Impact of Assumptions on DC Power Flow Model Accuracy

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Abstract—**The industry seems to be sanguine about the performance of dc power-flow models, but recent research has shown that the performance of different formulations is highly variable. Considering their pervasive use, the accuracy of dc power-flow models is of great concern. In this paper, three dc power-flow formulations are examined: the classical dc power-flow model, dc power-flow model with loss compensation and the so-called *α*-matching dc power-flow model. These three models are tested in three systems of different sizes, ranging from 10 buses to 62,000 buses. By comparing the dc power-flow results with the ac power-flow results, the paper concludes that the *α*-matching formulation has the highest accuracy among three dc power flow formulations.**

I. Introduction

Because of the recent upsurge in the use of the dc power flow for electricity-market application, such as auctions associated with transmission rights, real-time security-constrained dispatch, and day-ahead security-constrained unit commitment, a flurry of research aimed at improving and estimating the accuracy of dc power-flow formulations has occurred [1]-[8]. The dc power flow has found favor in real-time market-based application because of its robustness and speed [5], [9], [10]. While some authors have shown impressive accuracy with dc power-flow formulations, others are not so sanguine about the dc power-flow's accuracy [8].

The number of dc power-flow formulations available is legion. Several of the more accepted dc power-flow models are described in [2], [3], [8]-[11]. The different formulations are characterized by different definitions of active power injections, loss estimation and branch admittances [8], [10], [11]. One taxonomy for classifying dc power flow formulations starts by breaking the models into two categories: state-dependent and state-independent [3], [8]-[10]. The state-dependent dc power-flow models take into consideration the system losses and bus voltage values at a suitable state. The state-independent dc power-flow models assume a lossless network, one-per-unit voltage values, and small voltage angle differences [10]-[15]. These different assumptions have been analyzed and evaluated for accuracy acceptably assuming high X/R ratios and small voltage deviations from one per unit [3]. Clearly, the assumptions made will affect the accuracy of dc power-flow models' results. If anything, investigations into the accuracy of any formulation have shown that such analysis is not amenable to theoretical efforts, but must be assessed empirically, assessment that is system and case dependent.

Given the limits of theory and applied mathematics in formulation-acceptability analysis, this paper (like most papers of this genre) experimentally examines the accuracy of three dc power-flow models: one model that is state-independent and two models that are state-dependent. Errors introduced by these three dc power-flow models in the power-flow results are quantified and compared. The base-case simulations reported upon here use MATLAB and PowerWorld for three test systems: 10-bus system, IEEE 118-bus system, and the 62,000-bus Eastern Interconnection (EI) system. In addition, contingency analysis is performed to evaluate the change in accuracy of three models for operating conditions that deviate from the base case.

This paper is organized as follows. Section II reviews and provides a discussion of the classical dc model, dc power-flow model with loss compensation, and *α*-matching dc power-flow model. Section III presents the case studies. Section IV is used to draw conclusions from the experimental results.

II. Review of Dc Power-Flow Models

The dc-network-modeling process starts from the ac power-flow equations. For context, the ac power-flow model and its relation to the dc power-flow models are reviewed briefly.

For a transmission line spanning bus i and bus j the net active power injection at bus i and bus j are defined as p_i and p_j. [11], [12].

$$\begin{aligned} p_i &= \text{Re}\left\{ v_i\angle\delta_i\left[\left(g_{ij}+jb_{ij}\right)\left(v_i\angle\delta_i - v_j\angle\delta_j\right)\right]^*\right\} \\ &= g_{ij}(v_i^2 - v_iv_j\cos(\delta_i-\delta_j)) - v_iv_jb_{ij}\sin(\delta_i-\delta_j) \end{aligned} \quad (1)$$

$$p_j = -g_{ij}(v_j^2 - v_jv_i\cos(\delta_i-\delta_j)) - v_iv_jb_{ij}\sin(\delta_i-\delta_j) \quad (2)$$

where:

$r_{ij:}$ the resistance of the line *i-j*
$x_{ij:}$ the resistance of the line *i-j*
$g_{ij:}$ the conductance of the line *i-j*
$b_{ij:}$ the susceptance of the line *i-j*
v_i, $v_{j,}$: the magnitudes of bus voltage at bus *i* and bus *j*

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δ_i, δ_j: the bus voltage angles at bus i and bus j

It is easy to show that the sum of the terms of (1) and (2) involving the g coefficient represent the line losses:

$$l_i = g_{ij}(v_i^2 - v_i v_j \cos(\delta_i - \delta_j))$$
$$l_j = g_{ij}(v_j^2 - v_j v_i \cos(\delta_i - \delta_j)) \quad (3)$$

For our purposes, l_i may be viewed as (and modeled as) the loss contribution from bus i and l_j may be viewed as the loss contribution from bus j, though all we really know is that the sum of these two values represents the total line loss. The second terms in (1) and (2) are the identical, which, by default, must represent the active power flow on the line i-j in the ac model of the line [8]:

$$p_{ij} = -v_i v_j b_{ij} \sin(\delta_i - \delta_j) \quad (4)$$

With these interpretations in mind, an equivalent dc model of the line is shown in Fig. 1[8].

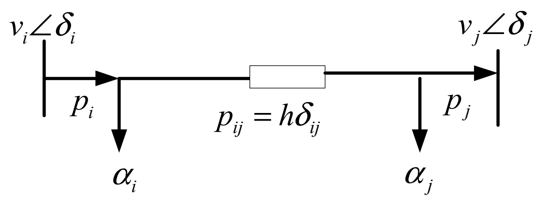


Fig. 1. dc equivalent model of branch

While it is tempting to equate the α's in the dc model with the l's in the ac model, we will see that, depending on how we model the series component in the dc model, to get an exact match between the ac and dc models will require some flexibility. Nevertheless, in the dc equivalent model, α_i and α_j represent the loss-approximating components of the line i-j at the sending end and the receiving end, respectively. The dc equivalent branch admittance is defined as h. Again, it is tempting to equate b from the ac model with h of the dc model (and this is one assumption that can be made) however the assignments of α and h are arbitrary, with a relationship between them (revealed later) that, if obeyed, allows the dc solution to exactly match the ac solution for any arbitrary base case. Furthermore, by selecting these values in one particular way, the dc model performs better than in selecting them another way.

Depending on the application of the dc model, for the "best" linear fit of the dc to the ac model over a practical operating range, h, α_i and α_i need to be evaluated at a "suitable" point [8]. For the purposes of comparison of accuracy, we choose as the base case the power-flow solution available with our data sets. Using the definitions above and Fig. 1, the three dc power-flow models are discussed below.

A. Classical Dc Power-Flow Model

In the classical dc power-flow model, a number of assumptions are made. Based on these assumptions, the classical dc power-flow model could be derived step by step from the equation (4).

First, by assuming that all voltage magnitudes are identical to one per unit, p_{ij}=-b_{ij}sin(δ_i-δ_j); second, if all the line resistances are ignored ($r_i\approx 0$), then p_{ij}=-sin(δ_i-δ_j)/x_{ij}; third, if small voltage angle differences are assumed, sin(δ_i-δ_j)$\approx$(δ_i-δ_j), then p_{ij}=-(δ_i-δ_j)/ x_{ij}.

Given these assumptions, this model is state-independent. No loss compensation is considered in this model; consequently α_i and α_j equal zero, and while the selection of h is quite arbitrary, it is traditional to select $h=1/x_{ij}$.

B. Dc Power-Flow Model with Loss Compensation

In this dc power-flow model, the network-modeling assumptions are the same as those for the classical model but to compensate for losses, a single multiplier (calculated as the ratio of the total generation to total load for the base case) is applied to each load, distributing the losses throughout the model. Since the losses of the system are obtained from the base case solution, this model is state-dependent. Clearly there are pros and cons to select the losses using a base case that may or may not be close to the conditions of interest.

C. α-Matching dc Power-Flow Model

Hot-start models are constructed when a solved ac power-flow solution is available, while cold-start models are built when an ac solution is unavailable. Hot-start models, which have the same MW losses as the ac solution, are often used in short-term operation and studies, e.g., real-time security constrained economic dispatch (SCED) [8], [13]. Cold-start models are often used in long-term planning studies, e.g., dc-model-based security constrained unit commitment [14].

When operating in a real-time environment where the current state of the system is known relatively accurately, the α-matching model is constructed by first specifying the branch parameter h (calculated by exactly matching the real power flow transmitted through line i-j for a particular base case in the absence of loses). Second, the loss-approximating components (α_i and α_i) are assigned so that the real power flows at the end of each line (including losses) exactly match [8]. A more detailed description of this follows.

1) Transmission line: The h parameter is defined by matching the p_{ij} in equation (4) and Fig. 1. Considering the active power at the sending end and the receiving end are p_i^0=α_i+$h\delta_{ij}^0$ and p_j^0=α_i+$h\delta_{ij}^0$, respectively, the loss-approximating components (α_i and α_j) can be obtained by matching p_i and p_j. The superscript 0 is used to indicate the base-case operating point.

The h, α_i and α_j equations are summarized as follows:

$$h = -v_i^\circ v_j^\circ b_{ij} \sin\delta_{ij}^\circ \big/ \delta_{ij}^\circ \quad (5)$$

$$\alpha_i = g_{ij}(v_i^{\circ 2} - v_i^\circ v_j^\circ \cos(\delta_i^\circ - \delta_j^\circ)) \quad (6)$$

$$\alpha_j = g_{ij}(v_j^2 - v_j v_i \cos(\delta_i - \delta_j)) \quad (7)$$

where:

v_i° : the magnitude of bus voltage at bus i at the base case

δ_i° : the bus voltage angle at bus i at the base case

2) Transformer/phase shifter: Considering the off-nominal transformer tap in the dc model, transformer/phase shifter is formulated differently from the transmission line. The quantity $t_i\angle\tau_i$ is a complex number representing the tap ratio and phase shift of transformer.

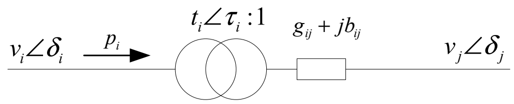


Fig. 2 .Transformer/phase shift diagram

The active power sending from bus *i*:

$$\begin{aligned} p_i &= \mathrm{Re}\left\{(v_i\angle\delta_i/t_i\angle\tau_i)\left[(g_{ij}+jb_{ij})\left((v_i\angle\delta_i/t_i\angle\tau_i)-v_j\angle\delta_j\right)\right]^*\right\} \\ &= g_{ij}((v_i/t_i)^2 - v_iv_j/t_i\cos(\delta_i-\delta_j-\tau_i)) - v_iv_jb_{ij}\sin(\delta_i-\delta_j-\tau_i)/t_i \end{aligned} \tag{8}$$

Therefore, based on a solved ac base-point solution, the branch admittance and loss-approximating components (α_i and α_j) for a transformer are derived as:

$$h_{ij}^{\circ} = -v_i^{\circ}v_j^{\circ}b_{ij}\sin(\delta_i^{\circ}-\delta_j^{\circ}-\tau_i^{\circ})/t_i^{\circ} \tag{9}$$

$$\alpha_i = g_{ij}\left(\left(v_i^{\circ}/t_i^{\circ}\right)^2 - v_i^{\circ}v_j^{\circ}/t_i^{\circ}\cos(\delta_i^{\circ}-\delta_j^{\circ}-\tau_i^{\circ})\right) \tag{10}$$

$$\alpha_j = g_{ij}(v_j^{\circ 2} - v_j^{\circ}v_i^{\circ}/t_i^{\circ}\cos(\delta_i^{\circ}-\delta_j^{\circ}-\tau_i^{\circ})) \tag{11}$$

D. Comparison of Models

Fig. 3 shows the relationship between the bus angle differences δ_{ij} and the real power injected at bus *i*. This is the solid line in the figure. The variables p_i^0 and δ_{ij}^0 represent the power injected at the sending end bus *i* and the bus-voltage-angle difference between the ends of the line at the base-case operating point. Line 1 indicates the behavior of the classical dc model and dc model with loss compensation. Because of the assumptions in these models, it is not surprising these models are far from the exact ac power-flow model. Note that the behavior of the p_i v. δ_{ij} model with loss compensation intersects the origin, which would normally be an indicator of doom; but by escalating loads to model losses, the losses are distributed throughout the system and this will lead to better performance than the classical model, as we will see in the following section. Compared to line 1, line 2 represents the α-matching dc power-flow model, which passes through the base-case operating point. As long as the changes to the operating point are within a limited range, the α-matching model is expected to perform the best among the three dc power-flow models, even though the α-matching model is state-dependent. How large this limited range might be is unclear.

III. Test Case Study

Three dc power-flow models discussed above have been tested on the 10-bus system, the 118-bus system and the 60,000-bus Eastern Interconnection (EI) system.

In these test-case studies, automatic generation control (AGC), LTC transformer control, phase shifter control and switched shunt control have been disabled. Two metrics were chosen to measure the accuracy of the dc model. The first is

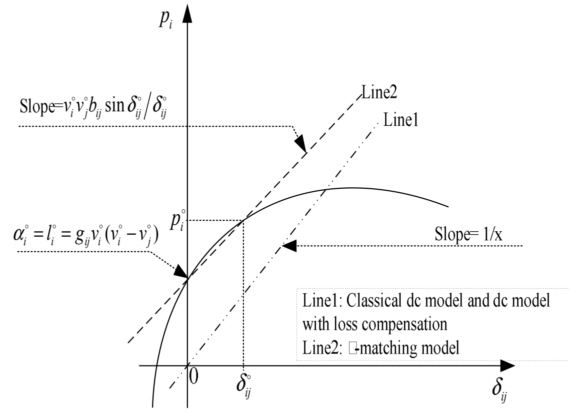
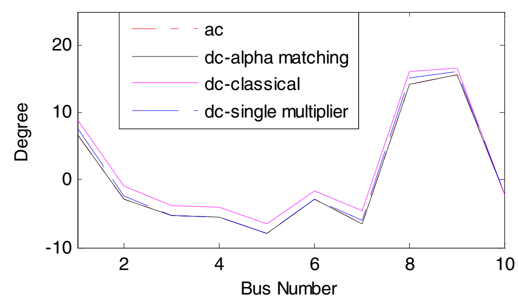


Fig. 3. Dc power-flow model approximation

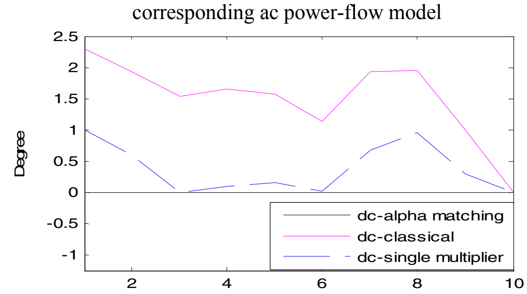
the absolute value of the difference between each corresponding branch flow (MW) from the dc and ac solution. The second metric is this value divide by (normalized by) the MVA limit of each respective line. Finally, the maximum values of these two metrics are used as a summative measure.

A. 10-bus system case

1) Voltage angles comparison: In this 10-bus test case, all the bus-voltage angles of these three dc models do not deviate far from the ac base-case angles, which is shown in Fig. 4 (a). Furthermore, the angle errors between dc power-flow models and the ac power-flow model are presented in Fig. 4 (b). The maximum voltage-angle error for the classical model is around 2.28°, while the maximum error for α-matching model is only 0.000337°, which is essentially perfectly.



(a) Solved bus voltage angles for the three dc power-flow models and the corresponding ac power-flow model



(b) Solved bus voltage angle errors between the three dc power-flow models and the corresponding ac power-flow model

Fig. 4. Angle differences between three dc power-flow models and the corresponding ac power-flow model

2)Line power-flow comparison:. In this 10-bus system, the line flows range from near 0 MW to150 MW. The MW flow errors for the classical and loss-compensated models are very similar with a maximum MW flow error of about 20 MW and an average MW flow error of about 7% (see Fig. 5. and TABLE I). These stand in sharp contrast to the α-matching model flow errors in MW and in percentage that are both measured in thousandths. For the 10-bus system, the α-matching model is superior (as it should be) since the h value is selected to exactly match the real-power-transfer flows for the bus angles of the base case and the α values are selected to exactly match the losses of the base case.

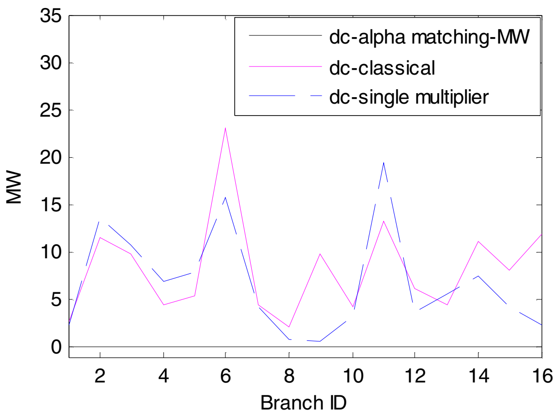


Fig. 5. Power flow errors for three dc models as measured against the ac power-flow solutions versus branch ID.

TABLE I
SUMMARY OF POWER FLOW COMPARISONS BETWEEN AC AND DC POWER-FLOW MODEL FOR 10-BUS TEST SYSTEM

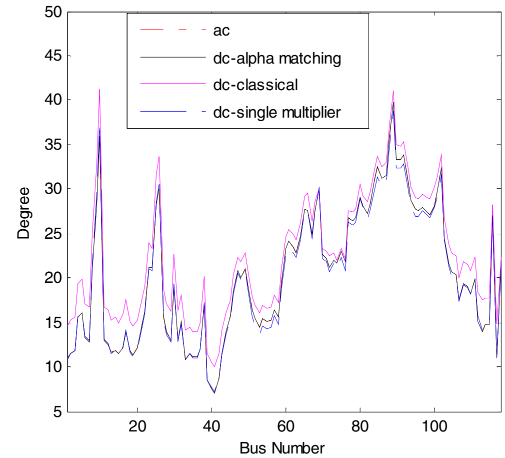
	Classical	Single multiplier	α-matching
Max Difference (MW)	23.1	17.7	0.0042
Average Difference (MW)	8.3	6.8	0.0012
Max Error (%)	19.2	21.7	0.0049
Average Error (%)	8.0	6.6	0.0012

B. *118-bus system case*

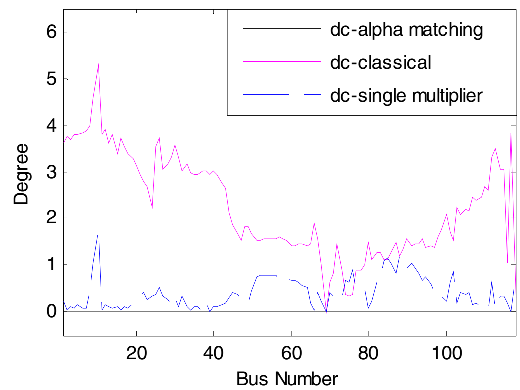
The IEEE 118-bus system case is a part of the Midwestern American Power Grid, which is commonly used as a test case [17]. Since no line MVA limits are provided in the data set, the error results cannot be shown as percentage values, only as MW values.

1)Voltage angles comparison: In Fig. 6 (a), the bus-voltage angles of the loss-compensated model and α-matching model match the angles of ac power-flow solution well. This is in contrast to the performance of the classical model, which deviates from the base point a farther, as shown in Fig. 6 (b). The maximum voltage-angle differences for the α-matching model is only 0.00082^o. In contrast, the maximum voltage-angle errors for the classical model is around 5.3^o and for the loss-compensated model is about 2.0^o.

2)Line power-flow comparison: In this 118-bus system, the branch power flows fall in the range from of +/- 450 MW. In Fig. 7 (a), it can be seen that the branch power flows of the three dc models are not far from the branch flows of the ac base-case. As shown in Fig. 7 (b), the maximum MW error of



(a) Solved bus voltage angles for the three dc power-flow models and the corresponding ac power-flow model



(b) Solved bus voltage angle errors between the three dc power-flow models and the corresponding ac power-flow model

Fig. 6. Angle differences between three dc power-flow models and the corresponding ac power-flow model

the classical dc power-flow is around 60 MW, while the maximum error for α-matching is 0.00757 MW. The large MW power-flow errors of the classical model occur on the lines that are located near the slack bus, which is expected since all of the reduction in (absence of) losses in the classical model must be compensated by reduced generation at the slack bus, a reduction that can become quite large for large systems. This reduced generation will change radically the branch flows near the slack bus. The average MW errors for the classical model and loss-compensated model are similar. For the base point, the accuracy of α-matching dc power-flow model is the best among these three dc power-flow models. The detailed simulation results are summarized in the Table II and Fig. 7.

TABLE II
SUMMARY OF POWER FLOW COMPARISONS BETWEEN AC AND DC POWER-FLOW MODELS FOR 118-BUS TEST

	Classical	Single multiplier	α-matching
Max Difference (MW)	60.5	22.0	0.076
Average Difference (MW)	5.4	4.7	0.0046

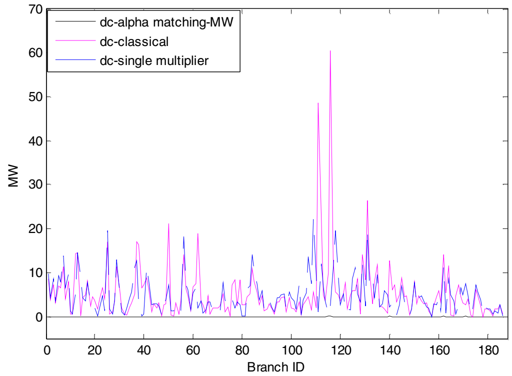


Fig. 7 Power flow errors for three dc models as measured against the ac power-flow solutions versus branch ID.

C. *62,000-bus EI system case*

In North America, the 62,000-bus Eastern Interconnection (EI) power grid is one of three major interconnections. In the EI, there are 62,013 buses, 76,437 branches (excluding all the open circuited lines), 8,190 generators and 24 HVDC lines.

Active power losses of the EI system are about 2.55% of the total generation.

1) Classical dc power-flow model: In this case, the power flow of classical dc power-flow model shows a considerable deviation from the ac power flow, which can be seen in Fig. 8. In this test case, the largest power flow difference between the classical dc power-flow model and ac power-flow model is 1525.684 MW, and the average error is 11.349 MW.

2) Dc power-flow model with loss compensations: In this case, active power losses are compensated in terms of a single multiplier, which is calculated as 1.0262. Compared to the classical dc model, the maximum errors and average errors are reduced by one third due to the improved loss modeling. The plot of flow errors versus branch ID is shown in Fig. 9.

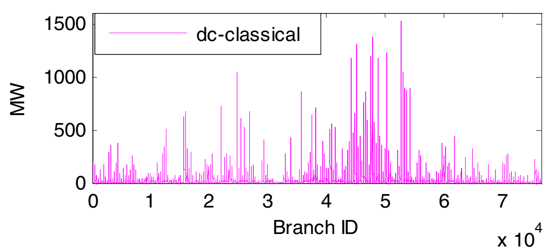


Fig. 8. MW branch-flow errors between the classical dc power-flow model and ac power-flow model

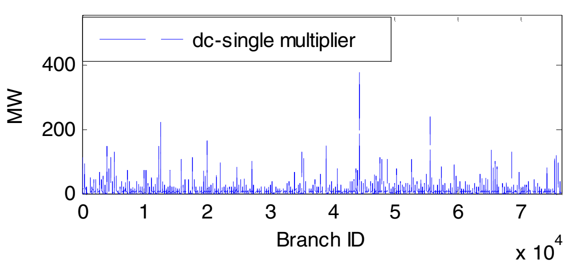


Fig. 9 MW branch-flow errors between the dc power-flow model with loss compensation and ac power-flow model

3) α-matching dc power-flow model: In Fig. 10, it can be seen that all of the MW flow errors of the α-matching model are less than 1MW. Not unexpectedly, the performance of α-matching dc power-flow model is the best among these three models. The plot of flow errors versus branch ID is shown in Fig. 10.

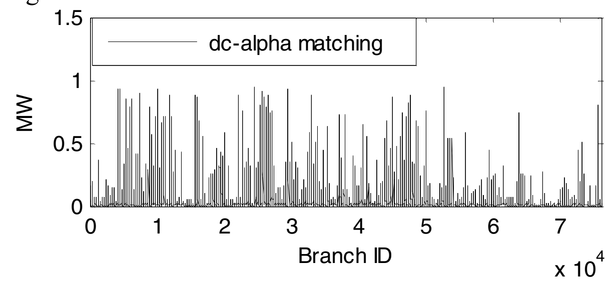


Fig. 10. MW branch-flow errors between the α-matching dc power-flow model and ac power-flow model

4) Summary: A summary of the simulation results are shown in Table III. The errors in percentage only include lines that had MVA limits in the data set. Compared to the pervious test cases, even though the percentage losses to not vary wildly between test systems, the total active losses of the EI system is considerably larger due to its size. As mentioned before, the classical dc model without loss compensation has the worst performance among these three dc power-flow model studies, one cause is the need for the slack bus to compensated for the absence of losses in the dc case. By compensating for losses by scaling loads at each bus, the dc power-flow model with loss compensation performs significantly better than the classical model, something not seen in the smaller systems, giving credence to the claim that the model accuracy is system dependent. By exactly matching the ac base case, the α-matching dc power-flow model provides essentially an exact fit to the ac power-flow model, as it should.

TABLE III
SUMMARY OF POWER FLOW COMPARISONS BETWEEN AC AND DC POWER-FLOW MODEL FOR THE EI TEST SYSTEM

	Classical	Single multiplier	α-matching
Max Difference (MW)	1525. 8	378.8	0.9
Average Difference (MW)	11.3	3.51	0.01
Max Error (%)	173.2	57.7	0.3
Average Error (%)	4.5	1.70	0.0058

D. *Effect of Contingencies on Accuracy*

For large-scale power systems, contingency analysis is rather limited due to computational demands [16], though contingency ranking algorithms can screen contingencies so that fewer need be considered. In this section, a number of random lines and generators (10 lines and 10 generators) at different voltage levels are selected to be outaged for testing the equivalents for change-cases. For the generator outages, the extra slack is picked up by the system slack bus. For line outages, no adjustments are made, even though system losses change. The simulation results are summarized in Table IV.

TABLE IV
SUMMARY OF CONTINGENCIES IN THE EI SYSTEM

	Classical	Single multiplier	α-matching
Max Difference (MW)	1525.5	379.4	9.23
Average Difference (MW)	11.35	3.5	0.02
Max Error (%)	174.4	57.7	2.30
Average Error (%)	4.6	1.7	0.18

Since the classical dc model is state-independent, its performance does not change much (c.f., Table III) due to the change in operating point. Even though the dc power-flow model with loss compensation is state-dependent, its performance does not change significantly from the base case. The α-matching model has the best performance. Since the α-matching model is constructed to match the based-case exactly, and since the contingency condition will often not be extremely different from the base case, it is expected that the errors will be smaller than those of the other models.

IV. CONCLUSION

This paper examines the classical dc power-flow model, dc power-flow model with loss compensation and α-matching dc power-flow model. Accuracy tests using these three dc models are conducted for three power systems: a 10-bus IEEE system, an IEEE 118-bus system, and the 62,000-bus Eastern Interconnection (EI) system. The classical dc model is state-independent and simple to construct. However, the accuracy of the classical model may not be satisfactory in most applications. Although the loss-compensated model performs similarly to the classical model in the 10-bus system case and 118-bus system case, its performance is much better than the classical model in the 62,000-bus system. The α-matching model performs the best among these three for the base case because it is structured to match the base case exactly; it localizes the losses and calculates the branch admittance to exactly match the base case.

In contingency analysis, the performance of classical and the loss-compensated models are similar. The α-matching model performs the best as expected.

V. ACKNOWLEDGMENT

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