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Line Capacity Expansion and Transmission Switching in Power Systems With Large-Scale Wind Power

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***Abstract*—In 2020 electricity production from wind power should constitute nearly 50% of electricity demand in Denmark. In this paper we look at optimal expansion of the transmission network in order to integrate 50% wind power in the system, while minimizing total fixed investment cost and expected cost of power generation. We allow for active switching of transmission elements to reduce congestion effects caused by Kirchhoff's voltage law. Results show that actively switching transmission lines may yield a better utilization of transmission networks with large-scale wind power and increase wind power penetration. Furthermore, it is shown that transmission switching is likely to affect the optimal line capacity expansion plan.**

***Index Terms*—Capacity planning, mathematical programming, smart grids, stochastic systems, transmission lines, wind power generation.**

I. Introduction

IN 2020 electricity generation from wind power is planned to constitute nearly 50% of demand for electricity in Denmark. This will primarily be achieved through a huge increase in the number of offshore wind farms.

As a consequence, massive changes are expected in the Danish electricity system in the years to come. Conventional power plants will close down, new market structures are expected to emerge as balancing needs and the requirement for flexible demand are going to increase. Transmission flows will change and the need for transmission capacity will increase.

In addition to the wind power development, it has been decided that all 132/150 kV overhead lines in Denmark shall be replaced by underground cables during the next 30 years. This constitutes a huge challenge for the Danish transmission system operator, Energinet.dk, however, it is also a great opportunity to redesign this part of the transmission grid as a whole.

Energinet.dk, is state owned and operates the grid on a nonprofit basis. In general, transmission investments are carried out at lowest cost while maintaining a certain high level of security of supply. Connections abroad and large domestic grid investments are considered to have significant societal economical impact, and hence the socio-economic welfare effect of such investments must be evaluated. Only investments that provide positive overall socio-economic impacts are promoted. To ensure robust decisions, the impact of any larger investment is evaluated in a context of different future scenarios, each describing likely developments of the society 20 years ahead in time.

Traditionally, in Denmark, investments have mainly been considered incrementally. However, considering the underground cabling and the rapid wind power development there might be huge gains by coordinating investments to find an optimal future grid. In this paper, we consider the stochastic line capacity expansion problem with transmission switching. This model can be applied to the development of the Danish grid and we show that active switching of transmission lines increases the utilization of the transmission grid.

The combinatorial complexity of the line capacity expansion problem makes it hard to solve. The deterministic version of this problem has been solved successfully using Benders decomposition [1], [2] and a commercial MIP solver (CPLEX) [3]. In [4] a stochastic scenario based formulation of the line capacity expansion problem for competitive markets is presented.

Adding new lines to an electricity network may increase the cost of power generation. This paradox is due to Kirchhoff's *voltage law* and is demonstrated for a three-node network by Bushnell and Stoft [5] and for a four-node network by Bjørndal and Jörnsten [6]. Conversely, removing lines may enable a more efficient use of network resources and decrease generation cost as shown by, e.g., Fisher *et al.* [7] for the IEEE 118-bus network [8]. Although this phenomenon is not typical for traditional power systems it may occur on a regular basis. In particular, congestions caused by Kirchhoff's voltage law are likely to occur in meshed networks consisting of different voltage levels. A cycle in the network graph containing both strong high voltage lines as well as weaker lower voltage lines is likely to cause congestion under particular load/production scenarios, if the impedance of one of the weak lines is relatively small compared to the impedance of one of the strong lines (considering the ratio between their thermal capacities). We believe that the situations with congestions due to Kirchhoff's voltage law are likely to increase significantly in the future due to increased wind power generation in parts of the network, that are far from demand centres requiring higher transmission of power. Furthermore, the intermittent nature of wind power requires the configuration of the network to be robust with respect to a large number of extreme production scenarios. Hence, any static configuration of the transmission network is likely to be sub-optimal in some situations.

Transmission switching [9] may alleviate congestion caused by Kirchhoff's voltage law by switching out transmission lines. In general transmission switching may increase security of a network [10] and decrease cost of generation [7], [11]. In particular, Fisher *et al.* [7] show that transmission switching may reduce the cost of generation in the IEEE 118-bus network by up to 25% for a peak demand period. Khodaei and Shahidehpour [12] propose a Benders decomposition approach for solving the dynamic line capacity expansion problem with transmission switching.

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In this paper we consider the expansion of an existing electricity network with transmission switching using the model proposed in [13]. In the planning phase we allow to invest in new transmission lines as well as advanced switches that enables automatic and frequent switching of existing as well as new lines. Transmission line investments are chosen from a candidate set of potential line investments each having predefined thermal capacities and impedance factors. This enables us to model, e.g., decisions of whether or not to strengthen an existing line by increasing its capacity as well as choices between different line types having different impedance factors. We show that allowing to switch existing and new transmission lines reduces operational costs and enables us to augment the network in a way that is cheaper—with respect to total cost—than what can be done for a static (non-switchable) network.

The problem is modeled as a two-stage stochastic mixed integer program with uncertain future generation capacities and demand. In the first stage, decisions on line capacity expansions and switch investments are made. In the second stage, operational decisions on power generation, line flows, and switching decisions are made in a number of scenarios reflecting variations in demand (e.g., peak/off-peak) and generation capacity (e.g., level of wind). The model will balance investment costs against potential reductions in operational cost yielding an optimal solution given the expectation of future generation and demand.

The contribution of this paper is two-fold. Firstly, we demonstrate that the line capacity expansion problem with transmission switching modeled as a two-stage stochastic mixed integer program can be solved efficiently for realistically sized networks using column generation. Secondly, we show that actively switching transmission elements in congested networks with large-scale wind power may reduce generation cost and increase the amount of wind power that can be integrated into the system. Furthermore, actively switching transmission lines may alter the optimal line capacity expansion plan.

We begin the paper by recalling a mixed integer programming formulation of the direct current approximated optimal power flow problem with transmission switching [7]. In Section III we state the line capacity expansion and switch investment problem as a two-stage stochastic program and provide the Dantzig-Wolfe reformulation [14] as proposed in [13]. Section IV provides computational results for a six-bus network, the IEEE 118-bus network and the Danish transmission network with large-scale wind power. Finally, concluding remarks and directions for future research are given in Section V.

II. Operational Dispatch

We assume a linear DC-approximation of the optimal power flow (see, e.g., [15] and [16]) with linear generation costs and no line losses. Consider the directed graph $G=(\mathcal{N},\mathcal{A})$, where the nodes represent the buses of a power transmission network, while the arcs represent the transmission lines and possibly transformers.

For each directed arc $a\in\mathcal{A}$, the reactance coefficient are given by r_a, while thermal transmission capacity is denoted by u_a. Let the set of arcs $\mathcal{F}(i)$, respectively, $\mathcal{T}(i)$ denote the set of arcs with tail, respectively, head i. Define by $\mathcal{D}$ the set of direct current (DC) lines, while $\mathcal{E}$ denotes the set of existing lines and $\mathcal{H}$ the set of switchable lines. The set $\mathcal{G}$ represents the set of generation units. The subset $\mathcal{G}(i)\subset\mathcal{G}$ denotes the set of generators located at node i. Each generator unit $g\in\mathcal{G}$ has marginal generation cost c_g and generation capacity b_g. Demand d_i must be met at all nodes i in $\mathcal{N}$.

In the operational dispatch problem, we have the following variables:

1) the generation q_g for each generator $g\in\mathcal{G}$;
2) the flow x_a on each transmission line $a\in\mathcal{A}$;
3) the voltage phase angle w_i for each node $i\in\mathcal{N}$;
4) a binary decision variable z_a for each switchable arc $a\in\mathcal{A}$ indicating whether line a is active $(z_a=0)$ or not. (An active line is one which is installed and not switched out.)

For a single time period the globally optimal state (q^*,x^*,w^*,z^*) of the system—yielding a maximum social surplus—may be found by solving the mixed integer linear program

$$\min\sum_{g\in\mathcal{G}} c_g q_g \tag{1}$$

subject to

$$q_g \le b_g \quad \forall g\in\mathcal{G} \tag{2}$$
$$x_a \le u_a(1-z_a) \quad \forall a\in\mathcal{A} \tag{3}$$
$$x_a \ge -u_a(1-z_a) \quad \forall a\in\mathcal{A} \tag{4}$$
$$\sum_{g\in\mathcal{G}_i} q_a + \sum_{a\in\mathcal{T}(i)} x_a = d_i + \sum_{a\in\mathcal{F}(i)} x_a \quad \forall i\in\mathcal{N} \tag{5}$$
$$z_a = 0 \Rightarrow r_a x_a = w_i - w_j \quad \forall a=(i,j)\in\mathcal{A}\setminus(\mathcal{S}\cap\mathcal{D}) \tag{6}$$
$$z_a\in\{0,1\} \quad \forall a\in\mathcal{A} \tag{7}$$
$$z_a = 0 \quad \forall a\in\mathcal{A}\setminus\mathcal{H} \tag{8}$$
$$q_g\ge 0 \quad \forall g\in\mathcal{G}. \tag{9}$$

The decision variables are chosen to minimize the total cost of operation expressed by the formula (1) subject to the constraints defined by (3)–(9). In more detail these are: capacity limit on power generation (2); capacity limits on line flows (3)–(4); conservation of flow (5) at each node; Kirchhoffs *voltage law* (6) for all active arcs. The *switching variables* z_a are binary (7) and fixed for lines that are not switchable (8).

The switching concept treated in this paper relies on automatic switching of lines. However, if this automation fails the operator needs to make corrective actions. The number of contingency situations caused by failure of automatic switching increases exponentially with the number of lines switched out at any given point in time (for instance if lines do not—as expected—automatically switch back in after having been switched out for a period of time). For this reason—and to reduce the computational complexity of the problem—we may restrict the number of lines that are being switched

$$\sum_{a\in\mathcal{E}} z_a \le k. \tag{10}$$

Equation (10) reduces the vulnerability of the system in an operational context by limiting the number of *existing* lines that may be switched out. Furthermore, the marginal economic benefit of switching one additional line decreases significantly with the number of lines already switched.

Note that constraints (6) may be linearized using a big-M construction

$$-Mz_a \le r_a x_a - w_i + w_j \le M z_a,\ \forall a=(i,j)\in\mathcal{A}\setminus(\mathcal{S}\cap\mathcal{D}) \tag{11}$$

where M is some sufficiently large number.

III. Two-Stage Stochastic Model With Transmission Switching

We now consider investment decisions for provisioning of switching capability and additional transmission capacity in order to reduce the total investment cost and expected operational cost over the lifetime of the investments. That is, our investment plan must accommodate uncertain outcomes with respect to generation cost and capacity as well as demand. In practice these outcomes will have a continuous probability distribution giving a two-stage stochastic program with integer recourse, where first stage decisions involve investment in switching equipment y_S and line capacity y_L, while the second stage models operational decisions (q, x, w, z) for dispatch and switching. This problem is typically reduced by sampling to a finite-dimensional problem (the sample average approximation [17]) having a finite number N of scenarios $\omega \in \Omega$. In the following, we will assume that sampling has been performed and that the resulting set of scenarios is available. To cover the case where the scenarios are constructed (rather than sampled) we assign them general probabilities $p(\omega)$. These will each take the value $1/N$ if sample average approximation is used.

In each scenario ω we have a realization of demand $d(\omega)$, generation cost $c(\omega)$, generation capacity $b(\omega)$, and transmission capacity $u(\omega)$. This enables us to vary parameters according to climatic conditions (e.g., high costs could model shortage of water in hydro stations, and low generation capacity model low wind outcomes for wind farms), outages of generation units and transmission lines, and different load levels. We assume that transmission switching and economic dispatch is carried out after these random outcomes are realized.

For each scenario $\omega \in \Omega$, let

$$\mathcal{Q}(\omega) = \{(q(\omega), x(\omega), w(\omega), z(\omega)) \mid (3) - (10)\}$$

corresponding to the set of feasible operational decisions in scenario ω. Let f_S be the fixed vector of switch installation costs and f_L be the fixed vector of line investment costs.

The model may now be formulated as

$$\min\ f_S^\top y_S + f_L^\top y_L + \sum_{\omega\in\Omega} p(\omega)c(\omega)^\top q(\omega) \tag{12}$$

$$\text{s.t.}\quad y_L - y_S + z(\omega) \le 1 \qquad \forall\omega \in \Omega \tag{13}$$

$$y_L + z(\omega) \ge 1 \qquad \forall\omega\in\Omega \tag{14}$$

$$(q(\omega), x(\omega), w(\omega), z(\omega)) \in \mathcal{Q}(\omega) \qquad \forall\omega\in\Omega \tag{15}$$

$$y_L, y_S \in \{0,1\}^{|\mathcal{A}|}. \tag{16}$$

In this model, the investment decisions y_L and y_S are the first stage decision variables, that are chosen prior to the realization of the random variables c, b, u, and d. The second stage decision variables q, x, w, and z models the economic dispatch and switching in each scenario subsequent to the realization of the random variables.

The objective (12) minimizes the hourly fixed and operational cost, while (13) ensures that switching of installed lines is only possible if a switch is also installed. Constraint (14) allows lines to be switched in only if they are also installed. We set $e_a^\top y_L = 1$ and $e_a^\top f_L = 0$ for existing lines a in $\mathcal{E}$, where e_a is the binary unit vector of $|\mathcal{A}|$ elements with the ath element being 1.

Note, that not installing a line corresponds to having the line switched out [i.e., $z(\omega) = 1$] in all scenarios $\omega \in \Omega$.

A. Dantzig-Wolfe Reformulation

The mathematical program (12)–(16) may be reformulated using Dantzig-Wolfe decomposition [14] and a branch-and-price algorithm may be applied to obtain optimal solutions to this reformulation.

The idea is to decompose the stochastic problem into a master problem and a number of subproblems—one for each scenario. We let the binary vector $z(\omega)$ define a *feasible switching plan* (FSP) for scenario ω if there exists $q(\omega)$, $x(\omega)$, $w(\omega)$ such that $(q(\omega), x(\omega), w(\omega), z(\omega)) \in \mathcal{Q}(\omega)$. Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of all FSPs for scenario ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can write any element in $Z(\omega)$ as

$$z(\omega) = \sum_{j\in J(\omega)} \varphi^j(\omega)\hat{z}^j(\omega)$$

$$\sum_{j\in J(\omega)} \varphi^j(\omega) = 1, \quad \varphi^j(\omega) \in \{0,1\},\ \forall j \in J(\omega).$$

Assume that for each feasible switching plan $\hat{z}^j(\omega)$ the corresponding optimal dispatch of generation and load shedding is given by $\hat{x}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{x}$ as

$$\text{MP} : \min\ f_L^\top y_L + f_S^\top y_S + \sum_{\omega\in\Omega}\sum_{j\in J_\omega} p(\omega)c(\omega)^\top \hat{q}^j(\omega)\varphi^j(\omega) \tag{17}$$

subject to

$$\sum_{j\in J(\omega)} \varphi^j(\omega) = 1\,[\mu(\omega)], \quad \forall\omega\in\Omega \tag{18}$$

$$y_L - y_S + \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) \le 1\,[\pi(\omega)], \quad \forall\omega\in\Omega \tag{19}$$

$$y_L + \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) \ge 1\,[\rho(\omega)], \quad \forall\omega\in\Omega \tag{20}$$

$$\varphi^j(\omega) \in \{0,1\},\ \forall j \in J(\omega) \tag{21}$$

$$y_L, y_S \in \{0,1\}^{|E|} \tag{22}$$

where $\mu(\omega)$, $\pi(\omega)$ and $\rho(\omega)$ denote the dual prices associated with the respective constraints.

The master problem MP is a two-stage stochastic integer program with integer variables in both stages. Although in general these are difficult to solve, the structure of MP is amenable to a branch-and-bound procedure by virtue of the following result.

Proposition 1: If y_L and y_S are chosen to be fixed binary integers, then the linear programming relaxation of MP has integer extreme points.

For a proof we refer the reader to [13].

It is convenient to consider only a subset $Z(\omega)' \subseteq Z(\omega)$ of feasible switching plans for each scenario ω in the master problem. We define this restricted master problem (RMP) by (17)–(22) with $J(\omega)$ replaced by $J(\omega)'$ the index set of $Z(\omega)'$. A column generation algorithm is applied to dynamically add feasible switching plans to the linear relaxation of the master problem. The algorithm is initialized by letting $Z(\omega)' = \{\hat{z}^0(\omega)\} = \{\mathbf{0}\}$, for all scenarios $\omega \in \Omega$. That is, initially no line may be switched out in either scenario. The corresponding operational costs $c(\omega)^\top\hat{q}^0(\omega)$ can easily be found by solving a linear program for each scenario. In each iteration of the algorithm, the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices μ, π, and ρ. A new column $(p(\omega)c(\omega)^\top\hat{q}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = p(\omega)c(\omega)^\top\hat{q}^j(\omega) + \pi(\omega)^\top\hat{z}^j(\omega) - \rho(\omega)^\top\hat{z}^j(\omega) - \mu(\omega)$ is negative.

A column for scenario ω may therefore be constructed by solving the subproblem:

$$\begin{aligned} \min \quad & p(\omega)c(\omega)^\top q + \pi(\omega)^\top z - \rho(\omega)^\top z - \mu(\omega) \\ \text{s.t.} \quad & (q, x, w, z) \in \mathcal{Q}(\omega) \end{aligned}$$

where $\mu(\omega)$, $\pi(\omega)$ and $\rho(\omega)$ are the dual prices returned from RMP-LP.

Any feasible solution $(q, x, w, z) \in \mathcal{Q}(\omega)$ with negative objective function gives rise to a potential candidate column for RMP-LP. If no columns with negative reduced cost exist then we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral, then (φ^*, y^*) is an optimal solution to the master problem (17)–(22) and y^* is the optimal switch investment strategy. Otherwise, we may resort to a branch-and-price framework for finding optimal integer solutions. Note that a fractional solution will always have at least one fractional y-value (see Proposition 1). Hence, we branch on one of the fractional y-variables and hope that this will resolve the fractionality. If not one may continue branching on y-variables until the fractionality is resolved.

IV. Computational Results

In this section experiments are performed on three different networks—a six-bus network, the IEEE 118-bus network and the Danish transmission network. Results for the six-bus network indicate that transmission switching alters the optimal line expansion plan and may reduce the need for grid reinforcement. Experiments with the IEEE 118-bus network with four scenarios suggests that transmission switching is beneficial for the integration of large-scale wind power. Also, these results justify the use of stochastic programming. Results for the Danish network with the expected development of off-shore wind power generation by 2020 confirm that allowing switching may reduce generation cost and increase the amount of wind power integrated in the system.

The purpose of this section is to illustrate the use of the model presented in Section III. We show that dynamically switching realistically sized networks is very likely to be beneficial in systems with large-scale wind power. The scenarios used in this paper are chosen so that they cover extreme combinations of realized wind power generation and demand (however for the six-bus network demand is constant). In practice, combinations of high/low wind and high/low demand are used with equal probabilities, however for the Danish network also intermediate levels of wind and demand are considered.

We note that the scenarios that are used in these examples are for illustration of the computational technique; they are not intended to capture all the variation in future (uncertain) parameters. In practice, a relatively small set of realistic scenarios can be generated using samples drawn from the *true distribution* that is estimated from historical data or provided by a calibrated simulation model, to create a model of the same form as (12)–(16). As mentioned above, this model is called a sample average approximation or SAA (see, e.g., [17]), and its solution determines a candidate investment policy. It can be shown [17] that the solution to SAA approaches the true solution as the sample size increases.

To test any candidate investment policy, we can easily simulate the operational dispatch over thousands of scenarios sampled from the true distribution in order to estimate the true expected cost. However, in the absence of a true model, we have chosen to estimate this expected cost using the scenarios that were used in the construction of the investment policy—an in-sample procedure that we recognize biases the estimate to be lower than the true value in expectation. Our intention in this paper is to illustrate how an investment policy computed from our model will improve over a deterministic solution. Further development will focus on the specification of the true distribution, from which the true expected cost can be estimated.

TABLE I
Demand and Generation in the Six-Bus Network.
$*$ Installed Wind Power Capacity

		Generation	
Node	Demand (MW)	Capacity (MW)	Marg. cost ($/MWh)
a	80	150	9
b	240	-	-
c	40	360	7
d	160	300 $*$	0
e	240	-	-
f	-	600	4

First, we consider a small six-bus instance with two scenarios occuring with equal probability. This example shows that switching may reduce the need for transmission capacity and increase the penetration of wind power. For the IEEE 118-bus network, four extreme equally weighted scenarios are considered. This example aims at providing insight into when and where switching is beneficial. The example shows that one cannot rely on finding a static configuration of switches that is optimal, and that dynamically switching the network is likely to be beneficial. For the Danish network we consider first six equally weighted scenarios, which yield a rather large benefit of transmission switching. Secondly, we consider a more realistic set of scenarios. Although, these are not generated from sampling it provides a good indication of the potential benefits for switching in the Danish network.

A. Six-Bus Network

We will first consider a modified version of the six-bus network first described by Garver [18]. The demand and generation units are detailed in Table I, while the transmission network properties are described in Table II. The network consists initially of six transmission lines. Another five lines may be added to the network each incurring an investment cost. We assume that the generator in node d is a 300 MW wind generator with variable generation output capacity. For the purpose of illustration, we consider only two scenarios, that differ only in the generation output from the wind generator in node d. In the first scenario $\omega = 0$, there is no wind and the output is therefore fixed to 0. In the second scenario $\omega = 1$, the wind generator may produce at its installed capacity of 300 MW with a marginal cost of 0. We allow for curtailment of wind power. Also, k is set to 11 to allow switching of all lines simultaneously. The two scenarios are assumed to occur with equal probability.

Without switching a total cost of 3151.96 is incurred. The corresponding optimal investment strategy will invest in all five new lines incurring an investment cost of 368. The solution is depicted in Figs. 1 and 2. Allowing to switch all transmission lines reduces the total cost to 3094.62. In this investment strategy only four new lines is installed incurring an investment cost of 338—the line between b and e is not installed. In scenario $\omega = 1$, transmission switching makes it possible to reduce generation output at node f by 17.75 MW, while increasing the

TABLE II
TRANSMISSION LINE PROPERTIES IN THE SIX-BUS NETWORK

Terminals	Thermal capacity (MW)	Reactance	Exists	Investment cost ($/h)
a - b	100	0.4	True	-
a - d	80	0.6	True	-
a - e	100	0.2	True	-
b - c	100	0.2	True	-
b - d	100	0.4	True	-
c - e	100	0.2	True	-
d - f	200	0.3	False	60
c - f	200	0.48	False	96
e - f	156	0.61	False	122
b - f	200	0.3	False	60
b - e	100	0.3	False	30

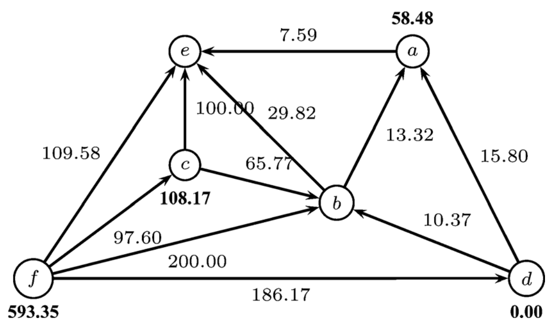

Fig. 1. Six-bus network. Operational decisions without switching in scenario $\omega = 0$. Node labels (bold) indicate generation injection, while arc labels show the corresponding transmission flows.

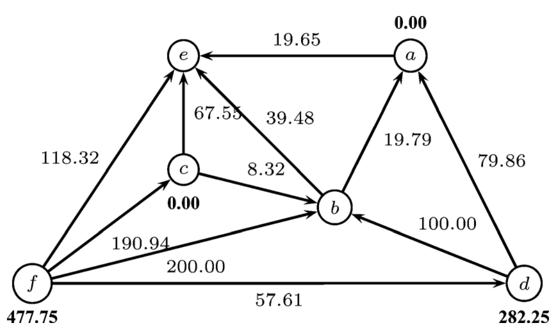

Fig. 2. Six-bus network. Operational decisions without switching in scenario $\omega = 1$. Node labels (bold) indicate generation injection, while arc labels show the corresponding transmission flows.

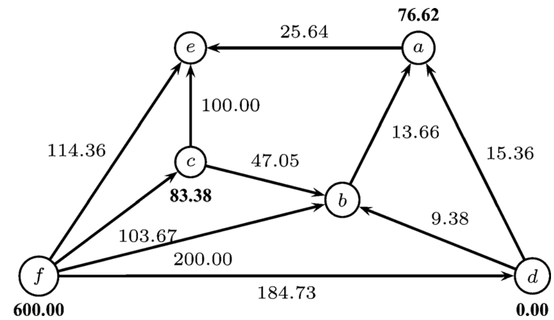

Fig. 3. Six-bus network. Operational decisions with switching in scenario $\omega = 0$. Node labels (bold) indicate generation injection, while arc labels show the corresponding transmission flows. No transmission lines are switched in this scenario.

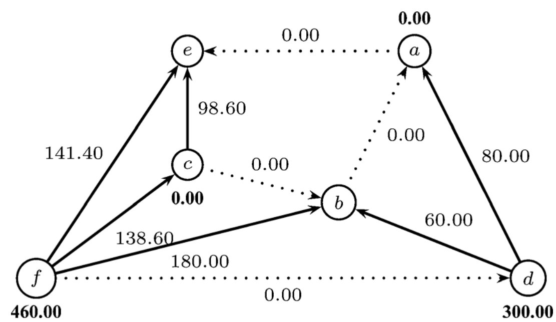

Fig. 4. Six-bus network. Operational decisions with switching in scenario $\omega = 1$. Node labels (bold) indicate generation injection, while arc labels show the corresponding transmission flows. Dotted arcs indicate lines that are switched out.

TABLE III
SUMMARY OF SCENARIOS FOR A SMALL INSTANCE OF THE SWITCH INVESTMENT PROBLEM

Scenario	Probability	Load		Windpower	
ω	$p(\omega)$		% of peak		capacity, MW
1	0.25	off-peak	59%	high	1600
2	0.25	peak	100%	high	1600
3	0.25	off-peak	64%	low	0
4	0.25	peak	99%	low	0

output of the wind generator in node d correspondingly—and thereby reduce the cost of generation and increase the amount of wind power in the system. The solution with switching is depicted in Figs. 3 and 4.

We note that—in this instance—the use of transmission switching reduces the need for investment in transmission capacity. Furthermore, transmission switching reduces curtailment of wind power in the second scenario. In fact generation output from the wind generator is increased from 282.25 MW to its maximum capacity of 300 MW due to the use of transmission switching.

B. IEEE 118-Bus Network

We will first study the IEEE 118-bus network [8] with network data described in [19]. This network has 185 lines, total peak load of 4519 MW, and a total thermal generator capacity of 5859 MW. We will consider a four-scenario instance of the switch investment problem presented in Section III with uncertain outcomes of demand and wind generation capacity. First stage decisions include only investment decisions in switching equipment. That is we assume $y_L = \mathbf{1}$ to be fixed. The results justify the use of stochastic programming and indicates that transmission switching is particularly beneficial in systems with large-scale wind power.

Four scenarios are defined with respect to the load level (peak/off-peak) and amount of wind power (high/low). The scenarios have equal probabilities and are summarized in Table III. The fixed amortized switch investment costs are arbitrarily set to $5/h for each switch. The scenarios considered are not likely to represent the true expectation of the future. Rather, the example serves to illustrate the concept of making strategic investment decisions under uncertainty in electricity networks employing transmission switching.

A 1600 MW intermittent wind power generator with varying supply capacity and 0 marginal cost is located at node 91. Generation from the windpower generator is not fixed so wind generation may be curtailed.

TABLE IV
OPTIMAL SWITCHING CONFIGURATIONS AND SAVED OPERATIONAL COSTS FOR A SMALL FOUR-SCENARIO INSTANCE OF THE SWITCH INVESTMENT PROBLEM ON THE IEEE 118-BUS NETWORK

ω	Switching configuration					Saved costs, \$/h
1	77-80	89-90		89-92		7.6
2	77-80	89-90		89-92		36.0
3	77-80	89-90			94-96	2.2
4	77-80		89-91		94-96	75.4
Total						121.2

TABLE V
WIND POWER GENERATION IN DIFFERENT SCENARIOS WITHOUT ($k = 0$) AND WITH ($k = 3$) SWITCHING

Scenario	$k = 0$			$k = 3$		
ω	91	5	26	91	5	26
1	499.7	479	453	633.13	697.49	395.28
2	538	918	383	737.43	937.25	595.08
3	0	0	0	0	0	0
4	0	0	0	0	0	0

Without switches the total generation cost incurred is \$1031.55/h. In the optimal switch investment strategy with $k = 3$ five switches are installed incurring a total investment cost of \$25/h and generation cost \$910.25/h. The total savings from switching is thus approximately 9%. With optimal switching the dispatched windpower is increased from 499 MW to 648 MW in scenario 1 and from 535 MW to 875 MW in scenario 2. Thus by employing active switching one can increase the amount of windpower in the system and decrease system cost. The optimal switching configurations and the corresponding saved operational costs for each scenario are shown in Table IV.

Since in general (1)–(10) is a difficult mixed integer program for $k \geq 1$, one might consider decoupling the scenarios and solve each scenario separately with amortized investment costs and subsequently piece the solutions together. While this approach might yield good solutions for some instances, we cannot rely on this in general. Applying this approach to our four-scenario instance described above by solving four smaller mixed integer programs, we obtain an investment strategy with nine switches and total operational cost of \$904/h. The net benefit (including switch installation costs) of switching is only \$82.6/h as opposed to \$96.3/h for the optimal switch investment strategy obtained by solving the integrated model. Hence, the value of switching is clearly lower when decoupling the scenarios completely.

We now consider an instance of the problem where we—in addition to having a wind power park at node 91—also have wind power parks in node 5 and node 26. All wind power parks are assumed to be relatively large (1600 MW installed capacity). The scenarios and network are unchanged.

Without switching the total expected generation cost is \$881.56/h. With switching ($k = 3$), this is decreased to \$750.45/h with a total of five installed switches leading to a net benefit of \$106.12/h or approximately 12%. This reduction in costs covers an increase in wind power on the three parks by a total of 430 MW in scenario 2 and 293 MW in scenario 1 (see Table V). The corresponding optimal switching configuration is shown in Table VI.

TABLE VI
OPTIMAL SWITCHING CONFIGURATIONS FOR A SMALL FOUR-SCENARIO INSTANCE OF THE SWITCH INVESTMENT PROBLEM ON THE IEEE 118 -BUS NETWORK

ω	Switching configuration				
1	77-80			23-25	
2	77-80		94-96	23-25	
3		89-91			38-37
4				23-25	

TABLE VII
SUMMARY OF PROJECTED INSTALLED OFF-SHORE WIND POWER CAPACITY IN THE YEAR 2020

name	capacity (MW)	area
Djursland	400	DK1
Horns Rev	1000	DK1
Læsø	600	DK1
Jammerbugt	800	DK1
Ringkøbing	1000	DK1
Kriegers Flak	800	DK2
Rønne Banke	400	DK2
S. Middelgrund	200	DK2

TABLE VIII
SUMMARY OF NETWORK DATA

no. of nodes (busses)	610
no. of transmission lines	529
no. of transformers	302
no. of generators	418
total gen. capacity (DK)	13530
total peak demand	6945

C. Danish Transmission Network

We will now consider the current Danish transmission network and potential line capacity expansions in a future setting with development of many new off-shore wind power plants.

Potential off-shore windpower development in Denmark in the period 2010–2025 is described in [20] and [21]. The projects considered are summarized in Table VII. We assume that all projects are realized.

Network and generation data is obtained from the Danish transmission system operator Energinet.dk. This data is confidential, but aggregated values are available in Table VIII. Potential line capacity expansion projects are likewise based on confidential data from Energinet.dk. Line investment costs for 400 kV lines are based on underground cable costs—overhead lines are not considered. This is in accordance with future expansion guidelines for the Danish electricity transmission grid [22]. The potential candidates for new transmission lines are limited to a set of 10 lines on the 400 kV level and 5 lines on 132 kV level. Operation and maintenance cost of transmission lines are neglected.

Neighboring areas (Norway, Sweden, Germany, and The Netherlands) are modeled in a very simplistic way with no demand and a generator with fixed marginal cost and capacity. This assumes that there is always excess generation capacity in the respective areas which can be supplied at constant cost.

The ability to switch a particular line incurs a fixed investment cost. This is assumed to cover any equipment needed to perform automatic switching of that line including the switch itself (if it needs to be upgraded) and any communication equipment if necessary. Here, we arbitrarily assumes a relatively small fixed cost of 1 DKK/h. In the experiments switching was allowed on high voltage (> 100 kV) transmission lines only.

TABLE IX
SUMMARY OF SCENARIOS. WIND CAPACITY IS THE SHARE OF INSTALLED CAPACITY

		Demand (MW)		Wind capacity	
ω	$p(\omega)$	DK1	DK2	on-shore	off-shore
0	0.16	4076	2869	0.90	0.95
1	0.16	4076	2869	0.50	0.50
2	0.17	4076	2869	0.00	0.00
3	0.17	1448	934	0.90	0.95
4	0.17	1448	934	0.50	0.50
5	0.17	1448	934	0.00	0.00

TABLE X
SUMMARY OF RESULTS FOR DIFFERENT LEVELS OF SWITCHING

	no switch	$k=0$	$k=1$	$k=2$	$k=3$
no. of installed lines	5	10	8	11	10
no. of installed switches	-	7	12	13	15
wind (MWh/h)	2649	2661	2687	2687	2689
fixed cost (DKK/h)	141	311	283	320	319
op. cost (DKK/h)	466829	454293	433982	430916	427294
total cost (DKK/h)	466970	454604	434265	431237	427613

TABLE XI
BENEFIT OF SWITCHING. VALUES ARE ABSOLUTE AND RELATIVE (IN %) DIFFERENCE AS COMPARED TO THE NON-SWITCHED NETWORK ($y_S = 0$)

	$k=0$		$k=1$		$k=2$		$k=3$	
	abs	rel	abs	rel	abs	rel	abs	rel
op. cost, DKK/h	-12536	-3	-32847	-7	-35913	-8	-39535	-8
fixed cost, DKK/h	170	120	142	100	179	127	178	126
total cost, DKK/h	-12366	-3	-32706	-7	-35734	-8	-39357	-8
wind (avg.), MWh/h	12	0.46	38	1.42	38	1.42	40	1.52
wind ($\omega = 0$), MWh/h	76	1.13	235	3.53	236	3.53	251	3.77

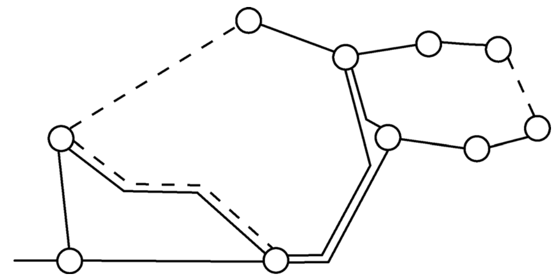

Fig. 5. Part of the existing Eastern 400 kV topology (solid) and potential expansion options (dashed).

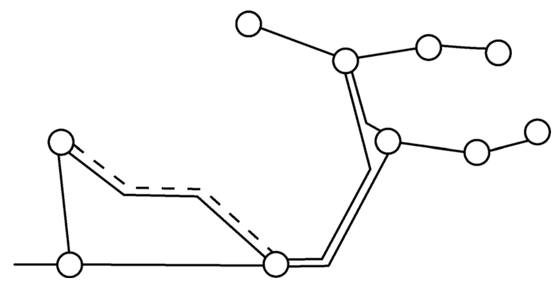

Fig. 6. Partial line capacity expansion plan without switching. The dashed line indicates new line investment.

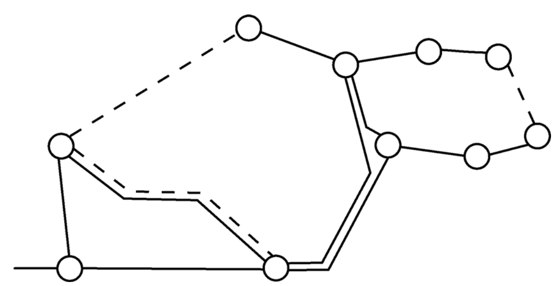

Fig. 7. Partial line capacity expansion plan ($k = 0$). Dashed lines indicate new line investments.

We shall now consider a particular six-scenario instance of the problem. In Table IX a summary of the scenarios is given. The scenarios and their probabilities are for illustration only, and do not reflect our true expectations for the future. Nevertheless, they do give some valuable insights—in particular with regard to the value of switching and its impact on investments in line capacity.

Five instances of the problem with different levels of switching is investigated: *no switch* and $k = 0, 1, \ldots, 3$. The *no switch* instance has fixed $y_S = 0$ and allows for investments in new lines only. The $k = 0$ instance allows switching on new lines only—no switching on existing lines is allowed. For the instances with $k = 1, 2, 3$ switching on all new lines and at most k existing lines is allowed. Table X gives a summary of results for the different instances, while Table XI gives an overview of the benefit of switching.

For the six scenarios described above, the optimal solution contains investments in 5 of the possible 15 new lines without switching (total cost of 466 970 DKK/h of which 141 DKK/h are investment costs). Allowing to switch new lines ($k = 0$), results in a total of 10 new lines installed of which 7 may be switched (Table X). This gives a net-benefit of 12 366 DKK/h (Table XI).

Fig. 5 shows part of the 400 kV network topology in Eastern Denmark, with proposed network expansions, while Fig. 6 shows the partial optimal line capacity expansion plan for that part of the network when no switching is allowed. The network shown in Fig. 7 depicts the optimal line capacity expansion plan when switching is allowed on new lines only ($k = 0$).

In the following experiment, we—in addition to allowing switching on new lines—also allow for switching of one existing line in each scenario ($k = 1$). This yields an investment plan with 8 new lines and 12 switches. The total net benefit of the solution is 32 706 DKK/h or 7% compared to the solution without switching. Fig. 8 depicts (part of) the corresponding optimal line capacity expansion plan.

By allowing switching the wind generation is increased by up to 251 MW in scenario 0 ($k = 3$). Also, operational costs are reduced significantly in the peak demand without wind (scenario 2) by switching to lower cost (thermal) generation.

The results obtained from experiments with the Danish transmission network with large-scale wind power suggests that transmission switching may reduce generation cost and increase wind power generation. As transmission switching acts to reduce congestion in the network, this reduction in cost is entirely due to relief of congestion in the peak demand scenarios.

More interestingly, the optimal line expansion plan is highly sensitive to the level of switching allowed. Actively switching transmission elements increases the number of installed transmission lines (roughly by a factor of 2). This is due to the fact that some lines may be beneficial in some scenarios, but restrictive in others.

The previous experiments include only a few *extreme* scenarios with equal probabilities. In addition, we now introduce a *medium* scenario with a high probability in order to investigate

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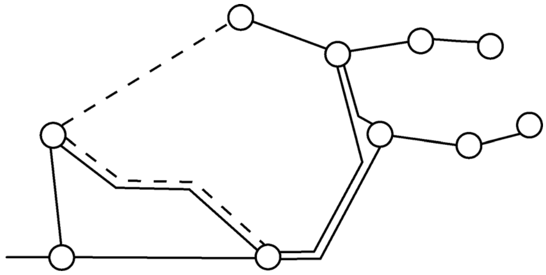

Fig. 8. Partial line capacity expansion plan ($k = 1$). Dashed lines indicate new line investments.

TABLE XII
SUMMARY OF SCENARIOS FOR THE INSTANCES WITH SEVEN SCENARIOS. WIND CAPACITY IS THE SHARE OF INSTALLED CAPACITY

		Demand (MW)		Wind capacity	
ω	$p(\omega)$	DK1	DK2	on-shore	off-shore
0	0.0833	4076	2869	0.90	0.95
1	0.0833	4076	2869	0.50	0.50
2	0.0833	4076	2869	0.00	0.00
3	0.0833	1448	934	0.90	0.95
4	0.0833	1448	934	0.50	0.50
5	0.0833	1448	934	0.00	0.00
6	0.5000	2869	1902	0.30	0.30

TABLE XIII
SUMMARY OF RESULTS FOR DIFFERENT LEVELS OF SWITCHING WITH SEVEN SCENARIOS

	no switch	$k=0$	$k=1$	$k=2$	$k=3$
no. of installed lines	6	10	8	8	7
no. of installed switches	-	7	10	10	12
wind (MWh/h)	2860	2861	2874	2874	2875
fixed cost (DKK/h)	254	332	300	300	323
op. cost (DKK/h)	339693	332965	322614	321094	319265
total cost (DKK/h)	339947	333297	322914	321394	319588

TABLE XIV
BENEFIT OF SWITCHING. VALUES ARE ABSOLUTE AND RELATIVE (IN %) DIFFERENCE AS COMPARED TO THE NON-SWITCHED NETWORK ($y_S = 0$)

	$k=0$		$k=1$		$k=2$		$k=3$	
	abs	rel	abs	rel	abs	rel	abs	rel
op. cost, DKK/h	-6728	-1.98	-17080	-5.03	-18599	-5.48	-20428	-6.01
fixed cost, DKK/h	78	30.76	46	18.11	46	18.11	69	27.15
total cost, DKK/h	-6650	-1.96	-17034	-5.01	-18553	-5.46	-20359	-5.99
wind (avg.), MWh/h	1	0.03	14	0.50	14	0.50	16	0.55
wind ($\omega = 0$), MWh/h	12	0.17	171	2.55	172	2.55	187	2.78

the proportionality of cost and wind power generation. The scenarios are summarized in Table XII. Also, the cost of adding a switch has been quadrupled. Both of these changes are expected to discourage the use of transmission switching.

Results for the seven scenario instances are summarized in Table XIII. We see that increasing the cost of switches and introducing a new scenario, results in different investment strategies for $k > 0$ with fewer (or the same) switches and line expansions.

Table XIV shows the benefit of allowing to switch transmission lines in the instances with seven scenarios. It is seen that increasing the cost of switches and introducing a *medium* scenario does reduce the benefit of switching considerably. However, the benefit is still significant. Switching allows a reduction in total cost of up to 6% and increases wind power generation in scenario $\omega = 0$ by up to 187 MW.

TABLE XV
COMPUTATIONAL RESULTS FOR SOLVING THE DANTZIG-WOLFE REFORMULATION USING BRANCH-AND-PRICE AND THE COMPACT FORMULATION USING CPLEX. GAP IS RELATIVE (IN %) FROM BEST KNOWN SOLUTION. ALL INSTANCES ARE SOLVED TO OPTIMALITY USING BRANCH-AND-PRICE

Instance		Branch-and-price				CPLEX	
		time (s)		price-	no.		
$\lvert\Omega\rvert$	k	total	master	passes	nodes	time (s)	gap (%)
6	-	740	131	134	3	8.4	0.00
6	0	126	11	20	1	431	0.00
6	1	965	68	35	1	2239	† 0.00
6	2	2592	58	32	1	8999	† 0.02
6	3	4795	57	31	1	10006	† 0.02
12	1	4094	201	62	1	-	-
24	1	11982	178	62	1	-	-

We acknowledge that the scenarios described here are not truly representative and that more work is necessary to identify a set of scenarios representing our true expectation of the future.

D. Running Times

Optimal solutions for the six-scenario instances described above was obtained using column generation. The model was implemented using the COIN-OR DIP framework [23] and instances were solved using default parameters except that each node was solved to optimality before branching ($\texttt{TailOffPercent} = 0$), compression of columns was turned off ($\texttt{CompressColumns} = 0$), and the master problems were solved to optimality ($\texttt{MasterGapLimit} = 0$) using interior point method (CPLEX 12.2 barrier). Subproblems were solved using CPLEX 12.2 MIP-solver. Table XV gives a summary of running times for different instances of the problem with branch-and-price (DIP) and CPLEX.

Except for the instance without switching all instances were solved to optimality in the root node—that is no branching was needed. For all instances with switching, column generation seems to be superior to solving the compact formulation using a commercial MIP solver (CPLEX). We were able to solve for 24 scenarios with $k = 1$ in less than 12 000 s. Solution times for the column generation approach seem to scale relatively well with the number of scenarios. However, the majority of the solution time is used to solve subproblems and this is prohibitive for the number of scenarios that can be solved in reasonable time—especially for values of k larger than 1.

V. CONCLUSION

In this paper we have treated the line capacity expansion problem with transmission switching under future uncertainty in demand and wind generation capacity. The problem is formulated as a two-stage stochastic program and the Dantzig-Wolfe decomposition is solved using column generation.

Results indicate that the topology of the transmission network is important for the dispatch of wind energy and that intermittent generation calls for a dynamically optimized topology. This can be achieved by actively switching transmission lines. Our results show that transmission switching may reduce curtailment of wind power with up to 250 MW in peak demand for the Danish network under study. Also, switching of transmission elements may influence the optimal line capacity expansion strategy, making it worthwhile to install more new transmission capacity. Solving the decomposed model makes it possible to solve instances for real networks in reasonable time and is superior to solving the compact formulation using a commercial MIP solver (CPLEX). Furthermore, the decomposition approach seems to scale well with increased number of scenarios.

The Danish network presented in this paper is isolated from the remaining European electricity transmission network. In order to obtain more realistic results further work is needed to represent the neighbouring areas in a better way. This is important as large scale wind power generation is also under way in other parts of Northern Europe.

The results presented here are only for a limited number of scenarios, that may not reflect our true expectation of the future. Further work is needed to identify realistic and representative scenarios. Other stochastic parameters may be relevant such as generation prices (depending on water values of hydro power generation units, oil prices, etc.). Also, geographically dependent wind power generation time series is highly relevant in order to capture periods of high wind power in one part of the network and low wind power in other parts of the network. Even though such outcomes may occur only with low probability (e.g., only for short periods of time), this may increase further the need for a dynamic network topology and the value of transmission switching.

In practice, expansion of transmission line capacity and investment in new off-shore wind power plants is performed over a planning period of many years. At each stage of the planning period the expectation of the future is changed as more information becomes available and so the optimal expansion plan may change as well. This model can be extended to a multi-stage formulation following the approach in [24]. In a multi-stage setting decisions on line capacity expansions may be made at any stage, while the expansion of wind power capacity may be subject to uncertainty.

REFERENCES

[1] G. Oliveira, A. Costa, and S. Binato, "Large scale transmission network planning using optimization and heuristic techniques," *IEEE Trans. Power Syst.*, vol. 10, no. 4, pp. 1828–1834, Nov. 1995.

[2] S. Binato, M. Pereira, and S. Granville, "A new Benders decomposition approach to solve power transmission network design problems," *IEEE Trans. Power Syst.*, vol. 16, no. 2, pp. 235–240, May 2001.

[3] N. Alguacil, A. Motto, and A. Conejo, "Transmission expansion planning: A mixed-integer LP approach," *IEEE Trans. Power Syst.*, vol. 18, no. 3, pp. 1070–1077, Aug. 2003.

[4] S. de la Torre, A. J. Conejo, and J. Contreras, "Transmission expansion planning in electricity markets," *IEEE Trans. Power Syst.*, vol. 23, no. 1, pp. 238–248, Feb. 2008.

[5] J. B. Bushnell and S. E. Stoft, "Electric grid investment under a contract network regime," *J. Regulatory Econ.*, vol. 10, pp. 61–79, 1996.

[6] M. Bjørndal and K. Jörnsten, "Investment paradoxes in electricity networks," in *Pareto Optimality, Game Theory And Equilibria*, ser. Springer Optimization and Its Applications, A. Chinchuluun, P. M. Pardalos, A. Migdalas, and L. Pitsoulis, Eds. New York: Springer, 2008, vol. 17, pp. 593–608.

[7] E. B. Fisher, R. P. O'Neill, and M. C. Ferris, "Optimal transmission switching," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1346–1355, Aug. 2008.

[8] Power System Test Case Archive, Electrical Engineering, University of Washington, 2010. [Online]. Available: http://www.ee.washington.edu/research/pstca/.

[9] H. Glavitsch, "Switching as means of control in the power system," *Int. J. Elect. Power Energy Syst.*, vol. 7, no. 2, pp. 92–100, 1985.

[10] G. Schnyder and H. Glavitsch, "Security enhancement using an optimal switching power flow," *IEEE Trans. Power Syst.*, vol. 5, no. 2, pp. 674–681, May 1990.

[11] K. W. Hedman, M. C. Ferris, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Co-optimization of generation unit commitment and transmission switching with N-1 reliability," *IEEE Trans. Power Syst.*, vol. 25, no. 2, pp. 1052–1063, May 2010.

[12] A. Khodaei and M. Shahidehpour, "Transmission switching in security-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 25, no. 4, pp. 1937–1945, Nov. 2010.

[13] J. C. Villumsen and A. B. Philpott, "Investment in electricity networks with transmission switching," *Eur. J. Oper. Res.*, vol. 222, no. 2, pp. 377–385, 2012.

[14] G. Dantzig and P. Wolfe, "Decomposition principle for linear programs," *Oper. Res.*, vol. 8, no. 1, pp. 101–111, 1960.

[15] R. Bohn, M. Caramanis, and F. Schweppe, "Optimal pricing in electrical networks over space and time," *Rand J. Econ.*, vol. 15, no. 3, pp. 360–376, 1984.

[16] H. Stigler and C. Todem, "Optimization of the Austrian electricity sector (control zone of Verbund APG) by nodal pricing," *Central Eur. J. Oper. Res.*, vol. 13, no. 2, p. 105, 2005.

[17] A. J. Kleywegt, A. Shapiro, and T. H. de Mello, "The sample average approximation method for stochastic discrete optimization," *SIAM J. Optimiz.*, vol. 12, no. 2, pp. 479–502, 2002.

[18] L. Garver, "Transmission network estimation using linear programming," *IEEE Trans. Power App. Syst.*, vol. PAS-89, no. 7, pp. 1688–1697, Sep. 1970.

[19] S. A. Blumsack, "Network topologies and transmission investment under electric-industry restructuring," Ph.D. dissertation, Carnegie Mellon Univ., Pittsburgh, PA, 2006.

[20] Energistyrelsen, Havmøllehandlingsplan 2008, Energistyrelsen, Tech. Rep., 2008.

[21] Energistyrelsen, *Stor-Skala Havmølleparker i Danmark—Opdatering af Fremtidens Havmølleplaceringer, Energistyrelsen, Tech. Rep.,* 2011.

[22] Energistyrelsen, *Nye Retningslinjer for Kabellægning og Udbygning af Transmissionsnettet, Energistyrelsen, Tech. Rep.,* 2008.

[23] M. Galati, "Decomposition in integer linear programming," Ph.D. dissertation, Lehigh Univ., Bethlehem, PA, 2009.

[24] K. J. Singh, A. B. Philpott, and R. K. Wood, "Dantzig-Wolfe decomposition for solving multistage stochastic capacity-planning problems," *Oper. Res.*, vol. 57, no. 5, pp. 1271–1286, 2009.

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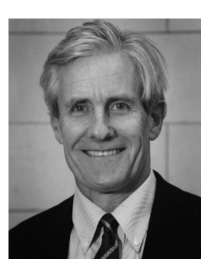

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