TECHNICAL NOTES

Plane Turbulent Wall Jets on Rough Boundaries with Limited Tailwater

S. A. Ead[1] and N. Rajaratnam, F.ASCE[2]

Abstract: Combining the results of a laboratory study of plane turbulent wall jets on rough boundaries with shallow tailwater, with the results of an earlier work of Rajaratnam on wall jets on rough boundaries with deep tailwater, this paper attempts to describe the effects of boundary roughness and tailwater depth on the characteristics of plane turbulent wall jets on rough beds, which are important in the field of hydraulic engineering. The time-averaged axial velocity profiles at different sections in the wall jet were found to be similar, with some difference from the profile of the classical plane wall jet. The normalized boundary layer thickness δ/b, where b is the length scale of the velocity profile, was equal to 0.35 for wall jets on rough boundaries compared to 0.16 for the classic wall jet. Two stages were seen to exist in the decay of the maximum velocity u_m as well as in the growth of the length scale, with the first stage corresponding to that of deep tailwater and the second stage to shallow tailwater. In the first stage, the decay of the maximum velocity u_m at any section in terms of the velocity u_0 at the slot, with the longitudinal distance x in terms of L which is the distance where $u_m=0.5U_0$, was described by one general function, for smooth as well as rough boundaries. The length scale L in terms of slot width decreases as the relative roughness of the boundary increases. The onset of the second stage was not affected significantly by the bed roughness. The growth rate of the length scale b of the wall jet increased from 0.076 for a smooth boundary to about 0.125 for a relative roughness k_s/b_0 in the range of 0.25 to 0.50, where k_s is the equivalent sand roughness and b_0 is the thickness of the jet at the slot.

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Introduction

For a plane turbulent wall jet issuing tangentially on a smooth boundary, submerged in an infinite expanse of the same fluid at rest (referred to herein as the classical or simple wall jet), it is known (Rajaratnam 1976) that the maximum velocity at any section in the growing wall jet $u_m \propto 1/\sqrt{x}$, where x is the longitudinal distance from the nozzle. If U_0 and b_0 are, respectively, the velocity of the jet and its thickness at the nozzle, it may be shown (Rajaratnam 1976) that

$$\left(\frac{u_m}{U_0}\right) = \frac{C_1}{\sqrt{(x/b_0)}} \tag{1}$$

where C_1= a constant equal to 3.50. Using L as the length scale, where L is defined as the value of x where u_m is equal to $0.5U_0$, Wu and Rajaratnam (1995) showed that

$$\left(\frac{u_m}{U_0}\right) = \frac{C_2}{\sqrt{(x/L)}} \tag{2}$$

where C_2=a constant equal to 0.50. They also found that L/b_0 is approximately equal to 60. Launder and Rodi (1981) found that the growth rate db/dx of the length scale b, which is equal to y, the normal distance from the boundary where the time-averaged longitudinal velocity $u=0.5u_m$ and the velocity gradient $\partial u/\partial y<0$, is equal to 0.073. Launder and Rodi (1981) also found that the growth rate $d\delta/dx$ of the boundary layer thickness δ, where δ is equal to y where $u=u_m$, is equal to 0.011.

For classical wall jets and the corresponding free jets, it has been generally assumed (Albertson et al. 1950; Rajaratnam 1976; Schlichting 1979) that the momentum flux would be preserved. In their study on the effect of shallow tailwater on the flow characteristics of plane turbulent wall jets on smooth boundaries, Ead and Rajaratnam (2001) found that the momentum flux in the forward flow region of the wall jet is not preserved [see also Schneider (1985) and Kotsovinos (1976)]. Even though the rise in the water surface at the location where the expanding wall jet surfaces (which constitutes the end of the surface eddy), is responsible for a significant portion of the momentum deficit, there is a noticeable loss due to the entrainment of the return flow, even before the end of the surface eddy. The velocity profiles in the wall jet were found to be similar. Ead and Rajaratnam (2002) also found that two stages existed in the decay of the maximum velocity. In the first stage, the decay of the normalized velocity scale could be described by Eq. (1) with C_1 equal to 4.00. The second stage of more rapid decay started at a distance x_0 from the gate

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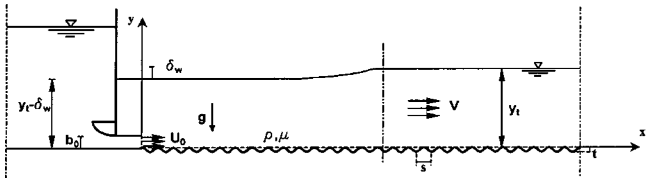

Fig. 1. Definition sketch for wall jets on corrugated beds

and the decay of the maximum velocity is described by

$$\left(\frac{u_m}{U_0}\right) = A\left(\frac{x}{b_0}\right) + B \tag{3}$$

where B=constant and A and x_0 were found to be functions of the tailwater depth ratio y_t/b_0. Ead and Rajaratnam (2002) also showed that the length scale of the jet b grew at a rate of 0.076 in the first stage and the rate of growth was much larger in the second stage. Based on this study, a tailwater depth ratio up to 50 would be considered shallow. The effect of the limited tailwater depth ratio was also observed for y_t/b_0 up to 100 in the study of plane turbulent surface jets in shallow tailwater by Ead and Rajaratnam (2001). These two studies show that most submerged flows in hydraulic engineering could be considered shallow. Rajaratnam (1967a) studied deeply submerged plane turbulent wall jets on rough boundaries and found that the velocity distribution was similar and that the decay of the maximum velocity was affected considerably by the boundary roughness.

In these studies, the effect of the limited tailwater and the effect of the boundary roughness on the flow characteristics of plane turbulent wall jets, were examined separately. This paper presents the results of an exploratory laboratory investigation on plane wall jets issuing tangentially on rough boundaries with limited tailwater and a general correlation for the decay of the maximum velocity on smooth and rough boundaries.

Theoretical Considerations

It is useful to present a brief theoretical discussion on plane turbulent wall jets with limited tailwater [from Ead and Rajaratnam (2002)]. For a plane turbulent wall jet of thickness b_0 with a flow rate per unit width of Q_0 and momentum flux per unit width of M_0 entering a rectangular channel, tangentially on a corrugated bed as shown in Fig. 1, let U_0 be the velocity of the jet at the slot (or nozzle). The downstream control gate is adjusted so that the tailwater depth y_t is large enough to make the water level immediately downstream of the gate (housing the slot) horizontal. Taking the tailwater surface elevation as a reference, our experiments showed a depression δ_w in the water surface elevation at the gate [see Figs. 2(a–d)]. The depression in the water surface elevation at the gate is created to produce the required pressure gradient to drive the return flow above the wall jet, for entrainment by the jet.

Assuming hydrostatic pressure distribution on the gate containing the slot and at the downstream section where the expanding wall jet reaches the water surface and occupies the whole depth and that the velocity distribution at the nozzle and the downstream section is uniform, and using the continuity and momentum equations, Ead and Rajaratnam (2002) found that

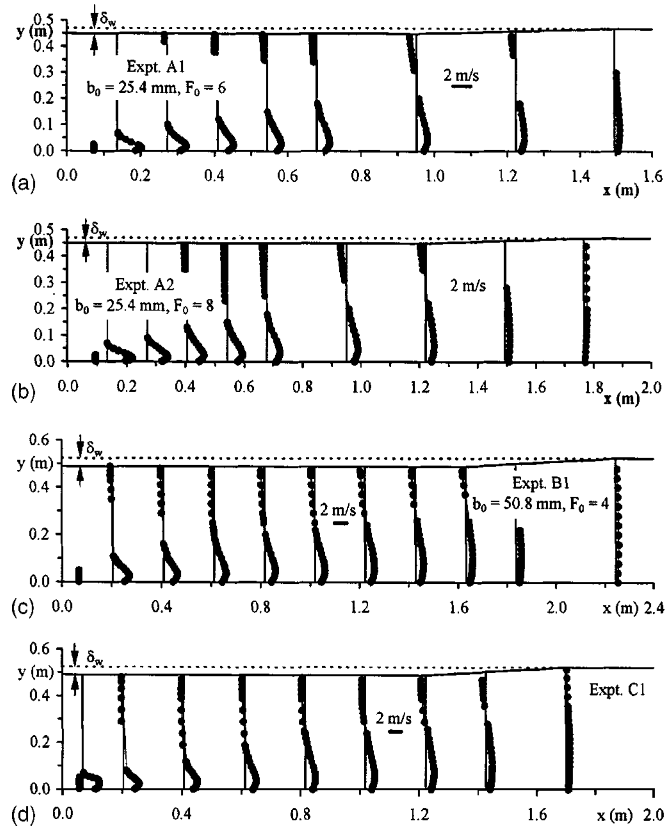

Fig. 2. (a)–(d) Velocity field with water surface profiles

$$\theta = \eta - \sqrt{\eta^2 - 2\left(\frac{\eta(1-\varepsilon)-1}{\eta}\right)\mathsf{F}_0^2} \tag{4}$$

In Eq. (4), $\theta = \delta_w/b_0$; η=tailwater depth ratio, equal to y_t/b_0; ε= shear force coefficient equal to F_τ/M_0; F_τ=integrated bed shear stress τ defined as $\mathsf{F}_\tau = \int_{x=0}^{x=L_e} \tau dx = \varepsilon M_0$; and L_e=length of the surface eddy. A study of Eq. (4) has shown that for any given F_0, θ decreases as η increases and shear stress on the bed also contributes to a decrease of θ. Further, θ increases as F_0 increases. Using the experimental measurements of θ, η, and F_0 in Eq. (4), Ead and Rajaratnam (2002) found that ε was in the range of 0.12–0.15 for their experiments conducted on wall jets on a smooth boundary. Using the experimental measurements of θ, η, and F_0 in Eq. (4), ε was found to be equal to 0.55, 0.72, 0.47, and 0.47 in Experiments A1, A2, B1, and C1 (of the present work), respectively.

Experimental Arrangement and Experiments

Wall jets were produced in a flume, 0.446 m wide, 0.60 m deep, and 7.6 m long, with Plexiglas sides. Corrugated aluminum sheets were installed on the bed of the flume in such a way that the crests of corrugations were at the same level as the upstream bed on which the supercritical stream was produced by a sluice gate (see Fig. 1). The corrugations acted as depressions in the bed, to create a system of turbulent eddies which might increase the Reynolds shear stress in the vicinity of the bed. Two corrugated sheets (I and II) with sinusoidal corrugations of wavelength, s, of 68 mm perpendicular to the flow direction, had amplitude t of 12.7 and 21.8 mm, respectively. Three pumps were used to supply the

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Table 1. Primary Details of Experiments

Experiment	Sheet	b_0 (mm)	s (mm)	t (mm)	t/b_0	U_0 (m/s)	F_0	Q_0 (m^3/s/m)	y_1 (m)	y_2 (m)	$\eta=y_1/b_0$	$\eta_f=y_2/b_0$	$S=\eta/\eta-1$	R
A1	I	25.4	68	12.7	0.50	3.00	6.0	76.074	0.470	0.203	18.50	8.00	1.31	76,074
A2	I	25.4	68	12.7	0.50	3.99	8.0	101.432	0.470	0.275	18.50	10.82	0.71	101,432
B1	I	50.8	68	12.7	0.25	2.82	4.0	143.447	0.525	0.263	10.33	5.18	1.00	143,447
C1	II	50.8	68	21.8	0.43	2.82	4.0	143.447	0.525	0.263	10.33	5.18	1.00	143,447

Note: t=corrugations height (from crest to trough); and s= corrugations wavelength.

head-tank feeding the flume and the discharges were measured by magnetic flowmeters located in the supply lines. Water entered the flume under a sluice gate with a streamlined lip thereby producing a wall jet of a thickness of b_0. A tailgate was used to control the tailwater depth in the flume.

A Prandtl tube with an external diameter of 3.0 mm, connected to a vertical manometer, was used to measure the time-averaged longitudinal velocity u. No corrections were made to the velocity observations to account for turbulence and the presence of air bubbles. Velocity profiles of the forward and backward flows were measured along vertical sections at different longitudinal distances from the nozzle producing the jet, in the centerplane of the flume, mostly on the crests of corrugations. A total of four experiments were conducted and the primary details of these experiments are shown in Table 1. Sheet I was used in Experiments A1, A2, and B1, whereas sheet II was used in Experiment C1. The slot width b_0, measured above the crest level of corrugations on the plane bed, was equal to 25.4 mm in Experiments A1 and A2, and 50.8 mm in Experiments B1 and C1. Values of b_0 and U_0 were selected to achieve a range of the Froude number, from 4 to 8. The submergence factor S (Rajaratnam 1967d), equal to $(y_r - y_2)/y_2$, ranged between 0.71 to 1.31, and y_2 is the subcritical sequent depth for the hydraulic jump obtained from the Belanger equation. The Reynolds number of the jet R, equal to $U_0 b_0/v$ was approximately in the range of 76,000 to 143,500 where v is the kinematic viscosity of the fluid. The ratio of the amplitude of the corrugations to the slot width, t/b_0 was equal to 0.25, 0.43, and 0.50.

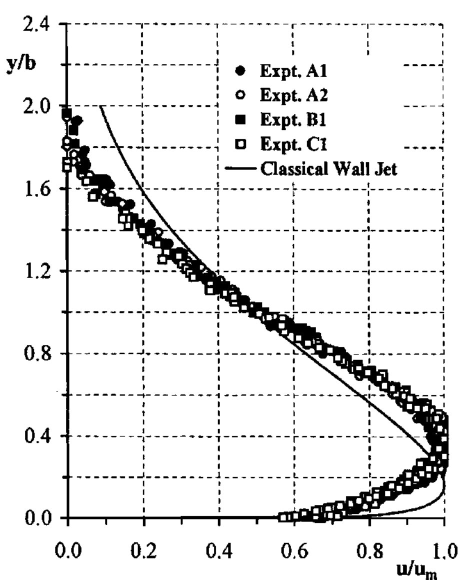

Fig. 3. Similarity profiles for the forward flow (all experiments)

Experimental Results and Analysis

Figs. 2(a–d) show the velocity profiles for the wall jets for the Experiments A1, A2, B1, and C1, at several sections with x/b_0 varying from 0 to about 70, and y is the distance above the crests of corrugations. In Figs. 2(a–d), it may be noticed that the water surface in the vicinity of the gate is approximately horizontal, but a rise in the water surface elevation may be noticed further downstream and the depression, δ_w, in the water surface elevation at the gate is also shown. The maximum reverse velocity at any station, was found to occur very near the water surface. The maximum reverse velocity in the surface eddy was found to occur at a distance of $L_e/2$ from the gate. The section at which the jet surfaces was found using dye injection. Water surface profiles for all the four experiments were measured in the vertical centerplane of the flume with a point gauge to an accuracy of 0.1 mm. These water surface profiles were used to determine the depression in the water surface elevation at the gate δ_w and the tailwater depth y_t.

To test for the similarity of the velocity profiles in the forward flow region, the maximum velocity, u_m, at any section was chosen as the velocity scale and the length scale, b, is the value of y at which $u=0.5u_m$ and $\partial u/\partial y$ is negative. Fig. 3 shows a consolidated plot of all the data for the four experiments, which clearly shows that the velocity profiles in the forward flow are similar but are somewhat different from the profile for the classical wall jet (Schwarz and Cosart 1961; Rajaratnam 1976). The thickness δ of the boundary layer part is about 0.35 times b, whereas for the classical wall jet the corresponding value is about 0.16.

Having found that the velocity profiles in the forward flow are similar, it is necessary to study the variation of the velocity scale u_m and the length scales b, L, and δ with the longitudinal distance x. Fig. 4 shows the variation of u_m/D_0 with x normalized with L. Fig. 4 shows the existence of two stages in the decay of the maximum velocity. The first stage represents the plane wall jet with large tailwater depth and the decay of the normalized velocity scale in this region can be described by Eq. (2) with the constant C_2 equal to 0.50. The advantage of using L as the length scale is that the constant C_2 in Eq. (2) is equal to 0.50 for both

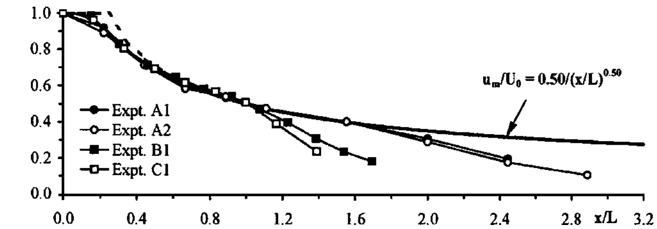

Fig. 4. Variation of the maximum velocity with distance

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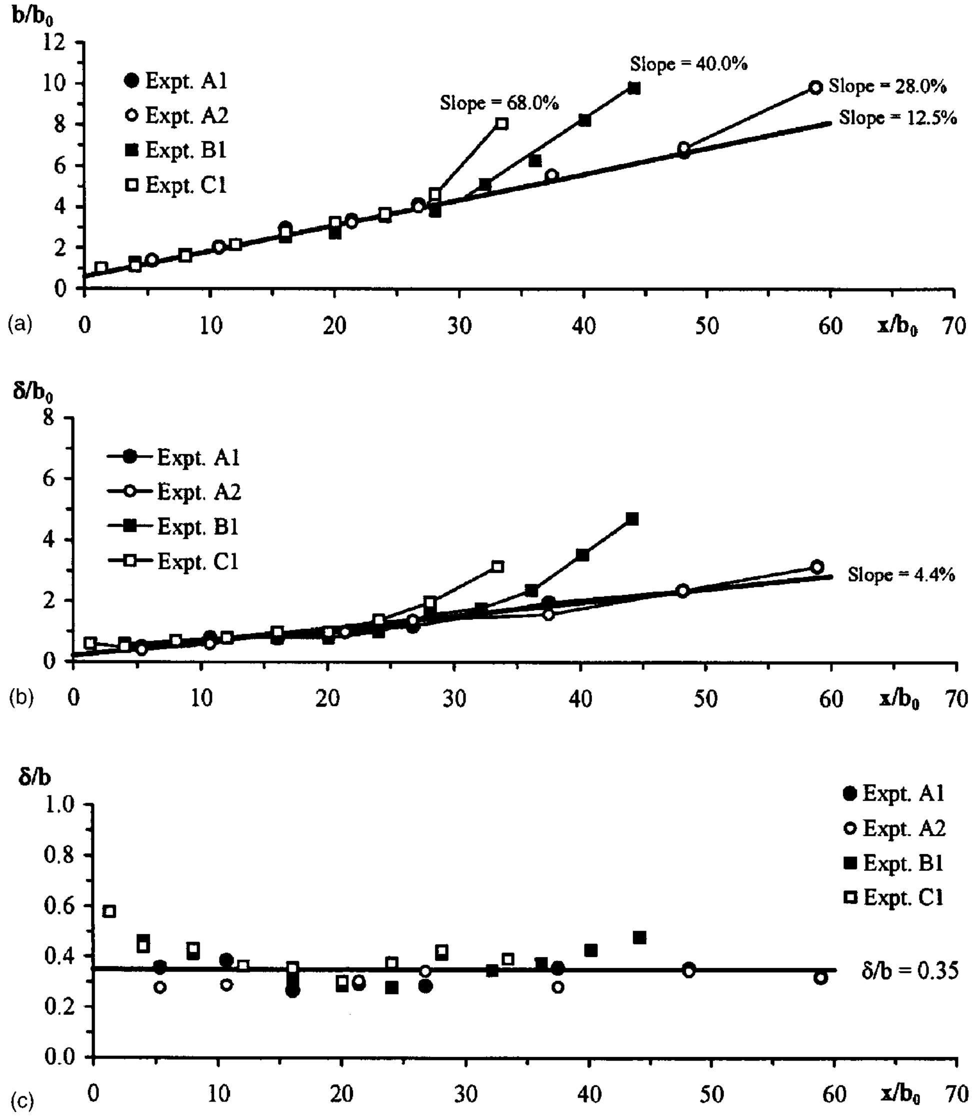

Fig. 5. (a)–(c) Variation of length scales with distance

smooth and rough boundaries. In the second stage of the maximum velocity decay, the observations deviate from the curve describing the first stage and the maximum velocity decays more rapidly. The decay in this second stage is almost linear as shown in Fig. 4. It is interesting to see that the curves for Experiments A1 and A2 deviated from the general trend of the maximum velocity decay at the same location as they both had the same relative roughness t/b_0 and tailwater depth ratio η Experiment C1 deviated from the general trend of the maximum velocity decay earlier than Experiment B1 as they both had the same tailwater depth ratio η but the boundary in Experiment C1 was rougher than that in Experiment B1.

The growth of the length scale b with distance is shown in Fig. 5(a). The growth rate was 0.125 for x/b_0 up to a particular value after which the length scale b grew at a rate of 0.28, 0.28, 0.40, and 0.68 in Experiments A1, A2, B1, and C1, respectively. This growth rate of 0.125 is larger than the value of 0.076 suggested by Ead and Rajaratnam (2002) for wall jets on smooth beds. The value of x/b_0 at which the breakdown in the growth rate of b occurred was about 48, 48, 31, and 27 in Experiments A1, A2, B1, and C1, respectively. Fig. 5(b) shows a similar situation for the growth rate of the normalized boundary layer thickness δ/b_0. Fig. 5(c) shows that δ/b is approximately equal to 0.35.

Since the main objective of this work was to study the effects of the tailwater shallowness and the boundary roughness on the main flow characteristics of plane turbulent wall jets, we examined the experimental results of Rajaratnam (1967a) on plane turbulent wall jets on rough boundaries. Rajaratnam (1967a) used six different roughnesses (four wire screens and two rubber floor mats), and used the "melted down thickness" k_e as a measure of roughness. To obtain the corresponding Nikuradse equivalent roughness k_s, the velocity Profiles in the boundary layer portion were analyzed [a similar procedure may be found in Ead et al. (2000)], the shear velocity u_* was obtained and then k_s was found by matching the observations with the Prandtl–Karman equation for rough turbulent flow

$$\frac{u}{u_*} = 5.75 \log\left(\frac{y}{k_s}\right) + 8.5 \tag{5}$$

For the corrugated sheets (used in the present study), Ead et al. (2000) found out that k_s is equal to the corrugation height t. The normalized equivalent roughness k_s/b_0 was in the range of 0.019–0.57 for the rough boundaries used by Rajaratnam (1967a)

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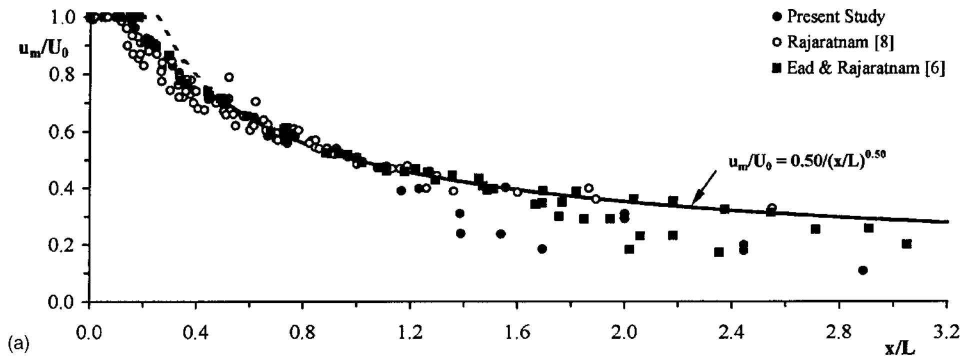

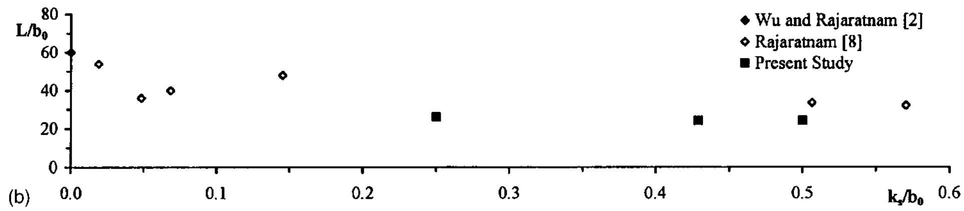

Fig. 6. (a) Variation of u_m/U_0 with x/L and (b) variation of L/b_0 with k_s/b_0

and in the range of 0.25–0.50 for the corrugated sheets.

Fig. 6(a) shows the variation of the normalized maximum velocity u_m/U_0 with the normalized longitudinal distance x/L. Experimental observations of Ead and Rajaratnam (2002) are for wall jets on smooth boundaries and shallow tailwater whereas those from the present study and that of Rajaratnam (1967a) are for wall jets on rough boundaries in shallow and deep tailwater, respectively. Fig. 6(a) shows that if the maximum velocity decay with the longitudinal distance normalized with L, all results come together until the effect of shallow tailwater triggers the faster decay, indicating the second stage of velocity decay. The distance x_0 of the section where the second stage starts was described by

$$\frac{x_0}{b_0} = 2.85(\eta - 2.0) \qquad (6)$$

for a smooth boundary [Ead and Rajaratnam (2002)]. This equation was found to predict approximately (within ±15%), the section where the faster decay sets in. It was observed that L/b_0 decreases with the relative roughness. Fig. 6(b) shows the variation of L/b_0 with the relative roughness k_s/b_0 wherein it may be seen that L/b_0 decreases from 60 for the smooth boundary to about 30 for the relative roughness k_s/b_0 equal to 0.25. The dimensionless length scale L/b_0 seems to be independent of the relative roughness when k_s/b_0 is greater than 0.25.

Conclusions

Based on a laboratory study of plane turbulent wall jets on rough boundaries for a range of Froude numbers from 4 to 8 and three values of the relative roughness from 0.25 to 0.50 and shallow tailwater, and the earlier work of Rajaratnam (1967a) on wall jets on rough boundaries with deep tailwater, the following conclusions are drawn. The axial velocity profiles at different sections in the wall jet were found to be similar, with some difference from the profile of the classical plane wall jet. The normalized boundary layer thickness δ/b, where b is the length scale of the velocity profile, was equal to 0.35 for wall jets on rough boundaries compared to 0.16 for the classic wall jet. Two stages were seen to exist in the decay of the maximum velocity as well as in the growth of the length scale, with the first stage corresponding to that of deep tailwater and the second stage to shallow tailwater. In the first stage, decay of the maximum velocity u_m at any section in terms of the velocity U_0 at the slot with the longitudinal distance x in terms of L which is the distance where $u_m = 0.5U_0$ was described by one function, for smooth as well as rough boundaries. The length scale L in terms of slot width was found to decrease with the relative roughness of the boundary. The onset of the second stage was not affected significantly by the bed roughness. The growth rate of the length scale b of the wall jet increased from 0.076 for a smooth boundary to about 0.125 for a relative roughness in the range of 0.25 to 0.50.

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Notation

The following symbols are used in this technical note:

A = function of the tailwater depth ratio;
B, C_1, C_2 = constants;
b = length scale equal to y where $u = 0.5u_m$ and $\partial u/\partial y < 0$;
b_0 = slot width;
F_0 = supercritical Froude number equal to $U_0/(gb_0)^{0.5}$;
F_τ = integrated bed shear stress, per unit width, over the eddy length;
g = acceleration due to gravity;
k_e = effective roughness;
k_s = Nikuradse equivalent sand roughness;

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L = length scale equal to the value of x where $u_m = U_0/2$;
L_e = length of surface eddy;
M_0 = wall jet momentum flux per unit width at the slot;
Q_0 = discharge per unit width;
R = Reynolds' number equal to $U_0 b_0/\nu$;
S = submergence ratio=$(y_t - y_2)/y_2$;
s = wavelength of corrugations;
t = corrugation height from crest to trough;
U_0 = velocity of the jet at the slot;
u = time-averaged longitudinal velocity at any point;
u_m = maximum value of u at any x station;
u_* = shear velocity;
x = longitudinal distance measured from the gate;
x_0 = function of the tailwater depth ratio;
y = normal distance from crest of corrugations;
y_t = tailwater depth;
y_2 = subcritical sequent depth of free hydraulic jump;
δ = boundary layer thickness;
δ_w = depression in the water surface elevation at the gate;
ε = shear force coefficient equal to F_τ/M_0;
η = tailwater depth ratio=$y_t b_0$;
ν = kinematic viscosity of the fluid μ/ρ;
θ = depression ratio=δ_w/b_0;
ρ = mass density of the fluid; and
τ = bed shear stress, also used as suffix.

References

Albertson, M. L., Dai, Y. B., Jensen, R. A., and Rouse, H. (1950). "Diffusion of submerged jets." *Trans. Am. Soc. Civ. Eng.*, 115, 639–664.
Ead, S. A., and Rajaratnam, N. (2001). "Plane turbulent surface jets in shallow tailwater." *J. Fluids Eng.*, 123(3), 121–127.
Ead, S. A., and Rajaratnam, N. (2002). "Plane turbulent wall jets in shallow tailwater." *J. Eng. Mech.*, 128(2), 143–155.
Ead, S. A., Rajaratnam, N., Katopodis, C., and Ade, F. (2000). "Turbulent open-channel flow in circular corrugated culverts." *J. Hydraul. Eng.*, 126(10), 750–757.
Kotsovinos, N. E. (1978). "A note on the conservation of the axial momentum of a turbulent jet." *J. Fluid Mech.*, 87(7), 55–63.
Launder, B. E., and Rodi, W. (1981). "The turbulent wall jet." *Prog. Aerosp. Sci.*, 19, 81–128.
Rajaratnam, N. (1967a). "Plane turbulent wall jets on rough boundaries." *I. Water Power*, 149–153.
Rajaratnam, N. (1967b). "II." *Water Power*, 196–201.
Rajaratnam, N. (1967d). "Hydraulic jumps." *Advances in hydroscience*, V. T. Chow ed., Academic, New York, Vol. 4, 197–280.
Rajaratnam, N. (1976). *Turbulent jets*, Elsevier, Amsterdam, The Netherlands.
Schneider, W. (1985). "Decay of momentum flux in submerged jets." *J. Fluid Mech.*, 154(5), 91-110.
Schlichting, H. (1979). *Boundary layer theory*, 4th Ed. McGraw–Hill, New York.
Schwarz, W. H., and Cosart, W. P. (1961). "Two-dimensional turbulent wall jet." *J. Fluid Mech.*, 10(4) 481–495.
Wu, S., and Rajaratnam, N. (1995). "Free jumps, submerged jumps, and wall jets." *J. Hydraul. Res.*, 33(2), 177-212.

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