Fuzzy-Model-Based Non-Fragile Control for Nonlinear Singularly Perturbed Systems with Semi-Markov Jump Parameters

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***Abstract*—This paper is concerned with the fuzzy-model-based non-fragile control problem for discrete-time nonlinear singularly perturbed systems with stochastic jumping parameters. The stochastic parameters are generated from the semi-Markov process. The memory property of the transition probabilities among subsystems is fully considered in the investigated systems. Consequently, the restriction that the transition probabilities are memoryless in widely used discrete-time Markov jump model can be removed. Based on the T-S fuzzy model approach and semi-Markov Kernel concept, several criteria ensuring δ-error mean square stability of the underlying closed-loop system are established. With the help of those criteria, the designed procedures which could well deal with the fragility problem in the implementation of the proposed fuzzy-model-based controller are presented. A technique is developed to estimate the permissible maximum value of singularly perturbed parameter for discrete-time nonlinear semi-Markov jump singularly perturbed systems. Finally, the validity of the established theoretical results is illustrated by a numerical example and a modified tunnel diode circuit model.**

***Index Terms*—Singularly perturbed systems, non-fragile fuzzy control, slow state variables feedback, semi-Markov jump systems.**

I. INTRODUCTION

SINGULARLY perturbed systems (SPSs) denote a class of dynamic systems displaying multiple-time scales (MTS) features. As a suitable modeling paradigm, SPSs are strongly capable of analyzing the dynamics behaviors of many physical systems with some *parasitic* parameters, such as transients in voltage regulators or machine reactances in power system models, time constants of actuators in industrial control systems, a small number of an enzyme in biochemical models, fast neutrons in nuclear reactor models [1]. It is readily explained why numerous research works have contributed to the study of linear and nonlinear SPSs. On the whole, most of the existing related results focus on the analysis and control of linear SPSs, an urgent question, therefore, is that how to extend such results to nonlinear SPSs. Realizing such a fact, some recent works have investigated the analysis and design of nonlinear SPSs by bond graph model approach [2], Euler's methodology [3], T-S fuzzy model approach [4]–[11] and so on.

As an important approach investigating the study of nonlinear SPSs, the fuzzy rule-based model approach has attracted increasing attention. The main reason lies much in its unique merits in solving nonlinear system identification and control problems. It has been recognized that such an approach could be successfully applied to represent many nonlinear dynamics [12]–[23]. In general, the underlying nonlinear singularly perturbed system is decomposed into many local linear SPSs in different regions. In this regard, by blending these local linear SPSs associated to the fixed nonlinear fuzzy membership functions, the approximating of the SPSs is accomplished, and the corresponding overall fuzzy model is presented. Along with this mechanism, many control issues of nonlinear SPSs have been addressed including, just to name a few, fuzzy $\mathcal{H}_\infty$ output feedback control [4], [24], static output feedback $\mathcal{H}_\infty$ controller design [5], robust sliding-mode control [6], multiobjective control [7], identification and trajectory tracking control [11] and filtering problem [25], [26].

As noted in [27]–[31], the abrupt changes of the parameter and structure can occur in many SPSs owing to the unavoidable phenomena. A typical example is power systems subject to component and interconnection failures. It is the main reason why many scholars focus on the SPSs with jumping parameters, where the jumping among different parameters in subsystems is assumed to be governed by a Markov process. In this context, for instance, the stability analysis for a class of stochastic hybrid nonlinear systems with singular perturbation was established in [32]; the stabilization problem for Markov jump SPSs was studied in [33], [34]; the stabilization bound by using the noise control method was addressed in [35]. It should be pointed out that [32], [33], [35] are very effective for linear Markov jump SPSs. By using T-S fuzzy model approach, [28] investigated the filtering problem of Markov jump nonlinear SPSs in continuous-time domains. However, they could not be

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easily extended to the discrete-time nonlinear Markov jump SPSs.

What's more, there are two shortcomings in some existing methods about Markov jump SPSs. On the one hand, they ignore the fragility problem according to an implicit assumption that the controller may be exactly implemented. Such an assumption, sometimes, is unavailable owing to the fact that uncertainties or inaccuracies may occur when a designed controller is implemented [36], [37]. On the other hand, it is widely acknowledged that the transition probabilities (TPs) play a significant role in determining the behavior and performance of Markov jump SPSs. The mentioned-above Markov jump SPSs are required to be subject to ideal, time-invariant and memoryless TPs. In this case, the sojourn time in Markov jump SPSs obey to geometric/exponential distribution in discrete-/continuous-time domains. In fact, such a memoryless restriction is not satisfied in many practical systems, one can refer to [38] for more details. In a more practical way, TPs should be considered to be time-varying with the *memory* property, and accordingly the necessity of semi-Markov jump SPSs with *memory* TPs is self-evident. Regrettably, inclusion of *memory* TPs in the study of nonlinear Markov jump SPSs has not been taken into account so far. As a consequence, there are still interesting questions requiring further investigation for the fuzzy-model-based control of discrete-time nonlinear SPSs as follows:

1) Is it possible to develop an analysis method to such a comprehensive system model, i.e., nonlinear discrete-time semi-Markov jump SPSs, where memory TPs are fully considered and the restriction that the sojourn time must obey to geometric distribution could be removed?

2) How can we design a fuzzy-model-based control strategy when only applying slow state variables and how to assess the effect of the singular perturbation parameter (SPP) upon the system performance?

3) How to cope with the fragility problem in the implementation of the proposed fuzzy-model-based controller and how to develop a technique to estimate the permissible maximum value of SPP for nonlinear discrete-time semi-Markov jump SPSs?

Although it has been recognized that seeking solutions to these problems is fairly necessary in the study of *nonlinear discrete-time semi-Markov jump SPSs*, no attempts have been made up to now. Therefore, this paper aims to shorten this gap, and the results are applied to the control of a modified tunnel diode circuit model [8].

In view of the above consideration, the fuzzy-model-based resilient control problem is addressed as the first attempt for discrete-time nonlinear semi-Markov jump SPSs in this work. The availability of the obtained results is finally illustrated by applying a numerical example and the control issue of a modified tunnel diode circuit model [8] as a practical example. The contributions of this paper are that: 1) Different from some previous works, a comprehensive system model, that is, nonlinear semi-Markov jump SPSs is investigated in this work. In particular, the *memory* property of the TPs is adequately taken into account. As such, the sojourn time is not requested to obey necessarily geometric distribution. 2) A fuzzy-model-based control strategy is established for nonlinear semi-Markov Jump SPSs only applying slow state variables by means of the T-S fuzzy model approach and semi-Markov Kernel (SMK) concept, and the fragility problem in the implementation of the proposed fuzzy-model-based controller is solved by designing a resilient controller. 3) The effect of the SPP upon the system performance is fully addressed, and a technique based on the convex optimization technique is developed to estimate the upper bound of SPP.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$: the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively; I_n and 0_n: $n\times n$-dimensional identity and zero matrix, respectively; $0_{n\times m}$: $n\times m$ zero matrix; $M>(<)0$: the matrix M is positive (negative) definite; M^T and M^{-1}: the transpose and inverse of the matrix M, respectively; $Sym\{M\}$: $M+M^T$; $(\Upsilon,\mathcal{F},\mathcal{P})$: a probability space where Υ is the sample space; $\mathcal{F}$ is the η_l-algebra of subsets of sample space; $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\begin{bmatrix} \coprod_1 & \coprod_2 \\ \star & \coprod_3 \end{bmatrix}$: $\begin{bmatrix} \coprod_1 & \coprod_2 \\ \coprod_2^T & \coprod_3 \end{bmatrix}$. If not explicitly stated, matrices under consideration are with compatible dimensions.

II. Problem Formulation

Consider a probability space $(\Upsilon,\mathcal{F},\mathcal{P})$, the nonlinear discrete-time slow sampling SPSs (Σ) with semi-Markov jump parameters can be described as follows:

Plant Rules p: **IF** $\xi_1(k)$ is ϑ_{p1}, and $\xi_2(k)$ is ϑ_{p1}, and ..., and $\xi_g(k)$ is ϑ_{pg}, **THEN**

$$\begin{cases} \zeta_1(k+1) = A_{11p}(\eta(k))\zeta_1(k) + \epsilon_r A_{12p}(\eta(k))\zeta_2(k) \\ \qquad\qquad + B_{1p}(\eta(k))u(k), \\ \zeta_2(k+1) = A_{21p}(\eta(k))\zeta_1(k) + \epsilon_r A_{22p}(\eta(k))\zeta_2(k) \\ \qquad\qquad + B_{2p}(\eta(k))u(k), \end{cases} \tag{1}$$

where $p\in\mathbb{W}_1\triangleq\{1,2,\ldots,w\}$ and w is the number of **IF-THEN** rules of system; $\zeta_1(k)\in\mathbb{R}^{n_s}$, $\zeta_2(k)\in\mathbb{R}^{n_f}$, $u(k)\in\mathbb{R}^{n_c}$ are, respectively, the slow state vector, the fast state vector and the control input; $\epsilon_r>0$ is the SPP; matrixes $A_{11p}(\eta(k))\in\mathbb{R}^{n_s\times n_s}$, $A_{12p}(\eta(k))\in\mathbb{R}^{n_s\times n_f}$, $A_{21p}(\eta(k))\in\mathbb{R}^{n_f\times n_s}$, $A_{22p}(\eta(k))\in\mathbb{R}^{n_f\times n_f}$, $B_{1p}(\eta(k))\in\mathbb{R}^{n_s\times n_c}$ and $B_{2p}(\eta(k))\in\mathbb{R}^{n_f\times n_c}$ are known real matrices, where $\{\eta(k)\}$, which denotes the mode of system at time k and takes values in a finite set $\mathbb{W}_2\triangleq\{1,2,\cdots,h\}$, is a semi-Markov chain and its evolution is generated by a SMK $\breve{\Pi}(\tau)=[\breve{\pi}_{ij}(\tau)]$, $i,j\in\mathbb{W}_2$ with

$$\begin{aligned} &\breve{\pi}_{ij}(\tau)\triangleq\Pr\left\{\Im\left(\bar{k}+1\right)=j, D_{\bar{k}+1}=\tau\middle|\Im\left(\bar{k}\right)=i\right\} \\ =\;& \frac{\Pr\left\{\Im\left(\bar{k}+1\right)=j,\Im\left(\bar{k}\right)=i\right\}}{\Pr\left\{\Im\left(\bar{k}\right)=i\right\}} \\ &\times\frac{\Pr\left\{\Im\left(\bar{k}+1\right)=j, D_{\bar{k}+1}=\tau,\Im\left(\bar{k}\right)=i\right\}}{\Pr\left\{\Im\left(\bar{k}+1\right)=j,\Im\left(\bar{k}\right)=i\right\}} \\ =\;& \vartheta_{ij}\phi_{ij}(\tau), \end{aligned} \tag{2}$$

where $\bar{k}$ stands for the time when system at $\bar{k}$-th jump; $\Im(\bar{k})$ denotes the system's mode at $\bar{k}$-th jump; $D_{\bar{k}+1}$ denotes the sojourn time between system at $\bar{k}$-th jump and the next jump; $\vartheta_{ij}\triangleq\Pr\left\{\Im\left(\bar{k}+1\right)=j\middle|\Im\left(\bar{k}\right)=i\right\}$ with $\vartheta_{ii}=0$

and $\phi_{ij}(\tau) \triangleq \Pr\{D_{\bar{k}+1} = \tau | \Im(\bar{k}+1) = j, \Im(\bar{k}) = i\}$. The cumulative density function of sojourn time for mode i is defined as $F(i,\tau) = \Pr\{D_{\bar{k}+1} \leqslant \tau | \eta(\bar{k}) = i\} = \sum_{r=1}^{\tau}\sum_{j\in\mathbb{W}_2}\breve{\pi}_{ij}(r)$, and without loss of generality, we suppose that $F(i,0) = \phi_{ij}(0) = 0$. To simplify the notation, for each $\eta(k) = i \in \mathbb{W}_2$, we denote $A_{11p,i}\triangleq A_{11p}(\eta(k))$, $A_{12p,i} \triangleq A_{12p}(\eta(k))$, $A_{21p,i} \triangleq A_{21p}(\eta(k))$, $A_{22p,i} \triangleq A_{22p}(\eta(k))$, $B_{1p,i} \triangleq B_{1p}(\eta(k))$ and $B_{2p,i} \triangleq B_{2p}(\eta(k))$.

Let $\zeta(k) = \begin{bmatrix} \zeta_1^T(k) & \zeta_2^T(k) \end{bmatrix}^T$, a compact presentation of the system (1) can be written as follows:

$$\zeta(k+1) = \sum_{p=1}^{w} h_p(\xi(k))\left[\mathbf{A}_{p,i}\mathbf{E}_\epsilon\zeta(k) + \mathbf{B}_{p,i}u(k)\right], \quad (3)$$

where

$$\mathbf{A}_{p,i} \triangleq \begin{bmatrix} A_{11p,i} & A_{12p,i} \\ A_{21p,i} & A_{22p,i} \end{bmatrix}, \mathbf{B}_{p,i} \triangleq \begin{bmatrix} B_{1p,i} \\ B_{2p,i} \end{bmatrix},$$
$$\mathbf{E}_\epsilon \triangleq diag\{I_{n_s}, \epsilon_r I_{n_f}\}.$$

Remark 1: It is worth pointing out that due to programming errors, fixed word length and round-off error numerical computation etc., there may exist inaccuracies or uncertainties during the controller implementation, which may result in the closed-loop system instability. Therefore, the designed controller should be insensitive to these inaccuracies or uncertainties. Taking this into account, in this paper, the following resilient slow state feedback controller is considered:

Control Rules p: **IF** $\xi_1(k)$ is ϑ_{p1}, and $\xi_2(k)$ is ϑ_{p1}, and $\ldots$, and $\xi_g(k)$ is ϑ_{pg}, **THEN**

$$\begin{aligned} u(k) &= [K_p(\eta(k)) + \Delta K_p(\eta(k))]\,\zeta_1(k) \\ &= (\mathcal{K}_{p,i} + \Delta\mathcal{K}_{p,i})\,\zeta(k), \end{aligned} \quad (4)$$

where $\mathcal{K}_{p,i} \triangleq \begin{bmatrix} \mathbf{K}_{p,i} & 0_{n_c\times n_f} \end{bmatrix}$; $\mathbf{K}_{p,i} \triangleq K_p(\eta(k)) \in \mathbb{R}^{n_c\times n_s}$ are the controller gains to be determined; $\Delta\mathcal{K}_{p,i} \triangleq \begin{bmatrix} \Delta\mathbf{K}_{p,i} & 0_{n_c\times n_f} \end{bmatrix}$; $\Delta\mathbf{K}_{p,i} \triangleq \Delta K_p(\eta(k)) \in \mathbb{R}^{n_c\times n_s}$ stand for gain variations of the controller which are assumed to be of the following form:

$$\Delta\mathbf{K}_{p,i} = M_{p,i}O_{p,i}(k)\,N_{p,i},$$

where $M_{p,i} \in \mathbb{R}^{n_c\times n_{o_1}}$, $N_{p,i} \in \mathbb{R}^{n_{o_2}\times n_s}$ are known real matrixes and $O_{p,i}(k) \in \mathbb{R}^{n_{o_1}\times n_{o_2}}$ is an unknown time-varying matrix function satisfying

$$O_{p,i}^T(k)\,O_{p,i}(k) \leqslant I.$$

Substituting (4) into (3) and denoting $x(k) \triangleq \begin{bmatrix} \zeta_1^T(k) & \epsilon\zeta_2^T(k) \end{bmatrix}^T$, one can get the closed-loop system

$$x(k+1) = \Xi_{pq,i}(k,h)\,x(k), \quad (5)$$

where

$$\Xi_{pq,i}(k,h) \triangleq \sum_{p=1}^{w}\sum_{q=1}^{w} h_p(\xi(k))\,h_q(\xi(k))\,\mathbb{A}_{pq,i},$$
$$\mathbb{A}_{pq,i} \triangleq \mathbf{E}_\epsilon\left[\mathbf{A}_{p,i} + \mathbf{B}_{p,i}(\mathcal{K}_{q,i} + \Delta\mathcal{K}_{q,i})\right].$$

To proceed further, we firstly introduce the following necessary definition and lemmas.

Definition 1: [38] Given a positive integer upper bound of sojourn time $T_{\max}^i \geqslant 1$, $\forall i \in \mathbb{W}_2$, the corresponding closed-loop system (5) is indicated to be δ-error mean square stable (δ-MSS) with

$$\delta \triangleq \sum_{i\in\mathbb{W}_2}\left|\ln\left(F\left(i, T_{\max}^i\right)\right)\right|,$$

if the condition in the following form holds for any initial state $x(0) \in \mathbb{R}^{n_s+n_f}$, $\forall i \in \mathbb{W}_2$:

$$\lim_{k\to\infty}\mathbb{E}\left\{\|x(k)\|^2\right\}\Big|_{x_0,\eta_0,D_{\bar{k}+1}\leqslant T_{\max}^i|_{\eta(\bar{k})=i}} = 0.$$

Lemma 1: [38] Consider a discrete-time stochastic switching system $x(k+1) = f(x(k),\eta(k))$, in which $\eta(k)$ and $x(k)$ denote the system mode index and state, respectively. Furthermore, the switching instants are represented by k_0, k_1, k_2, $\cdots$, k_s, $\cdots$, with $k_0 = 0$. The system is δ-MSS, if there exist three class $\mathcal{K}_\infty$ functions (ρ_1,ρ_2,ρ_3) and a group of $\mathcal{C}^1$ functions $V(x(k),\eta(k))$ such that for any initial condition $x_0 \in \mathbb{R}^{n_s+n_f}$, $\eta(k) \in \mathbb{W}_2$ and a fixed finite $\psi_i > 0$, $\forall\eta_s\triangleq\eta(k_s) = i$, $\eta_{s+1}\triangleq\eta(k_{s+1}) = j \in \mathbb{W}_2$, the following conditions are satisfied

$$\rho_1\|x(k)\| \leqslant V(x(k),i) \leqslant \rho_2\|x(k)\|, \quad (6)$$
$$V(x(k),i) \leqslant \psi_i V(x(k_s),i), k \in (k_s,k_{s+1}), \quad (7)$$
$$-\rho_3\|x(k)\| \geqslant \mathbb{E}\left\{V(x(k_{s+1}),j)|_{x_0,\eta_0,D_{s+1}\leqslant T_{\max}^i|\eta_s=i}\right\} - V(x(k_s),i). \quad (8)$$

Lemma 2: Given a finite constant $\psi_i > 0$ and a positive integer $T_{\max}^i$, then the slow state feedback controller (4) can guarantee that the system (5) is δ-MSS, if there exist matrices $G_{\alpha,i} \in \mathbb{R}^{(n_s+n_f)\times(n_s+n_f)} > 0$, $\forall\alpha \in \mathbb{W}_1$, $\forall i \in \mathbb{W}_2$ with $\mathcal{G}_{\alpha,i}(\tau) \triangleq \sum_{j\in\mathbb{W}_2}\pi_{ij}(\tau)\,G_{\alpha,j}/\sum_{\tau=1}^{T_{\max}^i}\sum_{j\in\mathbb{W}_2}\pi_{ij}(\tau)$ such that the following conditions hold for $\forall\alpha$, $\beta \in \mathbb{W}_1$, $\forall i \in \mathbb{W}_2$, $\forall t \in \{1,2,\cdots,T_{\max}^i - 1\}$, $\forall\tau \in \{1,2,\cdots,T_{\max}^i\}$

$$\Xi_i^T(k_s,h,t)\,G_{\alpha,i}\Xi_i(k_s,h,t) - \psi_i G_{\beta,i} < 0, \quad (9)$$
$$\sum_{\tau=1}^{T_{\max}^i}\Xi_i^T(k_s,h,\tau)\,\mathcal{G}_{\alpha,i}(\tau)\,\Xi_i(k_s,h,\tau) - G_{\beta,i} < 0, \quad (10)$$

with

$$\Xi_i(k_s,h,t) \triangleq \underbrace{\Xi_{pq,i}(k_s+t-1,h)\ldots\Xi_{yz,i}(k_s,h)}_{t},$$
$$\forall p,q,\ldots,y,z \in \mathbb{W}_1.$$

Proof: See Appendix A. ■

Lemma 3: Given a finite constant $\psi_i > 0$ and a positive integer $T_{\max}^i$, then the slow state feedback controller (4) can guarantee that the system (5) is δ-MSS, if there exist matrices $\Theta_{\alpha,i}(t,\varkappa) \in \mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, $\forall\alpha \in \mathbb{W}_1$, $\forall i \in \mathbb{W}_2$, $\forall t \in \{1,2,\cdots,T_{\max}^i - 1\}$, $\forall\varkappa \in \{0,1,\cdots,t-1\}$ with $\Theta_{\alpha,i} \triangleq \Theta_{\alpha,i}(t,t) > 0$, $\Phi_{\alpha,i}(\tau,\varrho) \in \mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, $\forall\alpha \in \mathbb{W}_1$, $\forall i \in \mathbb{W}_2$, $\forall\tau \in \{1,2,\cdots,T_{\max}^i\}$, $\forall\varrho \in \{0,1,\cdots,\tau-1\}$ with $\Phi_{\alpha,i}(\tau,\tau) \triangleq \sum_{j\in\mathbb{W}_2}\pi_{ij}(\tau)\,\Theta_{\alpha,j}/\sum_{\tau=1}^{T_{\max}^i}\sum_{j\in\mathbb{W}_2}\pi_{ij}(\tau)$ such that the following conditions hold for $\forall\alpha$, $\beta \in \mathbb{W}_1$, $\forall i \in \mathbb{W}_2$,

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$\forall t \in \{1,2,\cdots,T^i_{\max}-1\}$, $\forall \varkappa \in \{1,2,\cdots,t-1\}$, $\forall \tau \in \{1,2,\cdots,T^i_{\max}\}$, $\forall \varrho \in \{1,2,\cdots,\tau-1\}$

$$\begin{aligned} 0 &> \Xi^T_{pq,i}(k_s,h)\,\Theta_{\alpha,i}(t,\varkappa+1)\,\Xi_{pq,i}(k_s,h) \\ &\quad -\Theta_{\alpha,i}(t,\varkappa), \qquad (11)\\ 0 &> \Theta_{\alpha,i}(t,0)-\psi_i\Theta_{\beta,i}, \qquad (12)\\ 0 &> \Xi^T_{pq,i}(k_s,h)\,\Phi_{\alpha,i}(\tau,\varrho+1)\,\Xi^T_{pq,i}(k_s,h) \\ &\quad -\Phi_{\alpha,i}(\tau,\varrho), \qquad (13)\\ 0 &> \sum_{\tau=1}^{T^i_{\max}}\Phi_i(\tau,0)-\Theta_{\beta,i}. \qquad (14)\end{aligned}$$

Proof: From (11), one can derive that for all $\Xi_i(k_s,h,l)$, $l\in[0,t-1]$

$$\begin{aligned} 0 &> \Xi^T_i(k_s,h,l)\left[\Xi^T_{pq,i}(k_s,h)\,\Theta_{\alpha,i}(t,\varkappa+1)\right. \\ &\quad \left.\times\Xi_{pq,i}(k_s,h)-\Theta_{\alpha,i}(t,\varkappa)\right]\Xi_i(k_s,h,l), \quad (15)\end{aligned}$$

which implies that

$$\Xi^T_i(k_s,h,t)\,\Theta_{\alpha,i}(t,t)\,\Xi_i(k_s,h,t)-\Theta_{\alpha,i}(t,0)<0.$$

Then let $G_{\alpha,i}=\Theta_{\alpha,i}(t,t)\triangleq\Theta_{\alpha,i}$, $G_{\beta,i}=\Theta_{\beta,i}(t,t)\triangleq\Theta_{\beta,i}$ and combining (12), it is easy to find that (9) is guaranteed. Similarly, one can obtain (10) from (13)-(14). This completes the proof. ■

Lemma 4: [8] For given a scalar $\epsilon_M>0$ and matrices $\Lambda_l\,(l=1,2,3)$, if the following conditions hold simultaneously

$$\begin{aligned}\Lambda_1 &\geqslant 0,\\ \Lambda_3 &< 0,\\ \epsilon^2_M\Lambda_1+\epsilon_M\Lambda_2+\Lambda_3 &< 0,\end{aligned}$$

then the following condition holds for $\forall\epsilon_r\in[0,\epsilon_M]$

$$\epsilon^2_r\Lambda_1+\epsilon_r\Lambda_2+\Lambda_3<0.$$

Lemma 5: [37] Given matrices $\maltese_l\,(l=1,2,3)$ with appropriate dimensions, then

$$\maltese_1+\maltese_2 O(k)\maltese_3+\maltese^T_3 O^T(k)\maltese^T_2<0,$$

holds with $O(k)$ satisfying $O(k)O^T(k)\leqslant I$, if and only if there exists a positive scalar $\varepsilon>0$ such that

$$\maltese_1+\varepsilon^{-1}\maltese_2\maltese^T_2+\varepsilon\maltese^T_3\maltese_3<0.$$

III. Main Results

In this section, we will present the methods to evaluate the upper bound of SPP ϵ_M and obtain the controller gains.

Theorem 1: Given finite constants $\psi_i>0$, $\epsilon_M>0$, positive integer $T^i_{\max}$ and matrices $J_{1\alpha,i}$, $J_{2\alpha,i}$, $J_{3\alpha,i}$, if there exist constants ε_1, ε_2, matrices $F_i\in\mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, symmetric matrices $U_{\alpha i}(t,\varkappa)\in\mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, $\forall\alpha\in\mathbb{W}_1$, $\forall i\in\mathbb{W}_2$, $\forall t\in\{1,2,\cdots,T^i_{\max}-1\}$, $\forall\varkappa\in\{0,1,\cdots,t-1\}$ with $\mathcal{I}_3U_{\alpha i}(t,\varkappa)\mathcal{I}^T_3>0$, $U_{\alpha i}>0$ and $\mathcal{U}_{\alpha i}(\tau,\varrho)\in\mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, $\forall\alpha\in\mathbb{W}_1$, $\forall i\in\mathbb{W}_2$, $\forall\tau\in\{1,2,\cdots,T^i_{\max}\}$, $\forall\varrho\in\{0,1,\cdots,\tau-1\}$ with $\mathcal{I}_4\sum_{\tau=1}^{T^i_{\max}}\mathcal{U}_{\alpha i}(\tau,\varrho)\mathcal{I}^T_4>0$ such that the following inequalities hold for $\forall p$, q, α, $\beta\in\mathbb{W}_1$, $p<q$, $\forall i\in\mathbb{W}_2$, $\forall t\in\{1,2,\cdots,T^i_{\max}-1\}$, $\forall\varkappa\in\{0,1,\cdots,t-1\}$, $\forall\tau\in\{1,\cdots,T^i_{\max}\}$, $\forall\varrho\in\{0,1,\cdots,\tau-2\}$ and κ_1, $\kappa_2\in\{1,2\}$

$$\begin{bmatrix}\Psi^{\kappa_1,\kappa_2}_{1\alpha pp,i}(\chi_1,\chi_2) & \Psi_{2p,i}\\ \star & -\varepsilon_1 I\end{bmatrix}<0, \qquad (16)$$

$$\begin{bmatrix}\Psi^{\kappa_1,\kappa_2}_{3\alpha pq,i}(\chi_1,\chi_2) & \Psi_{4pq,i}\\ \star & \Psi_5\end{bmatrix}<0, \qquad (17)$$

$$\begin{bmatrix}\Omega^3_{\beta,i}(t) & I\\ I & \Omega^{4,\kappa_2}_{\alpha,i}(\chi_1)\end{bmatrix}<0, \qquad (18)$$

$$\begin{bmatrix}\Upsilon^l_{1\alpha pp,i} & \Upsilon_{2\alpha pp,i}\\ \star & \Upsilon_3\end{bmatrix}<0, \qquad (19)$$

$$\begin{bmatrix}\Upsilon^l_{1\alpha pq,i}+\Upsilon^l_{1\alpha qp,i} & \Upsilon_{4\alpha pq,i}\\ \star & \Upsilon_5\end{bmatrix}<0, \qquad (20)$$

where

$$\chi_1\triangleq(\kappa_2-1)\tau-(\kappa_2-2)t,\ \chi_2\triangleq(\kappa_2-1)\varrho-(\kappa_2-2)\varkappa,$$

$$\Psi^{\kappa_1,\kappa_2}_{1\alpha pp,i}(\chi_1,\chi_2)\triangleq\begin{bmatrix}\Omega^{\kappa_1,\kappa_2}_{1\alpha pp,i}(\chi_1,\chi_2) & \Omega_{2p,i}\\ \star & -\varepsilon_1 I\end{bmatrix},$$

$$\Psi_{2p,i}\triangleq\begin{bmatrix}\Psi_{21p,i}\\ 0\end{bmatrix},\ \Psi_{21p,i}\triangleq\begin{bmatrix}0\\ \varepsilon_1\mathbf{B}_{p,i}M_{p,i}\end{bmatrix},$$

$$\Psi^{\kappa_1,\kappa_2}_{3\alpha pq,i}(\chi_1,\chi_2)\triangleq\begin{bmatrix}\Psi^{\kappa_1,\kappa_2}_{31\alpha pq,i}(\chi_1,\chi_2) & \Omega_{2q,i}\\ \star & -\varepsilon_2 I\end{bmatrix},$$

$$\Psi^{\kappa_1,\kappa_2}_{31\alpha pq,i}(\chi_1,\chi_2)\triangleq\Omega^{\kappa_1,\kappa_2}_{1\alpha pq,i}(\chi_1,\chi_2)+\Omega^{\kappa_1,\kappa_2}_{1\alpha qp,i}(\chi_1,\chi_2),$$

$$\Psi_{4pq,i}\triangleq\begin{bmatrix}0 & F^T_i\mathcal{I}^T_3N_{p,i} & 0\\ \varepsilon_2\mathbf{B}_{p,i}M_{q,i} & 0 & \varepsilon_1\mathbf{B}_{q,i}M_{p,i}\\ 0 & 0 & 0\end{bmatrix},$$

$$\Psi_5\triangleq\operatorname{diag}\{-\varepsilon_2 I,-\varepsilon_1 I,-\varepsilon_1 I\},\Omega_{2p,i}\triangleq\begin{bmatrix}N_{p,i}\mathcal{I}_3F_i & 0\end{bmatrix}^T,$$

$$\Omega^{1,1}_{1\alpha pq,i}(t,\varkappa)\triangleq\begin{bmatrix}\Gamma^{1,1}_{\alpha,i}(t,\varkappa) & \Gamma^{1,2}_{pq,i}\\ \star & \Gamma^{1,3}_{\alpha,i}(t,\varkappa)\end{bmatrix},$$

$$\Omega^{1,2}_{1\alpha pq,i}(\tau,\varrho)\triangleq\begin{bmatrix}\Gamma^{2,1}_{\alpha,i}(\tau,\varrho) & \Gamma^{1,2}_{pq,i}\\ \star & \Gamma^{2,3}_{\alpha,i}(\tau,\varrho)\end{bmatrix},$$

$$\Omega^{2,1}_{1\alpha pq,i}(t,\varkappa)\triangleq\begin{bmatrix}\Gamma^{3,1}_{\alpha,i}(t,\varkappa) & \Gamma^{1,2}_{pq,i}\\ \star & \Gamma^{1,3}_{\alpha,i}(t,\varkappa)\end{bmatrix},$$

$$\Omega^{2,2}_{1\alpha pq,i}(\tau,\varrho)\triangleq\begin{bmatrix}\Gamma^{4,1}_{\alpha,i}(\tau,\varrho) & \Gamma^{1,2}_{pq,i}\\ \star & \Gamma^{2,3}_{\alpha,i}(\tau,\varrho)\end{bmatrix},$$

$$\Omega^3_{\beta,i}(t)\triangleq J_{3\beta,i}U_{\beta,i}J^T_{3\beta,i}-Sym\{J_{3\beta,i}\},$$

$$\Omega^{4,1}_{\alpha,i}(t)\triangleq-\psi_iU_{\alpha,i}(t,0),\ \Omega^{4,2}_{\alpha,i}(\tau)\triangleq-\sum_{\tau=1}^{T^i_{\max}}\mathcal{U}_{\alpha,i}(\tau,0),$$

$$\Gamma^{1,1}_{\alpha,i}(t,\varkappa)\triangleq J_{1\alpha,i}\mathcal{I}_1U_{\alpha,i}(t,\varkappa)\mathcal{I}_1J^T_{1\alpha,i}-Sym\{F^T_iJ^T_{1\alpha,i}\},$$

$$\Gamma^{2,1}_{\alpha,i}(\tau,\varrho)\triangleq J_{2\alpha,i}\mathcal{I}_1\mathcal{U}_{\alpha,i}(\tau,\varrho)\mathcal{I}_1J^T_{2\alpha,i}-Sym\{F^T_iJ^T_{2\alpha,i}\},$$

$$\Gamma^{1,2}_{pq,i}\triangleq F^T_i\mathbf{A}^T_{p,i}+F^T_i\mathcal{K}^T_{q,i}\mathbf{B}^T_{p,i},$$

$$\Gamma^{1,3}_{\alpha,i}(t,\varkappa)\triangleq-U_{\alpha,i}(t,\varkappa+1),\Gamma^{2,3}_{\alpha,i}(\tau,\varrho)\triangleq-\mathcal{U}_{\alpha,i}(\tau,\varrho+1),$$

$$\begin{aligned}\Gamma^{3,1}_{\alpha,i}(t,\varkappa)&\triangleq\Gamma^{1,1}_{\alpha,i}+Sym\{\epsilon_MJ_{1\alpha,i}\mathcal{I}_1U_{\alpha,i}(t,\varkappa)\mathcal{I}_2J^T_{1\alpha,i}\}\\ &\quad+\epsilon^2_MJ_{1\alpha,i}\mathcal{I}_2U_{\alpha,i}(t,\varkappa)\mathcal{I}_2J^T_{1\alpha,i},\end{aligned}$$

$$\Gamma^{4,1}_{\alpha,i}(\tau,\varrho) \triangleq \Gamma^{2,1}_{m,i} + Sym\left\{\epsilon_M J_{2\alpha,i}\mathcal{I}_1\mathcal{U}_{\alpha,i}(\tau,\varrho)\mathcal{I}_2 J^T_{2\alpha,i}\right\} + \epsilon^2_M J_{2\alpha,i}\mathcal{I}_2\mathcal{U}_{\alpha,i}(\tau,\varrho)\mathcal{I}_2 J^T_{2\alpha,i},$$

$$\Upsilon^l_{1\alpha pq,i} \triangleq \begin{bmatrix} \Upsilon^l_{\alpha,i} & \Gamma^{1,2}_{pq,i}\tilde{\mathcal{U}}_i(\tau) \\ \star & \tilde{U} \end{bmatrix},$$

$$\Upsilon_{2\alpha pq,i} \triangleq \begin{bmatrix} F^T_i\mathcal{I}_3 N^T_{q,i} & 0 \\ 0 & \varepsilon_1\left(\tilde{\mathcal{U}}_i(\tau)\right)^T\mathbf{B}_{p,i}M_{q,i} \end{bmatrix},$$

$$\Upsilon_3 \triangleq \mathrm{diag}\{-\varepsilon_1,-\varepsilon_1\},\ \Upsilon_{4\alpha pq,i} \triangleq \begin{bmatrix} \Upsilon^1_{4\alpha pq,i} & \Upsilon^2_{4\alpha pq,i} \end{bmatrix},$$

$$\Upsilon_5 \triangleq \mathrm{diag}\{-\varepsilon_2,-\varepsilon_2,\varepsilon_1,-\varepsilon_1\},$$

$$\Upsilon^1_{\alpha,i} \triangleq J_{3\alpha,i}\mathcal{I}_1 U_{\alpha,i}\mathcal{I}_1 J_{3\alpha,i} - Sym\left\{F^T_i J^T_{3\alpha,i}\right\},$$

$$\Upsilon^2_{\alpha,i} \triangleq \Upsilon^1_{\alpha,i} + Sym\left\{\epsilon_M J_{3\alpha,i}\mathcal{I}_1 U_{\alpha,i}\mathcal{I}_2 J^T_{3\alpha,i}\right\}, + \epsilon^2_M J_{3\alpha,i}\mathcal{I}_2 U_{\alpha,i}\mathcal{I}_2 J^T_{3\alpha,i},$$

$$\Upsilon^1_{4\alpha pq,i} \triangleq \begin{bmatrix} F^T_i\mathcal{I}_3 N^T_{q,i} & 0 \\ 0 & \varepsilon_2\left(\tilde{\mathcal{U}}_i(\tau)\right)^T\mathbf{B}_{p,i}M_{q,i} \end{bmatrix},$$

$$\Upsilon^2_{4\alpha pq,i} \triangleq \begin{bmatrix} F^T_i\mathcal{I}_3 N^T_{p,i} & 0 \\ 0 & \varepsilon_1\left(\tilde{\mathcal{U}}_i(\tau)\right)^T\mathbf{B}_{q,i}M_{p,i} \end{bmatrix},$$

$$\mathcal{U}_{0i} \triangleq \sum_{\tau=1}^{T^i_{\max}}\sum_{j\in\mathbb{W}_2}\pi_{ij}(\tau),\ \tilde{U} \triangleq \mathrm{diag}\{-U_{\alpha 1},-U\alpha_2,\cdots,-U_{\alpha h}\},$$

$$\tilde{\mathcal{U}}_i(\tau) \triangleq \begin{bmatrix} \sqrt{\frac{\pi_{i1}(\tau)}{\mathcal{U}_{0i}}}I & \sqrt{\frac{\pi_{i2}(\tau)}{\mathcal{U}_{0i}}}I & \cdots & \sqrt{\frac{\pi_{ih}(\tau)}{\mathcal{U}_{0i}}}I \end{bmatrix},$$

$$\mathcal{I}_1 \triangleq \mathrm{diag}\left\{I_{n_s},0_{n_f}\right\},\ \mathcal{I}_2 \triangleq \mathrm{diag}\left\{0_{n_s},I_{n_f}\right\},$$

$$\mathcal{I}_3 \triangleq \begin{bmatrix} I_{n_s} & 0_{n_s\times n_f}\end{bmatrix},\ \mathcal{I}_4 \triangleq \begin{bmatrix} 0_{n_f\times n_s} & I_{n_f}\end{bmatrix}.$$

Then for $\forall\epsilon_r\in[0,\epsilon_M]$, the closed-loop system (5) is σ-MSS.

Proof: See Appendix B. ■

Based on Theorem 1, we give a method to obtain the controller gains. For presentation convenience, we let

$$S \triangleq \begin{bmatrix} S_1 & S_2 \\ S_3 & S_4 \end{bmatrix},\ \bar{n} \triangleq n_s - n_f, \tag{21}$$

$$S_1 \triangleq I_{\min\{n_s,n_f\}},\ S_2 \triangleq 0_{\min\{n_s,n_f\}\times\max\{0,-\bar{n}\}},$$
$$S_3 \triangleq 0_{\max\{0,\bar{n}\}\times\min\{n_s,n_f\}},\ S_4 \triangleq 0_{\max\{0,\bar{n}\}\times\max\{0,-\bar{n}\}}.$$

Theorem 2: Given finite constants $\psi_i > 0$, c_1, c_2, positive integer $T^i_{\max}$ and matrices $J_{1\alpha,i}$, $J_{2\alpha,i}$, $J_{3\alpha,i}$, S, if there exist constants ε_1, ε_2, matrices $\tilde{K}_{p,i}\in\mathbb{R}^{n_u\times n_s}$, $F_i \triangleq \begin{bmatrix} c_1F_{11i} & c_2F_{11i}S \\ F_{21i} & F_{22i}\end{bmatrix}$ with $F_{11i}\in\mathbb{R}^{n_s\times n_s}$, $F_{21i}\in\mathbb{R}^{n_f\times n_s}$, $F_{22i}\in\mathbb{R}^{n_f\times n_f}$, symmetric matrices $U_{\alpha i}(t,\varkappa)\in\mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, $\forall\alpha\in\mathbb{W}_1$, $\forall i\in\mathbb{W}_2$, $\forall t\in\left\{1,2,\cdots,T^i_{\max}-1\right\}$, $\forall\varkappa\in\{0,1,\cdots,t-1\}$ with $\mathcal{I}_3U_{\alpha i}(t,\varkappa)\mathcal{I}^T_3 > 0$, $U_{\alpha i}(t,t) > 0$ and $\mathcal{U}_{\alpha i}(\tau,\varrho)\in\mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, $\forall\alpha\in\mathbb{W}_1$, $\forall i\in\mathbb{W}_2$, $\forall\tau\in\left\{1,2,\cdots,T^i_{\max}\right\}$, $\forall\varrho\in\{0,1,2,\cdots,\tau-1\}$ with $\mathcal{I}_4\sum_{\tau=1}^{T^i_{\max}}\mathcal{U}_{\alpha i}(\tau,\varrho)\mathcal{I}^T_4 > 0$ such that (18) and the following inequalities hold for $\forall p$, q, α, $\beta\in\mathbb{W}_1$, $p<q$, $\forall i\in\mathbb{W}_2$, $\forall t\in\left\{1,2,\cdots,T^i_{\max}-1\right\}$, $\forall\varkappa\in\{0,1,\cdots,t-1\}$, $\forall\tau\in\left\{1,\cdots,T^i_{\max}\right\}$, $\forall\varrho\in\{0,1,\cdots,\tau-2\}$, and $\kappa_2\in\{1,2\}$

$$\begin{bmatrix} \bar{\Psi}^{1,\kappa_2}_{1\alpha pq,i}(\chi_1,\chi_2) & \Psi_{2p,i} \\ \star & -\varepsilon_1 I\end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} \bar{\Psi}^{1,\kappa_2}_{3\alpha pq,i}(\chi_1,\chi_2) & \Psi_{4pq,i} \\ \star & \Psi_5\end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} \bar{\Upsilon}^1_{1\alpha pp,i} & \Upsilon_{2\alpha pp,i} \\ \star & \Upsilon_3\end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} \bar{\Upsilon}^l_{1\alpha pq,i} + \bar{\Upsilon}^l_{1\alpha qp,i} & \Upsilon_{4\alpha pq,i} \\ \star & \Upsilon_5\end{bmatrix} < 0, \tag{25}$$

where

$$\bar{\Psi}^{1,\kappa_2}_{1\alpha pq,i}(\chi_1,\chi_2) \triangleq \begin{bmatrix} \bar{\Omega}^{1,\kappa_2}_{1\alpha pq,i}(\chi_1,\chi_2) & \Omega_{2p,i} \\ \star & -\varepsilon_1 I\end{bmatrix},$$

$$\bar{\Psi}^{1,\kappa_2}_{3\alpha pq,i}(\chi_1,\chi_2) \triangleq \begin{bmatrix} \bar{\Psi}^{1,\kappa_2}_{31\alpha pq,i}(\chi_1,\chi_2) & \Omega_{2q,i} \\ \star & -\varepsilon_2 I\end{bmatrix},$$

$$\bar{\Psi}^{1,\kappa_2}_{31\alpha pq,i}(\chi_1,\chi_2) \triangleq \bar{\Omega}^{1,\kappa_2}_{1\alpha pq,i}(\chi_1,\chi_2) + \bar{\Omega}^{1,\kappa_2}_{1\alpha qp,i}(\chi_1,\chi_2),$$

$$\bar{\Omega}^{1,1}_{1\alpha pq,i}(t,\varkappa) \triangleq \begin{bmatrix} \Gamma^{1,1}_{\alpha,i}(t,\varkappa) & \bar{\Gamma}^{1,2}_{pq,i} \\ \star & \Gamma^{1,3}_{\alpha,i}(t,\varkappa)\end{bmatrix},$$

$$\bar{\Omega}^{1,2}_{1\alpha pq,i}(\tau,\varrho) \triangleq \begin{bmatrix} \Gamma^{2,1}_{\alpha,i}(\tau,\varrho) & \bar{\Gamma}^{1,2}_{pq,i} \\ \star & \Gamma^{2,3}_{\alpha,i}(\tau,\varrho)\end{bmatrix},$$

$$\bar{\Upsilon}^1_{1\alpha pq,i} \triangleq \begin{bmatrix} \Upsilon^1_{\alpha,i} & \bar{\Gamma}^{1,2}_{pq,i}\tilde{\mathcal{U}}_i(\tau) \\ \star & \tilde{U}\end{bmatrix},$$

$$\bar{\Gamma}^{1,2}_{pq,i} \triangleq F^T_i\mathbf{A}^T_{p,i} + c_1\mathcal{I}^T_3\tilde{K}^T_{q,i}\mathbf{B}^T_{p,i} + c_2\mathcal{I}^T_4 S^T\tilde{K}^T_{q,i}\mathbf{B}^T_{p,i},$$

and the other parameters are defined in Theorem 1. Then there exists a sufficient small constant $\epsilon_M > 0$ such that for $\forall\epsilon_r\in[0,\epsilon_M]$, the system (5) is σ-MSS with

$$K_{q,i} = \tilde{K}_{q,i}F^{-1}_{11i}.$$

Proof: Let $F_i \triangleq \begin{bmatrix} c_1F_{11i} & c_2F_{11i}S \\ F_{21i} & F_{22i}\end{bmatrix}$, $S \triangleq \begin{bmatrix} S_1 & S_2 \\ S_3 & S_4\end{bmatrix}$ and $\tilde{K}_{q,i} = K_{q,i}F_{11i}$, then, from Theorem 1, we can easy to derive that Theorem 2 is true. So the proof is omitted here. ■

Remark 2: Although Theorem 2 provides an approach to acquire the controller, it is independent of ϵ_M, which implies that the obtained controller by using Theorem 2 can not be directly applied since that it can not guarantee that the practical SPP ϵ_r meets $\epsilon_r\in(0,\epsilon_{M\max}]$, where $\epsilon_{M\max}$ is the permissible maximum value of SPP under the obtained controller by using Theorem 2. Once the ϵ_M is very small, the obtained controller by using Theorem 2 cannot guarantee the corresponding closed-loop system (5) is δ-MSS. Therefore, on the one hand, we need give a solution to obtain available controller based on Theorem 2, which is shown in Fig. 1. On the other hand, an alternative design method with less conservativeness should be proposed. Thus, we give the following theorem.

Theorem 3: Given finite constants $\psi_i > 0$, $\epsilon_M > 0$, c_1, c_2, positive integer $T^i_{\max}$ and matrices $J_{1\alpha,i}$, $J_{2\alpha,i}$, $J_{3\alpha,i}$, S, if there exist constants ε_1, ε_2, matrices $\tilde{K}_{p,i}\in\mathbb{R}^{n_u\times n_s}$, $F_i = \begin{bmatrix} c_1F_{11i} & c_2F_{11i}S \\ F_{21i} & F_{22i}\end{bmatrix}$ with

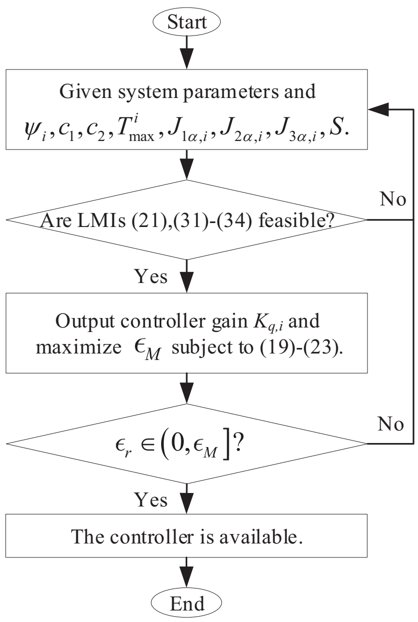

Fig. 1. Steps to obtain the available controller by using Theorem 2.

$F_{11i} \in \mathbb{R}^{n_s \times n_s}$, $F_{21i} \in \mathbb{R}^{n_f \times n_s}$, $F_{22i} \in \mathbb{R}^{n_f \times n_f}$, symmetric matrices $U_{\alpha i}(t, \varkappa) \in \mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, $\forall \alpha \in \mathbb{W}_1$, $\forall i \in \mathbb{W}_2$, $\forall t \in \{1, 2, \cdots, T^i_{\max} - 1\}$, $\forall \varkappa \in \{0, 1, \cdots, t-1\}$ with $\mathcal{I}_3 U_{\alpha i}(t, \varkappa) \mathcal{I}_3^T > 0$, $U_{\alpha i}(t, t) > 0$ and $\mathcal{U}_{\alpha i}(\tau, \varrho) \in \mathbb{R}^{(n_s+n_f)\times(n_s+n_f)}$, $\forall \alpha \in \mathbb{W}_1$, $\forall i \in \mathbb{W}_2$, $\forall \tau \in \{1, 2, \cdots, T^i_{\max}\}$, $\forall \varrho \in \{0, 1, 2, \cdots, \tau - 1\}$ with $\mathcal{I}_4 \sum_{\tau=1}^{T^i_{\max}} \mathcal{U}_{\alpha i}(\tau, \varrho) \mathcal{I}_4^T > 0$ such that (18) and the following inequalities hold for $\forall p$, q, α, $\beta \in \mathbb{W}_1$, $p < q$, $\forall i \in \mathbb{W}_2$, $\forall t \in \{1, 2, \cdots, T^i_{\max} - 1\}$, $\forall \varkappa \in \{0, 1, \cdots, t-1\}$, $\forall \tau \in \{1, \cdots, T^i_{\max}\}$, $\forall \varrho \in \{0, 1, \cdots, \tau - 2\}$ and κ_1, $\kappa_2 \in \{1, 2\}$

$$\begin{bmatrix} \bar{\Psi}^{\kappa_1,\kappa_2}_{1\alpha pq,i}(\chi_1, \chi_2) & \Psi_{2p,i} \\ \star & -\varepsilon_1 I \end{bmatrix} < 0, \tag{26}$$

$$\begin{bmatrix} \bar{\Psi}^{\kappa_1,\kappa_2}_{3\alpha pq,i}(\chi_1, \chi_2) & \Psi_{4pq,i} \\ \star & \Psi_5 \end{bmatrix} < 0, \tag{27}$$

$$\begin{bmatrix} \bar{\Upsilon}^l_{1\alpha pp,i} & \Upsilon_{2\alpha pp,i} \\ \star & \Upsilon_3 \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} \bar{\Upsilon}^l_{1\alpha pq,i} + \bar{\Upsilon}^l_{1\alpha qp,i} & \Upsilon_{4\alpha pq,i} \\ \star & \Upsilon_5 \end{bmatrix} < 0, \tag{29}$$

where

$$\bar{\Psi}^{\kappa_1,\kappa_2}_{1\alpha pq,i}(\chi_1, \chi_2) \triangleq \begin{bmatrix} \bar{\Omega}^{\kappa_1,\kappa_2}_{1\alpha pq,i}(\chi_1, \chi_2) & \Omega_{2p,i} \\ \star & -\varepsilon_1 I \end{bmatrix},$$

$$\bar{\Psi}^{\kappa_1,\kappa_2}_{3\alpha pq,i}(\chi_1, \chi_2) \triangleq \begin{bmatrix} \bar{\Psi}^{\kappa_1,\kappa_2}_{31\alpha pq,i}(\chi_1, \chi_2) & \Omega_{2q,i} \\ \star & -\varepsilon_2 I \end{bmatrix},$$

$$\bar{\Psi}^{\kappa_1,\kappa_2}_{31\alpha pq,i}(\chi_1, \chi_2) \triangleq \bar{\Omega}^{\kappa_1,\kappa_2}_{1\alpha pq,i}(\chi_1, \chi_2) + \bar{\Omega}^{\kappa_1,\kappa_2}_{1\alpha qp,i}(\chi_1, \chi_2),$$

$$\bar{\Omega}^{2,1}_{1\alpha pq,i}(t, \varkappa) \triangleq \begin{bmatrix} \Gamma^{3,1}_{\alpha,i}(t, \varkappa) & \bar{\Gamma}^{1,2}_{pq,i} \\ \star & \Gamma^{1,3}_{\alpha,i}(t, \varkappa) \end{bmatrix},$$

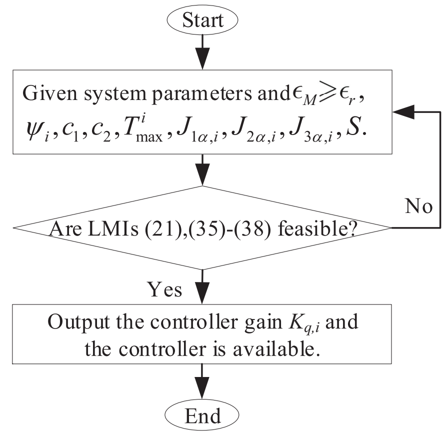

Fig. 2. Steps to obtain the available controller by using Theorem 3.

$$\bar{\Omega}^{2,2}_{1\alpha pq,i}(\tau, \varrho) \triangleq \begin{bmatrix} \Gamma^{4,1}_{\alpha,i}(\tau, \varrho) & \bar{\Gamma}^{1,2}_{pq,i} \\ \star & \Gamma^{2,3}_{\alpha,i}(\tau, \varrho) \end{bmatrix},$$

$$\bar{\Upsilon}^l_{1\alpha pq,i} \triangleq \begin{bmatrix} \Upsilon^l_{\alpha,i} & \bar{\Gamma}^{1,2}_{pq,i}\tilde{\mathcal{U}}_i(\tau) \\ \star & \tilde{U} \end{bmatrix},$$

and the other parameters have same expressions as those in Theorem 1 and Theorem 2. Then for $\forall \epsilon_r \in [0, \epsilon_M]$, the closed-loop system $(\check{\Sigma})$ with

$$K_{q,i} = \tilde{K}_{q,i} F^{-1}_{11i}$$

is σ-MSS.

Proof: The proof can be also easily obtained from Theorem 1 and omitted here. ■

Theorem 3 presents an alternative approach to obtain the controller and the detailed steps are shown in Fig. 2. Moreover, the permissible maximum value of SPP ϵ_M can be derived by the Algorithm 1.

Remark 3: From Fig. 1 and Fig. 2, it is readily observed that the controller design method presented in Theorem 3 is more convenient than the controller design method provided in Theorem 2. Additionally, compared with the controller design method provided in Theorem 2, the design method presented in Theorem 3 can be used to obtain the larger permissible maximum value of SPP, which will be demonstrated in Example 1 in Section IV.

Remark 4: To obtain the slow state feedback controller gains, [8] introduced extra slack variables $S^{il} = \begin{bmatrix} S_{11} & 0 \\ S^{il}_{21} & S^{il}_{22} \end{bmatrix}$. However, it should be noted that the element of S^{il} at the top right is chosen as zero matrix which leads to certain conservativeness. In Theorem 2 and Theorem 3, an improved matrix decoupling approach is proposed by introducing extra slack variables as $F_i \triangleq \begin{bmatrix} c_1 F_{11i} & c_2 F_{11i} S \\ F_{21i} & F_{22i} \end{bmatrix}$ where c_1, c_2 are constants and the expression of S is presented in (41). The key differences between the matrix decoupling approach in this work and [8] are: (I) the element of the introduced extra slack variables at the top right in this work is no longer zero matrix; (II) two adjustable constants (i.e. c_1 and c_2) are introduced. The existence of the two adjustable constants is

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Algorithm 1: Calculate $\epsilon_{M_{max}}$ based on Theorem 3

Input: ψ_i, c_1, c_2, $T^i_{\max}$, $J_{1\alpha,i}$, $J_{2\alpha,i}$, $J_{3\alpha,i}$, S, $\Delta\epsilon$ (computational accuracy), $\epsilon_{start} = 0$ and ϵ_{end} (large enough to make the LMI conditions (18), (26), (27) inconsistent)

Output: $\epsilon_{M_{max}}$

```
1  if ε_end − ε_start > Δε then
2      ε_M = (ε_end − ε_start)*0.5;
3      if (18), (26), (27) hold with ε_M then
4          ε_start = ε_M;
5      else
6          ε_end = ε_M;
7      end
8      Go back to 1;
9  else
10     if (18), (26), (27) hold with ε_end then
11         ε_Mmax = ε_end;
12     else
13         ε_Mmax = ε_start;
14     end
15 end
16 return ε_Mmax;
```

beneficial to get the gains of the controller and optimize the system performance, which will be verified in Example 1 in Section IV.

IV. Numerical Examples

In this section, two examples are used to illustrate the superiority of the proposed method. In Example 1, we first verify that the proposed controller design method is effective. Then, we prove the viewpoint proposed in Remark 3, i.e., compared with the controller design method provided in Theorem 2, the controller design method presented in Theorem 3 can be used to obtain the larger permissible maximum value of SPP. Next, we also discuss the relationship between gain variations of the controller, c_1, c_2 and the maximum permissible SPP in Example 1 and obtain some conclusions. In Example 2, a tunnel diode circuit modified from [8] is used to show the practicability of our proposed method.

Example 1: Consider the nonlinear semi-Markov jump discrete-time slow sampling SPSs (Σ) with two modes, two fuzzy rules and the following parameters:

$$
\begin{aligned}
A_{1,1} &= \begin{bmatrix} 0.8 & -1.6 \\ 1.7 & 0.18 \end{bmatrix}, A_{1,2} = \begin{bmatrix} 0.5 & 1.3 \\ 0.5 & 2.1 \end{bmatrix}, \\
A_{2,1} &= \begin{bmatrix} 1.2 & -0.7 \\ 1.5 & -1.2 \end{bmatrix}, A_{2,2} = \begin{bmatrix} 1.3 & -0.4 \\ 3.5 & 0.8 \end{bmatrix}, \\
B_{1,1} &= \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}^T, B_{1,2} = \begin{bmatrix} 0.1 & 1.0 \end{bmatrix}^T, \\
B_{2,1} &= \begin{bmatrix} 0.3 & 0.2 \end{bmatrix}^T, B_{2,2} = \begin{bmatrix} 0.1 & 1.0 \end{bmatrix}^T, \\
M_{1,1} &= M_{1,2} = M_{2,1} = M_{2,2} = \bar{\rho}, \\
N_{1,1} &= N_{1,2} = N_{2,1} = N_{2,2} = 0.33,
\end{aligned}
$$

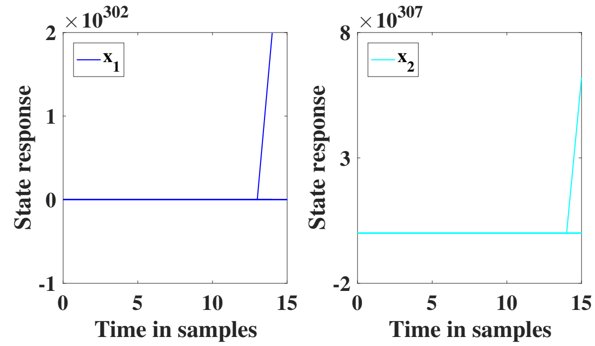

Fig. 3. State responses of open-loop system (100 realizations).

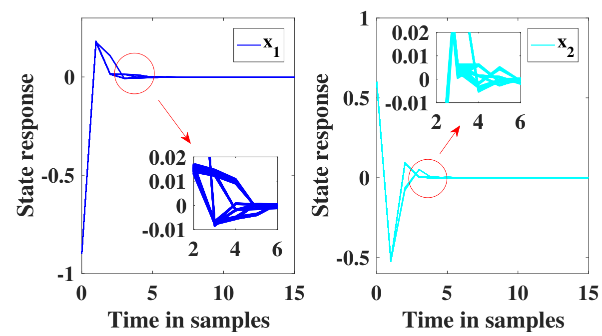

Fig. 4. State responses of closed-loop system (100 realizations).

where the SPP $\epsilon_r = 0.05$. Let $\vartheta_{12} = \vartheta_{21} = 1$, and $\phi_{11}(\tau) = \phi_{22}(\tau) = 0$, $\phi_{12}(\tau) = \frac{0.6^{\tau}\cdot 0.4^{10-\tau}\cdot 10!}{(10-\tau)!\cdot\tau!}$, $\phi_{21}(\tau) = 0.4^{\tau-1}\cdot 0.4^{\tau^{1.3}}$, one can calculate the SMK by (2).

Firstly, let $\bar{\rho} = 0.12$, $T^1_{\max} = 4$, $T^2_{\max} = 2$, and $x(0) = \begin{bmatrix} -0.8 & 0.6 \end{bmatrix}^T$, Fig. 3 shows the 100 realizations of the open-loop system's states responses. From Fig. 3, it is clear that the open-loop system is not σ-MSS. Fig. 4 shows the 100 realizations of the states response of the closed-loop system under the controller gains:

$$
\begin{aligned}
K_{1,1} &= -6.5670,\ K_{1,2} = -0.7619, \\
K_{1,2} &= -4.7476,\ K_{2,2} = -3.5552,
\end{aligned}
$$

which is obtained from Theorem 3 with $\psi_1 = \psi_2 = 2$, $\epsilon_M = 0.3$, $T^1_{\max} = 4$, $T^2_{\max} = 2$, $c_1 = 1$, $c_2 = 0.1$, $J_{1\alpha,i} = J_{2\alpha,i} = J_{3\alpha,i} = diag\{1,1\}$, $\forall\alpha \in \{1,2\}$, $\forall i \in \{1,2\}$. From Fig. 4, we can observe that the designed controller is effective.

Secondly, we verify that the controller design method presented in Theorem 3 can be used to obtain the larger permissible maximum value of SPP than the design approach presented in Theorem 2. Applying Theorem 2 and Theorem 3 with $\psi_1 = \psi_2 = 2$, $T^1_{\max} = 4$, $T^2_{\max} = 2$, $c_1 = 1$, $c_2 = 0.1$, $J_{1\alpha,i} = J_{2\alpha,i} = J_{3\alpha,i} = diag\{1,1\}$, $\forall\alpha \in \{1,2\}$, $\forall i \in \{1,2\}$, respectively, the corresponding controller gains and maximum permissible SPP $\epsilon_{M\max}$ for different $\bar{\rho}$ can be calculated. The corresponding computed results of Theorem 2 and Theorem 3 are listed in Table I and Table II, respectively.

From the Table I and II, one can readily observe that the maximum permissible SPP $\epsilon_{M\max}$ obtained by Theorem 3 is larger than that given by Theorem 2, which means that

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TABLE I
CONTROLLER GAINS AND MAXIMUM PERMISSIBLE SPP $\epsilon_{M\max}$ BY USING THEOREM 2 FOR DIFFERENT $\bar{\rho}$

	Controller gains $(K_{1,1}; K_{1,2}; K_{2,1}; K_{2,2})$	$\epsilon_{M\max}$
$\bar{\rho}=0$	$(-9.0486; -0.8288; -5.4564; -3.6291)$	0.3573
$\bar{\rho}=0.5$	$(-8.7163; -0.5840; -5.4305; -3.6032)$	0.3091
$\bar{\rho}=1.0$	$(-8.5082; -0.6104; -5.2123; -3.6248)$	0.2473
$\bar{\rho}=1.5$	$(-8.0176; -0.7569; -4.8553; -3.8132)$	0.1676

TABLE II
CONTROLLER GAINS AND MAXIMUM PERMISSIBLE SPP $\epsilon_{M\max}$ BY USING THEOREM 3 FOR DIFFERENT $\bar{\rho}$

	Controller gains $(K_{1,1}; K_{1,2}; K_{2,1}; K_{2,2})$	$\epsilon_{M\max}$
$\bar{\rho}=0$	$(-8.7565; -0.4791; -6.1949; -3.5559)$	0.3894
$\bar{\rho}=0.5$	$(-9.4413; -0.5868; -5.7666; -3.6372)$	0.3160
$\bar{\rho}=1.0$	$(-9.6489; -0.6711; -5.3700; -3.7124)$	0.2562
$\bar{\rho}=1.5$	$(-8.7052; -0.8141; -4.8498; -3.8857)$	0.1712

TABLE III
MAXIMUM PERMISSIBLE SPP $\bar{\epsilon}_{\max}$ FOR DIFFERENT $\bar{\rho}$ AND c_1, c_2

$\epsilon_{M\max}$	$\bar{\rho}=0$	$\bar{\rho}=0.5$	$\bar{\rho}=1.0$	$\bar{\rho}=1.5$
$c_1=1, c_2=0$	0.4529	0.3723	0.2915	0.2271
$c_1=0.8, c_2=-0.1$	0.3894	0.3160	0.2562	0.1712

the controller design method presented in Theorem 3 has less conservativeness than Theorem 2.

Next, we investigate the relationship between gain variations of the controller, c_1, c_2 and the maximum permissible SPP. Let $\psi_1=\psi_2=2$, $T^1_{\max}=4$, $T^2_{\max}=2$, $J_{1\alpha,i}=J_{2\alpha,i}=J_{3\alpha,i}=diag\{1,1\}, \forall\alpha\in\{1,2\}, \forall i\in\{1,2\}$. Then applying Theorem 3, we can obtain Table III.

From Table III, we can see that with $\bar{\rho}$ increasing, i.e., the gain variations are more fierce, the obtained permissible maximum value of SPP is decreasing, which means that the gain variations have a bad influence on the permissible maximum value of SPP. It indirectly shows the importance of taking the gain variations of the controller into account when designing the controller. Besides, one can also observe that the values of c_1, c_2 play an important role to optimize the maximum permissible SPP $\epsilon_{M\max}$, which is indirectly verified that compared with the matrix decoupling approach established in [8], an improved one is proposed in this work.

Example 2: In this example, a tunnel diode circuit modified from [8], which is shown in Fig. 5, is employed to illustrate the effectiveness of the proposed method. $i_D(t)$ and $V_D(t)$ are the diode current and diode voltage, respectively, and they satisfy that

$$i_D(t) = -0.2V_D(t) - 0.05V_D^3(t).$$

C and ε_L are the capacitor and inductance, respectively; R_1, R_2 and R_3 are resistances; $V_C(t)$ and $i_C(t)$ are the capacitor voltage and capacitor current, respectively; $V_L(t)$ and $i_L(t)$ are the inductance voltage and inductance current, respectively; $u(t)$ is the input voltage; the switch $S(t)$, in this paper, follows a semi-Markov chain. Let $x_1(t)=V_C(t)$ and $x_2(t)=i_L(t)$, then we can get that

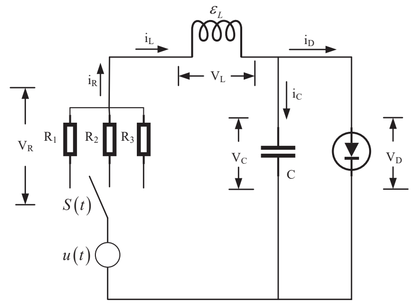

Fig. 5. Tunnel diode circuit.

$$\begin{cases} C\dot{x}_1(t) = 0.2x_1(t) + 0.05x_1^3(t) + x_2(t), \\ \varepsilon_L\dot{x}_2(t) = -x_1(t) - R_ix_2(t) + u(t), i=1,2,3. \end{cases} \quad (30)$$

Choosing $C=0.1F$, $\varepsilon_L=10^{-3}H$, $R_1=90\Omega$, $R_2=120\Omega$ and $R_3=150\Omega$, (30) can be rewritten as

$$\begin{cases} \dot{x}_1(t) = \frac{0.2}{C}x_1(t) + \frac{0.05}{C}x_1^3(t) + \frac{1}{C}x_2(t), \\ \varepsilon_r\dot{x}_2(t) = -0.1x_1(t) - 0.1R_ix_2(t) + 0.1u(t), \end{cases} \quad (31)$$

where $i=1,2,3$, $\varepsilon_r=10^{-4}$ is the SPP. Then, let $x(t)=\begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ and as noted in [8], assuming $|x(t)|\leqslant 3$, we can obtain the following T-S fuzzy model of (31):

$$\bar{E}_\epsilon\dot{x}(t) = \sum_{p=1}^{2} h_p(t)\left[A_{p,i}x(t) + B_{p,i}u(t)\right], \quad (32)$$

where

$$A_{1,1} = \begin{bmatrix} \frac{0.2}{C} & \frac{1}{C} \\ -0.1 & -0.1R_1 \end{bmatrix}, A_{2,1} = \begin{bmatrix} \frac{0.69}{C} & \frac{1}{C} \\ -0.1 & -0.1R_1 \end{bmatrix},$$

$$A_{1,2} = \begin{bmatrix} \frac{0.2}{C} & \frac{1}{C} \\ -0.1 & -0.1R_2 \end{bmatrix}, A_{2,2} = \begin{bmatrix} \frac{0.69}{C} & \frac{1}{C} \\ -0.1 & -0.1R_2 \end{bmatrix},$$

$$A_{1,3} = \begin{bmatrix} \frac{0.2}{C} & \frac{1}{C} \\ -0.1 & -0.1R_3 \end{bmatrix}, A_{2,3} = \begin{bmatrix} \frac{0.69}{C} & \frac{1}{C} \\ -0.1 & -0.1R_3 \end{bmatrix},$$

$$B_{1,1} = B_{1,2} = B_{1,3} = B_{2,1} = B_{2,2} = B_{2,3} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^T,$$
$$\bar{E}_\epsilon = diag\{1, \varepsilon_r\}, h_1(t) = 1 - \frac{x_1^2(t)}{9}, h_2(t) = 1 - h_1(t).$$

Then, choosing a sampling time $T_s=0.2s$, we can get the discrete time model of (32) as follows:

$$x(k+1) = \sum_{p=1}^{2} h_p(k)\left[\bar{A}_{p,i}\bar{E}_\epsilon x(k) + \bar{B}_{p,i}u(k)\right],$$

where

$$\bar{A}_{1,1} = \begin{bmatrix} 1.4590 & 1.6211 \\ -0.0162 & -0.0180 \end{bmatrix}, \bar{B}_{1,1} = \begin{bmatrix} 0.1528 \\ 0.0565 \end{bmatrix},$$

$$\bar{A}_{1,2} = \begin{bmatrix} 1.4672 & 1.2226 \\ -0.0122 & -0.0102 \end{bmatrix}, \bar{B}_{1,2} = \begin{bmatrix} 0.1166 \\ 0.0442 \end{bmatrix},$$

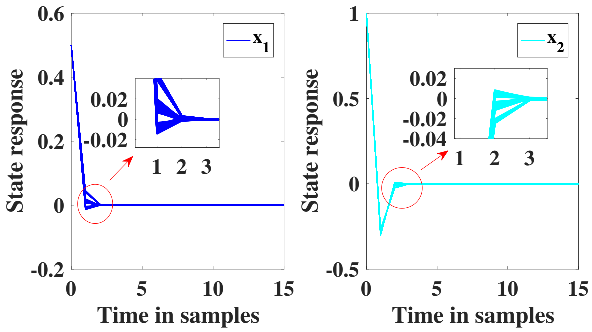

Fig. 6. State responses of closed-loop system in Example 2 (100 realizations).

$$\bar{A}_{1,3} = \begin{bmatrix} 1.4721 & 0.9814 \\ -0.0098 & -0.0065 \end{bmatrix}, \bar{B}_{1,3} = \begin{bmatrix} 0.0943 \\ 0.0362 \end{bmatrix},$$
$$\bar{A}_{2,1} = \begin{bmatrix} 3.8876 & 4.3192 \\ -0.0432 & -0.0480 \end{bmatrix}, \bar{B}_{2,1} = \begin{bmatrix} 0.2526 \\ 0.0498 \end{bmatrix},$$
$$\bar{A}_{2,2} = \begin{bmatrix} 3.9092 & 3.2575 \\ -0.0326 & -0.0271 \end{bmatrix}, \bar{B}_{2,2} = \begin{bmatrix} 0.1933 \\ 0.0403 \end{bmatrix},$$
$$\bar{A}_{2,3} = \begin{bmatrix} 3.9223 & 2.6147 \\ -0.0261 & -0.0174 \end{bmatrix}, \bar{B}_{2,3} = \begin{bmatrix} 0.1565 \\ 0.0337 \end{bmatrix}.$$

The SMK can be computed by (2) with the following parameters, which borrowed from [38]:

$$[\vartheta_{ij}] = \begin{bmatrix} 0 & 0.7 & 0.3 \\ 0.4 & 0 & 0.6 \\ 0.5 & 0.5 & 0 \end{bmatrix}, [\phi_{ij}(\tau)] = \begin{bmatrix} 0 & \Phi^{12} & \Phi^{13} \\ \Phi^{21} & 0 & \Phi^{23} \\ \Phi^{31} & \Phi^{32} & 0 \end{bmatrix},$$

where

$$\Phi^{12} = \frac{0.6^{\tau} \cdot 0.4^{10-\tau} \cdot 10!}{(10-\tau)! \cdot \tau!}, \quad \Phi^{13} = \frac{0.4^{\tau} \cdot 0.6^{10-\tau} \cdot 10!}{(10-\tau)! \cdot \tau!},$$
$$\Phi^{21} = 0.9^{(\tau-1)^2} - 0.9^{\tau^2}, \quad \Phi^{23} = \frac{5^{10} \cdot 10!}{(10-\tau)! \cdot \tau!},$$
$$\Phi^{31} = 0.4^{(\tau-1)^{1.3}} - 0.4^{\tau^{1.3}}, \quad \Phi^{32} = 0.3^{(\tau-1)^{0.8}} - 0.3^{\tau^{0.8}}.$$

The real matrixes $M_{p,i}$ and $N_{p,i}$ ($p \in \{1,2\}, i \in \{1,2,3\}$) are assumed as follows:

$$M_{p,i} = 0.12, N_{p,i} = 0.33.$$

Then applying Theorem 3 with $\psi_1 = 5.5$, $\psi_2 = 5.7$, $\psi_3 = 5.3$, $\epsilon_M = 0.1$, $T^1_{\max} = 4$, $T^2_{\max} = 3$, $T^3_{\max} = 2$, $c_1 = 1$, $c_2 = -0.1$, $J_{1\alpha,i} = J_{2\alpha,i} = J_{3\alpha,i} = diag\{1,1\}$, $\forall \alpha \in \{1,2\}$, $\forall i \in \{1,2,3\}$, the controller gains can be calculated as

$$K_{1,1} = -9.1659, \quad K_{1,2} = -12.5624, \quad K_{1,3} = -16.0073,$$
$$K_{2,1} = -13.0108, \quad K_{2,2} = -18.0432, \quad K_{2,3} = -23.9448.$$

Fig. 6 shows the 100 realizations of the states response of the closed-loop system under the calculated controller gains. From Fig. 6, we can observe that the designed controller is effective.

V. Conclusions

The fuzzy-model-based non-fragile control problem has been addressed for discrete-time nonlinear SPSs with stochastic jump parameters. A semi-Markov process has been employed to describe the stochastic jump parameters. Compared with the extensively used Markov jump model, the semi-Markov jump model is more general since that it is not required to be subject to ideal, time-invariant and memoryless transition probabilities. By employing the T-S fuzzy model approach and SMK concept, some δ-error mean square stability analysis criteria have been established and the resilient controller has been designed to solve the fragility problem when implementing fuzzy-model-based controller for nonlinear discrete-time semi-Markov jump SPSs. Moreover, an algorithm has been proposed to estimate the permissible maximum value of SPP. A numerical example and a modified tunnel diode circuit have been employed to illustrate the validity of the established theoretical results finally. Future work will be devoted to address the adaptive control problem for nonlinear discrete-time semi-Markov jump SPSs.

Appendix

A. Proof of Lemma 2

Proof: Pay attention to the multiple Lyapunov function as follows

$$V(x(k), \eta_s) = x^T(k) \sum_{\alpha \in \mathbb{W}_1} h_\alpha(k) G_{\alpha,i} x(k) \Big|_{\eta_s = i}, \quad \forall i \in \mathbb{W}_2.$$

Then, one can obtain that

$$\rho_1(\|x(k)\|) \leqslant V(x(k), \eta_s) \leqslant \rho_2(\|x(k)\|), \tag{33}$$

where

$$\rho_2(\|x(k)\|) \triangleq \sup_{\alpha \in \mathbb{W}_1, i \in \mathbb{W}_2} \{\lambda_{\max}(G_{\alpha,i})\} \|x(k)\|^2,$$
$$\rho_1(\|x(k)\|) \triangleq \inf_{\alpha \in \mathbb{W}_1, i \in \mathbb{W}_2} \{\lambda_{\min}(G_{\alpha,i})\} \|x(k)\|^2,$$

which guarantees the condition (6) in Lemma 1. Moreover, if the system's mode does not jump at k which means $\eta_s = i$, $k \in (k_s, k_{s+1})$, i.e., $k \in \{k_s + t, \forall t \in \{1, 2, \cdots, T^i_{\max} - 1\}\}$, then we have

$$\begin{aligned} & V(x(k), \eta_s) - \psi_i V(x(k_s), \eta_s) \\ \leqslant & \sum_{\alpha \in \mathbb{W}_1} \sum_{\beta \in \mathbb{W}_1} h_\alpha(\xi(k_s + t - 1)) h_\beta(\xi(k_s)) \{x^T(k_s) \\ & [\Xi_i^T(k_s, h, t) G_{\alpha,i} \Xi_i(k_s, h, t) - \psi_i G_{\beta,i}] x(k_s)\}. \end{aligned}$$

In light of (9), one can see that

$$V(x(k), \eta_s) - \psi_i V(x(k_s), \eta_s) < 0. \tag{34}$$

In addition, denoting τ as the sojourn time at the switching instant, for $\eta_s = i$, $\eta_{s+1} = j$, $\forall i \neq j \in \mathbb{W}_2$, it follows that

$$\begin{aligned} & \mathbb{E}[V(x(k_{s+1}), \eta_{s+1})]|_{x(0), r_0, D_{s+1} \leqslant T^i_{\max}|_{\eta_s = i}} \\ & -V(x(k_s), \eta_s) \\ \leqslant & \sum_{\alpha \in \mathbb{W}_1} \sum_{\beta \in \mathbb{W}_1} h_\alpha(\xi(k_s + \tau)) h_\beta(\xi(k_s)) \{x^T(k_s) \\ & \times \left[\sum_{\tau=1}^{T^i_{\max}} \Xi_i^T(k_s, h, \tau) \mathcal{G}_{\alpha,i}(\tau) \Xi_i(k_s, h, \tau) - G_{\beta,i}\right] \\ & \times x(k_s)\} \end{aligned}$$

$$\leqslant \quad -\rho_3 \left(\|x(k)\|\right), \tag{35}$$

where

$$\rho_3 \left(\|x(k)\|\right) \triangleq \inf_{\alpha,\beta\in\mathbb{W}_1, i\in\mathbb{W}_2} \left\{ -\lambda_{\max} \left[\sum_{\tau=1}^{T_{\max}^i} \Xi_i^T(k_s,h,\tau) \times \mathcal{G}_{\alpha,i}(\tau)\,\Xi_i(k_s,h,\tau) - G_{\beta,i} \right] \right\} \|x(k)\|^2 .$$

From (33)-(35) and Lemma 1, one could find that the δ-error mean-square stability of the corresponding closed-loop system (5) is ensured. This completes the proof. ■

B. Proof of Theorem 1

Proof: Firstly, when $\kappa_1 = 1$, $\kappa_2 = 1$, using Schur complement to (16), the following holds

$$\Omega_{1\alpha pq,i}^{1,1}(t,\varkappa) + \varepsilon_1^{-1}\Omega_{2p,i}\Omega_{2p,i}^T + \varepsilon_1^{-1}\Psi_{21p,i}\Psi_{21p,i}^T < 0, \tag{36}$$

which combining with Lemma 5 can derive that

$$\Omega_{1\alpha pq,i}^{1,1}(t,\varkappa) + \begin{bmatrix} 0 & F_i^T \Delta\mathcal{K}_{p,i}^T \mathbf{B}_{p,i}^T \\ \star & 0 \end{bmatrix} < 0.$$

Since $\Gamma_{\alpha,i}^{1,1}(t,\varkappa) \geqslant -F_i^T (\mathcal{I}_1 U_{\alpha,i}(t,\varkappa)\mathcal{I}_1)^{-1} F_i^T$, $\sum_{p=1}^{w} h_p(k_s) = 1$ and $h_p(k_s) \geqslant 0$ for $\forall p \in \mathbb{W}_1$, one can obtain that

$$\sum_{p=1}^{w} h_p^2(k_s) \begin{bmatrix} \Gamma_{\alpha,i}^{1,1}(t,\varkappa) & \Gamma_{pp,i}^{1,2} + F_i^T \Delta\mathcal{K}_{p,i}^T \mathbf{B}_{p,i}^T \\ \star & \Gamma_{\alpha,i}^{1,3}(t,\varkappa) \end{bmatrix} < 0. \tag{37}$$

Similarly, considering that $\sum_{q=1}^{w} h_q(k_s) = 1$, and $h_q(k_s) \geqslant 0$ for $\forall q \in \mathbb{W}_1$, when $\kappa_1 = 1$, $\kappa_2 = 1$, from (17), we can get the following condition holds for $\forall p < q$, $\alpha \in \mathbb{W}_1$, $\forall i \in \mathbb{W}_2$

$$\sum_{p=1}^{w}\sum_{q=1}^{w} h_p(k_s) h_q(k_s) \left\{ \begin{bmatrix} \Gamma_{\alpha,i}^{1,1}(t,\varkappa) & \Gamma_{pq,i}^{1,2} + F_i^T \Delta\mathcal{K}_{q,i}^T \mathbf{B}_{p,i}^T \\ \star & \Gamma_{\alpha,i}^{1,3}(t,\varkappa) \end{bmatrix} + \begin{bmatrix} \Gamma_{\alpha,i}^{1,1}(t,\varkappa) & \Gamma_{qp,i}^{1,2} + F_i^T \Delta\mathcal{K}_{p,i}^T \mathbf{B}_{p,i}^T \\ \star & \Gamma_{\alpha,i}^{1,3}(t,\varkappa) \end{bmatrix} \right\} < 0,$$

which combining with (37) implies that

$$\begin{bmatrix} \Gamma_{\alpha,i}^{1,1}(t,\varkappa) & \hat{\Xi}_{pq,i}(k_s,h) \\ \star & \Gamma_{\alpha,i}^{1,3}(t,\varkappa) \end{bmatrix} < 0, \quad \forall p,q,\alpha \in \mathbb{W}_1, \forall i \in \mathbb{W}_2, \tag{38}$$

where

$$\hat{\Xi}_{pq,i}(k_s,h) \triangleq \sum_{p=1}^{w}\sum_{q=1}^{w} h_p(k_s) h_q(k_s) \left\{ \Gamma_{pq,i}^{1,2} + F_i^T \Delta\mathcal{K}_{q,i}^T \mathbf{B}_{p,i}^T \right\}.$$

Additionally, when $\kappa_1 = 2$, $\kappa_2 = 1$, from (16) and (17), we can get

$$0 > \epsilon_M^2 \begin{bmatrix} J_{1\alpha,i}\mathcal{I}_2 U_{\alpha,i}(t,\varkappa)\mathcal{I}_2 J_{1\alpha,i}^T & 0 \\ \star & 0 \end{bmatrix} + \epsilon_M \begin{bmatrix} Sym\left\{J_{1\alpha,i}\mathcal{I}_1 U_{\alpha,i}(t,\varkappa)\mathcal{I}_2 J_{1\alpha,i}^T\right\} & 0 \\ \star & 0 \end{bmatrix} + \begin{bmatrix} \Gamma_{\alpha,i}^{1,1}(t,\varkappa) & \hat{\Xi}_{pq,i}(k_s,h) \\ \star & \Gamma_{\alpha,i}^{1,3}(t,\varkappa) \end{bmatrix}, \quad \forall p,q,\alpha \in \mathbb{W}_1, \forall i \in \mathbb{W}_2.$$

Thus, according to Lemma 4, for $\forall \epsilon_r \in [0, \epsilon_M]$

$$0 > \epsilon_r^2 \begin{bmatrix} J_{1\alpha,i}\mathcal{I}_2 U_{\alpha,i}(t,\varkappa)\mathcal{I}_2 J_{1\alpha,i}^T & 0 \\ \star & 0 \end{bmatrix} + \epsilon_r \begin{bmatrix} Sym\left\{J_{1\alpha,i}\mathcal{I}_1 U_{\alpha,i}(t,\varkappa)\mathcal{I}_2 J_{1\alpha,i}^T\right\} & 0 \\ \star & 0 \end{bmatrix} + \begin{bmatrix} \Gamma_{\alpha,i}^{1,1}(t,\varkappa) & \hat{\Xi}_{pq,i}(k_s,h) \\ \star & \Gamma_{\alpha,i}^{1,3}(t,\varkappa) \end{bmatrix}, \quad \forall p,q,\alpha \in \mathbb{W}_1, \forall i \in \mathbb{W}_2,$$

i.e.

$$\left[\begin{array}{c|c} J_{1\alpha,i}\mathbf{E}_\epsilon U_{\alpha,i}(t,\varkappa)\mathbf{E}_\epsilon J_{1\alpha,i}^T - Sym\left\{F_i^T J_{1\alpha,i}^T\right\} & \hat{\Xi}_{pq,i}(k_s,h) \\ \hline \star & \Gamma_{\alpha,i}^{1,3}(t,\varkappa) \end{array}\right] < 0, \quad \forall p,q,\alpha \in \mathbb{W}_1, \forall i \in \mathbb{W}_2. \tag{39}$$

Since $J_{1\alpha,i}\mathbf{E}_\epsilon U_{\alpha,i}(t,\varkappa)\mathbf{E}_\epsilon J_{1\alpha,i}^T - Sym\left\{F_i^T J_{1\alpha,i}^T\right\} \geqslant -F_i^T \left(\mathbf{E}_\epsilon U_{\alpha,i}(t,\varkappa)\mathbf{E}_\epsilon\right)^{-1} F_i$, one can obtain that

$$\begin{bmatrix} -F_i^T \left(\mathbf{E}_\epsilon U_{\alpha,i}(t,\varkappa)\mathbf{E}_\epsilon\right)^{-1} F_i & \hat{\Xi}_{pq,i}(k_s,h) \\ \star & \Gamma_{\alpha,i}^{1,3}(t,\varkappa) \end{bmatrix} < 0, \quad \forall p,q,\alpha \in \mathbb{W}_1, \forall i \in \mathbb{W}_2. \tag{40}$$

Let $U_{\alpha,i}(t,\varkappa) = \left(\mathbf{E}_\epsilon \Theta_{\alpha,i}(t,\varkappa)\mathbf{E}_\epsilon\right)^{-1}$, from (40) one can easily get that

$$\begin{bmatrix} -F_i^T \Theta_{\alpha,i}(t,\varkappa) F_i & \hat{\Xi}_{pq,i}(k_s,h) \\ \star & -\left(\mathbf{E}_\epsilon \Theta_{\alpha,i}(t,\varkappa+1)\mathbf{E}_\epsilon\right)^{-1} \end{bmatrix} < 0, \quad \forall p,q,\alpha \in \mathbb{W}_1, \forall i \in \mathbb{W}_2. \tag{41}$$

Pre- and post- multiply (41) by diag$\left\{F_i^{-T}, I\right\}$ and diag$\left\{F_i^{-1}, I\right\}$, respectively, then using Schur complement, one can see that the (11) in Lemma 3 is guaranteed.

Moreover, when $\kappa_2 = 1$, using Schur complement to (18), it results in that

$$\Omega_{\beta,i}^3(t) - \left[\psi_i U_{\alpha,i}(t,0)\right]^{-1} < 0.$$

Since $\Omega_{\beta,i}^3(t) \geqslant -U_{\beta,i}^{-1}$, we can get

$$-U_{\beta,i}^{-1} - \left[\psi_i U_{\alpha,i}(t,0)\right]^{-1} < 0.$$

Let $U_{\beta,i} = \left(\mathbf{E}_\epsilon \Theta_{\beta,i}\mathbf{E}_\epsilon\right)^{-1}$ and $U_{\alpha,i}(t,0) = \left(\mathbf{E}_\epsilon \Theta_{\alpha,i}(t,0)\mathbf{E}_\epsilon\right)^{-1}$, it is easy to see that (12) in Lemma 3 is guaranteed. In a similar way, we can prove that once the (16), (17) with $\kappa_1 = 1, 2$, $\kappa_2 = 2$, (18) with $\kappa_2 = 2$ and (19), (20) with $l = 1, 2$, are satisfied simultaneously, then conditions (13) and (14) hold. This completes the proof. ■

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