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Corporate social responsibility and decision analysis in a supply chain through government subsidy

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Corporate social responsibility and decision analysis in a supply chain through government subsidy

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Abstract: With rapid economic development, related social issues have become more prominent than before. How to effectively encourage enterprises to undertake more corporate social responsibility (CSR) and maintain the sustainable development of society has become an urgent task for managers and researchers to resolve. With respect to decision problems of a retailer-dominated supply chain considering CSR under government subsidy, based on game analysis technology, we establish a three-stage Stackelberg game model that consists of the government, a dominant retailer, and n suppliers, and exploit it for analyzing the optimal decisions either with or without government subsidy, and explore the impact of the government subsidy on the profits of the members of the supply chain, the CSR effort level and social welfare in the retailer-dominated supply chain, and determine the relationship between the optimal government subsidy rate and CSR level. Our results show that the higher the effort level is, the more positive effect on the profits the whole supply chain experiences. Compared with those situations under non-government subsidies, a certain range of government subsidies can promote the supply-chain members to undertake CSR and improve overall performance of the supply chain and the welfare of the society.

Key words: corporate social responsibility (CSR); sustainable supply chain; government subsidy; three-stage game; social welfare

1. Introduction

Social issues, such as environmental pollution, labor conflicts, food safety, etc., have become more and more prominent issues over time. Corporate social responsibility issues have become one of the hot topics of concern to all parties involved directly or indirectly. Chinese governments of various levels have been promulgating a series of policies, laws and regulations to motivate enterprises to take on CSR. AQSIQ and SAC jointly announced《Social Responsibility Guideline》(GB / T 36000-2015, referred to as GB 36000), which was put into effect on January 1, 2016.《Charity Law of the People's Republic of China》was passed at the Fourth Session of the Twelfth National People's Congress on March 16, 2016, and came into force on September 1 of the same year. This codocument has the corresponding guiding role in promoting the commitment of enterprises to public charities. On July 1, 2016,《Guiding Opinions on Better Performing Social Responsibilities of State-owned Enterprises》was issued by the State-owned Assets Supervision and Administration Commission of the State Council, and pointed out that the development of enterprises should take sustainable development as their core and that their operations should be integrated with social responsibilities. The introduction of these series of measures shows that corporate social responsibility has been elevated to the level of national strategic development. Meanwhile, it is clear that the Chinese national government has realized that the fulfillment of corporate social responsibility has an important influence on the coordinated development of economy and society. Therefore, how to strategically play the role of the government in the motivation of relevant enterprises to better fulfill CSR is of great significance for improving the performance of enterprises and the GDP benefits of the whole society.
In addition to the concerns of the governments of various levels, issues of consumer awareness of CSR is also becoming stronger than ever before. More and more consumers tend to buy environmentally friendly products with public welfare and begin to resist products produced by enterprises that are known to lack humanistic care and employee protection (Ageron et al., 2012). And stakeholders also hope enterprises to be more CSR for their operations (Ashby et al. 2012). Therefore, under the pressures from the governments and the public, enterprises have no choice but to pay additional attention to the issues around this area. Consequently, practicing CSR has become a new normal for many enterprises. To this end, some enterprises have carried out specific practices of CSR. For example, Carrefour (http://www.carrefour.com.cn/) has demonstrated its corporate social responsibility for food safety and humane care through its CSR Report. Wal-Mart and other retailers introduced CSR into their supply chain management by employing codes of conduct (Amaesh et al. 2008). CR Vanguard, a large retailer, has published its “Corporate Social Responsibility Report” for six consecutive years, while detailing the changes it has made to the society and environment, which effectively promoted the sustainable development process of its supply chain. All these real-life cases show that the decision-makers in supply chains are turning from their earlier narrow perspectives of interests to the so-called triple bottom line (Besiou and Van Wassenhove, 2015; Besiou and Van Wassenhove, 2016), that is, taking into account the balance of economy, environment and society. The balance of these three aspects of business depends on the ‘Game’ between government and enterprise. Therefore, this paper introduces government factors into the supply chain of concern to discuss the impact of government subsidy policy on corporate social responsibility decision-making in the supply chain and its relationship with corporate social responsibility so as to provide suggestions for the long-term strategic decision of the supply chain.

Government efforts to direct business behaviors toward certain socially desirable outcomes take a variety of forms and approaches. One approach that has been gathering substantial support in recent years is to provide subsidies. This paper will study how a CSR supply chain responds to government subsidy. Specially, this work attempts to answer the following three questions. Firstly, how can a three-stage game model be used to produce the optimal decisions of the supply chain considering CSR under government subsidy, and how does the government subsidy affect the decision-making of the supply chain. Secondly, to encourage a retailer to exhibit CSR, government would supply financial subsidy. Then, how does the government subsidy affect the decision-making of the retailer’s level of its CSR effort, and what is the relationship between them. Thirdly, because the level of government subsidy is determined from maximizing social welfare, how does the government balance between the profits of enterprises and social welfare (i.e. the goal of government), which can not only promote the enterprise to take additional responsibility, but also create no damage on social welfare.

The remainder of the paper is organized as follows. The related literature is reviewed in Section 2. Section 3 presents our model construction and the optimal solution to the supply chain considering CSR under anarchy and government subsidy. Sensitivity analysis and result interpretation are shown in Section 4. Section 5 draws key conclusions and points out some directions for future research.

2. Literature review

In the face of worsening environmental and social challenges, it has been an inevitable trend for each modern supply chain to choose a sustainable path. Base on this fact, there are many
literatures regarding CSR supply chains. At first, many scholars investigate the concept of CSR. For example, Bowen (1953) firstly proposed the concept of CSR, and pointed out that enterprises should not only pursue economic benefits unilaterally, but also pay attention to their own behaviors, so that they will not violate the social moral requirements and government regulations. After that, from different angles, more and more scholars defined the connotation and scope of CSR (Carrol, 1979; Carrol, 1999; Daklsrud, 2008). In summary, CSR means that enterprises should pay attention to not only economic benefits, but also the needs of the society, economy, environment and stakeholders. The core of CSR is to resolve and capture the most important concerns of the public on business and social relationship (Jamali, 2007; Carrol, 2015), such as employee benefits, environmental pollution and product quality (Zhang et al., 2012). Now CSR is regarded as the guarantee of sustainable development (Gimenez and Tachizawa, 2012; Besiou and Wassenhove, 2016; Wu, 2013). This paper is mainly related to three stream of the vast literature: supply chain decisions considering CSR, responsible supply chain coordination and the influence of government policy on a supply chain’s performance.

Decision analysis researches on supply chains mainly analyze the impact of CSR on the performance and the decision mechanism considering CSR. In terms of the impact of CSR on the supply chain of concern, Meng et al (2012) maintained that bearing CSR could help enterprises to win much more market share and increase the competitiveness of the supply chain due to product differentiation with CSR attributes. Song et al (2016) analyzed the influence of the CSR consciousness of the supply-chain members on the supply-chain’s decision and found that the stronger the awareness of CSR is, the more the consumer surplus is resulted, and a moderate CSR awareness could improve the overall performance of the supply chain. Hsueh (2014) designed a new revenue sharing contract to improve the CSR performance in the supply chain.

In terms of the decision mechanism of the supply chain considering CSR, by studying three kinds of CSR configuration modes, Ni et al (2015) found that supply chain members would automatically choose to undertake CSR when consumers had a positive response to the CSR behavior of the enterprises in the supply chain with the supply-chain members sharing CSR, which in turn leads to higher levels of economic and social benefits. Modak et al (2014) used consumer surplus to represent CSR and discussed the pricing decision mechanism under either decentralized or centralized decision making. By studying the purchasing decision of the manufacturer to undertake or not undertake CSR, Dai et al (2017) claimed that the purchase decision was related to the pricing difference of the suppliers and customers’ willingness to pay, and the smaller the pricing difference is, the greater customers are willingness to pay, and the more inclined the suppliers were to undertake CSR.

For the supply chain coordination considering CSR, such ideas as benefit distribution and incentive, cost sharing, price discount strategy, product quantity discount, and government subsidy have been used to design the coordination mechanism. In terms of profit distribution and incentive, Guo et al (2011) exploited Nash equilibrium, Stackelberg equilibrium and cooperative games to discuss the equilibrium and profit distribution of the supply chain. They pointed out that the revenue sharing contract could effectively realize the Pareto improvement of the supply chain. Modak et al (2016) discussed the channel coordination and benefit distribution in a two-level supply chain composed of a manufacturer with CSR and two competitive retailers. These scholars analyzed the influence of CSR on optimal decisions of the two competitive retailers, and they calimed that the two resultant pricing contracts could
effectively resolve the conflict. And then they extended the work to a three-level supply chain, and proposed a new revenue-sharing contract to resolve the channel conflict of the double marginal benefits. Panda (2014) analyzed optimal decisions of the manufacturer and retailer who undertake CSR, respectively, and then proposed a revenue-sharing contract to coordinate the supply chain. By combining CSR, product recovery and channel coordination to specify and quantify the CSR and its impact, Panda and others put forward a revenue-sharing contract to resolve the channel conflict of a supply chain that is dominated by the manufacturer (Panda et al., 2015; Panda and Modak, 2016; Panda et al., 2016; Panda et al., 2017). In terms of cost sharing, Hsueh (2008) discussed coordination problems of the supply chain network with CSR, where the network is composed of manufacturers, distributors and retailers, and realizes the optimization of the channels by using currency transfer to share CSR. Similarly, Cheng et al (2017) also expressed the idea of environmental responsibility sharing in his study.

In terms of price discount strategies, Ni et al (2010) analyzed how the downstream members of a supply chain share the cost of upstream members through the wholesale price contract, and provided a coordination strategy and mechanism of a two-level supply chain with CSR. In terms of quantity discounts of products, with respect to the problem of reducing the manufacturer’s profit due to insufficient efforts of supplier’s CSR, Wu et al (2017) put forward a flexible quantity discount contract and a wholesale price contract to encourage the suppliers to invest in CSR, which could in turn help improve the overall performance of the supply chain.

In terms of government subsidy, the literature mainly analyzed the influence of government policy and government subsidy on CSR, performance and decision of supply chains. In view of the relationship and effects between government policy subsidy and CSR, Atasu and Wassenhove (2012) and Chu (2013) respectively analyzed the impact of government policy and government subsidy incentives on CSR commitments, and also discussed the impact of government subsidy on decision- making of the members of a supply chain on CSR. But these authors didn’t explore the relationship between government subsidy and CSR. In terms of the influence of government subsidy on performance, Huang et al (2013) discussed the effect of government subsidy on environmental pollution control. Compared with situations of non-subsidy, and they found that government subsidy can promote the development of electric vehicles and the protection of the environment. Yi and Li summarized that a subsidy-based policy not only can improve the level of energy-saving, but can also reduce the level of carbon-emission. In terms of the impact of government subsidy on supply chain decisions concerning CSR, with the coexistence of the fossil fuel car and electric vehicle supply chains, Zhang and Wang (2017) discussed the influence of three kinds of government policies on two competing supply chains under either decentralized or centralized decision-making. However, these scholars didn’t explore the relationship between government subsidy and social welfare. Arya and Mittendorf (2015) studied the effect of government subsidy on charitable donations of retailers (a CSR behavior), and found that the subsidy could reduce the degree of double marginalization of the supply chain. He et al. (2016) analyzed the influence of consumers’ free-riding behavior on carbon emissions under dual channels, and found that government’s tax subsidy to e-retailers could effectively reduce free-riding behavior and total carbon emission. Hafezalkotob (2015) and Hafezalkotob et al (2016) analyzed the impact of government tax revenues (an environmental protection policy) on the supply-chain members’ optimal decisions through the government financial intervention, and found that government’s environmental protection policy and CSR could jointly reduce negative
business impact on the environment and stimulate green production of a green supply chain.

From the literature reviewed here, it can be seen that they mostly use two-stage game theory to analyze the impact of CSR on supply chain performance and the decision of the supply chain considering CSR, and discuss how to coordinate the supply-chain members. The research on supply chain decision-making based on government subsidy mechanism mainly focuses on closed-loop supply chains. However, how to design a reasonable subsidy mechanism from the perspective of social welfare maximization and how to stimulate the members of a supply chain to undertake CSR were not found so far. To better understand the relationship between government subsidy and CSR, and explore the influence of government subsidy on the decision of supply chains considering CSR, we exploit a three-stage Stackelberg game model to analyze the relationship between and decision process of government, a dominant retailer and suppliers, and obtain the optimal decisions of the supply chain. Our results provide suggestions for government officials to make subsidy policies and long-term strategic decisions for the dominant enterprises.

3. A decision model of the supply chain with CSR through government subsidy

In a retailer-dominated supply chain, such environmental and social problems as pollution, labor conflicts and food safety are becoming more and more serious than before, and consumers’ awareness of CSR, such as environmental protection, human rights, safety, etc., have also been gradually heightened. So some national governments from around the world have promulgated a series of policies, laws and regulations to encourage related enterprises to fulfill CSR by providing certain amounts of capital. (Here each government only subsidizes the retailer, while the retailer transfers the subsidy to the suppliers through certain measures). In this way, the decision system of the retailer-dominated supply chain is composed of the government, the dominant retailer and various suppliers, as shown in Fig. 1.

![Fig. 1. The decision system of a supply chain with CSR through government subsidy](image-url)

In the decision system, the government spends some money to subsidize the retailer for it to make business decisions with social needs in its mind. The government is in the dominant position, while the retailer is in a subordinate position. In the retailer-dominated supply chain, the retailer has the advantages of information resources, etc., and she can guide and coordinate the suppliers to make their individual decisions. Compared with the suppliers, the retailer is in the dominant position. And then, the decision process of the supply-chain system can be seen as a three-stage Stackelberg game, consisting of the government, the dominate retailer and suppliers. That is, to begin with, according to the principle of maximizing social welfare, the government supports the dominant retailer to undertake CSR with a certain rate of subsidy. Additionally, based on the rate of government subsidy and the goal of maximizing expected benefits, the retailer aims
to determine the level of CSR effort, order quantity and sales price. Finally, the suppliers decide their wholesale price on the basis of the retailer’s decision. Therefore, we exploit a Stackelberg game in our study of CSR and the optimal decisions of the supply chain through government subsidy.

3.1 Model construction

Assumption 1. A retailer-dominated supply chain and its decision system are composed of the government, a dominant retailer and homogeneous suppliers, and the government only gives coordination subsidies to the retailer. Let \( r, s_i(i = 1, 2, ..., n) \) represent the retailer and ith supplier, respectively, and they follow the Stackelberg game relationship with full information, and they are risk-neutral, and rational.

Assumption 2. There is no shortage and unsalable products in the supply chain considering CSR.

Assumption 3. Although the suppliers provide homogeneous products, their levels of CSR efforts, order quantities of products, selling prices and wholesale prices are different. Let \( e_i, q_i, p_i, w_i \) stand for the CSR effort level, order quantity, sale price and wholesale price in the process of the cooperation between the retailer and supplier \( i \), respectively, where \( e_i, q_i, p_i \) are the decision variables of the retailer and \( w_i \) a decision variable of the ith supplier. Because undertaking CSR can enhance corporate image and consequently improve the market demand of products, the demand function is affected by the product price and CSR. According to the law of diminishing marginal utility, the demand for supplier \( i \) decreases with the price, and the demand function can be written as \( q_i = a - bp_i + \beta e_i \), where \( \beta e_i \) is referred to as the increasing volume of the market demand as a result of undertaking CSR, and \( \beta \) stands for the impact coefficient of the retailer’s CSR on consumer demand, and \( a, b > 0 \).

Assumption 4. It is necessary to invest capital in CSR, the cost for CSR. The cost is closely related to the effort level of the suppliers. Let \( C(e_i) \) be the corresponding cost to undertake CSR in the process of the cooperation between the retailer and suppliers \( i \). According to Cobb-Douglas production function, this cost can be written as \( C(e_i) = \frac{k e_i^2}{2} \) (Ni et al., 2010), where \( k(k > 0) \) stands for the unit cost of CSR.

Assumption 5. The government subsidizes the dominant retailer according to its level of CSR effort. Let \( S, s(0 \leq s \leq 1) \) represent the amount of government subsidy and the rate of government subsidy, respectively, such that \( S = s \sum_{i=1}^{n} C(e_i) \).

The parameters and variables used in our model are as follows:

\( e_i \): The level of CSR effort of supplier \( i \);
$q_i$: The order quantity of supplier $i$;

$p_i$: The sale price of the retailer to supplier $i$;

$w_i$: The wholesale price of supplier $i$;

$c_i$: Unit production cost of supplier $i$;

$a$: Potential market capacity;

$\beta$: The impact coefficient of the retailer’s CSR on consumer demand;

$k$: The cost factor of CSR;

$s$: Subsidy coefficient;

$S$: The amount of government subsidy;

$CS$: The consumer surplus;

$\pi$: The objective function of the government;

$\pi_r$: The profits of the retailer;

$\pi_{si}$: The profit of the $i$th supplier;

$\pi_v$: The profit of the whole supply chain.

In the retailer-dominated supply chain through government subsidy, the government pursues after the maximization of social welfare, while the retailer and suppliers pursue after their individual self-interest maximization. Generally speaking, social welfare consists of the producers’ benefits, consumers’ benefits and subsidy costs (He, 2016; Sheu, 2011). According to the model assumption and notations, the basic model that relates these aspects is established as follows:

$$CS = \int_{p_{min}}^{p_{max}} qdp = \sum_{i} \int_{a-q_i+\beta e_i/b}^{a-q_i+\beta e_i/b} (a-bp_i+\beta e_i)dp_i = \sum_{i} q_i^2/2b$$

(1)

$$\pi = \pi_v + CS - S$$

(2)

$$\pi_r = \sum_{i=1}^{n} (p_i - w_i)q_i - C(e) + S$$

(3)

$$\pi_{si} = (w_i - c_i)q_i, i = 1, 2, ..., n$$

(4)

3.2 The optimal decisions of the supply chain considering CSR

In the supply chain considering CSR, if the government does not provide any subsidies to the supply chain members, then the amount of government subsidy is zero. Therefore, there exists a two-stage game between the retailer and suppliers in the supply chain. The retailer is the leader.
and firstly determines its level of CSR effort, sales prices and order quantities, and then the suppliers determine their wholesale prices. In view of this, based on the established Stackelberg game previously, we can analyze the optimal decisions of the supply chain as follows.

3.2.1 The optimal decisions of the retailer

If the government’s subsidy to the retailer is zero, that is, \( S = s = 0 \), then the expected profits of the retailer can be written as follows.

\[
\pi_r^N = \sum_{i=1}^{n} (p_i^N - w_i^N)q_i^N - C(e_i^N) = \sum_{i=1}^{n} (p_i^N - w_i^N)(a - bp_i^N + \beta e_i^N) - \frac{ke_i^{N^2}}{2}
\]  

(5)

**Theorem 1.** Assume that \( p_i^N, q_i^N, e_i^N, i = 1, 2, \ldots, n \), represent the selling price, order quantity, and the level of CSR effort in the process of the cooperation between the retailer and supplier \( i \) without government subsidy, respectively. If there exists a set of unique values \( p_i^{N^*}, q_i^{N^*}, e_i^{N^*} \) that maximize the expected profits of the retailer, and then the optimal decisions of the retailer without government subsidy can be determined as follows.

\[
p_i^{N^*} = \frac{a + bw_i^{N^*} + \beta e_i^{N^*}}{2b} \]

(6)

\[
q_i^{N^*} = \frac{a - bw_i^{N^*} + \beta e_i^{N^*}}{2} \]

(7)

\[
e_i^{N^*} = \frac{\beta(a - bw_i^{N^*})}{2bk - \beta^2} \]

(8)

\[
\pi_r^{N^*} = \frac{k(a - bw_i^{N^*})^2}{2(2bk - \beta^2)} \]

(9)

**Proof.** For the supply chain considering CSR, if there is no government subsidy, by computing the partial derivative of equation (5) with respect to variable \( p_i^N \), we can then obtain the first-order condition by equating this derivative to zero, that is,

\[
\frac{\partial \pi_r^N}{\partial p_i^N} = a + \beta e_i^N + (w_i^N - 2p_i^N) = 0
\]

Solving this equation, we can obtain \( p_i^N = \frac{a + bw_i^N + \beta e_i^N}{2b} \).

Putting the above formula into the demand function leads to the order quantity as follows:
\[ q_i^N = \frac{a - bw_i^N + \beta e_i^N}{2} \]

Similarly, we can obtain:

\[ \frac{\partial \pi_r^N}{\partial e_i^N} = \frac{\beta^2 e_i^N + \beta(a - bw_i^N) - 2kbe_i^N}{2b} = 0 \]

such that, the level of CSR effort can be determined as follows:

\[ e_i^N = \frac{\beta(a - bw_i^N)}{2bk - \beta^2} \]

And then by respectively computing the second-order partial derivatives of equation (5) with respect to \( p_i^N, q_i^N, e_i^N \), we can obtain:

\[ \frac{\partial^2 \pi_r^N}{\partial p_i^{N^2}} = -2b \]

\[ \frac{\partial^2 \pi_r^N}{\partial q_i^{N^2}} = \frac{2}{b} \]

\[ \frac{\partial^2 \pi_r^N}{\partial e_i^{N^2}} = -k \]

Due to \( b, k > 0 \), it is well known that the above equations are all negative. That is, \( \pi_r^N \) is a concave function of \( p_i^N, q_i^N, e_i^N \). So, there exist unique values \( p_i^{N^*}, q_i^{N^*}, e_i^{N^*} \) of \( p_i^N, q_i^N, e_i^N \), respectively, that maximize \( \pi_r^N \). So, the retailer’s optimal decisions can be determined. QED.

### 3.2.2 The optimal wholesale price decision of the suppliers

The expected profits of supplier \( i \) can be expressed as follows

\[ \pi_{si}^N = (w_i^N - c_i^{N^*})q_i^N = (w_i^N - c_i^{N^*})(a - bp_i^N + \beta e_i^N) \quad (10) \]

**Theorem 2.** Assume that \( w_i^N \) stands for the wholesale price of supplier \( i \). If there exists a uniquely determined value \( w_i^{N^*} \) that maximizes the expected profits of the supplier, then the optimal wholesale price decision for the supplier without government subsidy can be obtained as follows:

\[ w_i^{N^*} = \frac{a + bc_i^{N^*}}{2b} \quad (11) \]

**Proof:** For the supply chain considering CSR, if there is no government subsidy, we can substitute equations (6) and (8) into equation (10). Differentiating \( \pi_{si}^N \) with respect to variable \( w_i \) leads to the
following first-order condition
\[
\frac{\partial \pi^N_i}{\partial w^N_i} = \frac{bk\left(a + bc_i^N - 2bw^N_i\right)}{2bk - \beta^2} = 0
\]
So, we acquire:
\[
w^N_i = \frac{a + bc_i^N}{2b}
\]
And then, by computing the second-order partial derivatives of \(\pi^N_i\) with respect to \(w^N_i\), we can obtain as follows:
\[
\frac{\partial^2 \pi^N_i}{\partial w^{N^2}_i} = -\frac{2b^2k}{2bk - \beta^2} < 0
\]
That is, \(\pi^N_i\) is a concave function of \(w^N_i\), which shows that there is a unique value \(w^N_i\) that maximizes \(\pi^N_i\). If \(w^N_i\) represent this unique value of \(w^N_i\), then the optimal decision can be obtained. QED.

By substituting equation (11) into equations (6)-(10), the optimal decisions and profits of the retailer and supplier \(i\) without government subsidy can be determined, Table 1.

**Table 1**
The optimal decisions and profits of the supply chain without government subsidy

<table>
<thead>
<tr>
<th>(p^N_i)</th>
<th>(q^N_i)</th>
<th>(e^N_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3a + bc_i^N}{2b(2bk - \beta^2)})</td>
<td>(\frac{bk(a - bc_i^N)}{2(2bk - \beta^2)})</td>
<td>(\frac{\beta(a - bc_i^N)}{2(2bk - \beta^2)})</td>
</tr>
</tbody>
</table>

\[w^N_i = \frac{a + bc_i^N}{2}, \quad \pi^N_i = \frac{\sum_{i=1}^n k(a - bc_i^N)^2}{8(2bk - \beta^2)}, \quad \pi^N_s = \frac{k(a - bc_i^N)^2}{4(2bk - \beta^2)}\]

**Note:** We have \(2bk - \beta^2 > 0\) and \(a - bc < \frac{2(2bk - \beta^2)}{\beta}\) (due to \(0 < e_i < 1\))

### 3.3 The optimal decisions of the supply chain considering CSR through government subsidy

If there exists government subsidy, that is, \(S\) is greater than zero in equations (2) and (3), then a three-stage Stackelberg game, consisting of the government, a dominate retailer and suppliers, will come into being for the supply chain considering CSR. For such a case, according to social welfare maximization, the government plays the Stackelberg game with the retailer. According to the maximization of expected profits, the retailer and suppliers follow the Stackelberg game relationship.

#### 3.3.1 The optimal decisions of the dominant retailer

The expected profits of the dominant retailer through government subsidy can be rewritten as
follows.

$$
\pi^*_r = \sum_{i=1}^{n} (p_i^* - w_i^*) q_i^* - C(e_i^*) + S = \sum_{i=1}^{n} (p_i^* - w_i^*)(a - bp_i^* + \beta e_i^* - \frac{k(1-s)e_i^{s2}}{2})
$$

(12)

**Theorem 3.** Let $p_i^*$, $q_i^*$, $e_i^*$, $i = 1, 2, \ldots, n$, represent the selling price, order quantity, and the level of CSR effort in the process of the cooperation between the retailer and supplier $i$ through government subsidy. If there exists a set of uniquely determined values $p_i^*$, $q_i^*$, $e_i^*$ that maximize the expected profits of the retailer. The optimal decisions of the retailer are given as follows:

$$
p_i^* = \frac{a + bw_i^* + \beta e_i^*}{2b}
$$

(13)

$$
d_i^* = \frac{a - bw_i^* + \beta e_i^*}{2}
$$

(14)

$$
e_i^* = \frac{\beta(a - bw_i^*)}{2bk(1 - s^*) - \beta^2}
$$

(15)

$$
\pi^*_r = \sum_{i=1}^{n} \frac{k(1-s^*)}{2[2bk(1 - s^*) - \beta^2]}(a - bw_i^*)(a - bp_i^* - w_i^*)
$$

(16)

**Proof.** For the retailer-dominated supply chain considering CSR, if there exists government subsidy, then by computing the partial derivative of equation (12) with respect to variable $p_i^*$, we can obtain the first order condition as follows by equating the derivative to zero,

$$
\frac{\partial \pi^*_r}{\partial p_i^*} = a + \beta e_i^* - b(2p_i^* - w_i^*) = 0
$$

So, we can obtain

$$
p_i^* = \frac{a + bw_i^* + \beta e_i^*}{2b}
$$

Putting the above formula into the demand function gives us the order quantity as follows: (an equation is missing here)

Substituting the obtained value $p_i^*$ into the demand function leads to the order quantity as follows:
Similarly, differentiating equation (12) with respect to variable $e^i_\beta$ and equating the result to zero produce

$$\frac{\partial \pi^\beta_r}{\partial e^i_\beta} = \frac{\beta^2 e^i_\beta + \beta(a - bw^i_\beta) - 2be^i_\beta(1 - s^i_\beta)}{2b} = 0.$$ 

By solving this equation, we can obtain the level of CSR effort as follows:

$$e^i_\beta = \frac{\beta(a - bw^i_\beta)}{2bk(1 - s^{i*}) - \beta^2}.$$ 

And then by respectively computing the second-order partial derivatives of equation (12) with respect to $p^i_\beta$, $q^i_\beta$, $e^i_\beta$, we can acquire:

$$\frac{\partial^2 \pi^\beta_r}{\partial p^i_\beta^2} = -2b,$$

$$\frac{\partial^2 \pi^\beta_r}{\partial q^i_\beta^2} = -\frac{2}{b},$$

$$\frac{\partial^2 \pi^\beta_r}{\partial e^i_\beta^2} = -(1 - s^{i*})k.$$ 

Due to $b, k > 0, 0 < s < 1$, we see that these derivatives are all negative, which mean that $\pi^\beta_r$ is a concave function of $p^i_\beta$, $q^i_\beta$, $e^i_\beta$. Therefore, there exist uniquely determined values $p^{i*}_i, q^{i*}_i, e^{i*}_i$ of $p^i_\beta$, $q^i_\beta$, $e^i_\beta$, respectively, that maximize $\pi^\beta_r$. Hence, the retailer’s optimal decisions can be determined. QED.

3.3.2 The optimal wholesale price decisions of the suppliers

The expected profit function of supplier $i$ under government subsidy can be expressed as follows:

$$\pi^\beta_i = (w^i_\beta - c^i_\beta)q^i_\beta = (w^i_\beta - c^i_\beta)(a - bp^i_\beta + \beta e^i_\beta)$$

(17)

**Theorem 4.** Let $w^i_\beta$ represent the wholesale price of supplier $i$. If there exists a uniquely determined value $w^{i*}_i$ that maximizes the expected profits of the supplier, then the optimal wholesale price decision of the supplier under the government subsidy is given as follows:
\[ w_i^{b*} = \frac{a + bc_i^{b*}}{2b} \]

(18)

**Proof.** If there is government subsidy in the supply chain considering CSR, by substituting equations (13) and (15) into equation (17) and differentiating \( \pi_{si}^b \) with respect to variable \( w_i^b \), we obtain the following first-order condition by equating the derivative to zero,

\[ \frac{\partial \pi_{si}^b}{\partial w_i^b} = \frac{bk(1-s_i^b)(a + bc_i^{b*} - 2bw_i^b)}{2bk(1-s_i^b) - \beta^2} = 0 \]

By solving this equation, we obtain the wholesale price as follows

\[ w_i^b = \frac{a + bc_i^{b*}}{2b} \]

And by computing the second-order partial derivative of \( \pi_{si}^b \) with respect to \( w_i^b \), we acquire:

\[ \frac{\partial^2 \pi_{si}^b}{\partial w_i^{b*2}} = -\frac{2b^2k(1-s_i^b)}{2bk(1-s_i^b) - \beta^2} < 0, \]

According to this result, it follows that \( \pi_{si}^b \) is a concave function of \( w_i^b \), which show that there is a unique value \( w_i^{b*} \) of \( w_i^b \) that maximizes \( \pi_{si}^b \). Now the optimal decision can be obtained.

QED.

By substituting equation (18) into equations (13)-(17), the optimal decisions and the expected profits of the dominant retailer and suppliers under the government subsidy can be obtained, and the results are shown in Table 2.

**Table 2**
The optimal decisions and expected profits of the supply chain under government subsidy

<table>
<thead>
<tr>
<th>( P_i^{b*} )</th>
<th>( Q_i^{b*} )</th>
<th>( e_i^{b*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{(3a + bc_i^{b*})bk(1-s_i^{b*})-(a + bc_i^{b*})\beta^2}{2b[2bk(1-s_i^{b*})-\beta^2]} )</td>
<td>( \frac{bk(1-s_i^{b*})(a-bc_i^{b*})}{2[2bk(1-s_i^{b*})-\beta^2]} )</td>
<td>( \frac{\beta(a-bc_i^{b*})}{2[2bk(1-s_i^{b*})-\beta^2]} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( w_i^{b*} )</th>
<th>( \pi_r^{b*} )</th>
<th>( \pi_{si}^{b*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{a + bc_i^{b*}}{2} )</td>
<td>( \sum_{i=1}^{n} \frac{k(1-s_i^{b*})(a-bc_i^{b*})^2}{8[2bk(1-s_i^{b*})-\beta^2]} )</td>
<td>( \frac{k(1-s_i^{b*})(a-bc_i^{b*})^2}{4[2bk(1-s_i^{b*})-\beta^2]} )</td>
</tr>
</tbody>
</table>

**Note:** we have \( 2bk(1-s) - \beta^2 > 0 \), that is, \( 0 < s < 1 - \frac{\beta^2}{2bk} \).
3.3.3 The optimal subsidy rate decision of the government

In order to stimulate a better fulfillment of CSR, the government provides a certain amount of subsidy to the members of the supply chain. According to the goal of maximizing social welfare, the government determines the optimal subsidy rate, while its objective function can be expressed as the total revenue of the supply chain system plus consumer surplus minus government subsidy, that is

\[
\pi^g = \pi^g_u + \pi^g_s + CS^g - S = (p^g_i - w^g_i)q^g_i + \frac{ke^g_i}{2} + \frac{ke^g_i}{2} + (w^g_i - c^g_i)q + \frac{q^g_i}{2b} - \frac{s^g_i ke^g_i}{2} \quad (19)
\]

**Theorem 5.** Let \(s^{g^*}, \pi^{g^*}\) stand for the optimal government subsidy rate given to and the optimal level of social welfare produced by the supply chain considering CSR, respectively. If the government provides subsidy to the retainer, then the optimal government subsidy rate and the optimal level of resultant social welfare are given as follows:

\[
s^{g^*} = \frac{3bk - \beta^2}{5bk}
\]

(20)

\[
\pi^{g^*}_{si} = \frac{k(a - bc^g_i)^2[7bk(s^{g^*} - 1)^2 - \beta^2(3 - 2s^{g^*})]}{8[2bk(s^{g^*} - 1) + \beta^2]^2}
\]

(21)

**Proof.** If there is government subsidy in the supply chain considering CSR. By substituting the optimal decisions \(p^g_i, q^g_i, e^g_i\) into Equation (19) and differentiating \(\pi^g\) with respect to variable \(S\), we obtain the following first-order condition by equating the derivative to zero

\[
\frac{\partial \pi^g}{\partial S} = \frac{k \beta^2 (a - bc^g_i)^2 [bk(3 - 5s^g) - \beta^2]}{4[2bk(1 - s^g) - \beta^2]^2} = 0
\]

So, we acquire the optimal subsidy rate as follows:

\[
s^{g^*} = \frac{3bk - \beta^2}{5bk}
\]

And by substituting the optimal subsidy rate to Equation (19), we can determine the optimal level of resultant social welfare as follows.

\[
\pi^{g^*}_{si} = \frac{k(a - bc^g_i)^2[7bk(s^{g^*} - 1)^2 - \beta^2(3 - 2s^{g^*})]}{8[2bk(s^{g^*} - 1) + \beta^2]^2}
\]. QED.

By substituting Equations (20)-(21) into Table 2, the optimal decisions and expected profits of the government, the dominant retailer and suppliers through government subsidy are shown in Table 3.

**Table 3**

The optimal decisions of the supply chain through government subsidy

<table>
<thead>
<tr>
<th>(p^g_i)</th>
<th>(q^g_i)</th>
<th>(e^g_i)</th>
<th>(w^g_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\begin{array}{cccc}
 b(3a + bc_i^{b*}) - \beta^2 (a + 2bc_i^{b*}) & (2bk + \beta^2)(a - bc_i^{b*}) & 5\beta(a - bc_i^{b*}) & a + bc_i^{b*} \\
 b(4bk - 3\beta^2) & 2(4bk - 3\beta^2) & 2(4bk - 3\beta^2) & 2 \\
\end{array}
\]

\[
\begin{array}{cccc}
 s^{b*} & \pi_r^{b*} & \pi_{si}^{b*} & \pi^{b*} \\
 3bk - \beta^2 & \sum_{i=1}^{n} \frac{(2bk + \beta^2)(a - bc_i^{b*})^2}{8b(4bk - 3\beta^2)} & \frac{(2bk + \beta^2)(a - bc_i^{b*})^2}{4b(4bk - 3\beta^2)} & \frac{(7bk + \beta^2)(a - bc_i^{b*})^2}{8b(4bk - 3\beta^2)} \\
\end{array}
\]

Note: we have \(4bk - 3\beta^2 > 0\).

4. Sensitivity analysis

In this section, we will explore the impact of CSR factor on the decision of the supply chain, and analyze whether government subsidies can improve the performance of the supply chain to achieve Pareto optimization, and the relationship between government subsidy and CSR by using Maple 17.

4.1 The impact of CSR on the optimal decisions of the supply chain
4.1.1 Analysis of the impact of CSR cost factor on the optimal decisions

Because the levels of CSR effort of the supply-chain members are limited by their corresponding costs, we exploit mathematical methods to analyze the impact of CSR cost factor on the decision variables of the supply chain.

**Theorem 6.** Increasing CSR cost in the supply chain either with or without any government subsidy lowers the dominant retailer’s sales price, order quantity and level of CSR effort and profit, while the wholesale prices of the suppliers are not affected, but their profits as well as the total level of resultant social welfare will decrease.

**Proof.** For the supply chain considering CSR, if there is no government subsidy, by solving the partial derivatives of each decision variable in Table 1 on the CSR cost factor \(k\), we obtain the following.

\[
\frac{\partial p_i^{b*}}{\partial k} < 0, \frac{\partial q_i^{b*}}{\partial k} < 0, \frac{\partial e_i^{b*}}{\partial k} < 0, \frac{\partial \pi_i^{b*}}{\partial k} < 0, \frac{\partial \pi_{si}^{b*}}{\partial k} < 0, \frac{\partial \pi^{b*}}{\partial k} < 0.
\]

If there exists government subsidy, by computing the partial derivatives of each decision variable in Table 3 on the CSR cost factor \(k\), we acquire the following results.

\[
\frac{\partial p_i^{b*}}{\partial k} < 0, \frac{\partial q_i^{b*}}{\partial k} < 0, \frac{\partial e_i^{b*}}{\partial k} < 0, \frac{\partial \pi_i^{b*}}{\partial k} < 0, \frac{\partial \pi_{si}^{b*}}{\partial k} < 0, \frac{\partial \pi^{b*}}{\partial k} < 0. \text{ QED.}
\]

**Numerical example 1:** By taking the following parameter values \(n = 1, a = 100, b = 1, c = 10, \beta = 0.5\), respectively, we can determine \(s = \frac{3bk - \beta^2}{5bk}\), and then we can obtain the changing charts of the CSR cost factor \(k\) on the level CSR effort, profits of the supply chain members, and social welfare either with or without government subsidy, as shown in Fig. 2a-d, respectively.
According to Fig.1a-d, no matter under the condition of government subsidy or anarchy subsidy, the level of CSR effort, profits of the supply chain members, and the level of resultant social welfare decrease when the CSR cost factor $k$ increases. That is, they are negatively correlated with the CSR cost factor $k$. This phenomenon can be explained as follows: when the investment cost of the retailer is getting higher, it will reduce its level of CSR effort, and consumers will reduce their purchase of products with low CSR attribute, correspondingly resulting in a reduction in market demand for this kind of product and order quantity. Therefore, the retailer will reduce the sales price to stimulate consumer purchases, resulting in a decline in the total profit of the retailer. For the suppliers, their wholesale prices are unaffected because of not considering CSR, but their profits will also decrease due to the reduction in the retailer’s order quantities. Meanwhile, the consumer surplus decreases as market demand decreases, and the overall level of social welfare decreases. These results are consistent with Theorem 6.

4.1.2 Analysis of the impact of the level CSR effort on the optimal decisions

**Theorem 7.** The higher the level of CSR effort the dominant retailer is committed to, the higher the sale price, order quantities and profits will be for the suppliers, while the profits of the dominant retailer and the level of social welfare of the entire supply chain will increase firstly and
then decrease next.

**Proof.** Because the expression of wholesale prices is not related to \( c_i \), it can be treated be as a constant. Computing the partial derivative of Equations (6), (7), (13) and (14) with respect to the level \( c_i \) of CSR effort, and substituting Equation (6) into Equations (5) and (17) and then computing the partial derivatives of Equations (10) and (17) with respect to the level \( c_i \) of CSR effort produce:

\[
\frac{\partial \pi_{i,N}^s}{\partial c_i} > 0, \quad \frac{\partial q_{i,N}^s}{\partial c_i} > 0, \quad \frac{\partial \pi_{u,i}^{N*}}{\partial c_i} > 0, \quad \frac{\partial p_{i,N}^{b*}}{\partial c_i} > 0, \quad \frac{\partial q_{i,B}^{b*}}{\partial c_i} > 0, \quad \frac{\partial \pi_{u,i}^{B*}}{\partial c_i} > 0
\]

Since the first-order partial derivatives of the profit function of the retailer and the social welfare with respect to the level \( c_i \) of CSR effort are hard to directly judge in the two cases, we substitute Equation (6) into Equations (5), (12) and (19) and compute their second-order partial derivatives with respect to the level \( c_i \) of CSR effort as follows:

\[
\frac{\partial^2 \pi_{i,N}^{s*}}{\partial c_i^2} = \frac{2b - \beta^2}{2b}, \quad \frac{\partial^2 \pi_{i,B}^{b*}}{\partial c_i^2} = \frac{2b(1 - \frac{s}{N}) - \beta^2}{2b}, \quad \frac{\partial^2 \pi_{i,N}^{N*}}{\partial c_i^2} = \frac{4b - 3\beta^2}{4b}.
\]

Now, we can see that all the above second-order partial derivatives are less than zero based on the hypothesis that \( 2b - \beta^2 > 0, \quad 2b(1 - s) - \beta^2 > 0 \) and \( 4b - 3\beta^2 > 0 \). QED.

Theorem 7 shows that the higher the level of CSR effort the retailer is committed to, the higher the sale price, order quantities and profits will be for the suppliers. Intuitively, higher levels of CSR effort increase the market demand of the product with CSR attribute, and consumers are willing to purchase at a higher price. Meanwhile the profits of suppliers will increase accordingly due to the increase of order quantities, which is a "free riding behavior". The profit of the retailer and resultant level of social welfare will increase when the level of CSR effort of the retailer is improved within a certain range; however, excessive CSR investment will eat into the profits of the shareholders. Therefore, there is a certain limit on the amount of CSR investment, and the overall profits of the whole supply chain beyond this limit will be reduced, and consequently the level of resultant social welfare will also decline.

4.2 Analysis of the impact of government subsidy rate on the optimal decisions of the supply chain

**Theorem 8.** The more the government subsidizes the dominant retailer that assume the corporate social responsibility, the higher the level of CSR effort, sale price, order quantities and profit of the retailer are, and the higher the profits of the suppliers, but the level of social welfare, increase firstly and then decrease.

**Proof.** Let us compute the partial derivative of each decision variable in Table 2 with respect to the rate of government subsidy \( s \) as follows:

\[
\frac{\partial p_{i,N}^{b*}}{\partial s} > 0, \quad \frac{\partial q_{i,N}^{b*}}{\partial s} > 0, \quad \frac{\partial e_{i,N}^{b*}}{\partial s} > 0, \quad \frac{\partial \pi_{u,i}^{N*}}{\partial s} > 0, \quad \frac{\partial \pi_{u,i}^{B*}}{\partial s} > 0,
\]

and compute the partial derivative of Equation (19) with respect to the rate of government subsidy \( s \) by substituting the relevant variables in Table 2 into Equation (19) as follows:

\[
\frac{\partial \pi_{i,N}^{b*}}{\partial s} = \frac{k\beta^2(3bk - 5bks - \beta^2)(a - bc_i^{b*})^2}{4[2bk(1 - s) - \beta^2]}.
\]

When \( 3bk - 5bks - \beta^2 > 0 \), i.e. \( 0 < s < \frac{3}{5} \frac{\beta^2}{5bk} \), we have \( \frac{\partial \pi_{i,N}^{b*}}{\partial s} > 0 \). That is to say, the level
of resultant social welfare is positively related to the rate of government subsidy. QED

**Number example 2:** By taking the parameter values as $n = 1, a = 100, b = 1, c = 10, \beta = 0.5$, we can determine the optimal government subsidy rate as

$$s = 0.55$$

according to the range of the government subsidy rate $0 < s < \frac{3}{5} \frac{\beta^2}{5bk}$. Assume that $s \in [0.1, 0.6]$, then we can draw the trend charts of the level of CSR effort, profits of the supply chain members and the level of resultant social welfare on the government subsidy rate $s$, Figure 3a-d.

![Graphs showing the impact of government subsidy rate on CSR effort, retailer profits, supplier profits, and social welfare.](image)

**Fig.3.** The impact of government subsidy rate on the level of CSR effort (a), profits of the retailer (b), profits of the suppliers (c), and the social welfare (d)

According to Fig.3a-d, within the government subsidy rate range, the level of CSR effort and profits of the supply chain members increase with the government subsidy rate. The level of
resultant social welfare increases with the government subsidy rate at the beginning and then suddenly declines beyond a certain threshold value. This analysis shows that a certain government subsidy can enhance the level of CSR effort of the retailer and consumers are willing to purchase this kind of products with CSR attribute at a higher price. Ahat in turn result in increased market demand and order quantities, eventually the profit of the dominant retailer increases. For suppliers, their decisions are not affected by the government subsidy, so the wholesale prices remain unchanged. However, their profits will increase due to increasing order quantities. As the government makes policy decisions from the perspective of maximizing social welfare and cannot increase its subsidy indefinitely, that explains why only within a certain amount range of subsidy, the level of resultant social welfare will increase with increasing consumer surplus and the total profits of the supply chain, and then fall beyond this range. These results are consistent with Theorem 8.

Our analysis above also explains the following interesting phenomenon, where the suppliers do not have to take CSR and the government does not subsidize them, but their profits increase with increasing subsidy. What is worth noticing is that this phenomenon is actually a commonly seen free-rider behavior, and perhaps the government should punish those who do not actively take social responsibility, such as additional tax.

With regard to this conclusion, the reason why the government subsidy exceeds a certain range will lead to a decline in the level of resultant social welfare is that the government's excessive subsidies to the retailer can no longer raise its level of CSR effort and it will harm the level of the resultant social welfare by distorting allocation of resources away from consumers to the “dead weight loss” from CSR activities. This conclusion is consistent with the literature [45]. Or it can be explained that the government wants to increase the retailer's level of CSR effort through more subsidies, but raising the CSR level means higher CSR costs for the retailer. So the retailer will transfer this part of the cost to consumers for greater benefit. For consumers, they will reduce their amount of purchases of the particular product based on the maximization of their own utilities and instead they will look for alternatives, which results in reduced consumer surplus, and thus the level of resultant social welfare. This explanation is consistent with the literature (Hu, 2017).

4.3 Comparison of the performance of the supply chain between with or without government subsidy

Theorem 9. Compared to the situation without government subsidy, the market demand, the level of CSR effort of the dominant retailer, the profits of all members of the supply chain and the level of the resultant social welfare for the situation with government subsidy are greater.

Proof. For $2bk - \beta^2 > 0$ and $4bk - 3\beta^2 > 0$, the differences in the order quantities, the levels of CSR efforts, the profits of the retailer and suppliers and the levels of resultant social welfare in Table 1 and Table 3 can be calculated as follows:

$$
\Delta q = q_i^{\beta*} - q_i^{N*} = \frac{\beta^2(3bk - \beta^2)(a - bc_i^*)}{2(4bk - 3\beta^2)(2bk - \beta^2)} > 0
$$

$$
\Delta e = e_i^{\beta*} - e_i^{N*} = \frac{\beta(3bk - \beta^2)(a - bc_i^*)}{(4bk - 3\beta^2)(2bk - \beta^2)} > 0
$$
\[ \Delta \pi_r = \pi_r^{ws} - \pi_r^{ws} = \sum_{i=1}^{n} \frac{\beta^2 (3bk - \beta^2)(a - bc_i^*)^2}{8b(4bk - 3\beta^2)(2bk - \beta^2)} > 0 \]

\[ \Delta \pi_{s} = \pi_{s}^{ws} - \pi_{s}^{ws} = \frac{\beta^2 (3bk - \beta^2)(a - bc_i^*)^2}{4b(4bk - 3\beta^2)(2bk - \beta^2)} > 0 \]

\[ \Delta \pi = \pi^{ws} - \pi^{ws} = \frac{\beta^2 (3bk - \beta^2)(a - bc_i^*)^2}{8b(4bk - 3\beta^2)(2bk - \beta^2)} > 0 . \text{QED.} \]

**Number example 3:** By taking the parameter values as \( n = 1, a = 100, b = 1, c = 10, k = 1, \) we can compare the optimal decisions either with or without government subsidy under different parameter \( \beta \)-values, Table 4.

### Table 4
Comparison between the optimal decisions either with or without government subsidy

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( e )</th>
<th>( p )</th>
<th>( q )</th>
<th>( w )</th>
<th>( \pi_r )</th>
<th>( \pi_s )</th>
<th>( e )</th>
<th>( p )</th>
<th>( q )</th>
<th>( w )</th>
<th>( \pi_r )</th>
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</thead>
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<td>0.1</td>
<td>2.3</td>
<td>77.6</td>
<td>22.6</td>
<td>55</td>
<td>508.8</td>
<td>1017.6</td>
<td>5.7</td>
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<tr>
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<td>225</td>
<td>190</td>
<td>135</td>
<td>55</td>
<td>3037</td>
<td>6075</td>
</tr>
</tbody>
</table>

Table 4 details that the connection between subsidy and CSR is consistent with conventional wisdom. The retailer’s CSR effort is motivated by its desire to extract subsidy, and the retail price is boosted by increasing levels of CSR effort. That is to say, consumers need to pay more for the CSR effort of the retailer, and normally their purchasing power will decline with the price. However, on the contrary, the sales of the retailer increase with increasing level of CSR effort. We provide an explanation for why stronger consumers’ awareness of corporate social responsibility translates into their better willing to pay for products with CSR attributes (Chitra, 2007). For example, in 2005, the proportion of the European population that was willing to purchase expensive, environmentally-friendly products was 31% and rose to 75% in 2008 (Zhang et al., 2015). In addition, there is an interesting result, namely, the profits of suppliers under government subsidy are higher than those under no subsidy even though they do not demonstrate any CSR behavior. The reason for this end seems to be obvious that increase in order volumes results in their increasing profits. In fact, it is a free rider behavior, which for the time being has not been discussed.
Similarly, we can draw the trend charts for the level of CSR effort, profits of the supply chain members and the level of the resultant social welfare on the demand influence factor $\beta$ either with or without government subsidy, respectively, in Fig. 4a-d.

According to Fig. 4a-d, the level of CSR effort, the profits of the supply chain members and the level of resultant social welfare with government subsidy are higher than those without government subsidy, and all of the decision variables increase with the demand influence factor $\beta$, that is, they are positively correlated with the demand influence factor $\beta$. The reason for this end is that the greater the impact the retailer’ CSR exerts on consumer demand, the higher the level of CSR effort and the bigger the market demand is. For the retailer, government subsidy and increase in demand can overcompensate for the decline in profits caused by rising prices. So the profit of
the retailer increases with an increase in the demand influence factor $\beta$. For suppliers, as they do not participate in the CSR effort and do not receive government subsidy directly, their wholesale prices remain unchanged in both cases. However, their profits eventually increase with the increasing demand. This analysis also shows that a certain government subsidy can effectively encourage the dominant retailer to actively assume CSR, increase consumers’ demand for products with CSR attributes, improve the performance of the whole supply chain, increase the level of the resultant social welfare, and realize Pareto improvement of the supply chain system. These results are consistent with Theorem 9.

5. Conclusions and suggestions

With respect to decision problems of a retailer-dominated supply chain considering CSR under government subsidy, we exploit a three-stage Stackelberg game model to analyze the profits of all members, social benefits and consumer surplus, and explore the impact of CSR effort and government subsidies on decision variables and profits of the members, social welfare and consumer surplus, as well as the relationship between government subsidy rate and the level of CSR effort. The following summarizes our novel conclusions and suggestions:

(1) The higher the level of CSR effort the dominant retailer is committed to, the more beneficial it is to improve the profits of the members and the overall performance of the supply chain and the level of social welfare. Therefore, the leader of the supply chain should take on CSR actively in order to help with the overall performance of the supply chain and the maximization of the social welfare for sustainable development of the parties involved, and create a good corporate image so as to effectively guide other members of the supply chain to undertake CSR.

(2) As followers, the profits of suppliers under the government subsidy mechanism are all higher than that under anarchy subsidy and improve with increasing government subsidy, although they do not assume explicitly any CSR. It is a free-riding behavior that is not conducive to the maintenance of long-term relationships with the retailer. In order to create additional cooperation opportunities with giant retailers and enhance their own market competitiveness, suppliers should actively assume social responsibilities.

(3) As the dominant retailer assumes CSR, the higher the expenditure for the retailer to assume corporate social responsibility, the lower the level of CSR effort the retailer will be committed to. Therefore, the government should provide subsidy towards covering the corresponding cost incurred in taking social responsibility activities, such as research and development on technologies, application promotion and the constructions of complementary facilities and so on in order to enhance the motivation and effort level of enterprises.

(4) Similar to the study of Peng and Song (2016), the more subsidy the government provides for CSR, the higher the level of CSR effort the dominant retailer is committed to and the more profits supply chain members generate. However, Peng and Song didn’t consider the impact of subsidy on the level of the resultant social welfare, while we found that once the subsidy is beyond a certain range, the level of resultant social welfare will decline. So the government should not raise the level of subsidy indefinitely from the perspective of maximizing social welfare.

(5) Different from Xu et al (2014), where he only considered government subsidy as a parameter, this paper treats it as a decision variable, which is determined by the government (Huang et al., 2017). Based on this treatment, we found that the level of CSR effort, the performance of the supply chain and the level of the resultant social welfare under the government
subsidy mechanism are all higher than those under the anarchy subsidy. Therefore, in practical business operations, the government plays a key role in the sustainable development of the whole supply chain, and therefore it should join in the decision process as a decision maker (Its objective is the maximization of social welfare), rather than as a parameter considered in the profit functions of enterprises in their effort to coordinate the operation of the entire supply chain.

The conclusions of this study can provide reference and practical guidance for the government to formulate the CSR cost subsidy policy and for the members in the supply chain to make decisions. However, this paper points out that once the level of CSR effort is beyond a certain range, which will not only adversely affect the interests of socially responsible enterprise (here referring to the dominant retailer), but also have a negative impact on the level of the resultant social welfare, while it will exert a positive impact in those enterprises that do not consider CSR. Therefore, how to determine the range of the government subsidy rate to encourage enterprises to effectively assume social responsibility without adversely affecting the interests of shareholders, and how to formulate the corresponding punishment measures for those enterprises that do not consider CSR in order to avoid "free riding behavior" represent some of the very importance focuses of future research.

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