

# Decision making of a virtual power plant under uncertainties for bidding in a day-ahead market using point estimate method

Malahat Peik-Herfeh, H. Seifi\*, M.K. Sheikh-El-Eslami

Faculty of Electrical and Computer Engineering, Tarbiat Modares University, P.O. Box 14115-143, Tehran, Iran

## ARTICLE INFO

### Article history:

Received 19 October 2011  
Received in revised form 23 June 2012  
Accepted 5 July 2012  
Available online 9 August 2012

### Keywords:

Distributed energy resources  
Dispatchable load  
Point estimate method  
Price based unit commitment  
Virtual power plant

## ABSTRACT

Environmental concerns, improvements in renewable energy technologies, governmental incentives for the use of these resources, and increased T&D costs, are the main factors driving the energy sector into a new era, where considerable portions of electrical demand will be met through widespread installation of Distributed Energy Resources (DERs). The Virtual Power Plant (VPP) is a decentralized energy management system tasked to aggregate the capacity of some Distributed Generations (DGs), storage facilities, and Dispatchable Loads (DLs) for the purpose of energy trading and/or providing system support services. Due to the stochastic behavior of the prime sources of some DGs, such as wind speed and temperature, the steady state analysis of the systems with integration of such DG units requires a probabilistic approach. In this paper, a probabilistic Price Based Unit Commitment (PBUC) approach using Point Estimate Method (PEM) is employed to model the uncertainty in market price and generation sources, for optimal bidding of a VPP in a day-ahead electricity market. Also, the uncertainty of stochastic DGs generations is handled through increasing the amount of required reserve. The proposed model allows a VPP to decide on the unit commitment of its DERs, and the optimal sale/purchase bids to the day-ahead market. The proposed optimization algorithm is applied to an 18-buses system.

© 2012 Elsevier Ltd. All rights reserved.

## 1. Introduction

Due to the growth of energy consumption, the extensive use of conventional fossil fuels from the exhaustible resources and the environmental concerns, high penetration of distributed energy resources is considerably observed worldwide [1]. The integration of DERs in the electricity system imposes benefits and costs to electricity market players such as DERs, distribution and transmission networks operators. On the other hand, renewable energy technologies are playing an important role into the energy mix of future power systems. However, the intermittent nature of the output from these resources is frequently discussed as a potential barrier to larger scale application of these technologies.

Several definitions are available for describing distributed energy resources. Sometimes these definitions are not consistent. Distributed generation, can be defined as generation located at or near the load centers. CIGRE defines DG as the generation that has the following characteristics [2]: It is not centrally planned; it is not centrally dispatched at present; it is usually connected to the distribution network and it is smaller than 50–100 MW. Also, the DER portfolio includes not only the generators but also energy storage and load control.

DGs can generally be divided into two main groups: renewable-based and fueled-based technologies. Fuel-based technologies include conventional steam and combustion turbines, Internal Combustion Engine (ICE) generators, Micro-Turbines (MTs), and Fuel Cells (FCs). Renewable based technologies include Photovoltaic cells (PVs), Wind Turbines (WTs), and small-scale hydro-generation. High penetration of variable output generation such as wind and photovoltaic system exceeds the expected variability and uncertainty of the power system. REnewable Source (RES) based DGs cannot be regulated and their outputs are determined by the availability of the primary source, i.e., wind or sun radiation.

The issues of operations and planning of distribution networks in the presence of DERs have been studied and examined in some recent research. The long-term policies of distribution system expansions and the way they are affected in the new environment have been discussed in [3–6]. The short-term operations aspects of distribution systems in deregulated electricity markets have been investigated in [7–9]. In [10], a short-term scheduling procedure is adopted, composed of two stages: a day-ahead scheduler for the optimization of distributed resources production during the following day, and an intra-day scheduler which adjusts the scheduling every 15 min so that the operation requirements and constraints of the distribution network are taken into account. The integration of DERs into power system operation through participating in the ancillary service markets for providing regulation,

\* Corresponding author. Tel.: +98 2188220121.

E-mail address: [Seifi\\_ho@modares.ac.ir](mailto:Seifi_ho@modares.ac.ir) (H. Seifi).

## Nomenclature

$k$	set of hourly intervals	$RUP_{DG}$	ramp-up limit for DG unit (MW/h)
$DG$	set of dispatchable DGs active in VPP	$RDN_{DG}$	ramp-down limit for DG unit (MW/h)
$SG$	set of stochastic DGs active in VPP	$MUP_{DG}$	minimum up time limit for DG unit (h)
$GSP$	set of grid supply points	$MDN_{DG}$	minimum down time limit for DG unit (h)
$DL$	index for dispatchable load	$P_{VPP,GSP}^k$	active power exchange at the GSPs (MW)
$\lambda_{DM,GSP}^k$	day-ahead market price at the GSPs in hour $k$ (\$/MW h)	$S_{GSP}^{max}$	the rating of the GSP, for exchanging power with the main grid (MVA)
$\lambda_{Forecast,DM}^k$	forecasted day-ahead market price in hour $k$ (\$/MW h)	$S_{GSP}^k$	apparent power exchange at the GSPs (MVA)
$\rho_{DSO}^k$	price that is charged to local DSO customers in hour $k$ (\$/MW h)	$V_i^k$	bus $i$ voltage in hour $k$ (kV)
$P_{DL}^k$	the curtailment value of dispatchable load in hour $k$ (MW)	$V_i^{min}, V_i^{max}$	minimum and maximum limits on bus voltages, respectively (kV)
$P_{DL,max}^k$	upper limit for curtailing on $DL$ (MW)	$a, b$	model estimation parameters for distribution network demand
$C_{DL}^k$	cost of an interruptible consumer to curtail its load in hour $k$ (\$/MW h)	$P_{G,i}^k$	active power supplied through GSPs or by DG to bus $i$ in hour $k$ (MW)
$P_{SG}^k$	generation of stochastic DG in hour $k$ (MW)	$Q_{G,i}^k$	reactive power supplied through GSPs or by DG to bus $i$ in hour $k$ (MVar)
$P_{SG}^{max}$	installed capacity of stochastic DG (MW)	$P_{Dem,i}^k$	active power demand at bus $i$ in hour $k$ (MW)
$P_{SG,forecast}^k$	expected generation of stochastic DG in hour $k$ (MW)	$Q_{Dem,i}^k$	reactive power demand at bus $i$ in hour $k$ (MVar)
$P_{DG}^k$	generation of dispatchable DG in hour $k$ (MW)	$\theta_{ij}$	angles of complex Y-bus matrix elements (rad)
$P_{DG}^{max}$	maximum DG capacity limit for active power (MW)	$\delta_i$	voltage angle at bus $i$ (rad)
$P_{DG}^{min}$	minimum DG capacity limit for active power (MW)	$Y_{ij}$	magnitude of admittance matrix element
$SUC_{DG}$	start up cost of DG unit (\$)	$P_{Demand}^k$	total active power demand of the distribution network in hour $k$ (MW)
$SDC_{DG}$	shut down cost of DG unit (\$)	$P_{Loss}^k$	total active power loss of the distribution network in hour $k$ (MW)
$\alpha_{DG}^k$	binary decision variables for dispatchable DG unit status in hour $k$ (on = 1, off = 0)	$S_{ij}^k$	apparent power flow from bus $i$ to bus $j$ in hour $k$ (MVA)
$\alpha_{SG}^k$	binary decision variables for stochastic DG unit status in hour $k$ (on = 1, off = 0)	$S_{ij}^{max}$	upper limit for apparent power flow from bus $i$ to bus $j$ in hour $k$ (MVA)
$\beta_{DG}^k$	Binary decision variables for dispatchable DG unit start up decision in hour $k$		
$\gamma_{DG}^k$	binary decision variable for dispatchable DG unit shut down decision in hour $k$		
$C_{DG}, C_{SG}$	generation costs of dispatchable and stochastic DG units, respectively (\$/MW h)		

reserve services and voltage regulation are considered in [11–17]. In [18], the performance of customer-owned DG units is quantified from different perspectives through an uncertainty study. Implications and planning aspects of the interconnection of decentralized renewable resources into distribution grids are studied in [19–22]. In [23], a multi-period AC Optimal Power Flow (OPF) is used to determine the optimal accommodation of renewable DG in a way that the system energy losses are minimized. Optimum allocation of the maximum possible DG penetration in a distribution network is studied in [24]. A reconfiguration methodology based on an ant colony algorithm is proposed in [25] that aims at achieving the minimum power losses and increments load balance factor of radial distribution networks with DGs.

The future power system may have a large number of distributed generators and variable power generation from renewable energy resources. The challenges of designing a sustainable future power system with an integration of many distributed energy resources, especially renewable based units, are investigated in various research and projects [26]. However, the aggregation of many small-capacity generators into one large power generation project could improve the economics of DERs. If several DER units are linked together and are operated as one unit, the concept is often called a Virtual Power Plant (VPP).

VPP concept and its potential use in system operation, have been identified in some recent publications. The VPPs framework has been conceptually established in 1997 [27]. There are two types of a VPP. Commercial Virtual Power Plant (CVPP) is one type of VPP operation. From the commercial point of view, VPP as a

market agent seeks to obtain the maximum benefit from the generation and the demand portfolio without considering the network constraints. Technical Virtual Power Plant (TVPP) is another type of VPP operation. The TVPP takes into consideration also the operation of the grid. On the other hand, a TVPP consists of some DERs from the same geographic location. In this case, the impact of operation on the distribution network is also considered [28]. In other words, the CVPP optimizes its portfolio with reference to the wholesale markets, and passes DER schedules and operating parameters to the TVPP. The TVPP uses input from the CVPPs operating in its area to manage any local network constraints and determine the characteristics of the entire local network at the Grid Supply Points (GSPs) [28].

To determine the optimal operation of DERs for the next time horizon, a VPP should solve a Unit Commitment (UC) problem. On the other hand, in the context of liberalized markets, the UC problem is applied in two ways, namely, Security Constrained Unit Commitment (SCUC) and Price Based Unit Commitment (PBUC). The VPP UC problem is different from a normal UC problem by two main aspects. Firstly, the DER included in a VPP may be connected to various points in distribution network; so the network characteristics impact the decision making problem of a VPP. As a result, VPP considers the constraints of both network and DERs when bidding to the markets. Secondly, the uncertainties related to the output of the individual DERs, especially variable output ones, are the main limiting factors for the participation of individual small-scale DERs in the day-ahead market. Therefore, it is necessary to include these uncertain parameters in the problem formulation.

In [29], a market-based VPP model is proposed which provides individual DER units accesses to the current electricity markets. Also two operating scenarios for VPP operation, namely, general bidding scenario and price signal scenario are considered. An optimization algorithm is proposed in [30], to integrate some DGs into a VPP, capable of generating and selling both thermal and electrical energies. The objective function to be minimized is the variable cost associated with the supply of thermal and electric energies to the loads. A VPP including a wind turbine, a solar unit, a fuel cell and a storage battery; that can optimally operate the generation units, assuring the good functioning of equipment, including the maintenance, operation cost and the generation measurement and control; is considered in [31]. Optimized management of clustered CHP systems in the form of a VPP is used in [32]. Authors in [33], provide an optimization algorithm to manage a VPP composed of a large number of customers with thermostatically controlled appliances. The proposed algorithm, based on Direct Load Control (DLC), determines the optimal control schedules that an aggregator should apply to the controllable devices of the VPP, in order to optimize load reduction over a specified control period. A powerful tool for optimizing a coordinated operation of DG units in a VPP under uncertainty of power prices, power demand, and in-feed from renewable resources is proposed in [34]. A TVPP including dispatchable DGs, electrochemical storages, and interruptible loads is considered in [35,36]. The bidding problem faced by this VPP in a joint market of energy and spinning reserve service is discussed. The proposed bidding strategy is a non-equilibrium model based on the deterministic PBUC which takes the supply–demand balancing constraint. Authors in [37] propose a new methodology based on nodal pricing for optimal operation of a VPP in profit maximization of its owner.

To the best of our knowledge, VPP bidding problem is only reported in [38], later refined in [35,36]. On the other hand, most of the early works on the PBUC problem use a deterministic formulation. Deterministic PBUC uses a fixed and assigned input set of variables which can be formed by market prices and generation/load values. Since the precision of market price forecasting could have a direct impact on PBUC solution, it would be very important to consider the market price uncertainty in the PBUC problem formulation. On the other hand, due to the stochastic behavior of the prime sources of some DGs, such as wind speed and temperature, the uncertainties related to the output of these units are the main limiting factors for their participation in the day-ahead market. Therefore, it is also necessary to include these uncertain parameters.

Many engineering problems are subject to uncertainty, due to the inherent randomness of natural phenomena or to the implicit and inaccurate assumptions related to the modeling approach [39]. In order to take the uncertainties into consideration, different mathematical approaches for uncertainty analysis can be used. These techniques may be classified into the three main categories: Monte Carlo simulation, analytical methods, and approximate methods [39]. Uncertainty propagation studies based on sampling-based methods, such as Monte Carlo's, require several model runs that sample various combinations of input values. This technique has been widely used in power systems analysis to model uncertainty. The main drawback of the Monte Carlo method is the great number of simulations required to attain convergence. The analytical approach analyzes a system and its inputs using mathematical expressions, e.g. Probability Density Functions (PDFs), and obtains results also in terms of mathematical expressions. Analytical methods are computationally more effective, but they require some mathematical assumptions in order to simplify the problem. Likewise, convolution techniques are used to obtain a mathematical description of the behavior of output random variables. Approximate methods provide an approximate description

of the statistical properties of random output variables. Within these methods, point estimate approaches stand out. As Monte Carlo simulation, point estimate method uses deterministic routines for solving probabilistic problems; while requiring a lower computational burden. Furthermore, point estimate method overcomes the difficulties associated with the lack of perfect knowledge of the probability functions of stochastic variables, since these functions are approximated using only their first few statistical moments (i.e., mean, variance, skewness, and kurtosis). Therefore, a smaller level of data information is needed [39].

The aim of this paper is carrying out an optimal dispatch so that the operator of a VPP is capable of determining its economical optimum, in consideration of the relevant technical and economical constraints as well as some existing uncertainties. A TVPP similar to what defined in [28], is considered in this paper. A probabilistic PBUC with constraints for the inclusion of stochastic DG generation is proposed. The proposal uses a particular case of the point estimate method, known as Hong's Two-Point Estimate Method (TPEM), rather than the traditional approach based on Monte Carlo simulation. The main feature of the TPEM is that it only requires resolving  $2 \times m$  deterministic PBUC to obtain the behavior of  $m$  random variable. Since this paper focuses on the uncertainties involved by renewable generation sources and market prices, it is assumed that their statistical features are estimated or measured and there is no correlation between random input variables including market prices and generations. The impacts of generation of these resources are also modeled by increasing the amount of required reserve. Mixed Integer Nonlinear Programming (MINLP) is used for solving deterministic PBUC problems. The objective function of the above problem is to maximize the expected value of the profit for selling energy in the day-ahead market and covering DSO demand. The main contributions of this work are summarized as follows:

1. A probabilistic PBUC model is presented which allows a VPP to decide on the unit commitment of its DERs, and the optimal sale/purchase bids to the day-ahead market.
2. The stochastic behavior of the market price and stochastic DGs generations are modeled in the problem formulation using point estimated method.
3. The possibility of exchanging energy with the upstream network via various grid supply points is considered in this paper.

The rest of the paper is organized as follows: Section 3 presents a review on VPP. Problem formulation is described in Section 4. Section 5 describes the test system used in this paper. A brief summary of the simulation used to obtain the results, numerical results along with some observations and discussions are also included in this section. Finally, the contributions and conclusions of the paper are summarized in Section 6.

## 2. Virtual power plant

The increase in the amount of distributed energy resources, especially distributed generators based on renewable energy sources, is leading to many changes in a power system. In this situation, distributed generation and controllable demand may have the opportunity to participate in the operation of transmission and distribution networks. If all DG units individually present their outputs into an electricity market, due to expected penalty of not meeting the accepted schedule, the corresponding risk is high and DG operators will be dissuaded to access the market in individual form. Then, DG operators are obliged to accept relatively low prices for their energies and will not be able to take the advantage of participating in competitive electricity markets. This situa-

tion is worse particularly when the price of electricity is high and system needs more generation [40]. Therefore, penetration of DERs at main grids has caused emerging new concepts such as ‘Active distribution network’, ‘Cells’, ‘Microgrid’ and ‘Virtual power plant’. The VPP concept is based on the idea of aggregating the capacity of some distributed energy resources; generation, storage, or demand; in order to create a single operating profile, similar to a transmission-connected generator. VPPs are multi-technology and multi-site heterogeneous entities. In the scope of a VPP, producers can make sure their generators are optimally operated. At the same time, VPPs will be able to commit to a more robust generation profile, raising the value of non-dispatched generation technologies.

The following definition is provided by the European project FENIX: “A Virtual Power Plant (VPP) is a flexible representation of a portfolio of DERs that can be used to make contracts in the wholesale market and to offer services to the system operator”. There are two types of VPP, the Commercial VPP (CVPP) and the Technical VPP (TVPP). A DER can simultaneously be a part of both a CVPP and a TVPP [28]. The CVPP is a competitive market actor that manages the DER portfolio(s) to make optimal decisions on participation in electricity markets. The TVPP aggregates and models the response characteristics of a system containing DERs, controllable loads and networks within a single electric-geographical (grid) area. Therefore, the role of TVPP in distribution networks is the same as the transmission system operator’s role in transmission systems. The primary objective of optimal operation of VPP can vary. For example, economic optimization can either aim at minimizing the costs of producing energy and supplying it to the loads or maximizing the profits of a VPP owner.

### 3. Problem formulation

#### 3.1. Bidding strategy of VPP in a day-ahead market

In the context of liberalized markets, the UC problem is applied in two ways, namely, Security Constrained Unit Commitment (SCUC) and Price Based Unit Commitment (PBUC). The PBUC is a suitable approach for bidding in markets (energy and ancillary services) and can consider inter-temporal effects and integer variables such as minimum on/off times and ramping limits of generators [35].

The development of a bidding algorithm for a VPP, operating in a short-term electricity market, requires not only the usage of traditional unit commitment or economic-dispatch models, but also incorporating new market-modeling equations. In fact, the way market price is included in the model, plays an important role in optimal bidding of a VPP. Basically, the deterministic PBUC uses the fixed and assigned input set of variables which can be formed by market prices and generation/load values. Since the precision of market price forecasting could have a direct impact on PBUC solution, it would be very important to consider the market price uncertainty in the PBUC problem formulation. On the other hand, due to the stochastic behavior of the prime sources of some DGs, such as wind speed and temperature, the uncertainties related to the output of these units are the main limiting factors for their participation in the day-ahead market. Therefore, it is also necessary to include these uncertain parameters in the problem formulation.

In this paper, a VPP with a set of DERs is considered and assumed that the VPP wishes to submit a bid to the day-ahead electricity market. A bid contains information on how much power, in which area, and at what time a market participant is willing to buy or sell. We assume that VPP is a price-taker player that obtains its revenues by selling power at the market clearing price of the Pool-Co. The VPP optimizes its operation according to the forecasted

market prices, the bids received by the DG sources and dispatchable loads. It is assumed that the required data for the proposed model, including loads, renewable generations, and market prices can be estimated based on historical data.

There are significant points that should be also considered in the bidding problem of a VPP based on PBUC. Firstly, a VPP can be a producer or a consumer. It means, if the power produced by the DER sources is not enough or too expensive to cover the DSO demand, the VPP acts as a consumer and buys the energy from the day-ahead market and sells to its consumers. Secondly, DERs included in a VPP may be connected to various points in distribution network; so the network characteristics should be considered in the bidding problem of VPP. Thirdly, the uncertainty in market prices and generation of stochastic DGs should be taken into consideration in the bidding problem.

In following, some of the assumptions considered in the optimization problem have been given:

- VPP is assumed to be centrally controlled.
- A bid-based mechanism for curtailment of DLs is proposed wherein the customers submit their offers for load curtailment on an hourly basis.
- All DG units included in the VPP send offers to the VPP in form of blocks, for each hour, for active power.
- Both dispatchable and stochastic DGs are considered in the problem formulation. The start-up and shut-down costs can be considered if they are not negligible.
- VPP can exchange the energy with the upstream network via various GSPs. In this paper, the market prices at the GSPs are assumed to be different.
- Dispatchable DGs included in the VPP provide required reserve for the VPP due to possible variations in the stochastic DGs generations.
- The way that VPP shares the profit among its DER sources is not considered in this paper.

#### 3.1.1. Stochastic DG generation modeling

Variability and uncertainty are inherent characteristics of power systems. Demands and generator availability and performance all have some degree of variability and uncertainty. Stochastic DGs, cannot be regulated and their outputs are determined by the availability of the primary source, i.e., wind or sun radiation. In order to account for their productions in the optimization functions, Renewable Energy Sources (RESs) forecasting is required. Forecasting tools for generation of these resources are important issues, falling outside the scope of this paper. There are two approaches for handling uncertainties in the scheduling of the stochastic DG units: reserve requirement and mathematical approaches for uncertainty analysis [41]. In this paper, a combination of these two approaches is considered. The stochastic DGs power forecast is represented by an expected value ( $P_{SG}^{forecast}$ ) and an error ( $Er_{SG}$ ) that is modeled as a zero-mean normally-distributed random variable with a standard deviation of  $\sigma_{SDG}$ .

#### 3.1.2. Market price uncertainty modeling

In the context of liberalized markets, there are many optimization problems that search, for the next 24-h (or a smaller time horizon) solutions to determine the optimal production of generating resources. As discussed earlier, an electricity GENERATION COMPANY (GENCO) uses PBUC in order to maximize its profit. Given a forecasted price of electricity, the GENCO will optimize its scheduling and electricity generation to sell the power at the prevailing price [17]. On the other hand, the market price in the day-ahead market is highly variable and depends on some parameters such as demand, generator availabilities, and operational constraints. Therefore, once incorrect market price forecasts are used, this

deterministic approach may lead to a loss of profit that can be expected in a PBUC. Therefore, hourly market prices are modeled as normal distributions, with means equal to the forecasted market price at that hour, and with standard deviations equal to  $\sigma_{MCP}$  (\$/MWh).

Also, it is supposed that there is a relation between market prices and the amount of distribution networks demand. In this paper, distribution network is considered to be a price taker player. A linear model is used for the estimation of the relation between distribution network demands and the market prices.

$$P_{Demand}^k = a \cdot \lambda_{DM}^k + b \quad (1)$$

### 3.2. PBUC problem formulation

This paper applies a probabilistic PBUC with constraints for inclusion of stochastic DG generation. The impacts of generation of these resources are modeled by increasing the amount of required reserve. As discussed earlier, the stochastic trend in uncertainty of market price and generation sources are simulated by creating some concentrations that can be solved by deterministic PBUC. Mixed integer nonlinear programming is used for solving deterministic PBUC problems. The objective function of the above problem is to maximize the expected value of the profit. The VPP sells energy to the consumers of the network and also the excess production from the DG sources, if any, to the day-ahead market at the market price. The outputs of the optimization problem for bidding in the day-ahead market can be summarized as follows:

- Power exchanges at the GSP points.
- Power generated by DGs.
- Power generated by SGs.
- Load curtailment (unserved energy of customers).
- Reserve capacity

#### 3.2.1. Objective function

Maximize profit

$$profit = \sum_k (revenue^k - cost^k) \quad (2)$$

$$revenue^k = \sum_k (P_{Demand}^k + P_{Loss}^k) \cdot \rho_{DSO} + \sum_{GSP} P_{VPP,GSP}^k \cdot \lambda_{DM,GSP}^k \quad (3)$$

$$cost^k = \sum_{DG} (C_{DG} \cdot P_{DG}^k \cdot \alpha_{DG}^k + SUC_{DG} \cdot \beta_{dg}^k + SDC_{DG} \cdot \gamma_{DG}^k) + \sum_{SG} C_{SG} \cdot P_{SG}^k \cdot \alpha_{DG}^k + C_{DL}^k \cdot P_{DL}^k \quad (4)$$

The expected value of the profit in (2) is computed by expected revenues minus incurred operating costs for a given period. The first term of revenue in (3) is the income from selling energy to the consumers of the DSO and covering DSO losses. The second term of revenue in (3) is the income from selling the excess production of the DG sources, if any, to the day-ahead market at the GSPs. The first and the second components of costs in (4) are the cost of generation from DG and SG units, having different operational, start-up and shut-down costs. Also, the third component is the cost of curtailing dispatchable loads.

#### 3.2.2. Constraints

Constraints considered in this paper are as follows:

- (1) Network Equations:

$$P_{G,i}^k - P_{Dem,i}^k = \sum_j |V_i^k| |V_j^k| |Y_{ij}| \cos(\theta_{ij} + \delta_j^k - \delta_i^k) \quad (5)$$

$$Q_{G,i}^k - Q_{Dem,i}^k = -\sum_j |V_i^k| |V_j^k| |Y_{ij}| \sin(\theta_{ij} + \delta_j^k - \delta_i^k) \quad (6)$$

- (2) Bus voltage limits:

$$V_i^{\min} \leq V_i^k \leq V_i^{\max} \quad (7)$$

- (3) Capacity of interconnection with the main grid:

$$S_{GSP}^k \leq S_{GSP}^{\max} \quad (8)$$

- (4) Distribution line apparent power flow limits:

$$S_{ij} \leq S_{ij}^{\max} \quad (9)$$

- (5) Power balance: at least the DSO demand should be met, as expressed by:

$$\sum_{DG} P_{DG}^k + P_{DL}^k - \sum_{GSP} P_{VPP,GSP}^k \geq P_{Demand}^k + P_{Loss}^k \quad (10)$$

- (6) The limits on DLs:

$$P_{DL}^k \leq P_{DL}^{\max} \quad (11)$$

- (7) The upper and lower power limits of DGs:

$$P_{DG}^{\min} \leq P_{DG}^k \leq P_{DG}^{\max}, \quad P_{SG}^k \leq P_{SG,forecast}^k \quad (12)$$

- (8) Ramp limits for each DG:

$$P_{DG}^{k+1} - P_{DG}^k \leq RUP_{DG}, \quad P_{DG}^k - P_{DG}^{k+1} \leq RDN_{DG} \quad (13)$$

- (9) Minimum-up and down time constraints for each DG:

$$\sum_{l=1}^{MUP} \alpha_{DG}^{k+l} - 1 \geq MUP \quad \forall \beta_{DG}^k = 1, \quad (14)$$

$$\sum_{l=1}^{MDN} 1 - \alpha_{DG}^{k+l} \geq MDN \quad \forall \gamma_{DG}^k = 1$$

- (10) Reserve capacity: The reserve requirement should also be satisfied. Because of possible variations in the stochastic DGs generations, the system reserve requirement must increase as generation of these sources increases. Thus, in this paper, two components are considered in providing the operating reserve of the problem. The first one is considered as a percentage of the total generation of dispatchable units and curtailment option of the dispatchable loads ( $RSV_{Total}$ ) (e.g. 2%). The second one is called a surplus reserve ( $RSV_{SG}$ ) required to compensate for the error caused by the mismatch between the forecasted generation and the actual generation of SG units (e.g. 5%). So,

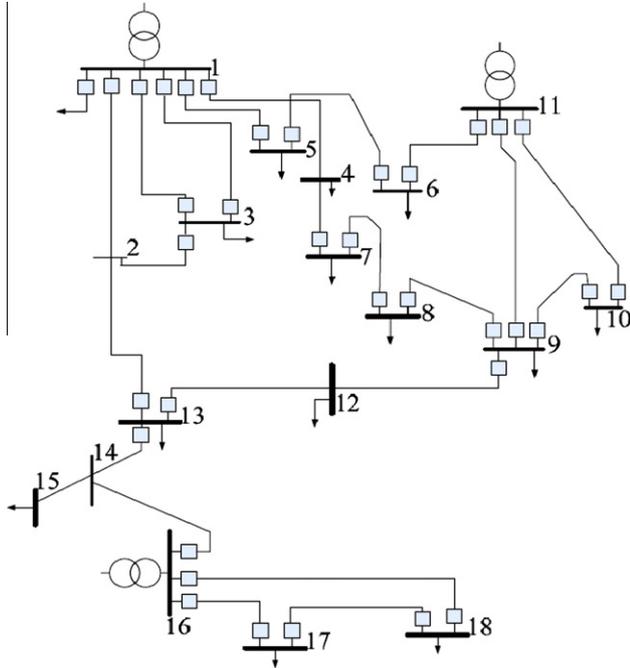
$$\sum_{DG} (P_{DG}^{\max} - P_{DG}^k) \cdot \alpha_{DG}^k + P_{DL}^k \geq RSV_{SG} \cdot \left( \sum_{SG} P_{SG}^k \cdot \alpha_{SG}^k \right) + RSV_{Total} \cdot \left( \sum_{DG} P_{DG}^k \cdot \alpha_{DG}^k + P_{DL}^k \right) \quad (15)$$

### 3.3. Implementation of point estimate method

Point estimate methods concentrate on the statistical information provided by the first few central moments of problem random input variables on  $s$  points for each variable, named concentration. By using these points and a function  $F$ , which relates input and output variables, information about the uncertainty associated with problem output random variables can be obtained [39]. This paper considers the existence of multiple uncorrelated random variables. For the particular case in which there is a correlation between random input variables, the techniques based on transformations can be used.

**Table 1**  
Main characteristics of DERs included in VPP portfolio.

DER No.	DER type	$p^{\min}$ (MW)	$p^{\max}$ (MW)	$C_{DG}$ (\$/MWh)	$RUP_{DG}$ (MW/h)	$RDN_{DG}$ (M/W/h)	SUC (\$)	SDC (\$)
DG2	DG	0	4	37	1	1	20	25
DG7	DG	0	5	40	1.25	1.25	20	25
DG8	DG	0	5.5	35	1.375	1.375	50	25
DG14	DG	0	7	45	1.75	1.75	50	25
SG15	SG	0	9	55				
SG18	SG	0	7	65				



**Fig. 1.** Schematic diagram of the 18-bus distribution system.

**Table 2**  
Characteristics of the stochastic DGs.

Hour	SG 15 (MW)		SG 18 (MW)	
	Mean	St. De	Mean	St. De
$k = 1, \dots, 4$	4	0.28	2	0.22
$k = 5, \dots, 8$	4.5	0.2	2.5	0.31
$k = 9, \dots, 12$	6.2	0.23	2.2	0.29
$k = 13, \dots, 16$	5.4	0.4	3.4	0.3
$k = 17, \dots, 20$	6.6	0.32	4.6	0.27
$k = 21, \dots, 24$	7.5	0.26	5.5	0.32

Suppose  $X = \{x_1, x_2, \dots, x_i, \dots, x_m\}$  is a random variable with a mean value  $\mu_{x_i}$  and standard deviation  $\sigma_{x_i}$ .  $Z$  is a random quantity in function of  $X$ :  $Z = F(X)$ . Each of the  $s^{\text{th}}$  concentrations of the variables  $x_i$  can be defined as a pair composed of a location  $x_{i,s}$  and a weight  $w_{i,s}$ . The proposal uses a particular case of the point estimate method, known as Hong's Two-Point Estimate Method (HTPEM). Using HTPEM, function  $F$  has to be evaluated only  $s$  times for each random input variable  $x_i$  at the points made up of the  $s^{\text{th}}$  location of the random input variable  $x_i$  and the mean ( $\mu_{x_i}$ ) of the remaining input variables. Therefore, the total number of evaluations is  $2 \times m$ . The location  $x_{i,s}$  to be determined is:

$$x_{i,s} = \mu_{x_i} + \zeta_{i,s} \cdot \sigma_{x_i} \quad (16)$$

**Table 3**  
Characteristics of the dispatchable loads and forecasted market prices.

Hour	$\lambda_{DM}^{\text{forecast}}(k)$ (\$/MW h)	$C_{DL}(k)$ (\$/ MW h)	$P_{DL}^{\max}(k)$ (MW)	Hour	$\lambda_{DM}^{\text{forecast}}(k)$ (\$/MW h)	$C_{DL}(k)$ (\$/ MW h)	$P_{DL}^{\max}(k)$ (MW)
1	50	50	0.575	13	85	87	0.698
2	40	30	0.540	14	75	80	0.663
3	42.5	35.5	0.549	15	65	70	0.628
4	43	41	0.551	16	55	65	0.593
5	50	50	0.575	17	57	65	0.600
6	60	65	0.610	18	56	80	0.596
7	64.5	68	0.626	19	72.5	80	0.654
8	70	75	0.645	20	80	85	0.680
9	75	78	0.663	21	87	89	0.705
10	76	76	0.666	22	72	75	0.652
11	65	65	0.628	23	52.5	65	0.584
12	80.5	85	0.682	24	60	65	0.610

where  $\zeta_{i,s}$  is the standard location of the random variable  $x_i$ . The standard locations and weights of random variable of  $x_i$  are computed by:

$$\zeta_{i,1} = \frac{\lambda_{i,3}}{2} + \sqrt{m + \left(\frac{\lambda_{i,3}}{2}\right)^2}, \quad \zeta_{i,2} = \frac{\lambda_{i,3}}{2} - \sqrt{m + \left(\frac{\lambda_{i,3}}{2}\right)^2}, \quad (17)$$

and,

$$w_{i,1} = -\frac{\zeta_{i,2}}{m(\zeta_{i,1} - \zeta_{i,2})}, \quad w_{i,2} = \frac{\zeta_{i,1}}{m(\zeta_{i,1} - \zeta_{i,2})} \quad (18)$$

where  $\lambda_{i,3}$  denotes the skewness of the random variable  $x_i$ :

$$\lambda_{i,3} = \frac{E[(x_i - \mu_{x_i})^3]}{(\sigma_{x_i})^3} \quad (19)$$

A deterministic PBUC must be run for each concentration. The solution of a PBUC problem is:

$$Z_{i,s} = F(x_{i,1}, x_{i,2}, \dots, x_{i,s}, \dots, x_{m,s}) \quad (20)$$

where  $Z_{i,s}$  is the vector of random output variables associated with the  $s^{\text{th}}$  concentration of random input variable and represents the nonlinear relation between input and output variables in the PBUC problem. The raw moments of the output random variables to be determined are:

$$E(Z) \cong E(Z) + \sum_s w_{i,s} \cdot Z_{i,s} \quad (21)$$

The solution steps are proposed as follows:

- Step 1: Set the first and second moment of  $s^{\text{th}}$  output random variables to zero:  $E(Z) = 0$ .
- Step 2: Select the input random variable  $x_i$ .
- Step 3: Compute  $\lambda_{i,3}$ ,  $\zeta_{i,s}$ ,  $w_{i,s}$  based on (17–19).
- Step 4: determine the two estimated locations of  $x_{i,s}$ .
- Step 5: Solve deterministic PBUC for each concentration.
- Step 6: Update the raw moments of the output variables.
- Step 7: Repeat steps 2–6 until all the concentrations of all input random variables are taken into account.
- Step 8: Compute statistical information of the output random variables.

#### 4. Tests and results

The methodology presented in this paper is tested using a network with 18 buses (Fig. 1). For simplification of the problem, we suppose that all of the DGs included in VPP are located in the same distribution network. The system is connected to the main grid through three substation transformers at bus 1, 11 and 16. This system has been extracted from the well-known IEEE-30 buses

**Table 4**  
Optimal dispatch of DERs included in VPP in case I.

Hour	DG2 (MW)	DG7 (MW)	DG8 (MW)	DG 14 (MW)	SG 15 (MW)	SG 18 (MW)	DL(MW)
1	4	5	5.5	3.078			0.575
2	3	3.75	4.125	1.328			0.515
3	4	5	5.5	3.078			0.549
4	4	5	5.5	4.828			0.55
5	4	5	5.5	6.578			0.575
6	4	5	5.5	6.309	5.5		
7	4	5	5.5	6.309	5.5		
8	4	5	5.5	6.137	5.5	3.5	
9	4	5	5.5	6.069	6.2	4.2	0.662
10	4	5	5.5	6.069	6.2	4.2	0.666
11	4	5	5.5	6.275	6.2		0.627
12	4	5	5.5	6.069	6.2	4.2	
13	4	5	5.5	6.147	5.4	3.4	0.697
14	4	5	5.5	6.147	5.4	3.4	
15	4	5	5.5	6.314	5.4		
16	4	5	5.5	6.43	3.036		
17	4	5	5.5	6.255	6.6		
18	4	5	5.5	6.255	6.6		
19	4	5	5.5	6.029	6.6	4.6	
20	4	5	5.5	6.029	6.6	4.6	
21	4	5	5.5	5.941	7.5	5.5	0.704
22	4	5	5.5	5.941	7.5	5.5	0.652
23	4	5	5.5	6.578			
24	4	5	5.5	6.211	7.5		
Sum	95.00	118.75	130.63	136.40	109.44	43.10	6.77

**Table 5**  
Hourly bids of VPP to the day-ahead market, total costs, revenues and profits in case I.

Hour	GSP 11 (MW)	GSP 16 (MW)	Revenue (\$)	Cost (\$)	Profit (\$)
1	6.644		924.28	707.78	216.50
2	2.405		423.69	480.62	-56.93
3	7.142		785.58	698.51	87.07
4	8.855		873.83	780.35	93.48
5	10.134		1108.00	865.28	242.72
6	11.884	2.184	1614.18	1126.90	487.28
7	11.622	2.132	1734.39	1126.90	607.50
8	11.294	5.403	2114.13	1346.68	767.45
9	11.689	6.651	2416.16	1479.26	936.90
10	11.634	6.639	2448.43	1478.20	970.23
11	12.245	2.765	1833.92	1204.64	629.28
12	10.705	6.587	2536.05	1427.59	1108.47
13	11.113	5.044	2609.51	1395.80	1213.71
14	10.999	5.159	2249.77	1335.12	914.65
15	11.589	2.035	1741.55	1121.62	619.93
16	12.08		1351.30	996.79	354.50
17	12.098	3.222	1593.70	1184.97	408.73
18	12.157	3.234	1565.91	1184.97	380.94
19	11.183	7.425	2340.52	1473.82	867.10
20	10.747	7.339	2581.33	1473.82	1107.51
21	11.071	8.937	3018.82	1640.55	1378.27
22	11.895	9.11	2497.52	1626.75	870.77
23	9.384		1131.25	836.53	294.72
24	11.953	4.01	1728.50	1232.49	496.01
Sum	112.38	150.34	42509.00	28239.68	14269.33

system by considering only the 33 kV networks [42]. There are four dispatchable DGs at buses 4, 7, 8 and 14 and two stochastic DGs at buses 15 and 18. The bid price of each DG is based on its Levelized Cost Of Electricity (LCOE). This LCOE is evaluated on the basis of the installed capital cost, operation and maintenance cost, time of operation and lifetime of the DG [43,44]. In all cases, the LCOE should also consider the expenses for the communication and control infrastructures which are essential for the coordinated control of DGs in VPP operation. All DG units are simulated with their required active powers, and constrained by the maximum and the minimum values. The producers present proposals on the sale of energy, based on the generation of the previous days and on the

meteorological forecasts. Forecast errors for the quantity of electricity generation depend on the DG's technology as well as the timing of the forecast. As mentioned earlier, the market prices at the GSPs are assumed to be different. Therefore, the market price at the GSPs 1, 11 and 16 are assumed to be 95, 105, and 100 percent of  $\lambda_{Forecast,DM}^k$ . The main characteristics of DGs and characteristics of dispatchable load are summarized in Tables 1–3. Upper limit for curtailing on dispatchable load is considered to be five percent of total forecasted demand of DSO.  $\rho_{DSO}^k$  is assumed to be equal to the day-ahead market price at GSP 16.  $RSV_{Total}$  and  $RSV_{SG}$  in (15) are considered to be 2% and 5%, respectively. Amount of  $a$  and  $b$  in (1) are also considered to be 0.07 and 8, respectively. On the other hand,  $\sigma_{MCP}$  is considered to be 4 along whole 24-h optimization horizon. Moreover, it is assumed that the capacity of transformers substation at buses 1, 11 and 16 are 40, 24 and 12 MW, respectively.

The problem formulation is implemented in GAMS [45]. In order to clearly illustrate the effectiveness of the proposed method, a comparison among the results of three different cases: (I) ignoring the uncertainty in input parameters, (II) observing two uncertain parameters (market price, and DG15 generation), and (III) observing 3 uncertain parameters (market price, and DG15 and DG18 generations), is presented. In the following, the simulation results are described. The optimal management of VPP in these three cases can be analyzed from Tables 4–11 and Figs. 2 and 3.

#### 4.1. Case I: Ignoring the uncertain parameters

In case I, only mean value of the market price and stochastic DGs forecasted generation are considered in the PBU. The forecasted price profile is presented in Table 3. The optimal dispatch of DGs and curtailment of dispatchable load for bidding in the day-ahead market and covering the DSO demand for this case, are given in Table 4. Hourly bids to the day-ahead market, costs, revenues and profits of the VPP are also summarized in Table 5. As seen from Tables 4 and 5, during all hours of the optimization horizon, due to the market price profile and DSO demand, all the dispatchable DG units are "on". On the other hand, according to (15), DG14 with the highest production cost among dispatchable

**Table 6**  
Mean and standard deviation values of optimal dispatch of DERs included in VPP in case II.

Hour	DG2 (MW)		DG7 (MW)		DG8 (MW)		DG14 (MW)		SG15 (MW)		SG18 (MW)		DL (MW)	
	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De
1	4		4.749	0.938	5.5	0.687	4.388	2.056					0.536	0.144
2	3.49	0.487	3.002	1.499	4.81		2.115	1.848					0.515	
3	3.9	0.276	4.529	1.243	5.5		4.002	2.219					0.549	
4	4		4.37	1.085	5.5		4.487	1.741					0.412	0.239
5	4		4.88	0.368	5.5		6.284	0.905					0.52	0.169
6	4		5		5.5		6.002	0.48	5.5	0.3			0.128	0.249
7	4		5		5.5		6.245	0.099	5.315	0.716	1.49	1.73	0.266	0.309
8	4		5		5.5		6.156	0.055	5.5	0.3	3.12	1.084	0.253	0.315
9	4		5		5.5		6.069	0.021	6.2	0.43	4.2		0.365	0.33
10	4		5		5.5		6.069	0.021	6.2	0.43	4.2		0.666	
11	4		5		5.5		6.242	0.078	6.2	0.43	0.67	1.534	0.627	
12	4		5		5.5		6.069	0.021	6.2	0.43	4.2		0.092	0.233
13	4		5		5.5		6.147	0.029	5.4	0.6	3.4		0.697	
14	4		5		5.5		6.147	0.029	5.4	0.6	3.4		0.171	0.29
15	4		5		5.5		6.274	0.077	5.4	0.6	0.81	1.445	0.148	0.267
16	4		5		5.5		6.47	0.091	2.207	1.86			0.076	0.198
17	4		5		5.5		6.247	0.048	6.6	0.52	0.16	0.838	0.021	0.109
18	4		5		5.5		6.304	0.119	5.595	2.427				
19	4		5		5.5		6.029	0.025	6.6	0.52	4.6		0.025	0.125
20	4		5		5.5		6.029	0.025	6.6	0.52	4.6		0.137	0.273
21	4		5		5.5		5.941	0.023	7.5	0.46	5.5		0.625	0.223
22	4		5		5.5		5.941	0.023	7.5	0.46	5.5		0.334	0.326
23	4		5		5.5		6.516	0.138	1.272	2.815				
24	4		5		5.5		6.242	0.08	6.859	1.636			0.218	0.292
Sum	95.4		116.53		131		138.42		108.05		45.84		7.381	

**Table 7**  
Hourly bids of VPP to the day-ahead market, total costs, revenues and profits in case II.

Hour	GSP11 (MW)		GSP16 (MW)		Revenue (\$)		Cost (\$)		Profit\$	
	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De
1	7.66	2.62			986.03	184.81	754.72	123.63	231.31	61.63
2	3.62	3.03			475.11	152.96	528.17	136.58	-53.06	16.48
3	7.49	3.23			811.86	194.28	717.37	147.45	94.49	46.85
4	7.75	2.80			832.43	184.18	734.08	128.75	98.35	59.52
5	9.67	1.24			1087.65	126.76	844.50	63.90	243.15	64.69
6	12.00	0.13	1.89	0.51	1605.60	117.17	1121.46	34.78	484.14	85.18
7	11.88	0.26	3.38	1.88	1837.15	207.23	1228.74	142.91	608.41	80.07
8	11.55	0.25	5.05	1.04	2111.15	165.34	1342.03	79.63	769.12	86.31
9	11.39	0.30	6.65	0.39	2393.15	113.53	1456.06	34.29	937.09	96.12
10	11.63	0.18	6.64	0.39	2448.40	100.81	1478.20	22.70	970.20	96.25
11	12.24	0.18	3.40	1.48	1879.02	185.22	1246.48	98.99	632.54	91.00
12	10.80	0.10	6.59	0.39	2544.51	118.45	1435.44	30.15	1109.07	95.79
13	11.11	0.18	5.04	0.55	2609.48	103.40	1395.80	31.68	1213.68	92.82
14	11.17	0.18	5.16	0.55	2263.91	117.78	1348.76	39.25	915.15	90.75
15	11.74	0.16	2.80	1.46	1805.51	190.24	1182.54	113.90	622.97	84.62
16	11.11	1.33	0.26	0.69	1314.92	181.12	957.99	107.07	356.93	77.67
17	12.12	0.07	3.37	0.90	1606.16	151	1196.23	65.72	409.93	89.62
18	11.71	0.90	2.73	1.25	1517.38	195	1131.92	128.16	385.46	73.84
19	11.21	0.06	7.43	0.48	2343.17	113	1475.82	29.22	867.36	97.91
20	10.88	0.15	7.34	0.48	2593.56	124	1485.48	35.94	1108.09	97.89
21	10.99	0.09	8.94	0.42	3012.18	127	1633.46	31.37	1378.71	103.09
22	11.58	0.33	9.11	0.42	2473.71	116	1602.93	34.46	870.78	103.46
23	9.91	0.97	0.68	1.51	1203.41	218.80	903.68	148.59	299.72	74.18
24	12.15	0.21	3.43	1.46	1710.25	175	1212.80	93.70	497.46	83.24
Sum	253.34		89.89		43465.70		28414.66		14422.55	

DGs provides required reserve for the VPP. During hour 2, in which the forecasted price of day-ahead market is relatively low, subject to dispatchable DGs ramp down limits, dispatchable DGs remain “on” but their generations remain at the minimum levels. Also, total profit of the VPP, obtained from bidding to the day-ahead market and covering DSO demand, reaches the minimum levels in this hour. However, during hours 3–24, the generations of dispatchable DGs reach the upper levels. Due to low market prices, during hours 1–5, the stochastic DGs also remain “off”. During hours 6–22, and

24, DG15 is “on” and its generation remains at the upper level. Also, during hours 8–10, 12–14 and 19–22, maximum capability of DG18 for providing energy are applied. During hours 1–5, 9–11, and 21–22, according to the bid price of dispatchable loads, market price and the DSO demand, more profit is made by curtailing the loads, and selling their powers to the market. Also, as seen from Table 5, Due to higher market prices at GSP11 and 16, more profit is made by exchanging energy with the upstream network via these points.

**Table 8**  
Mean and standard deviation values of optimal dispatch of DERs included in VPP in case III.

Hour	DG2 (MW)		DG7 (MW)		DG8 (MW)		DG14 (MW)		SG 15 (MW)		SG18 (MW)		DL (MW)	
	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De
1	4		4.66	1.07	5.5		3.60	1.82	0.73	1.29			0.52	0.17
2	3.31	0.44	3.20	1.31	4.58	0.65	1.75	1.47					0.52	0.00
3	3.89	0.28	4.56	1.20	5.5		3.43	1.85					0.55	0.00
4	4		4.59	0.91	5.5		4.60	1.43					0.46	0.21
5	4		4.87	0.37	5.5		6.22	0.94	0.69	1.27			0.51	0.18
6	4		5		5.5		6.12	0.41	5.50	0.31			0.09	0.22
7	4		5		5.5		6.28	0.09	5.26	0.80	0.83	1.49	0.15	0.27
8	4		5		5.5		6.16	0.06	5.50	0.31	3.11	1.28	0.14	0.27
9	4		5		5.5		6.07	0.04	6.20	0.52	4.20	0.56	0.50	0.29
10	4		5		5.5		6.07	0.03	6.20	0.43	4.20	0.50	0.50	0.29
11	4		5		5.5		6.25	0.07	6.20	0.47	0.56	1.43	0.50	0.25
12	4		5		5.5		6.07	0.03	6.20	0.46	4.20	0.49	0.08	0.22
13	4		5		5.5		6.15	0.04	5.40	0.60	3.40	0.58	0.53	0.30
14	4		5		5.5		6.15	0.04	5.40	0.64	3.40	0.58	0.11	0.25
15	4		5		5.5		6.29	0.07	5.40	0.63	0.55	1.25	0.10	0.23
16	4		5		5.5		6.45	0.08	2.67	1.58			0.07	0.19
17	4		5		5.5		6.28	0.11	5.68	1.66	0.35	1.22	0.05	0.16
18	4		5		5.5		6.30	0.11	5.74	2.28				
19	4		5		5.5		6.03	0.04	6.60	0.53	4.60	0.47	0.05	0.18
20	4		5		5.5		6.03	0.04	6.60	0.52	4.60	0.56	0.10	0.24
21	4		5		5.5		5.94	0.03	7.50	0.46	5.50	0.53	0.62	0.22
22	4		5		5.5		5.94	0.04	7.50	0.50	5.50	0.56	0.48	0.29
23	4		5		5.5		6.53	0.13	1.03	2.58				
24	4		5		5.5		6.24	0.08	6.93	1.60			0.13	0.25
Sum	95.2		116.88		131		136.92		108.92	45.00		6.76		

**Table 9**  
Hourly bids of VPP to the day-ahead market, total costs, revenues and profits in case III.

Hour	GSP11 (MW)		GSP16 (MW)		Revenue (\$)		Cost (\$)		Profit (\$)	
	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De	Mean	St. De
1	7.48	3.50			981.86	252.73	754.42	176.64	227.44	76.97
2	3.05	2.41			453.71	133.46	505.24	110.25	-51.53	23.21
3	6.94	2.84			786.81	182.03	692.59	130.44	94.22	52.82
4	8.13	2.29			848.13	159.88	750.10	106.42	98.04	58.61
5	10.27	1.85			1122.26	175.17	878.50	102.60	243.76	73.14
6	11.97	0.12	2.00	0.46	1609.95	111.11	1124.31	30.28	485.64	84.44
7	11.76	0.20	2.70	1.66	1786.20	212.89	1176.25	125.62	609.95	95.99
8	11.44	0.20	5.03	1.22	2102.08	175.11	1332.79	87.30	769.29	94.76
9	11.52	0.24	6.65	0.71	2403.85	146.89	1466.45	49.79	937.40	122.79
10	11.47	0.24	6.64	0.62	2435.78	147.40	1465.55	44.59	970.24	126.01
11	12.12	0.18	3.30	1.40	1863.82	185.71	1231.47	97.25	632.35	92.63
12	10.79	0.12	6.59	0.63	2543.77	127.15	1434.75	43.58	1109.02	100.44
13	10.94	0.25	5.04	0.78	2594.90	146.97	1380.93	54.89	1213.98	115.16
14	11.11	0.16	5.16	0.80	2259.18	122.86	1344.09	53.50	915.08	91.06
15	11.69	0.14	2.56	1.30	1785.83	175.20	1163.27	100.52	622.55	84.86
16	11.56	1.08	0.25	0.67	1340.04	170.49	982.34	92.05	357.71	80.12
17	12.11	0.08	2.72	2.00	1574.15	253	1161.29	132.88	412.86	124.43
18	11.77	0.84	2.80	1.19	1524.63	187	1139.59	120.57	385.05	77.43
19	11.24	0.11	7.43	0.66	2345.48	156	1477.93	42.88	867.55	136.01
20	10.85	0.15	7.34	0.72	2590.44	129	1482.44	49.17	1108.00	98.82
21	10.99	0.12	8.94	0.66	3012.09	140	1633.39	45.82	1378.71	111.52
22	11.72	0.33	9.11	0.71	2485.09	232	1613.57	49.14	871.52	214.78
23	9.81	0.89	0.55	1.38	1189.95	204.82	890.77	136.14	299.19	75.03
24	12.06	0.15	3.49	1.43	1708.24	170	1210.69	88.76	497.55	87.13
Sum	252.77		88.29		43348.25		28292.70		14422.55	

4.2. Case II: Observing two uncertain parameters

In this case, uncertainty of market price and generation of DG15 are considered in the problem formulation. Table 6 shows the mean and standard deviation values of generation of all units included in VPP and curtailment of dispatchable load for bidding in the day-ahead energy market. The mean and standard deviation values of hourly bids to the day-ahead market, costs, revenues and profits of VPP for this case are also summarized in Table 7. On observing both Tables 6 and 7 along the whole 24-h optimiza-

tion horizon, it is clear that generations of DG and SG units differ from case I. As seen, compared to the former case, generations of

**Table 10**  
Comparison of three different cases.

	Case I	Case II	Case III
Total generation of DERs in 24-h (MW)	640.09	642.9	640.76
Objective function (\$)	14996.77	15051.04	15055.56

**Table 11**  
Comparison of distribution network losses for three different cases.

Hour	Case I (MW)	Case 11 (MW)		Case 111 (MW)	
		Mean	St. De	Mean	St. De
1	0.009	0.013	0.005	0.015	0.011
2	0.004	0.006	0.004	0.005	0.003
3	0.010	0.013	0.006	0.011	0.005
4	0.014	0.013	0.005	0.013	0.004
5	0.019	0.018	0.003	0.022	0.008
6	0.040	0.040	0.001	0.040	0.001
7	0.040	0.040	0.001	0.039	0.002
8	0.040	0.040	0.001	0.040	0.001
9	0.042	0.042	0.002	0.042	0.002
10	0.042	0.042	0.002	0.042	0.002
11	0.042	0.042	0.001	0.042	0.002
12	0.041	0.041	0.001	0.041	0.002
13	0.038	0.038	0.002	0.038	0.002
14	0.039	0.039	0.002	0.039	0.002
15	0.039	0.040	0.002	0.040	0.002
16	0.035	0.030	0.008	0.032	0.007
17	0.044	0.044	0.002	0.042	0.004
18	0.044	0.041	0.009	0.041	0.009
19	0.044	0.044	0.002	0.044	0.002
20	0.043	0.043	0.002	0.043	0.002
21	0.047	0.047	0.002	0.047	0.002
22	0.049	0.049	0.002	0.049	0.002
23	0.019	0.024	0.011	0.023	0.010
24	0.048	0.046	0.005	0.046	0.005
Sum	1.321	0.835		0.836	

dispatchable DGs during hours 1–5 are decreased. Also, the number of hours that SG units are “on” is greater than the former case. Also, the curtailed loads in this case is greater than case I. In this case, curtailing the loads is applied during hours 1–17, 19–22, and 24. The total curtailed loads along the whole 24-h of optimization horizon in this case is 9% higher than case I. On the other hand, because of considering market price uncertainty, generation of DG18 during hours 7, 11, 15, and 17 is higher than case I. During hours 7, 16, 18, and 24, generation of DG15 is lower than case I. This is because, in this case, the uncertainty of DG15 generation is taken into consideration.

4.3. Case III: Observing three uncertain parameters

The uncertainty of market prices and two stochastic DGs generation is considered in this case. Tables 8 and 9 show the results of this case. In this case, similar to case II, generation of dispatchable DGs during hours 1–5, is lower than case I. However, total generation of dispatchable DGs along the 24-h is 0.32% than case II. This is because, in this case, the number of concentrations is increased. Compared to case I, during hours 1, 5 and 23, DG15 is “on” but its generation during hours 7, 16–18, and 24 is lower than case I.

Total dispatch of DERs along the whole 24-h optimization horizon is shown in Fig. 2. Also, hourly sale/purchase bids of VPP to the day-ahead market for these three different cases are shown in

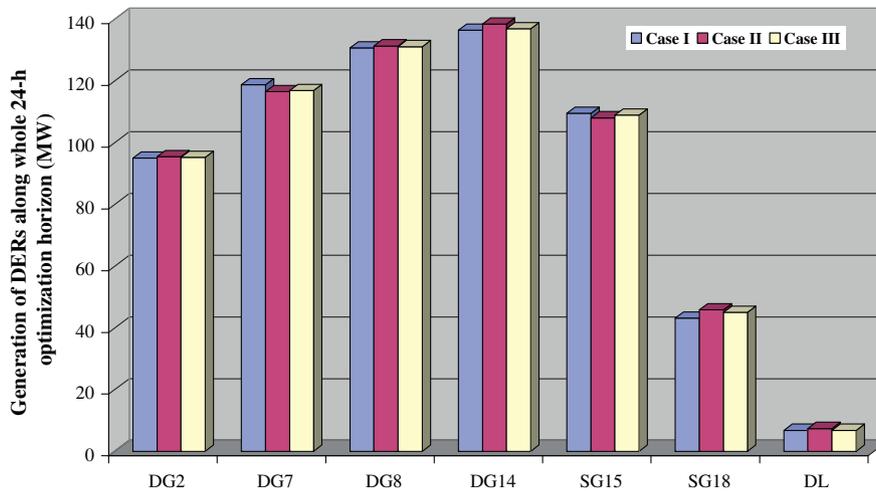


Fig. 2. Dispatch of DERs along the whole 24-h optimization horizon.

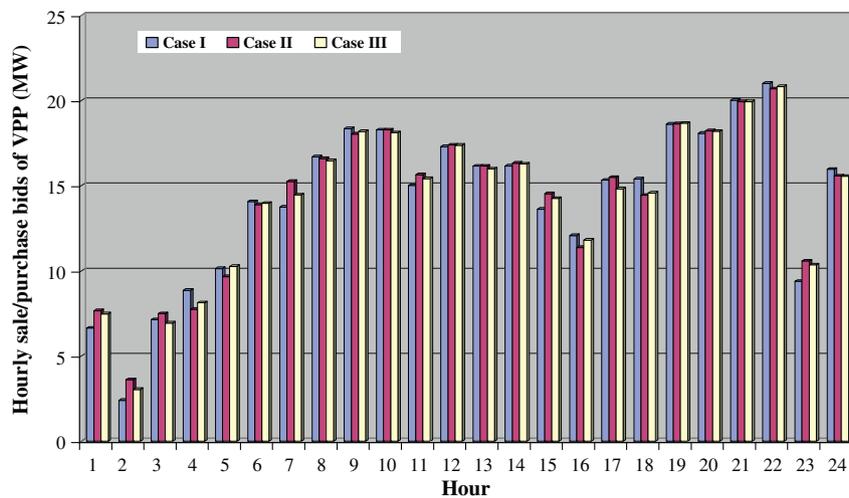


Fig. 3. Hourly sale/purchase bids of VPP to the day-ahead market for three different cases.

Fig. 3. In Table 10, some results of these three cases are compared. The difference between the total profits in these cases is a criterion to compare the possible additional costs or cost saving caused by observing uncertainty of input parameters. Generally, VPP makes proposals to exchange power with the day-ahead market based on the values obtained for the mean and standard deviations.

The distribution network losses for three cases are also shown in Table 11. On observing Table 11 along the whole 24-h optimization horizon, it is clear that total losses in cases II and III are lower than case I. This is because, in cases II and III, the amount of curtailing dispatchable load is more than case I.

## 5. Conclusions

In this paper, a procedure was presented for a VPP to optimally manage several DERs when bidding in a day-ahead electricity market. A probabilistic PBUC model was proposed to allow VPP to decide on the unit commitment of its DERs and the optimal sale/purchase bids to the day-ahead market. The main objective of the algorithm was to maximize the expected profit of the VPP owner from its involvement in the day-ahead market and covering the DSO demand. In this model, network constraints, technical constraints of DERs, uncertainty in market price and generation of stochastic DGs, and also economical aspects were taken into consideration. The stochastic trends in the uncertainty of the input parameters were simulated using point estimate method by creating concentrations that could be solved by deterministic methods. Mixed integer nonlinear programming was used for solving deterministic PBUC problems by the developed program elaborated in GAMS.

In order to evaluate the proposed model, the results were analyzed for three cases. The application of the methodology to a test case, demonstrated the effectiveness and the robustness of the proposal. The results of simulation studies showed that the local operation of the DERs would be highly sensitive to the market price variations. On the other hand, because of intermittent nature of some of DGs included in a VPP, the choice of the adequate reserve and considering the uncertainty of generation of these units were crucial for VPP success in the context of competitive electricity markets.

## References

- [1] Leão RPS, Barroso GC, Sampaio RF, Almada JB, Lima CFP, Rego MCO, et al. The future of low voltage networks: moving from passive to active. *Int J Elect Power Energy Syst* 2011;33:1506–12.
- [2] Jenkins N. Secretary of the CIRED working group no. 4 on dispersed generation. Preliminary discussion at CIRED; 1999.
- [3] Moradi MH, Abedini M. A combination of genetic algorithm and particle swarm optimization for optimal DG location and sizing in distribution systems. *Int J Elect Power Energy Syst* 2012;34:66–74.
- [4] Rotaru F, Chicco G, Grigoras G, Cartina G. Two-stage distributed generation optimal sizing with clustering-based node selection. *Int J Elect Power Energy Syst* 2012;40:120–9.
- [5] Porkar S, Poure P, Abbaspour-Tehrani-fard A, Saadate S. A novel optimal distribution system planning framework implementing distributed generation in a deregulated electricity market. *Electr Power Syst Res* 2010;80:828–37.
- [6] Ochoa LF, Dent CJ, Harrison GP. Distribution network capacity assessment: variable DG and active networks. *IEEE Trans Power Syst* 2010;25:87–95.
- [7] Houwing M, Ajah AN, Heijnen PW, Bouwmans I, Herder PM. Uncertainties in the design and operation of distributed energy resources: the case of micro-CHP systems. *Energy* 2008;33:1518–36.
- [8] Algarni AAS, Bhattacharya K. Disco operation considering DG units and their goodness factors. *IEEE Trans Power Syst* 2009;24:1831–40.
- [9] Keane A, Zhou Q, Bialek JW, O'Malley M. Planning and operating non-firm distributed generation. *IET Renew Power Gen* 2009;3:455–64.
- [10] Borghetti A, Bosetti M, Grillo S, Massucco S, Nucci CA, Paolone M, et al. Short-term scheduling and control of active distribution systems with high penetration of renewable resources. *IEEE Syst J* 2010;4:313–22.
- [11] Jakus D, Goic R, Krstulovic J. The impact of wind power plants on slow voltage variations in distribution networks. *Electr Power Syst Res* 2011;81:589–98.
- [12] Malekpour AR, Niknam T. A probabilistic multi-objective daily volt/var control at distribution networks including renewable energy sources. *Energy* 2011;36:3477–88.
- [13] Hong YY, Pen KL. Optimal VAR planning considering intermittent wind power using markov model and quantum evolutionary algorithm. *IEEE Trans Power Del* 2010;25:2987–96.
- [14] Raineri R, Rios S, Schiele D. Technical and economic aspects of ancillary services markets in the electric power industry: an international comparison. *Energy Policy* 2006;34:1540–55.
- [15] Rebours YG, Kirschen DS, Trotignon M, Rossignol S. A survey of frequency and voltage control ancillary services – Part II: Economic features. *IEEE Trans Power Syst* 2007;22:358–66.
- [16] Lopes JAP, Hatziargyriou N, Mutale J, Djapic P, Jenkins N. Integrating distributed generation into electric power systems: a review of drivers, challenges and opportunities. *Electr Power Syst Res* 2007;77:1189–203.
- [17] Ackermann T. Distributed resources and re-regulated electricity markets. *Electr Power Syst Res* 2007;77:1148–59.
- [18] Zangiabadi M, Feuillet R, Lesani H, Hadj-Said N, Kvaløy JT. Assessing the performance and benefits of customer distributed generation developers under uncertainties. *Energy* 2011;36:1703–12.
- [19] Hemdan NGA, Kurat M. Interconnection of decentralized renewable resources into distribution grids: implications and planning aspects. *Electr Power Syst Res* 2011;81:1410–23.
- [20] Brunetto C, Tinab G. Wind generation imbalances penalties in day-ahead energy markets: the Italian case. *Electr Power Syst Res* 2011;81:1446–55.
- [21] Hong YY, Chang HL, Chiu CS. Hour-ahead wind power and speed forecasting using simultaneous perturbation stochastic approximation (SPSA) algorithm and neural network with fuzzy inputs. *Energy* 2010;35:3870–6.
- [22] Hamidi V, Li F, Yao L. Value of wind power at different locations in the grid. *IEEE Trans Power Del* 2011;26:526–37.
- [23] Ochoa LF, Harrison GP. Minimizing energy losses: optimal accommodation and smart operation of renewable distributed generation. *IEEE Trans Power Syst* 2011;26:198–205.
- [24] Koutroumpetzis GN, Safigianni AS. Optimum allocation of the maximum possible distributed generation penetration in a distribution network. *Electr Power Syst Res* 2010;80:1421–7.
- [25] Wu YK, Lee CY, Liu LC, Tsai SH. Study of reconfiguration for the distribution system with distributed generators. *IEEE Trans Power Del* 2010;25:1678–85.
- [26] Coll-Mayora D, Pagetb M, Lightner E. Future intelligent power grids: analysis of the vision in the European Union and the United States. *Energy Policy* 2007;35:2453–65.
- [27] Awerbuch S, Preston A. The virtual utility: accounting, technology & competitive aspects of the emerging industry. MA, USA: Kluwer Academic Publishers; 1997.
- [28] The FENIX vision: The virtual power plant and system integration of distributed energy resources. Contract No: SES6 - 518272, Deliverable 1.4.0; 2006.
- [29] You S, Treholt C, Poulsen B. A market-based virtual power plant. In: *Proc clean elect power int conf capri*; 2009. p. 460–5.
- [30] Caldón R, Patria AR, Turri R. Optimization algorithm for a virtual power plant operation. In: *Proc 39th int universities power eng conf*, vol. 3; 2004. p. 1058–62.
- [31] Morais H, Kadar P, Faria P, Vale ZA, Khodr HM. Optimal scheduling of a renewable micro-grid in an isolated load area using mixed-integer linear programming. *Renew Energy* 2010;35:151–6.
- [32] Wille-Haussmann B, Erge T, Wittwer C. Decentralised optimisation of cogeneration in virtual power plants. *Solar Energy* 2010;84:604–11.
- [33] Ruiz N, Cobelo I, Oyarzabal J. A direct load control model for virtual power plant management. *IEEE Trans Power Syst* 2009;24:959–66.
- [34] Handschin E, Neise F, Neumann H, Schultz R. Optimal operation of dispersed generation under uncertainty using mathematical programming. In: *15th PSCC*; 2005.
- [35] Mashhour E, Moghaddas-Tafreshi S. Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part I: problem formulation. *IEEE Trans Power Syst* 2011;26:949–56.
- [36] Mashhour E, Moghaddas-Tafreshi S. Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part II: numerical analysis. *IEEE Trans Power Syst* 2011;26:957–64.
- [37] Peikherfeh M, Seifi H, Sheikh-Aleslami MK. Optimal decision making for virtual power plant operation. In: *Proc 9th int power and energy conf*; 2010. p. 625.
- [38] Mashhour E, Moghaddas-Tafreshi S. Integration of distributed energy resources into low voltage grid: a market-based multiperiod optimization model. *Electr Power Syst Res* 2010;80:473–80.
- [39] Morales JM, Pérez-Ruiz J. Point estimate schemes to solve the probabilistic power flow. *IEEE Trans Power Syst* 2007;22:1594–601.
- [40] Sotkiewicz PM, Vignolo JM. Nodal pricing for distribution networks: efficient pricing for efficiency enhancing DG. *IEEE Trans Power Syst* 2006;21:1013–4.
- [41] Albadi MHH. On techno-economic evaluation of wind-based DG. PHD Thesis. Department of Elect and Computer Eng, Univ Waterloo; 2010.
- [42] [http://www.ee.washington.edu/research/pstca/pf30/pg\\_tca30bus.htm](http://www.ee.washington.edu/research/pstca/pf30/pg_tca30bus.htm).
- [43] IEA. Report of international energy agency; 2002.
- [44] Wade. Report of world alliance for decentralized energy; 2003.
- [45] GAMS Webpage: <<http://www.gams.com/>>.