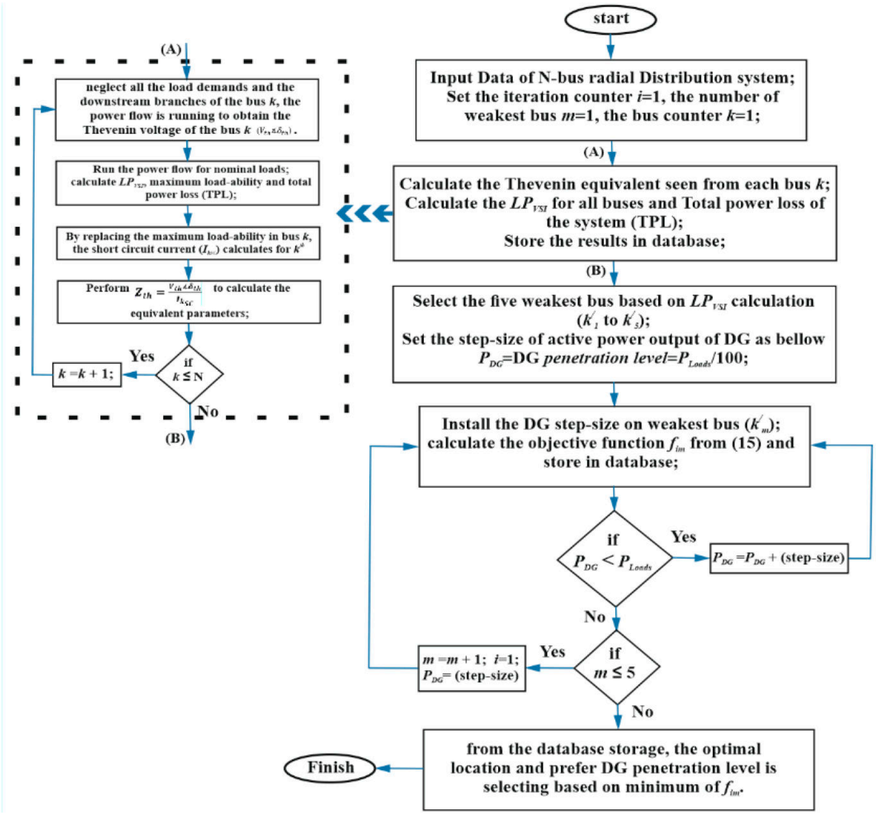

Fig. 4 *Flowchart of the proposed algorithm*

loadability, while keeping the voltage profiles in the network within specified limit.

3.1 Impact of DG allocation on power loss, voltage profile and voltage stability

3.1.1 Reduction of power loss: Choosing the appropriate size and location of DGs can reduce the total losses of the system. In order to calculate the optimal DG with respective to the total active power, the following equation is used:

$$\mathrm{TPL} = [R_{abc}]_{ij} * \left|[\boldsymbol{I}_{abc}]_j\right|^2, \quad i, j = 1, \ldots, N; \quad i < j \tag{13}$$

where R_{abc} is the electrical resistance of each branch in each phase, $\boldsymbol{I}_{abc}$ is the current of the jth node of each phase and N is the number of nodes in the network.

3.1.2 Impact on voltage profile: With reduction of the active and reactive losses through injection of active power by DGs, the voltage profile will also be improved. To analyse effects of the DGs' size and location on the voltage profile for all the three phases in multiphase unbalanced systems, the inter-line mutual impedances will be considered in the computations

$$[\boldsymbol{V}_{abc}]_j = [\boldsymbol{V}_{abc}]_i - [\boldsymbol{Z}_{abc}]_{ij} * [\boldsymbol{I}_{abc}]_j, \quad i, j = 1, \ldots, N; \; i < j \tag{14}$$

where $\boldsymbol{V}_{abc}$ is the phase voltage of the ith and jth nodes, $\boldsymbol{Z}_{abc}$ is the impedance matrix of each branch and $\boldsymbol{I}_{abc}$ is the phases' currents of the node j.

In this paper, an unbalanced backward/forward sweep power flow is performed to assess the voltage stability of an unbalanced distribution system. To update the voltage of the unbalanced distribution system buses in the sweep forward, the branch impedance matrix is used considering the mutual impedances. It is quite compatible with the balanced BFS power flow, which is based on the positive sequence analysis. As a result, the proposed voltage stability index can be easily used in the balanced systems. It is important to note that in order to detect the weakest buses from the voltage stability point of view in the unbalanced distribution systems, the maximum value of the $\mathrm{LP_{VSI}}$s in the three phases for each bus is considered.

3.1.3 Voltage stability improvement: The maximum loadability of the system is considered as the limit for voltage stability assessment of the system. Using the proposed $\mathrm{LP_{VSI}}$ indicator, performance of the two scenarios of the daily peak load and gradually incremental load are tracked until the voltage becomes unstable.

3.2 Proposed algorithm for determining maximum penetration level of DG

To analyse the penetration level of DGs in a multiphase unbalanced distribution network, an iterative algorithm is proposed with the objectives of reduction of the total system losses, improvement of the voltage profile and increasing the system loadability. Proposed algorithm and the proper DG penetration level in optimal location are depicted in flowchart of Fig. 4.

The objective function for the proposed iterative algorithm is shown in the following equation:

$$f = \min \left\{ \frac{\left(\sum_{i=j=1,\; i \neq j}^{N} [R_{abc}]_{ij} * \left|[\boldsymbol{I}_{abc}]_j\right|^2 \right)}{\mathrm{TPL}_{(\text{without DG})}} + \mathrm{LP}_{\mathrm{VSI}_j} \right\} \tag{15}$$

subject to

$$P_{\mathrm{DG}} \leq \sum_{i=1}^{N} P_{\mathrm{Load}_i} \tag{16}$$

$$V_{j,\min} \leq V_j \leq V_{j,\max}, \quad j = 2, \ldots, N \tag{17}$$

where N is the number of buses and the voltage constraints for $V_{j\max}$ and $V_{j\min}$ are 1.05 and 0.9 p.u., respectively.

So far, by using a preliminary BFS power flow and the proposed $\mathrm{LP_{VSI}}$, the weakest nodes are recognised with respect to the voltage instability. Then, by connecting a DG to this critical node and according to the increase in the DG's penetration level,

Table 2 Two-bus system loadability by increasing simultaneously the coefficient λ of active and reactive powers

λ	PV curve		LP_{VSI}	CAT-VSI [36]	A-VSI [14]
	V_{max}, p.u.	V_{min}, p.u.			
1.000	0.8536	0.1464	0.5000	0.1250	0.8750
1.359	0.7831	0.2169	0.6795	0.0801	0.8301
1.697	0.6946	0.3054	0.8485	0.0379	0.7879
2.000	0.5000	0.5000	1.0000	0.0000	0.7500

Table 3 Two-bus system loadability by only increasing active power (P_a)

λ	PV curve		LP_{VSI}	CAT-VSI [36]	A-VSI [14]
	V_{max}, p.u.	V_{min}, p.u.			
1.000	0.8536	0.1464	0.5000	0.1250	0.8750
1.162	0.8298	0.1704	0.5653	0.1087	0.0000
1.718	0.7287	0.2745	0.7937	0.0519	—
2.213	0.5037	0.5037	1.0000	0.0000	—

Table 4 Two-bus system loadability by only increasing active power (Q_a)

λ	PV curve		LP_{VSI}	CAT-VSI [36]	A-VSI [14]
	V_{max}, p.u.	V_{min}, p.u.			
1.000	0.8536	0.1464	0.5000	0.1250	0.8750
1.163	0.8469	0.1526	0.5168	0.1209	0.0000
3.266	0.7443	0.2876	0.7915	0.0555	—
4.660	0.5325	0.5325	1.0000	0.0000	—

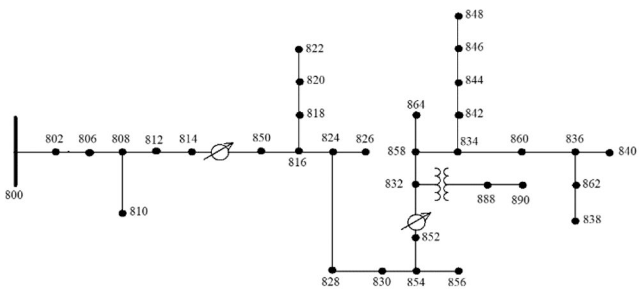

Fig. 5 *IEEE 34 Node Test Feeder*

the optimal size of the DG is obtained, as formulated in the following equation:

$$\text{DG penetration level} = \frac{P_{DG}}{P_{Loads}} \times 100 \tag{18}$$

where P_{DG} is the active power provided by DG and P_{Loads} the total power consumption of the system under study.

4 Numerical results

4.1 Assessment of validity of LP_{VSI}

For comparison, the proposed LP_{VSI} has been tested on the Thevenin equivalent system as Fig. 1. The LP_{VSI} is compared with two practical indices CAT-VSI [36] and A-VSI [14]

$$CAT_{VSI(k)} = \left(P_a R_{th} + Q_a X_{th} - 0.5|V_{th}|^2\right)^2 - Z_{th}^2\left(P_a^2 + Q_a^2\right) \tag{19}$$

$$A-VSI_k = |V_{th}|^4 - 4|V_{th}|^2\left(\theta|V_{th}||V_k| - |V_k|^2\right) - 4\left(|V_{th}||V_k|\delta_k\right)^2 \tag{20}$$

Assuming $V_{th}\measuredangle\delta_{th} = 1.00\measuredangle 0\,(\text{p.u.})$, $R_{th} + jX_{th} = 0.1 + j0.05\,(\text{p.u.})$ and $P_a + jQ_a = 1 + j0.5\,(\text{p.u.})$. Also, $\lambda = 1.00$ refers to nominal load in bus k. The results are derived for three cases as follows: (i) increase in the loadability by increasing simultaneously the coefficient λ of active and reactive powers (Table 2), (ii) increase in the loadability by only increasing active power of loads and (iii) increase in the loadability by only increasing reactive power of loads.

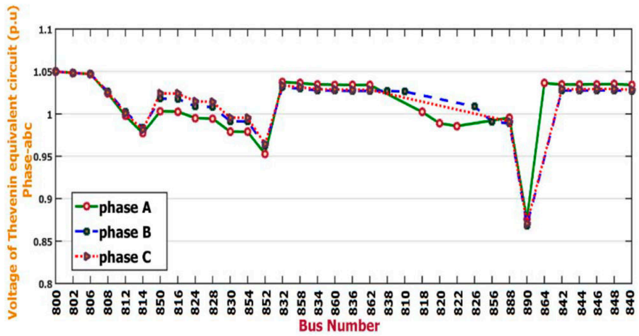

Fig. 6 *Voltage profile for IEEE 34 Node Test Feeder*

Tables 3 and 4 show that when the loadability increases only in active/reactive power, the A-VSI voltage stability index cannot detect the maximum system's loadability.

4.2 Assessment of voltage stability

Voltage stability assessment is performed on the IEEE 34 Node Test Feeder, illustrated in Fig. 5. This test feeder, described in [54], is a part of a real distribution network located in Arizona. The network has been implemented in the *m-file* environment of MATLAB software. Due to the long length and the unbalanced nature of the feeders, it is one of the benchmarks used to validate voltage stability indices. The total active and reactive power loads of the feeder are modified in Table 1.

The resulted voltage profile of each phase of the tested system for nominal loads are shown in Fig. 6. As can be seen in this figure, nodes 890 and 852 have the minimum voltage magnitudes. Due to unbalanced nature of the system, some nodes are disconnected in some phases.

From Fig. 7, it is observed that LP_{VSI} is maximum at node 890; thus, this node has a highest sensitivity to the voltage collapse. The node 852 might be assumed as the second highest sensitive bus to the voltage collapse under the nominal load, but it is not always true. It can be easily concluded that when the system operating

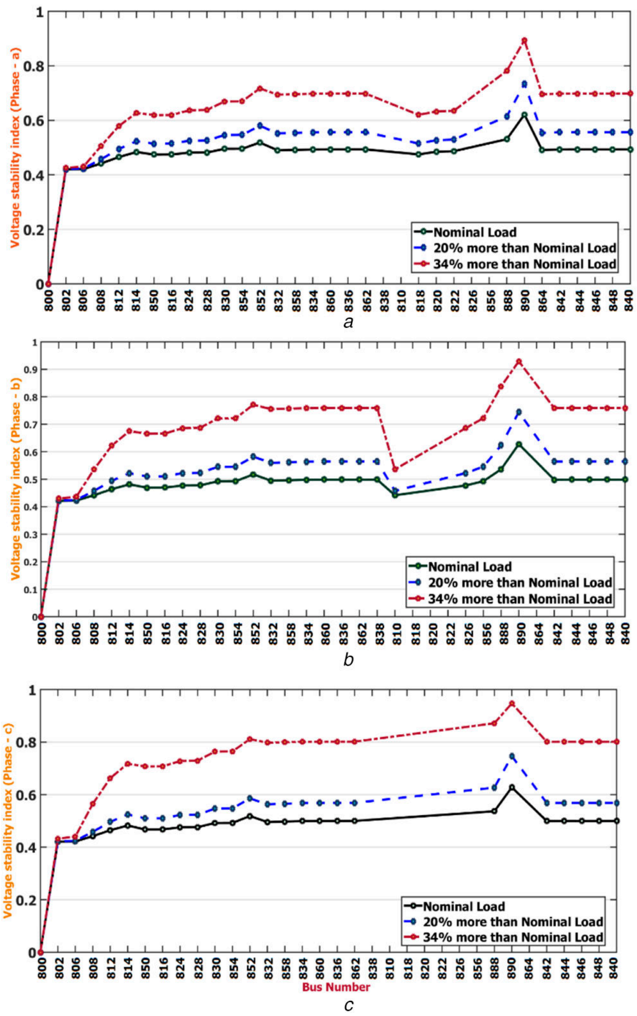

Fig. 7 *Voltage stability index for IEEE 34 Node Test Feeder*
(a) Phase A, ***(b)*** Phase B, ***(c)*** Phase C

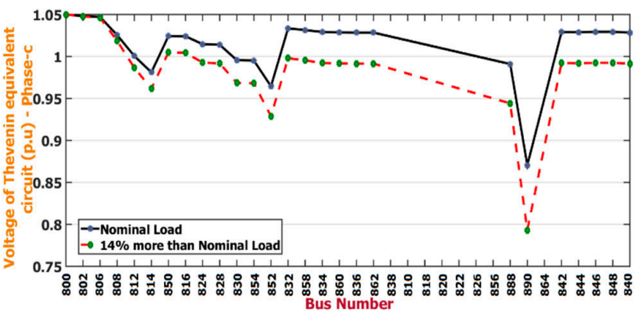

Fig. 8 *Voltage profile for phase C of IEEE 34 Node Test Feeder in peak load demand*

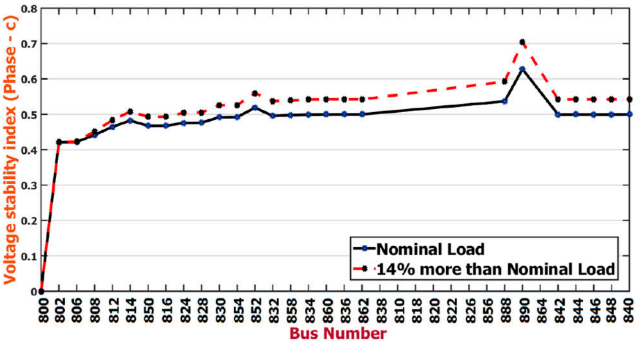

Fig. 9 *Voltage stability index for phase C of IEEE 34 Node Test Feeder in peak load demand*

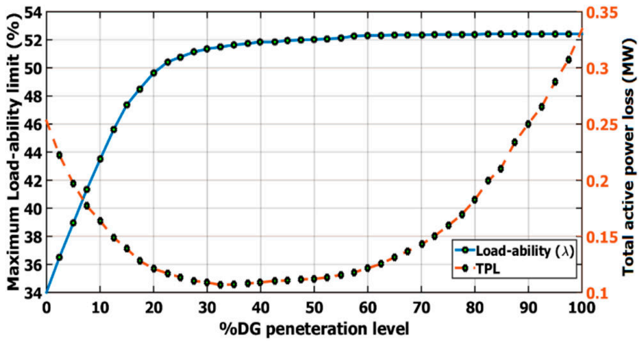

Fig. 10 *Effect of increasing of DG penetration connected in Node 890, LL percentage and total active power loss (MW)*

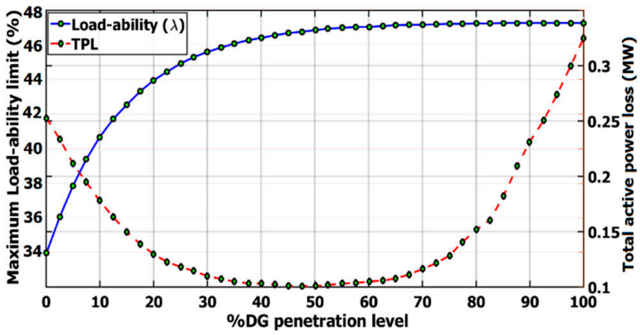

Fig. 11 *Effect of increasing of DG penetration level in Node 852, LL percentage and total active power loss (MW)*

point (power demand) increases, the sensitivity to the voltage collapse at the node 888 is more than that of the node 852.

Figs. 8 and 9 show, respectively, the voltage profile and LP_{VSI} for phase c under the nominal and peak loading condition, according to Table 1. It is observed that with an increase of load to 1.14 times the nominal load at 22:00 (peak time at night), in spite of experiencing more stresses in the system's voltages, the system maintains the voltage stability. The maximum value of the LP_{VSI} is 0.7 and the lowest voltage amplitude of the buses is just below 0.8 p.u.

Fig. 10 shows the system loadability for increasing the penetration level of the DG connected to the node 890 as it is the weakest bus with respect to the voltage stability. According to the figure, the loadability has increased significantly up to the penetration level of 30%. When the penetration level of DG reaches the span between 30 and 40%, the rate of loadability is smaller than that penetration level of 30%. Also, in Fig. 10, the total system losses are plotted versus the increase of the DG's penetration level. It is observed that the lower losses in the system occur when the penetration level of the DG is between 30 and 40% of the total system load. Altogether, from Fig. 10, it can be concluded that the best penetration level for connection of DGs in the studied network is in the range between 30 and 40% of the total system power consumption.

Fig. 11 illustrates the maximum loadability and the total power loss of the network, respectively, considering the penetration level of a DG unit connected to the critical node 852. It can be seen in these figures that when the DG's penetration level is 45% of the total network consumption, the lowest total power loss (0.102 MW) at the maximum loadability of 1.467 is achievable in order to avoid voltage collapse at the network level.

4.3 Comparison of simulation results with the other DG placement methods

In order to show the performance and accuracy of the proposed algorithm, the results of the simulation of the 34-nodes unbalanced multiphase system have been compared with those of [5, 12]. In [5], a two-dimensional multi-resolution modelling method has been proposed considering the voltage fluctuations as constraints in their algorithm. Juanuwattanakul and Masoum [12] use a similar

Table 5 Comparison of simulation results for DG placement and sizing with other references

Method	DG location	DG penetration, %	Maximum LL	Total loss, MW
load pattern based proposed algorithm (LP_{VSI})	890	33	1.515	0.1067
2D multi-resolution modelling algorithm [5]	832	30	1.7	0.1128
iterative algorithm for DG placement [12]	890	36	3.575	0.0965

algorithm to find the optimal DG locations. The results of the comparison with these two methods are presented in Table 5.

According to Table 5, the penetration level of DGs compared to the results of our study is slightly higher in [12] and slightly lower in [5]. This can be due to the applying of load pattern based voltage stability index and system's LL. Also, detection of the maximum loadability in system is conveniently close to that obtained in [5]. On the other hand, the total power loss of the system, here, is estimated to be 0.1067 MW, which when compared with [5, 12], is roughly the same. This, in turn, validates the accuracy of our proposed algorithm.

4.4 Computational time

The computational advantage of the proposed method is that there is no need to repeat power flow. Therefore, the LP_{VSI} index can be very efficient in the practical applications and voltage stability monitoring in the large power systems as it can be updated easily by using deviation active and reactive power loads. Since in the actual distribution systems it is possible to create conditions such as reconfiguration, therefore, more research is still needed in practical systems in the future when using the proposed index.

5 Conclusion

A new load pattern based voltage stability index is presented to identify weak buses. This new index is introduced for online estimation of loadability margin. After finding Thevenin equivalent circuit seen from a load point under nominal load, it is observed that the proposed indicator is only dependent on the bus voltage and active and reactive power deviation. The effectiveness of the proposed voltage stability index for unbalanced multiphase distribution network is demonstrated using IEEE 34 Node Test Feeder.

Based on iterative algorithm and proposed LP_{VSI}, our work focused on finding optimal DG placement in an unbalanced distribution network. It is observed that by comparing with other DG placement approaches in [5, 12], it was demonstrated that the proposed technique provides better interpretation in results for LLs and lower grid losses.

The penetration level of the DG is evaluated for the maximum loadability and minimum power loss of the system using the proposed voltage stability index. The results indicate that in order to accurately determine the DGs' penetration levels within the networks, besides studying the power loss reduction, voltage profile improvement and the voltage stability index, the variations of the system loadability of the studied network are also required to be considered.

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