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Markov Chain MINLP Model for Reliability Optimization of System Design and Maintenance

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Abstract

The need for optimization tools for reliability design considering operational factors is motivated by the significance of availability of process systems and the lack of systematic and rigorous tools. In response to this need, this paper introduces a systematic approach to model the stochastic process of system failures and repairs as a continuous-time Markov chain, in which the impact of maintenance is incorporated in order to find the optimal selection of parallel units. An illustrative example is shown.

Keywords: Reliability, Availability, Maintenance, MINLP, Markov Chain

1. Introduction

Plant availability has been a critical consideration for chemical processes, as it represents the fraction of normal operating time, which directly impacts profitability. In practice, discrete-event simulation tools are used to examine the availability of a few selected designs of different redundancy levels under various maintenance policies (Sharda and Bury, 2008). However, the best plan selected through simulation is usually suboptimal because the list of candidates is not exhaustive. Thus, there is a strong motivation for systematic optimization tools for redundancy design considering operational factors.

Literature review of research in reliability engineering can be found in Ye et al. (2017) where a general mixed-integer framework is proposed to select standby units to maximize availability and minimize cost. However, for a more comprehensive optimization, it is also important to consider the impacts of operational factors such as maintenance. In particular, preventive maintenance (PM) is a major strategy to improve the availability of units (Ding and Kamaruddin, 2015). Pistikopoulos et al. (2001) and Goel et al. (2003) formulate an MILP model for the selection of units and production and maintenance planning for a fixed system configuration. Markov chain is a powerful mathematical tool being extensively used to capture the stochastic process of systems transitioning among different states. Bloch-Mercier (2002) models the deterioration process of a system as continuous-time Markov chain to optimize inspection intervals. Lin et al. (2012) model a simple utility system using Markov chain and carry out RAM (reliability, availability & maintainability) analysis iteratively to decide the optimal reliability design.

Given the aforementioned research gaps and knowledge basis, this work extends our recent mixed-integer framework Ye et al. (2017) by introducing a systematic approach to model the stochastic process of system failures and repairs as a continuous-time Markov chain, for which the impact of maintenance is incorporated in order to find the optimal selection of parallel units.

2. Problem statement

We define a general modelling framework for production systems with underlying serial structures for availability evaluation as shown in Figure 1. For each stage k, a set of potential parallel units J_k are available for selection at the design phase. The goal is to determine which one or several potential parallel units to install, as well as the length of inspection intervals t_k^i, in order to maximize the system availability (i.e. probability that the system performs without failures), while minimizing the total cost of the system.

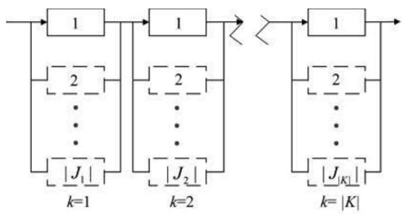

Figure 1 - A serial system

Each stage k has a set of potential units $j \in J_k$, for which the following is given:

- Availability parameters, i.e. Failure rate $\lambda_{k,j}$ and repair rate $\mu_{k,j}$.
- Operating priority within stage k (indicated by the order of j). A unit becomes active if and only if all the selected units with higher priorities have failed.
- Cost rates, including installation, inspection, maintenance and repair.

For each stage k, inspections are scheduled for active equipment at a certain time period to be determined, t_k^i, called inspection interval. If the inspection indicates that the equipment has a deterioration, a predictive maintenance task will be carried out in time. In that case, there will be enough time to order the spare parts and hence reduce the shipping costs, with which a maintenance task is cheaper than the repair upon failure. A deterioration can be detected by scheduled inspections in a certain period before it happens, called delay time (Christer, 1999), or PF-interval (Moubray, 1997) of length T_k^d.

Based on the parameters provided above, the availability of stage k will depend on the selection of parallel units $y_{k,j}$ and the inspection intervals of each processing stage t_k^i.

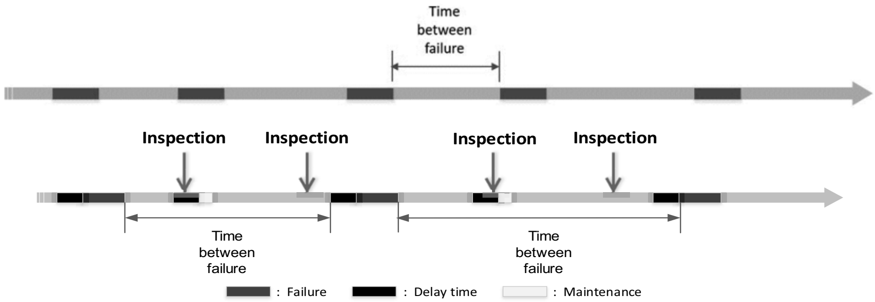

Figure 2 – Timeline of inspection/maintenance/repair

3. Modeling

3.1. Availability calculation

A continuous-time Markov chain is characterized by its transition rate matrix, the "**Q** Matrix", which can be used to solve for the stationary probability vector $\boldsymbol{\pi}$ through the linear equations, $\boldsymbol{\pi}^{\mathrm{T}}\mathbf{Q} = \mathbf{0}$ and $\boldsymbol{\pi}^{\mathrm{T}}\mathbf{1} = 1$. For the failure-repair system that we are considering, the availability can be obtained by adding up the probabilities of the non-failure states. Next, we show how to construct the "**Q** Matrix" of the system, and model the impact of the selection of units and inspection intervals on the "**Q** Matrix", and therefore on $\boldsymbol{\pi}$ and system availability.

First, for single stages, as we are making selections among the potential parallel units within stage *k*, there are several exclusive designs indexed by *h*, and each of them has a corresponding "**Q** Matrix" constructed as follows:

- A state space $T_{k,h}$ is enumerated where the transition from state *i* to state *j* ($i, j \in T_{k,h}$) is due to the happening of a failure or a repair. For example, the design decision of installing two units has the following 4 states:

Table 1 - State space example

Unit 1	Active	Active	Being repaired	Being repaired
Unit 2	Standby	Being repaired	Active	Being repaired

- Then the **Q** Matrix of design h is a 4×4 matrix where the element in row *i* and column *j* is equal to the failure rate or repair rate of the transition from state *i* to state *j*.
- And the element on the diagonal is equal to the opposite number of the sum of the other elements in the same row.

After the **Q** Matrices for the designs in stage *k* are formulated, they are put together to form a block-diagonal matrix $\mathbf{QM}_k$ called the "pseudo **Q** matrix" of stage *k,* which include all possible states for stage *k*. Following from that, the "pseudo **Q** matrix" of the system, $\mathbf{WM}_k$ is calculated using the following formula, where ⊗ is the Kronecker product and satisfies the associative law of addition.

$$WM = I_{n_{|K|}n_{|K|-1}\cdots n_2} \otimes QM_1 + I_{n_{|K|}n_{|K|-1}\cdots n_3} \otimes QM_2 \otimes I_{n_1} + I_{n_{|K|}n_{|K|-1}\cdots n_4} \otimes QM_3 \otimes I_{n_2 n_1} + \cdots$$
$$+ I_{n_{|K|}} \otimes QM_{|K|-1} \otimes I_{n_{|K|-2}n_{|K|-3}\cdots n_1} + QM_{|K|} \otimes I_{n_{|K|-1}n_{|K|-2}\cdots n_1}$$

The following constraints are used to manipulate the elements in $\mathbf{WM}_k$. The values of binary variables $y_{k,j}$ decide which columns and rows are eliminated from $\mathbf{WM}_k$ to find the actual $\mathbf{Q}$ matrix through equations (1)-(11). The variables t_k^i will affect the failure rates of single units, and therefore the values of the elements in $\mathbf{WM}_k$ ((12)-(18)).

$$instCost = \sum_{k\in K}\sum_{j\in J_k} y_{k,j} c\_inst_{k,j} \quad (1)$$

$$\sum_{j\in J_k} y_{k,j} \geq MIN_K, \qquad \forall k \in K \quad (2)$$

$$Z_{k,h} \Leftrightarrow \bigwedge_{\alpha_{j,k,h}=1} Y_{k,j} \bigwedge_{\alpha_{j,k,h}=0} \neg Y_{k,j}, \qquad \forall k \in K, h \in H_k \quad (3)$$

$$\bigvee_{h\in H_k} Z_{k,h}, \qquad \forall k \in K \quad (4)$$

$$ZZ_{k,s} \Leftrightarrow Z_{k,h}, \qquad \forall k \in K \quad (5)$$

$$\bar{Z}_{\bar{h}} \Leftrightarrow \bigwedge_{k\in K, h\in H_k, |h|=hc_{k,\bar{h}}} Z_{k,h}, \qquad \forall \bar{h} \in \bar{H} \quad (6)$$

$$\overline{ZZ}_{\bar{s}} \Leftrightarrow \bar{Z}_{\bar{h}}, \qquad \forall \bar{h} \in \bar{H}, \bar{s} \in \bar{T}_{\bar{h}} \quad (7)$$

$$\begin{bmatrix} \overline{ZZ}_{\bar{s}} \\ 0 \leq \pi_{\bar{s}} \leq 1 \end{bmatrix} \vee \begin{bmatrix} \neg\overline{ZZ}_{\bar{s}} \\ \pi_{\bar{s}} = 0 \end{bmatrix}, \qquad \forall \bar{s} \in \bar{S} \quad (8)$$

$$\begin{bmatrix} \overline{ZZ}_{\bar{r}} \\ \sum_{\bar{s}} \pi_{\bar{s}} WM(\bar{s},\bar{r}) = 0 \end{bmatrix} \vee \begin{bmatrix} \neg\overline{ZZ}_{\bar{r}} \\ \sum_{\bar{s}} \pi_{\bar{s}} WM(\bar{s},\bar{r}) < \infty \end{bmatrix}, \qquad \forall \bar{r} \in \bar{R} \quad (9)$$

$$\sum_{\bar{s}} \pi_{\bar{s}} = 1 \quad (10)$$

$$A = 1 - \sum_{\bar{s}\in\bar{S}^f} \pi_{\bar{s}} \quad (11)$$

$$\lambda_{k,j}^0 - \lambda_{k,j} = \frac{e^{-\lambda_{k,j}^0 t_k^i} - e^{-\lambda_{k,j}^0 \left(t_k^i + T_k^d\right)}}{t_k^i}, \qquad \forall k \in K, j \in J_k \quad (12)$$

$$\sum_{l\in L} x_{k,l} = 1, \qquad \forall k \in K \quad (13)$$

$$\sum_{l\in L} x_{k,l} T_l^i = t_k^i, \qquad \forall k \in K \quad (14)$$

$$inspCost \geq \sum_{k\in K} \frac{T}{t_k^i} c\_insp_k \quad (15)$$

$$repaCost \geq T \sum_{\bar{s}\in\bar{S}^f} -WM(\bar{s},\bar{s})\pi_{\bar{s}} \sum_{k\in K_{\bar{s}}^f} c\_repa_k \quad (15)$$

$$prevTimes_k \geq \frac{y_{k,j}\left(\lambda_{k,j}^0 - \lambda_{k,j}\right)}{\lambda_{k,j}}, \qquad \forall j \in J_k \quad (16)$$

$$prevCost \geq T \sum_{\bar{s}\in\bar{S}^f} -WM(\bar{s},\bar{s})\pi_{\bar{s}} \sum_{k\in K_{\bar{s}}^f} prevTimes_k c\_prev_k \quad (17)$$

$$prevTime \geq T \sum_{\bar{s}\in\bar{S}^f} -WM(\bar{s},\bar{s})\pi_{\bar{s}} \sum_{k\in K_{\bar{s}}^f} prevTimes_k T\_prev_k \quad (18)$$

$$A^{net} = A - \frac{prevTime}{T} \quad (19)$$

3.2. Economic dependence

The net profit is the sum of the revenue, penalty and bonus minus the sum of costs. The total revenue is considered proportional to the availability of the system. Generally, in the contract between the plant and the customer, two reference bounds are set for the availability of the plant. If the actual availability of the plant does not meet the lower bound, the plant will be charged proportional to the difference. Conversely, if the actual availability exceeds the upper bound, the customer reward the plant with a bonus that is also proportional to the difference.

$$\max NP = RV - PN + BN - instCost - repaCost - inspCost - prevCost \quad (20)$$

$$RV = rvA^{net} \quad (21)$$

$$W_1 \veebar W_2 \veebar W_3 \quad (22)$$

$$\begin{bmatrix} W_1 \\ A^{net} \leq A\_lo \\ PN = (A\_lo - A^{net})pn \\ BN = 0 \end{bmatrix} \vee \begin{bmatrix} W_2 \\ A\_lo \leq A^{net} \leq A\_up \\ PN = 0 \\ BN = 0 \end{bmatrix} \vee \begin{bmatrix} W_1 \\ A^{net} \geq A\_up \\ PN = 0 \\ BN = (A^{net} - A\_up)bn \end{bmatrix} \quad (23)$$

4. Example

In this section, a small example is shown featuring the system in Figure 3 with 2 serial stages, where stage 1 has 3 potential units and stage 2 has 2 potential units. Both of the two stages require at least 1 unit to function properly. Table 2 displays the parameters for the case study. A 10-year time horizon is considered.

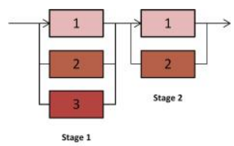

Figure 3 - Two stage-two unit system

Table 2 - Parameters

Stage	**Unit**	MTBF (day)	MTTR (day)	Fixed cost (k$)	Repair cost rate (k$ per time)	Inspection cost rate (k$ per time)	Maintenance cost rate (k$ per time)	Maintenance time (day)	Delay time (day)
1	1	50	7	200	12	0.1	0.6	1	10
	2	45.5	7.7	160					
	3	41.7	8.3	120					
2	1	66.7	2.6	240	10	0.1	0.5	1	12
	2	50	2.8	200					

Revenue rate (k$)	Penalty rate (k$)	Bonus rate (k$)	Availability lower bound	Availability upper bound
700	1000	10000	0.988	0.998

The MINLP model without maintenance is solved with BARON in 91.26 CPUs ((1)-(9) and (17) -(19)). There are 44921 equations and 44737 variables with 40 binary variables. The optimal design is to have all potential units installed. The expected system availability is 0.999, and the net profit is $6039.6 k, with a revenue of $6990.1 k and a bonus of $5.83 k. $920 k is spent on unit investment, and $36.3 k is spent on repair. The entire MINLP model with maintenance ((1)-(19)) is solved with BARON in 14924.15 CPUs, which has 44938 equations, 44458 variables with 49 binary variables. The optimal design is to install the last two units for stage 1 and both two units for stage 2. The expected system availability is 0.994. The expected net profit is $6131.6 k, with a revenue of $6959.9 k and no bonus or penalty. $720 k is spent on unit investment, $52.1 k on inspections, $2.7 k on maintenance, and $53.4 k on repair. Other results are shown in Table 3.

Table 3 - Optimization results

Stage	Inspection interval (day)	Equivalent MTBF (day)
1	14	Unit2: 98.0, Unit 3: 86.2
2	14	Unit1: 181.8, Unit 2: 117.6

From the above results we can see that when maintenance is considered, the model suggest additional costs on inspection and maintenance, while less is to spend on the unit investment, which leads to an overall higher net profit.

5. Conclusion

In this paper, we have proposed a general modeling framework to represent the failure-repair process of a multi-unit system as a continuous-time Markov Chain, and to incorporate the design decision of selecting one or more standby units, as well as the operation decision of determining inspection intervals. The resulting MINLP model was implemented and solved for an illustrative example.

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