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A novel dropout compensation scheme for control of networked T–S fuzzy dynamic systems ☆

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Abstract

This paper considers the problem of H_∞ state feedback control for networked nonlinear systems under unreliable communication links with packet dropouts. The nonlinear plant in this paper is described by a Takagi–Sugeno (T–S) fuzzy model. The packet dropouts in both sensor-to-controller (S/C) and controller-to-actuator (C/A) channels are considered, which are modeled by Bernoulli processes. A new compensation scheme for the estimation of missing packets is proposed, and a piecewise state feedback controller is designed so that the resulting closed-loop control system is stochastically stable with guaranteed H_∞ performance. Then the results are extended to the case when both network-induced delays and packet dropouts exist in communication links. Finally, some numerical examples are given to illustrate the procedure of the proposed approaches and the optimal H_∞ performance in comparison to the existing approaches.

Keywords: T–S fuzzy models; Networked control systems; State feedback; Dropouts; Time-delay; Piecewise Lyapunov functions

1. Introduction

In recent decades, great effort has been devoted to model-based fuzzy control systems [1–5]. In particular, Takagi–Sugeno (T–S) fuzzy models [6] have been widely studied. This model describes a nonlinear system by a group of local linear systems, which are blended by several IF-THEN rules [7,8]. Therefore, T–S fuzzy models provide a basis for systematic stability analysis and synthesis of nonlinear control systems by applying conventional control theory. At the early stage of the research on T–S fuzzy control systems, some basic stability analysis and controller design results are proposed [9–11]. The basic idea of these methods is to design a feedback controller for each local

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model and construct a global controller from the local controllers to guarantee the closed-loop performance. However, the stability criterion is mostly based on common quadratic Lyapunov functions, which is proved to be conservative when dealing with highly complex nonlinear systems [12]. Some less conservative methods are then proposed, which are based on piecewise or fuzzy quadratic Lyapunov functions [13,14]. Piecewise quadratic Lyapunov functions are represented by several Lyapunov functions in different regions according to the partition method to the controlled plant. It is noted that common quadratic Lyapunov functions are a special case of piecewise quadratic Lyapunov functions when the same Lyapunov function is utilized in different regions. Another typical less conservative Lyapunov function used in T–S fuzzy control is the fuzzy quadratic Lyapunov function, which is grouped by several local Lyapunov functions via the same membership function as the controlled plant. Obviously, fuzzy Lyapunov functions will reduce to common Lyapunov functions when the same local Lyapunov functions are selected.

Recently, networked control systems (NCSs) have attracted great attention because of their theoretical and practical significance. NCSs are feedback control systems in which the control loops are closed through a real-time network [15,16]. The actuators, controller and sensors of a physical plant are distributed in a large physical space in NCSs, so they have giant advantages over traditional control systems, such as lower costs and more convenience for installation, extension and remote control [17,18]. However, communication links are usually unreliable due to media access constraint, data rate constraint, network-induced delays, packet dropouts and so on, which will degrade the performance of the closed-loop systems or even make them unstable in some cases [15]. To deal with these problems, numerous methods have been proposed [19–30].

Packet dropout is one of the critical problems of controller design for NCSs, which occurs if there exist packet collisions, buffer overflows or other network congestions [16]. Many researchers have studied the modeling and control of the systems with packet dropouts, and several approaches on the modeling have been reported, such as those based on switching systems [31], asynchronous dynamic systems [18] or jump linear systems with Markov chains [19]. These methods for modeling packet dropout phenomenon can be classified into two categories. One can be called *zero strategy* [18,32,33], in which the missing packet is set to be zero without any compensation. This strategy has a relatively simple mathematical expression for analysis and synthesis of control systems, but the overall system will be open loop for a period of time if packets are lost continuously. Obviously, it will degrade the system performance or even make it unstable if the packet-loss rate is high. The other is called *hold strategy* [19,31], in which the data at last sampling time are held when the current packet is lost during the transmission. Different from the zero strategy, it does not result in the open-loop scenario. However, the controller performance in this case might not be satisfactory if multiple packet dropouts phenomena occur, because the system control inputs are not updated frequently enough. Therefore, to achieve better system performance, a better strategy is needed to deal with the packet dropout phenomenon. Moreover, most of the results on NCSs with packet dropouts reported in literature consider only linear plants [15–17,19,34]. Though there are some works on fuzzy-model-based nonlinear NCSs, they are all based on zero/hold strategy [18,35]. It is thus significant for us to consider a new strategy in order to achieve better performance of T–S fuzzy systems under the unsatisfactory network environment in practice, which motives our research.

In this paper, we propose a new compensation method for fuzzy-model-based nonlinear NCSs to deal with packet dropouts in both S/C and C/A channels. The nonlinear physical plant is described by a T–S fuzzy model. By utilizing a piecewise Lyapunov function method, the H_∞ controller is obtained. The result is then extended to the case when both network-induced delay and packet dropout phenomena exist. Finally, two examples are given to show that our strategy is effective and able to achieve better performance in comparison to the zero/hold strategies. The contributions of this paper can be summarized as follows: (1) *a new approach to solving the H_∞ control design problem of the T–S fuzzy control system with packet dropouts in both S/C and C/A channels is proposed*; (2) *the optimal H_∞ performance is achieved by utilizing the proposed compensation approach*; (3) *the proposed approaches can deal with the case when both packet dropouts and network-induced delay phenomena exist.*

The remainder of the paper is organized as follows. Section 2 is devoted to problem formulation and the description of the novel compensation strategy. Section 3 presents the H_∞ analysis and synthesis results based on a piecewise quadratic Lyapunov function. Section 4 presents an extension to the case when both network-induced delay and packet dropout phenomena exist. In Section 5, two simulation examples are given to illustrate the effectiveness of the proposed approaches and show better performance in comparison with the existing methods. Finally, a conclusion is drawn in Section 6.

Notations: The notations utilized in this paper are standard. The superscript "T" stands for matrix transpose; $\Re^n$ denotes the n-dimensional Euclidean space; $\Re^{m\times n}$ is the set of all real m by n matrices. We use an asterisk (*) to

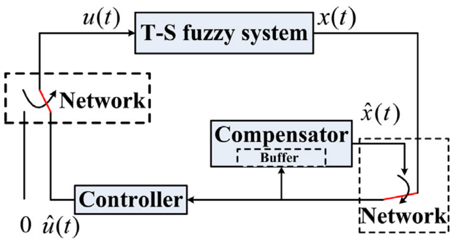


Fig. 1. Structure of the networked T–S fuzzy system.

represent a term that can be induced by symmetry in symmetric matrices. diag{...} stands for a block-diagonal matrix. The notation $P > 0$ means that P is real symmetric and positive definite. **I** and **0** denote the identity matrix and zero matrix, respectively. In addition, $\mathbb{E}\{x\}$ and $\mathbb{E}\{x \mid y\}$ denote expectation of x and x conditional on y, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Model description and problem formulation

In this paper, we focus on a class of T–S fuzzy systems with packet dropouts in communication links as illustrated in Fig. 1. Note that the packet dropout phenomenon exists both in S/C and C/A channels. Therefore, the inputs to the controller $\hat{x}(t)$ are not the same as the states of the controlled plant $x(t)$, and the control inputs to the plant $u(t)$ are also different from the outputs of the controller $\hat{u}(t)$. Now, we model the physical plant, dynamic compensator and controller mathematically.

2.1. Physical plant

The T–S fuzzy-model-based physical plant in this paper is given by:
Plant rule $\mathscr{R}^l$: If $\theta_1(t)$ is F_1^l and $\theta_2(t)$ is F_2^l and $\cdots$ and $\theta_v(t)$ is F_v^l, then

$$\begin{cases} x(t+1) = A_l x(t) + B_l u(t) + D_l w(t), \\ z(t) = L_l x(t) + H_l u(t), \quad l \in \mathscr{I} := \{1, 2, \ldots, r\}, \end{cases} \tag{1}$$

where $\mathscr{R}^l$ denotes the lth fuzzy inference rule, r the number of inference rules, F_φ^l ($\varphi = 1, 2, \ldots, v$) the fuzzy sets, $x(t) \in \Re^{n_x}$ the state vector, $u(t) \in \Re^{n_u}$ the input vector, $z(t) \in \Re^{n_z}$ the regulated output vector, $w(t) \in \Re^{n_w}$ the disturbance input vector, $[\theta_1(t), \theta_2(t), \ldots, \theta_v(t)]$ the premise variables, which are some measurable variables of the system such as the state variables, and $(A_l, B_l, D_l, L_l, H_l)$ denotes the matrices of the system's lth local model.

By using a standard fuzzy inference method, which includes a singleton fuzzifier, product fuzzy inference and center-average defuzzier, the T–S fuzzy system in (1) can be inferred as,

$$\begin{cases} x(t+1) = A(\mu)x(t) + B(\mu)u(t) + D(\mu)w(t), \\ z(t) = L(\mu)x(t) + H(\mu)u(t), \end{cases} \tag{2}$$

where

$$A(\mu) = \sum_{l=1}^{r} \mu_l A_l, \qquad B(\mu) = \sum_{l=1}^{r} \mu_l B_l, \qquad D(\mu) = \sum_{l=1}^{r} \mu_l D_l,$$
$$L(\mu) = \sum_{l=1}^{r} \mu_l L_l, \qquad H(\mu) = \sum_{l=1}^{r} \mu_l H_l, \tag{3}$$

and μ_l is the normalized membership function satisfying

$$\mu_l = \frac{\zeta_l(\theta)}{\sum_{\varphi=1}^{r} \zeta_\varphi(\theta)}, \qquad \zeta_l(\theta) = \prod_{\varphi=1}^{v} F_\varphi^l(\theta_\varphi), \qquad \mu_l \geqslant 0, \qquad \sum_{l=1}^{r} \mu_l = 1 \tag{4}$$

with $F_\varphi^l(\theta_\varphi)$ representing the grade of membership of θ_φ in the fuzzy set F_φ^l.

In order to investigate the robust H_∞ state feedback control problem for system (1) based on piecewise Lyapunov functions, the premise variable space is partitioned into a number of polyhedral regions $\{S_i\}_{i\in\bar{\mathscr{I}}} \subseteq \Re^{v}$, which are divided into crisp and fuzzy regions with the following definitions, respectively,

$$S_{i_c} = \{\theta(t) \mid \mu_m(\theta(t)) = 1,\ m \in \aleph(i_c)\}, \quad i_c \in \bar{\mathscr{I}}, \tag{5}$$

and

$$S_{i_f} = \{\theta(t) \mid 0 \leqslant \mu_m(\theta(t)) < 1,\ m \in \aleph(i_f)\}, \quad i_f \in \bar{\mathscr{I}}, \tag{6}$$

where $\aleph(i_c), \aleph(i_f)$ are sets containing the indices of rules in each region, and $\bar{\mathscr{I}}$ is the set of polyhedral regions. Local models act in crisp or fuzzy regions: in crisp regions the dynamic system is described by one local model and in fuzzy regions the dynamic system is determined by a blending of several local models. The partition method is not unique, one feasible approach is to set crisp regions when the dynamic system is governed by one local model, and set different fuzzy regions when the dynamics are determined by different local models according to the membership functions. Therefore, the rules of T–S fuzzy systems induce polyhedral partition regions. An example to set polyhedral regions for a practical system will be described in Example 5.1 in Section 5.

With such a partition method, the global fuzzy control system (2) is rewritten in each region as

$$\begin{cases} x(t+1) = \mathcal{A}_i x(t) + \mathcal{B}_i u(t) + \mathcal{D}_i w(t), \\ z(t) = \mathcal{L}_i x(t) + \mathcal{H}_i u(t), \quad x(t) \in S_i, i \in \bar{\mathscr{I}}, \end{cases} \tag{7}$$

where

$$\begin{aligned} &\mathcal{A}_i = \sum_{m\in\aleph(i)} \mu_m A_m, \qquad \mathcal{B}_i = \sum_{m\in\aleph(i)} \mu_m B_m, \qquad \mathcal{D}_i = \sum_{m\in\aleph(i)} \mu_m D_m, \\ &\mathcal{L}_i = \sum_{m\in\aleph(i)} \mu_m L_m, \qquad \mathcal{H}_i = \sum_{m\in\aleph(i)} \mu_m H_m \end{aligned} \tag{8}$$

with $0 \leqslant \mu_m(\theta(t)) \leqslant 1$, $\sum_{m\in\aleph(i)} \mu_m(\theta(t)) = 1$.

In order to carry out the controller design based on piecewise Lyapunov functions, we also define a set $\mathscr{T}$ that represents all possible transitions among regions as follows:

$$\mathscr{T} := \{(i,j) \mid \theta(t) \in S_i,\ \theta(t+1) \in S_j,\ \forall i, j \in \bar{\mathscr{I}}\}. \tag{9}$$

The states remain in the same region S_i in the case of $j = i$, $(i,j) \in \mathscr{T}$. Otherwise, the states transit from region S_i to S_j.

2.2. Dynamic compensator

Before presenting the compensation results, it is assumed in this section that there do not exist network-induced delay effects in communication links, and only packet dropout phenomenon is considered in the unreliable transmission [17].

The dynamic compensator in S/C channel has the following form,

$$\hat{x}(t+1) = \alpha(t)\bar{A}_i x(t) + (1-\alpha(t))\bar{A}_s \hat{x}(t), \quad i, s \in \bar{\mathscr{I}}, \tag{10}$$

where $\hat{x}(t)$ is the compensated state, $\bar{A}_i$, $\bar{A}_s$, $i, s \in \bar{\mathscr{I}}$ are compensator gains in different regions to be determined, and $\{\alpha(t)\}$ is a Bernoulli process representing the unreliable transmission in S/C channel. Moreover, $\alpha(t) = 0$ when the data are lost, while $\alpha(t) = 1$ means a successful transmission.

A natural assumption on $\alpha(t)$ can be made as follows:

$$\mathrm{Prob}\{\alpha(t) = 1\} = \mathbb{E}\{\alpha(t)\} = \bar{\alpha}, \qquad \mathrm{Prob}\{\alpha(t) = 0\} = 1 - \bar{\alpha}. \tag{11}$$

For the compensated state $\hat{x}(t)$, we also define a set $\hat{\mathscr{T}}$ to describe the region transitions:

$$\hat{\mathscr{T}} = \{(s,k) \mid \hat{x}(t) \in S_s,\ \hat{x}(t+1) \in S_k,\ s, k \in \bar{\mathscr{I}}\}. \tag{12}$$

Remark 2.1. There is a memory in the compensator node such that the missing states $x(t+1)$ can be estimated as $\hat{x}(t+1)$ by historical states stored in the memory according to (10).

Remark 2.2. The dynamic compensator is event driven, that is, the compensated state $\hat{x}(t+1)$ will be estimated by $x(t)$ if the packet carrying $x(t)$ arrives at the compensator node at sampling time t, otherwise, $\hat{x}(t)$ is utilized for the estimation. Obviously, the compensator is still able to estimate the state when the multiple packet dropouts phenomenon occurs.

Remark 2.3. It is noted that the states of a physical plant are available at the physical plant node instantly, but not always available in the compensation node due to the unreliable transmission. Therefore, the compensated states are utilized for determining the region where the compensator stays when the packet dropout phenomenon occurs, and the region information of $x(t)$ is no longer needed at the compensator node. In this case, the plant states and the compensated states might not be in the same region, thus the region transitions of the physical plant and compensator might not be synchronized, which are defined in (9) and (12), respectively. Therefore, the compensator in (10) utilizes compensation gains in different regions according to the plant states and compensated ones. More specifically, $\bar{A}_i$ and $\bar{A}_s$ are compensation gains in regions S_i and S_s, respectively.

Remark 2.4. It is noted from [36] that the compensator will be identical to a linear compensator if we design a fuzzy compensator based on conventional parallel distributed compensation (PDC) [22] when the premise variables between the physical plant and the designed fuzzy filter are different. Therefore, the piecewise compensator in (10) is designed, which is less conservative.

Remark 2.5. The proposed compensation scheme is more general than the traditional hold and zero strategies. More specifically, hold strategy is a special case when $\bar{A}_i = \bar{A}_s = \mathbf{I}$ in (10), and zero strategy is a special case when $\bar{A}_i = \bar{A}_s = \mathbf{0}$.

Remark 2.6. Methods utilizing the hold strategy need an assumption that the maximum dropout step is bounded when dealing with multiple packet dropouts [19,37]. Actually, we are not always able to estimate the maximum packet-loss upper bound beforehand in practical situation. However, this assumption is not needed in our approach.

2.3. Controller

Based on the compensator given in (10), the following piecewise controller is considered,

$$\hat{u}(t) = \alpha(t)K_i x(t) + \big(1-\alpha(t)\big)K_s\hat{x}(t), \quad i, s \in \bar{\mathscr{I}}, \tag{13}$$

where $K_i, K_s, i, s \in \bar{\mathscr{I}}$ are controller gains in different regions to be determined.

Remark 2.7. Similar to Remark 2.3, it is also noted that the states of physical plant are not always available in the controller node because of the unreliable transmission. Therefore, the compensated state $\hat{x}(t)$ will be employed for the controller design if the packet carrying $x(t)$ is dropped during the transmission. Otherwise, $x(t)$ is utilized. It is also noted that the plant states and the compensated states might not be in the same region. Therefore, the controller in (13) utilizes controller gains in different regions according to the plant states and compensated ones, which is similar to the compensator given in (10). More specifically, K_i and K_s are controller gains in regions S_i and S_s, respectively.

Remark 2.8. The controller in (13) is rather different from those given in some existing results, such as [33,38], which utilize a parallel distributed compensator (PDC) with the assumption that the premise variables of the physical plant are always available at the controller node. However, this assumption is unpractical under network-based circumstances. Moreover, it is also assumed that there exists a perfect communication link so that the region index information of the physical plant can be sent to the receiving node in [37]. However, this assumption is also not needed in this paper, which is more practical.

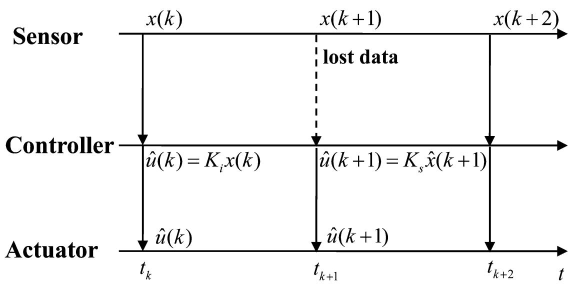


Fig. 2. The time-sequence diagram of the signals in the closed-loop system.

2.4. Closed-loop system

We model the packet dropouts in C/A channel as follows,

$$u(t)=\beta(t)\hat{u}(t), \tag{14}$$

where $\{\beta(t)\}$ is a Bernoulli process representing the unreliable transmission in C/A channel, which is independent of $\{\alpha(t)\}$. More specifically, $\beta(t)=0$ when the packet is dropped, while $\beta(t)=1$ indicates a successful transmission.

Similar to (11), $\beta(t)$ is assumed to have the following stochastic properties:

$$\text{Prob}\{\beta(t)=1\}=\mathbb{E}\{\beta(t)\}=\bar{\beta}, \qquad \text{Prob}\{\beta(t)=0\}=1-\bar{\beta}. \tag{15}$$

Additionally, we define $\tilde{\alpha}(t)=\alpha(t)-\bar{\alpha}$ and $\tilde{\beta}(t)=\beta(t)-\bar{\beta}$. It is clear that

$$\mathbb{E}\{\tilde{\alpha}(t)\}=0, \qquad \mathbb{E}\{\tilde{\alpha}(t)\tilde{\alpha}(t)\}=\sigma_1^2, \qquad \mathbb{E}\{\tilde{\beta}(t)\}=0, \qquad \mathbb{E}\{\tilde{\beta}(t)\tilde{\beta}(t)\}=\sigma_2^2, \tag{16}$$

where $\sigma_1=\sqrt{\bar{\alpha}(1-\bar{\alpha})}$, $\sigma_2=\sqrt{\bar{\beta}(1-\bar{\beta})}$.

Then based on (10), (13) and (14), the physical plant (7) is rewritten as,

$$\begin{cases} x(t+1)=\big(\mathcal{A}_i+\alpha(t)\beta(t)\mathcal{B}_iK_i\big)x(t)+\big(1-\alpha(t)\big)\beta(t)\mathcal{B}_iK_s\hat{x}(t)+\mathcal{D}_iw(t), \\ z(t)=\big(\mathcal{L}_i(t)+\alpha(t)\beta(t)\mathcal{H}_iK_i\big)x(t)+\big(1-\alpha(t)\big)\beta(t)\mathcal{H}_iK_s\hat{x}(t), \quad i,s\in\bar{\mathscr{I}}. \end{cases} \tag{17}$$

From (10) and (17), we have the following augmented closed-loop system,

$$\begin{cases} \xi(t+1)=\mathscr{A}_{is}(t)\xi(t)+\mathscr{D}_iw(t), \\ z(t)=\mathscr{L}_{is}(t)\xi(t), \quad i,s\in\bar{\mathscr{I}}, \end{cases} \tag{18}$$

where

$$\begin{aligned}
&\xi(t)=\begin{bmatrix} x^T(t) & \hat{x}^T(t)\end{bmatrix}^T, \qquad \mathscr{D}_i=\begin{bmatrix}\mathcal{D}_i^T & \mathbf{0}\end{bmatrix}^T, \\
&\mathscr{A}_{is}(t)=\tilde{\alpha}(t)\mathscr{A}_{is1}+\tilde{\beta}(t)\mathscr{A}_{is2}+\tilde{\alpha}(t)\tilde{\beta}(t)\mathscr{A}_{is3}+\mathscr{A}_{is4}, \mathscr{L}_{is}(t)=\tilde{\alpha}(t)\mathscr{L}_{is1}+\tilde{\beta}(t)\mathscr{L}_{is2}+\tilde{\alpha}(t)\tilde{\beta}(t)\mathscr{L}_{is3}+\mathscr{L}_{is4}, \\
&\mathscr{A}_{is1}=\begin{bmatrix}\bar{\beta}\mathcal{B}_iK_i & -\bar{\beta}\mathcal{B}_iK_s \\ \bar{A}_i & -\bar{A}_s\end{bmatrix}, \qquad \mathscr{A}_{is2}=\begin{bmatrix}\bar{\alpha}\mathcal{B}_iK_i & (1-\bar{\alpha})\mathcal{B}_iK_s \\ \mathbf{0} & \mathbf{0}\end{bmatrix}, \\
&\mathscr{A}_{is3}=\begin{bmatrix}\mathcal{B}_iK_i & -\mathcal{B}_iK_s \\ \mathbf{0} & \mathbf{0}\end{bmatrix}, \qquad \mathscr{A}_{is4}=\begin{bmatrix}\mathcal{A}_i+\bar{\alpha}\bar{\beta}\mathcal{B}_iK_i & (1-\bar{\alpha})\bar{\beta}\mathcal{B}_iK_s \\ \bar{\alpha}\bar{A}_i & (1-\bar{\alpha})\bar{A}_s\end{bmatrix}, \\
&\mathscr{L}_{is1}=\begin{bmatrix}\bar{\beta}\mathcal{H}_iK_i & -\bar{\beta}\mathcal{H}_iK_s\end{bmatrix}, \qquad \mathscr{L}_{is2}=\begin{bmatrix}\bar{\alpha}\mathcal{H}_iK_i & (1-\bar{\alpha})\mathcal{H}_iK_s\end{bmatrix}, \\
&\mathscr{L}_{is3}=\begin{bmatrix}\mathcal{H}_iK_i & -\mathcal{H}_iK_s\end{bmatrix}, \qquad \mathscr{L}_{is4}=\begin{bmatrix}\mathcal{L}_i+\bar{\alpha}\bar{\beta}\mathcal{H}_iK_i & (1-\bar{\alpha})\bar{\beta}\mathcal{H}_iK_s\end{bmatrix}.
\end{aligned} \tag{19}$$

The time-sequence diagram of the signals in the closed-loop system is illustrated in Fig. 2.

2.5. Problem formulation

Before proceeding further, some basic definitions are introduced as follows.

Definition 2.1. (See [37].) The closed-loop fuzzy system in (18) is said to be stochastically stable in the mean square sense if, when $w(t) \equiv 0$ and for any initial condition $\xi(0)$, there is a finite matrix $Y > 0$ such that

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\left|\xi(t)\right|^2 \,\middle|\, \xi(0)\right\} < \xi^T(0)Y\xi(0). \tag{20}$$

Definition 2.2. (See [37].) The closed-loop fuzzy system in (18) is stochastically stable with guaranteed H_∞ performance γ if the following two conditions are satisfied:

2.5.1. Stochastic stability

The closed-loop fuzzy system (18) is stochastically stable in the sense of Definition 2.1.

2.5.2. H_∞ performance

Under zero-initial conditions, the controlled output $z(t)$ satisfies

$$\|z\|_E \leqslant \gamma\|w\|_2, \tag{21}$$

where $\|z\|_E := \mathbb{E}\{\sqrt{\sum_{t=0}^{\infty}|z(t)|^2}\}$, and $\gamma > 0$ is a prescribed scalar.

The problem to be addressed in this paper is described as follows.

***H_∞* Dropout Compensator and Controller Design Problem.** Consider the fuzzy system in (1) and suppose that the network parameters $\bar{\alpha}$ and $\bar{\beta}$ are given. Design a compensator and controller in the form of (10) and (13) respectively such that the augmented system (18) is stochastically stable with guaranteed H_∞ performance γ by Definition 2.2.

3. Main results

In this section, the piecewise compensator and H_∞ state feedback controller analysis and synthesis problem of the fuzzy system (7) is investigated, which is solved by a linear matrix inequality (LMI) approach based on piecewise Lyapunov functions.

The following lemma presents a condition to guarantee the stochastic stability and H_∞ performance of the closed-loop system (18).

Lemma 3.1. *Consider the system* (1) *and suppose that the compensator and controller gain matrices $\bar{A}_s$ and $K_s, s \in \bar{\mathscr{I}}$ of the local compensators in* (10) *and controllers in* (13) *are given. The augmented system* (18) *is stochastically stable with guaranteed H_∞ performance γ, if there exist matrices $X_{is} = X_{is}^T > 0, i, s \in \bar{\mathscr{I}}$ satisfying*

$$\begin{bmatrix} \Gamma_1 & \mathbf{0} & \Gamma_2 X_{is} & \mathbf{0} \\ * & \Gamma_3 & \Gamma_4 X_{is} & \Gamma_5 \\ * & * & -X_{is} & \mathbf{0} \\ * & * & * & -I \end{bmatrix} < 0, \quad i, s \in \bar{\mathscr{I}}, \tag{22}$$

where

$$\begin{aligned} &\Gamma_1 = diag\{-\gamma^2 I, -\gamma^2 I, -\gamma^2 I, -\gamma^2 I\}, \qquad \Gamma_2 = \begin{bmatrix} \sigma_1\mathscr{L}_{is1}^T & \sigma_2\mathscr{L}_{is2}^T & \sigma_1\sigma_2\mathscr{L}_{is3}^T & \mathscr{L}_{is4}^T \end{bmatrix}^T, \\ &\Gamma_3 = diag\{-X_{jk}, -X_{jk}, -X_{jk}, -X_{jk}\}, \qquad \Gamma_4 = \begin{bmatrix} \sigma_1\mathscr{A}_{is1}^T & \sigma_2\mathscr{A}_{is2}^T & \sigma_1\sigma_2\mathscr{A}_{is3}^T & \mathscr{A}_{is4}^T \end{bmatrix}^T, \\ &\Gamma_5 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathscr{D}_i^T \end{bmatrix}^T. \end{aligned} \tag{23}$$

Proof. Consider the following piecewise Lyapunov function,

$$V(t) = \xi^T(t)X_{is}^{-1}\xi(t), \quad i, s \in \bar{\mathscr{I}}, \tag{24}$$

where $X_{is}^T = X_{is} > 0$ are Lyapunov matrices to be determined.

It is known that the closed-loop system in (18) can be demonstrated stochastically stable in the mean square sense with H_∞ performance γ under zero initial conditions by proving the following index J is negative:

$$J = \mathbb{E}\{V(t+1) \mid \eta(t)\} + \mathbb{E}\{\gamma^{-2} z^T(t) z(t) \mid \eta(t)\} - \xi^T(t) X_{is}^{-1} \xi(t) - w^T(t) w(t), \tag{25}$$

where $\eta(t) = [\, \xi^T(t) \;\; w^T(t) \,]^T$.

From (19), we have

$$\begin{aligned} J &= \mathbb{E}\left\{\eta^T(t) \begin{bmatrix} \mathscr{A}_i^T \\ \mathscr{D}_i^T \end{bmatrix} X_{jk}^{-1} [\, \mathscr{A}_i \quad \mathscr{D}_i \,] \eta(t) \,\middle|\, \eta(t)\right\} \\ &\quad + \mathbb{E}\{\gamma^{-2} \xi^T(t) (\tilde{\alpha}(t) \mathscr{L}_{is1}^T + \tilde{\beta}(t) \mathscr{L}_{is2}^T + \tilde{\alpha}(t)\tilde{\beta}(t) \mathscr{L}_{is3}^T + \mathscr{L}_{is4}^T) \\ &\quad \times (\tilde{\alpha}(t) \mathscr{L}_{is1} + \tilde{\beta}(t) \mathscr{L}_{is2} + \tilde{\alpha}(t)\tilde{\beta}(t) \mathscr{L}_{is3} + \mathscr{L}_{is4}^T) \xi(t) \mid \eta(t)\} - \eta^T(t) \mathrm{diag}\{X_{is}^{-1}, \mathrm{I}\} \eta(t) \\ &= \eta^T(t)\{-[\, \Gamma_4 \quad \Gamma_5 \,]^T \Gamma_3^{-1} [\, \Gamma_4 \quad \Gamma_5 \,] - [\, \Gamma_2 \quad \mathbf{0} \,]^T \Gamma_1^{-1} [\, \Gamma_2 \quad \mathbf{0} \,] - \mathrm{diag}\{X_{is}^{-1}, \mathrm{I}\}\} \eta(t). \end{aligned} \tag{26}$$

Then by the Schur complement and (22), we have $J < 0$. The proof is thus completed. □

Remark 3.1. A piecewise quadratic Lyapunov function is utilized in Lemma 3.1, which is defined in (24). It is noted that (24) will reduce to the common quadratic Lyapunov function if $X_{is} = X$ for any $i, s \in \bar{\mathscr{I}}$.

In terms of Lemma 3.1, now we present the controller synthesis result in the following theorem.

Theorem 3.1. *Consider system* (1). *The closed-loop system* (18) *is stochastically stable with guaranteed H_∞ performance γ, if there exist matrices $X_{is} = X_{is}^T > 0$, Q_s, $\bar{G}$, R_s, $i, s \in \bar{\mathscr{I}}$ such that the following linear matrix inequalities are satisfied*:

$$\begin{bmatrix} \Gamma_1 & \mathbf{0} & \bar{\Gamma}_2 & \mathbf{0} \\ * & \Gamma_3 & \bar{\Gamma}_4 & \bar{\Gamma}_5 \\ * & * & X_{is} - \bar{G} - \bar{G}^T & \mathbf{0} \\ * & * & * & -I \end{bmatrix} < 0, \quad i, s \in \bar{\mathscr{I}}, \tag{27}$$

where

$$\begin{aligned} X_{is} &= \begin{bmatrix} X_{is11} & X_{is12} \\ * & X_{is22} \end{bmatrix}, \qquad \bar{G} = \begin{bmatrix} G & G \\ G & G \end{bmatrix}, \qquad \bar{D}_m = \begin{bmatrix} D_m \\ \mathbf{0} \end{bmatrix}, \\ \bar{\Gamma}_2 &= \begin{bmatrix} \sigma_1 \Pi_{mis5} \\ \sigma_2 \Pi_{mis6} \\ \sigma_1\sigma_2 \Pi_{mis7} \\ \Pi_{mis8} \end{bmatrix}, \qquad \bar{\Gamma}_4 = \begin{bmatrix} \sigma_1 \Pi_{mis1} \\ \sigma_2 \Pi_{mis2} \\ \sigma_1\sigma_2 \Pi_{mis3} \\ \Pi_{mis4} \end{bmatrix}, \qquad \bar{\Gamma}_5 = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \bar{D}_m \end{bmatrix}, \\ \Pi_{mis1} &= \begin{bmatrix} \bar{\beta} B_m Q_i - \bar{\beta} B_m Q_s & \bar{\beta} B_m Q_i - \bar{\beta} B_m Q_s \\ R_i - R_s & R_i - R_s \end{bmatrix}, \qquad \Pi_{mis3} = \begin{bmatrix} B_m Q_i - B_m Q_s & B_m Q_i - B_m Q_s \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \\ \Pi_{mis2} &= \begin{bmatrix} \bar{\alpha} B_m Q_i + (1-\bar{\alpha}) B_m Q_s & \bar{\alpha} B_m Q_i + (1-\bar{\alpha}) B_m Q_s \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \\ \Pi_{mis4} &= \begin{bmatrix} A_m G + \bar{\alpha}\bar{\beta} B_m Q_i + (1-\bar{\alpha})\bar{\beta} B_m Q_s & A_m G + \bar{\alpha}\bar{\beta} B_m Q_i + (1-\bar{\alpha})\bar{\beta} B_m Q_s \\ \bar{\alpha} R_i + (1-\bar{\alpha}) R_s & \bar{\alpha} R_i + (1-\bar{\alpha}) R_s \end{bmatrix}, \\ \Pi_{mis5} &= [\, \bar{\beta} H_m Q_i - \bar{\beta} H_m Q_s \quad \bar{\beta} H_m Q_i - \bar{\beta} H_m Q_s \,], \qquad \Pi_{mis7} = [\, H_m Q_i - H_m Q_s \quad H_m Q_i - H_m Q_s \,], \\ \Pi_{mis6} &= [\, \bar{\alpha} H_m Q_i + (1-\bar{\alpha}) H_m Q_s \quad \bar{\alpha} H_m Q_i + (1-\bar{\alpha}) H_m Q_s \,], \\ \Pi_{mis8} &= [\, L_m G + \bar{\alpha}\bar{\beta} H_m Q_i + (1-\bar{\alpha})\bar{\beta} H_m Q_s \quad L_m G + \bar{\alpha}\bar{\beta} H_m Q_i + (1-\bar{\alpha})\bar{\beta} H_m Q_s \,]. \end{aligned} \tag{28}$$

Moreover, the controller and compensation gains are respectively given by

$$K_i = Q_i G^{-1}, \qquad \bar{A}_i = R_i G^{-1}, \quad i \in \bar{\mathscr{I}}. \tag{29}$$

Proof. According to Lemma 3.1, if there exist matrices $X_{is} > 0, i, s \in \bar{\mathscr{I}}$ satisfying (22), the closed-loop fuzzy system (18) is stochastically stable with guaranteed H_∞ performance γ. Based on (8), the left hand side of (22) can be rewritten as $\sum_{m\in\aleph(i)} \mu_m \Psi_{mjkis}$, $j, k, i, s \in \bar{\mathscr{I}}$, where

$$\Psi_{mjkis} = \begin{bmatrix} \Gamma_1 & \mathbf{0} & \breve{\Gamma}_2 X_{is} & \mathbf{0} \\ * & \Gamma_3 & \breve{\Gamma}_4 X_{is} & \bar{\Gamma}_5 \\ * & * & -X_{is} & \mathbf{0} \\ * & * & * & -\mathrm{I} \end{bmatrix},$$

$$\breve{\Gamma}_2 = \begin{bmatrix} \sigma_1 \bar{\mathscr{L}}^T_{mis1} & \sigma_2 \bar{\mathscr{L}}^T_{mis2} & \sigma_1\sigma_2 \bar{\mathscr{L}}^T_{mis3} & \bar{\mathscr{L}}^T_{mis4} \end{bmatrix}^T,$$

$$\breve{\Gamma}_4 = \begin{bmatrix} \sigma_1 \bar{\mathscr{A}}^T_{mis1} & \sigma_2 \bar{\mathscr{A}}^T_{mis2} & \sigma_1\sigma_2 \bar{\mathscr{A}}^T_{mis3} & \bar{\mathscr{A}}^T_{mis4} \end{bmatrix}^T,$$

$$\bar{\mathscr{A}}_{mis1} = \begin{bmatrix} \bar{\beta} B_m K_i & -\bar{\beta} B_m K_s \\ \bar{A}_i & -\bar{A}_s \end{bmatrix}, \qquad \bar{\mathscr{A}}_{mis2} = \begin{bmatrix} \bar{\alpha} B_m K_i & (1-\bar{\alpha}) B_m K_s \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\bar{\mathscr{A}}_{mis3} = \begin{bmatrix} B_m K_i & -B_m K_s \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \qquad \bar{\mathscr{A}}_{mis4} = \begin{bmatrix} A_m + \bar{\alpha}\bar{\beta} B_m K_i & (1-\bar{\alpha})\bar{\beta} B_m K_s \\ \bar{\alpha} \bar{A}_i & (1-\bar{\alpha}) \bar{A}_s \end{bmatrix},$$

$$\bar{\mathscr{L}}_{mis1} = \begin{bmatrix} \bar{\beta} H_m K_i & -\bar{\beta} H_m K_s \end{bmatrix}, \qquad \bar{\mathscr{L}}_{mis2} = \begin{bmatrix} \bar{\alpha} H_m K_i & (1-\bar{\alpha}) H_m K_s \end{bmatrix},$$

$$\bar{\mathscr{L}}_{mis3} = \begin{bmatrix} H_m K_i & -H_m K_s \end{bmatrix}, \qquad \bar{\mathscr{L}}_{mis4} = \begin{bmatrix} L_m + \bar{\alpha}\bar{\beta} H_m K_i & (1-\bar{\alpha})\bar{\beta} H_m K_s \end{bmatrix}. \tag{30}$$

We can see from (30) that the compensation matrices are coupled with the Lyapunov matrices, which is difficult for the controller synthesis. To facilitate the controller design, we introduce an additional slack matrix $\bar{G} = [\begin{smallmatrix} G & G \\ G & G \end{smallmatrix}]$.

Then, post- and pre-multiplying Ψ_{mjkis} by $diag\{\mathrm{I}, \mathrm{I}, X_{is}^{-1}\bar{G}, \mathrm{I}\}$ and its transpose, respectively, lead to

$$\begin{bmatrix} \Gamma_1 & \mathbf{0} & \breve{\Gamma}_2 \bar{G} & \mathbf{0} \\ * & \Gamma_3 & \breve{\Gamma}_4 \bar{G} & \bar{\Gamma}_5 \\ * & * & -\bar{G}^T X_{is}^{-1} \bar{G} & \mathbf{0} \\ * & * & * & -\mathrm{I} \end{bmatrix} < 0, \quad i, s \in \bar{\mathscr{I}},\ m \in \aleph(i). \tag{31}$$

Note that

$$X_{is} - \bar{G} - \bar{G}^T + \bar{G}^T X_{is}^{-1} \bar{G} = \left(X_{is} - \bar{G}^T\right) X_{is}^{-1} (X_{is} - \bar{G}) \geqslant 0, \tag{32}$$

which implies

$$-\bar{G}^T X_{is}^{-1} \bar{G} \leqslant X_{is} - \bar{G} - \bar{G}^T. \tag{33}$$

Based on (33), the following inequality implies (31)

$$\begin{bmatrix} \Gamma_1 & \mathbf{0} & \breve{\Gamma}_2 \bar{G} & \mathbf{0} \\ * & \Gamma_3 & \breve{\Gamma}_4 \bar{G} & \bar{\Gamma}_5 \\ * & * & X_{is} - \bar{G} - \bar{G}^T & \mathbf{0} \\ * & * & * & -\mathrm{I} \end{bmatrix} < 0, \quad i, j \in \bar{\mathscr{I}},\ m \in \aleph(i). \tag{34}$$

We define $Q_i = K_i G$, $R_i = \bar{A}_i G$. It then follows that (27) implies (34).

Based on Lemma 3.1, the fuzzy system (18) is stochastically stable with H_∞ performance γ, and the proof is thus completed. □

Remark 3.2. If we let $\bar{A}_i = \bar{A}_s = \mathbf{I}$ or $\bar{A}_i = \bar{A}_s = \mathbf{0}$, the proposed dropout compensation results will reduce to hold or zero strategy-based ones, respectively. In hold strategy, the last packet $\hat{x}(t-1)$ stored in the buffer will be utilized for the controller design when the packet carrying $x(t)$ is lost. It is noted that $\hat{x}(t-1)$ and $x(t)$ may be in different regions, thus the asynchronous control method is also utilized [35]. In zero strategy, the inputs of the controller are assumed to be zero when packet dropout phenomenon occurs. In this case, the asynchronous control problem is not involved [33].

4. Extensions

Besides packet dropout phenomenon, network-induced delay is another important issue in study of networked control systems [16]. In order to deal with this issue, the following assumption is needed.

Assumption 4.1. The delays and consecutive steps of packet dropouts in S/C channel are bounded, say, less than N [39].

Based on the compensator in (10), we have the following compensation scheme:

$$\hat{x}(t) = \hat{x}(t \mid t_d), \quad t_d \leqslant t, \tag{35}$$

where $\hat{x}(t)$ is the compensated state at sampling time t if the current packet $x(t)$ does not arrive, and $\hat{x}(t \mid t_d)$ is the estimated state compensated by $x(t_d)$, which is the latest system state stored in the buffer.

We define $d(t) = t - t_d$, which is the time-delay of the packet in the controller caused by the network-induced delay and packet dropout phenomena in S/C channel, and it is assumed to satisfy $0 \leqslant d(t) \leqslant N$. Obviously, the time-delay in the buffer $d(t)$ is only related to $d(t-1)$ and the current packet received at time t.

Remark 4.1. A time stamp is added to the packet before it is transmitted into the network links in both S/C and C/A channels, and network delay $d(t)$ is measurable by comparing the time stamp of the latest packet with the current time instant.

Remark 4.2. It is noted that any packet which does not arrive in time will be compensated no matter it is delayed or dropped. When a delayed packet $x(t - d(t))$ arrives at sampling time t, it will be utilized for the estimation of $\hat{x}(t)$ if it is newer than the packet stored in the buffer. Otherwise, the packet $x(t - d(t))$ will be dropped and the original latest packet stored in the buffer will be utilized. Therefore, network-induced delays, packet dropouts and the packets out of sequence can be treated in the unified model simultaneously.

The dynamic compensator has the following form when the packet is delayed or lost during the transmission:

$$\begin{cases} \hat{x}(t+1 \mid t) = \bar{A}_{i_0} x(t), \\ \hat{x}(t+1 \mid t-1) = \bar{A}_{i_1} \hat{x}(t \mid t-1), \\ \quad \vdots \\ \hat{x}(t+1 \mid t-N) = \bar{A}_{i_N} \hat{x}(t \mid t-N), \end{cases} \tag{36}$$

where $\hat{x}(t \mid t-1), \ldots, \hat{x}(t \mid t-N)$ are states of the dynamic compensator estimated by the latest states stored in the buffer. $\bar{A}_{i_0}, \ldots, \bar{A}_{i_N}, i_0, \ldots, i_N \in \bar{\mathscr{I}}$ are compensator gains in different regions to be determined.

Remark 4.3. It is noted that the plant states and the estimated states compensated by different states stored in the buffer might not be in the same region, thus the compensator in (36) utilizes compensation gains in different regions according to the plant states and compensated ones. More specifically, $\bar{A}_{i_1}, \ldots, \bar{A}_{i_N}$ are compensation gains in region $S_{i_1}, \ldots, S_{i_N}$, respectively.

The following piecewise controller is utilized:

$$u(t) = \alpha_{p0}(t) K_{i_0} x(t) + \alpha_{p1}(t) K_{i_1} \hat{x}(t \mid t-1) + \cdots + \alpha_{pN}(t) K_{i_N} \hat{x}(t \mid t-N), \tag{37}$$

where

$$\begin{aligned} &\alpha_{pq}(t) = \alpha\big(d(t) = q, d(t-1) = p\big), \\ &\alpha_{pq}(t) = \begin{cases} 1, & d(t) = q, d(t-1) = p, \\ 0, & d(t) \neq q, d(t-1) = p. \end{cases} \end{aligned} \tag{38}$$

and $K_{i_0}, \ldots, K_{i_N}, i_0, \ldots, i_N \in \bar{\mathscr{I}}$ are the controller gains in different regions to be determined.

Remark 4.4. The proposed controller in (37) is similar to the model in [40]. However, [40] employs the hold strategy, where a delayed state $x(t-d(t))$ is utilized for the controller design at sampling time t. If we let $\hat{x}(t)=x(t-d(t))$ specifically, our compensation scheme will reduce to the model in [40].

Remark 4.5. Similar to the model in the second section, the premise variables are also not available at the compensator and controller nodes with the existence of packet dropout and network-induced delay phenomena. Therefore, the asynchronous approach is also employed.

We model the delays and packet dropouts in C/A channel as follows,

$$u(t)=\beta(t)\hat{u}(t), \tag{39}$$

where $\beta(t)=1$ when the packet carrying $\hat{u}(t)$ arrives at the actuator node in time. Otherwise, $\beta(t)=0$.

Natural assumptions can be made as follows:

$$\mathrm{Prob}\{\alpha_{pq}(t)=1\}=\mathbb{E}\{\alpha_{pq}(t)\}=\pi_{pq}, \qquad \mathrm{Prob}\{\beta(t)\}=\mathbb{E}\{\beta(t)\}=\beta, \tag{40}$$

and the transition probability matrix is

$$\mathbb{T}=\begin{bmatrix} \pi_{00} & \pi_{01} & \cdots & \pi_{0N} \\ \pi_{10} & \pi_{11} & \cdots & \pi_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{N0} & \pi_{N1} & \cdots & \pi_{NN} \end{bmatrix}. \tag{41}$$

Additionally, we define $\tilde{e}_{pq}(t)=e_{pq}(t)-\bar{e}_{pq}$, $p,q=0,1,\ldots,N$, where $e_{pq}(t)=\alpha_{pq}(t)\beta(t)$ and $\bar{e}_{pq}=\mathbb{E}\{e_{pq}(t)\}=\pi_{pq}\beta$. It is clear that

$$\mathbb{E}\{\tilde{e}_{pq}(t)\}=0, \qquad \mathbb{E}\{\tilde{e}_{pq}(t)\tilde{e}_{pq}(t)\}=\sigma_{pq}^2 \tag{42}$$

with $\sigma_{pq}=\sqrt{\bar{e}_{pq}(1-\bar{e}_{pq})}$.

Based on (1), (36), (37) and (39), the closed-loop fuzzy control system is expressed as follows:

$$\begin{cases} \xi(t+1)=\Omega_i(t)\xi(t)+\bar{\mathcal{D}}_i w(t), \\ z(t)=\Phi_i(t)\xi(t), \end{cases} \tag{43}$$

where

$$\begin{aligned}
&\xi(t)=\begin{bmatrix} x^T(t) & \hat{x}^T(t\mid t-1) & \cdots & \hat{x}^T(t\mid t-N+1) & \hat{x}^T(t\mid t-N) \end{bmatrix}^T, \\
&\Omega_i(t)=\tilde{e}_{p0}(t)\Omega_{i0}+\cdots+\tilde{e}_{pN}(t)\Omega_{iN}+\Omega_{ia}, \\
&\Phi_i(t)=\tilde{e}_{p0}(t)\Phi_{i0}+\cdots+\tilde{e}_{pN}(t)\Phi_{iN}+\Phi_{ia}, \qquad \bar{\mathcal{D}}_i=\begin{bmatrix} \mathcal{D}_{i_0}^T & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}^T, \\
&\Omega_{i0}=\begin{bmatrix} \mathcal{B}_{i_0}K_{i_0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \ldots, \quad \Omega_{iN}=\begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathcal{B}_{i_0}K_{i_N} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}, \\
&\Omega_{ia}=\begin{bmatrix} \mathcal{A}_{i_0}+\tilde{e}_{p0}\mathcal{B}_{i_0}K_{i_0} & \cdots & \tilde{e}_{pN-1}\mathcal{B}_{i_0}K_{i_{N-1}} & \tilde{e}_{pN}\mathcal{B}_{i_0}K_{i_N} \\ \bar{A}_{i_0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \cdots & \bar{A}_{i_{N-1}} & \mathbf{0} \end{bmatrix}, \\
&\Phi_{i0}=\begin{bmatrix} \mathcal{H}_{i_0}K_{i_0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \ldots, \quad \Phi_{iN}=\begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathcal{H}_{i_0}K_{i_N} \end{bmatrix}, \\
&\Phi_{ia}=\begin{bmatrix} \mathcal{L}_{i_0}+\hat{e}_{p0}\mathcal{H}_{i_0}K_{i_0} & \cdots & \hat{e}_{pN-1}\mathcal{H}_{i_0}K_{i_{N-1}} & \hat{e}_{pN}\mathcal{H}_{i}K_{i_N} \end{bmatrix}.
\end{aligned} \tag{44}$$

Based on the model above, we have the following stability analysis result.

Lemma 4.1. *Consider fuzzy system* (7) *and suppose that the compensator and controller gain matrices* $\bar{A}_{i_\delta}$ *and* $K_{i_\delta}, i_\delta \in \bar{\mathscr{I}}, \delta = 0, 1, \ldots, N$ *of the local compensators in* (36) *and controllers in* (37) *are given. The closed-loop system* (43) *is stochastically stable with guaranteed* H_∞ *performance* γ, *if there exists a set of matrices* $X_{i_0 \ldots i_{N-1} i_N} = X^T_{i_0 \ldots i_{N-1} i_N} > 0, i_0, \ldots, i_{N-1}, i_N \in \bar{\mathscr{I}}$ *satisfying*:

$$\begin{bmatrix} \Lambda_1 & \mathbf{0} & \Lambda_2 X_{i_0 \ldots i_{N-1} i_N} & \mathbf{0} \\ * & \Lambda_3 & \Lambda_4 X_{i_0 \ldots i_{N-1} i_N} & \Lambda_5 \\ * & * & -X_{i_0 \ldots i_{N-1} i_N} & \mathbf{0} \\ * & * & * & -I \end{bmatrix} < 0, \quad i, s \in \bar{\mathscr{I}}, \tag{45}$$

where

$$\begin{aligned} &\Lambda_1 = \operatorname{diag}\{-\gamma^2 I, \ldots, -\gamma^2 I, -\gamma^2 I\}, \qquad \Lambda_2 = \begin{bmatrix} \sigma_{p0} \Phi^T_{i0} & \cdots & \sigma_{pN} \Phi^T_{iN} & \Phi^T_{ia} \end{bmatrix}^T, \\ &\Lambda_3 = \operatorname{diag}\{-X_{j_0 \ldots j_{N-1} j_N}, \ldots, -X_{j_0 \ldots j_{N-1} j_N}, -X_{j_0 \ldots j_{N-1} j_N}\}, \\ &\Lambda_4 = \begin{bmatrix} \sigma_{p0} \Omega^T_{i0} & \cdots & \sigma_{pN} \Omega^T_{iN} & \Omega^T_{ia} \end{bmatrix}^T, \qquad \Lambda_5 = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & D^T_{i_0} \end{bmatrix}^T. \end{aligned} \tag{46}$$

Proof. The proof is omitted for its similarity to Lemma 3.1. □

Remark 4.6. A piecewise quadratic Lyapunov function is also utilized in Lemma 4.1, which will reduce to the common quadratic Lyapunov function if $X_{i_0 \ldots i_{N-1} i_N} = X$ for any $i_0, \ldots, i_N \in \bar{\mathscr{I}}$ in (45).

In terms of Lemma 4.1, now we present the controller synthesis result in the following theorem.

Theorem 4.1. *Consider fuzzy system* (7). *The closed-loop system* (43) *is stochastically stable with guaranteed* H_∞ *performance* γ, *if there exist matrices* $X_{i_0 \ldots i_{N-1} i_N} = X^T_{i_0 \ldots i_{N-1} i_N} > 0$, R_{i_δ}, $Q_{i_\delta}, i_\delta \in \bar{\mathscr{I}}$ *and* $\bar{G}$ *such that the following linear matrix inequalities are satisfied for all* δ, $p = \{0, 1, 2, \ldots, N\}$:

$$\begin{bmatrix} \Lambda_1 & \mathbf{0} & \bar{\Lambda}_2 & \mathbf{0} \\ * & \Lambda_3 & \bar{\Lambda}_4 & \bar{\Lambda}_5 \\ * & * & X_{i_0 \ldots i_{N-1} i_N} - \bar{G} - \bar{G}^T & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad m \in \aleph(i_0),\ i_0, \ldots, i_N \in \bar{\mathscr{I}}, \tag{47}$$

where

$$\bar{G} = \begin{bmatrix} G & \cdots & G & G \\ \vdots & \ddots & \vdots & \vdots \\ G & \cdots & G & G \\ G & \cdots & G & G \end{bmatrix}, \qquad \bar{D}_m = \begin{bmatrix} D_m \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \qquad \bar{\Lambda}_2 = \begin{bmatrix} \sigma_{p0} \Theta_{mi0} \\ \vdots \\ \sigma_{pN} \Theta_{miN} \\ \Theta_{mia} \end{bmatrix},$$

$$\bar{\Lambda}_4 = \begin{bmatrix} \sigma_{p0} \Xi_{mi0} \\ \vdots \\ \sigma_{pN} \Xi_{miN} \\ \Xi_{mia} \end{bmatrix}, \qquad \bar{\Lambda}_5 = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ D_m \end{bmatrix},$$

$$\Xi_{mi0} = \begin{bmatrix} B_m Q_{i_0} & \cdots & B_m Q_{i_0} & B_m Q_{i_0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \ldots, \quad \Xi_{miN} = \begin{bmatrix} B_m Q_{i_N} & \cdots & B_m Q_{i_N} & B_m Q_{i_N} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\Xi_{mia} = \begin{bmatrix} \Xi_{mia(1)} & \cdots & \Xi_{mia(1)} & \Xi_{mia(1)} \\ R_{i_0} & \cdots & R_{i_0} & R_{i_0} \\ \vdots & \ddots & \vdots & \vdots \\ R_{i_{N-1}} & \cdots & R_{i_{N-1}} & R_{i_{N-1}} \end{bmatrix}, \qquad \Xi_{mia(1)} = A_m G + \bar{e}_{p0} B_m Q_{i_0} + \cdots + \bar{e}_{pN} B_m Q_{i_N},$$

$$\begin{aligned}
&\Theta_{mi0}=\begin{bmatrix} H_m Q_{i_0} & \cdots & H_m Q_{i_0} & H_m Q_{i_0}\end{bmatrix}, \quad \ldots, \quad \Theta_{miN}=\begin{bmatrix} H_m Q_{i_N} & \cdots & H_m Q_{i_N} & H_m Q_{i_N}\end{bmatrix},\\
&\Theta_{mia}=\begin{bmatrix} \Theta_{pia(1)} & \cdots & \Theta_{pia(1)} & \Theta_{pia(1)}\end{bmatrix}, \quad \Theta_{pia(1)}=L_p G+\hat{e}_{m0}H_p Q_{i_0}+\cdots+\hat{e}_{mN}H_p Q_{i_N}.
\end{aligned} \tag{48}$$

Moreover, the controller gains and the compensation matrices are respectively given by

$$K_{i_\delta}=Q_{i_\delta}G^{-1}, \qquad \bar{A}_{i_\delta}=R_{i_\delta}G^{-1}, \quad \delta=0,1,\ldots,N-1. \tag{49}$$

Proof. The proof is omitted for its similarity to Theorem 3.1. □

5. Simulation examples

In this section, we use two examples to demonstrate the effectiveness of the compensator and controller design methods proposed in this paper.

Example 5.1. In this example, we use a nonlinear inverted pendulum on a cart [18] to demonstrate the improvement of the proposed compensation method in comparison with the existing results.

$$\begin{aligned}
&\dot{x}_1=x_2,\\
&\dot{x}_2=\frac{g\sin(x_1)-amlx_2^2\sin(2x_1)/2-a\cos(x_1)u}{4l/3-aml\cos^2(x_1)}+w,
\end{aligned}$$

where x_1 denotes the angle of the pendulum from the vertical axis, x_2 is the angular velocity, $g=9.8$ m/s^2 is the gravity constant, m is the mass of the pendulum, $a=1/(m+M)$, M is the mass of the cart, $2l$ is the length of the pendulum, u is the force applied to the cart, and w is the external disturbance. In this example, we choose $m=2$ kg, $M=8$ kg, and $2l=1$ m.

Then, we linearize the plant around the origin, $x=(\pm 60°,0)$, and $x=(\pm 88°,0)$. After choosing sampling period $T=0.01$ s, we obtain the discrete-time T–S fuzzy system as follows:

Plant rule $\mathscr{R}^l$: If $|x_1|$ is F^l, then

$$\begin{cases} x(t+1)=A_l x(t)+B_l u(t)+D_l w(t),\\ z(t)=L_l x(t)+H_l u(t), \quad l\in\mathscr{I}:=\{1,2\}, \end{cases} \tag{50}$$

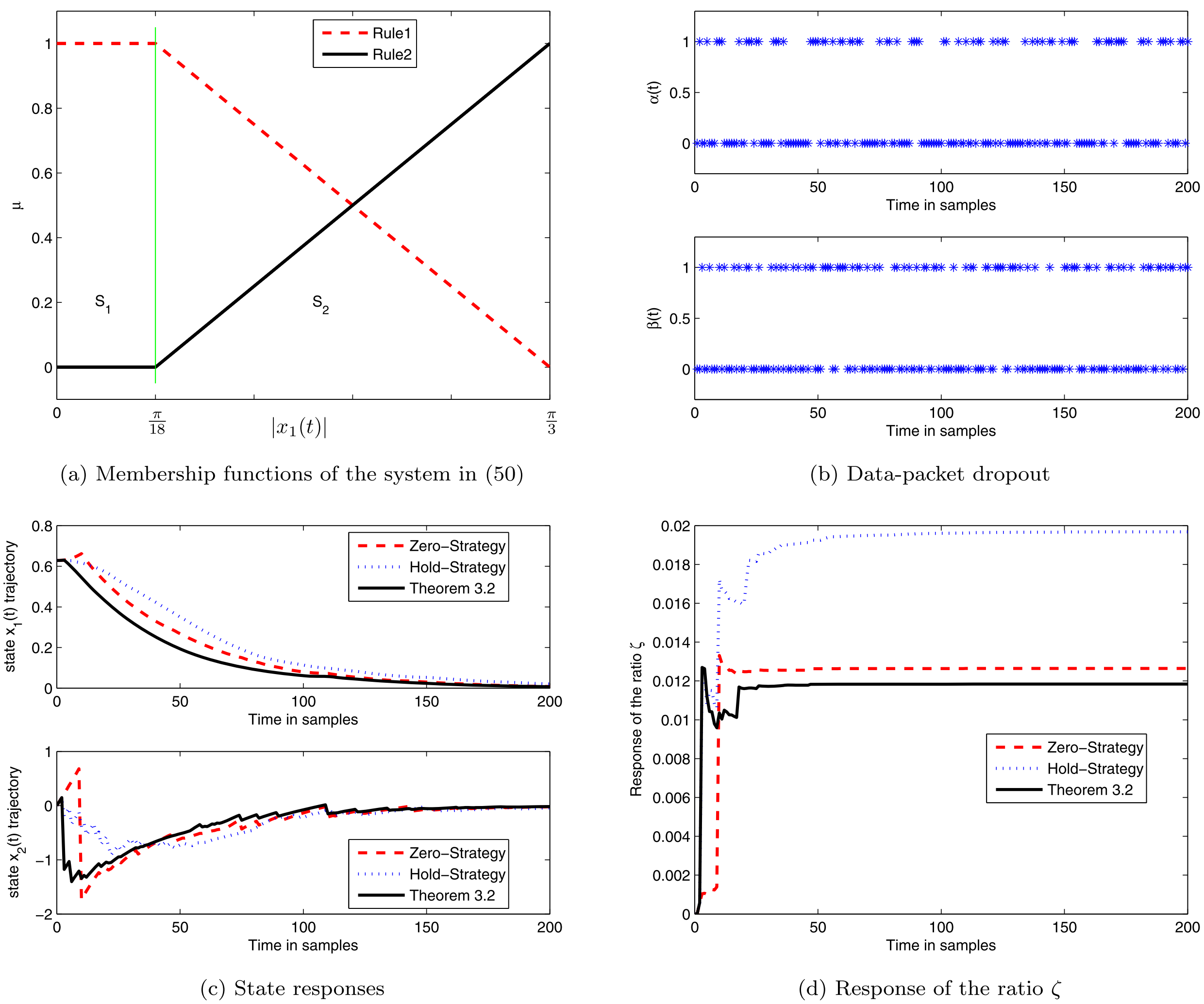
where

$$\begin{aligned}
&A_1=\begin{bmatrix}1 & 0.01\\ 0.1729 & 1\end{bmatrix}, \qquad B_1=\begin{bmatrix}0\\ -0.0018\end{bmatrix}, \qquad D_1=\begin{bmatrix}0\\ 0.01\end{bmatrix}, \qquad L_1=\begin{bmatrix}0.5 & 0.1\end{bmatrix},\\
&H_1=0.005,\\
&A_2=\begin{bmatrix}1 & 0.01\\ 0.0585 & 1\end{bmatrix}, \qquad B_2=\begin{bmatrix}0\\ -0.0007792\end{bmatrix}, \qquad D_2=\begin{bmatrix}0\\ 0.01\end{bmatrix}, \qquad L_1=\begin{bmatrix}0.5 & 0.1\end{bmatrix},\\
&H_2=0.01.
\end{aligned}$$

The membership functions are shown in Fig. 3(a). It is noted that the fuzzy system is governed by one linear dynamic when $0\leqslant|x_1|<\frac{\pi}{18}$ according to Fig. 3(a), and a fuzzy blending of two local dynamics when $\frac{\pi}{18}\leqslant|x_1|\leqslant\frac{\pi}{3}$. Applying the partition method given in the second section, the physical plant can be partitioned into two regions, which are described as follows:

$$\begin{aligned}
&S_1:=\left\{x \;\middle|\; 0\leqslant|x_1|<\frac{\pi}{18}\right\},\\
&S_2:=\left\{x \;\middle|\; \frac{\pi}{18}\leqslant|x_1|\leqslant\frac{\pi}{3}\right\}.
\end{aligned}$$

Similar to the case in the physical plant node, the states in the compensator and controller nodes are partitioned into two regions as follows:



(a) Membership functions of the system in (50)

(b) Data-packet dropout

(c) State responses

(d) Response of the ratio ζ

Fig. 3. Simulation results of Example 5.1.

$$S_1 := \left\{ \hat{x} \,\middle|\, 0 \leqslant |\hat{x}_1| < \frac{\pi}{18} \right\},$$
$$S_2 := \left\{ \hat{x} \,\middle|\, \frac{\pi}{18} \leqslant |\hat{x}_1| \leqslant \frac{\pi}{3} \right\}.$$

With such a partition, the fuzzy system is rewritten in each region as follows:

$$\begin{cases} x(t+1) = \mathcal{A}_i x(t) + \mathcal{B}_i u(t) + \mathcal{D}_i w(t), \\ z(t) = \mathcal{L}_i x(t) + \mathcal{H}_i u(t), \quad x(t) \in S_i, \ i \in \{1, 2\}, \end{cases}$$

where

$$\mathcal{A}_1 = A_1, \qquad \mathcal{B}_1 = B_1, \qquad \mathcal{D}_1 = D_1, \qquad \mathcal{L}_1 = L_1, \qquad \mathcal{H}_1 = H_1,$$
$$\mathcal{A}_2 = \sum_{m=1}^{2} \mu_m A_m, \qquad \mathcal{B}_2 = \sum_{m=1}^{2} \mu_m B_m, \qquad \mathcal{D}_2 = \sum_{m=1}^{2} \mu_m D_m, \qquad \mathcal{L}_2 = \sum_{m=1}^{2} \mu_m L_m, \qquad \mathcal{H}_2 = \sum_{m=1}^{2} \mu_m H_m.$$

It is noted that the open-loop system is unstable. The objective is to design a piecewise fuzzy compensator and state feedback controller of the form (10) and (13) so that the closed-loop system (18) is stochastically stable with H_∞ performance γ.

Table 1
Minimum H_∞ performance $\gamma_{\min}$ in each situation of Example 5.1.

Models	$\bar{\alpha}/\bar{\beta}$	0.0/0.0	0.2/0.2	0.3/0.3	0.4/0.4	0.5/0.5	0.6/0.6	1.0/1.0
Zero-strategy	$\gamma_{\min}$	∞	∞	7.9846	0.7368	0.4367	0.3139	0.0956
Hold-strategy	$\gamma_{\min}$	∞	∞	∞	0.8109	0.3916	0.2778	0.1401
Theorem 3.1	$\gamma_{\min}$	∞	0.5551	0.3678	0.2872	0.2398	0.2076	0.1354

When $\bar{\alpha} = 0.2$ and $\bar{\beta} = 0.2$, *there are no feasible solutions utilizing the zero or hold strategies*, while we obtain $\gamma_{\min} = 0.5551$ by applying Theorem 3.1. The feasible controller and compensator gains are

$$K_1 = \begin{bmatrix} -2.3634 & 0.8538 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -2.3633 & 0.8538 \end{bmatrix},$$
$$\bar{A}_1 = 10^{-3} \times \begin{bmatrix} -0.0882 & 0.3245 \\ -0.2131 & 0.7617 \end{bmatrix}, \qquad \bar{A}_2 = 10^{-3} \times \begin{bmatrix} -0.0852 & 0.3237 \\ -0.2088 & 0.7608 \end{bmatrix}.$$

With the purpose of illustrating the state responses, $\bar{\alpha} = 0.4$ and $\bar{\beta} = 0.4$ are assumed such that *there are feasible solutions by different strategies*. We assume the initial conditions of the system to be $\hat{x}(0) = x(0) = \begin{bmatrix} \frac{\pi}{5} & 0 \end{bmatrix}^T$. The data missing is randomly generated according to the probability shown in Fig. 3(b). The state responses are illustrated in Fig. 3(c), where the external disturbance $w(t) = 0$. It can be observed that the states of the closed-loop system converge to zero.

In order to illustrate the H_∞ performance, we assume the initial conditions to be zero, and the external disturbance $w(t) = \mathrm{e}^{-0.1t}\cos(2t)$. Fig. 3(d) shows the response of the ratio ζ of the closed-loop system, where

$$\zeta = \frac{\sqrt{\sum_{i=0}^{k} z^T(i) z(i)}}{\sqrt{\sum_{i=0}^{k} w^T(i) w(i)}}.$$

From Fig. 3 we can observe that *better H_∞ performance and system response can be achieved by the proposed compensation method in comparison with other strategies.*

The more detailed comparison between the existing strategies and the compensation strategy proposed in this paper on the minimum H_∞ performance $\gamma_{\min}$ under various scenarios of $\bar{\alpha}$ and $\bar{\beta}$ is illustrated in Table 1, which clearly demonstrates that much better H_∞ performance can be achieved by utilizing the proposed strategy than those existing ones.

Remark 5.1. It is noted from Table 1 that the proposed strategy does not achieve the best system performance in the special case when $\bar{\alpha} = \bar{\beta} = 1$, which implies that there are no packet dropouts throughout the transmission. That is because the compensator incurs some unnecessary effects in that special case. In this case, a normal fuzzy controller would be used instead if there are no packet dropouts, which can be found in some existing results, see [2] and references therein.

Example 5.2. In this example, we consider both packet dropout and network-induced delay problems, aiming to illustrate the effectiveness of Theorem 4.1.

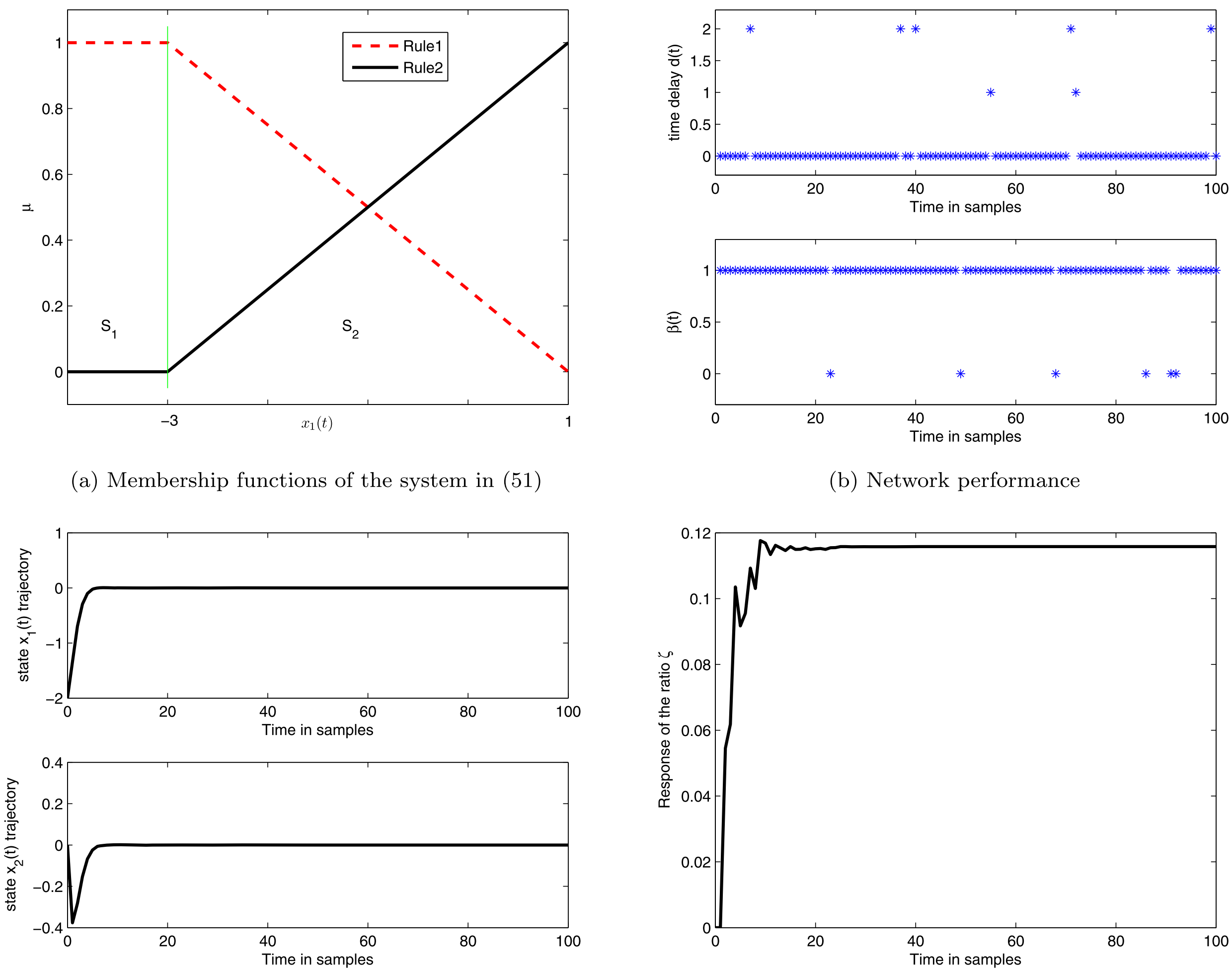
Consider the following discrete-time fuzzy system of form (7) with two rules.

Rule $\mathscr{R}^l$: If x_1 is F_1^l, then

$$\begin{cases} x(t+1) = A_l x(t) + B_l u(t) + D_l w(t), \\ z(t) = L_l x(t) + H_l u(t), \quad l \in \mathscr{I} := \{1, 2\}, \end{cases} \tag{51}$$

where

$$A_1 = \begin{bmatrix} 1.5 & -0.5 \\ 1 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \qquad L_1 = \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}, \qquad H_1 = 0.4,$$
$$A_2 = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}, \qquad H_2 = 0.2.$$



(a) Membership functions of the system in (51)

(b) Network performance

(c) State responses

(d) Response of the ratio ζ

Fig. 4. Simulation results of Example 5.2.

The membership functions are shown in Fig. 4(a). According to the partition method given in the second section, there are two regions of the physical plant, which are illustrated as follows:

$$S_1 := \{x_1 \mid -\infty < x_1 \leqslant -3\},$$
$$S_2 := \{x_1 \mid -3 < x_1 \leqslant 1\}.$$

Similar to the case in the physical plant node, the states in the compensator and controller nodes are partitioned into following two regions:

$$S_1 := \{\hat{x}_1 \mid -\infty < \hat{x}_1 \leqslant -3\},$$
$$S_2 := \{\hat{x}_1 \mid -3 < \hat{x}_1 \leqslant 1\}.$$

With such a partition, the fuzzy system is rewritten in each region as follows:

$$\begin{cases} x(t+1) = \mathcal{A}_i x(t) + \mathcal{B}_i u(t) + \mathcal{D}_i w(t), \\ z(t) = \mathcal{L}_i x(t) + \mathcal{H}_i u(t), \quad x(t) \in S_i, \ i \in \{1, 2\}, \end{cases}$$

where

$$\mathcal{A}_1 = A_1, \qquad \mathcal{B}_1 = B_1, \qquad \mathcal{D}_1 = D_1, \qquad \mathcal{L}_1 = L_1, \qquad \mathcal{H}_1 = H_1,$$
$$\mathcal{A}_2 = \sum_{m=1}^{2} \mu_m A_m, \qquad \mathcal{B}_2 = \sum_{m=1}^{2} \mu_m B_m, \qquad \mathcal{D}_2 = \sum_{m=1}^{2} \mu_m D_m, \qquad \mathcal{L}_2 = \sum_{m=1}^{2} \mu_m L_m, \qquad \mathcal{H}_2 = \sum_{m=1}^{2} \mu_m H_m.$$

In this example, the largest consecutive steps of network-induced delays and packet dropouts $N = 2$ is assumed. Additionally, $\bar{\beta} = 0.95$, and the transition probability matrix is given by

$$\mathbb{T} = \begin{bmatrix} 0.95 & 0.03 & 0.02 \\ 0.95 & 0.03 & 0.02 \\ 0.95 & 0.03 & 0.02 \end{bmatrix}.$$

By applying Theorem 4.1, the corresponding controller and compensation gains are obtained as follows,

$$K_1 = \begin{bmatrix} -0.8153 & 0.1178 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -0.8117 & 0.0779 \end{bmatrix},$$
$$\bar{A}_1 = 10^{-3} \times \begin{bmatrix} 1.3262 & -0.8298 \\ 1.6967 & 4.4351 \end{bmatrix}, \qquad \bar{A}_2 = 10^{-3} \times \begin{bmatrix} -0.5474 & 1.5152 \\ -4.3911 & 5.9701 \end{bmatrix},$$

and the minimum H_∞ performance $\gamma_{\min} = 0.2040$. However, it is also noted that for this example there is no feasible solution by utilizing the hold strategy.

To illustrate the closed-loop performance, some simulations have been done with the initial conditions of the system as $\hat{x}(0) = x(0) = [\,-2\;\;0\,]^T$. The network-induced delays and data missing are randomly generated according to the probability shown in Fig. 4(b). The state responses of the closed-loop system are illustrated in Fig. 4(c), where the external disturbance $w(t) = 0$. It can be observed that the states of the closed-loop system converge to zero.

In order to illustrate the H_∞ performance, we assume the initial conditions to be zero, and the external disturbance $w(t) = \mathrm{e}^{-0.1t} \cos(2t)$. Fig. 4(d) shows the response of the ratio ζ of the closed-loop system.

From Fig. 4 we can observe that the system is stochastically stable with guaranteed H_∞ performance, which shows the effectiveness of Theorem 4.1.

6. Conclusion

The H_∞ control problem of fuzzy-model-based nonlinear NCSs with packet dropouts is discussed in this paper. A new compensation method for packet dropouts is proposed, and a piecewise state feedback controller is designed. It is shown that the closed-loop system is stochastically stable with guaranteed H_∞ performance with the existence of packet dropouts in both S/C and C/A channels. Moreover, the proposed compensation method can achieve better control performance in comparison to the existing approaches such as zero and hold strategies. Moreover, the results are extended to the case where both packet dropouts and network-induced delay phenomena exist. The developed results are illustrated by two simulation examples.

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