DC Power Flow Revisited

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***Abstract*—Linear MW-only "dc" network power flow models are in widespread and even increasing use, particularly in congestion-constrained market applications. Many versions of these approximate models are possible. When their MW flows are reasonably correct (and this is by no means assured), they can often offer compelling advantages. Given their considerable importance in today's electric power industry, dc models merit closer scrutiny. This paper attempts such a re-examination.**

***Index Terms*—Congestion revenue rights, contingency analysis, dc power flow, economic dispatch, financial transmission rights, LMP pricing, unit commitment.**

I. INTRODUCTION

THIS paper addresses so-called "dc" MW-only power flow modeling, which is of increased interest today because of recent upsurges in its use—mostly in LMP-based market applications where prices are constrained by network congestion. Such applications include real-time security-constrained dispatch (SCED), day-ahead security-constrained unit commitment (SCUC), and the auctions and allocations associated with transmission rights (FTR-CRR-TCC). And more traditionally, dc models are widely used in contingency screening, transmission loading relief, transfer analysis, and medium-to-long term transmission planning.

Many dc power flow model versions are available, but we have found nothing in the literature that identifies and categorizes them. Papers that describe dc power flow applications frequently do not specify exactly which dc model was used.

Dc power flow models are inherently approximate, and it is well known that their accuracies are very system and case dependent. At the same time, hard documentary data about this is sparse and often contradictory—few large-scale dc model accuracy tests have been reported.

Given the above, this paper offers two main contributions. Firstly, it reviews dc power flow methods by identifying and classifying different model versions—both the presentation and some of the dc models are novel. Secondly, it summarizes the results of extensive, large-scale dc model testing, whose purpose was to investigate accuracy trends among the dc modeling variants.

This paper covers dc modeling only at its fundamental level. It does not deal with other forms of linearization or the impact of any dc power flow model on any specific application.

II. WHY DC MODELS?

The linear, bilateral, non-complex, often state-independent, properties of a dc-type power flow model have considerable analytical and computational appeal. The use of such a model is limited to those MW-oriented applications where the effects of network voltage and VAr conditions are minimal (a very difficult-to-judge criterion). But then, as opposed to using the ac power flow model, the perceived advantages of a dc model are as follows.

(a) Its solutions are non-iterative, reliable and unique.
(b) Its methods and software are relatively simple.
(c) Its models can be solved and optimized efficiently, particularly in the demanding area of contingency analysis.
(d) Its network data isminimal and relatively easy to obtain.
(e) Its linearity fits the economic theory on which much of transmission-oriented market design is based.
(f) Its approximated MW flows are reasonably accurate, at least for the heavily loaded branches that might constrain system operation.

These are powerful attractions and, with exceptions to be noted later, items (a)–(e) are mostly valid. However, the big uncertainty is proposition (f), and this complicates the choice between dc and ac models in any given application. On the other hand, sometimes there may be no viable alternative to the use of a dc model, for example when:

(i) only linear theory and/or calculation techniques are available for certain (often market) applications;
(ii) reliable voltage-VAr control data isnot available to support stable, meaningful ac power flow solutions;
(iii) certain applications in large markets involve volumes of computing that would be prohibitive with ac modeling;
(iv) a dc model is needed for cross-compatibility between two or more related applications.

III. DC POWER FLOW—A BRIEF BACKGROUND

The term "dc" power (or load) flow comes from the old dc network analyzer [1], [2], in which each network branch was represented by a resistance proportional to its series reactance and each dc current was proportional to a MW flow. In the digital era this model becomes a simple, real (non-complex) nodal admittance matrix equation in terms of bus voltage angles and MW injections.

The different dc model versions are distinguished by the definitions of the injections and admittances in this equation and, as will be shown here, minor variations in them can have big effects on model performance. Nevertheless, the original "classical" series-reactance version is still widely regarded as *the* dc power flow method. It is the version presented and derived in books dealing with power flow, for example [3]–[7]. Its admittance matrix—the same as matrix $\boldsymbol{B'}$ in the fast decoupled

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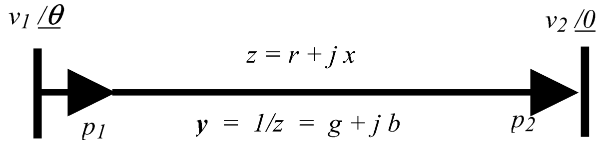

Fig. 1. Series transmission line.

power flow [8]—yields MW sensitivities that have been applied very extensively by the industry [9]–[13], [21], [22].

Attempts at the theoretical error analysis of dc models have been made [14], [15]. However, it seems that a dc model's performance on a non-small network can only be assessed experimentally, by comparing the MW flows obtained from dc and ac power flow solutions. A literature search reveals several reports of such experiments, all of them relatively recent [16]–[22].

Reference [16] compared dc power flow with other linear models on small networks. Reference [17] investigated ac-versus-dc LMP calculations on realistically large (13 000 bus) systems. Reference [18] and [19] describe systematic Monte Carlo simulations performed on small networks to investigate the influences on the classical dc model of various parameters such as line r/x ratios. Reference [20] studied LMP calculations with ac and dc models. Reference [21] studied the accuracy of power transfer distribution factors (PDTFs) on the European network. Reference [22] measured PDTF errors in small and large networks. In general, the above results tend to be more optimistic about dc model MW-flow accuracy than those from our own tests, to be described later.

IV. DC POWER FLOW BASICS

A. Notations

Each network scalar is italic and lowercase. Thus, p is a branch MW flow, θ is the angle in radians across the branch, z and its inverse y are series branch parameters on a 1 MVA per-unit base, and v is a bus voltage magnitude in per-unit. The bus MW and angle vectors $\boldsymbol{p}$ and $\boldsymbol{\theta}$ are shown bold, italic and lowercase. The dc-approximation branch admittance is h, and the corresponding dc nodal admittance matrix is $\boldsymbol{H}$.

B. AC Branch-Flow Model

The dc modeling process starts from the familiar ac transmission line shown in Fig. 1.

The exact expressions for the sending and receiving end MW flows in the line are

$$p_1 = +gv_1(v_1 - v_2\cos\theta) - v_1v_2b\sin\theta \qquad (1a)$$
$$p_2 = -gv_2(v_2 - v_1\cos\theta) - v_1v_2b\sin\theta. \qquad (1b)$$

For a transformer, each voltage magnitude v is reflected to the impedance side of its turns ratio. If the transformer has a phase shift ϕ, angle θ becomes $(\theta + \phi)$. The terms involving conductance g represent series branch losses. Branch shunt conductances are not shown here—their inclusion is trivial.

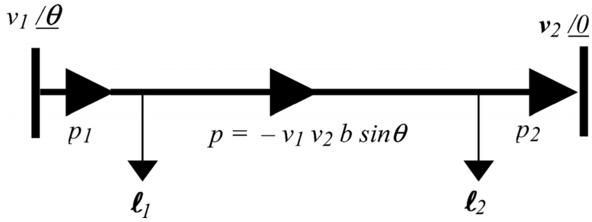

Fig. 2. Exact ac equivalent model of line.

C. Classical DC Model Derivation

A very typical textbook dc power flow derivation [3]–[7] reduces the exact branch MW flow (1) via a sequence of approximations; thus

$$\text{loss} \approx 0 \rightarrow p_1 = p_2 = p = -v_1v_2b\sin\theta \qquad (2)$$
$$\sin\theta \approx \theta \rightarrow \qquad p = -v_1v_2b.\theta \qquad (3)$$
$$v_1, v_2 \approx 1 \rightarrow \qquad p = -b.\theta \qquad (4)$$
$$-b \approx 1/x \rightarrow \qquad p = \theta/x. \qquad (5)$$

Equation (5) is the classical dc power flow model. As the basis for understanding and predicting dc model accuracy, this kind of derivation can be misleading. In contradiction to (2), it is essential to include an estimate of MW losses for a non-small power system. Then the approximations in (3)–(5) above perform surprisingly better than might at face value be expected. This is explained briefly in the Appendix.

D. More General DC Modeling

The dc modeling problem can be presented in a more general form as follows (this treatment is possibly novel). In the exact ac model of (1), let us designate

$$\boldsymbol{\ell}_1 = gv_1(v_1 - v_2\cos\theta) \qquad (6a)$$
$$\boldsymbol{\ell}_2 = gv_2(v_2 - v_1\cos\theta) \qquad (6b)$$

where the branch series loss is $\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2$. Then (1) can be rewritten as

$$p = -v_1v_2b\sin\theta \qquad (7)$$
$$p_1 = \boldsymbol{\ell}_1 + p \qquad (8a)$$
$$p_2 = -\boldsymbol{\ell}_2 + p \qquad (8b)$$

and its equivalent circuit is shown in Fig. 2. Functions p_1 and p_2 would be sinusoidal if v_1 and v_2 remained constant.

A dc branch model linearly approximates (7) and (8). Since this model must be bilateral, it is restricted to the form

$$p = h \cdot \theta \qquad (9)$$
$$p_1 = \alpha_1 + p \qquad (10a)$$
$$p_2 = -\alpha_2 + p. \qquad (10b)$$

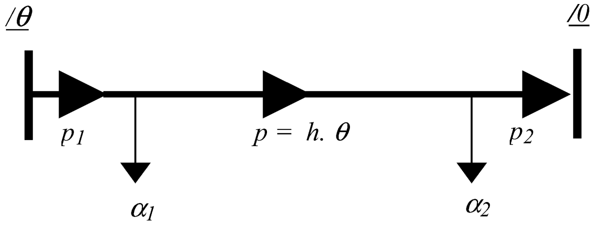

Fig. 3. Dc equivalent model of line.

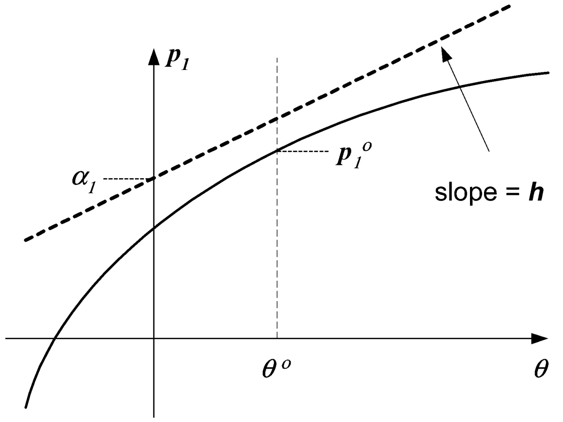

Fig. 4. Graphic of (1a) showing dc model.

In (9) h is the fixed dc equivalent branch admittance. In (10) α_1 and α_2 are fixed injections whose sum approximates the branch loss at some chosen operating point. This translates to the dc equivalent circuit of Fig. 3.

E. DC Modeling Challenge

Fig. 4 depicts a section of the true nonlinear curve p_1 versus θ. The intercept on the p axis is $gv_1 \cdot (v_1 - v_2)$, which for small θ remains close to $\boldsymbol{\ell}_1$. The intercept on the θ-axis is roughly $(r/x).(v_1 - v_2)/v_2$. That is, the curve's asymmetric displacement from the origin is strongly a function of both the branch's impedance and its state-varying voltage drop. The curve p_2 versus θ is very similar.

The dashed straight line in Fig. 4 represents the dc model approximation to the true curve of p_1 versus θ. Ideally, we would like to find values of h, α_1 and α_2 for which (9) and (10) provide the best linear fit to the true curves over the expected range of practical operation. Unfortunately, identifying a good such model is complicated and ends up being largely empirical.

Candidate definitions for admittance h are suggested directly by (3)–(5) (assuming certain fixed voltages v° in the first of these); or h could be chosen as a value in between $-b$ and $1/x$, such as $\mathrm{sign}(x).|y|$; or we could obtain h from the slope of the tangent to the true curve at a suitable point

$$p = -v_1^\circ v_2^\circ b \cos\theta^\circ.\theta. \tag{11}$$

Likewise, by comparing (10) with (8), the most obvious choices for loss-approximating injections α_1 and α_2 would appear to be $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$, evaluated at a suitable point.

Intuitively, it seems desirable, particularly in real-time/online applications, for the dc model to be as accurate as possible at the "base point" where it is constructed. Then any model errors will tend to increase only to the extent that the power system operating state moves away from this base point.

F. DC Power Flow Matrix Equation

Equations (9) and (10) lead to a dc nodal admittance equation with a sparse numerically symmetric matrix $\boldsymbol{H}$

$$\boldsymbol{p} = \boldsymbol{p}_G - \boldsymbol{p}_D - \boldsymbol{p}_S - \boldsymbol{p}_H - \boldsymbol{p}_\phi - \boldsymbol{p}_L = \boldsymbol{H} \cdot \boldsymbol{\theta} \tag{12}$$

where the components of the bus MW vector $\boldsymbol{p}$ are

$\boldsymbol{p}_G =$ generation;
$\boldsymbol{p}_D =$ demand (load);
$\boldsymbol{p}_S =$ bus shunt losses;
$\boldsymbol{p}_H =$ hvdc infeeds;
$\boldsymbol{p}_\phi =$ injections representing phase shifts;
$\boldsymbol{p}_L =$ injections representing branch losses.

V. DC POWER FLOW MODEL CATEGORIES

A. Explicit Models

These sparse non-incremental dc models are mostly used for the pre-contingency state in security-constrained applications. They can also be used for post-contingency power flow, although incremental versions are then more common. Here we introduce the terms "hot and cold start."

a) Hot-Start Models: This type of model is constructed at a solved ac power flow base point. It is often used in real-time security constrained economic dispatch (SCED), run from a state estimator solution [23]–[25]. It can also be used in short and medium term operation and planning studies where an initial ac solution is available.

b) Cold-Start Models: A dc model has to be constructed in "cold-start" mode when a reliable base-point ac power flow solution is unavailable (usually due to lack of good voltage/VAr data). This situation typically arises in dc-model-based security constrained unit commitment (SCUC) [26]–[28], FTR-CRR auctions and allocations [29]–[31], and longer-term planning studies [32], [33].

B. Incremental Models

These models compute changes from a known ac or dc base-point state. By definition they "fit" the initial base point. There are two types.

a) Sparse Models: These models are used for real-time SCED and any other application where a solved base-point ac or dc model is available.

b) Sensitivity Factor Models: These factors are generated from the sparse dc network matrix. When used directly in system contingency monitoring and remedial control action, they are pre-computed. Their names vary.

VI. HOT-START DC MODELS

An initial solved ac power flow solution is very helpful as a base point from which to construct an explicit dc model. This section describes several of the many possible model variants. Series and shunt MW losses and ZIP loads are evaluated from the base point ac solution and generally remain fixed thereafter in (12). At the base point, all hot-start models have the same total MW losses as the ac solution, and therefore their reference bus injections are correct.

A. Net Loss Dispersal

A dispersed-loss dc model ignores (10) and their constants α_1 and α_2. It can be constructed at an ac base point by semi-arbitrarily distributing the known net losses as additional bus injections at the load and/or generation buses to form vector $\boldsymbol{p}_L$ in (12). After solving for the bus angles, (9) provides the branch flows. Such a hot-start model has little attraction, however, because the exact amounts and locations of the initial losses are known and usable, as follows.

B. Localized Loss Modeling

Branch constants α_1 and α_2 in (10) can be defined from the known individual losses in different ways, for instance:

1) as $\boldsymbol{\ell}_1^\circ$ and $\boldsymbol{\ell}_2^\circ$, respectively, evaluated from (6) at the initial ac solution point (by comparing Figs. 2 and 3);
2) as 50% each of the initial branch loss.

The α's form vector $\boldsymbol{p}_L$. After solving (12) for the bus angles, the branch flows are given by (9) and (10).

C. Base-Point Matching

When the dashed line in Fig. 4 passes through the $(p_1^\circ, \theta^\circ)$ point, the localized branch loss model matches the MW flows and angles of the ac base point perfectly. Thus (this seems to be unpublished) we will equate (10a) with (8a) at this point thus:

$$p_1^\circ = \alpha_1 + h \cdot \theta^\circ = \boldsymbol{\ell}_1^\circ - v_1^\circ v_2^\circ b \sin\theta^\circ \tag{13a}$$

and we equate (10b) with (8b) as thus:

$$p_2^\circ = -\alpha_2 + h \cdot \theta^\circ = -\boldsymbol{\ell}_2^\circ - v_1^\circ v_2^\circ b \sin\theta^\circ. \tag{13b}$$

Two ways of achieving this matching are as follows.

a) α-Matching: With any specified h, (13) can be solved to provide matching values for α_1 and α_2. Note that their sum remains equal to the initial branch loss $\boldsymbol{\ell}_1^\circ + \boldsymbol{\ell}_2^\circ$. This is also equivalent to adding a fixed matching phase shift to each network branch.

b) h-Matching: For any given α_1 and α_2 either (13a) or (13b) can be solved for the matching value for h as follows:

$$h = -v_1^\circ v_2^\circ b \sin\theta^\circ / \theta^\circ \tag{14}$$

where $\sin\theta^\circ/\theta^\circ \to 1$ as $\theta \to 0$. This model was used (by one of the present authors) in the software of [34].

VII. COLD-START DC MODELS

The absence of an initial ac power flow solution makes constructing a reliable base-point dc model much more difficult than in the hot-start case.

A. Net Loss Dispersal

This commonly-used approach is similar to that in Section VI-A, except that here the net loss has to be estimated, for instance as a percentage of net load. In routine operational simulations on a given system, net losses can be guessed fairly well. But in any case, the loss dispersal is so arbitrary that these loss estimates have second-order effects on MW flow accuracies (sometimes even a zero estimate is used).

The net estimated loss is distributed as injections at the load and/or generation buses to form vector $\boldsymbol{p}_L$ in (12). Island net MWs are then balanced by scaling or otherwise adjusting the generations and/or the loads. (*Caveat*: if most units start at maximum, any net generation increase could end up unevenly allocated to just a few units.)

The choice of dc branch admittance h is usually limited to a state-independent value such as $-b$ or $1/x$. ZIP loads may be evaluated at nominal or typical voltages. Shunts are evaluated likewise or are simply lumped with the net loss.

Then (12) is solved for the dc-model base-point angles, and (9) gives the branch flows.

B. Loss Redistribution

Once constructed and solved, the dispersed-loss dc model of Section VII-A can be refined by redistributing the losses to the individual branches as follows.

(a) Calculate loss components $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$ for each branch, either from (6) with $v = 1$ or by setting both as $\frac{1}{2}p^2 r$ [4].
(b) Scale each $\boldsymbol{\ell}$ in order to keep the net system loss at its original estimated value.
(c) Set loss terms α_1 and α_2 in (10) as $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$, respectively, establishing a new vector $\boldsymbol{p}_L$.
(d) Solve (12) and obtain new branch flows from (9).

The above process can be iterated, and convergence is rapid. A variant is to omit step 2), allowing the net branch losses to change, in which case the system MWs have to be rebalanced at each iteration. This variant [20] usually converges but the net system loss has no anchor value and is free to drift.

C. Fixed-Voltage AC Solution Start

Another approach to cold-start modeling is to obtain the solution of a simplified ac power flow problem, to which a matched hot-start dc model can be fitted.

All buses in this simplified ac formulation have their voltage magnitudes fixed at nominal or typical values. That is, all buses become designated as PV with no VAr limits.

The basic "flat-voltage" version, requiring no voltage/VAr data whatever, sets all voltages and taps to 1 per-unit. This ac solution's VAr flows are completely wrong, of course. However, there is some hope that its MW flows, net losses and loss distribution will be better than those in Section VII-A.

VIII. INCREMENTAL DC MODELS

A. Sparse Matrix Models

The incremental version of (9) is

$$\Delta p = p - p^o = h \cdot \Delta\theta \tag{15}$$

where superscript ^{o} signifies the base-point value. In matrix form, this becomes the incremental version of (12)

$$\boldsymbol{\Delta p} = \boldsymbol{H} \cdot \boldsymbol{\Delta\theta}. \tag{16}$$

By substituting $\boldsymbol{p}-\boldsymbol{p}^{\circ} = \boldsymbol{\Delta p}$ and $\boldsymbol{\theta}-\boldsymbol{\theta}^{\circ} = \boldsymbol{\Delta\theta}$ into this equation, we get the equivalent explicit dc model

$$\boldsymbol{p} - [\boldsymbol{p}^{o} - \boldsymbol{H} \cdot \boldsymbol{\theta}^{o}] = \boldsymbol{H} \cdot \boldsymbol{\theta} \tag{17}$$

which is identical to the α-matched model of Section VI-C (a).

For calculations (e.g., contingency analysis) involving changes in the network, (16) can be written

$$\boldsymbol{\Delta p} = [\boldsymbol{H} + \boldsymbol{\Delta H}] \cdot \boldsymbol{\Delta\theta}. \tag{18}$$

Branch and bus outages (single or multiple) are handled efficiently by compensation or factor-updating techniques [35]. These techniques can also handle more complex topology changes; otherwise matrix re-factorization is undertaken.

B. Sensitivity Factor Models

Sensitivity factors are very widely known and used [5], [9]–[13], [21], [22]. Their pure dc versions derive directly from (16) and (18). The factors of main interest are usually known as:

— PTDF (power transfer distribution factor) = the MW change in a branch flow for a 1 MW exchange between a bus and the point or distributed reference;

— OTDF (outage transfer distribution factor) = the post-contingency MW change in a branch for a 1 MW pre-contingency bus-to-reference exchange;

— LODF (line outage distribution factor) = the MW change in a branch flow due to the outage of a branch with 1 MW pre-outage flow.

The PTDF and OTDF factors have been used explicitly by system operators for transmission loading relief (TLR) procedures. Most network security-constrained optimizations internally calculate and use these factors.

The LODFs can provide very fast dc contingency screening. Once computed, it is necessary to store and use only the non-small factors, which are relatively very few in number. That is, only the transmission elements that are sufficiently sensitive to a given contingency are monitored. As long as the network topology does not change, the factors remain the same and they can be used repeatedly and rapidly. But whenever the network changes, they have to be recalculated.

OTDF and LODF factors can readily be derived and applied for contingencies comprising multiple line outages. However, the efficiency of the LODF approach is lost when any contingency involves changes other than simple non-islanding series reactance outages—for instance the outage of a phase shifter, or any outage requiring bus MW redistribution.

IX. SOME OTHER DC MODEL ISSUES

A. Phase Shifting Transformers

The dc equivalent model for a phase shifting transformer with fixed angle ϕ and admittance h is trivial. It comprises a pair of MW injections $\pm h \cdot \phi$ at the branch terminals. Variation of h with angle is usually not modeled.

The big modeling problem occurs for a phase angle regulator (PAR), where ϕ is automatically adjusted to maintain a scheduled MW flow s. When ϕ is in range, the simplest (not necessarily best) way of modeling this is as an open circuit with terminal injections of $\pm s$. However, when the angle reaches a limit, the device reverts to its fixed-angle version. The network model and its sensitivities therefore change each time an angle is fixed on or backs off a limit, and the now-discontinuous linear dc model has to be solved iteratively. Iteration can only be avoided by ignoring phase angle limits, but this is likely to produce grossly unrealistic MW flows.

In an optimizing calculation, one attempt to circumvent the problem is to designate each phase shift angle as an optimization variable, and constrain the MW flow. This brings its own complications, in terms of costs of shift, target ranges, binding constraints and solution uniqueness.

Thus, the modeling of PARs with angle limits has huge analytical and computational consequences. It makes pre-calculated sensitivities virtually unusable, at least for the pre-contingency network state (post-contingency phase shifts are often represented as fixed at their pre-contingency values).

Similar considerations apply to HVDC and FACTs devices that automatically control MWs.

B. Security-Constrained Optimization

The biggest use of dc-type models is in linear SCOPF/SCED—security-constrained optimal power flow or dispatch [23]–[25]. Similar models are used in security constrained unit commitment [26]–[28] and financial transmission rights applications [29]–[31]. This is not the subject of the present paper, but a few comments on the dc modeling aspects are appropriate.

Formulation: A dc-based SCOPF formulation can be expressed in its simplest form as the minimization of

$$-\text{a (piecewise) linear function} \quad c(\boldsymbol{p}) \tag{19}$$

subject to linear sets of constraints, for $\mathrm{i} = 0 \ldots \mathrm{n}$

$$-\text{power flow equations} \quad \boldsymbol{f}^{\boldsymbol{i}}(\boldsymbol{p}, \boldsymbol{\theta}) = 0 \tag{20a}$$

$$-\text{inequalities} \quad \boldsymbol{m}^{\boldsymbol{i}}(\boldsymbol{p}, \boldsymbol{\theta}) \leq 0 \tag{20b}$$

where superscript $\mathrm{i} = 0$ signifies the pre-contingency state and otherwise refers to contingency case i.

Discontinuities: As described in Section IX-A, iteration is required whenever the continuity of the linear dc model is interrupted. This sacrifices some of the advantages of linear modeling and makes solutions more complicated, time-consuming, path-dependent and non-unique. Apart from PAR angle limiting, other model discontinuity examples are when 1) a MW outage or islanding requires MW rebalancing within generator limits, 2) secondary switching is modeled, or 3) market-specific precedence and/or infeasibility rules are invoked. Normally, a dc-modeled SCOPF formulation would have to be simplified to the point of extreme impracticality before it could be fed into a general-purpose optimization package.

Consistency: Given the vast numbers of linear network constraints in (20), the SCOPF/SCED solution process needs to monitor them in an "outer-loop," which feeds any violated or near-violated constraints to the redispatch engine. Quick-and-dirty implementations perform this outer loop once only, ignoring the fact that redispatch to address some but not all insecurities may create even worse insecurities. Any trustworthy SCOPF must iterate outer-loop monitoring until no insecurities remain. Such iteration (generally taking just a few passes) requires consistent pre- and post-contingency dc models. Otherwise, constraints enforced in one pass will not be on their limits in the next pass, and oscillatory solutions can result.

X. ASSESSING DC MODEL ACCURACY

A. General

It is axiomatic that dc power flow approximations vary enormously in accuracy for different systems, loadings, flow patterns and individual transmission elements. In this volatile, analytically confusing, heuristic area, there is no prospect for simply ranking dc models in order of goodness.

Nevertheless, by running thousands of cases on dozens of networks, we have tried to accumulate some general insights into the expected accuracies of different dc models. Most dc models find their use within some form of security-constrained optimization. Therefore it seemed appropriate to focus on a model's accuracy:

- at the "base point" where the model is constructed;
- after contingencies;
- after redispatch.

Fig. 5 outlines our test procedure. In this field, small differences in methodology can lead to big differences in results. Therefore we try to explain our test procedures in some detail.

Every pre- and post-contingency power flow solution was run with both ac and dc models, making sure that the comparisons are realistic. In particular, the slack power in every ac solution was shared among all generators, so that MW-loss changes do not accumulate at the reference bus.

B. MW-Flow Comparisons

The scope of our tests was very basic—to measure the errors in the dc-modeled MW flows. We did not address the techniques for imposing limits (mostly quoted in MVA or amperes) on these flows. Typical such methods try to account for the MVAr flows that are absent from dc models—for example, a power factor is imputed to each branch flow. Reference [36] describes a more sophisticated approach.

C. MW-Flow Accuracy Measurement

The critical flows in a network are the potentially congestive ones—those that can substantially affect system dispatch and pricing. Thus, in all our tests, we monitored branches loaded above 70% of rating (and we ignored all flows below 50 MW).

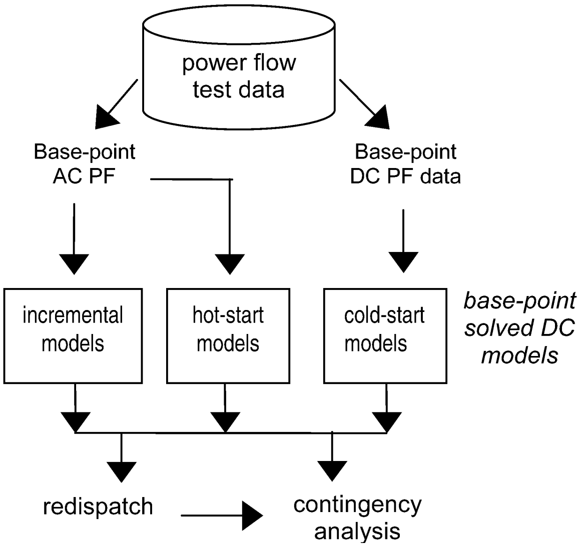

Fig. 5. Test procedures.

TABLE I
POWER FLOW TEST SYSTEMS

	region	bus count, by kV range				G W	loss %	load level
		total	%LV < 115	%HV	%EHV ≥ 345			
1	N Amer. E	48K	53	44	3	675	2.5	H
2	N Amer. W	14K	51	47	2	160	3.6	H
3	N Amer. S	5K	45	51	4	53	2.2	H
4	W Europe	5K	7	65	28	160	1.8	M
5	Brazil	4K	45	44	11	70	5.8	H
6	Brazil	4K	45	44	11	42	4.7	L

The error in each monitored dc flow was expressed as a percentage of the corresponding "exact" ac MW power flow.

At no stage in the tests were we able to discern any statistical patterns in the dc-flow error scatters. This defeated all our attempts to find concise, meaningful indices with which to characterize and display dc-model accuracies. We ended up simply tabulating the ranges (i.e., the extremes) and the averages of the dc-model flow errors. We separately recorded the average positive errors (dc MWs too high) and the average negative (dc MWs too low) errors—under-estimating critical flows compromises system security, while over-estimating them leads to transmission under-utilization and congestion over-pricing.

D. Test Systems

Tests were carried out on dozens of power systems. Table I lists the six systems for which illustrative results are given here. The voltage ranges defined as Low, High, and Extra-High (LV, HV, EHV) are of course somewhat arbitrary. H, M, and L, respectively, stand for Heavy, Medium, and Light load.

Each such system is a large modern power grid, reflecting the tendency towards centralized modeling and large markets. It also exposes dc modeling to a diverse range of network characteristics. The data for each system comes as a solved ac power flow case that has been widely used by the industry, either in its entirety (as tested here) or in reduced form. We filtered out of

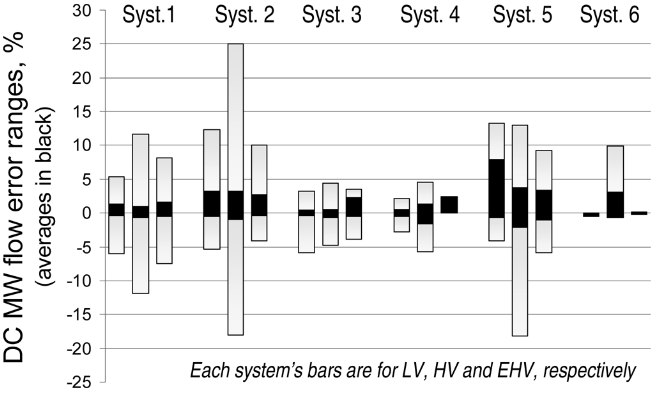

Fig. 6. Base-point cold-start dispersed-loss model.

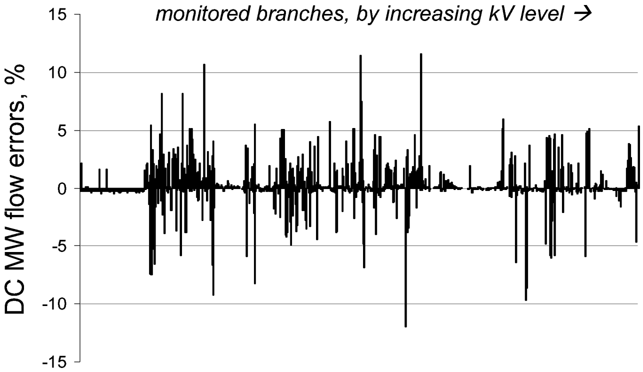

Fig. 7. Detailed error spread corresponding to System 1 in Fig. 6.

our error analyses a few data items that were obviously wrong, such as base-point branch flows in Systems 1 and 2 that were 100% overloaded.

XI. Test-Based Observations

This section offers comments on the dc-modeling trends that we have observed in our tests, with illustrative results.

A. Base-Point Modeling

As previously mentioned, the base-point accuracy of a dc model is important—large errors at the model's base point will rarely diminish much when the operating point changes.

Loss Dispersal Models (Sections VI-A and VII-A): The most common explicit dc models are those that distribute net losses in some semi-arbitrary manner among the network buses (or ignore losses completely). By their nature, these models are not accurate in large power systems.

Fig. 6 shows the dc MW errors for a cold-start model with $h = -b$ (generally a bit better than $h = 1/x$ or $\pm|y|$). In this specific case, net losses, underestimated by 10% relative to the exact ac reference solution, were dispersed to the load buses. In Fig. 6, for the bar corresponding to a given test system and kV range, the gray portion shows the maximum MW errors. The black portion depicts the average of such errors.

These results are little different from those of the corresponding dispersed-loss hot-start model, constructed with accurate net losses, etc. They are not much worse when the losses are estimated as zero. The average errors in Fig. 6 seem reasonable, but the extreme ones in several of the test systems are certainly not. The large errors are not just isolated cases—they are found throughout the network at the different voltage levels. This is illustrated in Fig. 7, which plots the full set of monitored flow errors in the results of Fig. 6 from System 1.

On systems with long-distance generator-to-load flows, the dispersal of losses to the generator buses worked slightly better than dispersal to load buses, but not by much. Loss dispersal to all buses likewise had minor effects.

Localized-Loss Hot-Start Models (Sections VI-B and C): Compared with the dispersed-loss models of Section VI-A, the localized-loss versions of Section VI-B produced huge improvements (e.g., halved errors). In the latter, incorporating the initial voltages from (3) into the dc admittance h often halved the MW flow errors again. A small further improvement was to define h as the tangent to the initial curve per (11). The α definitions in Section VI-B (a) performed amazingly better than those in Section VI-B (b).

This is useful dc modeling insight, but from a hot-start perspective, it seems always preferable to use the matched versions of Section VI-C, which fit the ac base point MW-flow and angle solution exactly. Depending on the implementation, this is also true of the incremental models of Section VIII.

Cold-Start Model Enhancements (Sections VII-B and C): We found that the loss-redistributing steps *1)–4)* of Section VII-B improved the cold-start dc model significantly. A single iteration was sufficient. Disappointingly, System 1 and 5 each had one persistent large MW-flow error, attributed to flow-path r/x-ratio differences.

Extremely similar results were obtained using the "flat-voltage ac" technique of Section VII-C, including the identical large anomalous errors. The associated dc model average and worst errors are shown in Fig. 8, which is directly comparable with Fig. 6. (*Note:* in a fixed-voltage ac formulation, flows on certain branches with impractically high r/x ratios are infeasible. Temporarily reducing these ratios to 1 overcame any convergence problems.)

Omitting step 2) in Section VII-B lets the losses "find their own level" and sometimes gives small further improvements. But in other cases a big positive feedback effect takes place, and with iteration the errors increase enormously on several high-loss branches. Extra logic might take care of this.

Sources of Large Errors: In all results, considerable efforts were made to pinpoint the sources of the largest dc MW flow errors. By far the most common situation is when heavy flows divide themselves between local paths whose r/x ratios differ widely. Branches with very large transmission angles can be another problem area. Significantly, we also often saw large errors on branches with no "bad" characteristics, indicating propagation from the poor linear modeling of other branch flows.

B. Post-Contingency Accuracies

Contingencies Simulated: On each test system we outaged every branch whose base-point loading exceeded 70%. In the ac solutions, post-contingency controls on taps, phase shifts and switched shunts were blocked. Any non-convergent outage

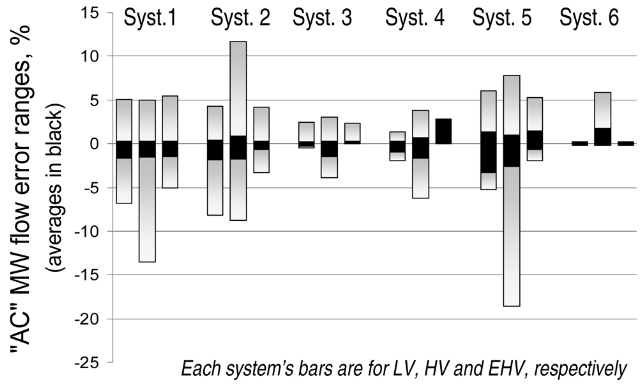

Fig. 8. Base-point flat-voltage ac start.

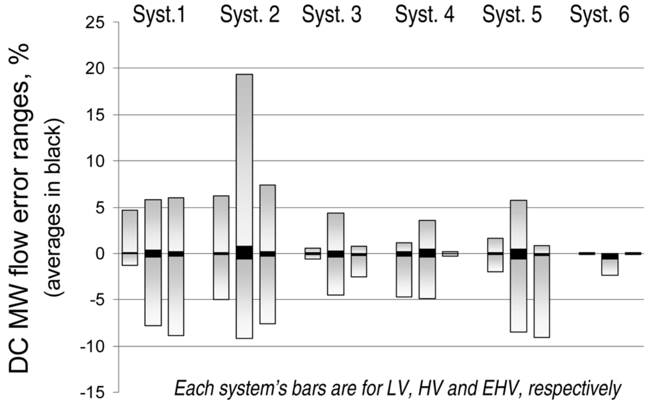

Fig. 9. Hot-start contingency error ranges.

case was necessarily ignored. Likewise, those few outages producing implausible ac voltage drops or overloads were ignored, on the basis that they would most likely be accompanied by supplementary control actions. When in the dc solutions the losses of outaged branches were also outaged, they had to be distributed—otherwise they were occasionally large enough to distort the flows near the reference bus.

Hot-Start Contingency Modeling: Modeling errors that are contingency-specific can most easily be seen by starting from a hot-start dc model that perfectly matches the ac flows. The following results are based on the α-matching version of Section VI-C. The h-matched version gave very similar results. Similar or identical results are obtained from the incremental versions of Section VIII.

Fig. 9 gives results using the definition for h in (11), which generally performs best. However, its state-dependent h seems less suitable for LODF versions, whose factors are intended for multiple reuse. The gray area of each bar in the figure represents the worst errors over all contingency cases. The black portion of the bar delineates the average of the worst errors among all contingency cases. These averages are very small. They are dwarfed by error outliers that seem to be due to flow division between parallel paths with very different nonlinearities, principally different r/x ratios.

Another lesser cause of post-contingency errors was due to the dc model's fixed-loss modeling. Occasionally, local pre-contingency branch losses are high, and after the contingency they become very low, or vice versa.

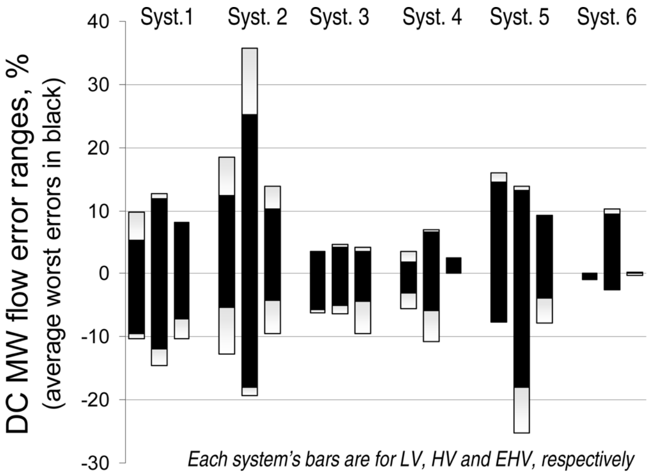

Fig. 10. Cold-start contingency error ranges.

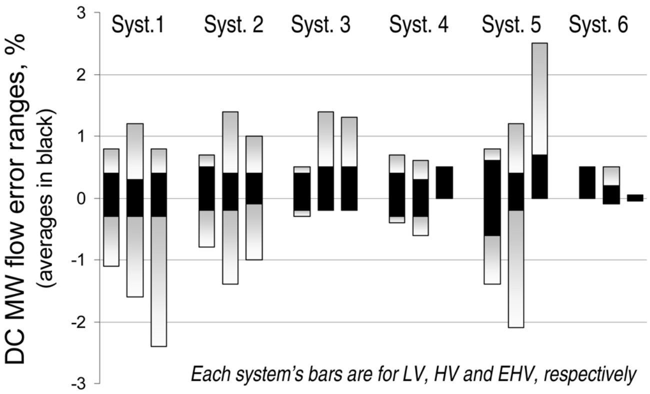

Fig. 11. Hot-start re-dispatch.

Cold-Start Contingency Modeling: With a standard cold-start model as per Section VII-A, the average-worst and globally-worst post-contingency errors tend to be very high. This is illustrated in Fig. 10, which very non-rigorously may be thought of as a combination of the results in Figs. 6 and 9. These errors can be partially mitigated by starting with a flat-voltage ac solution, for instance.

C. Post-Redispatch Accuracies

Our interest here was in observing the changes in accuracy of the base-point dc models after non-traumatic operating changes such as routine constrained re-dispatch during load following. There are unlimited alternatives for performing such simulations. In our case, we simply ran each system with its load level reduced by 5%, randomly re-dispatching each generator within a bound of 25% of its output.

Fig. 11 gives error results for the hot-start base-point model with α-matching and h as tangent per (11). It is seen that for such non-traumatic operating changes, the errors are relatively small. Obviously, different random re-dispatching produces slightly different numbers. Of the hot-start models, this version tends to be the most accurate one. Incremental model results are quite similar.

When the same load-reduction/re-dispatch exercise is conducted on a cold-start dc model, the model's base-point error

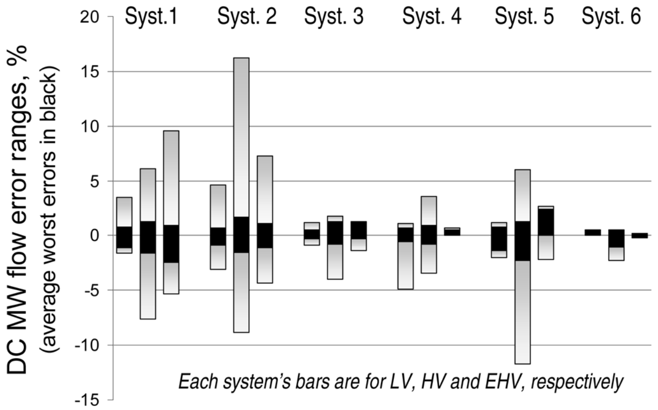

Fig. 12. Hot-start re-dispatch + contingency analysis.

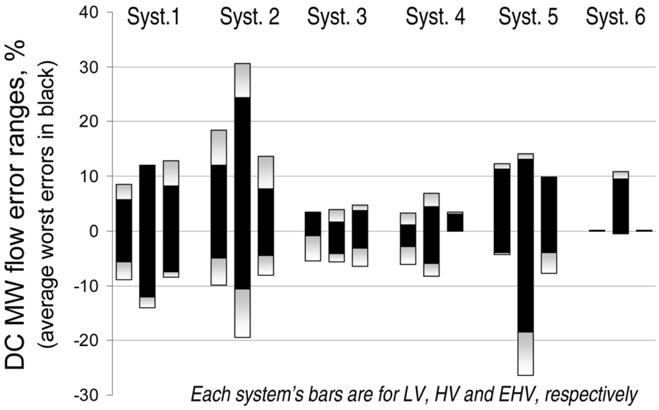

Fig. 13. Cold-start re-dispatch + contingency analysis.

results (not shown here) dominate, looking much like Fig. 6. However, they were fractionally smaller, attributable to reduced nonlinearity because of lower load.

The results for contingency analysis at the hot-start re-dispatched points of Fig. 11 are shown in Fig. 12.

The corresponding results for contingency analysis following re-dispatch of a cold-start model are shown in Fig. 13. This is very similar to Fig. 10, and once again illustrates the problem of constructing viable cold-start models.

D. Reliability

Ac power flow fails to converge in sufficiently severe contingency and re-dispatch cases. On the other hand, dc power flow inherently always solves. Such a dc property can clearly be very dangerous. However, it may actually be valuable in some network-constrained applications.

Consider security-constrained dispatch with an ac network model. Whenever the power flow solution fails (pre- or post-contingency) the relevant security limits cannot be monitored. And network constraints cannot easily then be imposed on the dispatch process in order to avoid operation in a region that is unstable algorithmically, analytically and/or in the real-life system. A dc network model does not have this problem—it always provides the dispatch process with meaningful (if not necessarily accurate) MW-related constraints.

Among converged ac solutions we encountered occasional situations where a bus voltage (generally at LV) was near collapse. Then the difference between the ac and dc flows can be considerable. In contingency analysis, this was deemed most likely to be a result of naïve ac modeling. Paradoxically, the dc results in these extreme situations sometimes seemed more realistic than the ac results. This is a very gray area, and it underscores the need for detailed analysis of dc modeling in any specific power system and network application.

Extremely occasionally, cases of system near-separation arise, and the dc-model angle across a branch becomes impractically large. These pathological cases are detectable.

XII. GENERAL COMMENTS

As shown here, a spectrum of dc modeling variants is possible, with accuracies that vary enormously over different networks, transmission elements, and loading levels. Inevitably, however, certain dc model versions are superior to others in given circumstances, and trends can be observed.

As previously noted, dc-model accuracy is of great interest because network constraints that erroneously become congestive, or fail to do so, might have huge impacts on market costs, as well as on system security monitoring. When assessing the accuracy of a dc power flow model against an ac model, it must be recognized that the latter has statistical and possibly gross errors. This does not affect the basic assessment.

Our test-based observations indicate that critical (potentially binding) dc MW flows are on average wrong by plus or minus a few percent, with little overall bias towards over- or under-estimation. It is very clear from our results, however, that the worst errors on such critical flows can sometimes be alarmingly high. (We have carefully verified the correctness of our calculations with different totally independent software packages.) How often, and by how much, such error outliers will distort system dispatch and pricing is a subject for analysis outside the present paper's scope.

Clearly, there is a big accuracy gap between hot-start and cold-start models. In the former, matched and incremental dc models work best, particularly in moderate-change redispatch and contingency analysis. On the other hand, cold-start models are seen, even from the few results shown, to be vulnerable to huge inaccuracies on certain critically-loaded branches. This seems not to be widely recognized [28].

It seems important to identify the trouble spots in a network where dc-model error extremes can appear. It might then be possible to correct data anomalies, use better dc models or apply special mitigations. Sometimes it is expedient to omit lower voltages from the network model, or at least not to monitor offending but unimportant circuits. Best-practice ac power flow modeling at all voltage levels is imperative to minimize such trouble spots. We ourselves noted and fixed a number of evident data problems in various power flow models. Certain types of network reduction produce highly unrealistic equivalents, and should be avoided.

Obviously, the big enemy of dc modeling is nonlinear network MW behavior. An excellent illustration of the nonlinearity associated with network stress is the difference in accuracies between Test Systems 5 and 6, which represent the same system under heavy and light loading, respectively.

Compared with classic textbook dc power flow, which looks almost trivial to code, many dc versions require careful implementation. For instance, a branch's sending and receiving dc flows may be unequal, and balancing an island's MWs (including slack redistribution) is often needed.

Locally-controlled phase shifter, HVDC, and FACTs devices considerably complicate the use of dc models, because recursive solutions are needed to impose or back off their limits. The same applies to any other discontinuities, such as post-contingency MW redistribution to generator units within their limits. Adding to this the fact that contingency constraints are so numerous that they must be handled in outer calculation loops, there seems to be little scope for solving realistic dc-modeled problems at one pass with a general-purpose linearly-constrained solver.

XIII. CONCLUSION

The electric power industry has invested heavily in the use of dc-type power flow models for network security and pricing calculations. However, our results underscore that the accuracy of a particular dc model, or indeed of dc modeling in general, should never be taken for granted in any given power system and application.

Dc-model testing of the kind described here is not very complicated. When it reveals large MW flow errors on potentially binding network elements, there is a strong case for investigating the source of such errors and how they impact LMP prices and/or security as appropriate (this requires analytical software that supports both ac and dc network models).

As long as certain power system applications continue to rely on linear network models, dc-type modeling will remain of high interest. We hope that the present paper might stimulate further development, testing, and verification in this area.

APPENDIX
COMMENTS ON DC MODELING APPROXIMATIONS

The approximations used in deriving (2) to (5) provide certain insights but also some false impressions about the dc modeling process.

Firstly, neglecting the losses as per (2) on a branch implies a typical error of only a few percent. However, in a non-incremental dc mode, such errors are cumulative—they show up at the reference bus. For a non-small power system the MW flows on the branches in the bus's vicinity can be wrong by hundreds of percent. The dc model must then preserve the system generation-load-loss MW balance by representing losses as equivalent injections, either dispersed among the buses or local to each branch.

Secondly, at first sight it would seem that the approximations in (3)–(5) can sometimes introduce enormous errors into the dc model. To illustrate:

1) At 40° the difference in (3) between $\sin\theta$ and θ is 8.6%. (Angles as high as this may be found on very long lines.)
2) If $v_1 = v_2 = 1.2$ per-unit, the approximation producing (4) has a 44% error. (Typical NERC-MMWG models have voltages in the range 0.75 to 1.4 per-unit.)
3) When substituting (5) for (4), the percentage error in $-b$ is $100(r/x)^2$. Thus, for $r/x = 1/3, 1$ and 3, respectively, the errors are 11%, 100%, and 900%.

In practice, the errors in the dc flows tend to be *much* lower than would be predicted from 1)–3) above. The reason is that the dc model is essentially a direct-current dividing network, allowing MWs to flow under Ohms and Kirchhoff's laws according to the *relative* values of the dc branch admittances h. These admittances only need to be in approximately correct *proportion* to each other. Thus, if all voltages are 1.5 per-unit, the actual MW flow error associated with 2) above is zero. If the line is radial, no MW flow error apart from losses will arise as a result of 1) or 3). The same is true for any radial chain of branches (in the absence of ZIP loads).

By the same token, when dc MW flows divide themselves among network paths whose true ac models are substantially different in nonlinearity, large errors can result. Moreover, these errors can be unpredictably self-cancelling or cumulative. In the latter case, they can propagate round network loops and degrade the MW flow accuracies on branches whose individual dc models are excellent.

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