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# LRFD for assessment of deteriorating existing structures

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## Abstract

The deterioration of infrastructures is a widespread problem in many countries. In order to assess such structures for continued future service, simple and practical tools need to be developed for evaluating the time-dependent reliability and performance of the structures. This paper describes the concept of a resistance reduction factor due to degradation of a component to approximate a time-dependent reliability problem as a time-independent one. With the factor the time-varying resistance can be equivalently replaced with a time-invariant resistance, and load and resistance factors can be determined for the assessment applying simple AFOSM. An approximation method to determine the factor is proposed, and a numerical example shows that the target reliability level can be achieved fairly accurately using an approximate reduction factor. It is also demonstrated that the approximation method can be applied to estimate a resistance reduction factor including the cases when the strength of a component is restored to some extent by means of inspection and repair, and, accordingly, to evaluate the effect of inspection and repair on the reliability of the component. © 2002 Elsevier Science Ltd. All rights reserved.

*Keywords:* Structural reliability; LRFD; Deterioration; Existing structures; Inspection and repair

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## 1. Introduction

The continued use of existing structures is of great importance because the built environment is a huge economic and political asset [1]. Also the importance of preserving the earth environment is well recognized. A major concern in evaluating structures for continued service is in ensuring that their capacity to withstand extreme events has not deteriorated from structural aging during their previous service history.

The safety problem in structural engineering can be treated more rationally with probabilistic methods. These methods provide basic tools for evaluating structural safety quantitatively [2]. Uncertainties in loads, material properties, and construction practice, which have been traditionally dealt with by empirical safety factors, can be taken into account explicitly and consistently in a

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probabilistic safety assessment. Such methods have been used to develop the first-generation of limit state design codes, in which the safety checks are associated with a specified limit state probability or reliability [3]. However, in the development of the first-generation codes the focus has been put on new construction and there is no presumption that structural resistance decreases and periodic inspection and repair would be carried out to restore the resistance.

In order to assess an existing structure for continued future service, one of the authors developed methodologies for evaluating time-dependent reliability and performance of a structural component with resistance varying in time due to aging and/or aggressive service environments [4]. Enright and Frangopol [5] extended these methodologies for the system reliability assessment of deteriorating concrete bridges considering load redistribution. Vu and Stewart [6] used the Monte-Carlo simulation method to estimate time-dependent reliability for RC bridge decks subjected to corrosion resulting from chlorides taking the interaction between cracking, diffusion of chlorides, and corrosion initiation into account. However, such an assessment requires a substantial investment in reliability analysis and acquisition of supporting data, which may not be feasible for the assessment of the majority of buildings for possible retrofit [7]. Vrouwenvelder and Schießl [8] proposed to provide two sets of partial factors within the framework of current design code in order to take the deterioration of structural components into account. However, the conditions of existing structures differ from structure to structure, and cannot be classified into only a few categories. Moreover, the effect of inspection and repair should be taken into account in the assessment for continued service. Simple but versatile reliability tools need to be developed for use in practice.

This paper describes the concept of a resistance reduction factor reflecting the impact of degradation of a component on its performance in order to approximate a time-dependent reliability problem as a time-independent one. With the factor the time-varying resistance can be equivalently replaced with a time-invariant resistance, and load and resistance factors can be determined for the assessment of an existing structure applying simple Advanced first-order second-moment method (AFOSM). In order to avoid time-dependent reliability analysis to determine the resistance reduction factor, an approximation method is proposed and a numerical example shows that the target reliability level can be achieved fairly accurately using an approximate reduction factor. It is also demonstrated that the approximation method can be applied to determine a resistance reduction factor including the cases when the strength of a component is restored to some extent by means of inspection and repair.

## 2. Structural degradation

Time-dependent effects on in situ strength must be considered in assessing the effect of aging and possible structural deterioration of new or existing structures. The strength of a structure or a component may degrade in time due to aging and/or environmental stressors according to,

$$R(t) = R_0 \cdot G(t) \quad (1)$$

in which  $R_0$  is the component capacity in the undegraded (original) state, and  $G(t)$  is a time-dependent degradation function defining the fraction of initial strength remaining at time  $t$ . The

degradation mechanisms are uncertain, experimental data are lacking, and thus the function  $G(t)$  should be treated as stochastic. However, as it has been found that the variability in  $G(t)$  is of minor importance when compared to mean degradation and load process characteristics [4], it is assumed that  $G(t)$  is deterministic and equal to mean  $E[G(t)] = g(t)$ .

Conceptually, a degradation function  $g(t)$  can be associated with each environmental stressor [9]. For reinforced concrete structures, most significant resistance deterioration mechanisms have been identified qualitatively [9,10]. Corrosion of reinforcement is one of the most damaging mechanisms affecting the strength of reinforced or prestressed concrete structures over time, and followed by sulfate attack, freeze-thaw cycling, and reactive aggregate reactions within the concrete. Quantitative degradation models are available for some of degradation mechanisms [6,11,12].

It is assumed in the following that a degradation function is independent of the load history; with this assumption, the subsequent formulation can address deterioration due to corrosion, sulfate attack, and similar environmental effects.

### 3. Stochastic load models

Events resulting in significant structural loads occur randomly in time and are random in their intensity and are in general modeled as a stochastic process,  $S(t)$ . Examples of relatively simple models of the overall temporal variation in structural loads are a Poisson impulse (PI) process and a Poisson square wave (PSW) process [2,13,14].

A PI process consists of impulses which occur according to a Poisson process with a mean occurrence rate,  $\lambda_{PI}$ . In a Poisson process, the probability that the  $N(t)$  load events occur within the time interval  $(0, t)$  is expressed as,

$$P[N(t) = n] = \frac{(\lambda_{PI} \cdot t)^n \cdot \exp\{-\lambda_{PI} \cdot t\}}{n!}; \quad n = 0, 1, 2, \dots \quad (2)$$

in which  $P[\bullet]$  is the probability of event  $\bullet$ .

A PSW process consists of pulses; the changes of the intensity of the pulses occur according to a Poisson process with a mean occurrence rate,  $\lambda_{PSW}$ . The duration of each pulse is a random variable described by an exponential distribution function with a mean value,  $1/\lambda_{PSW}$ .

Sequence of load intensity  $S_j, j = 1, 2, \dots, n$  in the respective PI and PSW processes are assumed to be identically distributed and statistically independent random variables. It is also assumed that the intensity of each pulse in a PSW process is constant during its duration. Load events that randomly occur with very short duration, such as wind load and earthquake load, are generally modeled as a PI process, while load events to which a structure is always subjected and whose intensity changes randomly, such as sustained live load, are generally modeled as a PSW process.

The combination of a PSW process and a PI process along with a time-invariant dead load is considered in the following (see Fig. 1).

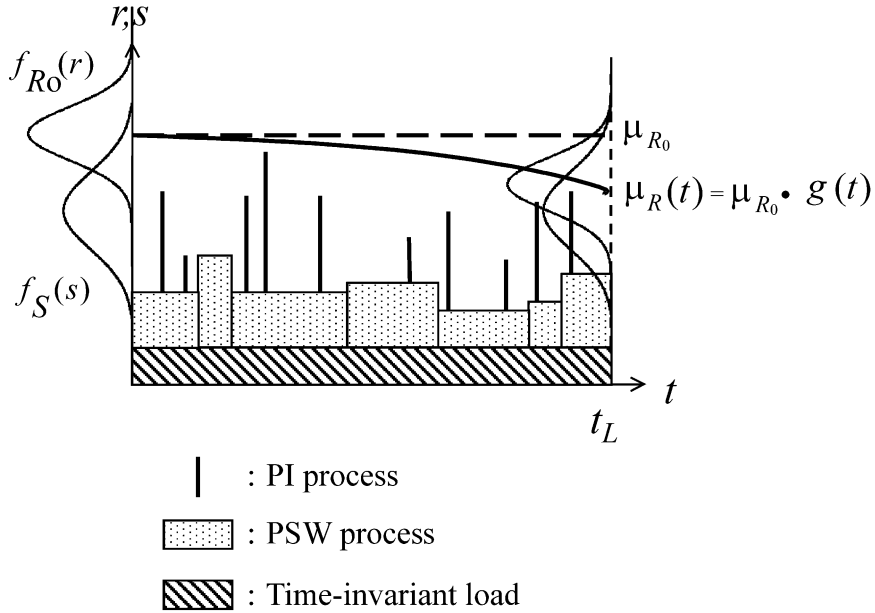


Fig. 1. Schematic representation of load process and resistance degradation.

#### 4. LRFD for deteriorating component

The design criteria of a deteriorating structural component subjected to dead load, sustained live load (PSW process), and either wind or seismic load (PI process) can be expressed as,

$$P[g(t) \cdot R_0 < D + Ls(t) + W(t); \text{ for } 0 < t \leq t_L] \leq P_{f_T} \tag{3}$$

in which  $P_{f_T}$  is the target failure probability,  $D$  is the intensity of time-invariant dead load, and  $Ls(t)$  and  $W(t)$  are the intensity of sustained live load and either wind or seismic load at time  $t$ .  $R_0$  in Eq. (3) is expressed in units that are dimensionally consistent with  $D$ ,  $L(t)$ , and  $W(t)$ . As a lot of computational effort is in general required to evaluate the failure probability of the deteriorating structure, Eq. (3) is not appropriate for use in practice.

If one can appropriately evaluate an equivalent resistance reduction factor  $g^*$  which makes the following design criteria equivalent to Eq. (3), the reliability analysis can be simplified.

$$P\left[g^* \cdot R_0 < \max_{0 < t < t_L} \{D + Ls(t) + W(t)\}\right] \leq P_{f_T} \tag{4}$$

Applying Turkstra's rule [15] to evaluate the failure probability in Eq. (4) considering  $W(t)$  as the primary load, the design criteria can be further simplified as,

$$P\left[g^* \cdot R_0 < D + Ls_{\text{apt}} + \max_{0 < t < t_L} \{W(t)\}\right] \leq P_{f_T} \tag{5}$$

in which  $L_{s_{apt}}$  is the intensity of arbitrary point-in-time sustained live load.

AFOSM can now be applied to evaluate the failure probability in Eq. (5) and also the checking point can be determined. Then the design criterion can be reduced to a familiar load and resistance factor format expressed as,

$$\phi \cdot g^* \cdot R_n > \gamma_D \cdot D_n + \gamma_L \cdot L_{s_n} + \gamma_W \cdot W_n \tag{6}$$

in which  $\phi$  is the resistance factor,  $\gamma_{\bullet}$  is the load factor for load  $\bullet$ , and  $\bullet_n$  is the nominal value of  $\bullet$ .

### 5. Estimation of resistance reduction factor

#### 5.1. Time-dependent reliability analysis

When a structural component subjected to stochastic load events modeled as a PI process and when  $g(t)$  is independent of the load history, the probability of failure in a time interval  $(0, t_L)$  is expressed as [4],

$$F(t_L) = 1 - \int_0^\infty \exp \left[ -\lambda \cdot t_L \cdot \left[ 1 - \frac{1}{t_L} \int_0^{t_L} F_S \{ r \cdot g(t) \} dt \right] \right] \cdot f_{R_0}(r) dr \tag{7}$$

in which  $\lambda$  is the mean occurrence rate of load events,  $F_S(s)$  is the probability distribution function (cdf) of load intensity,  $S$ , and  $f_{R_0}(r)$  is the probability density function (pdf) of  $R_0$ .

Assuming that the variability in the intensity of dead load and sustained live load is relatively small, the failure probability of a deteriorating structural component subjected to the combination of  $D$ ,  $L(t)$ , and  $W(t)$  during the time interval  $(0, t_L)$  can be evaluated as,

$$F(t_L) = 1 - \int_0^\infty \exp \left[ -\lambda_W \cdot t_L \left[ 1 - \frac{1}{t_L} \int_0^{t_L} F_W \{ r \cdot g(\tau) - \mu_{DL} \} d\tau \right] \right] f_{R_0}(r) dr \tag{8}$$

in which  $\lambda_W$  and  $F_W(w)$  are the mean occurrence rate and the cdf of the intensity of the primary load,  $W(t)$ , respectively, and  $\mu_{DL}$  is the mean value of  $D + L_{s_{apt}}$ .

Using a resistance reduction factor,  $g^*$ , the failure probability can be evaluated as,

$$F(t_L) = 1 - \int_0^\infty \exp \left[ -\lambda_W \cdot t_L \left[ 1 - F_W \{ r \cdot g^* - \mu_{DL} \} \right] \right] f_{R_0}(r) dr \tag{9}$$

A resistance reduction factor can be determined so that the failure probability estimated by Eq. (8) may be equal to that by Eq. (9).

#### 5.2. Sensitivity of $g^*$

The resistance reduction factor,  $g^*$ , would be dependent on the target reliability level, the rate and shape of a degradation function, the variability in resistance, and the degree of dominance,

Table 1  
Degradation model

Shape of the degradation function	Corresponding degradation mechanism
Linear: $g(t) = 1 - at$	Corrosion
Parabolic: $g(t) = 1 - at^2$	Sulfate attack
Square root: $g(t) = 1 - a\sqrt{t}$	Diffusion controlled degradation

the mean occurrence rate, and the statistical characteristics of the intensity of the primary load. The degree of the dependence of  $g^*$  on these factors is investigated using a simple degradation model defined by

$$g(t) = 1 - a \cdot t^b \quad (10)$$

in which  $a$  is the degradation rate, and  $b$  is dependent on the degradation mechanism. Example of the value of  $b$  for typical degradation mechanisms of a reinforced concrete component are presented in Table 1 [9].

Fig. 2(a)–(g) illustrates the resistance reduction factor  $g^*$  as a function of the degradation level,  $g(t_L)$ , at the end of the reference period of 100 years using the degradation model and the probability model shown in Tables 1 and 2, respectively. The values in the parentheses in Table 2 are used in the following unless otherwise noted.

Fig. 2(a)–(d) show the dependence of  $g^*$  on the coefficient of variation (c.o.v.) of resistance,  $V_R$ , the mean number of the primary load events during the reference period,  $\lambda_W \cdot t_L$ , the c.o.v. of the primary load intensity,  $V_W$ , and the ratio of the mean value of the primary load intensity to that of dead load intensity,  $\mu_W/\mu_D$ , respectively, assuming that the primary load intensity is lognormally distributed. Fig. 2(e)–(g) shows the dependence of  $g^*$  on the target reliability index,  $\beta_T$ , assuming that the primary load intensity is described by (e) lognormal distribution function, (f) Type I extreme value distribution function, and (g) Type II extreme value distribution function.

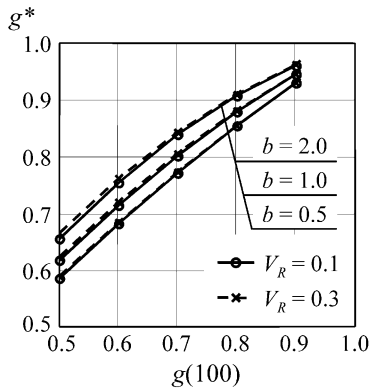
It can be seen from Fig. 2(a)–(g) that a resistance reduction factor is fairly lightly dependent on  $V_R$ ,  $\lambda_W t_L$ ,  $\mu_W/\mu_D$ , and  $\beta_T$ , but strongly dependent on the behavior of the degradation function and the statistical characteristics of the primary load intensity.

### 5.3. Approximation method for $g^*$

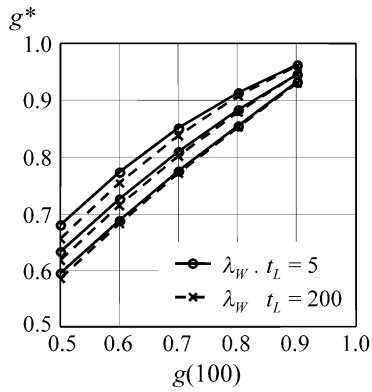
The procedure of determining a resistance reduction factor described in Section 5.1. requires a time-dependent reliability analysis, which is not suitable for use in practice. In this section an approximation method is proposed to estimate  $g^*$  without losing accuracy.

Taking the first order approximation, Eqs. (8) and (9) can be rewritten by Eqs. (11) and (12), respectively.

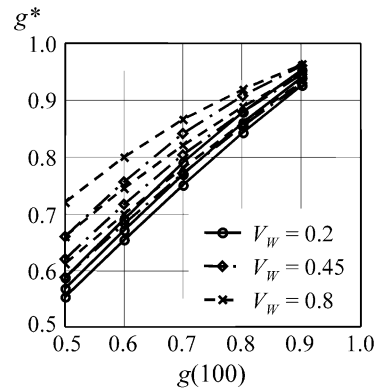
$$F(t_L) \simeq 1 - \exp \left[ -\lambda_W \cdot t_L \left[ 1 - \frac{1}{t_L} \int_0^{t_L} F_W \{ \mu_{R_0} \cdot g(\tau) - \mu_{DL} \} d\tau \right] \right] \quad (11)$$



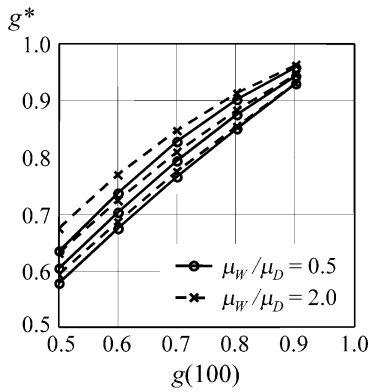
(a) Dependence on  $V_R$   
(  $W \sim \text{Lognormal}$  )



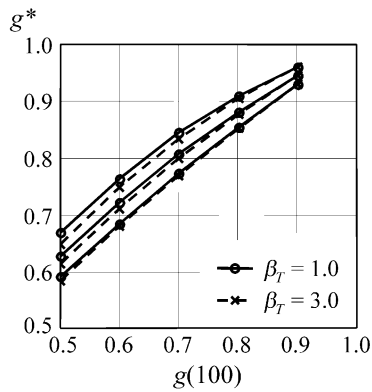
(b) Dependence on  $\lambda_W \cdot t_L$   
(  $W \sim \text{Lognormal}$  )



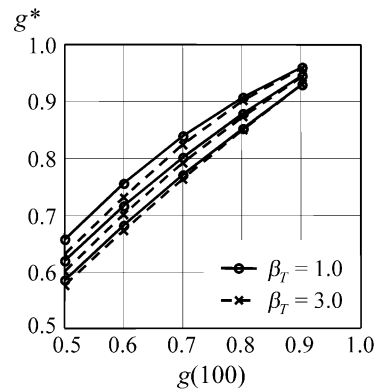
(c) Dependence on  $V_W$   
(  $W \sim \text{Lognormal}$  )



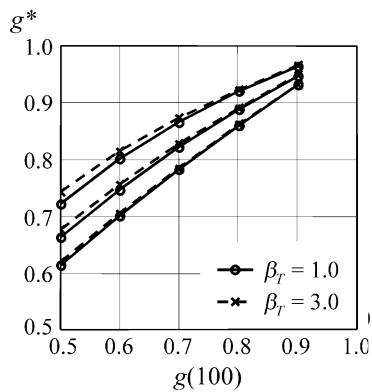
(d) Dependence on  $\mu_W / \mu_D$   
(  $W \sim \text{Lognormal}$  )



(e) Dependence on  $\beta_T$   
(  $W \sim \text{Lognormal}$  )



(f) Dependence on  $\beta_T$   
(  $W \sim \text{Type I}$  )



(g) Dependence on  $\beta_T$   
(  $W \sim \text{Type II}$  )

Fig. 2. Degradation level  $g(t_L)$  vs. resistance reduction factor  $g^*$ .

Table 2  
Probability model of load and resistance

	pdf	Mean*	c.o.v.	Process	$\lambda$ (year <sup>-1</sup> )
$D$	Normal	$\mu_D$	0.10	–	–
$L_S$	LN	$0.3\mu_D$	0.40	PSW	1/8
$W$	Type I, II, lognormal	$0.5\mu_D-2.5\mu_D(1.0\mu_D)$	0.20–1.00 (0.45)	PI	1/20–2.0(1.0)
$R$	Lognormal	$\beta_T=1.0-3.0$ in 100 years (2.0)	0.1–1.3 (0.15)	–	–

$$F(t_L) \simeq 1 - \exp[-\lambda_W \cdot t_L [1 - F_W\{\mu_{R_0} \cdot g^* - \mu_{DL}\}]] \quad (12)$$

Letting Eq. (11) equal Eq. (12), the resistance reduction factor,  $g^*$ , can be evaluated by

$$g^* = \frac{F_W^{-1} \left[ \frac{1}{t_L} \int_0^{t_L} F_W\{\mu_{R_0} \cdot g(\tau) - \mu_{DL}\} d\tau \right] + \mu_{DL}}{\mu_{R_0}} \quad (13)$$

in which  $F_W^{-1}(w)$  is the inverse of the cdf  $F_W(w)$ , and  $\mu_{R_0}$  is the mean value of the initial resistance. It can be seen from Eq. (13) that  $g^*$  is a function of the statistical characteristics of the primary load intensity and the degradation function, which agrees with the fact observed in the sensitivity analysis in Section 5.2.

When designing a structural component taking time-varying resistance into account, two unknown values in Eq. (13), i.e.  $\mu_{R_0}$  and  $g^*$ , must be determined. This task can be achieved solving Eq. (13) along with Eq. (14) simultaneously in an iterative manner.

$$\mu_{R_0} = \mu_R / g^* \quad (14)$$

in which  $\mu_R$  is the mean value of the resistance of a non-degrading component with the same target reliability level, which can be determined by AFOSM or load and resistance factors evaluated by the simplified method provided in the draft recommendations of limit state design by AIJ [16] (see Appendix A).

As  $g^*$  is fairly insensitive to the mean and variance of the initial resistance as seen in Fig. 2(a) and Fig. 2(e)–(g), only one loop of iteration could provide a good approximation.  $\mu_{R_0}$  for Eq. (13) can be estimated by Eq. (14) replacing  $g^*$  with  $\bar{g}$ , the temporal mean value of the degradation function over the time interval  $(0, t_L]$ , evaluated by

$$\bar{g} = \frac{1}{t_L} \int_0^{t_L} g(\tau) d\tau \quad (15)$$



## 6. Numerical example

### 6.1. Accuracy of approximate $g^*$

In Figs. 3(a)–(c) and 4(a)–(c), the reduction factors estimated by Eq. (13) are illustrated as a function of  $g(100)$  using the degradation model and probability model in Tables 1 and 2. It is assumed that  $V_W$  equals 0.2 [Fig. 3(a)–(c)] or 0.8 [Fig. 4(a)–(c)]. The solid lines, chain lines, and dashed lines are the approximate  $g^*$  for the degradation function with  $b$  in Eq. (10) equals 0.5, 1.0, and 2.0, respectively. The resistance reduction factors evaluated accurately as described in Section 5.1 are also plotted in the figures. The intensity of the primary load is described by (a) lognormal distribution function, (b) Type I extreme value distribution function, and (c) Type II extreme value distribution function, respectively.

Fig. 5(a)–(c) illustrates the effect of error in the estimation of  $g^*$  on the reliability index assuming that the degradation level at 100 years after construction,  $g(100)$ , equals 0.7 and that

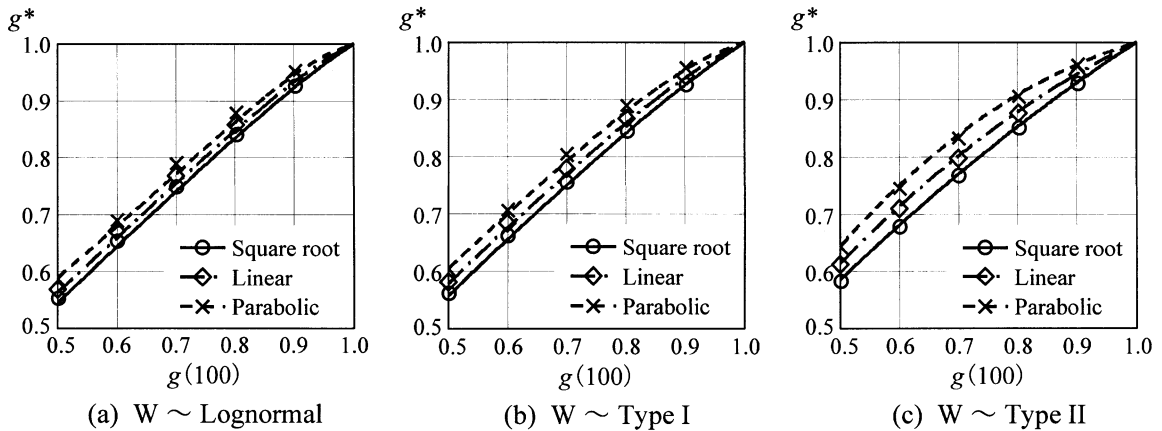


Fig. 3. Accuracy of approximate  $g^*$  ( $V_W=0.2$ ).

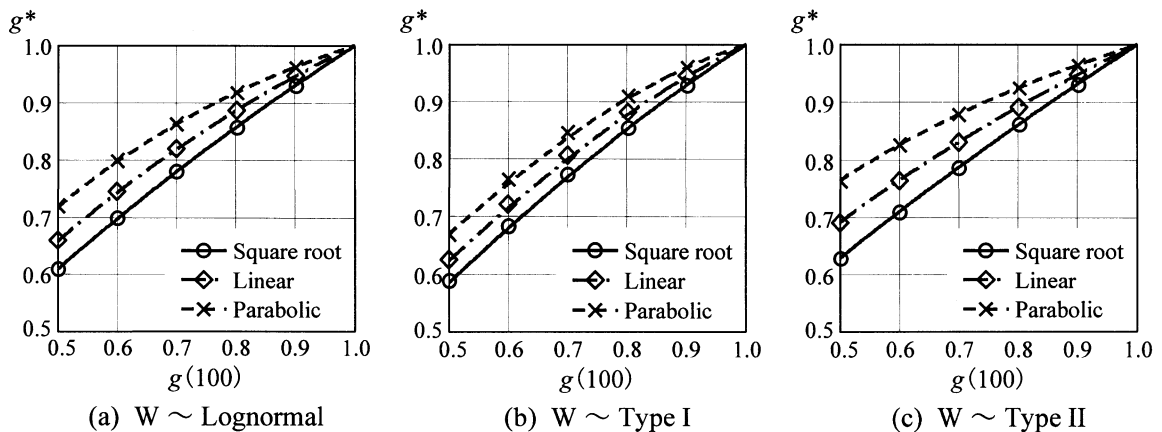
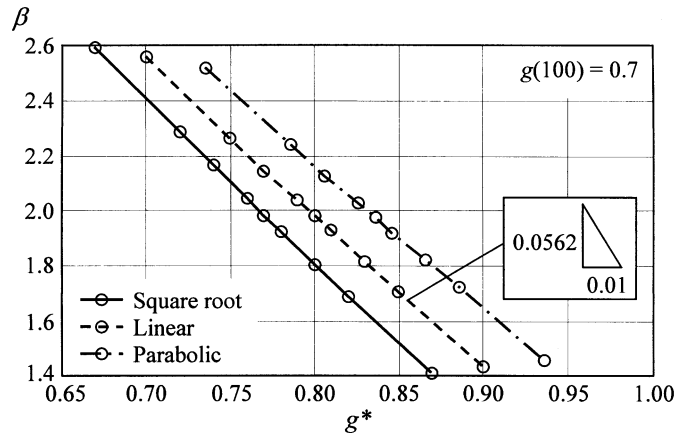
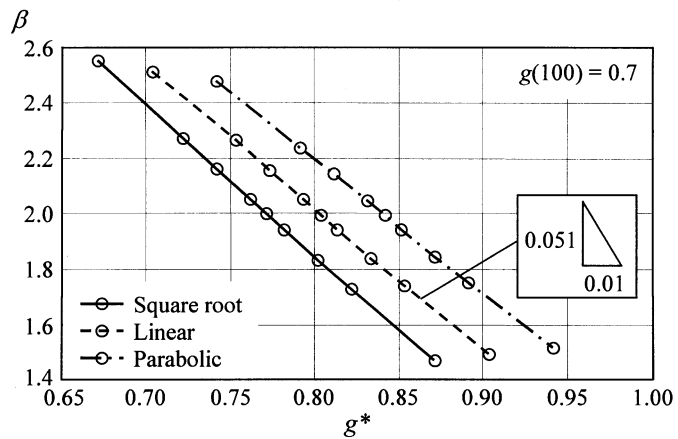


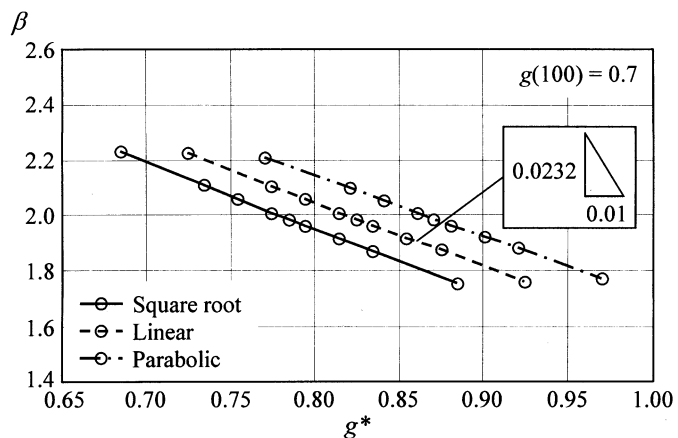
Fig. 4. Accuracy of approximate  $g^*$  ( $V_W=0.8$ ).



(a)  $W \sim \text{Type I}$



(b)  $W \sim \text{Lognormal}$



(c)  $W \sim \text{Type II}$

Fig. 5. Sensitivity of reliability index to  $g^*$ .

the primary load intensity is described by (a) lognormal distribution function, (b) Type I extreme value distribution function, (c) Type II extreme value distribution function. In any case, the reliability index  $\beta$  decreases almost linearly as  $g^*$  increases within the range of  $\beta$  considered in the example. The slope of the line is dependent on the type of the cdf of the primary load intensity but nearly independent of the shape of the degradation function. The slope is about 5/1 when the primary load intensity is described by a lognormal or a Type I distribution function, yielding about 5% error in the estimation of  $\beta$  with the error of 0.02 in the estimation of  $g^*$ . When the load intensity is described by a Type II distribution function, the slope is about 2.5/1 yielding only 2.5% error. Therefore, the error observed in Fig. 4(a)–(c) would not have much importance in practice.

### 6.2. Reliability level achieved using approximate $g^*$

The reliability level achieved using  $g^*$  estimated by Eqs. (12)–(15) is illustrated in Fig. 6(a)–(f) in terms of the ratio of the actual failure probability of a degrading component designed using approximate  $g^*$  to the target failure probability. Load and resistance factors by the simple method provided in the draft recommendations of limit state design by AIJ is used changing the reference period from 50 years to 100 years to determine  $\mu_R$  in Eq. (14). In order to avoid the error other than using an approximate  $g^*$  in designing a component, the maximum intensity of the combination of time-varying load processes is accurately evaluated using the method proposed by Mori and Murai [17], and probability analysis using FFT [18] is applied for accurate and efficient reliability analysis. Time-dependent reliabilities are evaluated by Monte Carlo simulation. The number of samples is  $\beta_T = 1.0$  and 2.0, and  $10^6$  for  $\beta_T = 3.0$ .

Fig. 6(a)–(c) illustrates the dependence of the achieved level on  $g(100)$  considering (a) square root, (b) linear, and (c) parabolic degradation functions. Fig. 6(d)–(f) illustrates the dependence of the achieved level on (d)  $V_R$ , (e)  $V_W$ , and (f)  $\mu_W/\mu_D$ . In any case, the target reliability level is achieved fairly accurately using an approximate  $g^*$ .

### 6.3. In-service inspection and repair

Periodic in-service inspection followed by suitable repair may restore a degraded structural component to near-original condition. Since inspection and repair is costly, there are tradeoffs between the frequency, extent and accuracy of inspection, required level of reliability, and cost. To design the optimum inspection/repair strategy, one must solve the non-linear optimization problem including time-dependent reliability analysis, which requires substantial computational effort [19]. However, using the resistance reduction factor, the optimization problem can be reduced into two simpler problems neither of which requires time-dependent reliability analysis: determination of  $g^*$  required to achieve the target reliability level and determination of schedule and accuracy of inspection and repair which satisfies  $g^*$ .

In order to illustrate the role of inspection and repair in maintaining the reliability of a deteriorating component and to demonstrate how an approximate resistance reduction factor works to assess the reliability of the component with inspection and repair, two simple inspection and repair strategies are considered: (1) infrequent inspection and repair carried out at 30 and 65 years after the construction with strength restored to its original level following repair, and (2)

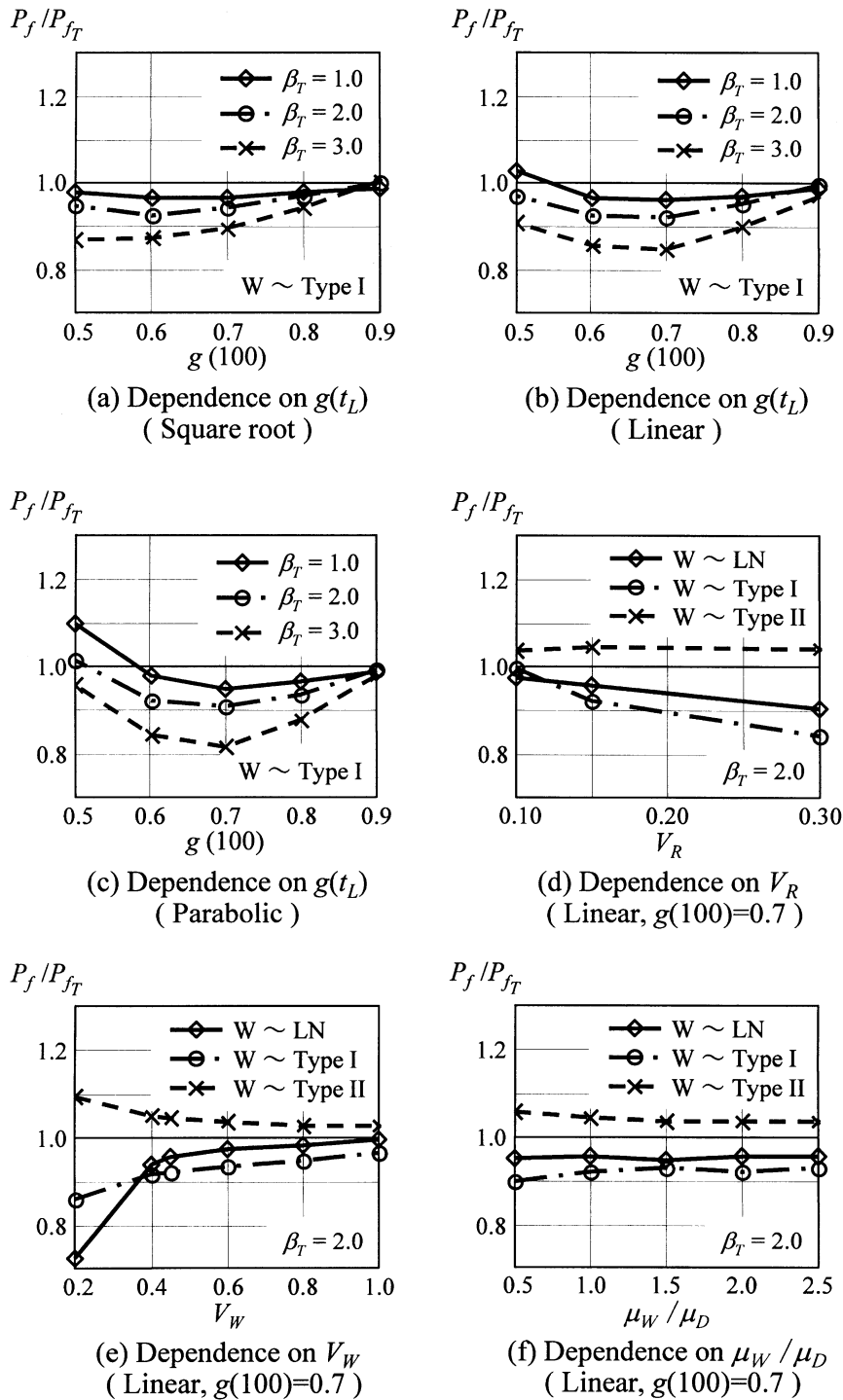
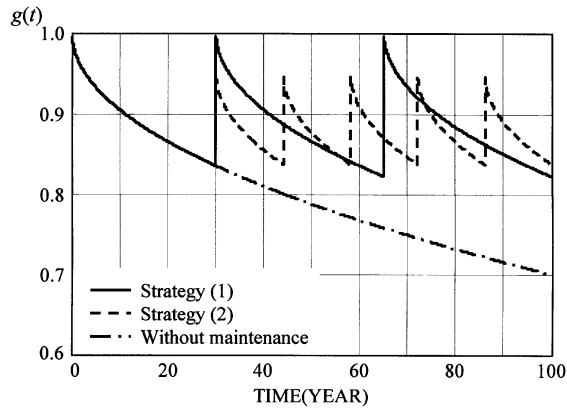
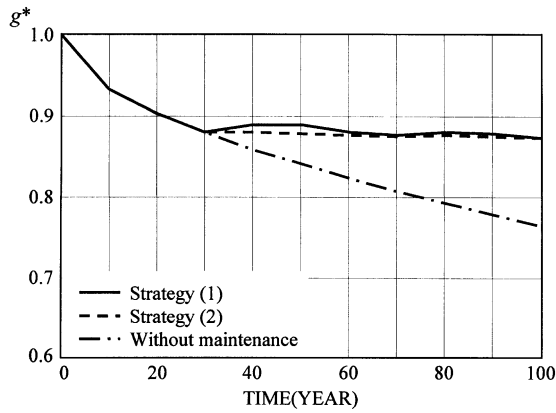


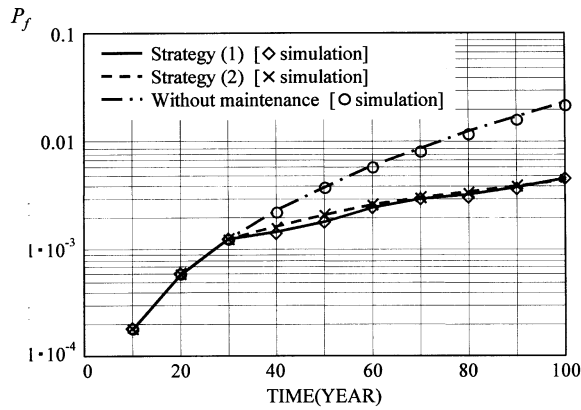
Fig. 6. Reliability level achieved using approximate  $g^*$ .



(a) Degradation function



(b) Resistance reduction factor



(c) Failure probability

Fig. 7. Role of inspection and repair.

frequent inspection and repair performed at 30, 44, 58, 72 and 86 years with strength restored to only 95% of its original level following repair. Without inspection/repair it is assumed that resistance of the component degrades as,

$$g(t) = 1 - 0.03\sqrt{t} \quad (16)$$

It is also assumed that following repair the resistance degrades as,

$$g(t) = c - 0.03\sqrt{t - t_R} \quad (17)$$

in which  $c$  is the level of the resistance restored by the repair and  $t_R$  is the time of inspection/repair. The degradation functions, the resistance reduction factors, and failure probabilities of a component associated with these strategies are illustrated in Fig. 7(a)–(c), respectively. The failure probability using approximate  $g^*$  is evaluated applying the method proposed by Mori and Murai for load combination and probability analysis using FFT in order to avoid the error other than using the reduction factor. The time-dependent reliabilities evaluated by the Monte-Carlo simulation is also plotted in Fig. 7(c). After the first inspection/repair, the resistance reduction factor is kept relatively constantly by the following periodic inspection/repair. At any time of the reference period the failure probability is fairly accurately estimated using an approximate  $g^*$ . At the time of inspection and repair, the failure probability changes its slope; this change is more distinct when the component is inspected more thoroughly. If  $P_{JT} = 5 \times 10^{-3}$  for the reference period of 100 years, both strategies would be acceptable in terms of risk. The selection of strategy, then, will be based on the cost associated with each strategy.

## 7. Conclusions

The concept of a resistance reduction factor to take into account the deterioration of a structural component is introduced to approximate a time-dependent reliability problem as a time-independent one. It is found that the resistance reduction factor is fairly lightly dependent on the mean number of occurrences of the primary load during the reference period, the ratio of the mean intensity of the primary load to that of the dead load, and target reliability level. An approximation method is proposed to estimate the reduction factor simply for use in practice. Numerical example shows that the target reliability level can be achieved fairly accurately using an approximate reduction factor. It is also demonstrated that the approximation method can be applied to estimate a resistance reduction factor including the cases when the strength of a component is restored to some extent by means of inspection and repair, and, accordingly, to evaluate the effect of inspection and repair on the reliability of the component. Thus, using a resistance reduction factor, the optimal inspection/repair strategy to maintain structural reliability at an acceptable level during the remaining service life of an existing structure with minimal cost can be determined without time-dependent reliability analysis.

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### Appendix A

Conceptually structural engineers could control the reliability level of a structure within the framework of a limit state design method. However, in most of the first-generation of limit state design codes, only one set of load and resistance factors are provided for each load combination, and thus the reliability level is basically fixed using these factors. If an engineer would like to control the reliability level, they have to conduct a reliability analysis by themselves. Recently, a simple procedure to determine load and resistance factors is provided in the draft recommendation of limit state design, AIJ taking the variability in loads and resistance and target reliability level into account [16]. The procedure is described in the following.

Load and resistance factor design format can generally be expressed as,

$$\phi \cdot \mu_R = \sum_i \gamma_i \cdot \mu_{S_i} \tag{A1}$$

in which  $\mu$  is the mean value of , and  $\phi$  and  $\gamma_i$  are a resistance factor and a load factor for load  $S_i$ , respectively, to be multiplied to the respective mean value.

When all the basic random variables are lognormally distributed, the load and resistance factors can be estimated fairly accurately by [20],

$$\phi = \frac{1}{\sqrt{1 + V_R^2}} \exp(-\alpha_R \cdot \beta_T \cdot \sigma_{\ln R}) \tag{A2}$$

$$\gamma_i = \frac{1}{\sqrt{1 + V_{S_i}^2}} \exp(\alpha_i \cdot \beta_T \cdot \sigma_{\ln S_i}) \tag{A3}$$

in which  $\sigma_{\bullet}$ ,  $V_{\bullet}$  are standard deviation and coefficient of variation (c.o.v) of  $\bullet$ , respectively,  $\beta_T$  is the target reliability index, and  $\alpha_{\bullet}$  is the separation function of variable  $\bullet$ , which can be approximately estimated by

$$\alpha_R = \frac{\tilde{\sigma}_R}{\sqrt{\tilde{\sigma}_R^2 + \sum \sigma_{S_i}^2}} \tag{A4}$$

$$\alpha_{S_i} = \frac{\sigma_{S_i}}{\sqrt{\tilde{\sigma}_R^2 + \sum \sigma_{S_i}^2}} \tag{A5}$$

in which  $\tilde{\sigma}_R = V_R \cdot \sum \mu_{S_i}$ .

Table A1  
Coefficient for Eqs. (A6) and (A7) for annual maximum value

Type of a year max value	Eq. (A6)				Eq. (A7)			
	$e_0$	$e_1$	$e_2$	$e_3$	$s_0$	$s_1$	$s_2$	$s_3$
Normal	0.00	0.00	0.00	0.00	0.01	0.85	-0.49	0.14
Type I	0.00	-0.16	-0.01	0.00	0.02	1.13	-0.67	0.20
Type II	0.00	-0.28	-0.05	0.07	0.00	1.44	-0.98	0.26

Table A2  
Coefficient for Eqs. (A6) and (A7) for 50-year maximum value

Type of a year max value	Eq. (A6)				Eq. (A7)			
	$e_0$	$e_1$	$e_2$	$e_3$	$s_0$	$s_1$	$s_2$	$s_3$
Normal	0.02	1.89	-1.05	0.30	0.02	0.34	-0.32	0.11
Lognormal	-0.01	2.34	-1.01	0.16	0.00	0.61	-0.13	0.00
Type I	0.04	2.32	-1.43	0.43	0.05	0.59	-0.60	0.22
Type II ( $\beta_T \leq 2.6$ )	0.01	2.82	-2.15	0.62	0.00	1.44	-0.98	0.26
Type II ( $\beta_T > 2.6$ )	0.01	2.28	-2.15	0.62	0.00	1.68	-1.14	0.30

When the basic random variables are not lognormally distributed, ‘equivalent’ lognormal random variates can be used. In the recommendation, it is presumed that the statistical characteristics of the annual maximum value of each load are given and the reference period is considered to be 50 years. The statistics of the equivalent lognormal random variate of the primary load are determined so that the 50 and 99% values of the 50-year maximum value of the primary load intensity and those of the equivalent lognormal random variate may be equal to each other except for the case when the target reliability level is higher than 2.6 and the annual maximum value is described by a Type II distribution function. The statistics of an equivalent lognormal random variate of the secondary load is determined so that the 50 and 99% values of the annual maximum value of the secondary load intensity and that of the equivalent lognormal random variate may be equal to each other.

The following regression formulas are provided in the recommendation to estimate the statistics of the equivalent lognormal random variate,  $Seq_i$ , easily. The parameters in Eqs. (A6) and (A7) are presented in Tables A1 and A2.

$$\mu_{\ln Seq_i} = e_0 + e_1 \cdot V_{S_i} + e_2 \cdot V_S + e_3 \cdot V_S + \ln(\mu_{S_i}) \tag{A6}$$

$$\sigma_{\ln Seq_i} = s_0 + s_1 \cdot V_{S_i} + s_2 \cdot V_S + s_3 \cdot V_S \tag{A7}$$

The c.o.v., mean value, and standard deviation of the equivalent lognormal random variate are estimated using  $\mu_{\ln Seq_i}$  and  $\sigma_{\ln Seq_i}$  as,



$$V_{Seq_i} = \sqrt{\exp(\sigma_{\ln Seq}) - 1} \quad (\text{A8})$$

$$\mu_{Seq_i} = \exp(\mu_{\ln Seq_i}) \cdot \sqrt{1 + V_{Seq_i}^2} \quad (\text{A9})$$

$$\sigma_{Seq_i} = \mu_{Seq_i} \cdot V_{Seq_i} \quad (\text{A10})$$

Load and resistance factors can now be determined by Eqs. (A2)–(A5) using the statistics of the equivalent lognormal random variates evaluated by Eqs. (A6)–(A10).

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