Comparisons on State Space Models of Doubly Fed Induction Genertors (DFIG) for Power System Research

Hao Jiang, Hui Liu, Linlin Wu, Yunhong Li
State Grid Jibei Electric Power Research Institute
Beijing, China
tommy_jiang@foxmail.com

***Abstract*—State space models of doubly fed induction generators (DFIG) are useful for small-signal stability study in power system research. They can also be used for identifying different oscillation modes (mechanical, electromechanical, electro-magnetic) between DFIG and power systems. This paper compares different state space models of DFIG for power system study. Firstly, the modeling details of the turbine, drive train, pitch controller, induction machine and the controllers of both the Rotor Side Converter (RSC) and the Grid Side Converter (GSC) of DFIG are introduced. Then we compare different DFIG state space models in literatures. The conclusions and suggestions on DFIG state space models for different research topics in power systems are given in the last part.**

***Index Terms*-- DFIG, state space, power system**

I. INTRODUCTION

Wind power has been growing fast all over the world and is playing a more and more important role in power systems nowadays. With a high penetration level, the impacts of wind power on power system operation and stability need to be studied, which requires the modelling of wind power in simulation software and other tools [1], [2].

The modelling of wind power can be done by organizing blocks representing different components in simulation software [3]. The model built in simulation software can be used for time-domain simulation, which is useful for power system studies. However, the mechanism of the impacts of wind power on power systems cannot be directly revealed.

State space models are widely used in power system studies, including small signal stability, subsynchronous oscillation and other topics. The DFIG state space model is also required for power system study. Some early works considered no control or only open loop control for DFIG [4], [5]. In fact, the DFIG is always operated under closed loop control. Therefore, the closed loop DFIG state space model needs to be studied.

The structure of this paper is as follows: The complete state space model of DFIG including turbine, drive train, pitch controller, induction generator, GSC/RSC controller and the GSC dynamics is introduced in Section II. In Section III, we discuss and compare different DFIG state space models for different research topics in power systems. Section IV concludes the whole paper.

II. THE COMPLETE STATE SPACE MODEL OF DFIG

The mechanical parts (the turbine and the shaft) and the electrical parts (the induction generator, the RSC (Rotor Side Converter) and the GSC (Grid Side Converter)) of DFIG are shown in Figure 1:

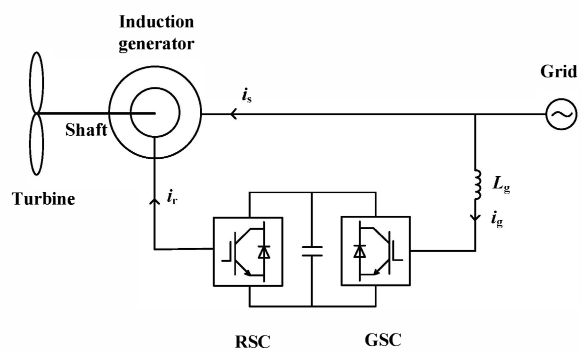

Figure 1. Major parts of DFIG

Note that the controllers including the RSC/GSC controller and the pitch controller are not shown in Figure 1. The model of each part in DFIG is discussed in this section. All the equations are in p.u. if no further description is presented. The linearized state space model can be obtained by linearization under the operating point of the DFIG.

A. Turbine

The mechanical torque T_m of the turbine in one DFIG can be represented as follows:

$$T_{\mathrm{m}} = \frac{0.5\rho\pi R^2 C_p(\lambda,\beta)v_{\mathrm{w}}^3}{\omega_{\mathrm{t}}} \tag{1}$$

978-1-5090-5417-6/16/ $ 31.00 ©2016 IEEE

where ρ is the air density, R is the blade length, v_w is the wind speed, ω_t is the turbine speed. The details of $C_p(\lambda,\beta)$ can be found in [6]. Note that there are no state variables in the turbine model. Equation (1) is in actual value instead of p.u. value.

B. Drive Train

For the drive train in DFIG, the most common model is the two-mass model, which models the turbine and the induction generator independently. The model of the drive train is as follows:

$$\begin{bmatrix} \frac{\mathrm{d}\omega_t}{\mathrm{d}t} \\ \frac{\mathrm{d}\omega_r}{\mathrm{d}t} \\ \frac{\mathrm{d}T_g}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} \frac{-D_t - D_{tg}}{2H_t} & \frac{D_{tg}}{2H_t} & \frac{-1}{2H_t} \\ \frac{D_{tg}}{2H_g} & \frac{-D_t - D_{tg}}{2H_g} & \frac{1}{2H_g} \\ K_{tg} & -K_{tg} & 0 \end{bmatrix} \begin{bmatrix} \omega_t \\ \omega_r \\ T_g \end{bmatrix} + \begin{bmatrix} \frac{T_m}{2H_t} \\ \frac{-T_e}{2H_g} \\ 0 \end{bmatrix} \tag{2}$$

where ω_r is the generator speed, T_g is the torsional torque between the generator and the turbine, D_t and D_{tg} are the damping coefficients, K_{tg} is the torque coefficient, T_e is the electrical torque, H_t and H_g are the inertias of the turbine and the generator.

Note that with the above two-mass model, there are three state variables. Some papers use only one-mass or three-mass model for the drive train, the number of state variables is one and five, respectively.

C. Pitch Controller

The model of the pitch controller is shown in Figure 2 [7].

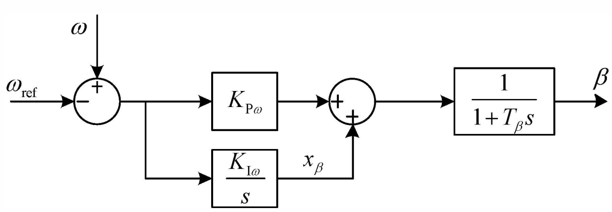

Figure 2. Model of the pitch controller

The model shown in Figure 2 can be represented as follows:

$$\begin{bmatrix} \frac{\mathrm{d}x_\beta}{\mathrm{d}t} \\ \frac{\mathrm{d}\beta}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1}{T_\beta} & -\frac{1}{T_\beta} \end{bmatrix} \begin{bmatrix} x_\beta \\ \beta \end{bmatrix} + \begin{bmatrix} K_{I\omega}(\omega - \omega_{ref}) \\ \frac{K_{P\omega}}{T_\beta}(\omega - \omega_{ref}) \end{bmatrix} \tag{3}$$

There are two state variables in the model shown in Figure 2. Some papers omit the later first-order block so that there is only one state variable in the pitch controller. Note that this controller is only activated when the wind speed is greater than the rated wind speed.

This work is supported by the State Grid Project “”.

D. Induction Generator

The electrical parts of DFIG are usually modelled in dq frame. The relationship between the abc frame and the dq frame in this paper is shown in Figure 3.

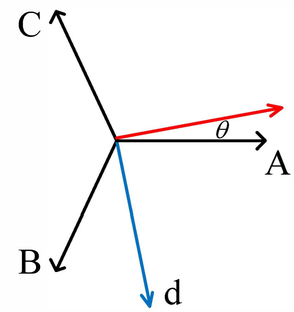

Figure 3. The relationship between the abc frame and the dq frame

As can be seen in Figure 3, the q axis leads A axis *theta* and the d axis lags q axis 90°.

In this paper, we choose the stator and the rotor current. Other variables such as the flux or the transient voltage can also be chosen as state variables.

The model of the induction generator is as follows:

$$\begin{bmatrix} \frac{\mathrm{d}i_{qs}}{\mathrm{d}t} \\ \frac{\mathrm{d}i_{ds}}{\mathrm{d}t} \\ \frac{\mathrm{d}i_{qr}}{\mathrm{d}t} \\ \frac{\mathrm{d}i_{dr}}{\mathrm{d}t} \end{bmatrix} = -B\cdot A \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} + B^{-1} \begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{qr} \\ u_{dr} \end{bmatrix} \tag{4}$$

$$A = \begin{bmatrix} R_s & L_s & 0 & L_m \\ -L_s & R_s & -L_m & 0 \\ 0 & (1-\omega_r)L_m & R_r & (1-\omega_r)L_r \\ -(1-\omega_r)L_m & 0 & -(1-\omega_r)L_r & R_r \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix} \tag{6}$$

where $i_{qs}, i_{ds}, i_{qr}, i_{dr}$ are the q axis and d axis stator and rotor current, $u_{qs}, u_{ds}, u_{qr}, u_{dr}$ are the q axis and d axis stator and rotor voltage, R_s and L_s are the stator resistance and inductance, R_r and L_r are the rotor resistance and inductance, L_m is the mutual inductance.

Note that we consider the three-phase balance case here, therefore there are four state variables for the induction generator. If this condition is not satisfied, the 0 axis must be considered.

E. GSC Controller

The model of GSC controller is shown in Figure 4.

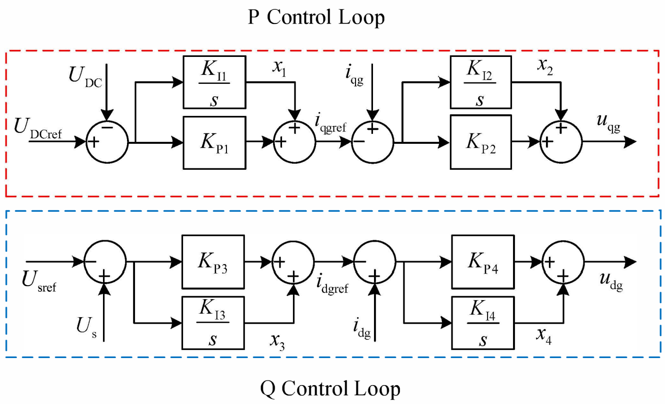

Figure 4. The model of the GSC controller

As can be seen in Figure 4, the GSC controller includes two control loops with the cascading PI structure. Usually the P control loop regulates the voltage of the DC capacitor. The Q control loop can regulate the terminal voltage of the DFIG U_s or the reactive power of GSC Q_g. However, the form of the equations are similar regardless of the input control variables.

The model of the GSC controller shown in Figure 4 is as follows:

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = \frac{1}{K_{I1}}\left(U_{DCref} - U_{DC}\right) \tag{7}$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = \frac{1}{K_{I2}}\left(i_{qg} - \left(x_1 + K_{P1}\left(U_{DCref} - U_{DC}\right)\right)\right) \tag{8}$$

$$\frac{\mathrm{d}x_3}{\mathrm{d}t} = \frac{1}{K_{I3}}\left(U_s - U_{sref}\right) \tag{9}$$

$$\frac{\mathrm{d}x_4}{\mathrm{d}t} = \frac{1}{K_{I4}}\left(i_{dg} - \left(x_3 + K_{P3}\left(U_s - U_{sref}\right)\right)\right) \tag{10}$$

$$u_{qg} = x_2 + K_{P2}\left(i_{qg} - \left(x_1 + K_{P1}\left(U_{DCref} - U_{DC}\right)\right)\right) \tag{11}$$

$$u_{dg} = x_4 + K_{P4}\left(i_{dg} - \left(x_3 + K_{P3}\left(U_s - U_{sref}\right)\right)\right) \tag{12}$$

where x_1, x_2, x_3, x_4 are artificial state variables, i_{qg}, i_{dg} are q axis and d axis current of the GSC.

Note that each integration block introduces one state variable, so there are in total four state variables in the GSC controller. The last two equations are algebraic with no state variables.

F. RSC Controller

The structure of the RSC controller is similar to the GSC controller. The model of the RSC controller is shown in Figure 5:

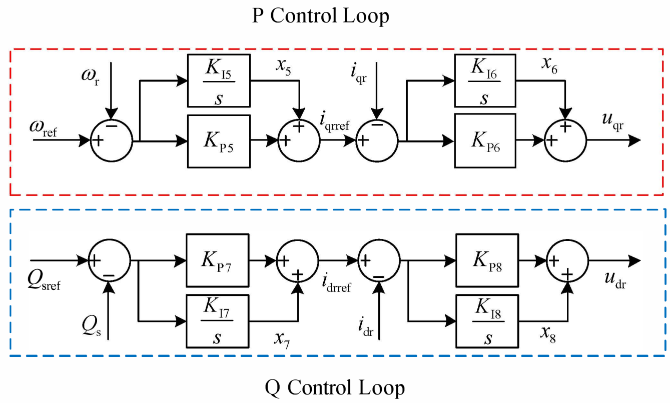

Figure 5. The model of the RSC controller.

Note that the control variable of the Q control loop in RSC is not restricted to Q_s. The model of the GSC controller shown in Figure 5 is as follows:

$$\frac{\mathrm{d}x_5}{\mathrm{d}t} = \frac{1}{K_{I5}}\left(\omega_{ref} - \omega_r\right) \tag{13}$$

$$\frac{\mathrm{d}x_6}{\mathrm{d}t} = \frac{1}{K_{I6}}\left(-i_{qr} + \left(x_5 + K_{P5}\left(\omega_{ref} - \omega_r\right)\right)\right) \tag{14}$$

$$\frac{\mathrm{d}x_7}{\mathrm{d}t} = \frac{1}{K_{I7}}\left(Q_{sref} - Q_s\right) \tag{15}$$

$$\frac{\mathrm{d}x_8}{\mathrm{d}t} = \frac{1}{K_{I8}}\left(-i_{dr} + \left(x_7 + K_{P7}\left(Q_{sref} - Q_s\right)\right)\right) \tag{16}$$

$$u_{qr} = x_6 + K_{P6}\left(-i_{qr} + \left(x_5 + K_{P5}\left(\omega_{ref} - \omega_r\right)\right)\right) \tag{17}$$

$$u_{dr} = x_8 + K_{P8}\left(-i_{dr} + \left(x_7 + K_{P7}\left(Q_{sref} - Q_s\right)\right)\right) \tag{18}$$

where x_5, x_6, x_7, x_8 are artificial state variables, Q_s is the stator reactive power.

G. GSC Inductance Dynamics

The inductance L_g between the GSC and the grid is used to regulate the output power of the GSC as well as a filter for the output current. The model of the GSC dynamics is as follows:

$$\begin{bmatrix} \dfrac{\mathrm{d}i_{qg}}{\mathrm{d}t} \\ \dfrac{\mathrm{d}i_{dg}}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_{qg} \\ i_{dg} \end{bmatrix} + \frac{1}{L_g} \begin{bmatrix} u_{qs} - u_{qg} \\ u_{ds} - u_{dg} \end{bmatrix} \tag{19}$$

There are two state variables in equation (19). If the dynamics of the GSC inductance is ignored, equation (19) turns into the following form:

$$\begin{bmatrix} i_{qg} \\ i_{dg} \end{bmatrix} = \frac{1}{L_g} \begin{bmatrix} -u_{ds} + u_{dg} \\ u_{qs} - u_{qg} \end{bmatrix} \tag{20}$$

In this case, since the dynamics are ignored, there are no state variables for the GSC inductance L_g.

H. Summary

In this subsection, we summarize the complete DFIG state space model introduced in subsection A throughout subsection G. The details of the complete state space model of DFIG is shown in Table I.

Table I. Details of the complete state space model of DFIG

Component	Number of state variables
Turbine	0
Drive train	3, 5
Pitch controller	0, 1, 2
Induction generator	4
GSC controller	1, 2, 3,4
RSC controller	2, 3, 4
GSC dynamics	0, 2

As can be seen in Table I, the number of state variables in the complete DFIG state space model can be different due to different component model. For example, the number of state variables in the GSC controller ranges from 1 to 4. Comparisons of different DFIG state space models for different research topics in power systems are discussed in the following section.

III. Comparisons of State Space Models in Literatures

In this section we compare the state space models used in literatures for different research topics in power systems, including modal analysis of the DFIG-infinite bus system, small signal stability analysis, sub-synchronous oscillation analysis and so on [7]-[19]. The details of different state space models are shown in Table II. We compare the number of drive train masses, GSC/RSC PI blocks, pitch controller blocks and state variables of the GSC dynamics.

A. Modal Analysis of the DFIG-Infinite Bus System

As can be seen in Table II, the state space model of DFIG was first used in modal analysis of the DFIG-infinite bus system to identify different oscillation modes in the system. For example, the stator mode, the electromechanical mode, the and so on. The controller parameters are also designed to maximum the worst damping ration or the worst oscillation modes.

For this type of study, the number of PI blocks in GSC and RSC differs from only one to all four. The reasons for this simplification are as follows:

(1) The inner loops of RSC or GSC controller are sometimes ignored due to their fast dynamics. This is because the fast dynamics have little impacts on the oscillation modes of the whole system.

(2) The outer loops of RSC or GSC controller are sometimes ignored. This is because DFIG is always controlled in unity power factor mode which means Q_{ref} is always set to zero. This allows the calculation of i_{drref} directly. Therefore, the outer loop of the Q loop in the RSC or GSC controller can be omitted and the number of state variables in RSC is reduced to three or even less.

The modelling of the pitch controller also differs in papers for modal analysis. In fact, the pitch controller uses mechanical components so that its dynamics are relatively slow. Moreover, the dynamics of the pitch controller is decoupled from other system dynamics and is non-oscillatory. Therefore, it is ignored in some research papers.

B. Small Signal Stability in Power Systems

The study of mall signal stability in power systems refers to the study of the impacts of DFIGs on the oscillation frequency and dampings of power systems. The difference between this study and the modal analysis study is the size of the studied power system. The study of small signal stability always uses a power system including more than three generators distributed in more than one area.

As can be seen in Table II, the state space model for small signal stability study is almost as complete as the model introduced in Section II. This is because each control block would have some impacts on the power system since there are many state variables as well as system modes. For example, reference [12] points out that the voltage control block in the RSC controller has great impacts on the inter-area oscillation mode of the system while has few impacts on the inner-area oscillation modes. Therefore, the complete state space model is preferred in small signal stability study.

C. Subsynchronous oscillation in power systems

In this subsection we discuss the subsynchronous oscillation between DFIG and the power system. Note both subsynchronous resonance and torsional vibration are included in subsynchronous oscillation.

As can be seen in Table II, the DFIG state space models for subsynchronous oscillation study share an identical structure with complete GSC/RSC controller and no pitch controller. This is because the concerned frequency range in subsynchronous oscillation lies between 10 to 30 Hz, which is much faster than the dynamics of the pitch controller. Note that reference [19] uses the three masses drive train model for torsional vibration study to study the oscillation modes between each draft in DFIG with the power system.

IV. Conclusions

In this paper, we review research papers of the state space model of DFIG and compare the model differences for different research topics. We introduce the complete DFIG state space model with each component first. Then we discuss and compare different DFIG state space models for different research topics in power systems. The main differences are concluded as follows:

(1)Modal analysis: the model used in modal analysis differs from each other with different number of GSC/RSC PI

Table II. Details of different state space models in literatures

Paper	Research topic	Drive train masses	GSC PI block number	RSC PI block number	Pitch controller block number
Yang[7]	Modal analysis	2	1	2	2
Wu[8]	Modal analysis	2	3	4	1
Mishra[9]	Modal analysis	2	2	4	0
Mishra[10]	Modal analysis	2	2	4	0
Yang[11]	Modal analysis	2	3	3	1
Yang[12]	Small signal stability	2	3	4	2
Wei[13]	Small signal stability	3	4	4	1
Li[14]	Small signal stability	2	4	4	1
Fan[15]	Subsynchronous oscillation	2	4	4	0
Ugalde-Loo[16]	Modal analysis	2	4	4	0
Mohammadpour [17]	Subsynchronous oscillation	2	4	4	0
Huang[18]	Subsynchronous oscillation	2	4	4	0
Lu[19]	Torsional vibration	3	3	4	0

blocks and pitch blocks. However, this has little impacts on identifying the dominant oscillation modes including the mechanical and electromechanical modes in the DFIG-infinite bus system.

(2)Small signal stability: the state space model for small signal stability study is the most complete one to study the impacts of different control blocks on the system modes.

(3)Subsynchronous oscillation: pitch controller is ignored due to its slow dynamics. Drive train masses need to be modelled in detail for torsional vibration study.

Future work will be focused on comparisons of impacts on oscillation modes in power systems with different state space models.

ACKNOWLEDGMENT

The authors gratefully acknowledge the State Grid Project "The operation characteristics and control of large scale renewable energy with series compensated transmission system (52010116000S)" and the Wind-Solar-Energy Storage Hybrid Power Generation Technology Laboratory.

REFERENCES

[1] Y. V. Makarov, C. Loutan, J. Ma, et al. "Operational impacts of wind generation on California power systems," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 1039-1050, May. 2009.

[2] D. Gautam, V. Vittal, T. Harbour. "Impact of increased penetration of DFIG-based wind turbine generators on transient and small signal stability of power systems, " *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1426-1434, Aug. 2009.

[3] M. Asmine, J. Brochu, J. Fortmann, et al. "Impact of increased penetration of DFIG-based wind turbine generators on transient and small signal stability of power systems," *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1426-1434, Aug. 2009.

[4] F. Mei, B. C. Pal. "Modal analysis of grid-connected doubly fed induction generators," *IEEE Trans. Energy Convers.*, vol. 22, no. 3, pp. 728-736, Sep. 2007.

[5] J. Li, W. S. Wang, J. H. Song. "Linearized dynamic model of doubly-fed induction generator and analysis of its operating performance (in Chinese)," *Power System Technology*, vol. 28, no. 13, pp. 13-17, Jul. 2004.

[6] L. P. Kunjumuhammed, B. C. Pal, C. Oates, et al. "Electrical oscillations in wind farm systems: analysis and insight based on detailed modeling," *IEEE Trans. Sustain. Energy*, vol. 7, no. 1, pp. 51-62, Jan. 2016.

[7] L. H. Yang, Z. Xu, J. Østergaard, et al. "Oscillatory stability and eigenvalue sensitivity analysis of a DFIG wind turbine system," *IEEE Trans. Energy Convers.*, vol. 26, no. 1, pp. 328-339, Mar. 2011.

[8] F. Wu, X. P. Zhang, K. Godfrey, et al. "Small signal stability analysis and optimal control of a wind turbine with doubly fed induction generator," *IET Gener. Trans. Distrib.*, vol. 5, no. 1, pp. 751-760, Sep. 2007.

[9] Y. Mishra, S. Mishra, F. X. Li, et al. "Small signal stability analysis of a DFIG-based wind power system under different modes of operation," *IEEE Trans. Energy Convers.*, vol. 24, no. 3, pp. 972-982, Sep. 2009.

[10] Y. Mishra, S. Mishra, M. Tripathy, et al. "Improving stability of a DFIG-based wind power system with tuned damping controller," *IEEE Trans. Energy Convers.*, vol. 24, no. 3, pp. 650-660, Sep. 2009.

[11] L. Yang, G. Y. Yang, Z. Xu, et al. "Optimal controller design of a doubly-fed induction generator wind turbine system for small signal stability enhancement," *IET Gener. Trans. Distrib.*, vol. 4, no. 5, pp. 579-597, May. 2010.

[12] L. H. Yang, X. K. Ma. "Impact of doubly fed induction generator wind turbine on power system low-frequency oscillation characteristic (in Chinese)," *Proceedings of the CSEE*, vol. 31, no. 10, pp. 19-25, Apr. 2011.

[13] W. Wei, Y. Liu, L. J. Ding, et al. "An improved small perturbation model for doubly fed wind power system (in Chinese)," *Power System Technology*, vol. 37, no. 10, pp. 2904-2911, Oct. 2013.

[14] H. Li, H. W. Chen, C. Yang, et al. "Modal analysis of the low-frequency oscillation of power systems with DFIG-based wind farms (in Chinese)," *Proceedings of the CSEE*, vol. 33, no. 28, pp. 17-24, Oct. 2013.

[15] L. L. Fan, C. X. Zhu, Z. X. Miao, et al. "Modal analysis of a DFIG-based wind farm interfaced with a series compensated network," *IEEE Trans. Energy Convers.*, vol. 26, no. 4, pp. 1010-1020, Dec. 2011.

[16] C. E. Ugalde-Loo, J. B. Ekanayake, N. Jenkins. "State-space modeling of wind turbine generators for power system studies," *IEEE Trans. Indus. Appl.*, vol. 49, no. 1, pp. 223-232, Jan. 2013.

[17] H. A. Mohammadpour, A. Ghaderi, E. Santi. "Analysis of sub-synchronous resonance in doubly-fed induction generator-based wind farms interfaced with gate-controlled series capacitor," *IET Gener. Trans. Distrib.*, vol. 8, no. 12, pp. 1998-2011, Dec. 2013.

[18] P. H. Huang, M. S. El Moursi, W. D. Xiao, et al. "Subsynchronous resonance mitigation for series-compensated DFIG-based wind farm by using two-degree-of-freedom control strategy," *IEEE Trans. Power Syst.*, vol. 30, no. 3, pp. 1442-1454, May. 2015.

[19] Y. P. Lu, D. Xie, J. B. Sun, et al. "Small signal modeling and simulation for torsional vibration of wind farms (in Chinese)," *Power System Technology*, vol. 40, no. 4, pp. 1120-1127, Apr. 2016.