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Banking crises and liquidity in a monetary economy☆

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ABSTRACT

This study investigates banks' liquidity provision using the Lagos and Wright model of monetary exchange. With aggregate uncertainty, we show that banks sometimes exhaust their cash reserves and fail to satisfy their depositors' needs for consumption smoothing. We also show that banking crises can be eliminated by a rate-of-return-equalizing policy under perfect risk sharing, but the first-best outcome can be only achieved with the Friedman rule. These results cannot be obtained with other monetary models (e.g., overlapping generations models). We also derive a rich array of non-trivial effects of inflation on equilibrium deposits, the probability of banking crises, and banks' portfolios.

1. Introduction

This study investigates banks' liquidity provision within a Lagos and Wright (2005) economy of monetary exchange. We aim to provide a monetary view of the implications of banking technologies à la Diamond and Dybvig (1983). We explicitly demonstrate the impact of money on the occurrence of banking fragility. With aggregate uncertainty, we show that banks sometimes exhaust their cash reserves and fail to satisfy their depositors' need for consumption smoothing in monetary equilibria.

In the spirit of Kiyotaki and Wright (1989, 1991), our starting point is that the essential role of money emerges owing to market frictions in which agents face idiosyncratic consumption opportunities (which cause the well-known double coincidence of wants problem). Given their uncertain consumption needs, agents can benefit from a liquidity pool or may demand callable bank deposits in case they wish to consume and need money for their purchases. If no such immediate need for money emerges, agents can use credit backed by their capital claims and receive matured returns.

More specifically, we extend (Williamson, 2012) monetary model of banking to allow for aggregate uncertainty. Following Williamson (2012), we assume that meetings in the decentralized market can be either non-monitored, such that money is required for payment, or monitored, such that other payment instruments can be used as well. Thus, the

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idiosyncratic need for money motivates banks to provide liquidity insurance. In this study, we introduce aggregate uncertainty regarding the fraction of non-monitored exchanges, and, hence, uncertainty regarding the total demand for money into Williamson (2012) model. Unlike Williamson (2012), we show that states exists in which banks exhaust their cash reserves and suspend the convertibility of deposits into cash; we refer to such states as *banking crises*.

Our monetary equilibrium with bank deposits has the following properties. On one hand, banking crises occur more often when the proportion of non-monitored exchanges in higher or when banks' money reserves are lower. On the other hand, banking crises generate liquidity premia such that buyers engaged in non-monitored exchanges are forced to consume lower amounts than those engaged in monitored exchanges. We therefor view banking crises as monetary phenomena in both of these senses. We show that our model generates a unique welfare consequence that is more in line with standard modern monetary economics and that is difficult to obtain using other models.

In our model, as in those of Champ et al. (1996), Smith (2002), and Boyd et al. (2004), and many others, a banking crisis entails bank reserve shortages and suspensions of convertibility rather than banking insolvency. Calomiris and Gorton (1991) state that "*a banking panic occurs when bank debt holders at all or many banks in the banking system suddenly demand that banks convert their debt claims into cash (at par) to such an extent that the banks suspend convertibility of their debt into cash*" (p.112). The notion of a banking crisis used in this study seems to capture traditional bank runs driven by the withdrawal of retail depositors rather than by the withdrawal of repurchase agreements, as in the 2007–2008 financial crisis.

Several explanations can justify the aggregate uncertainty in our setup. The first potential explanation is seasonal variation. In an agricultural economy, because workers must be paid at the time of harvest and crops must be shipped before crop revenue has materialized, the aggregate demand for money is high during harvest season. Thus, to the extent that harvest seasons are stochastic and are not easy for individuals to predict, large seasonal pressures can cause banking crises (see, e.g., Calomiris and Gorton, 1991; Miron, 1986; Sprague, 1910). The second potential explanation for aggregate uncertainty is an imperfect credit system. A high cost of acquiring asset quality information makes it difficult for sellers to recognize the value of collateral, rendering even secured credit unacceptable, as Lester et al. (2012) discuss. The costs of acquiring information about certain assets (e.g., asset-backed securities) can be very volatile and hard to predict, especially across periods of higher and lower financial fragility. The third potential explanation is that unexpected events, such as large-scale natural disasters, blackouts, and the event of September 11, 2001, can damage the social infrastructure and communication tools necessary for credit transactions, suddenly increasing the aggregate demand for money.

Banks' portfolio choices take into account that money has a lower rate of return and is more liquid than capital. Given the higher rate of return of capital, banks invest a positive fraction of their deposits in capital. In addition, because banks anticipate trade opportunities in which their depositors can use capital claim, they do not hold enough money to satisfy their depositors' liquidity needs in every possible state. Thus, banking crises occur with positive probability. With the policy of rate-of-return-equality, which is not associated with the Friedman rule, the higher rate of return of capital disappears, and banks hold all their deposits as money. In this situation, banks have sufficient money reserves to guarantee perfect consumption smoothing to their depositors; in other words, banking crises never occur. This allocation, however, is below the efficient level. The latter distortion can be eliminated only with the Friedman rule under which the costs of holding money disappear.

Our result sharply contrasts with the optimal monetary policy under an overlapping generations (OLG) model like those of Champ et al. (1996) and Smith (2002, 2003), all of whom show that the Friedman rule is not optimal. Smith (2002, p.133) states "when intermediation is taken seriously, pursuing the Friedman rule either is not optimal or leads to massive indeterminacies, or both." We confirm the optimality of the Friedman rule in a modern monetary model in which intermediation is taken seriously. Further, the literature on the OLG model attempts to obtain the first-best allocation by combing of the Friedman rule and extensive discount window lending (see, e.g., Antinolfi and Keister, 2006; Haslag and Martin, 2007). We also discuss a version of our model with discount window lending. In our model, however, although the central bank serving as the lender of last resort is unambiguously beneficial, it is not necessary to implement the first-best allocation. Our result is in the spirit of standard modern monetary models, and we believe that our approach enhances the understanding of the monetary view of banking crises.

Our model has several additional features. It generates the endogenous dependence of the probability of banking crises on inflation. Empirical studies (e.g., Boyd et al., 2014; Demirguc-Kunt and Detragiache, 1998; Demirguc-Kunt and Detragiache, 2005; Hardy and Pazarbasioglu, 1999) show that banking crises and inflation are positively correlated. In particular, this positive correlation is more prominent in developing and emerging market economies than in developed countries (Joyce, 2011; Davis et al., 2011; Duttagupta and Cashin, 2011). Our theoretical predictions are consistent with these empirical findings.

In addition, our model suggests a rich array of non-trivial effects of inflation. For example, we show that the effect of inflation on deposit levels depends on the risk aversion parameter. When risk aversion is low, agents reduce deposits as inflation increases. When risk aversion is high, agents attempt to mitigate consumption inequality across non-monitored and monitored exchanges by increasing the resources transferred to future periods, and, thus, deposits increase with inflation. Risk aversion also impacts the effect of inflation on banks' portfolio choices. Two economic forces are at work. On one hand, an increase in the inflation rate induces banks to shift their resources from money to capital. On the other hand, the total amount of deposits determines the levels of both assets. These potentially conflicting effects lead to a non-monotonic relationship between inflation and banks' equilibrium portfolios.

In the literature on micro-founded money, some studies investigate money and collateralized credit, claims backed by capital, or capital as a medium of exchange (e.g., Lagos and Rocheteau, 2008; Ferraris and Watanabe, 2008; Ferraris and Watanabe, 2012, Gu, Mattesini, and Wright, 2016), and others investigate banking and credit (e.g., Bencivenga and Camera, 2011; Berentsen et al., 2007; Ferraris and Watanabe, 2008; Ferraris and Watanabe, 2011, Gu et al., 2013), but none of these studies addresses the issue of banking crises. Among the studies that investigate banking and liquidity, Williamson (2012) study is the closest to ours. The cases in which no banking crisis occurs, which comprise a subset of the parameter space in our model, have essentially the same properties as in Williamson (2012) model. Our innovation is that we generalize his banking problem to allow for cases in which a banking crisis does occur, and we show that such cases can indeed survive in a monetary equilibrium with aggregate money-demand uncertainty, even when banks' cash reserve ratios and depositors' deposit levels are chosen endogenously. In a recent study, Andolfatto et al. (2017) consider financial stability, but they take a very different approach. In their study, banking crises are modeled as banking insolvencies, whereas we modeled them as reserve shortages at banks and suspensions of convertibility. Furthermore, in their model, the probability of a crisis is sunspot-driven and is treated as an exogenous constant, whereas we treat the probability of a crisis as endogenous and influenced by inflation.

A number of studies focus on liquidity, banking, and monetary policy using OLG models with random relocation and spatial separation, such as the studies of Champ et al. (1996), Antinolfi et al. (2001), Smith (2002), Smith (2003), Boyd et al. (2004), Antinolfi and Keister (2006), Gomis-Porqueras and Smith (2003), Gomis-Porqueras and Smith (2006), Jiang (2008), and Matsuoka (2012), and with nominal bank deposit contracts and a short-horizon economy, such as the studies of Chang and Velasco (2000), Diamond and Rajan (2006), Skeie (2008), and Allen et al. (2014). Although we use a related definition of a banking crisis and find some related results, our model differs fundamentally from their models in terms of the notion of money, the role of money in the provision of liquidity, and the occurrence of banking crises, and the welfare consequences. Other non-monetary models of banking crises include, for example, those of Wallace (1990), Allen and Gale (1998), Allen and Gale (2004), Green and Lin (2003), Peck and Shell (2003), Andolfatto et al. (2007), Ennis and Keister (2006), Ennis and Keister (2009), and Ennis and Keister (2010).

The reminder of this paper is organized as follows. Section 2 describes the model environment. Section 3 derives the monetary equilibrium with bank deposits and studies its dependence on inflation and monetary policy. Section 4 discusses the model's implications for historical episodes, empirical evidence, and the discount window, and Section 5 concludes. All proofs can be found in the appendix.

2. Environment

This model builds on a version of Lagos and Wright (2005) model. Time is discrete and infinite. Each period is divided into two subperiods, called day and night. A market is open in each subperiod. There are two types of a [0,1] continuum of infinitely-lived agents. Agents of the same type are homogeneous. One type of agents, called sellers, has technology to produce a perishable and divisible good, referred to as a special good, during the day. The other type of agents, called buyers, does not have any production technology during the day but can consume the special good. Other divisible goods, referred to as general goods, are produced and consumed during the night. There is also an intrinsically worthless good that is perfectly divisible and storable called fiat money. Agents discount future payoffs at the rate $\beta \in (0, 1)$ across periods, but there is no discounting between the two subperiods.

We assume that the instantaneous utility functions of buyers and sellers are given by $u(q^b) + U(x) - h$ and $-q^s + U(x) - h$, respectively, where q^b represents a buyer's consumption of special goods, q^s is a seller's production of special goods, and h is nighttime work hours. We assume constant marginal production costs and that the utility function $u(q)$ is a strictly increasing, strictly concave, and twice continuously differentiable function with $u(0) = 0, u'(0) = \infty, u'(\infty) = 0$. We assume that there exists some $\hat{q} > 0$ such that $u(\hat{q}) = \hat{q}$. For analytical tractability, we assume that $\xi \equiv -\frac{qu''(q)}{u'(q)}$ is a positive constant. If an agent consumes x general goods, then his utility is given by $U(x)$. We assume that $U(x)$ is strictly increasing, concave, and twice continuously differentiable.

During the day, buyers and sellers can trade special goods in a decentralized market (DM); this trade involves bilateral random matching. Following Williamson (2012) (see also Section 5 of Williamson and Wright, 2010), we assume that in the DM, a fraction $\alpha \in (0, 1)$ of sellers are engaged in *non-monitored* exchanges, and the remaining fraction $1 - \alpha$ of sellers are engaged in *monitored* exchanges. At the beginning of the day, sellers meet with their counterparts, and buyers learn whether they will trade with sellers in non-monitored or monitored meetings. In the DM, exchanges are anonymous, and trading histories are private knowledge. Thus, claims to capital can be easily counterfeited in non-monitored exchanges, and because the meetings are random, sellers must receive money as immediate compensation for their products. In contrast, fake claims are confiscated with certainty in monitored exchanges, and buyers can use their claims to capital as a means of payment as well as money. Let c denote the claims to capital used by each individual monitored buyer in the DM. This randomness in terms of the medium of exchange faced by buyers plays the role of a "liquidity preference shock." This shock is similar in spirit to that used in Diamond and Dybvig (1983) to motivate banks' risk sharing role. In any meeting, we assume for simplicity that buyers make take-it-or-leave-it offers to sellers.

We assume that the fraction of monitored meetings is a random variable. It is publicly observable and identically distributed over time. Let $F = F(\alpha)$ represent its distribution function, which is assumed to be continuous, differentiable, and strictly increasing, and let $f = f(\alpha) > 0$ represent the associated density function. As mentioned earlier, this randomness can be interpreted as seasonal fluctuations or credit-system collapses, and plays a key role in our model.

At night, general goods are traded in the centralized market (CM), which is Walrasian. Two additional economic activities take place during the night. First, buyers can access a storage technology whereby storing one unit of general goods stored, called capital, generates $R > 1$ units of general goods in the next period. The return must satisfy $\beta R \leq 1$ because the solution would explode otherwise. Capital is not mobile, and, as mentioned above, claims to capital can be costlessly counterfeited in the DM. Second, agents form banks in the CM at night. A bank offers each of its depositors a deposit contract that stipulates the repayment plan specified below. The depositors are buyers who deposit general goods in the CM and require money or capital in the following DM. Banks have m money and k capital stocks in the CM. Any credit contracts in the DM are settled in the CM in the same period.

Money is issued by the central bank. M denotes the stock of money available in a period, and it grows (or shrinks) at a constant rate $\pi > \beta$ (i.e., $M_{+1} = \pi M$) through lump-sum injections to (or withdrawals from) buyers in the CM, where the subscript $+1$ indicates the next period. Let ϕ denote the price of money in terms of general goods.

Finally, the money market clears each period, and the return to money must equal the inverse of the inflation rate, $\frac{\phi_{+1}}{\phi} = \frac{1}{\pi}$, in a stationary monetary equilibrium.

The first-best solution in our economy is straightforward. The socially optimal level of capital, denoted by k^*, is indeterminate if $\beta R = 1$ and is given by $k^* = 0$ if $\beta R < 1$. The socially optimal levels of consumption, denoted by q_b^*, q_s^*, are given by $q_b^* = q_s^* = q^* \equiv u^{-1\prime}(1)$, that is, the marginal utility of special goods consumption $(= u'(q^*))$ equals the marginal cost $(= 1)$.

3. Monetary equilibrium

3.1. Banks' payment schedules

We start with the banks' optimal repayment plan. First, because banks can do anything that buyers can do, we assume, without loss of generality, that buyers deposit all of their general goods that they want to bring to the next period during the night. Let $d \geq 0$ denote the amount of general goods that a buyer deposits in a representative bank. Note that although our infinite horizon problem involves aggregate uncertainty, buyers deposit the same quantity in each period given the Lagos and Wright formulation of the quasi-linear utility in the CM (see below).

At the beginning of each day, before buyers learn their meeting types, banks choose their payment schedules given their cash m and capital k stocks selected in the previous CM (see below). These payments can be contingent on the realized aggregate state. Let $q^n = q^n(\alpha)$ ($q^m = q^m(\alpha)$) denote a payment to a non-monitored (monitored) buyer, and let $\theta_1 = \theta_1(\alpha)$ ($\theta_2 = \theta_2(\alpha)$) denote the fraction of cash reserves that a bank pays to non-monitored (monitored) buyers. Given values of m and k and the bank's balance sheet constraint,

$$\phi m + k = d,$$

a bank's optimal choice of θ_1, θ_2 and individual claims $c = c(\alpha)$ determine the optimal q^n and q^m. We assume competitive banks with free entry such that each maximizes the sum of the expected value of its representative depositor (i.e., buyer) and net wealth at the beginning of the CM, $(1 - \theta_1 - \theta_2)m\phi_{+1} + Rk + (1 - \alpha)c$, both of which are measured in terms of general goods. Without loss of generality, we assume that the remaining assets that are not used in the DM are rebated to monitored buyers. Thus, given the realized values $\alpha \in (0, 1)$, a bank's problem in the DM can be written as

$$\max_{\theta_1, \theta_2 \in [0,1], c \geq 0} \alpha u(q^n) + (1 - \alpha)\left\{u(q^m) + \frac{(1 - \theta_1 - \theta_2)m\phi_{+1} + Rk}{1 - \alpha} - c\right\}, \tag{1}$$

subject to

$$\alpha q^n = \theta_1 m\phi_{+1}, \tag{2}$$

$$(1 - \alpha)q^m = \theta_2 m\phi_{+1} + (1 - \alpha)c, \tag{3}$$

$$(1 - \alpha)c \leq (1 - \theta_1 - \theta_2)m\phi_{+1} + Rk, \tag{4}$$

and $\theta_1 + \theta_2 \leq 1$. Constraint (2) indicates that each individual non-monitored buyer receives $\frac{\theta_1 m}{\alpha}$ units of cash from the bank and, given that buyers make take-it-or-leave-it offers, exchanges this cash with the matched seller for $\frac{\theta_1 m\phi_{+1}}{\alpha}$ units of special goods. Constraint (3) similarly indicates that the total real payment to monitored buyers equals the sum of the bank's cash reserves that are not repaid to non-monitored buyers and the total capital claims received by monitored buyers, $(1 - \alpha)c$. Constraint (4) indicates that the total capital claims used in the DM cannot exceed the total value of the bank's remaining cash reserves and capital in the following CM, $(1 - \theta_1 - \theta_2)m\phi_{+1} + Rk$.

Discussion: The above problem of a bank features needs of different liquid assets across different depositors. It may be useful to clarify a few things. First, one interpretation of constraint (4) is that the debt of each individual monitored buyer, c, to the corresponding seller cannot exceed the real value of the buyer's remaining money and capital holdings,

$\frac{(1-\theta_1-\theta_2)m\phi_{+1}+Rk}{1-\alpha}$. In monitored exchanges, fake claims cannot be made, and, thus, individual buyers can use their capital as collateral to credibly promise payments to sellers. Alternatively, this constraint could be interpreted as bank credit such that all monitored claims are managed by banks and each bank uses its capital as collateral to promise payments from monitored buyers to sellers. Thus, each of the individual buyers makes a bank loan that cannot exceed the real value of the available bank capital. Indeed, our main insight holds for any form of individual secured credit, bank credit, or even bank notes that can be used in monitored but not in non-monitored exchanges.[1] Second, although it is not made explicit here, depositors could have an incentive problem because the meeting types of individual depositors do not need to be observed by their banks. However, monitored buyers have no incentive to misrepresent their types because $q^m \geq q^n$, and non-monitored buyers similarly have no such incentive because bank payments in the form of credit are useless for them in the DM (see the proof of Theorem 1). Thus, such an incentive constraint would never bind in the optimal solution. Third, the suspension of convertibility is embedded in our setup in the sense that the bank refuses to liquidate capital prematurely and only pay out reserves selected in the previous CM during the day. Unlike in Diamond and Dybvig's model, liquidated capital cannot be consumed, and the capital claims are not used in non-monitored exchanges. Fourth, no self-fulfilling bank runs occur in this model. Because we assume that banks' cash reserves are distributed equally among those who announce that they are non-monitored buyers instead of assuming a sequential service constraint. To see why, suppose that an individual monitored buyer thinks that all monitored buyers will misrepresent their trading types and will try to withdraw money like non-monitored buyers. If this buyer also tries to withdraw money, he can obtain m units of money and consume $m\phi_{+1}$ units of special goods in the DM. However, if he announces his trading type honestly, his bank will give only him capital claims, allowing him to consume at the efficient level, q^*. Thus, it is always optimal for monitored buyers to announce their types honestly, implying that no self-fulfilling bank runs occur in equilibrium. Finally, because non-monitored and monitored buyers cannot trade with each other, there is no concern that side trading may potentially achieve a superior allocation to the current bank arrangement.

The first-order conditions are

$$u'(q^n) - u'(q^m) \geq 0 \quad (\text{with} = \text{ if } \theta_1 < 1)$$
$$u'(q^m) \geq 1 \quad (\text{with} = \text{ if } (1-\alpha)c < (1-\theta_1-\theta_2)m\phi_{+1} + Rk).$$

The first condition relates to the choice of θ_1 and shows that two situations are possible. The first is $\theta_1 < 1$, which implies $q^n = q^m$, that is, consumption smoothing. The second is $\theta = 1$, which implies that the bank exhausts all its cash reserves and fails to achieve consumption smoothing, $q^n < q^m$. We refer to this event as *a banking crisis*. The second condition relates to the choice of c and shows that the monitored allocation q^m can be the first best, $q^* = u^{-1\prime}(1)$, when the remaining reserves and capital are abundant.

Let $\gamma \equiv \frac{\phi m}{d}$ denote the ratio of a bank's cash reserves to deposits. Because γ is chosen earlier in the previous CM, the optimal solution to the bank's problem can be written in terms of α, d, and γ, with $m = \frac{\gamma d}{\phi}$, $k = (1-\gamma)d$, and $\frac{\phi_{+1}}{\phi} = \frac{1}{\pi}$, as summarized by the following proposition.

Theorem 1 (Bank's Payment Plan)**.** *Define the following critical values:*

$$\alpha_{AB} \equiv 1 - \frac{R(1-\gamma)d}{q^*}, \quad \alpha_{BC} \equiv \frac{\gamma}{\gamma + (1-\gamma)R\pi}, \quad \alpha_{AD} \equiv \frac{\gamma d}{\pi q^*}, \quad \gamma_{CD} \equiv \frac{R\pi - \frac{\pi q^*}{d}}{R\pi - 1}, \quad \gamma_0 \equiv \frac{\pi q^*}{d}.$$

Given $\alpha \in (0, 1)$, $d > 0$ *and* $\gamma \in [0, 1]$, *a bank's optimal payment plan choice* $(q^n, q^m) \leq (q^*, q^*)$ *can be described as follows:*

- *Region A. For* $\alpha \in (\max\{\alpha_{AB}, \alpha_{AD}\}, 1]$,
$$q^n = \frac{\gamma d}{\alpha\pi} < q^m = q^*;$$
- *Region B. For* $\alpha \in [\alpha_{BC}, \alpha_{AB}]$,
$$q^n = \frac{\gamma d}{\alpha\pi} < q^m = \frac{R(1-\gamma)d}{1-\alpha} < q^*;$$
- *Region C. For* $\alpha \in [0, \alpha_{BC})$, *and for* $\gamma \in [\gamma_{CD}, 1]$ *if* $R\pi > 1$ *and* $\gamma \in [0, \min\{\gamma_{CD}, 1\}]$ *if* $R\pi \leq 1$,
$$q^n = q^m = d\left\{\frac{\gamma}{\pi} + (1-\gamma)R\right\} < q^*;$$
- *Region D. For* $\alpha \in [0, \min\{\alpha_{AD}, 1\}]$, *and for* $\gamma \in [0, \min\{\gamma_{CD}, 1\})$ *if* $R\pi > 1$ *and* $\gamma \in (\gamma_{CD}, 1]$ *if* $R\pi \leq 1$,
$$q^n = q^m = q^*.$$

The outcomes in the case of $R > \frac{1}{\pi}$ and $R \leq \frac{1}{\pi}$ are illustrated in Figs. 1 and 2, respectively. For low realizations of α or high values of the reserve ratio γ, which correspond to Regions C and D, banks have enough cash reserves relative

[1] In the OLG model with random relocation, credit transactions are not modeled explicitly. Smith (2002, p.129) states that "Agents who are not relocated remain in contact with their bank. Moreover, the agents they transact with can also contact their bank. Hence, agents who are not relocated can make purchases with checks, credit cards, or other credit instruments." However, in Smith (2002) model, non-relocated agents simply consume the asset returns from their banks directly when they are old, and there is no time lag between goods consumption and repayments. This explicit model of credit transactions is one of the main departures of our model from the OLG model.

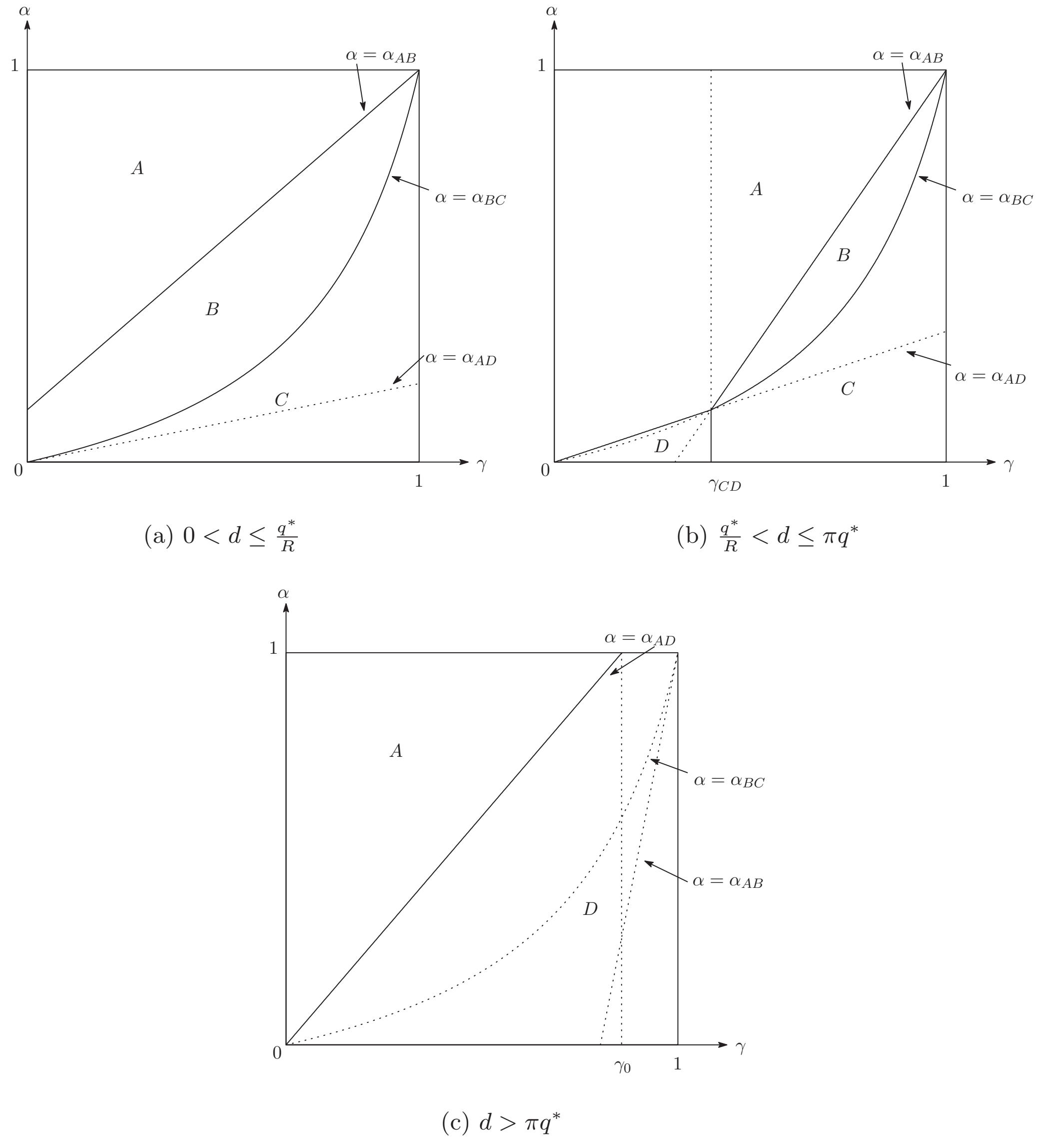

Fig. 1. Bank's Payment Plan when $R > \frac{1}{\pi}$.

to the cash needs of their depositors. In these regions, banks pay out only a fraction of their reserves to non-monitored buyers (i.e., $\theta_1 < 1$), and depositors receive the same payments irrespective of the type of meetings, and therefore enjoy consumption smoothing (i.e., $q^n = q^m$). Thus, banks offer a perfect risk sharing service. In Region D, the trades are efficient (i.e., $q^n = q^m = q^*$). In particular, for $\gamma \in [\gamma_0, 1]$, trades are made with cash only (i.e., $c = 0$), and the remaining cash reserves go to monitored buyers (i.e., $\theta_1 + \theta_2 < 1$). In other words, consumption smoothing is achieved in any realized state. For higher values of α or lower values of γ, reserves are tighter relative to depositors' needs. In Regions A and B, because all of the reserves are paid out to non-monitored buyers (i.e., $\theta_1 = 1$), non-monitored buyers receive lower payments than monitored buyers do (i.e., $q^n < q^m$), and consumption smoothing fails. Thus, risk sharing does not work perfectly here. In these regions, no money is left for monitored buyers, and, thus, payments to monitored buyers are drawn from capital claims. In particular, in Region A, although each monitored exchange is efficient (i.e., $q^m = q^*$), the total volume of monitored exchanges, $(1-\alpha)q^m$, is so low that part of the available credit prepared for monitored buyers is left unused. Because in this region either α is very high or γ is very low, each non-monitored buyer receives a very low payment, leading to a very large difference in consumption levels. Note that the total deposit size also matters; Region D disappears when $d \le q^* \min\{\frac{1}{R}, \pi\}$, whereas Regions B and C disappear when $d \ge q^* \max\{\frac{1}{R}, \pi\}$. The three upward-sloping curves that divide the regions meet $\alpha_{AB} = \alpha_{BC} = \alpha_{AD} \in (0, 1)$ at $\gamma = \gamma_{CD} \in (0, 1)$.

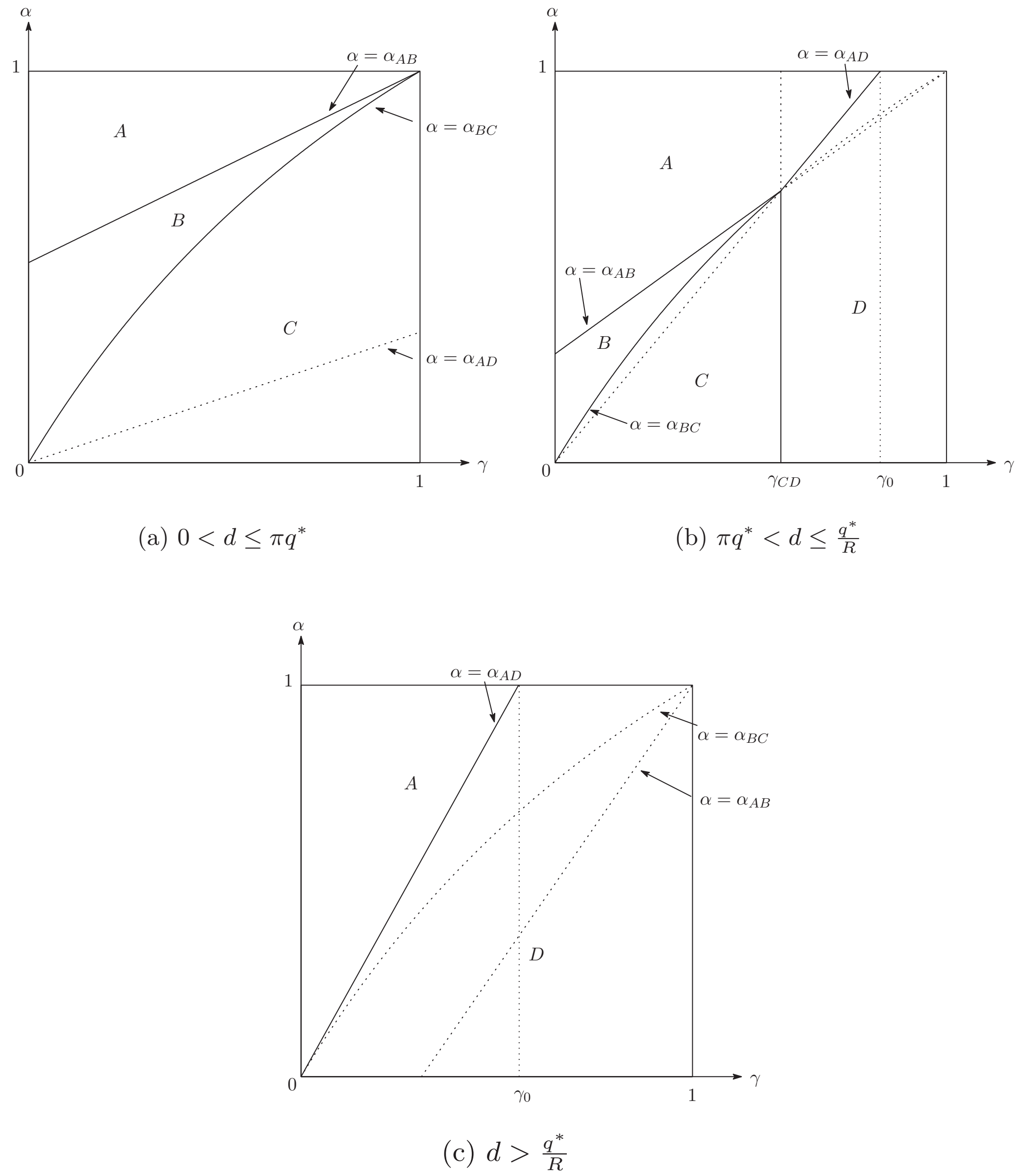

Fig. 2. Bank's Payment Plan when $R \leq \frac{1}{\pi}$.

As mentioned earlier, the characteristics of a banking crisis in this model are consistent with Calomiris and Gorton's definition. This definition aims to capture the situation in which a significant number of retail depositors suddenly demand to redeem bank debt for cash, leading to a shortage in the overall amount of reserves in the banking system and the suspension of convertibility. This analysis is highly relevant to the crises in the National Banking Era in the U.S. (1863–1913) and the recent crises in Cyprus in 2013 and in Greece in 2015 rather than the 2007–2008 financial crisis.

3.2. Monetary equilibrium with bank deposits

We now establish a stationary monetary equilibrium with bank deposits by deriving banks' optimal portfolios and the buyers' optimal deposits. Our main interest lies in determining whether a banking crisis, as described by Regions A or B above, can occur with aggregate uncertainty. We focus on the case of $R > \frac{1}{\pi}$ in the following discussion (the other case follows similarly).

Given the value of d and the repayment plan described by Theorem 1, the bank chooses a reserve ratio γ in the CM. We denote the optimal solution by $\gamma(d)$. The bank's objective function differs depending on the value of d. For $d \leq \pi q^*$, we have two cases. Suppose that $\gamma = \gamma(d) \geq \gamma_{CD}$. Then, because $d \leq \frac{q^*}{R} \Longleftrightarrow \gamma_{CD} \equiv \frac{\pi R - \frac{\pi q^*}{d}}{\pi R - 1} \leq 0$, this case lies in either Fig. 1 (a) or the right half of Fig. 1 (b), that is, we are in either Region A, B, or C depending on the value of α. Thus, the bank's problem can be written as

$$\begin{aligned}\tilde{V}(d \mid \gamma) = \max_{\gamma \in [\gamma_{CD}, 1]} & \int_{\alpha_{AB}}^{1} \left[\alpha u(q^n) + (1-\alpha)\left\{u(q^*) + W(0, k', q^*)\right\}\right] f(\alpha) d\alpha \\ & + \int_{\alpha_{BC}}^{\alpha_{AB}} \left[\alpha u(q^n) + (1-\alpha)\left\{u(c) + W(0, k', c)\right\}\right] f(\alpha) d\alpha \\ & + \int_{0}^{\alpha_{BC}} \left[u(q^n) + (1-\alpha) W(0, k', c)\right] f(\alpha) d\alpha + \int_0^1 \alpha W(0,0,0) f(\alpha) d\alpha, \end{aligned} \tag{5}$$

where $W(m', k', c)$ is the expected value of a buyer who enters the CM with holdings of $m' = \frac{(1-\theta_1-\theta_2)\gamma d}{1-\alpha}$ remaining cash reserves, $k' = \frac{(1-\gamma)d}{1-\alpha}$ in capital, and c units of claims, as the bank capital is split equally among the monitored buyers. The first integral represents the depositors' expected utility in Region A, where $q^n = \frac{\gamma d}{\alpha \pi}$ and $q^m = c = q^*$; the second integral represents that in Region B, where $q^n = \frac{\gamma d}{\alpha \pi}$ and $q^m = \frac{R(1-\gamma)d}{1-\alpha}$; and the third integral represents that in Region C, where $q^n = q^m = d\{\frac{\gamma}{\pi} + (1-\gamma)R\}$. In the latter two regions, $c = \frac{R(1-\gamma)d}{1-\alpha}$. The critical values are $\max\{\alpha_{AB}, \alpha_{AD}\} = \alpha_{AB}$ and α_{BC}.

Next, suppose that $\gamma = \gamma(d) < \gamma_{CD}$ $(= \min\{\gamma_{CD}, \gamma_0\} < 1)$. Then, $\alpha_{BC} > \alpha_{AB}$ and, thus, Region B is empty. This case therefore lies in the left half of Fig. 1 (b), that is, it lies in either Region A or Region D. The bank's problem can be written as

$$\begin{aligned}\tilde{V}(d \mid \gamma) = \max_{\gamma \in [0, \gamma_{CD}]} & \int_{\alpha_{AD}}^{1} \left[\alpha u(q^n) + (1-\alpha)\left\{u(q^*) + W(0, k', q^*)\right\}\right] f(\alpha) d\alpha \\ & + \int_{0}^{\alpha_{AD}} \left[u(q^*) + (1-\alpha) W(0, k', c)\right] f(\alpha) d\alpha + \int_0^1 \alpha W(0,0,0) f(\alpha) d\alpha. \end{aligned} \tag{6}$$

The first integral represents the expected utility of depositors in Region A, as stated before, and the second integral represents that in Region D, where $q^n = q^m = q^*$ and $c = \frac{q^* - \frac{\gamma d}{\pi}}{1-\alpha} > 0$. The critical value is $\max\{\alpha_{AB}, \alpha_{AD}\} = \alpha_{AD}$. Non-monitored buyers have no capital and no debt claims and, thus, their value at night is always given by $W(0, 0, 0)$, as is summarized by the last term.

The case $d > \pi q^*$, lies in Fig. 1 (c), that is, either Region A or Region D. For $\gamma = \gamma(d) < \gamma_0$ $(= \min\{\gamma_{CD}, \gamma_0\} < 1)$, the bank's problem is the same as in (6), whereas for $\gamma \in [\gamma_0, 1]$, because $\alpha_{AD} \geq 1$, the problem is

$$\hat{V}(d \mid \gamma) = \int_0^1 \left[u(q^*) + \alpha W(0,0,0) + (1-\alpha) W\left(m', k', 0\right)\right] f(\alpha) d\alpha. \tag{7}$$

Note that the capital claims are not used (i.e., $c = 0$), and some cash reserves are rebated to monitored buyers (i.e., $\theta_1 + \theta_2 < 1$).

We now specify the value at night, $W(\cdot)$. In the CM, buyers produce and consume general goods, invest in capital, and settle payments that they promised during the day (if any). The expected value of a buyer entering the night market in a given period with holdings k' of capital and c of debt from the DM, denoted by $W(k', c)$, satisfies

$$W(m', k', c) = \max_{x, h, d_{+1} \geq 0} U(x) - h + \beta V(d_{+1}),$$

subject to

$$x + d_{+1} + c = h + \phi_{+1} m' + R k' + \phi T,$$

where T denotes monetary transfers (or taxes) and $V(d_{+1})$ denotes the expected value in the next DM with deposits d_{+1}. The usual non-negativity constraints also hold. Note that banks must use these resources to stock money and capital during the night, as we describe in detail later.

Substituting for h, we obtain

$$W(m', k', c) = \phi_{+1} m' + R k' + \phi T - c + \max_{x > 0}\{U(x) - x\} + \max_{d_{+1} > 0}\{-d_{+1} + \beta V(d_{+1})\},$$

implying that d_{+1} is independent of wealth $\phi_{+1} m' + R k' + \phi T - c$. This property is standard in the Lagos and Wright model. It is especially important in our model because, with aggregate uncertainty, the buyer's claims to capital c are contingent on the realization of the aggregate state. However, owing to this property, the deposit choice in each period does not depend on the distribution of c. The first-order condition yields the consumption of general goods $x^* = U^{-1\prime}(1)$. The same solution applies to sellers, who are passive in the CM and do not bring anything to the next period, because sellers do not consume anything in the DM and do not have storage technology. Clearly, then, they do not make deposits in banks.

We can now solve the bank's portfolio choice problem. We prove in the Appendix that this problem has a unique solution that is characterized as follows.

Theorem 2 (Bank's Portfolio Choice). *For $R > \frac{1}{\pi}$, the bank's optimal reserve ratio is given by a unique value $\gamma(d) \in (0, 1)$ for any $d > 0$. Further, $\gamma(d)$ is monotonically decreasing in d. For $R \le \frac{1}{\pi} < \frac{1}{\beta}$, the bank's optimal reserve ratio is given by $\gamma(d) = 1$ for any $d > 0$.*

For $R > \frac{1}{\pi}$, at the optimum, the marginal expected utility of cash reserves is equal to their marginal cost. The optimal solution should satisfy $\gamma(d) < 1$, especially because, with a higher cash-reserve rate, monitored buyers suffer from reductions in repayments in some realized states and reductions in capital holdings. These reductions create the possibility of banking crises. In addition, $\gamma(d)$ is decreasing in d because banks increasingly care about the rate of return dominance, $R > \frac{\phi_{+1}}{\phi} = \frac{1}{\pi}$, rather than about liquidity benefits as d increases. For $R \le \frac{1}{\pi}$, because money dominates capital, banks hold only cash reserves. When deposits are sufficiently large, that is, $d > \pi q^*$, some cash reserves are unused in the DM, and these reserves are transferred to monitored buyers.

The last step is to determine buyers' optimal deposits, given the deposit contracts offered by banks described above. The buyer's problem in the CM is

$$\max_{d \in [0,\infty)} \{-d + \beta V(d)\}, \tag{8}$$

where $V(d) = \max_{\gamma \in [0,1]} \tilde{V}(d \mid \gamma)$, as derived above.

Definition 1. A stationary monetary equilibrium with bank deposits is described by a market-clearing-price of money, $\phi > 0$, a deposit level $d \ge 0$ that satisfies (8), the bank's reserve ratio $\gamma(d) \in [0, 1]$ given by Theorem 2, and the repayment plan $(q^n, q^m) \le (q^*, q^*)$ given by Theorem 1.

Consider first the case in which $R > \frac{1}{\pi}$. As $\gamma = \gamma(d)$ is a monotonically decreasing function of d, there exists a unique $\tilde{d} \in (\frac{q^*}{R}, \pi q^*)$ such that (see the proof of Proposition 1):

- for $d \in (0, \tilde{d}]$, $\gamma(d) \in [\gamma_{CD}, 1)$, and its associated value $\tilde{V}(d \mid \gamma(d))$ is given by (5);
- for $d \in (\tilde{d}, \infty]$, $\gamma(d) \in (0, \min\{\gamma_{CD}, \gamma_0\})$, and its associated value $\tilde{V}(d \mid \gamma(d))$ is given by (6).

For $d \in (0, \tilde{d}]$, the Euler equation is

$$\frac{\pi}{\beta} = \int_{\alpha_{BC}}^{1} u'\left(\frac{\gamma d^*}{\alpha \pi}\right) f(\alpha) d\alpha + u'\left(d^*\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC}),$$

with $\gamma = \gamma(d^*)$, whereas for $d > \tilde{d}$,

$$V'(d) = R.$$

This result reflects the concavity of the value function $V(d)$, and the marginal value of deposits reaches the lower bound R for high values of d. The deposit choice in this case is illustrated by Fig. 3(a).

Next, consider the case of $R \le \frac{1}{\pi}$. Similarly, we obtain that

- for $d \in (0, \pi q^*]$, $\gamma(d) = 1$, and its associated value $\tilde{V}(d \mid \gamma(d))$ is given by
$$\tilde{V}(d \mid \gamma(d)) = u\left(\frac{d}{\pi}\right) + W(0, 0, 0);$$
- for $d \in (\pi q^*, \infty]$, $\gamma(d) = 1$, and its associated value $\tilde{V}(d \mid \gamma(d))$ is given by
$$\tilde{V}(d \mid \gamma(d)) = u(q^*) + \frac{1}{\pi}(d - \pi q^*) + W(0, 0, 0).$$

For $d \in (0, \pi q^*]$, $\gamma(d) = 1$, the Euler equation is

$$\frac{\pi}{\beta} = u'\left(\frac{d^{**}}{\pi}\right),$$

and for $d \in (\pi q^*, \infty]$, we obtain $V'(d) = \frac{1}{\pi}$. The deposit choice in this case is illustrated by Fig. 3(b).

In a stationary monetary equilibrium, the aggregate real demand for money and the supply of money must be equal. These values are both constant over time, that is, $\phi M_{+1} = \gamma(d) d = \phi_{+1} M_{+2}$ for any given period, leading to $\frac{\phi_+}{\phi} = \frac{1}{\pi}$. Note that, in our model, because α is not realized until after the market in which money is traded for goods is closed, the equilibrium price level is unaffected by the realization of α.

We therefore obtain the following result.

Proposition 1 (Monetary Equilibrium with Bank Deposit). *A monetary equilibrium with bank deposits exists.*

- *If $\frac{1}{\pi} < R < \frac{1}{\beta}$, there exists a unique outcome $d^* \in (0, \tilde{d})$ with $\tilde{d} \in (\frac{q^*}{R}, \pi q^*)$ in which a banking crisis occurs with positive probability. This result satisfies $\gamma(d) \in (\gamma_{CD}, 1)$ and $q^n, q^m \le q^*$ in Region A, B, or C.*

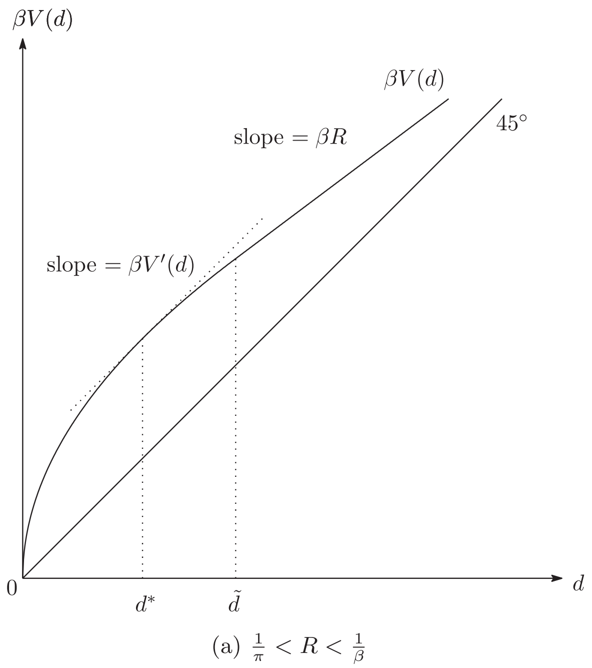

Fig. 3. Deposit Choice.

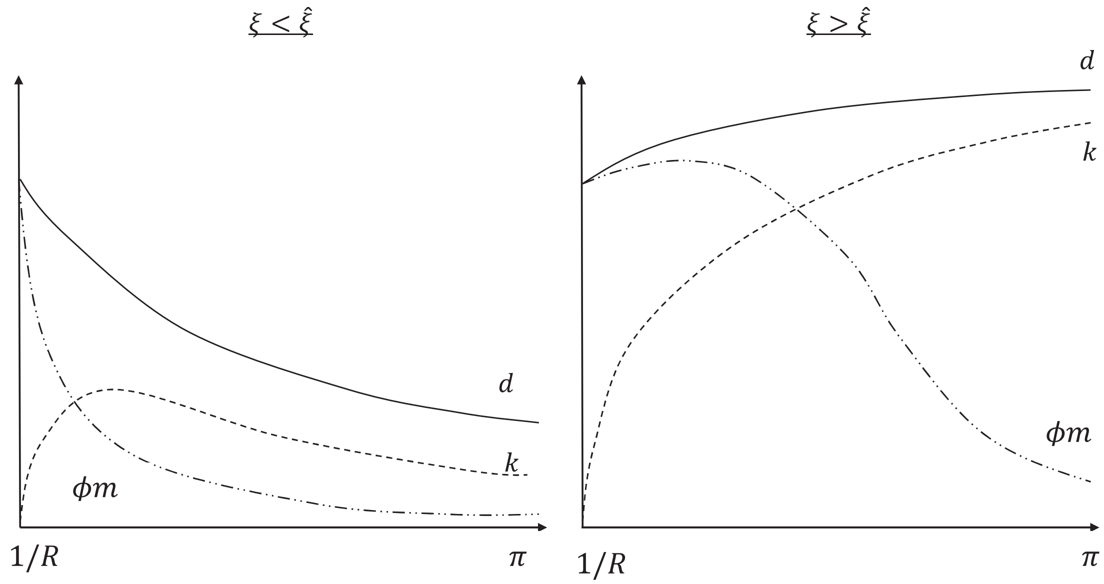

Fig. 4. Equilibrium Deposit and Bank's Portfolio.

- *If $R \leq \frac{1}{\pi} < \frac{1}{\beta}$, there exists a unique outcome $d^{**} \in (0, \pi q^*)$ in which no banking crisis occurs. This result satisfies $\gamma(d) = 1$ and $q^n = q^m = \frac{d^{**}}{\pi} \leq q^*$ in Region C.*
- *If $\beta R = 1$, the equilibrium is indeterminate such that any $d \in [\tilde{d}, \infty)$ leads to banking crises and efficient risk sharing. The multiple equilibria involve $\gamma(d) \in (0, \min\{\gamma_{CD}, 1\}]$ and q^n, $q^m \leq q^*$ in Region A or D.*

The proposition shows that, with aggregate uncertainty, a banking crisis can occur in equilibrium with positive probability when $\frac{1}{\pi} < R \leq \frac{1}{\beta}$. When $\frac{1}{\pi} < R < \frac{1}{\beta}$, the equilibrium is unique with relatively low deposits. With relatively high realizations of α, the available liquidity offered by banks is insufficient to guarantee consumption smoothing, leading to $q^n < q^m$ (Regions A and B in Fig. 1), although consumption smoothing is guaranteed for relatively low realizations of α, that is $q^n = q^m < q^*$ (Region C in Fig. 1). In other words, banks expect that capital claims can be used as a payment with positive probability, and, thus, do not pool sufficient liquidity to provide their depositors with perfect risk sharing. When $R \leq \frac{1}{\pi} < \frac{1}{\beta}$, money dominates capital, and the banks hold only cash reserves. In this case, consumption inequality and the possibility of a banking crisis are completely eliminated, but the quantities are not efficient. Similar logic applies to the case of $\beta R = 1$ with a continuum of equilibrium deposits that encompasses banking crises (Region A) and efficient risk sharing $q^n = q^m = q^*$ (Region D). Note that $\beta R = 1$ implies that holding capital is costless, and, thus, agents (and their banks) are indifferent about how much capital they invest, leading to the indeterminacy of deposit levels.

The following proposition explains the occurrence of perfect risk sharing and the efficient allocation in our monetary equilibrium with bank deposits.

Proposition 2 (Risk sharing/Efficiency)**.** *Consider the monetary equilibrium with bank deposit.*

- *If $\beta R < 1$, the rate-of-return-equalizing policy, $\pi \to \frac{1}{R}$, implements perfect risk sharing, $q^n = q^m$, but the equilibrium allocation is efficient if and only if $\pi \to \beta$ at the Friedman rule.*
- *If $\beta R = 1$, then the rate-of-return-equalizing policy that is identical to the Friedman rule, $\pi \to \frac{1}{R} = \beta$, the equilibrium allocations are efficient.*

The rate-of-return-equalizing policy equates the rates of return to money and capital and, thus, banks hold their entire deposits as money owing to its liquidity benefit, that is, $\gamma(d) \to 1$. Banks can therefore offer perfect risk sharing to their depositors, yielding consumption smoothing (i.e., $q^n = q^m$). If $\beta R = 1$, these payments are efficient. However, if $\beta R < 1$, these payments fall short of the efficient level because, given that $\pi > \beta$, the deposit choice takes into account the cost of holding money, leading to $d < \frac{q^*}{R}$. The latter costs are eliminated only at the Friedman rule, where $d \to \frac{q^*}{R}$ as $\pi \to \beta$, yielding efficient risk sharing $q^n = q^m \to q^*$. This result is in clear contrast to the optimal monetary policy in the OLG model along the lines of Smith (2002) who show that the Friedman rule is not optimal. We demonstrate the optimality of the Friedman rule in a modern monetary model in which intermediation is taken seriously. We discuss this point further in Section 4.

The effects of inflation on the equilibrium outcomes are summarized as follows.

Proposition 3 (Effects of inflation)**.** *Consider the monetary equilibrium with bank deposits, and suppose that $\frac{1}{\pi} < R < \frac{1}{\beta}$ and $\xi \equiv -\frac{u''(q)q}{u'(q)} < 1$. Then, in equilibrium,*

1. *The bank cash reserve rate γ is monotonically decreasing in inflation.*
2. *The probability of a banking crisis is monotonically increasing in inflation.*
3. *The equilibrium deposit level d is monotonically decreasing in inflation when risk aversion is low $\xi < \hat{\xi}$ and is monotonically increasing in inflation when risk aversion is high $\xi > \hat{\xi}$ for some critical value of risk aversion $\hat{\xi} \in (0, 1)$.*
4. *Capital is monotonically increasing in inflation when $\xi > \hat{\xi}$ and it increases under low inflation and decreases under high inflation when $\xi < \hat{\xi}$.*
5. *Money holdings are monotonically decreasing in inflation when $\xi < \hat{\xi}$, and they increase under low inflation and decrease under high inflation when $\xi > \hat{\xi}$.*

The bank's equilibrium rate of cash reserves γ decreases with inflation, reflecting the usual increasing costs of holding money. This result leads to the positive effect of inflation on the probability of a banking crisis. The effect of inflation on deposit levels depends on the risk aversion parameter ξ. When ξ is low, the equilibrium deposit level decreases with inflation, whereas, when ξ is high, buyers greatly care about consumption smoothing. In this situation, buyers attempt to mitigate the inequality in consumption between non-monitored and monitored exchanges by increasing the total resources transferred to future periods, and, thus, deposits increase with inflation. Risk aversion also impacts the effect of inflation on banks' portfolio choices. Two economic forces are at work. On one hand, an increase in the inflation rate induces banks to shift their resources from money to capital. On the other hand, total deposits determine the levels of both assets. When ξ is low, the former (latter) effect is dominant for low (high) inflation in determining capital holdings and, thus, capital investments are non-monotonic in inflation. When ξ is high, the latter (former) effect is dominant for low (high) inflation in determining money holdings and, thus, money holdings are non-monotonic in inflation. These results are summarized in Fig. 4 and have not been reported by existing studies of OLG models. In most OLG models with random relocation, agents deposit their entire endowments in banks, that is, the level of deposits is exogenous. The endogenous choice of deposits is another way in which this model departs from OLG models.

4. Discussions

The following discussion provides historical backgrounds, empirical support for our key results, and the discount window.

Historical Backgrounds: First, we argue that some aspects of our model can be connected to historical banking crises that occurred during the National Banking Era and in emerging market economies. According to Sprague (1910) and Chari (1989), the National Banking System of the U.S. had two key features that played important roles in banking crises. One is that banks were not able to operate across states and issue notes freely, and the other is that banks could deposit their reserves in New York City banks at a high interest rate. The former feature implied that the banks could not diversify against regional and seasonal liquidity risks and that repayments had to be made from banks' own liquid reserves. The latter feature meant that aggregate liquidity reserves experienced shortage across the whole banking system during crises because New York City banks invested their deposits into illiquid assets to earn the interest demanded. A committee appointed by the New York Clearing House Association in 1873 argued, "The aggregate [reserves] held by all the national banks of the United States does not finally much exceed 10 per cent of their direct liabilities, without reference to the large amount of debt which is otherwise dependent upon the same reserves" and "The abandonment of the practice of paying interest upon deposits will remove a great inducement to divide these reserves between cash in hand and deposits in cities, and make the banks throughout the country what they should always be, financial outposts to strengthen the general situation" (Sprague, 1910, p. 96–97). During the period from 1864 to 1933, seven economy-wide crises occurred (in 1873, 1884, 1890, 1893, 1907, 1930, and 1933), and four of them (1873, 1893, 1907, and 1933) led to large suspensions of the convertibility of deposits into currency.

The Argentinean crisis of 2001 is also a relevant episode for our analysis (see Ennis and Keister, 2009). In the late 1990s, Russia's sovereign default created a credit collapse and a deep recession in Argentina. From November 28, to 30 in 2001, a panics occurred, and many depositors rushed to withdraw their funds from all banks simultaneously. Banks were under severe pressure from these massive withdrawals, and the government declared that deposits would be frozen for 90 days beginning on December 1. During this freeze, depositors could withdraw only up to 1000 pesos per month from each account. The crises in Cyprus in 2013 and in Greece in 2015 were quite similar because they also involved a shortage of currency and the suspension of convertibility. The suspension of convertibility (often called a deposit freeze or a banking holiday) is the common policy response to such crises even today.

Our model illustrates several important features of these episodes. First, a sudden aggregate currency demand (due to seasonal variations or a credit collapse) causes a shortage of aggregate reserves, leading to a banking crisis. Second, banks are restricted from using other repayment methods (e.g., issuing bank notes). Without this feature, a crisis can be eliminated, as in the Canadian banking system during the National Banking Era. Third, the convertibility of deposits into currency is often suspended during a crisis. This study explores the nature of banking failures in an economy in which currency is used as a medium of exchange.

Empirical Implications: Substantial evidence, such as that provided by Demirguc-Kunt and Detragiache (1998, 2005), Hardy and Pazarbasioglu (1999), Joyce (2011), Duttagupta and Cashin (2011), Davis et al. (2011), and Boyd et al. (2014) shows that banking crises and inflation are positively correlated. Gomis-Porqueras and Smith (2006) use data spanning the years 1890 to 1913 during the National Banking Era and find that the reserve-deposit ratio decreases and the probability of a banking crisis increases as the nominal interest rate increases. These results are consistent with the first two parts of Proposition 3. In addition, the other three results (in the case of low ξ) imply that the relationship between inflation and real economic activities (investment and output) involves a threshold effect. Bullard and Keating (1995), King and Watson (1997), Bruno and Easterly (1998), Ahmed and Rogers (2000), Crosby and Otto (2000), Khan and Senhadji (2001), and Kremer et al. (2013), among others, show that inflation has almost no (or a slightly positive) effect on real economic activity (e.g., investment, output, and growth) when the inflation rate is low but that its impact becomes significantly negative when the inflation rate exceeds a certain threshold. Boyd et al. (2001) and Khan et al. (2006) identify the inflation threshold in the relationship between inflation and financial market activities (i.e., bank lending activities and stock market development). Clearly, these findings also support our results.

Discount Window Lending: One may wonder if the central bank acting as the lender of last resort (LLR) could lead to a better outcome, even to achieve the first-best outcome, as actually shown in the literature of OLG models and banking panics, such as Antinolfi et al. (2001), Antinolfi and Keister (2006), and Haslag and Martin (2007). Our companion paper, Matsuoka and Watanabe (2019), introduces the discount window lending offered by the central bank in the following way.[2] After the realization of α, each bank determines the nominal amount $b \in [0, \bar{b}]$ that it would like to borrow from the discount window at a nominal interest rate $r > 1$, where $\bar{b}$ and r are set by the LLR. Discount window loans are a short-term (intraday) currency loans that are fully collateralized by a bank's asset values at the beginning of the CM. We can show that if the central bank can make large enough loans (i.e., $\bar{b}$ is sufficiently large) at a zero nominal interest rate (i.e., $r \to 1$), both consumption inequality and banking crises are eliminated. In this situation, if $\beta R = 1$, the first-best equilibrium allocation is achieved. However, if $\beta R < 1$, the Friedman rule is a necessary and sufficient condition for the first-best allocation. The intuition is as follows. When $R > \frac{1}{\pi}$, the liquidity support provided by the LLR is beneficial because the discount window loans are substitutes for reserves and banks can earn higher returns by increasing their capital investments. Then, if $\bar{b}$ is sufficiently large and r is sufficiently close to unity, a bank's reserves become zero but banking crises are eliminated; otherwise a crisis occurs with positive probability. When $R \le \frac{1}{\pi} < \frac{1}{\beta}$, money dominates capital, and banks invest their deposits only in cash reserves. In this case, no banking crises occur, and perfect risk sharing is achieved, implying that the LLR function is not necessary. However, banks still perceive the opportunity cost of holding reserves because the nominal interest rate is positive (i.e., $i \equiv (\pi - \beta)/\beta > 0$). Thus, the Friedman rule (i.e., $\pi \to \beta$) is necessary and sufficient for the first-best allocation.

[2] The main issue addressed in Matsuoka and Watanabe (2019) is a moral hazard problem associated with the LLR. We extend the model presented here by introducing a risky asset and a positive interest rate on discount window loans in order to examine the effect of the LLR on banks' incentives to take risks. We show that while the LLR function is welfare improving, it reduces the banks' ex ante incentive to hold reserves, which increases the probability of a crisis, and leads to higher risk-taking. We also show that high (penalty) lending rates do not have much impact on moral hazard under some conditions.

In our model, the discount window policy is not a necessary condition for the first-best allocation. This result is in contrast to the results of the OLG models. Antinolfi and Keister (2006) and Haslag and Martin (2007) show that the combination of the Friedman rule and extensive discount window lending leads to the first-best allocation. In their models, two policies are necessary and play the two different roles. The Friedman rule eliminates consumption inequality but transfers some resources to old agents who simply consume rather than investing. Thus, extensive discount window lending is necessary to use all of the economy's resources for productive investment. Our model does not have such intergenerational transfers, and, thus, the Friedman rule does all the job for efficiency.

5. Conclusion

We studied banks' liquidity provision using the Lagos and Wright model of monetary exchange. With aggregate uncertainty, we showed that banks sometimes exhaust their cash reserves, leading to banking crises, and fail to satisfy their depositors' needs for consumption smoothing. Banking crises can be eliminated through rate-of-return-equalizing policy, which equates the rates of return of money and capital, motivating banks to hold sufficient liquidity to ensure perfect risk sharing. However, the total amount of deposits is insufficient for the equilibrium allocation to be efficient. This distortion can be eliminated only at the Friedman rule. In our monetary equilibrium, the probability of a banking crisis is endogenously determined. We find that this probability is increasing in the rate of inflation, as is consistent with the empirical evidence. The model offers a rich array of non-trivial effects of inflation on equilibrium deposits and banks' portfolios.

The model is fairly tractable. We think that the following extensions would be interesting. First, in the presence of information frictions, liquidity support from the LLR would distort the banks' ex-ante investment decisions across assets with different returns and different liquidities, inducing them to invest in risky assets (i.e., creating moral hazard). It is important to investigate the optimal design of the LLR taking moral hazard into account. Second, although a perfect interbank market should allow banks to reallocate liquidity ex post and hedge liquidity risk completely, we could study whether an imperfect interbank market, such as that supported by repo contracts, reduces or facilitates banking crises. Third, it would be interesting to investigate the effect of a deposit insurance system on banks' portfolio choices and the level of deposits. This question could be addressed by introducing both a tax on deposits (i.e., an insurance premium) and liquidity injections to banks during crises. Finally, by assuming that capital can be scraped and that the scraped capital can be used in non-monitored exchanges with a haircut, we can investigate why some banking crises are associated with no output losses, whereas other crises cause serious recessions. We leave these questions for future research.

Appendix A

Proof of Theorem 1. Applying $m=\frac{\gamma d}{\phi}$, we can write $q^n=\frac{\theta_1\gamma d}{\alpha\pi}$ and $q^m=\frac{\theta_2\gamma d}{(1-\alpha)\pi}+c$. Applying $k=(1-\gamma)d$, we obtain the Lagrangian function,

$$\mathfrak{L}=\alpha u\left(\frac{\theta_1\gamma d}{\alpha\pi}\right)+(1-\alpha)u\left(\frac{\theta_2\gamma d}{(1-\alpha)\pi}+c\right)+(1+\mu_0)\left\{(1-\theta_1-\theta_2)\frac{\gamma d}{\pi}+R(1-\gamma)d-(1-\alpha)c\right\}$$
$$+\mu_1 c+\mu_2\theta_2+\mu_3(1-\theta_1-\theta_2),$$

where $\mu_0, \mu_1, \mu_2, \mu_3\geq 0$ are the Lagrangian multipliers. Note that $\theta_1=0$, which leads to $q^n=0$, cannot be a solution by the Inada condition. The first-order conditions are

$$\frac{\partial\mathfrak{L}}{\partial\theta_1}=\frac{\gamma d}{\pi}\{u'(q^n)-(1+\mu_0)\}-\mu_3=0, \tag{A.1}$$

$$\frac{\partial\mathfrak{L}}{\partial\theta_2}=\frac{\gamma d}{\pi}\{u'(q^m)-(1+\mu_0)\}+\mu_2-\mu_3=0, \tag{A.2}$$

$$\frac{\partial\mathfrak{L}}{\partial c}=(1-\alpha)\left[u'(q^m)-(1+\mu_0)\right]+\mu_1=0. \tag{A.3}$$

Region A: $\theta_1=1$, $\theta_2=0$, **and** $c<\frac{R(1-\gamma)d}{1-\alpha}$. In this case, $\mu_0=0$, and, because $\theta_1=1$, $c>0$ must hold, implying that $\mu_1=0$ ($c=0$, leading to $q^m=0$, cannot be a solution by the Inada condition). Thus, from (A.3), $q^m=q^*\equiv u^{-1\prime}(1)$. This result further implies that

$$c=q^*<\frac{R(1-\gamma)d}{1-\alpha}\iff\alpha>1-\frac{R(1-\gamma)d}{q^*}\equiv\alpha_{AB}.$$

On the other hand, using $\mu_2, \mu_3\geq 0$ into (A.1) and (A.2) yields $u'(q^n)\geq u'(q^m)\iff q^m=q^*\geq q^n=\frac{\gamma d}{\alpha\pi}\iff$

$$\alpha\geq\frac{\gamma d}{\pi q^*}\equiv\alpha_{AD}.$$

Region B: $\theta_1 = 1,\ \theta_2 = 0,$ **and** $c = \frac{R(1-\gamma)d}{1-\alpha}$. (A.1) and (A.2) imply that $u'(q^n) \ge u'(q^m) \Longleftrightarrow q^m \ge q^n \Longleftrightarrow$

$$\alpha \ge \frac{\gamma}{\gamma + (1-\gamma)R\pi} \equiv \alpha_{BC}.$$

Using (A.3) and $\mu_1 = 0$, we obtain $\alpha \le \alpha_{AB}$.
Region C: $\theta_1 < 1,\ \theta_2 = 1 - \theta_1$ **and** $c = \frac{R(1-\gamma)d}{1-\alpha}$. In this case, $\mu_1 = \mu_2 = 0$. By (A.1) and (A.2), we have $q^n = q^m$. Applying the expressions for q^n and q^m, we obtain

$$\theta_1 = \alpha\left(1 + \frac{1-\gamma}{\gamma}R\pi\right) < 1 \Longleftrightarrow \alpha < \alpha_{BC}.$$

(A.3) and $\mu_1 = 0$ imply that $u'(q^m) \ge 1 \Longleftrightarrow$

$$1 - \left(\frac{\gamma}{\pi} + (1-\gamma)R\right)\frac{d}{q^*} \ge \alpha\left\{1 - \left(\frac{\gamma}{\pi} + (1-\gamma)R\right)\frac{d}{q^*}\right\}.$$

When $R > \frac{1}{\pi}$, this inequality holds if

$$\gamma \ge \frac{\pi R - \frac{\pi q^*}{d}}{\pi R - 1} \equiv \gamma_{CD},$$

and when $R \le \frac{1}{\pi}$, it holds if $\gamma \le \gamma_{CD}$.
Region D: $\theta_1 < 1,\ \theta_2 \le 1 - \theta_1,$ **and** $0 \le c < \frac{R(1-\gamma)d}{1-\alpha}$. There are two possible cases. Consider first the case of $\theta_2 = 1 - \theta_1$, and $0 < c < \frac{R(1-\gamma)d}{1-\alpha}$. In this case, $\mu_0 = \mu_2 = 0$. By (A.1) and (A.2), we have $q^n = q^m$. For $c > 0$ ($\Longleftrightarrow \mu_1 = 0$), (A.3) implies that $q^m = q^*$, leading to $q^n = q^* \Longleftrightarrow$

$$\theta_1 = \frac{\alpha\pi q^*}{\gamma d}.$$

Thus, $\theta_1 < 1 \Longleftrightarrow \alpha < \alpha_{AD}$. Applying this expression to $q^m = q^*$, we obtain

$$c = \frac{q^* - \frac{\gamma d}{\pi}}{1-\alpha}.$$

We therefore have $c < \frac{R(1-\gamma)d}{1-\alpha} \Longleftrightarrow \gamma < \gamma_{CD}$ if $R > \frac{1}{\pi}$ and $\gamma > \gamma_{CD}$ if $R \le \frac{1}{\pi}$. In addition, $c > 0 \Longleftrightarrow \gamma < \frac{\pi q^*}{d} \equiv \gamma_0$.

Consider the next case of $\theta_2 \le 1 - \theta_1$, and $c = 0$. In this case, we again obtain $q^n = q^m = q^*$, which yields

$$\theta_1 = \frac{\alpha\pi q^*}{\gamma d} \quad \text{and} \quad \theta_2 = \frac{(1-\alpha)\pi q^*}{d\gamma}.$$

Thus, we obtain $\theta_1 < 1 \Longleftrightarrow \alpha < \alpha_{AD}$ and $\theta_2 \le 1 - \theta_1 \Longleftrightarrow \gamma \ge \gamma_0$. This case therefore holds when $\alpha < \min\{\alpha_{AD},\ 1\}$ and $\gamma < \min\{\gamma_{CD},\ 1\}$ if $R > \frac{1}{\pi}$ and $\gamma \in (\gamma_{CD},\ 1]$ if $R \le \frac{1}{\pi}$. The above discussion covers all possible cases.

Finally, we check buyers' incentive constraints. Because $q^n \le q^m$, monitored buyers have no incentive to misrepresent their type. Remember that a monitored buyer receives $\frac{\theta_2\gamma d}{(1-\alpha)\pi}$ in money and c in capital claims. If a non-monitored buyer claims to be a monitored buyer, he can use the monetary payment in the DM but the claims are completely useless. Thus, the incentive constraint of non-monitored buyers is

$$u(q^n) \ge u\left(\frac{\theta_2\gamma d}{(1-\alpha)\pi}\right), \tag{A.4}$$

where the bank optimally holds on allocating capital $\frac{k}{1-\alpha}$ if the deviator does not use the allocated claim in the DM. In Regions A and B, $\theta_1 = 1$, and, thus, (A.4) is satisfied. In Region C, $q^n = q^m = d(\frac{\gamma}{\pi} + R(1-\gamma))$, implying that $q^n > \frac{\theta_2\gamma d}{(1-\alpha)\pi}$ for any $c > 0$. Finally, in Regions D and E, the above argument can be applied, because $q^n = q^m = q^*$. Thus, we have shown that non-monitored buyers have no incentive to misrepresent their type, and the proof of Theorem 1 is complete. □

Proof of Theorem 2. We first consider the case of $R > \frac{1}{\pi}$.

⊙ Case $d \le \pi q^*$. For $\gamma \ge \gamma_{CD}$, the bank's problem can be written as

$$\max_{\gamma\in[\gamma_{CD},1]} \int_{\alpha_{AB}}^{1}\left[\alpha u\left(\frac{\gamma d}{\alpha\pi}\right) + (1-\alpha)u(q^*) - (1-\alpha)q^*\right]f(\alpha)d\alpha$$
$$+ \int_{\alpha_{BC}}^{\alpha_{AB}}\left[\alpha u\left(\frac{\gamma d}{\alpha\pi}\right) + (1-\alpha)u\left(\frac{R(1-\gamma)d}{1-\alpha}\right)\right]f(\alpha)d\alpha$$
$$+ \int_{0}^{\alpha_{BC}} u\left(d\left(\frac{\gamma}{\pi} + (1-\gamma)R\right)\right)f(\alpha)d\alpha + (1-F(\alpha_{AB}))R(1-\gamma)d.$$

The first-order condition is given by

$$\Phi(\gamma, d, \pi) \equiv \int_{\alpha_{BC}}^{1} \frac{d}{\pi} u'\left(\frac{\gamma d}{\alpha\pi}\right) f(\alpha)d\alpha - \int_{\alpha_{BC}}^{\alpha_{AB}} Rdu'\left(\frac{R(1-\gamma)d}{1-\alpha}\right) f(\alpha)d\alpha$$
$$- d\left(R - \frac{1}{\pi}\right)u'\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right)F(\alpha_{BC}) - Rd(1 - F(\alpha_{AB})) = 0. \tag{A.5}$$

Observe that $\Phi(1, \cdot) = -d(R - \frac{1}{\pi})u'(\frac{d}{\pi}) < 0$ and that the second order condition is satisfied:

$$\frac{\partial\Phi(\gamma, \cdot)}{\partial\gamma} = \int_{\alpha_{BC}}^{1} \frac{d^2}{\alpha\pi^2} u''\left(\frac{\gamma d}{\alpha\pi}\right) f(\alpha)d\alpha + \int_{\alpha_{BC}}^{\alpha_{AB}} \frac{(Rd)^2}{1-\alpha} u''\left(\frac{R(1-\gamma)d}{1-\alpha}\right) f(\alpha)d\alpha$$
$$+ d^2\left(R - \frac{1}{\pi}\right)^2 u''\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right)F(\alpha_{BC}) < 0.$$

Thus, the solution to (A.5) must be some unique $\gamma(d) \in [\gamma_{DC}, 1)$ (if it exists).

For $\gamma \le \min\{\gamma_{CD}, \gamma_0\} = \gamma_{CD}$ (<1), the bank's problem can be written as

$$\max_{\gamma\in[0,\gamma_{CD}]} \int_{\alpha_{AD}}^{1} \left[\alpha u\left(\frac{\gamma d}{\alpha\pi}\right) + (1-\alpha)(u(q^*) - q^*)\right] f(\alpha)d\alpha + \int_{0}^{\alpha_{AD}} \left[u(q^*) - \left(q^* - \frac{\gamma d}{\pi}\right)\right] f(\alpha)d\alpha + R(1-\gamma)d.$$

Differentiating with respect to γ yields the first-order condition

$$\Psi(\gamma, d, \pi) \equiv \int_{\alpha_{AD}}^{1} \frac{d}{\pi} u'\left(\frac{\gamma d}{\alpha\pi}\right) f(\alpha)d\alpha - d\left(R - \frac{F(\alpha_{AD})}{\pi}\right) = 0. \tag{A.6}$$

Note that $\Psi(0, \cdot) > 0$, implying that $\gamma = 0$ is never optimal. The second order condition,

$$\frac{\partial\Psi}{\partial\gamma} = \int_{\alpha_{AD}}^{1} \frac{d^2}{\alpha\pi^2} u''(q^n) f(\alpha)d\alpha < 0,$$

is satisfied. Thus, a solution to (A.6) has to be some unique $\gamma(d) \in (0, \gamma_{DC}]$ (if it exists).

To combine the above two subcases, observe that for $d \le \frac{q^*}{R}$, $\gamma_{CD} \equiv \frac{\pi R - \frac{\pi q^*}{d}}{\pi R - 1} \le 0$, and, thus, the solution is determined by (A.5). For $d > \frac{q^*}{R}$, at $\gamma = \gamma_{CD} > 0$ it holds that

$$\alpha_{AB} = \alpha_{BC} = \alpha_{AD} = \frac{Rd - q^*}{q^*(\pi R - 1)} \equiv \hat{\alpha}(d) \in [0, 1],$$

that is, all three critical values of α coincide, and, thus, the bank's two objective functions and their derivatives must coincide. The bank's objective function is therefore continuous and differentiable for $\gamma \in [0, 1]$, and it holds that, for $d \le \pi q^*$, there exists a unique solution $\gamma(d) \in (0, 1)$ determined either by (A.5) or by (A.6).

⊙ Case $d > \pi q^*$. In this case, the critical values defined in Theorem 1 satisfy $\alpha_{AB} < \alpha_{BC}$ and $\gamma_{CD} > \gamma_0$. Further, for $\gamma \ge \gamma_0 \equiv \frac{\pi q^*}{d}$, $\alpha_{AD} \ge 1$ must hold. Thus, by setting $\gamma(d) = \gamma_0$ the bank can guarantee that the efficient level of risk-sharing is achieved, that is, $q^n = q^m = q^* = \phi_+ m$ with $c = 0$. However, this result cannot be a solution. For $\gamma \le \min\{\gamma_{CD}, \gamma_0\} = \gamma_0$ (<1), the bank's problem is identical to the above problem for $\gamma \le \min\{\gamma_{CD}, \gamma_0\} = \gamma_{CD}$ when $d \le \pi q^*$, and the first-order condition is given by (A.6). Because $\Psi(\gamma_0, d, \pi) = -d\left(R - \frac{1}{\pi}\right) < 0$, the solution must be some unique $\gamma(d) \in (0, \gamma_0]$ (if it exists).

Thus, for a given $d > 0$, there exists a unique solution $\gamma(d) \in (0, 1)$ determined either by (A.5) or by (A.6). The first part of Theorem 2 is proven.

⊙ The dependence of $\gamma(d)$ on d.
Differentiation yields

$$\frac{\partial\Psi}{\partial d} = \int_{\alpha_{AD}}^{1} \frac{\gamma d}{\alpha\pi^2} u''\left(\frac{\gamma d}{\alpha\pi}\right) f(\alpha)d\alpha < 0$$

$$\frac{\partial\Phi}{\partial d} = (1-\xi)\int_{\alpha_{BC}}^{1} \frac{d}{\pi} u'\left(\frac{\gamma d}{\alpha\pi}\right) f(\alpha)d\alpha - (1-\xi)\int_{\alpha_{BC}}^{\alpha_{AB}} Rdu'\left(\frac{R(1-\gamma)d}{1-\alpha}\right) f(\alpha)d\alpha$$
$$- (1-\xi)d\left(R - \frac{1}{\pi}\right)u'\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right)F(\alpha_{BC}) - R(1 - F(\alpha_{AB}))$$
$$= -\xi R(1 - F(\alpha_{AB})) < 0,$$

where $\xi \equiv -\frac{qu''(q)}{u'(q)} > 0$ is a positive constant, and in the last equality, we apply (A.5). Thus, because $\frac{\partial\Psi}{\partial\gamma}, \frac{\partial\Phi}{\partial\gamma} < 0$, we obtain $\frac{\partial\gamma(d)}{\partial d} < 0$.

Finally, we consider the case of $R \le \frac{1}{\pi}$.

⊙ Case $d \le \pi q^*$. The bank's problem is the same as in the case of $R > \frac{1}{\pi}$, and the first-order condition is given by (A.5). Because $\Phi(1, \cdot) = -d(R - \frac{1}{\pi})u'(\frac{d}{\pi}) \ge 0$, it is optimal for the bank to set $\gamma(d) = 1$.

⊙ Case $d > \pi q^*$. For $\gamma > \gamma_{CD}$, the first order condition of the bank's problem is given by (A.6). From this condition, we obtain that $\Psi(\gamma_0, \cdot) = -d(R - \frac{1}{\pi}) \geq 0$, showing that the bank offers the efficient consumption plan, $q^n = q^m = q^*$. For $\gamma \in [\gamma_0, 1]$, the objective function becomes

$$\max_{\gamma \in [\gamma_0, 1]} \; u(q^*) - q^* + \frac{\gamma d}{\pi} + R(1-\gamma)d,$$

and the solution is $\gamma(d) = 1$, completing the proof of Theorem 2. □

Proof of Proposition 1. We first consider the case of $\frac{1}{\pi} < R < \frac{1}{\beta}$ and start by determining which of the conditions, (A.5) or (A.6), leads to the optimal $\gamma(d)$ depending on the values of $d \leq \pi q^*$. Recall from the proof of Theorem 2 that for $d \leq \frac{q^*}{R}$, we obtain $\gamma_{CD} \equiv \frac{\pi R - \frac{\pi q^*}{d}}{\pi R - 1} \leq 0$, and, thus, the solution is determined by (A.5). For $d \geq \frac{q^*}{R}$, at $\gamma = \gamma_{CD} > 0$ ($\Longleftrightarrow d \geq \frac{q^*}{R}$), all three critical values of α coincide and take the value $\hat{\alpha}(d) \equiv \frac{Rd - q^*}{q^*(\pi R - 1)} \in [0, 1]$, and the conditions (A.6) and (A.5) become $\Psi(\gamma_{CD}; \cdot) = \Phi(\gamma_{CD}; \cdot) = \frac{d}{\pi}\Upsilon(\hat{\alpha})$, where

$$\Upsilon(\hat{\alpha}) \equiv \int_{\hat{\alpha}}^{1} u'\left(\frac{\hat{\alpha} q^*}{\alpha}\right) f(\alpha) d\alpha + F(\hat{\alpha}) - R\pi.$$

Observe that $\Upsilon(0) = +\infty > 0$; $\Upsilon(1) = 1 - R\pi < 0$; $\Upsilon'(\cdot) = \int_{\hat{\alpha}}^{1} \frac{q^*}{\alpha} u''\left(\frac{\hat{\alpha} q^*}{\alpha}\right) f(\alpha) d\alpha < 0$. Thus, there exists a $\tilde{d} \in (\frac{q^*}{R}, \pi q^*)$ such that $\Upsilon(\hat{\alpha}(\tilde{d})) = 0$, $\Upsilon(\hat{\alpha}(d)) > 0$ if $d < \tilde{d}$, and $\Upsilon(\hat{\alpha}(d)) < 0$ if $d > \tilde{d}$. Because $\gamma(d)$ is strictly decreasing in d, it follows that $\gamma(d) = \gamma_{CD}$ if $d = \tilde{d}$, $\gamma(d) > \gamma_{CD}$ if $d < \tilde{d}$, and $\gamma(d) < \gamma_{CD}$ if $d > \tilde{d}$.

To summarize the analysis so far,

- for $d \in (0, \tilde{d}]$, $\gamma(d) \in [\gamma_{CD}, 1)$ is determined by (A.5), and its associated value $\tilde{V}(d \mid \gamma(d))$ is given by (5),
- for $d \in (\tilde{d}, \pi q^*]$, $\gamma(d) \in (0, \gamma_{CD})$ is determined by (A.6), and its associated value $\tilde{V}(d \mid \gamma(d))$ is given by (6),
- for $d \in (\pi q^*, \infty)$, $\gamma(d) \in (0, \gamma_0)$ is determined by (A.6), and its associated value $\tilde{V}(d \mid \gamma(d))$ is given by (6).

We now compute the derivative of the value function. For $d \in (0, \tilde{d}]$, the differentiation of (5) yields

$$\begin{aligned}\tilde{V}'(d \mid \gamma(d)) = &\int_{\alpha_{BC}}^{1} \frac{\gamma}{\pi} u'\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha + \int_{\alpha_{BC}}^{\alpha_{AB}} R(1-\gamma) u'\left(\frac{R(1-\gamma)d}{1-\alpha}\right) f(\alpha) d\alpha \\ &+ \left(\frac{\gamma}{\pi} + R(1-\gamma)\right) u'\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC}) + R(1-\gamma)(1 - F(\alpha_{AB})) \\ = &\frac{1}{\pi}\left[\int_{\alpha_{BC}}^{1} u'\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha + u'\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC})\right],\end{aligned} \tag{A.7}$$

with $\gamma = \gamma(d)$, where the second equality follows from (A.5). $\tilde{V}'(d \mid \gamma(d)) = R$ is satisfied if $d = \tilde{d}$. For $d \in (\tilde{d}, \infty)$, the differentiation of (6) yields

$$\tilde{V}'(d \mid \gamma(d)) = \int_{\alpha_{BC}}^{1} \frac{\gamma}{\pi} u'\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha + \frac{\gamma}{\pi} F(\alpha_{AD}) + R(1-\gamma) = R$$

with $\gamma = \gamma(d)$, where the second equality follows from (A.6).

We now derive the monetary equilibrium with bank deposits. The equilibrium deposit $d > 0$ must satisfy the optimality condition,

$$1 = \beta V'(d).$$

If $\beta R < 1$, $V'(d) > R$ must hold. The only possibility is $d < \tilde{d}$. Using (A.7), we can write the fixed-point condition of the equilibrium as

$$\frac{\pi}{\beta} = \int_{\alpha_{BC}}^{1} u'\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha + u'\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC}) \equiv \Omega(d), \tag{A.8}$$

with $\gamma = \gamma(d)$. Observe that: $\Omega(0) = +\infty > \frac{\pi}{\beta}$; $\Omega(\tilde{d}) = \int_{\hat{\alpha}}^{1} u'\left(\frac{\hat{\alpha} q^*}{\alpha}\right) f(\alpha) d\alpha + F(\hat{\alpha}) = \pi R < \frac{\pi}{\beta}$ (the second equality follows from (A.5));

$$\begin{aligned}\Omega'(d) = &\int_{\alpha_{BC}}^{1} \frac{\gamma}{\alpha \pi} u''\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha + \left(\frac{\gamma}{\pi} + R(1-\gamma)\right) u''\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC}) \\ &+ \frac{\partial \gamma}{\partial d}\left[\int_{\alpha_{BC}}^{1} \frac{d}{\alpha \pi} u''\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha - d\left(R - \frac{1}{\pi}\right) u''\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC})\right].\end{aligned}$$

Because $\frac{\partial \gamma}{\partial d} < 0$, if

$$\gamma + d\frac{\partial \gamma}{\partial d} > 0 \tag{A.9}$$

holds, then $\Omega'(d) < 0$. This result can be shown by the following steps. First, we can show that (A.9)$\Longleftrightarrow \frac{\frac{\partial \Phi}{\partial d}}{\frac{\partial \Phi}{\partial \gamma}} < \frac{\gamma}{d} \Longleftrightarrow \xi R(1 - F(\alpha_{AB})) < -\frac{\gamma}{d}\frac{\partial \Phi}{\partial \gamma}$ (see the proof of Theorem 2 for these derivative expressions). Second, using the expression $\xi \equiv -\frac{qu''(q)}{u'(q)}$, we can write

$$\begin{aligned}\frac{\partial \Phi}{\partial \gamma} &= \int_{\alpha_{BC}}^{1} \frac{d^2}{\alpha \pi^2} u''\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha + \int_{\alpha_{BC}}^{\alpha_{AB}} \frac{(Rd)^2}{1-\alpha} u''\left(\frac{R(1-\gamma)d}{1-\alpha}\right) f(\alpha) d\alpha + d^2 (R - \frac{1}{\pi})^2 u'' \\ &\quad \times \left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC}) \\ &= -\xi\left[\int_{\alpha_{BC}}^{1} \frac{d}{\pi \gamma} u'\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha + \int_{\alpha_{BC}}^{\alpha_{AB}} \frac{Rd}{1-\gamma} u'\left(\frac{R(1-\gamma)d}{1-\alpha}\right) f(\alpha) d\alpha + \frac{d(R-\frac{1}{\pi})^2}{\frac{\gamma}{\pi} + R(1-\gamma)} u' \right. \\ &\quad \left. \times \left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC})\right].\end{aligned}$$

Finally, using the last expression, we obtain

$$\begin{aligned}\pi R(1 - F(\alpha_{AB})) &< \int_{\alpha_{BC}}^{1} u'\left(\frac{\gamma d}{\alpha \pi}\right) f(\alpha) d\alpha + \int_{\alpha_{BC}}^{\alpha_{AB}} \frac{\pi R \gamma}{1-\gamma} u'\left(\frac{R(1-\gamma)d}{1-\alpha}\right) f(\alpha) d\alpha \\ &\quad + \frac{\pi \gamma (R - \frac{1}{\pi})^2}{\frac{\gamma}{\pi} + R(1-\gamma)} u'\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC})\end{aligned}$$

$\Longleftrightarrow$

$$-\int_{\alpha_{BC}}^{\alpha_{AB}} \frac{\pi R}{1-\gamma} u'\left(\frac{R(1-\gamma)d}{1-\alpha}\right) f(\alpha) d\alpha < \frac{R\left(R - \frac{1}{\pi}\right)}{\frac{\gamma}{\pi} + R(1-\gamma)} u'\left(d\left(\frac{\gamma}{\pi} + R(1-\gamma)\right)\right) F(\alpha_{BC}),$$

which is true (we use (A.5) to obtain the last expression). We therefore have shown that there exists a unique $d \in (0, \tilde{d})$ such that the fixed point condition (A.8) is satisfied. Given this solution, $\gamma(d) \in (\gamma_{CD}, 1)$ is determined by (A.5), and $(q^n, q^m) \leq (q^*, q^*)$ is determined as described in Theorem 1 (Region A, B, or C). This result satisfies all the equilibrium conditions and, thus, describes a monetary equilibrium with bank deposits.

Next consider the case of $R \leq \frac{1}{\pi}$. Similarly, we know that

- for $d \in (0, \pi q^*]$, $\gamma(d) = 1$, and its associated value $V(d)$ is given by

$$V(d) = u\left(\frac{d}{\pi}\right) + W(0, 0, 0),$$

- for $d \in (\pi q^*, \infty]$, $\gamma(d) = 1$, and its associated value $V(d)$ is given by

$$V(d) = u(q^*) + \frac{1}{\pi}(d - \pi q^*) + W(0, 0, 0).$$

Because $\beta V'(d) = \frac{\beta}{\pi} < 1$ for $d \in (\pi q^*, \infty]$, we obtain an interior solution $d^{**} \in (0, \pi q^*]$ that satisfies

$$\beta V'(d^{**}) = \frac{\beta}{\pi} u'\left(\frac{d^{**}}{\pi}\right) = 1. \tag{A.10}$$

In a monetary equilibrium with $d^{**} \in (0, \pi q^*]$, we obtain $q^n = q^m = \frac{d^{**}}{\pi} < q^*$ in Region C.

Finally, if $\beta R = 1$, $V'(d) = R$ must hold, which is the case if and only if $d \geq \tilde{d}$. Thus, a monetary equilibrium with bank deposit exists, but it is indeterminate for any $d \in [\tilde{d}, \infty)$. In equilibria with $d \in [\tilde{d}, \infty)$, we have $\gamma(d) \in (0, \min\{\gamma_{CD}, 1\})$ is determined by (A.6) and $(q^n, q^m) \leq (q^*, q^*)$ in Region A or D. The proof of Proposition 1 is complete. □

Proof of Proposition 2. Suppose that $\beta R < 1$. The equilibrium $\gamma(d) \in (\gamma_{CD}, 1)$ and $d \in (0, \tilde{d})$, with $\tilde{d} \in (\frac{\pi}{R}, \pi q^*)$, are jointly determined by (A.5) and (A.8).

$\odot$ The limit as $\pi \to \frac{1}{R} > \beta$. For any value of $\gamma = \gamma(d)$, it must hold that

$$\alpha_{BC} \equiv \frac{\gamma}{\gamma + (1-\gamma)\pi R} \to 1 \quad \text{as} \quad \pi \to \frac{1}{R}.$$

Furthermore,

$$\alpha_{AB} \equiv 1 - \frac{R(1-\gamma)d}{q^*} \to 1 \quad \text{as} \quad \pi \to \frac{1}{R}$$

is the only solution to (A.5), which implies that

$$\gamma = \gamma(d) \to 1 \quad \text{as} \quad \pi \to \frac{1}{R}.$$

Finally, given all these limiting values, (A.8) becomes

$$\frac{1}{\beta R} = u'(dR) \quad \text{as} \quad \pi \to \frac{1}{R}, \tag{A.11}$$

which holds true if and only if

$$d < \frac{q^*}{R} \quad \text{as} \quad \pi \to \frac{1}{R}.$$

$\odot$ The limit as $\pi \to \frac{1}{R}$ and $\pi \to \beta$. The outcome is the same as that when $\pi \to \frac{1}{R} > \beta$, except that by (A.11),

$$d \to \pi q^* \quad \text{as} \quad \pi \to \frac{1}{R} \text{ and } \pi \to \beta.$$

Combining the above two limiting results proves the first claim.

Next, suppose that $\beta R = 1$. In this case, the equilibrium $\gamma(d) \in (0, \min\{\gamma_{CD}, 1\})$ is determined by (A.6) and $d > \tilde{d}$ is indeterminate.

$\odot$ The limit as $\pi \to \frac{1}{R} = \beta$. For any value of $\gamma = \gamma(d)$, it must hold that

$$\alpha_{AD} \equiv \frac{\gamma d}{\pi q^*} \to 1 \quad \text{as} \quad \pi \to \frac{1}{R} = \beta,$$

from (A.6), implying that

$$\gamma = \gamma(d) \to \gamma_0 \text{ and } q^n \to q^* \quad \text{as} \quad \pi \to \frac{1}{R} = \beta.$$

The second claim and Proposition 2 are therefore proven. □

Proof of Proposition 3.. Recall that for a given d and π, (A.5) determines $\gamma = \gamma(d, \pi) \in (\gamma_{CD}, 1)$ satisfying

$$\Phi(\gamma, d, \pi) = 0.$$

To reflect the dependence of the equilibrium on π, the fixed-point condition (A.7) can be written as

$$\Theta(\gamma, d, \pi) \equiv \Omega(\gamma, d, \pi) - \frac{\pi}{\beta} = 0.$$

These implicit equations determine two continuous and differentiable functions $d = d(\pi)$ and $\gamma = \gamma(\pi)$, which are characterized as follows:

$$\begin{aligned} \begin{pmatrix} \frac{\partial \gamma(\pi)}{\partial \pi} \\ \frac{\partial d(\pi)}{\partial \pi} \end{pmatrix} &= -\begin{pmatrix} \frac{\partial \Phi}{\partial \gamma} & \frac{\partial \Phi}{\partial d} \\ \frac{\partial \Theta}{\partial \gamma} & \frac{\partial \Theta}{\partial d} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial \Phi}{\partial \pi} \\ \frac{\partial \Theta}{\partial \pi} \end{pmatrix} \\ &= -\frac{1}{\Lambda} \begin{pmatrix} \frac{\partial \Theta}{\partial d} & -\frac{\partial \Phi}{\partial d} \\ -\frac{\partial \Theta}{\partial \gamma} & \frac{\partial \Phi}{\partial \gamma} \end{pmatrix} \begin{pmatrix} \frac{\partial \Phi}{\partial \pi} \\ \frac{\partial \Theta}{\partial \pi} \end{pmatrix} \end{aligned} \tag{A.12}$$

where

$$\Lambda \equiv \det \begin{pmatrix} \frac{\partial \Phi}{\partial \gamma} & \frac{\partial \Phi}{\partial d} \\ \frac{\partial \Theta}{\partial \gamma} & \frac{\partial \Theta}{\partial d} \end{pmatrix} = \frac{\partial \Phi}{\partial \gamma}\frac{\partial \Theta}{\partial d} - \frac{\partial \Phi}{\partial d}\frac{\partial \Theta}{\partial \gamma} = \frac{\partial \Phi}{\partial \gamma}\left(\frac{\partial \Theta}{\partial d} + \frac{\partial \gamma}{\partial d}\frac{\partial \Theta}{\partial \gamma}\right) > 0,$$

because $\frac{\partial \Phi}{\partial \gamma} < 0$ (see the proof of Theorem 2) and $\frac{\partial \Theta}{\partial d} + \frac{\partial \gamma}{\partial d}\frac{\partial \Theta}{\partial \gamma} = \Omega'(\cdot) < 0$ (see the proof of Proposition 1).

$\odot$ The effect on $\gamma(\pi)$ and $1 - F(\alpha_{BC})$. Now, from (A.12), we obtain

$$\Lambda \frac{\partial \gamma(\pi)}{\partial \pi} = -\frac{\partial \Theta}{\partial d}\frac{\partial \Phi}{\partial \pi} + \frac{\partial \Phi}{\partial d}\frac{\partial \Theta}{\partial \pi}. \tag{A.13}$$

Partial differentiation in (A.7) yields

$$\begin{aligned}\frac{\partial\Theta}{\partial d} &= \int_{\alpha_{BC}}^{1}\frac{\gamma}{\alpha\pi}u''\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha + \left(\frac{\gamma}{\pi}+R(1-\gamma)\right)u''\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})\\ &= -\frac{\xi}{d}\left[\int_{\alpha_{BC}}^{1}u'\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha + u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})\right]\\ &= -\frac{\xi}{d}\frac{\pi}{\beta},\end{aligned}$$

where we apply $\xi \equiv -\frac{u''(q)q}{u'(q)}$ in the second equality and (A.7) in the third equality. Similarly,

$$\begin{aligned}\frac{\partial\Theta}{\partial\pi} &= -\int_{\alpha_{BC}}^{1}\frac{\gamma d}{\alpha\pi^2}u''\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha - \frac{d\gamma}{\pi^2}u''\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC}) - \frac{1}{\beta}\\ &= \frac{\xi}{\pi}\left[\int_{\alpha_{BC}}^{1}u'\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha + \frac{\gamma}{\gamma+(1-\gamma)\pi R}u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})\right] - \frac{1}{\beta}\\ &= -\frac{1-\xi}{\beta} - \frac{\xi(1-\gamma)R}{\gamma+(1-\gamma)\pi R}u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC}).\end{aligned}$$

We know from the proof of Theorem 2 that $\frac{\partial\Phi}{\partial d} = -\xi R(1-F(\alpha_{AB}))$. In addition,

$$\begin{aligned}\frac{\partial\Phi}{\partial\pi} &= -\int_{\alpha_{BC}}^{1}\frac{d(1-\xi)}{\pi^2}u'\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha - \frac{d}{\pi}\left(\frac{\xi\gamma(R-\frac{1}{\pi})}{\gamma+(1-\gamma)\pi R}+\frac{1}{\pi}\right)u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})\\ &= -\frac{d(1-\xi)}{\pi\beta} - \frac{d}{\pi}\frac{\xi R}{\gamma+(1-\gamma)\pi R}u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC}).\end{aligned}$$

Applying these expression to (A.13), we get

$$\begin{aligned}\Lambda\frac{\partial\gamma(\pi)}{\partial\pi} &= -\frac{\partial\Theta}{\partial d}\frac{\partial\Phi}{\partial\pi} + \frac{\partial\Phi}{\partial d}\frac{\partial\Theta}{\partial\pi}\\ &= -\frac{\xi(1-\xi)}{\beta}\left(\frac{1}{\beta}-R(1-F(\alpha_{AB}))\right) - \frac{\xi^2 R}{\gamma+(1-\gamma)\pi R}\left(\frac{1}{\beta}-R(1-F(\alpha_{AB}))(1-\gamma)\right)u'\\ &\quad\times\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC}) < 0\end{aligned}$$

for all $\xi \in (0, 1)$, proving Claim 1. This result also proves Claim 2 because it leads to a decreasing critical value of banking crises $\alpha_{BC} = \frac{\gamma}{\gamma+(1-\gamma)\pi R}$ and an increasing probability of banking crises, $1 - F(\alpha_{BC})$.

$\odot$ The effect on $d(\pi)$. Now, from (A.12), we obtain

$$\Lambda\frac{\partial d(\pi)}{\partial\pi} = \frac{\partial\Theta}{\partial\gamma}\frac{\partial\Phi}{\partial\pi} - \frac{\partial\Phi}{\partial\gamma}\frac{\partial\Theta}{\partial\pi}. \tag{A.14}$$

Partial differentiation in (A.8) yields

$$\begin{aligned}\frac{\partial\Theta}{\partial\gamma} &= \int_{\alpha_{BC}}^{1}\frac{d}{\alpha\pi}u''\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha - d\left(R-\frac{1}{\pi}\right)u''\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})\\ &= -\frac{\xi}{\gamma}\left[\int_{\alpha_{BC}}^{1}u'\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha - \frac{(R\pi-1)\gamma}{\gamma+(1-\gamma)\pi R}u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})\right]\\ &= -\frac{\xi}{\gamma}\left[\frac{\pi}{\beta} - \frac{\pi R}{\gamma+(1-\gamma)\pi R}u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})\right].\end{aligned}$$

The derivative expression from the proof of Proposition 1 can be rearranged as

$$\begin{aligned}\frac{\partial\Phi}{\partial\gamma} &= -\xi\Bigg[\int_{\alpha_{BC}}^{1}\frac{d}{\pi\gamma}u'\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha + \int_{\alpha_{BC}}^{\alpha_{AB}}\frac{Rd}{1-\gamma}u'\left(\frac{R(1-\gamma)d}{1-\alpha}\right)f(\alpha)d\alpha\\ &\quad+\frac{d(R-\frac{1}{\pi})^2}{\frac{\gamma}{\pi}+R(1-\gamma)}u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})\Bigg]\\ &= -\frac{\xi d}{1-\gamma}\Bigg[\int_{\alpha_{BC}}^{1}\frac{1}{\pi\gamma}u'\left(\frac{\gamma d}{\alpha\pi}\right)f(\alpha)d\alpha\\ &\quad-\frac{R-\frac{1}{\pi}}{\gamma+(1-\gamma)\pi R}u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC}) - R(1-F(\alpha_{AB}))\Bigg]\end{aligned}$$

$$= -\frac{\xi d}{\gamma(1-\gamma)}\left[\frac{1}{\beta} - \frac{R}{\gamma+(1-\gamma)\pi R}u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC}) - R\gamma(1-F(\alpha_{AB}))\right],$$

where we apply (A.5) in the second equality and (A.8) in the third equality. Applying these derivative expressions, we obtain

$$\gamma(d\xi)^{-1}\Lambda\frac{\partial d(\pi)}{\partial\pi}$$
$$= \left[\frac{1}{\beta} - \frac{Ru'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})}{\gamma+(1-\gamma)\pi R}\right]\left[\frac{1-\xi}{\beta} + \frac{\xi Ru'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})}{\gamma+(1-\gamma)\pi R}\right]$$
$$-\left[\frac{1}{\beta} - \frac{Ru'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})}{\gamma+(1-\gamma)\pi R} - R\gamma(1-F(\alpha_{AB}))\right]\left[\frac{1-\xi}{\beta(1-\gamma)} + \frac{\xi Ru'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})}{\gamma+(1-\gamma)\pi R}\right].$$

Observe that this equation is linear in ξ. As $\xi \to 1$, this equation approaches

$$R\gamma(1-F(\alpha_{AB}))\frac{Ru'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})}{\gamma+(1-\gamma)\pi R} > 0,$$

and as $\xi \to 0$, this equation approaches

$$-\left[\frac{1}{\beta} - \frac{Ru'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})}{\gamma+(1-\gamma)\pi R} - R(1-F(\alpha_{AB}))\right]\frac{\gamma}{\beta(1-\gamma)} < 0,$$

because

$$1 - \frac{R\beta u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)F(\alpha_{BC})}{\gamma+(1-\gamma)\pi R} - R\beta(1-F(\alpha_{AB})) > F(\alpha_{AB}) - F(\alpha_{BC})\frac{u'\left(d\left(\frac{\gamma}{\pi}+R(1-\gamma)\right)\right)}{\gamma+(1-\gamma)\pi R}$$
$$> F(\alpha_{AB}) - F(\alpha_{BC}) > 0,$$

where the last inequality follows from $u'(q) \to u'(q^*) = 1$ for any $q \in (0, q^*)$ as $\xi \to 0$. Thus, there exists a unique cutoff value $\hat{\xi} \in (0, 1)$ such that $\frac{\partial d(\pi)}{\partial\pi} < 0$ for $\xi < \hat{\xi}$ and $\frac{\partial d(\pi)}{\partial\pi} > 0$ for $\xi > \hat{\xi}$, proving Claim 3.

⊙ The effect on $k = d(1-\gamma)$. Observe that

$$\frac{\partial k}{\partial\pi} = \frac{\partial d}{\partial\pi}(1-\gamma) - d\frac{\partial\gamma}{\partial\pi}.$$

Thus, because $\frac{\partial\gamma}{\partial\pi} < 0$, when ξ is sufficiently high that $\frac{\partial d}{\partial\pi} > 0$, we obtain $\frac{\partial k}{\partial\pi} > 0$. When ξ is sufficiently low that $\frac{\partial d}{\partial\pi} < 0$, the final effect depends on the rate of inflation π. When π is low (around the limit $\pi \to \frac{1}{R}$), we find that $\gamma \to 1$ (which implies $k \to 0$) and, thus, that $\frac{\partial k}{\partial\pi} > 0$.

To investigate the effect for high inflation, observe that as $\pi \to \infty$,

$$\alpha_{BC} \equiv \frac{\gamma}{\gamma+(1-\gamma)\pi R} \to 0 \quad \text{as} \quad \pi \to \infty.$$

Further, we observe that $\gamma \to 0$ and/or $d \to 0$ is the only possible solution to (A.5) and (A.8). If $d \to 0$ (which implies that $k \to 0$) as $\pi \to \infty$, then $\frac{\partial k}{\partial\pi} < 0$ must hold as $\pi \to \infty$. If $d > 0$ (which implies $k \to 0$) as $\pi \to \infty$, then $\gamma \to 0$ must hold. Because $\frac{\gamma}{d} \to 0$ as $\pi \to \infty$, observe that

$$|\gamma(d\xi)^{-1}\Lambda\frac{\partial\gamma(\pi)}{\partial\pi}| \to 0,$$

whereas

$$|\gamma(d\xi)^{-1}\Lambda\frac{\partial d(\pi)}{\partial\pi}| \neq 0,$$

as $\pi \to \infty$. Thus,

$$\text{sign}\left[\frac{\partial k}{\partial\pi}\right] \to \text{sign}\left[\frac{\partial d}{\partial\pi}(1-\gamma)\right] = \text{negative}$$

as $\pi \to \infty$. Thus, Claim 4 is proven.

⊙ The effect on $\phi m = d\gamma$. We follow similar steps to those described above. Observe that

$$\frac{\partial\phi m}{\partial\pi} = \frac{\partial d}{\partial\pi}\gamma + d\frac{\partial\gamma}{\partial\pi}.$$

Thus, because $\frac{\partial \gamma}{\partial \pi} < 0$, when ξ is sufficiently low that $\frac{\partial d}{\partial \pi} < 0$, we obtain $\frac{\partial k}{\partial \pi} < 0$. When ξ is sufficiently high that $\frac{\partial d}{\partial \pi} > 0$, the final effect depends on the rate of inflation π.

When π is low (around the limit $\pi \to \frac{1}{R}$), we obtain $\gamma \to 1$ and, thus,

$$|\gamma(d\xi)^{-1}\Lambda\frac{\partial d(\pi)}{\partial \pi}| \to \infty > |\gamma(d\xi)^{-1}\Lambda\frac{\partial \gamma(\pi)}{\partial \pi}|.$$

Thus,

$$\text{sign}\left[\frac{\partial \phi m}{\partial \pi}\right] \to \text{sign}\left[\frac{\partial d}{\partial \pi}\gamma\right] = \text{positive}$$

as $\pi \to \frac{1}{R}$. When π is high, we observe that $\gamma \to 0$ and/or $d \to 0$ are the only possible solution to (A.5) and (A.8) as $\pi \to \infty$. In any case, $\phi m = d\gamma \to 0$ as $\pi \to \infty$, and, thus, $\frac{\partial \phi m}{\partial \pi} < 0$ as $\pi \to \infty$ must hold, proving Claim 5.

The proof of Proposition 3 is complete. □

References

Ahmed, S., Rogers, J.H., 2000. Inflation and the great ratios: long term evidence from the US. J. Monet. Econ. 45 (1), 3–35.
Allen, F., Carletti, E., Gale, D., 2014. Money, financial stability and efficiency. J. Econ. Theory 149, 100–127.
Allen, F., Gale, D., 1998. Optimal financial crises. J. Finance 53 (4), 1245–1284.
Allen, F., Gale, D., 2004. Financial intermediaries and markets. Econometrica 72 (4), 1023–1061.
Andolfatto, D., Berentsen, A., Martin, F. M., 2017. Money, banking and financial markets. Federal Reserve Bank of St. Louis Working Paper Series, (2017-023).
Andolfatto, D., Nosal, E., Wallace, N., 2007. The role of independence in the green-lin diamond-dybvig model. J. Econ. Theory 137, 709–715.
Antinolfi, G., Huybens, E., Keister, T., 2001. Monetary stability and liquidity crises: the role of the lender of last resort. J. Econ. Theory 99 (1–2), 187–219.
Antinolfi, G., Keister, T., 2006. Discount window policy, banking crises, and indeterminacy of equilibrium. Macroecon. Dyn. 10 (01), 1–19.
Bencivenga, V.R., Camera, G., 2011. Banking in a matching model of money and capital. J. Money Credit Bank. 43 (s2), 449–476.
Berentsen, A., Camera, G., Waller, C., 2007. Money, credit and banking. J. Econ. Theory 135 (1), 171–195.
Boyd, J.H., De Nicolo, G., Smith, B.D., 2004. Crises in competitive versus monopolistic banking systems. J. Money Credit Bank. 487–506.
Boyd, J.H., Gomis-Porqueras, P., Kwak, S., Smith, B.D., 2014. A user's guide to banking crises. Ann. Econ. Finance 15 (2), 800–892.
Boyd, J.H., Levine, R., Smith, B.D., 2001. The impact of inflation on financial sector performance. J. Monet. Econ. 47 (2), 221–248.
Bruno, M., Easterly, W., 1998. Inflation crises and long-run growth. J. Monet. Econ. 41 (1), 3–26.
Bullard, J., Keating, J.W., 1995. The long-run relationship between inflation and output in postwar economies. J. Monet. Econ. 36 (3), 477–496.
Calomiris, C.W., Gorton, G., 1991. The origins of banking panics: models, facts, and bank regulation. In: Financial markets and financial crises. University of Chicago Press, pp. 109–174.
Champ, B., Smith, B.D., Williamson, S.D., 1996. Currency elasticity and banking panics: theory and evidence. Canad. J. Econ. 828–864.
Chang, R., Velasco, A., 2000. Financial fragility and the exchange rate regime. J. Eecon. Theory 92 (1), 1–34.
Chari, V.V., 1989. Banking without deposit insurance or bank panics: lessons from a model of the US national banking system. Federal Reserve Bank Minneapol. Q.Rev. 13 (3), 3–19.
Crosby, M., Otto, G., 2000. Inflation and the capital stock. J. Money Credit Bank. 236–253.
Davis, E.P., Karim, D., Liadze, I., 2011. Should multivariate early warning systems for banking crises pool across regions? Rev. World Econ. 147 (4), 693–716.
Demirguc-Kunt, A., Detragiache, E., 1998. The determinants of banking crises in developing and developed countries. Staff Papers 45 (1), 81–109.
Demirguc-Kunt, A., Detragiache, E., 2005. Cross-country empirical studies of systemic bank distress: a survey. Natl. Inst. Econ. Rev. 192 (1), 68–83.
Diamond, D.W., Dybvig, P.H., 1983. Bank runs, deposit insurance, and liquidity. J. Polit. Econ. 91 (3), 401–419.
Diamond, D.W., Rajan, R.G., 2006. Money in a theory of banking. Am. Econ. Rev. 96 (1), 30–53.
Duttagupta, R., Cashin, P., 2011. Anatomy of banking crises in developing and emerging market countries. J. Int. Money Finance 30 (2), 354–376.
Ennis, H.M., Keister, T., 2006. Bank runs and investment decisions revisited. J. Monet. Econ. 53 (2), 217–232.
Ennis, H.M., Keister, T., 2009. Bank runs and institutions: the perils of intervention. Am. Econ. Rev. 99 (4), 1588–1607.
Ennis, H.M., Keister, T., 2010. Banking panics and policy responses. J Monet. Econ. 57 (4), 404–419.
Ferraris, L., Watanabe, M., 2008. Collateral secured loans in a monetary economy. J. Econ. Theory 143 (1), 405–424.
Ferraris, L., Watanabe, M., 2011. Collateral fluctuations in a monetary economy. J. Econ. Theory 146 (5), 1915–1940.
Ferraris, L., Watanabe, M., 2012. Liquidity constraints in a monetary economy. Int. Econ. Rev. 53, 255–277.
Gomis-Porqueras, P., Smith, B.D., 2003. Seasonality and monetary policy. Macroecon. Dyn. 7 (04), 477–502.
Gomis-Porqueras, P., Smith, B.D., 2006. The seasonality of banking failures during the late national banking era. Canad. J. Econ./Revue Canad. D'economique 39 (1), 296–319.
Green, E.J., Lin, O., 2003. Implementing efficient allocations in a model of financial intermediation. J. Econ. Theory 109, 1–23.
Gu, C., Mattesini, F., Monnet, C., Wright, R., 2013. Banking: a new monetarist approach. Rev. Econ. Stud. 80 (636), 62.
Gu, C., Mattesini, F., Wright, R., 2016. Money and credit redux. Econometrica 84 (1), 1–32.
Hardy, D.C., Pazarbasioglu, C., 1999. Determinants and leading indicators of banking crises: further evidence. IMF Econ. Rev. 46 (3), 247–258.
Haslag, J.H., Martin, A., 2007. Optimality of the friedman rule in an overlapping generations model with spatial separation. J. Money Credit Bank. 39 (7), 1741–1758.
Jiang, J.H., 2008. Banking crises in monetary economies. Canad. J. Econ./Revue Canad. d'economique 41 (1), 80–104.
Joyce, J.P., 2011. Financial globalization and banking crises in emerging markets. Open Econ. Rev. 22 (5), 875–895.
Khan, M.S., Senhadji, A.S., 2001. Threshold effects in the relationship between inflation and growth. IMF Staff Papers 48 (1), 1–21.
Khan, M.S., Senhadji, A.S., Smith, B.D., 2006. Inflation and financial depth. Macroecon. Dyn. 10 (2), 165–182.
King, R.G., Watson, M.W., 1997. Testing long-run neutrality. Econ. Q.-Federal Res. BankRichmond 83 (3), 69.
Kiyotaki, N., Wright, R., 1989. On money as a medium of exchange. J. Polit. Econ. 97 (4), 927–954.
Kiyotaki, N., Wright, R., 1991. A contribution to the pure theory of money. J. Econ. Theory 53 (2), 215–235.
Kremer, S., Bick, A., Nautz, D., 2013. Inflation and growth: new evidence from a dynamic panel threshold analysis. Empir. Econ. 1–18.
Lagos, R., Wright, R., 2005. A unified framework for monetary theory and policy analysis. J. Polit. Econ. 113 (3), 463–484.
Lagos, R., Rocheteau, G., 2008. Money and capital as competing media of exchange. J. Econ. Theory 142 (1), 247–258.
Lester, B., Postlewaite, A., Wright, R., 2012. Information, liquidity, asset prices, and monetary policy. Rev. Econ. Stud. 79 (3), 1209–1238.
Matsuoka, T., 2012. Imperfect interbank markets and the lender of last resort. J. Econ. Dyn. Control 36 (11), 1673–1687.
Matsuoka, T., Watanabe, M., 2019. Banking panics and the lender of last resort in a monetary economy. Tinbergen Institute Discussion Paper 2019-002/VII.
Miron, J.A., 1986. Financial panics, the seasonality of the nominal interest rate, and the founding of the fed. Am. Econ. Rev. 76 (1), 125–140.
Peck, J., Shell, K., 2003. Equilibrium bank runs. J. Polit. Econ. 111 (1), 103–123.
Skeie, D.R., 2008. Banking with nominal deposits and inside money. J. Financ. Intermediat. 17 (4), 562–584.

Smith, B.D., 2002. Monetary policy, banking crises, and the friedman rule. Am. Econ. Rev. 92 (2), 128–134.
Smith, B.D., 2003. Taking intermediation seriously. J. Money Credit Bank. 35 (6), 1319–1357.
Sprague, O.M.W., 1910. History of Crises Under the National Banking System (vol. 538). US Government Printing Office.
Wallace, N., 1990. A banking model in which partial suspension is best. federal reserve bank of minneapolis. Q. Rev. 14 (Fall), 11–23.
Williamson, S.D., 2012. Liquidity, monetary policy, and the financial crisis: a new monetarist approach. Am. Econ. Rev. 102 (6), 2570–2605.
Williamson, S.D., Wright, R., 2010. New monetarist economics: models. In: Friedman, M., Woodford, M. (Eds.), Handbook of Monetary Economics, vol. 2. Benjamin, North-Holland.