

## A study of correlated failures on the network reliability of power transmission systems

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### ABSTRACT

This paper studies the performance evaluation of a power transmission system in terms of its network topology, where edges represent transmission lines and nodes represent subsidiary stations. The power transmission network is modeled as a stochastic-flow network (SFN) due to the possibility of failure, partial failure, and maintenance of edges (transmission lines). Furthermore, correlation poses a particular concern for such an SFN because the simultaneous failure of multiple components can dangerously degrade performance. We develop a method to measure the impact of correlated failures on network reliability, which is defined as the probability of demand satisfaction. Experimental results show that correlation may produce a significantly negative impact on network reliability, especially when there is a high level of network demand. Thus, the proposed approach captures the influence of correlation on network reliability and offers a method to quantify the utility of reducing correlation.

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### 1. Introduction

Due to the possibility of failure, partial failure, and maintenance, a power transmission system may be characterized as a stochastic-flow network (SFN), which exhibits multiple levels of performance. In the context of power transmission networks, each edge models a transmission line, while each node represents a subsidiary station of an electric power generation and distribution system. This infrastructure serves as the foundation for electric power transmission. The reliability and stability of power transmission networks are indispensable to modern society [1]. Thus, measuring the capability of power transmission networks is a crucial task. Several works [2–7] have characterized power transmission systems in terms of a binary-state network, consisting of edges and nodes to evaluate network reliability. More recently, Lin and Yeh [8,9] suggested a more detailed approach, where the power transmission network is stochastic because each edge (transmission line) is a combination of several physical lines and is therefore more aptly modeled by multi-state components. For instance, the  $i$ th edge  $e_i$  in the power transmission network contains  $w_i$  identical physical lines, suggesting that  $w_i + 1$  capacity levels are possible. The lowest level (0) corresponds to complete malfunction, while  $w_i$  denotes the highest level of operation. Thus, a power transmission network characterized by such multi-state edges also demonstrates stochastic capacities and is suitable for modeling as an SFN.

Lin's works [10,11] formulated an SFN problem with multi-state edges, computing network reliability in terms of minimal paths (MPs), where an MP is a path whose proper subsets are no longer paths. A great many studies have also devoted a considerable amount of research [8–20] to evaluate the network reliability of SFN. These studies define the network reliability as the probability that the SFN can send a requested demand  $d$  from the source to sink, which requires that the transmission capacity of the SFN be measured in terms of its ability to satisfy this demand. Despite the significant amount of research on the topic of SFN, virtually all of the previous research assumes that the capacity of each edge is stochastic with a given probability distribution [10–17]. A smaller amount of research [8,9,18,19] derives probability distributions for the capacities, assuming each physical line of an edge is binary state, reliable or failed. This latter approach characterizes the capacity probability distribution of each edge  $e_i$  as an independent binomial distribution with parameters  $r_i$  for the reliability of each physical line and  $w_i$  to denote the maximal capacity of each edge. These previous works fail to consider the possibility of correlated failures, where two or more physical lines comprising an edge of the network fail simultaneously. Thus, previous SFN models fail to consider the impact correlated failures could have on the network reliability.

A technique to explicitly integrate correlation into stochastic-flow network reliability models is broadly needed because several real-world scenarios can lead to correlated failure. One commonly occurring example of a correlated failure specific to power transmission networks is a large tree limb falling on a power line during stormy weather. Another less common but equally dangerous

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example is a car accident that damages a telephone pole so severely that the wires for transmitting electricity are torn during the collapse of the pole. Given that these serious failures can lead to severely degraded network reliability, enhanced stochastic-flow network models incorporating correlated failures are needed to quantify the impact such events could have on network reliability. This modeling technique will also be useful for measuring the reliability benefits obtainable through reductions in the occurrence of correlated failures with strategies such as physically shielding vulnerable portions of the network. The approach should also prove effective for conducting tradeoff studies to compare alternative network topologies, which can identify designs that will be more resilient to correlated failures.

Following the reliability evaluation model proposed by Lin [10], this paper develops a method to evaluate the network reliability of an SFN, where the failure of the physical lines comprising the edges of the network may experience correlated failures. The network reliability is defined as the probability that the network can transmit  $d$  units of electric power from a power generator (source) to a specific area (sink). We employ a correlated binomial distribution [21] to characterize the performance distribution of the edges. This distribution assumes that the reliability of the physical lines are identically distributed, but share a common failure correlation parameter  $\rho$ . The assumption of identically distributed reliabilities is justifiable because the wires comprising power transmission networks are typically homogeneous in nature, but deployed with different degrees of redundancy to enable a desired level of capacity. We restrict our consideration of correlation to the physical lines in a single edge. Thus, we do not consider the case where the physical lines in two adjacent edges may be correlated. This more general case will be necessary for larger scale events such as hurricanes and earthquakes that can cause widespread damage to the edges comprising the network.

## 2. Notations and assumptions

Let  $G = (\mathbf{N}, \mathbf{E}, \mathbf{W})$  denote an SFN with source node  $s$  and sink node  $t$ , where  $\mathbf{N}$  is the set of nodes,  $\mathbf{E} = \{e_i | i = 1, 2, \dots, n\}$  represents the set of edges, and  $\mathbf{W} = \{w_i | i = 1, 2, \dots, n\}$  the vector of the maximal capacities  $w_i$  of  $e_i$ . Each edge  $e_i$  consists of  $w_i$  physical lines of reliability  $r_i$ . Let  $\rho_i$  denote the correlation between the failures of the physical lines in edge  $e_i$ . The capacity vector  $X = (x_1, x_2, \dots, x_n)$  is defined as the system state of  $G$ , where  $x_i$  represents the current capacity of edge  $e_i$ . Such a  $G$  is assumed to further satisfy the following assumptions:

1. Each node is perfectly reliable. The case where nodes are unreliable can also be solved in terms of MPs, redefined to be ordered sequences of edges and nodes.
2. The capacities of different edges are statistically independent.
3. Flow in  $G$  has to satisfy the so-called flow-conservation law [22].

Assumption 3 indicates that flow in such a  $G$  satisfies Kirchhoff's current law. That is, the sum of flow at a node is equal to the sum of flow out of another node during transmission. Such an assumption is necessary to be considered for a power transmission system while evaluating its network reliability [8,9,23]. Same as the previous studies [8,9] not addressing Kirchhoff's voltage law, we primarily focus on the capacity of the power transmission network to satisfy a certain demand for electric power.

## 3. Stochastic-flow network model with correlated failures

This section presents a method to assess the network reliability of an SFN, considering correlated failures. The development is

conducted in a hierarchical manner. We first formalize concepts including network reliability in terms of the minimal paths as a function of the flows in the edges of the network. Next, an efficient algorithm to generate all minimal capacity vectors of an SFN is given. Finally, a correlated binomial distribution is discussed. This latter approach quantifies the impact correlation will have on the performance distribution of each edge of the network. These performance distributions may then be used to evaluate the probability that the minimal capacity vectors are satisfied. The subsequent network reliability estimate is then formed from these minimal capacity vectors, directly providing an assessment that incorporates the impact of correlated failures in the edges on the overall network reliability.

We construct a power transmission system as an SFN to compute the network reliability as follows. Let  $P_1, P_2, \dots, P_k$  be the  $k$  minimal paths of the SFN. This SFN can be described with capacity vector  $X = (x_1, x_2, \dots, x_n)$  and a flow vector  $F = (f_1, f_2, \dots, f_k)$ , where  $x_i$  represents the capacity of  $e_i$  and  $f_j$  the flow on the  $j$ th MP. Therefore, the capacity of  $e_i$  may be determined in terms of flow vectors [10] by

$$x_i = \sum_{j=1}^k \{f_j | e_i \in P_j\}, \quad (1)$$

Subject to:

$$\sum_{j=1}^k \{f_j | e_i \in P_j\} \leq w_i, \text{ where } w_i \text{ is the maximal capacity of edge } e_i. \quad (2)$$

### 3.1. Network reliability evaluation

Given demand  $d$ , the network reliability  $R_d$  is the probability that the network capacity is sufficient to transmit the requested electric power from source to sink. Thus, the network reliability is  $\Pr\{X|V(X) \geq d\}$ , where  $V(X)$  is defined to be the maximum demand that can be sent given network state  $X$ . It is computationally inefficient to find all  $X$  such that  $V(X) \geq d$  and then sum their probabilities to obtain  $R_d$ . The minimal capacity vectors  $Y_1, Y_2, \dots, Y_h$  in the set  $\{X|V(X) \geq d\}$  constitute a more effective approach to compute the reliability of the network under demand  $d$ . A capacity vector  $Y$  is said to be minimal for  $d$  if and only if (i)  $V(Y) \geq d$  and (ii)  $V(Y') < d$  for any capacity vectors  $Y'$  such that  $Y' < Y$ . Given,  $Y_1, Y_2, \dots, Y_h$ , the set of minimal capacity vectors capable of satisfying demand  $d$ , the network reliability  $R_d$  is

$$R_d = \Pr\left\{\bigcup_{v=1}^h D_v\right\}, \quad (3)$$

where  $D_v = \{X|X \geq Y_v\}$ ,  $v = 1, 2, \dots, h$ . Several methods such as the Recursive Sum of Disjoint Products (RSDP) algorithm [8,9,18–20], inclusion–exclusion method [10,11,16,17], disjoint-event method [13,14], and state-space decomposition [12,15] may be applied to compute  $\Pr\left\{\bigcup_{v=1}^h D_v\right\}$ . The inclusion–exclusion method is often intractable, especially for large networks. In practice, the RSDP algorithm demonstrates better computational efficiency than the state-space decomposition approach for large networks [20]. Hence, the RSDP algorithm is chosen as the method to derive the network reliability herein.

### 3.2. Algorithm to generate all minimal capacity vectors

Given  $k$  minimal paths (MPs)  $P_1, P_2, \dots, P_k$ , with flow on the  $j$ th MP  $f_j$ , the minimal capacity vectors for  $d$  can be generated with the following steps [10].

Step 1. Find all feasible solutions of flow vector  $F = (f_1, f_2, \dots, f_k)$  satisfying  $\sum_{j=1}^k f_j = d$  subject to

$$\sum_{j=1}^k \{f_j | e_i \in P_j\} \leq w_i. \tag{4}$$

Step 2. Transform each flow vector  $F$  into capacity vector  $X = (x_1, x_2, \dots, x_n)$  according to

$$x_i = \sum_{j=1}^k \{f_j | e_i \in P_j\} \quad \text{for } i = 1, 2, \dots, n. \tag{5}$$

Step 3. Find minimal capacity vectors among the  $X$  obtained in Step 2. Suppose  $Y_1, Y_2, \dots, Y_h$  are the minimal capacity vectors for  $d$ .

Using these minimal capacity vectors obtained from the algorithm, the network reliability may be computed according to the capacity probability distribution for edges considering the correlated failures.

3.3. Reliability of edges with correlated failures

Consider an edge  $e_i$  consisting of  $w_i$  physical lines of reliability  $r_i$ , and let  $\rho_i$  represent the correlation between the failures of the physical lines. The probability that  $\gamma$  ( $1 \leq \gamma \leq w_i$ ) or more lines are reliable [21] is

$$\Pr\{x_i \geq \gamma\} = \frac{1}{\beta_i} \sum_{\alpha=\gamma}^{w_i} \binom{w_i}{\alpha} (r_i \beta_i)^\alpha (1 - r_i \beta_i)^{w_i - \alpha}, \tag{6}$$

where  $\beta_i = (1 + \frac{\rho(1-r_i)}{r_i})$ , while the probability that edge  $i$  exhibits performance level  $\alpha$  ( $1 \leq \alpha \leq w_i$ ) is simply

$$\Pr\{x_i = \alpha\} = \frac{1}{\beta_i} \binom{w_i}{\alpha} (r_i \beta_i)^\alpha (1 - r_i \beta_i)^{w_i - \alpha}, \tag{7}$$

and the probability that all  $w_i$  physical lines fail is

$$\Pr\{x_i = 0\} = 1 - \frac{1}{\beta_i} \sum_{\alpha=1}^{w_i} \binom{w_i}{\alpha} (r_i \beta_i)^\alpha (1 - r_i \beta_i)^{w_i - \alpha}. \tag{8}$$

The following numerical example illustrates these equations in the context of an edge of a stochastic flow network consisting of three physical lines ( $w_i = 3$ ), where each physical line delivers one unit of performance with probability  $r_i = 0.9$ . Table 1 shows how correlation alters the performance distribution of the edge.

Note that increasing the correlation between the failures of the physical lines increases the probability that the edge exhibits the lowest (0) and highest (3) performance levels. This occurs because higher levels of correlation result in simultaneous failure and reliability more often. Thus, correlation can increase the probability of the highest performance level. This will improve network reliability when the edge performance must equal its full capacity. For example, when  $\rho = 0$  the probability that the edge performance is 3 is only 0.729, but increases to 0.9 for  $\rho = 1.0$ . When the required performance is less than the full capacity, however, correlation lowers the probability that the minimum performance level is met. For example, when the edge capacity must be two or greater

Table 1 Impact of correlation on edge performance.

Edge performance	Correlation ( $\rho$ )				
	0	0.1	0.5	0.9	1.0
0	0.001	0.01171	0.05275	0.09091	0.1
1	0.027	0.02187	0.00675	0.00027	0.0
2	0.243	0.22113	0.12825	0.02673	0.0
3	0.729	0.74529	0.81225	0.88209	0.9

Table 2 Impact of correlation for different system types.

System type	Reliability				
	$\rho = 0$	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.9$	$\rho = 1.0$
Parallel	0.999	0.98829	0.94725	0.90909	0.9
2-out-of-3	0.972	0.96642	0.94050	0.90882	0.9
Series	0.729	0.74529	0.81225	0.88209	0.9

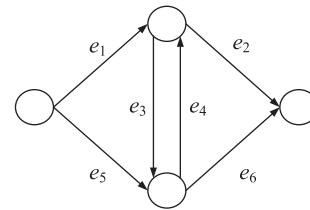


Fig. 1. Benchmark network.

Table 3 Edges data for example 1.

Edge ( $e_i$ )	# Of physical lines ( $w_i$ )	Reliability ( $r_i$ )
1	2	0.950
2	3	0.969
3	2	0.994
4	3	0.968
5	3	0.984
6	3	0.951

the probability is 0.972 for  $\rho = 0$ , but decreases to 0.9 when  $\rho = 1.0$ . This second situation is undesirable because it reduces SFN reliability because lower edge performances render the network incapable of delivering the specified demand.

In the general case, correlation improves the reliability of a series system and lowers the reliability of a  $k$ -out-of- $n$  system when  $k$  is close to  $n$  [21]. When all physical lines must be reliable to deliver the required performance this is equivalent to a series system. Edges that do not need all of the physical lines to satisfy a minimal capacity vector can be viewed as  $k$ -out-of- $n$  systems, explaining why correlation can improve the performance distribution of some edges, but lower the performance of others. Table 2 illustrates this by computing the reliability of the series, parallel, and 2-out-of-3 system for each correlation.

Note that in practice, correlations between the failures of the physical lines may be estimated from historical data. For example,

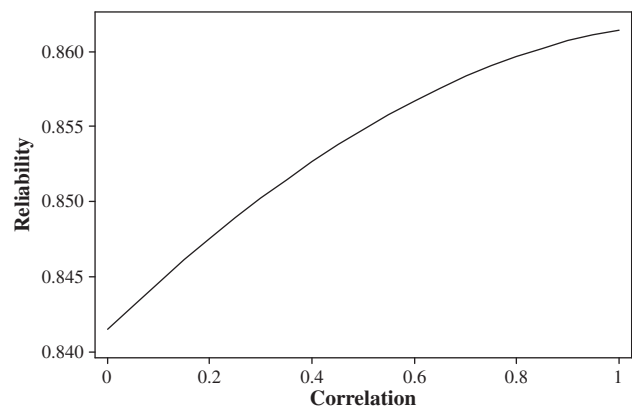


Fig. 2. Impact of correlation on network reliability.

**Table 4**  
Sensitivity analysis for correlations.

Edge ( $e_i$ )	Change in network reliability	Ranking
1	0.02214	6
2	-0.00672	2
3	0.00000	4
4	-0.00169	3
5	0.01379	5
6	-0.01218	1

many regional power distribution companies monitor the performance of their network to measure safety and quality of service as well as identify strategies to improve reliability and availability. The accuracy of these estimates will be specific to different providers and the amount of data and details recorded by their monitoring infrastructure. Consider the situation where a power distribution company records outage statistics on the physical

lines of their network. These logs can then be processed to estimate the correlation in the outages of each set of physical lines comprising the edges of the network.

**4. Numerical examples**

*4.1. Benchmark network*

The benchmark network, shown in Fig. 1, was originally given by Lin [10] to demonstrate his solution procedure to obtain a set of minimal capacity vectors satisfying demand  $d$ . This network illustrates a power transmission system composed of two areas [24]. However, this previous study assumed arbitrary probability distributions for the performance of the edges to illustrate the network reliability evaluation process. Similar to this earlier work, we utilize the network topology and potential capacity levels of each edge, but quantify the reliability of each physical line and the



**Fig. 3.** Taiwan power transmission network.

**Table 5**  
Edges data for example 2.

Edge ( $e_i$ )	# Of physical lines ( $w_i$ )	Reliability ( $r_i$ )	Edge ( $e_i$ )	# Of physical lines ( $w_i$ )	Reliability ( $r_i$ )
1	3	0.943	30	4	0.975
2	5	0.959	31	3	0.958
3	3	0.957	32	4	0.965
4	4	0.956	33	2	0.973
5	3	0.953	34	2	0.995
6	3	0.958	35	4	0.961
7	5	0.975	36	3	0.971
8	5	0.983	37	4	0.954
9	4	0.965	38	2	0.981
10	2	0.951	39	5	0.990
11	2	0.982	40	4	0.989
12	3	0.977	41	4	0.987
13	3	0.955	42	3	0.978
14	5	0.979	43	2	0.985
15	4	0.964	44	5	0.968
16	2	0.983	45	4	0.964
17	2	0.951	46	3	0.981
18	3	0.956	47	2	0.968
19	3	0.966	48	3	0.981
20	5	0.945	49	3	0.983
21	5	0.972	50	2	0.953
22	4	0.960	51	3	0.962
23	4	0.967	52	4	0.968
24	3	0.985	53	3	0.951
25	5	0.956	54	4	0.976
26	2	0.982	55	2	0.964
27	3	0.957	56	3	0.951
28	3	0.972	57	3	0.975
29	5	0.952	58	2	0.958

**Table 6**  
Network reliability at different demand levels and correlations.

Demand <sup>a</sup>	Network reliability						
	$\rho = 0.0$	$\rho = 0.1$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$	$\rho = 1.0$
0	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
100	0.9999999	0.9999963	0.9999126	0.9996099	0.9989561	0.9978312	0.9970584
200	0.9999999	0.9999686	0.9997255	0.9992481	0.9985431	0.9976114	0.9970584
300	0.9999996	0.9996702	0.9972951	0.9926298	0.9857809	0.9769445	0.9718593
400	0.9999827	0.9984209	0.9921753	0.9821794	0.9687980	0.9523009	0.9429533
500	0.9995044	0.9898441	0.9675948	0.9428994	0.9172252	0.8917339	0.8793425
600	0.9913773	0.9560463	0.8906593	0.8307138	0.7747121	0.7215562	0.6957938
700	0.9134448	0.8597414	0.7634304	0.6806366	0.6098297	0.5495032	0.5228116
800	0.5544850	0.5354672	0.4969611	0.4581122	0.4192718	0.3807976	0.3618101
900	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

<sup>a</sup> The unit of demand is MW (megawatt).

correlation between them with the method discussed in Section 3.3. Table 3 provides the number of physical lines  $w_i$  in  $e_i$ , and the reliability of these physical lines  $r_i$ .

When the demand is five units ( $d = 5$ ), and the correlation parameter  $\rho_i$  for each set of physical lines is set to zero, the network reliability, calculated in terms of the minimal capacity vectors  $Y_1 = (2, 2, 0, 0, 3, 3)$  and  $Y_2 = (2, 3, 0, 1, 3, 2)$ , is 0.841481. Larger values of correlation increase network reliability because  $e_1$  and  $e_5$  must operate at their highest performance level to satisfy the specified demand. This requires that all of the physical lines in these edges are reliable. Thus, positive correlation among the failures also increases the probability that these physical lines are reliable simultaneously. For these larger values of correlation, the negative impact of correlation on the performance distribution of the other edges is small. Hence, the benefit to reliability provided by correlation between the failures of the edges requiring high performance outweighs the negative impact of correlation in the other edges raising the overall network reliability. The network reliability increases to 0.861436 when the value of correlation is 1.0. Thus, the proposed approach can quantify the impact of correlated fail-

ures on network reliability. Fig. 2 illustrates this influence of correlated failures in the physical lines on the network reliability.

To illustrate the flexibility of the approach to identify the correlations lowering network reliability most significantly [25], we set the correlation  $\rho_i$  for the physical lines in edge  $e_i$  to 0.5, while holding all the other correlations constant at 0, ( $\rho_j = 0, j \neq i$ ). These sensitivity analyses, shown in Table 4, indicate the change in the network reliability and rank the edges according to their criticality from a network reliability standpoint. Increasing the correlation in edges  $e_1$  and  $e_5$  improves network reliability. These improvements occur because the minimal capacity vector requires that all physical lines in these edges are reliable. Furthermore, Table 1 demonstrated that correlation increases the probability of the high performance level. Thus, positive correlation improves the probability that the minimum performance is satisfied. However, Table 1 also showed that increasing correlation lowers the probability of the performance levels 2 through  $w_i - 1$ . Since the minimal capacity vector for the other edges do not require all  $w_i$  physical lines to be reliable to satisfy demand  $d$ , correlation lowers the probability that these other edges satisfy their respective minimum

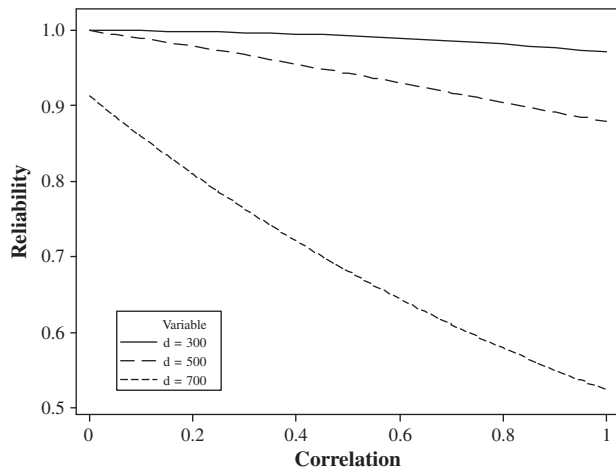


Fig. 4. Impact of correlation on TPTN reliability.

performance requirements. This decrease in probability lowers the network reliability.

#### 4.2. Case study for TPTN

We employ the TPTN with 58 edges, shown in Fig. 3 [8,9], to demonstrate the utility of the approach for assessing larger SFN. In the TPTN, the voltage is transformed from high to low or the reverse using transformers. The first and the second nuclear power stations located at north Taiwan mainly supply electric power to Taipei City. However, the third nuclear power plant at Pingtung County in south Taiwan is needed to transmit the electric power to Taipei City due to high regional demand [8,9]. The Taiwan Power Company thus pays close attention to evaluate the network reliability of the TPTN to transmit electric power from Pingtung County to Taipei City. In this example, each edge is composed of several physical lines and each physical line provides two possible capacities, 100 MW (megawatt) and 0 MW. Since the lines are provided by different suppliers, the capacity of each edge follows a distinct probability distribution. For each edge  $e_i$ , Table 5 provides the number of physical lines  $w_i$  in  $e_i$  and the reliability of these physical lines  $r_i$ . Note that the reliabilities of physical lines in this example have been lowered to more clearly illustrate the impact of correlated failures under different demand levels. In most real-world systems, however, physical lines are highly reliable, often demonstrating reliability 0.9999 or higher. Nevertheless, the proposed network reliability evaluation procedure can be applied to networks composed of physical lines demonstrating any level of reliability, including such highly reliable lines.

Table 6 summarizes the results of network reliability assessment for different levels of demand and correlation. Increasing demand or correlation lowers the network reliability of TPTN. Fig. 4 illustrates the impact of correlated failures in the physical lines on the network reliability for several levels of demand. This figure clearly shows that larger values of correlation can lower SFN reliability noticeably. It is also apparent that this negative impact of correlation becomes even more significant for higher levels of demand.

## 5. Conclusion

This paper models power transmission system as a stochastic-flow network, rather than a binary-state network because the capacity of transmission lines is typically multi-state. The network reliability, probability of demand satisfaction, is evaluated to

assess the capability of the power transmission network. Following the reliability evaluation model proposed by Lin [10], we treat the power transmission network as an SFN and develop a method to measure the impact of correlated failures on the reliability of SFN. We first find the minimal capacity vectors, which form the combinations of edge performances that can satisfy a given level of demand. Correlation in the failures of the physical lines was modeled with a correlated binomial distribution. Two experiments were performed, including a large-scale case study of the Taiwan Power Transmission Network (TPTN). Our results reveal that correlated failures can exert a substantially negative impact on the network reliability of large SFN, especially for higher levels of demand. Thus, this modeling technique is more detailed than the traditional approaches, which assume physical lines in the edges fail independently. This greater detail will allow network designers to explicitly consider the problems posed by correlation when designing networks to ensure that user demands can be satisfied with high probability.

Several problems for future research exist. The first is to consider correlation between the physical lines of different edges and nodes. Further design-oriented studies should conduct detailed sensitivity analysis to identify the most important edge improvements, which would significantly improve the network reliability of large SFN. In addition, this paper assumed that flow in the power transmission system satisfies the flow-conservation law [22], where no flow increases or decreases during transmission. However, loss of load must be considered in future work to make our approach more applicable to real-world systems [24,26,27]. Thus, we will extend our approach to evaluate reliability indices, including LOLP (Loss of Load Probability), LOLF (Loss of Load Frequency), and EENS (Expected Energy Not-Supplied).

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