Usefulness of DC Power Flow for Active Power Flow Analysis

Konrad Purchala, *Student Member, IEEE*, Leonardo Meeus, Daniel Van Dommelen, *Senior Member, IEEE* and Ronnie Belmans, *Fellow, IEEE*

***Abstract* — In recent days almost every study concerning the analyses of power systems for market related purposes uses DC power flow. DC power flow is a simplification of a full power flow looking only at active power flows. Aspects as voltage support and reactive power management are not considered. However, such simplifications cannot always be justified and might sometimes be unrealistic. In this paper authors analyze the assumptions of DC power flow, and make an attempt at quantifying these using indexes. Among other, the paper answers the question of how low the X/R ratio of line parameters can be, and what is the maximal deviation from the perfect flat voltage which still allows DC power flow to be acceptably accurate.**

***Index Terms* — power systems, power system analysis, power flow, DC power flow.**

I. INTRODUCTION

STATIC power system analysis has always been performed using full power flow. It is one of the fundamental tools for power system analysis and is used in the operational as well as planning stages. Vertically integrated companies have used it to control their systems, as well as to plan the optimal economic operation of generation resources, either by means of optimal power flow or unit commitment. It is therefore extremely important to solve the load flow problem as efficiently as possible. Since the invention and widespread of computers, in the 1950's and 1960's, many methods for solving the load flow problem have been developed [1],[2].

Full power flow allows for management of both active and reactive power flows. Recently, with the liberalization of electricity markets, active power and reactive power are treated as different products. Active power is a tradable commodity, while reactive power is rather regarded as an ancillary service that has to be provided by the system operator and its costs are socialized among all users of the system. Due to the separation of these products, methods looking only at the active power flow become of increasing interest.

DC power flow is of the variations of the Newton method, very similar to fast decoupled method [3],[4]. It is a simplification of a full AC power flow and looks only at active power flows, neglecting voltage support, reactive power management and transmission losses. Thanks to its simplicity, and even more to the fact that DC power flow problem is linear, it is very often used for techno-economic studies of power systems for assessing the influence of commercial energy exchanges on active power flows in the transmission network [11],[6]. The method as such is well-known and its fundamentals have been discussed in many research papers [7],[8].

DC power flow can be applied if a number of assumptions are satisfied. However, it is not always evident how these assumptions should be understood. Take the one stating that line resistances have to be negligible. As it is obvious that the line resistances will not be infinitely small, there is somewhere a border value for X/R ratio that guarantees a given accuracy. However, where this border can be put is still an open question. Moreover, the sensitivity of the DC power flow solution to these assumptions has not been addressed. It seems that this method is often taken for granted [7],[10],[11], and the fact that it has been established 30 years ago can lead to the misuse of it and misinterpretation of its assumptions.

DC power flow is indeed an interesting alternative to classic power flow for techno-economic related purposes. Moreover, the method has been almost reinvented with the liberalization of electricity and the need for simple power system analysis tools. However, care should be taken while interpreting the results produced by this method. They can be a very good approximation of active power flows only if the assumptions underlying the method are met; otherwise the errors in estimation of active power flows become significant.

In this paper authors review the assumptions of DC load flow, aiming at quantifying up the criteria that have to be met in order to guarantee an acceptable accuracy of the abovementioned method. The paper is structured as follows. First the formulation of a DC power flow problem is recalled. Then each of the assumptions underlying the method is discussed, and the sensitivity of the power flow solution is analyzed. The latter involves variations of line parameters of a test network introduced by means of Monte Carlo simulations, and comparison of the results of a classic power flow and its DC simplified version. Based on these tests a number of criteria are set up that should guarantee the accuracy of DC

K. Purchala (e-mail: konrad.purchala@esat.kuleuven.ac.be), L. Meeus (e-mail: leonardo.meeus@esat.kuleuven.ac.be), D. Van Dommelen (e-mail: D.VanDommelen@esat.kuleuven.ac.be) and R. Belmans (e-mail: ronnie.belmans@esat.kuleuven.ac.be) are with the Electrical Engineering Department of University of Leuven, Kasteelpark Arenberg 10, 3001 Leuven, Belgium.

power flow to be within a 5% error margin. Finally the established criteria are tested on randomly generated test networks of varying topology and size.

II. FORMULATION OF DC POWER FLOW

The classic power flow problem consists of active and reactive power flow and it can be formulated using four variables per each node - voltage angle, voltage magnitude, active and reactive power injections. Active power losses are not known in advance as they depend on active power injection pattern and voltage profile. Other variables are also interdependent, which makes the problem non-linear. This is why it is often made linear and the solution is iterated. The losses are re-estimated at each iteration based on all other variables.

DC power flow, on the contrary, is a linear problem. It neglects active power losses, and assumes that magnitudes of nodal voltages are equal. Furthermore, voltage angle differences are assumed to be small. The only variables are voltage angles and active power injections. Due to the fact that losses are neglected, all active power injections are known in advance. Therefore the problem becomes linear and there is no need for iterations. For each node *i* in the system, the following set of equations must hold:

$$P_i = \sum_{j=1}^{nr\_lines} B_{ij}(\delta_i - \delta_j) \tag{1}$$

$$\sum_{i=1}^{n}\left[P_{G,i} - P_{L,i} - P_i\right] = 0 \tag{2}$$

Where:

P_i active power leaving node *i*.
$P_{G,i}$ active power injected at node *i*;
$P_{L,i}$ active power withdrawn from node *i*;

III. ASSUMPTIONS OF DC LOADFLOW

In order to simplify the power flow problem and make it linear, a number of assumptions are made:

- Voltage angle differences are small i.e. $sin(\delta) = \delta$
- Line resistance is negligible i.e. $R<<X$, thus lossless lines
- Flat voltage profile

However, such assumptions are not always realistic. Firstly, the *X/R* ratio condition can be difficult to guarantee. The influence of resistance increases with the decrease of voltage, which means that only the high voltage transport networks can withstand this condition. Moreover, voltages will most likely not be flat but will vary among busses, causing the voltage profile to be different from the assumed one. Each of these assumptions has some influence on the accuracy of the power flow calculations.

In the subsequent paragraphs, the sensitivity of the DC power flow results to the breach of assumptions underlying the method will be analyzed. First, the assumptions are examined using a 30-node network included in a power flow package MatPower [12]. Based on these tests, set of rules of thumb is developed that guarantee the accuracy of DC power flow estimation to be within 5% compared to the classic power flow. Finally, the developed rules are tested on a number of randomly generated networks, with changing topology, size and system load.

A. Voltage angle differences

The assumption of small voltage angle differences allows the *sine* to be replaced by its argument. It results in the following approximation:

$$\sin(\delta_i - \delta_j) \approx (\delta_i - \delta_j) \tag{3}$$

$$\cos(\delta_i - \delta_j) \approx 1 \tag{4}$$

It is often said that that such approximations can only be justified for weakly loaded networks. However, if voltage angle differences are sufficiently small, such approximation should not lead to significant errors as far as active power estimation is concerned. In order to check the actual values of voltage angle differences in real power system, the example of Belgian high voltage grid consisting of over 900 lines, with rated voltages from 70 kV to 380 kV, is used. The scenario adopted is the winter peak of 13 GW.

Fig. 1 shows the voltage angle differences experienced in Belgian HV network. The highest angle differences lies in the range of 6-7°, however, in 94 % of lines the voltage angle differences are lower than 2°.

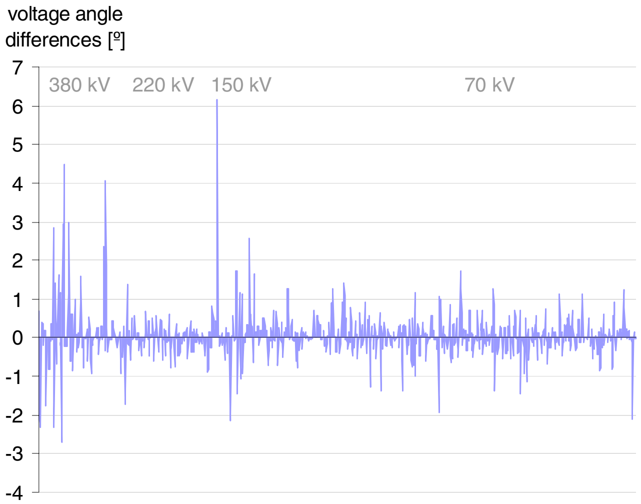

Figure 1. Line voltage angle differences in the Belgian HV grid, 13GW winter peak (950 lines at 70, 150, 220 and 380 kV)

Fig. 2 shows the error arising from assumptions (3-4). Comparison to voltage angles experienced in the Belgian HV grid shows, that abovementioned assumption should not cause any significant error.

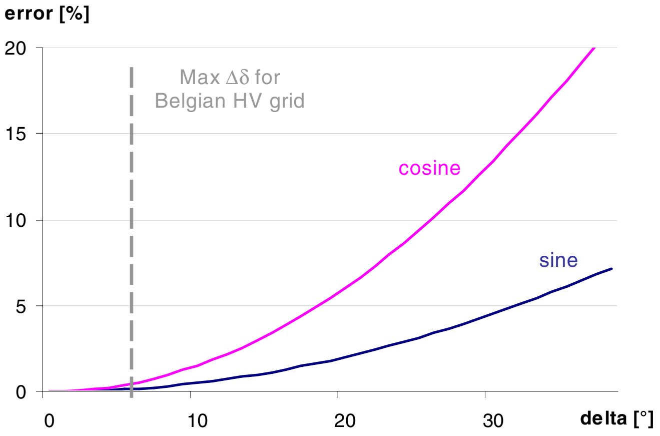

Figure 2. Consequences of *sine* and *cosine* approximations.

B. *Line resistance*

Another assumption playing a major role in the accuracy of DC power flow is the one of negligible line resistance. However, in actual electric networks this is not always the case.

As an example of typical values of X/R ratio for electricity transmission system the Belgian system has been chosen. Table I shows, for different voltage levels, minimal, maximal and average *X/R* ratios, as well as minimal and maximal values of resistances and reactances in the Belgian high voltage grid. Obviously, *X/R* ratio of a transmission line can vary depending on the voltage level, for Belgium being it from 0.8 to 12.5. The assumption of negligible resistance is therefore impossible to be guaranteed.

TABLE I
X AND *R* VALUES FOR BELGIAN HIGH VOLTAGE GRID [Ω/KM]

[kV]	min R	max R	avg R	min X	max X	avg X	min X/R	max X/R	avg X/R
380	0.025	0.038	0.031	0.278	0.353	0.325	8.4	12.5	10.5
220	0.038	0.088	0.067	0.184	0.429	0.364	3.5	8.0	5.5
150	0.018	0.292	0.090	0.071	1.458	0.374	1.0	12.0	4.2
70	0.034	0.425	0.174	0.034	0.756	0.360	0.8	9.0	2.1

In order to check the consequences of neglecting a non-negligible resistance a number of simulations have been made, using the 30-node network included in the MatPower power flow simulations package, which topology is shown in Fig. 3. However, it has to be stressed that it is only the topology which is relevant, as for the sake of the simulations line parameters are randomly set based on Monte Carlo method. Each line is assigned a randomly generated value of impedance, ranging from the minimal to maximal of reference values from Table I. In order to generalize the findings, all lines have the same impedance and X/R ratio for a given sample. Line voltage is set to 380 kV.

Fig. 4 shows, averaged over all lines, an active power estimation error P_{error} in function of *X/R* ratio, for a given value of resistance. P_{error} is defined as follows (5):

$$P_{error} = \frac{P_{ac} - P_{dc}}{P_{ac}} \cdot 100\% \qquad (5)$$

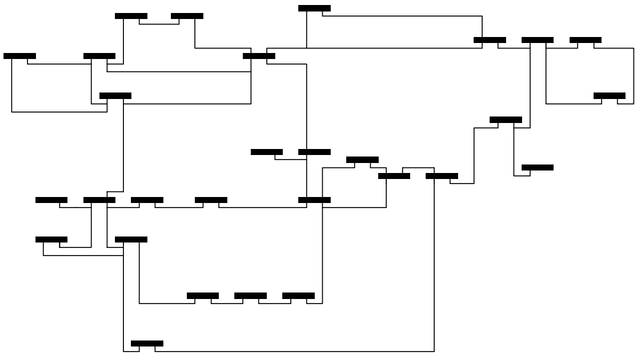

Figure 3. Topology of the 30-node test network (MatPower, case30.m).

As expected, the size of *R* has an influence on the active power estimation error; the higher the resistance, the higher the P_{error}. As DC power flow neglects active power losses, this assumption obviously introduces an error rising with increase of resistance. However, the rise of P_{error} is not very significant. For any tested combination of line parameters, even for very low *X/R* ratios and high resistance, 5% error margin is virtually never exceeded. For low values of resistance, being below 5Ω, the error is quite independent of the *X/R* ratio. The higher the resistance gets, the more significant the influence of *X/R* ratio becomes. This can be explained by an increased reactive power consumption of highly inductive lines, leading to a worse voltage profile. However, if *X/R* ratio is higher than 2, P_{error} is always smaller than 5%.

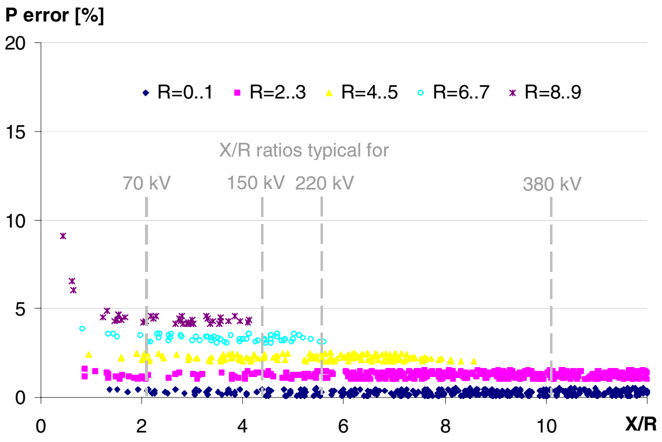

Figure 4. Influence of *X/R* ratio on active flow estimation error P_{error} for a given range of resistance *R* (5000 samples)

Fig. 5 shows the influence of reactance *X* on the P_{error}. It can be noticed that the curve is different from the former. For each value of reactance *X*, there is an increase of the active power estimation error for lower values of *X/R*. This implies, that not so much the reactance, but rather the *X/R* ratio is a decisive factor. Though the increase of line reactance causes an increase in P_{error}, it is insignificant for high values of *X/R*, and becomes considerable for low *X/R* ratios.

To check whether the both line parameters are interdependent, their influence has been plotted against each other in Fig. 6.

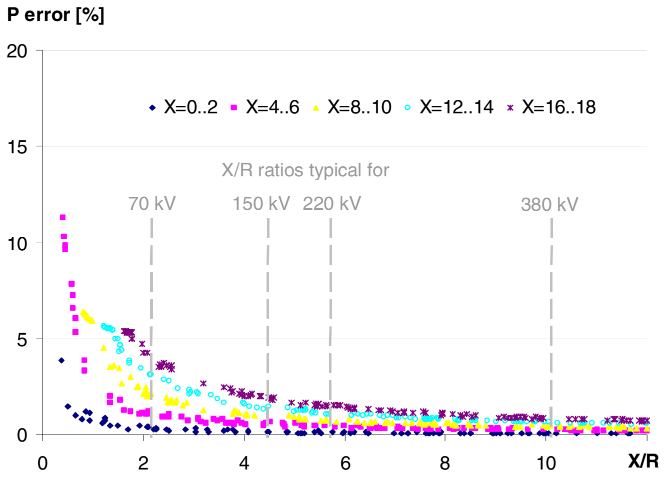

Figure 5. Influence of *X/R* ratio on active flow estimation error P_{error} for a given range of reactance *X* (5000 samples)

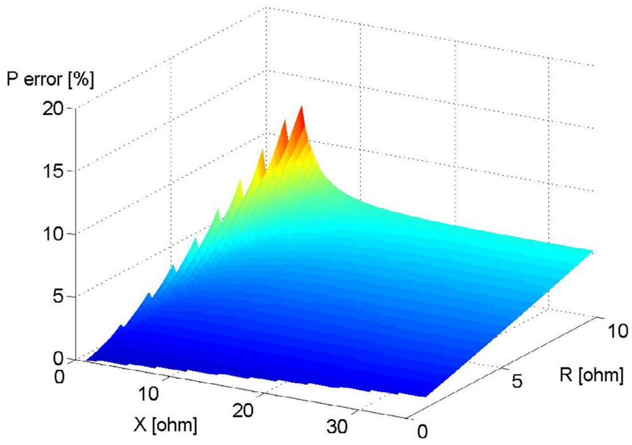

Figure 6. Influence line parameters on active flow estimation error P_{error} (5000 samples)

The following conclusions can be drawn:

- For low values of the resistance the size of the reactance, and consequently *X/R* ratio, is almost of no importance, as there is virtually no change of P_{error} with varying reactance.
- The higher the resistance, the more important *X/R* becomes. Even for the highest values of the resistance, provided that reactance is high enough, DC load flow does not introduce unacceptable active power estimation error.

As a general conclusion it can be said that it is the line resistance which is the decisive factor for the feasibility of DC power flow approximation. The smaller the line resistance, the better the DC approximation of power flows. Additionally, *X/R* ratio greater than 4 should be enough to limit the P_{error} increase in case of higher values of the line resistance.

C. *Influence of voltage variations (bad voltages)*

One of the assumptions of DC power flow is the flat voltage profile meaning that, in per unit terms, all voltages are equal. It is often asserted that these should be as close to 1 p.u. as possible. However, it is not the absolute voltage magnitude that matters, but deviations from the predefined value. If the nodal voltages are scattered round 1.1. p.u. instead of 1 p.u. DC power flow gives actually a better approximation of the power flows, as higher line voltages decrease losses. Voltage deviations, on the other hand, lead to line voltage differences that cannot be accounted for in DC power flow, which in turn influences the active power estimation error. From Fig. 7 it can be seen that the P_{error} increases with the increase of voltage deviations measured by means of standard deviation s_U (6).

$$s_U = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(U_i - \bar{U})^2} \qquad (6)$$

Though for most cases the average P_{error} is limited to 5%, the maximal error *MAX* P_{error}, almost perfectly correlated to the average, is over 8 times higher. Therefore the flat voltage profile is of extreme importance for the accuracy of DC power flow.

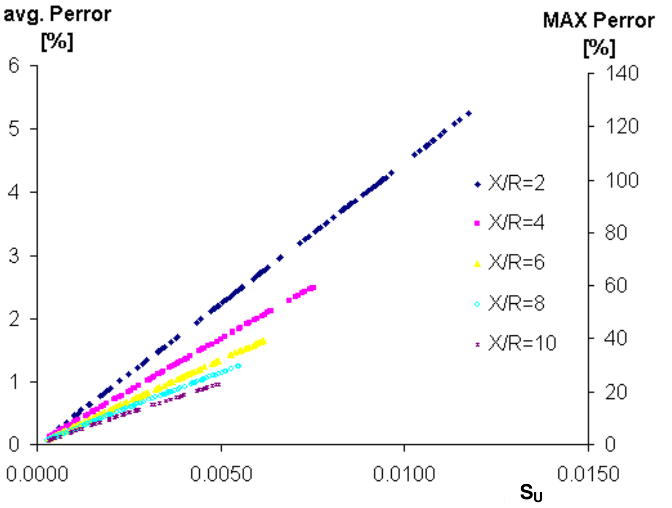

Figure 7. Influence of voltage fluctuations (standard deviation of the voltage) on active flow estimation error (1000 samples)

In the actual power system however it is quasi impossible to keep all voltages constant, avoiding voltage fluctuations. To check the likelihood of a favorable voltage profile, voltages in the Belgian high voltage network are taken as an example. have been analyzed. Fig. 8 presents the nodal voltage magnitudes in the Belgian high voltage transmission grid, thus 70-380 kV, for the 13 GW winter peak scenario. As noticed, voltage profile is not very flat, the standard deviation being $s_U = 0.0166$. As seen in Fig. 7, P_{error} is very sensitive to voltage deviations, and care should therefore be taken while interpreting the results. Realistic example of voltages in the actual power system shows that the assumption of perfect voltage profile is the most critical one and voltage profile is the biggest source of active power estimation error.

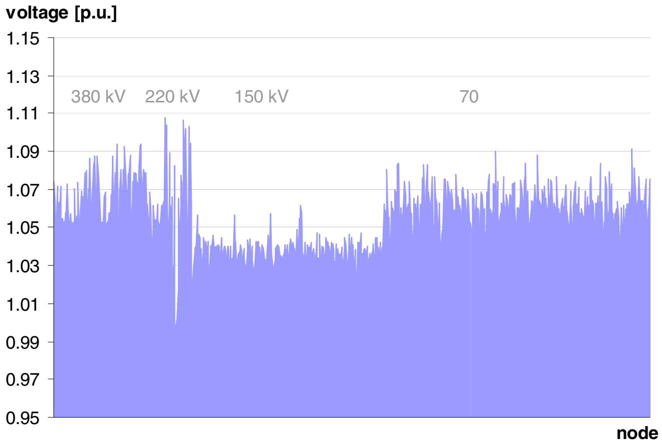

Figure 8. Voltage magnitude in the Belgian high voltage grid, 13 GW winter peak scenario. 713 nodes.

D. *Assumptions of DC load flow – conclusions*

In the previous paragraphs the assumptions underlying DC power flows have been analyzed using a test network of a fixed topology and varying line parameters. The main observation is that not all networks can be analyzed with the same precision using this technique, which is not at all surprising. From all the assumptions the one of perfect, flat voltage profile seems to be the most critical. Typically, a high voltage transmission grid is operated at voltages higher than rated as it decreases transmission losses and allows better absorption of rapid load changes. Moreover, nodes having controllable voltage are usually set above nominal voltage to account for voltage drops along transmission lines, while the voltages on other nodes depend on constantly changing system conditions. These voltage deviations affect the accuracy of DC power flow calculations as the higher the standard deviation s_U, the higher the P_{error} becomes. However, if voltage fluctuations can be limited, the performance of DC power flow becomes very good. Note that an error in estimation of active power flows on a given line can be higher than the average P_{error}.

Another important factor is the line resistance R as it influences both total line impedance, a decisive factor for the power flow pattern, and active power losses. Therefore, DC power flow calculation performed for networks having lines of X/R ratio lower than 4, are the most likely to be incorrect. Lines with X/R ratios higher that 4 on the contrary, usually introduce less error. There exist modifications which attempt to overcome these difficulties [13]. However, in this paper authors would like to evaluate pure DC power flow solutions as the present techno-economic studies employ the original method.

Based on the simulations from the previous paragraphs, the assumptions underlying DC power flow are quantified. This implies that the assumptions are extended with indexes, allowing for their better understanding:

- Negligible line resistance means that $X/R > 4$
- Flat voltage profile means that the standard deviation of voltages $s_U < 0.01$

IV. TEST OF THE FINDINGS

In the previous paragraph the assumptions of DC power flow were analyzed. Based on results of the comparison between the full and simplified power flow methods and errors introduced by the latter, the indexes quantifying the assumptions of DC power flow have been proposed. In this paragraph these indexes will be further examined using randomly generated networks, in to be able to generalize the conclusions and avoid being network specific.

A. *Test on randomly generated networks*

A number of networks have been randomly generated in order to evaluate the accuracy of DC load flow. The number of nodes, lines as well as the network topology have been randomly chosen. Line parameters are also generated randomly and each line can have a different resistance and reactance. X/R ratio of each line is always greater than 4, as established in the previous paragraphs to be a prerequisite for acceptable average P_{error} produced by DC power flow. Fig. 9 and Fig. 10 present the P_{error} and corresponding active power flow P_{flow} for each line of the randomly generated networks.

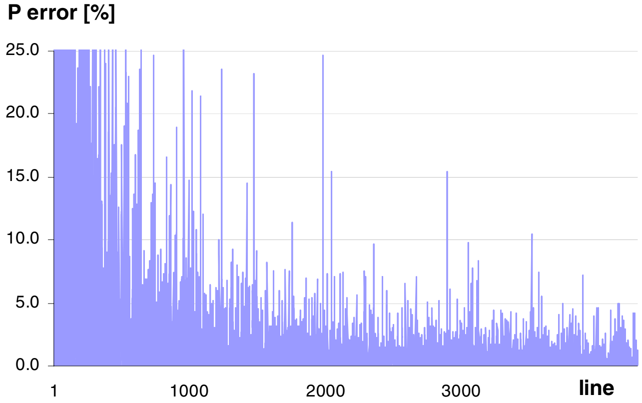

Figure 9. Active power estimation error P_{error} per line for randomly generated test cases

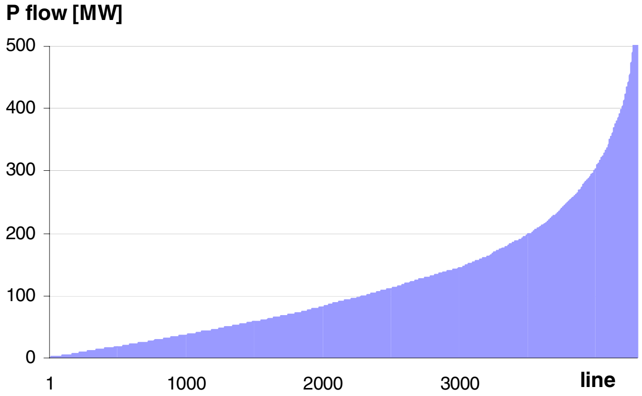

Figure 10. Active power flow per line for randomly generated test cases

From the above figures it is seen, that even though all conditions for acceptable accuracy of DC load flow, developed in chapter III are met, the active power estimation error P_{error} on a given line can sometimes exceed the predefined 5% limit. However, most lines with significant P_{error} are very weakly loaded, and consequently the absolute value of the P_{error} is insignificant. Nevertheless, even for large flows, there are some cases of P_{error} exceeding 5%. Yet the frequency of occurrence of these anomalies is rather low. If one limits the analysis to lines transferring 22MW or more, in 95% of the cases the P_{error} is within limits, and averages modest 1.5%.

V. CONCLUSIONS

This paper has examined DC power flow as a power system analysis tool. The method is increasingly used for techno-economic studies, related to electricity markets. The authors identified indexes that quantify the assumptions underlying the method. There are number of factors affecting the accuracy of DC power flow. First of all, the voltage profile has to be as flat as possible, meaning that there should be as little voltage deviations as possible. The higher they become the higher the active power estimation error. The notion of standard deviation s_U is used in the paper to depict the influence of voltage deviations on the accuracy of power flow solution. Secondly, the *X/R* ratio should be high enough, otherwise the assumption of negligible resistance is violated. The proposed border value is set at *X/R=4*.

On the whole DC power flows can give a good approximation of active power flows in the network. However, even if all assumptions to limit the average P_{error} to 5% are fulfilled, errors on individual lines can occasionally be significant, especially if voltage profile is not sufficiently flat. Therefore care should be taken when drawing conclusions based on simulations performed using this technique, as not every network is suitable for DC power flow calculations.

VI. REFERENCES

[1] W. F. Tinney and C. E. Hart, “Power Flow Solution by Newton’s Method”, *IEEE Transactions on Power Apparatus and Systems*, vol. 86, n. 11, Nov. 1967, pp. 1449-1460.

[2] B. Stott, “Review of Load-Flow Calculation Methods”, *Proceedings of the IEEE*, Vol. 2, No. 7, July 1974, pp. 916-929.

[3] B. Stott, O. Alsac, “Fast decoupled load flow”, *IEEE Transactions on Power Apparatus and Systems*, May-June 1974, p859-869.

[4] K. Behnam-Goilani, "Fast Decoupled Load Flow: The Hybrid Model", *IEEE Transactions on Power Systems*, vol. 3, n. 2, May 1998, pp. 734-742

[5] ETSO - European Association of Electricity Transmission System Operators, “Coordinated auctioning of Transmission Capacity in Meshed Networks”, Discussion Paper, Mar. 2001, http://www.etso-net.org

[6] ETSO - European Association of Electricity Transmission System Operators, “Reconciliation of market splitting with coordinated auction concepts”, Discussion Paper, Feb. 2002, http://www.etso-net.org

[7] S. Deckmann, A.C. Pizzolante, A.J. Monticelli, B. Stott, O. Alsac, “Numerical testing of power system load flow equivalents”, *IEEE Transactions on Power Apparatus and Systems*, vol. 6, 1999, pp. 2292-2300.

[8] S. Deckmann, A.C. Pizzolante, A.J. Monticelli, B. Stott, O. Alsac, “Studies on power system load flow equivalents”, *IEEE Transactions on Power Apparatus and Systems*, vol. 6, 1999, pp. 2301-2310.

[9] Grainger J.J., Stevenson W.D.,Jr. *Power system analysis*, McGraw-Hill, Inc., 1994.

[10] Syed A. Nasar, Trutt F.C., *Electric power systems*, CRC Press, USA, 1999.

[11] U.G. Knight, *Power Systems in Emergencies*, John Wiley & Sons; 1st edition, February 15, 2001

[12] R. D. Zimmerman, D. Gan, MatPower – Matlab Power System Simulation Package, School of Electrical Engineering, Cornell University, http://www.pserc.cornell.edu/matpower/matpower.html

[13] D. Rajicic, A. Bose, "A modification to the fast decoupled power flow for networks with high R/X ratios”, *IEEE Transactions on Power Systems*, vol. 3, n. 2, Aug. 1988, pp. 743-746.

VII. BIOGRAPHIES

Konrad Purchala graduated in 1999 as electrotechnical engineer from Warsaw University of Technology, Poland. Since 2000 he is working as a research assistant at Katholieke Universiteit Leuven (K.U.Leuven). He is a member of KULeuven Energy Institute and Electrical Energy research group (ELECTA) of the department of Electrical Engineering, where he is writing his Ph.D. on technical and economic aspects of congestion management. His research interests include congestion management, techno-economic aspects of power systems and electricity markets.

Leonardo Meeus graduated in 2002 as commercial engineer from the Katholieke Universiteit Leuven (K.U.Leuven). He is a member of the KULeuven Energy Institute and of the Electrical Energy research group (ELECTA) of the department of Electrical Engineering of the KULeuven, where he is working towards a Ph.D. on electricity market design. His research interests include techno-economic aspects of power systems, congestion management and security of power supply.

Daniel Van Dommelen is electrotechnical engineer from the K.U.Leuven (1964) and has an M.Sc. in Electrical Engineering (Univ. of Wisc., Madison, U.S.A., 1966) and a Ph.D. from the K.U.Leuven in 1971. Since 1977 he is a full professor at the K.U.Leuven at the research topics of Power Systems, High Voltage and Electroheat. He was visiting professor at the Univ. of British Columbia in Vancouver from July 1979 till March 1980. In 1981, he founded the European EMTP Users Group and chaired the Leuven EMTP Center till 1993. He is also chairman and Belgian representative in the Electroheat Research and Education (ERE) Committee of the UIE and has been chairman of the IEEE Benelux Section, and is a member of the IEEE PES (SM ’78), CIGRE, SEE and national electrical engineering societies.

Ronnie Belmans received the M.S. degree in electrical engineering in 1979 and the Ph.D. degree in 1984, both from the K.U.Leuven, Belgium, the Special Doctorate in 1989 and the Habilitierung in 1993, both from the RWTH, Aachen, Germany. From 1979 to 1985, he was a member of the staff of the K.U.Leuven. Currently, he is a full professor with the K.U.Leuven, teaching electric power and energy systems. His research interests include techno-economic aspects of power systems, power quality and distributed generation. He was the Director of the NATO Advanced Research Workshop on Vibrations and Audible Noise in Alternating Current Machines (August 1986). He was with the Laboratory for Electrical Machines of the RWTH, Aachen, Germany, as a Von Humboldt Fellow (October 1988-September 1989). From October 1989 to September 1990, he was a visiting associate professor at Mc Master University, Hamilton, Ont., Canada. During the academic year 1995-1996 he occupied the Chair at the London University, offered by the Anglo-Belgian Society. From June 2002 he is chairman of the board of directors of ELIA, the Belgian transmission grid operator.