Systems & Control Letters 65 (2014) 89–95

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Global stabilization of high-order nonlinear time-delay systems by state feedback☆

Xu Zhang, Yan Lin *

School of Automation, Beijing University of Aeronautics and Astronautics, Beijing, 100191, China

ARTICLE INFO

Article history:
Received 20 June 2012
Received in revised form
1 July 2013
Accepted 27 December 2013
Available online 2 February 2014

Keywords:
High-order nonlinear systems
Time-delay
State feedback
Dynamic gain-based backstepping

ABSTRACT

We consider the problem of global stabilization by state feedback for a class of high-order nonlinear systems with time-delay. By developing a novel dynamic gain-based backstepping approach, a state feedback controller independent of the time-delay is explicitly constructed with the help of appropriate Lyapunov–Krasovskii functionals. The precise knowledge (even the upper bound) of the time-delay is not required. It is proved that the states of the nonlinear time-delay systems can be regulated to the origin while all the closed loop signals are globally bounded. Finally, both physical and academic examples are given to illustrate the applications of the proposed scheme.

1. Introduction

In this paper, we consider the state feedback control for a class of high-order nonlinear time-delay systems via the Lyapunov–Krasovskii method:

$$\begin{aligned}
&\dot{x}_i(t) = x_{i+1}^{p_i}(t) + g_i(\bar{x}_i(t)) + f_i(\bar{x}_i(t-d)), \\
&\quad i = 1, \dots, n-1, \\
&\dot{x}_n(t) = u(t) + g_n(x(t)) + f_n(x(t-d)), \\
&x(\tau) = \zeta(\tau), \quad \tau \in [-d, 0],
\end{aligned} \tag{1}$$

where $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$ is the state vector, $\bar{x}_i = [x_1, \dots, x_i]^T$, $u \in \mathbb{R}$ is the system input, p_i are odd integers, g_i and f_i are locally Lipschitz functions and not necessary to be completely known, the nonnegative constant d denotes the unknown time-delay, and $\zeta(\tau) \in \mathbb{R}^n$ is a continuous function vector defined on $[-d, 0]$.

High-order nonlinear system (1) in the absence of time-delay has been widely studied in [1–3]. It is pointed out in [1] that global stabilization of the system (1) without time-delay can be achieved by state feedback under a high-order version of feedback linearizable condition.

When $p_i = 1, 1 \le i \le n-1$, the system (1) reduces to the well-known strict feedback system with time-delay. In [4], the author extended the backstepping method [5,6] to such a nonlinear time-delay system and designed a memoryless state feedback controller using the Lyapunov–Krasovskii method which, however, was proved incorrect later by [7,8]. The problem existing in [4] is that the Lyapunov functions in the traditional backstepping design are no longer effective for the nonlinear time-delay systems and the virtual control signals are too hard to design. By employing the idea of changing supply functions [9], this problem was solved for a class of lower-triangular systems in [10] with the knowledge of the upper bound of the time-delay. A more general class of nonlinear time-delay systems was investigated in [11], but the delay amplitude knowledge was still required. When some restrictive growth conditions [12,13] are imposed on the system nonlinearities, global output feedback stabilization can be achieved. Note that the problem in [4] can be avoided for these output feedback schemes because the construction of the Lyapunov–Krasovskii functional is necessary in one step.

When $p_i > 1$, the system (1) is in the high-order strict-feedback form [1] with time-delay. While the Jacobian linearization is uncontrollable, the mentioned system is not feedback linearizable. If we apply adding a power integrator technique [1] to the controller design by using the Lyapunov–Krasovskii method, the same problem in [4] will occur and become more complicated for such high-order systems. Under restrictive growth conditions, output feedback control was considered in [14,15] for high-order time-delay systems which, by the same reason as those for $p_i = 1$ [12,13], can also avoid the problem in [4]. Indeed, compared with output feedback control, *state feedback control* for the nonlinear

☆ Work supported by the NSF of China under Grant 61273141 and the National Basic Research Program of China (973 Program) under Grant 2014CB046406.
* Corresponding author.
E-mail addresses: zx350304@126.com (X. Zhang), linyanee2@yahoo.com.cn (Y. Lin).

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http://dx.doi.org/10.1016/j.sysconle.2013.12.015

time-delay systems is a more challenging issue. It remains unclear and open how a memoryless state feedback controller can be constructed for the system (1) to achieve global asymptotic stabilization in the sense that, for any initial conditions $x(t_0)$, $-d \le t_0 \le 0$, the state x converges to the origin and all the closed loop signals are bounded. Hence, a new tool is required for the global stabilization problem.

The main objective of this paper is to give a solution to global state feedback stabilization of the high-order nonlinear time-delay system (1) and in particular, to circumvent the long-standing problem in [4]. For this purpose, we develop a novel dynamic gain-based backstepping approach which, in addition, introduces a dynamic gain in each step of the recursive design. A key feature of the proposed dynamic gain-based backstepping technique is that the Lyapunov function is chosen in a new recursive manner with appropriately designed gain-based Krasovskii functionals. Owing to the introduction of the dynamic gains, additional negative terms appear in the derivative of the Lyapunov function, which can be used to counteract stronger system nonlinearities. As a result, a memoryless state feedback controller can be constructed. Moreover, the upper bound of the time-delay is not required to be known a priori. It is shown that the system states can be regulated to the origin while all the closed loop signals are globally bounded. Finally, numerical examples are given to show the effectiveness of the proposed scheme.

2. Preliminary lemmas

In this section, we shall introduce two technical lemmas which are useful for the controller design and stability analysis.

Lemma 1. *For any positive integers* m, n *and any positive function* $f(x, y) : \mathbb{R}^2 \to \mathbb{R}^+$, *the following inequality holds*

$$|x|^m |y|^n \le \frac{m}{m+n} f(x, y) |x|^{m+n} + \frac{n}{m+n} f^{-m/n}(x, y) |y|^{m+n}. \quad (2)$$

Proof. The reader may refer to [1] for the proof of this lemma.

Lemma 2. *For any nonnegative smooth function* $f(x_1, \ldots, x_n)$ *and positive integer* k, *there exist nonnegative smooth functions* $f_i(x_1, \ldots, x_i)$, $1 \le i \le n$, *such that*

$$f(x_1, \ldots, x_n) \sum_{i=1}^{n} |x_i|^k \le \sum_{i=1}^{n} f_i(x_1, \ldots, x_i) |x_i|^k. \quad (3)$$

Proof. Define a nonnegative continuous function[1]

$$\bar{f}(r) = \max_{x \in B_r} |f(x)|, \quad B_r = \left\{x : \|x\|^2 \le r,\ r \ge 0\right\}, \quad (4)$$

where $x = [x_1, \ldots, x_n]^T$. It is known that any continuous function $\bar{f}(r) : [0, +\infty) \to \mathbb{R}$ can be dominated by a smooth nondecreasing function $f^*(r)$, i.e., $\bar{f}(r) \le f^*(r)$. Note that $f(x) \le \bar{f}\left(\|x\|^2\right)$. Then, we can obtain that

$$\begin{aligned} f(x) &\le \bar{f}\left(\|x\|^2\right) \\ &\le f^*\left(\|x\|^2\right) \\ &= f^*\left(x_1^2 + \cdots + x_n^2\right) \\ &\le f^*\left(nx_1^2\right) + \cdots + f^*\left(nx_n^2\right). \end{aligned} \quad (5)$$

For $1 \le i, j \le n$, we have

$$\begin{aligned} f^*\left(nx_i^2\right)\left|x_j\right|^k &= \left[f^*\left(nx_i^2\right) - f^*(0)\right]\left|x_j\right|^k + f^*(0)\left|x_j\right|^k \\ &\le f_{ij}(x_i)|x_i|\left|x_j\right|^k + f^*(0)\left|x_j\right|^k \\ &\le \bar{f}_{ij}(x_i)|x_i|^{k+1} + \left|x_j\right|^{k+1} + f^*(0)\left|x_j\right|^k, \end{aligned} \quad (6)$$

where to obtain the last inequality, Lemma 1 has been used by considering $f_{ij}(x_i)|x_i|$ as one part, and $f_{ij}(x_i)$ and $\bar{f}_{ij}(x_i)$ are some nonnegative smooth functions. With (5) and (6), it is not difficult to prove that Lemma 2 holds.

3. Main results

To achieve global stabilization of the nonlinear time-delay systems, the following assumptions have been made for (1).

A.1: $p_1 \ge \cdots \ge p_{n-1} \ge p_n = 1$.
A.2: The nonlinearities $g_i(\bar{x}_i)$ and $f_i(\bar{x}_{id})$, $1 \le i \le n$, satisfy[2]

$$\begin{aligned} |g_i(\bar{x}_i)| &\le \gamma_i(\bar{x}_i)\left(|x_1|^{p_i} + \cdots + |x_i|^{p_i}\right), \\ |f_i(\bar{x}_{id})| &\le \sigma_i(\bar{x}_{id})\left(|x_{1d}|^{p_i} + \cdots + |x_{id}|^{p_i}\right), \end{aligned} \quad (7)$$

for known nonnegative smooth functions γ_i and σ_i.

Remark 1. Without considering the time-delay, the above hypotheses can be viewed as a high-order version of feedback linearizable condition, whose global stabilization problem has been solved in [1] by means of adding a power integrator. However, the presence of time-delay makes it impossible to implement the conventional recursive design. Specifically, the virtual controllers cannot be well defined to counteract the time-delay nonlinearities, which may result from the utilization of inappropriate Lyapunov functions. Therefore, it is necessary to find appropriate Lyapunov functions for global stabilization of the nonlinear time-delay systems. In this paper, a new recursive design method is developed by introducing one dynamic gain and the corresponding gain-based Lyapunov function at each step. Together with Krasovskii functionals, the designed controller guarantees global stability of the closed loop system.

Remark 2. It should be pointed out that when the functions $g_i(\bar{x}_i) + f_i(\bar{x}_{id})$ are replaced by $f_i(x, x_d)$ in the right-hand side of (1), the condition

$$|f_i(x, x_d)| \le \sigma_i(\bar{x}_i, \bar{x}_{id}) \sum_{j=1}^{i}\left(\left|x_j\right|^{p_i} + \left|x_{jd}\right|^{p_i}\right) \quad (8)$$

is equivalent to (7). In fact, according to the inequalities (5) and (6) in the proof of Lemma 2, we have $|f_i(x, x_d)| \le \sigma_{i1}(\bar{x}_i)\sum_{j=1}^{i}\left|x_j\right|^{p_i} + \sigma_{i2}(\bar{x}_{id})\sum_{j=1}^{i}\left|x_{jd}\right|^{p_i}$ for some nonnegative smooth functions $\sigma_{i1}(\bar{x}_i)$ and $\sigma_{i2}(\bar{x}_{id})$. Here, the expressions $g_i(\bar{x}_i) + f_i(\bar{x}_{id})$ are used only for the sake of a better understanding of our proposed method.

Now, we are ready to present the main results of the paper.

Theorem 1. *Consider the nonlinear time-delay system* (1) *satisfying the hypotheses* A.1 *and* A.2. *Then, a* $(n-1)$*th-order memoryless state feedback controller can be constructed to globally asymptotically stabilize the mentioned system in the sense that, for any continuous initial conditions* $x(t_0)$, $-d \le t_0 \le 0$, *the state* x *converges to the origin while all the signals of the closed loop system are bounded.*

[1] Throughout the paper, we use $\|\cdot\|$ to denote the Euclidean norm of a vector or the corresponding induced norm of a matrix.

[2] In this paper, for simplicity, we let ς_d denote the corresponding delay term $\varsigma(t-d)$. For instance, $x_{1d} = x_1(t-d)$ and $f_i(\bar{x}_{id}) = f_i(\bar{x}_i(t-d))$.

Proof. The memoryless state feedback controller is designed based on a dynamic gain-based backstepping approach.

Step 1: The derivative of $S_1 = x_1$ by viewing (1) is computed as

$$\dot{S}_1 = x_2^{p_1} + g_1(x_1) + f_1(x_{1d}). \tag{9}$$

Let $S_2 = x_2 - \alpha_1$ with

$$\alpha_1 = -\beta_1^{1/p_1}(S_1) S_1, \tag{10}$$

where β_1 is a positive smooth design function. From (10), it can be seen that the difference between our proposed method and [11] is that the upper bound knowledge of the time-delay is not required in the virtual controller design. The same feature is also possessed by the subsequent recursive design. Consider the Lyapunov function

$$V_1 = \frac{S_1^2}{2} + \frac{S_1^2}{2l_1}, \tag{11}$$

where l_1 is a dynamic gain updated by

$$\dot{l}_1 = S_1^{p_1-1}\max\left\{-2l_1^2 + l_1\rho_1(S_1), 0\right\}, \quad l_1(0) = 1 \tag{12}$$

with ρ_1 a positive smooth function to be determined. Note that the right-hand side of the differential equation (12) is locally Lipschitz in (S_1, l_1). Some important properties of l_1 to be used in the following are listed here

$$\begin{aligned} &0 \le \dot{l}_1 \le l_1\rho_1 S_1^{p_1-1},\\ &\dot{l}_1 \ge -2l_1^2 S_1^{p_1-1} + l_1\rho_1 S_1^{p_1-1},\\ &l_1 \ge l_{1d} \ge 1. \end{aligned} \tag{13}$$

Using (13), the derivative of V_1 is computed as

$$\begin{aligned} \dot{V}_1 &= \left(1+\frac{1}{l_1}\right) S_1\left(x_2^{p_1} - \alpha_1^{p_1} - \beta_1 S_1^{p_1} + g_1 + f_1\right) - \frac{\dot{l}_1}{2l_1^2}S_1^2\\ &\le -(\beta_1 - 1) S_1^{p_1+1} + 2\left|S_1\left(x_2^{p_1} - \alpha_1^{p_1}\right)\right|\\ &\quad + 2|S_1 g_1| + 2|S_1 f_1| - \frac{\rho_1}{2l_1}S_1^{p_1+1}. \end{aligned} \tag{14}$$

By Lemma 1 and the assumption A.2, the following inequalities hold

$$\begin{aligned} 2\left|S_1\left(x_2^{p_1} - \alpha_1^{p_1}\right)\right| &= 2\left|S_1 S_2 \sum_{i=1}^{p_1} x_2^{p_1-i}\alpha_1^{i-1}\right|\\ &\le S_1^{p_1+1} + \phi_2(S_1, S_2) S_2^{p_1+1},\\ 2|S_1 f_1| \le S_1^{p_1+1} &+ \bar{\sigma}_1(S_{1d}) S_{1d}^{p_1+1} \end{aligned} \tag{15}$$

for some nonnegative smooth functions ϕ_2 and $\bar{\sigma}_1$. From (15) it follows that

$$\begin{aligned} \dot{V}_1 &\le -(\beta_1 - 3 - 2\gamma_1) S_1^{p_1+1} + \phi_2 S_2^{p_1+1}\\ &\quad + \bar{\sigma}_1(S_{1d}) S_{1d}^{p_1+1} - \frac{\rho_1}{2l_1}S_1^{p_1+1}. \end{aligned} \tag{16}$$

Define the Lyapunov–Krasovskii functional

$$V_{1\mathrm{KL}} = V_1 + \int_{t-d}^{t} \bar{\sigma}_1(S_1(\tau)) S_1^{p_1+1}(\tau)\, d\tau, \tag{17}$$

whose time derivative by viewing (16), satisfies

$$\dot{V}_{1\mathrm{KL}} \le -(\beta_1 - \Upsilon_1) S_1^{p_1+1} + \phi_2 S_2^{p_1+1} - \frac{\rho_1}{2l_1}S_1^{p_1+1}, \tag{18}$$

where $\Upsilon_1(S_1) = 3 + 2\gamma_1 + \bar{\sigma}_1$. It is seen that by introducing the dynamic gain and the corresponding gain-based Lyapunov function, an additional term $-\rho_1 S_1^{p_1+1}/(2l_1)$ is generated in $\dot{V}_1$, which can be used to counteract the terms associated with α_1 appearing in the next steps.

Step 2: The derivative of $S_2 = x_2 - \alpha_1$ by viewing (1) and (10), is computed as

$$\dot{S}_2 = x_3^{p_2} + \bar{g}_2 + \bar{f}_2, \tag{19}$$

where

$$\begin{aligned} \bar{g}_2 &= g_2(x_1, x_2) - \frac{\partial\alpha_1}{\partial x_1}\left[x_2^{p_1} + g_1(x_1)\right],\\ \bar{f}_2 &= f_2(x_{1d}, x_{2d}) - \frac{\partial\alpha_1}{\partial x_1} f_1(x_{1d}). \end{aligned} \tag{20}$$

Let $S_3 = x_3 - \alpha_2$ with

$$\alpha_2 = -l_1^{1/p_2}\beta_2^{1/p_2}(S_1, S_2) S_2, \tag{21}$$

where β_2 is a positive smooth design function. Consider the Lyapunov function

$$V_2 = V_{1\mathrm{KL}} + \frac{S_2^{p_1-p_2+2}}{(p_1-p_2+2)\, l_1} + \frac{S_2^{p_1-p_2+2}}{(p_1-p_2+2)\, l_1 l_2}. \tag{22}$$

The dynamic gain l_2 is updated by

$$\begin{aligned} \dot{l}_2 = &\, l_1 S_2^{p_2-1}\max\left\{-(p_1-p_2+2)\, l_2^2\right.\\ &\left. + l_2\rho_2(l_1, S_1, S_2), 0\right\}, \quad l_2(0) = 1 \end{aligned} \tag{23}$$

with a positive smooth function ρ_2 to be determined. From (23), l_2 owns similar properties to l_1. Moreover, it can be proved by contradiction[3] that if (l_1, S_1, S_2) are bounded on the right maximum time interval $[0, T_f)$ for some $T_f \in (0, +\infty]$, then l_2 is bounded on $[0, T_f)$. Differentiating V_2 yields

$$\begin{aligned} \dot{V}_2 &\le -(\beta_1 - \Upsilon_1) S_1^{p_1+1} + \phi_2 S_2^{p_1+1} - \frac{\rho_1}{2l_1}S_1^{p_1+1}\\ &\quad + \frac{1}{l_1}\left(1+\frac{1}{l_2}\right) S_2^{p_1-p_2+1}\left(x_3^{p_2} - \alpha_2^{p_2} - l_1\beta_2 S_2^{p_2} + \bar{g}_2 + \bar{f}_2\right)\\ &\quad - \frac{\dot{l}_1 S_2^{p_1-p_2+2}}{(p_1-p_2+2)\, l_1^2} - \frac{\left(\dot{l}_1 l_2 + l_1\dot{l}_2\right) S_2^{p_1-p_2+2}}{(p_1-p_2+2)\, l_1^2 l_2^2}\\ &\le -(\beta_1 - \Upsilon_1) S_1^{p_1+1} - \frac{\rho_1}{2l_1}S_1^{p_1+1} - (\beta_2 - 1 - \phi_2) S_2^{p_1+1}\\ &\quad + 2\left|S_2^{p_1-p_2+1}\left(x_3^{p_2} - \alpha_2^{p_2}\right)\right| + 2\left|S_2^{p_1-p_2+1}\bar{g}_2\right|\\ &\quad + \frac{2}{l_1}\left|S_2^{p_1-p_2+1}\bar{f}_2\right| - \frac{\rho_2}{(p_1-p_2+2)\, l_2}S_2^{p_1+1}, \end{aligned} \tag{24}$$

where the properties of l_1 and l_2 have been used. According to Lemmas 1–2, the assumptions A.1–A.2 and the fact $l_1 \ge l_{1d} \ge 1$, we have

$$\begin{aligned} &2\left|S_2^{p_1-p_2+1}\left(x_3^{p_2} - \alpha_2^{p_2}\right)\right| \le S_2^{p_1+1} + \phi_3(l_1, S_1, S_2, S_3) S_3^{p_1+1},\\ &2\left|S_2^{p_1-p_2+1}\bar{g}_2\right| \le S_1^{p_1+1} + \bar{\gamma}_2(S_1, S_2) S_2^{p_1+1}, \end{aligned}$$

[3] Note that $l_2(t)$ is a positive nondecreasing function of time t. Suppose $\lim_{t\to T_f} l_2(t) = \infty$. Then, from (23), the smoothness of $\rho_2(l_1, S_1, S_2)$ and the boundedness of (l_1, S_1, S_2), there must exist a time $T_1 \in (0, T_f)$ such that $\dot{l}_2 \equiv 0$ on (T_1, T_f). Therefore, $l_2(t) = l_2(T_1) + \int_{T_1}^{t}\dot{l}_2(s)ds = l_2(T_1)$ on (T_1, T_f), a contradiction.

$$\begin{aligned}\frac{2}{l_1}\left|S_2^{p_1-p_2+1}\bar{f}_2\right| &\le \gamma_2^*(S_1,S_2)S_2^{p_1+1}+\frac{1}{l_{1d}}\bar{\sigma}_{21}(x_{1d})x_{1d}^{p_1+1}\\&\quad+\frac{1}{l_{1d}}\bar{\sigma}_{22}(x_{1d},x_{2d})x_{2d}^{p_1+1}\\&\le \gamma_2^*(S_1,S_2)S_2^{p_1+1}+\frac{1}{l_{1d}}\bar{\rho}_{21}(S_{1d})S_{1d}^{p_1+1}\\&\quad+\bar{\rho}_{22}(S_{1d},S_{2d})S_{2d}^{p_1+1}\end{aligned}\tag{25}$$

for some nonnegative smooth functions $\phi_3,\bar{\gamma}_2,\gamma_2^*,\bar{\sigma}_{2i}$ and $\bar{\rho}_{2i}$, $i=1,2$. Define the Lyapunov–Krasovskii functional

$$V_{2\mathrm{KL}}=V_2+\int_{t-d}^{t}\left[\frac{1}{l_1(\tau)}\bar{\rho}_{21}(S_1(\tau))S_1^{p_1+1}(\tau)+\bar{\rho}_{22}(S_1(\tau),S_2(\tau))S_2^{p_1+1}(\tau)\right]d\tau,\tag{26}$$

whose time derivative by viewing (24) and (25), satisfies

$$\begin{aligned}\dot{V}_{2\mathrm{KL}}\le&-(\beta_1-\Upsilon_1-1)S_1^{p_1+1}-(\beta_2-\Upsilon_2)S_2^{p_1+1}+\phi_3S_3^{p_1+1}\\&-\frac{\rho_1}{2l_1}S_1^{p_1+1}+\frac{1}{l_1}\bar{\rho}_{21}S_1^{p_1+1}-\frac{\rho_2}{(p_1-p_2+2)l_2}S_2^{p_1+1},\end{aligned}\tag{27}$$

where $\Upsilon_2(S_1,S_2)=2+\phi_2+\bar{\gamma}_2+\gamma_2^*+\bar{\rho}_{22}$. Because $\bar{\rho}_{21}$ is related to β_1, the term $l_1^{-1}\bar{\rho}_{21}S_1^{p_1+1}$ cannot be eliminated by choosing β_1. However, the introduction of the dynamic gain l_1 makes it possible to eliminate $l_1^{-1}\bar{\rho}_{21}S_1^{p_1+1}$ by the term $-\rho_1S_1^{p_1+1}/(2l_1)$.

Step i ($3\le i\le n-1$): The derivative of $S_i=x_i-\alpha_{i-1}$ by viewing (1) and α_{i-1} given by the previous step, is computed as

$$\dot{S}_i=x_{i+1}^{p_i}+\bar{g}_i+\bar{f}_i,\tag{28}$$

where

$$\begin{aligned}\bar{g}_i&=g_i(\bar{x}_i)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left[x_{j+1}^{p_j}+g_j(\bar{x}_j)\right]-\sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial l_j}\dot{l}_j,\\\bar{f}_i&=f_i(\bar{x}_{id})-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j(\bar{x}_{jd}).\end{aligned}\tag{29}$$

Let $S_{i+1}=x_{i+1}-\alpha_i$ with

$$\alpha_i=-(l_1\cdots l_{i-1})^{1/p_i}\beta_i^{1/p_i}(l_1,\ldots,l_{i-2},S_1,\ldots,S_i)S_i,\tag{30}$$

where β_i is a positive smooth design function. Suppose by induction that at step $i-1$, there are a positive definite and radially unbounded Lyapunov function $V_{i-1\mathrm{KL}}$ and a change of coordinates

$$\begin{aligned}S_1&=x_1,\\S_2&=x_2-\alpha_1,\ \alpha_1=-\beta_1^{1/p_1}(S_1)S_1,\\&\vdots\\S_i&=x_i-\alpha_{i-1},\ \alpha_{i-1}=-(l_1\cdots l_{i-2})^{1/p_{i-1}}\\&\quad\times\beta_{i-1}^{1/p_{i-1}}(l_1,\ldots,l_{i-3},S_1,\ldots,S_{i-1})S_{i-1},\end{aligned}\tag{31}$$

such that

$$\begin{aligned}\dot{V}_{(i-1)\mathrm{KL}}\le&-\sum_{j=1}^{i-1}\left(\beta_j-i+j+1-\Upsilon_j\right)S_j^{p_1+1}+\phi_iS_i^{p_1+1}\\&-\sum_{j=1}^{i-1}\frac{\rho_j}{(p_1-p_j+2)l_j}S_j^{p_1+1}\\&+\sum_{j=1}^{i-2}\sum_{k=j+1}^{i-1}\frac{1}{l_j}\bar{\rho}_{kj}S_j^{p_1+1}.\end{aligned}\tag{32}$$

Then, consider the Lyapunov function

$$V_i=V_{(i-1)\mathrm{KL}}+\frac{S_i^{p_1-p_i+2}}{(p_1-p_i+2)l_1\cdots l_{i-1}}+\frac{S_i^{p_1-p_i+2}}{(p_1-p_i+2)l_1\cdots l_i}.\tag{33}$$

The dynamic gain l_i is updated by

$$\begin{aligned}\dot{l}_i=&\,l_1\cdots l_{i-1}S_i^{p_i-1}\max\{-(p_1-p_i+2)l_i^2\\&+l_i\rho_i(l_1,\ldots,l_{i-1},S_1,\ldots,S_i),0\},\quad l_i(0)=1\end{aligned}\tag{34}$$

with a positive smooth function ρ_i to be determined. Differentiating V_i yields

$$\begin{aligned}\dot{V}_i\le&-\sum_{j=1}^{i-1}\left(\beta_j-i+j+1-\Upsilon_j\right)S_j^{p_1+1}\\&-\sum_{j=1}^{i-1}\frac{\rho_j}{(p_1-p_j+2)l_j}S_j^{p_1+1}+\sum_{j=1}^{i-2}\sum_{k=j+1}^{i-1}\frac{1}{l_j}\bar{\rho}_{kj}S_j^{p_1+1}\\&-(\beta_i-1-\phi_i)S_i^{p_1+1}+2\left|S_i^{p_1-p_i+1}\left(x_{i+1}^{p_i}-\alpha_i^{p_i}\right)\right|\\&+2\left|S_i^{p_1-p_i+1}\bar{g}_i\right|+\frac{2}{l_1\cdots l_{i-1}}\left|S_i^{p_1-p_i+1}\bar{f}_i\right|\\&-\frac{\rho_i}{(p_1-p_i+2)l_i}S_i^{p_1+1}.\end{aligned}\tag{35}$$

Similar to (25), it can be checked that

$$\begin{aligned}&2\left|S_i^{p_1-p_i+1}\left(x_{i+1}^{p_i}-\alpha_i^{p_i}\right)\right|\\&\quad\le S_i^{p_1+1}+\phi_{i+1}(l_1,\ldots,l_{i-1},S_1,\ldots,S_{i+1})S_{i+1}^{p_1+1},\\&2\left|S_i^{p_1-p_i+1}\bar{g}_i\right|\\&\quad\le\sum_{j=1}^{i-1}S_j^{p_1+1}+\bar{\gamma}_i(l_1,\ldots,l_{i-2},S_1,\ldots,S_i)S_i^{p_1+1},\\&\frac{2}{l_1\cdots l_{i-1}}\left|S_i^{p_1-p_i+1}\bar{f}_i\right|\le\gamma_i^*(l_1,\ldots,l_{i-2},S_1,\ldots,S_i)S_i^{p_1+1}\\&\quad+\frac{1}{l_{1d}\cdots l_{i-1d}}\sum_{j=1}^{i}\bar{\sigma}_{ij}\left(x_{1d},\ldots,x_{jd}\right)x_{jd}^{p_1+1}\\&\quad\le\gamma_i^*(l_1,\ldots,l_{i-2},S_1,\ldots,S_i)S_i^{p_1+1}\\&\quad+\sum_{j=1}^{i-1}\frac{1}{l_{jd}}\bar{\rho}_{ij}\left(l_{1d},\ldots,l_{j-1d},S_{1d},\ldots,S_{jd}\right)S_{jd}^{p_1+1}\\&\quad+\bar{\rho}_{ii}(l_{1d},\ldots,l_{i-2d},S_{1d},\ldots,S_{id})S_{id}^{p_1+1}\end{aligned}\tag{36}$$

for some nonnegative smooth functions $\phi_{i+1},\bar{\gamma}_i,\gamma_i^*,\bar{\sigma}_{ij}$, and $\bar{\rho}_{ij}$, $1\le j\le i$. Note that $\bar{\rho}_{ij}$, $1\le j\le i-1$, only depends on $\alpha_1,\ldots,\alpha_j$. Define the Lyapunov–Krasovskii functional

$$\begin{aligned}V_{i\mathrm{KL}}=V_i+\int_{t-d}^{t}\Bigg[&\sum_{j=1}^{i-1}\frac{1}{l_j(\tau)}\bar{\rho}_{ij}\left(l_1(\tau),\ldots,l_{j-1}(\tau),S_1(\tau),\ldots,S_j(\tau)\right)S_j^{p_1+1}(\tau)+\bar{\rho}_{ii}(l_1(\tau),\ldots,\\&l_{i-2}(\tau),S_1(\tau),\ldots,S_i(\tau))S_i^{p_1+1}(\tau)\Bigg]d\tau,\end{aligned}\tag{37}$$

whose time derivative by viewing (35) and (36), satisfies

$$\dot{V}_{i\mathrm{KL}} \le -\sum_{j=1}^{i}\left(\beta_j - i + j - \Upsilon_j\right) S_j^{p_1+1} + \phi_{i+1} S_{i+1}^{p_1+1} - \sum_{j=1}^{i} \frac{\rho_j}{(p_1 - p_j + 2)\, l_j} S_j^{p_1+1} + \sum_{j=1}^{i-1}\sum_{k=j+1}^{i} \frac{1}{l_j} \bar{\rho}_{kj} S_j^{p_1+1}, \tag{38}$$

where $\Upsilon_i(l_1, \ldots, l_{i-2}, S_1, \ldots, S_i) = 2 + \phi_i + \bar{\gamma}_i + \gamma_i^* + \bar{\rho}_{ii}$.

Step n: The derivative of $S_n = x_n - \alpha_{n-1}$ by viewing (1) and (30) with $i = n-1$, is computed as

$$\dot{S}_n = u + g_n(x) + f_n(x_d) - \sum_{i=1}^{n-2} \frac{\partial \alpha_{n-1}}{\partial l_i} \dot{l}_i - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i}\left[x_{i+1}^{p_i} + g_i(x_i) + f_i(x_{id})\right]. \tag{39}$$

Finally, the actual control u is chosen as

$$u = -l_1 \cdots l_{n-1} \beta_n (l_1, \ldots, l_{n-2}, S_1, \ldots, S_n) S_n \tag{40}$$

with β_n a positive smooth design function. Consider the Lyapunov functions

$$V_n = V_{(n-1)\mathrm{KL}} + \frac{S_n^{p_1+1}}{(p_1+1)\, l_1 \cdots l_{n-1}},$$

$$V_{n\mathrm{KL}} = V_n + \int_{t-d}^{t} \left[\sum_{i=1}^{n-1} \frac{1}{l_i(\tau)} \bar{\rho}_{ni}(l_1(\tau), \ldots, l_{i-1}(\tau), S_1(\tau), \ldots, S_i(\tau)) S_i^{p_1+1}(\tau) + \bar{\rho}_{nn}(l_1(\tau), \ldots, l_{n-2}(\tau), S_1(\tau), \ldots, S_n(\tau)) S_n^{p_1+1}(\tau)\right] d\tau, \tag{41}$$

where $\bar{\rho}_{ni}, 1 \le i \le n$, are nonnegative smooth functions. The derivative of $V_{n\mathrm{KL}}$ satisfies

$$\dot{V}_{n\mathrm{KL}} \le -\sum_{i=1}^{n}(\beta_i - n + i - \Upsilon_i) S_i^{p_1+1} - \sum_{i=1}^{n-1} \frac{\rho_i}{(p_1 - p_i + 2)\, l_i} S_i^{p_1+1} + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \frac{1}{l_i} \bar{\rho}_{ji} S_i^{p_1+1}, \tag{42}$$

where $\Upsilon_n(l_1, \ldots, l_{n-2}, S_1, \ldots, S_n)$ is a nonnegative smooth function. Choosing the design functions β_i, ρ_i such that

$$\beta_i \ge n - i + \Upsilon_i + c, \quad 1 \le i \le n,$$

$$\rho_i \ge (p_1 - p_i + 2) \sum_{j=i+1}^{n} \bar{\rho}_{ji}, \quad 1 \le i \le n-1 \tag{43}$$

for a positive constant c, we arrive at

$$\dot{V}_n \le -c \sum_{i=1}^{n} S_i^{p_1+1}, \tag{44}$$

which implies that $S_1, S_i^{p_1 - p_i + 2}/(l_1 \cdots l_{i-1}), 2 \le i \le n$, are bounded on the right maximum time interval $[0, T_f)$. Note that if $l_1, \ldots, l_{i-1}, S_1, \ldots, S_i$ are bounded, then it can be proved from the updated law (34) that l_i is bounded. Therefore, the boundedness of $S_1, l_1, S_2, l_2, \ldots, S_{n-1}, l_{n-1}, S_n$ can be proved one by one, which follows that x is bounded. Hence, $T_f = +\infty$. Moreover, (44) implies that $S_1, \ldots, S_n$ converge to zero asymptotically. Using the change of coordinates $S_1 = x_1, S_i = x_i - \alpha_{i-1}, 2 \le i \le n$, the convergence of x can be inferred, i.e., global asymptotic stabilization of system (1) can be achieved via the proposed approach. This completes the proof. ■

Remark 3. From the above analysis, it is clear that the main difference between the proposed dynamic gain-based backstepping approach and the conventional backstepping approach is that, by introducing a dynamic gain at each design step, the Lyapunov function is constructed in a new recursive manner. Such a design procedure produces additional negative terms in the derivative of the Lyapunov function which, compared with the conventional backstepping design, is able to eliminate the effects of stronger system nonlinearities. Notice that the dynamic gains do not converge to the origin but a positive constant. Therefore, one may call the considered stabilization problem a regulation problem.

Remark 4. When $p_i = 1, 1 \le i \le n-1$, as a special case, our proposed approach gives a solution to the state feedback control of the strict feedback system [5,6] with time-delay. In particular, the difficulty encountered in [4] can be circumvented by using our controller design.

Remark 5. In this paper, the time-delay d is an unknown constant and the upper bound of d is not required in the controller design. It should be pointed out that if the time-delay is nonnegative bounded time-varying function $d(t)$, the main results still hold under the condition $\dot{d} \le \bar{d} < 1$ for a known constant $\bar{d}$.

Remark 6. The design functions $\beta_1, \rho_1, \beta_2, \rho_2, \ldots, \beta_{n-1}, \rho_{n-1}, \beta_n$ are chosen recursively such that (43) holds. The functions Υ_i and $\bar{\rho}_{ji}$ depend on the system nonlinearities and can be directly determined by using the assumptions A.1–A.2, Lemmas 1–2, (15), (25) and (36).

4. Illustrative examples

In this section, we use both academic and physical examples to illustrate the applications of our scheme.

Example 1. Consider the following second-order plant

$$\dot{x}_1(t) = x_2^3(t),$$
$$\dot{x}_2(t) = u(t) + 0.5 x_2^2(t-d), \tag{45}$$

where d denotes the unknown time-delay. Obviously, (45) satisfies the assumptions A.1–A.2 and therefore, it can be globally stabilized by the memoryless state feedback controller given in Section 3. Following the design procedure, the controller is constructed as follows.

Step 1: Let $S_1 = x_1$ and $S_2 = x_2 - \alpha_1$ with $\alpha_1 = -\beta_1^{1/3}(S_1) S_1$. The dynamic gain l_1 is updated by

$$\dot{l}_1 = S_1^2 \max\left\{-2l_1^2 + l_1 \rho_1(S_1), 0\right\}, \quad l_1(0) = 1. \tag{46}$$

The derivative of $V_1 = S_1^2/2 + S_1^2/(2l_1)$ is computed as

$$\dot{V}_1 \le -(\beta_1 - 1) S_1^4 + 2\left|S_1\left(x_2^3 - \alpha_1^3\right)\right| - \frac{\rho_1}{2l_1} S_1^4 \le -(\beta_1 - 2) S_1^4 + \phi_2(S_1, S_2) S_2^4 - \frac{\rho_1}{2l_1} S_1^4, \tag{47}$$

where ϕ_2 is determined by (15). Choose $\beta_1 = 2.5$. Then, we have

$$\dot{V}_1 \le -0.5 S_1^4 + 120 S_2^4 - \frac{\rho_1}{2l_1} S_1^4. \tag{48}$$

Since the first differential equation of (45) has no time-delay, we can simply let $V_{1\mathrm{KL}} = V_1$.

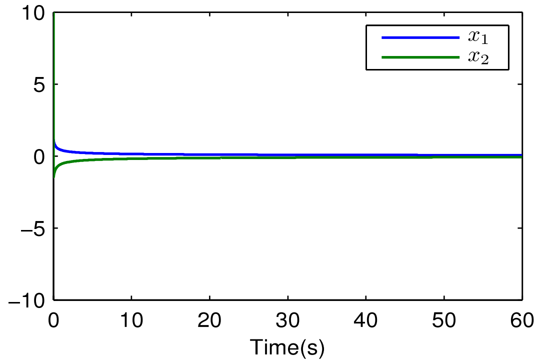

Fig. 1. System states (x_1, x_2).

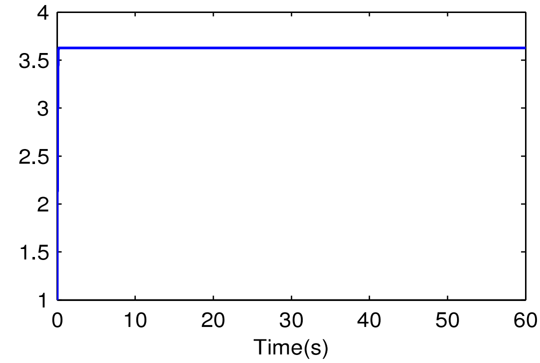

Fig. 2. Dynamic gain l_1.

Step 2: By (40), the actual control u is designed as

$$u = -l_1\beta_2 (S_1, S_2) S_2, \tag{49}$$

where the positive smooth design function $\beta_2 (S_1, S_2)$ will be determined later. The derivative of $V_2 = V_{1\mathrm{KL}} + S_2^4/4l_1$ is computed as

$$\begin{aligned}\dot V_2 &\le -0.5S_1^4 - \frac{\rho_1}{2l_1}S_1^4 - (\beta_2 - 120) S_2^4 \\ &\quad + \frac{1}{l_1}\left|S_2^3 \left(0.2x_{2d}^2 - 2.5^{1/3}x_2^3\right)\right| \\ &\le -0.5S_1^4 - \frac{\rho_1}{2l_1}S_1^4 - (\beta_2 - 120) S_2^4 \\ &\quad + 40S_2^4 + 9S_2^6 + \frac{5}{l_1}S_1^6 + 0.25S_{2d}^8 + \frac{2}{l_1}S_{1d}^8.\end{aligned} \tag{50}$$

Define the Lyapunov–Krasovskii functional

$$V_{2\mathrm{KL}} = V_2 + \int_{t-d}^{t}\left[\frac{2}{l_1(\tau)}S_1^8(\tau) + 0.25S_2^8(\tau)\right]d\tau, \tag{51}$$

whose time derivative satisfies

$$\begin{aligned}\dot V_{2\mathrm{KL}} &\le -0.5S_1^4 - \frac{1}{2l_1}\left(\rho_1 - 10S_1^2 - 4S_1^4\right)S_1^4 \\ &\quad - \left(\beta_2 - 160 - 9S_2^2 - 0.25S_2^4\right)S_2^4.\end{aligned} \tag{52}$$

Choosing $\rho_1 = 10S_1^2 + 4S_1^4$ and $\beta_2 = 160.5 + 9S_2^2 + 0.25S_2^4$, we have $\dot V_{2\mathrm{KL}} \le -0.5S_1^4 - 0.5S_2^4$ and the final form of the controller

$$\begin{aligned}u &= -l_1[160.5 + 9(x_2 + 2.5^{1/3}x_1)^2 + 0.25(x_2 + 2.5^{1/3}x_1)^4] \\ &\quad \times (x_2 + 2.5^{1/3}x_1), \\ \dot l_1 &= x_1^2 \max\{-2l_1^2 + l_1(10x_1^2 + 4x_1^4), 0\}, \quad l_1(0) = 1.\end{aligned} \tag{53}$$

With $d = 2$ and the initial conditions $(x_1(t_0), x_2(t_0)) = (1, 10)$ for $t_0 \in [-2, 0]$, Figs. 1–2 show the simulation results, which accord with the analysis in Section 3.

Example 2. Let us consider the cascade chemical system [10] with two reactors

$$\begin{aligned}\dot x_1(t) &= -k_1x_1(t) - \frac{1}{\theta_1}x_1(t) - \frac{1}{\theta_1}x_1(t-d) \\ &\quad + \frac{1-R_2}{V_1}x_2(t) + \delta_1(x_1(t-d)), \\ \dot x_2(t) &= -k_2x_2(t) - \frac{1}{\theta_2}x_2^2(t) + \frac{R_1}{V_2}x_1(t-d) - \frac{1}{\theta_2}x_2(t) \\ &\quad + \frac{R_2}{V_2}x_2(t-d) + \frac{F}{V_2}u(t) + \delta_2(x_2(t-d)),\end{aligned} \tag{54}$$

where $x_1(t)$ and $x_2(t)$ are the compositions, d is the unknown time-delay, $R_i, i = 1, 2$, are the recycle flow rates, θ_i are the reactor residence times, k_i are the reaction constants, F is the feed rate, V_i are reactor volumes, and δ_i are nonlinear functions for describing the system uncertainties and external disturbances. A global stabilizer was designed in [10] for system (54) but requiring the knowledge of the upper bound of the time-delay. When the time-delay d is completely unknown, existing results based on the Lyapunov–Krasovskii method cannot be applied to global stabilization of the time-delay system. However, noting that the system is in the form (1) with $p_1 = p_2 = 1$, a memoryless state feedback can be designed for (54) according to Theorem 1.

To proceed the simulation, we choose the same parameters as those of [10]: $\theta_i = 2$, $k_i = 0.5$, $R_i = 0.5$, $V_i = 0.5$, and $F = 0.5$. The uncertainties δ_i are the functions: $\delta_1(x_1(t-d)) = 0.5x_1(t-d)$ and $\delta_2(x_2(t-d)) = 0.5x_2^2(t-d)e^{0.01x_2(t-d)}$. Then, the state feedback controller is given by

$$\begin{aligned}u &= -l_1\beta_2 (S_1, S_2) S_2, \\ \dot l_1 &= \max\left\{-2l_1^2 + l_1\rho_1(S_1), 0\right\}, \quad l_1(0) = 1,\end{aligned} \tag{55}$$

where $S_1 = x_1$, $S_2 = x_2 - \alpha_1 = x_2 + \beta_1(S_1)S_1$. Consider the Lyapunov–Krasovskii functional

$$\begin{aligned}V_{2\mathrm{KL}} &= \frac{S_1^2}{2} + \frac{S_1^2}{2l_1} + \frac{S_2^2}{2l_1} + \int_{t-d}^{t}\left[S_1^2(\tau) + \frac{1}{4l_1(\tau)}S_1^2(\tau)\right]d\tau \\ &\quad + \int_{t-d}^{t}\frac{0.01}{l_1(\tau)}\beta_1^4(S_1(\tau))\,e^{-0.01\beta_1(S_1(\tau))S_1(\tau)}S_1^4(\tau)\,d\tau \\ &\quad + \int_{t-d}^{t}\frac{0.0001}{l_1(\tau)}\beta_1^2(S_1(\tau))\,e^{-0.04\beta_1(S_1(\tau))S_1(\tau)}S_1^2(\tau)\,d\tau \\ &\quad + \int_{t-d}^{t}\left[\frac{1}{4}e^{0.02S_2(\tau)}S_2^4(\tau) + 0.01e^{0.04S_2(\tau)}S_2^2(\tau)\right]d\tau,\end{aligned} \tag{56}$$

whose time derivative satisfies

$$\begin{aligned}\dot V_{2\mathrm{KL}} &\le -(\beta_1 - 3.5)S_1^2 - \frac{1}{2l_1}(\rho_1 - 5)S_1^2 \\ &\quad + \frac{0.01}{l_1}\beta_1^4e^{-0.01\beta_1S_1}S_1^4 + \frac{0.0001}{l_1}\beta_1^2e^{-0.04\beta_1S_1}S_1^2 \\ &\quad - \left(\beta_2 - 8 - 1.75S_2^2\right)S_2^2 + \left(\frac{1}{4}\beta_1^4 + \frac{1}{4}\beta_1^2 + \frac{9}{8}\beta_1\right)S_2^2 \\ &\quad + \frac{1}{4}e^{0.02S_2}S_2^4 + 0.01e^{0.04S_2}S_2^2.\end{aligned} \tag{57}$$

From (57), the design functions β_1, ρ_1 and β_2 are chosen as $\beta_1 = 4$, $\rho_1 = 5 + 2e^{-0.04S_1}S_1^2 + 0.004e^{-0.16S_1}$ and $\beta_2 = 81 + 1.75S_2^2 + 0.25e^{0.02S_2}S_2^2 + 0.01e^{0.04S_2}$, respectively. Then, we have $\dot V_{2\mathrm{KL}}$

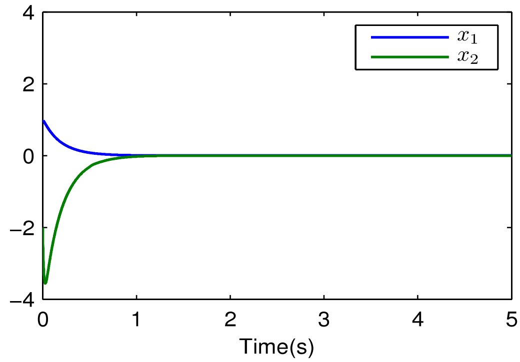

Fig. 3. System states (x_1, x_2).

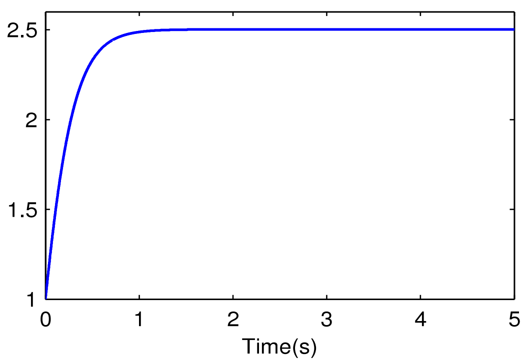

Fig. 4. Dynamic gain l_1.

$\leq -0.5S_1^2 - 0.5S_2^2$ and the final form of the controller

$$\begin{aligned} u &= -l_1[81 + 1.75\,(x_2 + 4x_1)^2 + 0.25e^{0.02(x_2+4x_1)}\,(x_2 + 4x_1)^2 \\ &\quad + 0.01e^{0.04(x_2+4x_1)}]\,(x_2 + 4x_1)\,, \\ \dot{l}_1 &= \max\{-2l_1^2 + l_1(5 + 2e^{-0.04x_1}x_1^2 \\ &\quad + 0.004e^{-0.16x_1}), 0\}, \quad l_1\,(0) = 1. \end{aligned} \tag{58}$$

With $d = 0.5$ and the initial conditions $(x_1\,(t_0)\,, x_2\,(t_0)) = (1, -2)$ for $t_0 \in [-0.5, 0]$, the simulation results are shown in Figs. 3–4, which illustrate the effectiveness of our scheme.

5. Conclusion

In this paper, the problem of global stabilization by state feedback for a class of high-order nonlinear systems with time-delay has been addressed. With new Lyapunov–Krasovskii functionals, a memoryless state feedback controller is explicitly constructed via the novel dynamic gain-based backstepping approach. The precise knowledge of the time-delay is not necessary for the controller design and stability analysis. It has been proved that the states of the nonlinear time-delay systems converge to the origin while all the closed loop signals are globally bounded. Both physical and academic examples have been given to illustrate the applications of the proposed scheme.

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