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# Centralized allocation of human resources. An application to public schools

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ABSTRACT

Centralized resource allocation requires planning how to implement changes in order to adapt resources to the allocated budget without losing outputs. This paper presents an alternative model to reallocate human resources in a public education network based on the so-called centralized data envelopment analysis. This stream has received less attention in the efficiency literature and evaluates the overall efficiency of a set of decision making units controlled by a central authority. In this study, an extended centralized data envelopment analysis dealing with non-transferable inputs and environmental factors is proposed to assess the global efficiency of a centralized education network. We then design an iterative procedure capable of reallocating resources without jeopardizing the level of efficiency. The proposed model is applied in a real case of public schools from Catalonia (north-east Spain). The results indicate the network could be improved to optimally redistribute educational resources without falls in the level of aggregated outputs. The study also provides useful information for accountability and decision making when implementing improvement programs in public schools.



## 1. Introduction

In recent decades, interest in efficiency in education has increased from both practitioner and academic points of view [50]. One of the reasons for the emergence of studies in this field is the increasing importance of the education sector in the economy since it provides intellectual training for the population, better quality human capital, and increased labor productivity [15]. In addition, education is considered essential to enhance a country's economic growth [74]. In the wake of the current global economic crisis, countries face the challenges of making public finances sustainable. Publicly funded sectors are under pressure to deliver more for less and none more so than the education sector. This environment requires an education system that is efficient in translating resources into educational outcomes [69]. In Spain, the economic reality is currently undergoing social and political debate. In the public sector, the pressure to increase performance implies that any action to improve efficiency becomes an economic policy priority. Under the current budget constraints the continuity of any public entity is a decision variable [10].

Performance measurement is important not only in the private sector, but also in the public sector since it can highlight strengths and weaknesses in current practices, reveal directions for improvement, and ultimately may lead to better use of the resources spent on providing public services [5]. In the Spanish case, the government rationalized education spending through an 11% budget cut to save 3,000 million euros by implementing certain measures at the regional level, such as increasing the student-teacher ratio, expanding the range of increases in university fees, delaying teacher replacements, and even forcing the closure of several schools. For instance, in Catalonia more than seven schools have been closed in recent academic years and the regional government is planning to eliminate several elementary education groups due to lack of resources. In this environment, schools are forced to make additional savings in personnel budgets to keep expenditure low. These budgetary restrictions have provoked growing interest in rationalizing the allocation of available resources [108].

Nevertheless, in spite of these global budget cutbacks, the current education system does not encourage schools to work effectively. For this to occur, reorganization is needed that will motivate education institutions to achieve good results efficiently. In the scenario of cutbacks and closures, it appears that one suggestion for increasing efficiency is to merge education institutions [52,69]. From a theoretical perspective, a merger may accrue

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efficiency benefits from returns to scale, as a consequence of increased administrative, economic and academic efficiency, or returns to scope if the merging institutions have complementary activities [104,51,61]. Previous quantitative studies on the empirical effect of merging on efficiency demonstrate that the impact is positive [64,69,87].

Consequently, the approach proposed in this paper consists of creating an internal performance-based scheme that stimulates an effective level of performance in public schools while complying with the budget constraints imposed by the government without losing quality. The introduction of incentives can help schools to be more efficient and sustain efficiencies over time [18,63]. The main purpose of this paper is to propose a method to assess and re-design a group of schools in a public education network to make additional savings in the education budget. We also aim to reallocate resources through a new education network to optimize overall efficiency without jeopardizing the level of educational quality.

To develop the approach we extend a specific nonparametric frontier technique known as Centralized Data Envelopment Analysis (CDEA), initially proposed by Lozano and Villa [79] and modified by Mar-Molinero et al. [88]. We extend the CDEA model in several ways. First, we include a specific constraint to deal with the inclusion of non-discretionary factors (environmental variables) in a centralized scenario. We need to bear in mind that the performance of public schools can also be affected by exogenous or environmental factors, which in the context of our study are represented mainly by the characteristics of the students, families and the nearest school environment [91]. As these variables are not under the control of either the decision making units (*DMU*s, in our case schools) or the decision maker (i.e., the Department of Education), we need to include them in our efficiency analysis in a different way.

Second, we incorporate additional constraints referring to transferable and non-transferable inputs. To do so, we design an efficiency model that encourages good educational practices and penalizes unsatisfactory results. Following Yu et al. [112] the actions to improve results are determined by three main policies that regulate government powers. The first is a short-term policy under which the decision maker cannot dismiss any teachers (permanent or non-permanent) but can transfer non-permanent teachers from one school to another, while maintaining the status of permanent teachers in all schools, and the original number of schools. Secondly, the middle-term policy grants the government restricted power to dismiss only non-permanent teachers, but it can transfer permanent teachers between schools and change the original number of them. This is the alternative we propose in this paper. Thirdly, the long-term policy refers to the possibility of dismissing any permanent or non-permanent teacher from any school. This most extreme policy is not taken into account in this paper.

After designing the extended CDEA, we propose an iterative method capable of re-designing the public education network by reallocating resources among schools with reception capacity, taking into account environmental conditions. To that end, we apply a conditional nonparametric efficiency approach to better know which schools were the worst performers (i.e., the candidates to be merged). The conditional model is known to be a better approach than traditional models, such as two-stage models, to account for environmental factors (see [101,27] for an overview). The validity of these traditional models is limited because they assume the separability condition between the input-output space and the space of environmental factors. Therefore, we use the so-called conditional nonparametric approach [19,30–32] which avoids this restrictive separability assumption.

Previous studies on resource allocation in schools have rarely considered the issue of centralized human resource adjustment scenarios (e.g., [99,57,88,53]). In addition, prior empirical papers have revealed a significant and positive relationship between human resources practices and performance (e.g., [65,24,112]). According to Chen and Huang [24], the appropriate allocation of human resources has a positive effect on organizational performance. Yu et al. [112] also argue that periodic organizational change is necessary to improve organizational performance. A new model is therefore needed to resize the education network in order to provide objective solutions to the general cutbacks proposed by the government (which affect all schools regardless of their performance); in other words, to create a computational model capable of determining the poorly performing schools overall, and to design a mechanism to resize the network without penalizing the best performers.

The results show that overall efficiency can be improved without losing outputs and quality, but this improvement depends on the objectives of the Department of Education. First, 12.7% of resources can be saved without changing the current network. Second, it would be possible to save 17.2% of resources if the network composition was changed by resizing the number of schools operating.

Our work supplements previous research in several ways. First, the extended CDEA we propose overcomes the problem of including non-transferable inputs and non-discretionary factors that were not included in previous research. Second, our analysis may be particularly relevant from a policy perspective because it establishes the actions needed to optimize the network through budget reallocation in a real case study. Third, the implementation of the proposed model in a real application provides valuable information for public authorities and facilitates the implementation of improvement programs in schools, which contributes to higher levels of quality, motivation, and fairness within the system.

Following this introduction, Section 2 reviews the related research on the CDEA model and resource allocation. Section 3 explains the methodology, and Section 4 details the data and variables we use. Section 5 summarizes and discusses the results, and finally, Section 6 concludes.

## 2. Empirical evidence on efficiency and resource allocation

Schools are multidimensional in nature, consisting of different functions that are difficult to quantify. The education sector is non-for-profit, there is an absence of output and input prices, and schools produce multiple outputs from multiple inputs. It is therefore a complex task to define and estimate the production technology that students use to acquire knowledge [109].

Several methodological approaches have been employed to solve the problem of efficiency measurement in the education context (see [68,70], and references therein). However, the efficiency in education literature has mainly used frontier methods in two variants: nonparametric models (such as Data Envelopment Analysis (DEA) [22], Free Disposal Hull (FDH) [38], order-*m* frontiers [19] and parametric models like Stochastic Frontier Analysis (SFA) [3]. A review of the advantages and shortcomings of different frontier analysis techniques can be found in Fried, Lovell and Schmidt [46].

In this environment, DEA has become very popular in empirical studies on the efficiency in education, since it can easily handle multiple dimensions of performance and is less vulnerable to the misspecification problems that can affect econometric models [109,78]. This method can also handle multiple inputs and outputs without requiring the specification of an ungranted functional form of the input-output relationship [102]. Hence, DEA is presented as a method of competitive benchmarking in terms of learning from the best practice and searching for weaknesses in an effort to increase efficiency of business processes in the future [81].

Since DEA first emerged, its applications to the field of efficiency in education have grown steadily [41]. Many papers in the literature have applied DEA to assess efficiency in education (e.g., [12,23, 13,14,103,97,71,100,89,91,1,48,92,110,95,105,96], among others). This line of research is based on empirical studies designed to estimate the magnitude of the impact of schools' results promoted by internal inputs and environmental factors [109]. Since its inception, findings from this research stream have contributed to improve the knowledge and understanding of which educational elements affect students' development, thus providing information for decision making in the classroom, the school, and the education system.

Although there are numerous applications of DEA to measure school efficiency, most approaches consider the *DMU*s separately, providing a relative efficiency index for each unit as compared to the rest. However, relatively few studies have applied an approach in which units are studied jointly and simultaneously projected to the efficiency frontier with an overall objective. There are situations in which the *DMU*s operate under a common centralized direction. This type of scenario is common when all units belong to the same organization that provides the resources needed to achieve results, such as bank branches, hospitals, public schools, supermarket chains or police stations [88].

As a response to these situations, Lozano and Villa [79] proposed the CDEA model. These authors present different centralized resource allocation models in a decision-making environment. The idea behind this formulation is to globally reduce the total use of inputs or globally increase the productions of all outputs, as opposed to the philosophy of traditional DEA models, which optimize the functioning of each *DMU* separately [4].

Many DEA applications can be found in this centralized scenario, although not all of them use the CDEA perspective: supermarket branches [111,42,73], hospitals [76], universities or schools [53,55,57,88,99], police stations [5], airports [112], public entities [4,43,6,82–84] or branches of private companies [107,39,45,49]. Other examples with simulated data include Lozano and Villa [80]; Nesterenko and Zelenyuk [93] and Li and Cui [75]. In all these examples, the central authority, as well as being interested in the efficiency of each unit, is also concerned about the total consumption of inputs by different *DMU*s and the overall production of outputs.

Some extensions of CDEA models have appeared since Lozano and Villa [79]. For instance, Asmild et al. [4] reconsider one of the centralized models proposed by Lozano and Villa [79,80] and suggest modifying it to only consider adjustments of previously inefficient units. Asmild et al. [5] propose a novel way to reallocate personnel resources between tasks within a specific unit as well as between similar units using a CDEA model. They show reallocations in six different scenarios to offer solutions depending on different circumstances or policy objectives that may exist in the organizations. Fang [42] introduces a new generalized centralized resource allocation model which extends Lozano and Villa's and Asmild et al.'s models to a more general case. In addition, the paper considers situations in which some *DMU*s are geographically dispersed or for which it may be impossible to reallocate inputs or transfer outputs across *DMU*s because of regulation or indivisibilities. Consequently, inputs may be reallocated or outputs may be transferred, but only across some *DMU*s. Yu et al. [112] present an alternative approach to reallocating human resources by constructing a modified CDEA model combined with a Russell measure and applied to three different human resource reallocation policies in Taiwanese airports. Finally, Mar-Molinero et al. [88] developed a simplified version of the model by Lozano and Villa [79,80] that makes the model easier to implement in several situations. These authors also consider the scenario in which some units could disappear because the reallocation of inputs to other units could improve the overall efficiency of the group.

In our paper we propose a further extension of CDEA developed by Mar-Molinero et al. [88] that includes transferable and non-transferable inputs and takes into account the role of environmental factors. To the best of our knowledge this new combined approach has not been considered in previous work on efficiency in education.

## 3. Analytical models for efficiency analysis and resource allocation

### 3.1. The extended CDEA model

In this section we detail the methodology followed to achieve our main goal. We assume that all schools are operating under the control of the decision maker represented by the Department of Education.

Following the education policies detailed in Section 1, our CDEA model can have different orientations. We run the first two policies, namely short- and middle-term policies since the long-term policy is consider too extreme for our scenario. Thus, we assume the decision maker cannot dismiss permanent teachers in any case, but can transfer them under the middle-term policy. Therefore, following Fang [42], permanent teachers are treated differently in the restrictions of CDEA model. Another difference between short-term and middle-term policies is the treatment of non-permanent teachers, who can be transferred in both policies and made redundant only under the middle-term policy. In this case, following Mar-Molinero et al.'s model, we will know the number of non-permanent teachers who are candidates for dismissal.

Another improvement in our model concerns the treatment of non-discretionary factors in CDEA models. According to Asmild et al. [4] and Fang [42], when considering organizations that operate under a multi-level management hierarchy, it is important to understand that certain factors are outside the control of the local *DMU*s. Previous research in education has considered this situation, typically by designating these factors as non-discretionary (e.g., [91,27,28,29,33]). The treatment of non-discretionary factors in these examples is appropriate; however, it does not take into account the existence of non-controllable factors in a centralized model. In many cases such factors are in fact exogenously determined and beyond the control of both local and central management. Therefore, it is relevant to consider them within the analysis in order to allocate the transferable resources to similar environmental conditions.[1]

Certain assumptions need to be clarified before going any further. First, due to the different nature of the contracts, non-permanent teachers cannot replace permanent teachers and vice versa. Second, we assume there is no cost in transferring permanent teachers and/or dismissing non-permanent teachers.[2] Finally, we consider data for only one academic year, which precludes any knowledge of how the simulation of the reallocation process

[1] Here we follow Banker and Morey [9] to include the role of external factors in the CDEA model. We thank the anonymous referee for pointing this out.

[2] We are aware this assumption is severe; nevertheless, it is justified because it is contemplated in the Spanish law. On the one hand, non-permanent teachers have temporary contracts that expire at the end of each academic year. Therefore, if the mergers are carried out, the educational system can adjust the number of temporary contracts depending on need. On the other hand, under special circumstances, the Spanish law allows permanent teachers to be reallocated among different schools without any economic incentive for teachers. Similar assumptions have been made in previous papers dealing with resource allocation (e.g., [112]).

might affect the efficiency of schools after this procedure. However, as we stated in the introduction, previous empirical studies have proved that mergers have a positive impact on efficiency [104,51,61,64,69,87]. For instance, Gordon and Knight [51] apply a merger estimator to explain political integration in a wave of school district mergers in the state of Iowa during the 1990s. The authors conclude highlighting the importance of state financial incentives for consolidation (mergers) and the benefits in terms of economies of scale. In addition, Johnes [69] deals with the effects on efficiency of merged universities in the UK. She states that merger activity in English higher education institutions seems not to be a reaction to a crisis in efficiency in the merging institutions. Specifically, a merged higher education institution is significantly more efficient than either pre-merger or non-merging institutions, suggesting that, on average, merging is a positive activity. This evidence leads us to suspect that whether or not the government undertakes these mergers, the overall efficiency of the network will be reinforced.

That said the process is developed in several stages. First, we obtain the overall efficiency of the current education network under the short-term policy (the government cannot dismiss any teachers, but can transfer non-permanent teachers). Second, we test the possibility of optimizing the performance of the network. To do this, we apply the middle-term policy and we find the optimum number of schools that the network should have to increase the overall efficiency ($n \neq n^*$). This step not only shows the optimal size of the network, but also reveals the number of schools that are candidates to be merged with other schools with reception capacity located in a similar area.

Having explained the intuition of our CDEA model, let us define $j=1, 2, \ldots, J$: sub-index for each *DMU*; $i=1, 2, \ldots, I$: sub-index for each input; $k=1, 2, \ldots, K$: sub-index for each output; $x_{ij}$=amount of input $i$ consumed by *DMUj*; $y_{kj}$=amount of output produced by the *DMUj*; $\theta$=technical efficiency ratio; $(\lambda_1, \lambda_2, \ldots, \lambda_n)$=intensity vector of the inputs and outputs of each *DMUj*; $t$ symbolizes the transferable inputs ($t=1, \ldots, T$); while $nt$ shows the non-transferable inputs ($nt=1, \ldots, NT$). Lastly, $nd$ represents the non-discretionary factors ($nd=1, \ldots, ND$). Assuming that all *DMUs* are under the control of the decision maker that aims to minimize the total input consumption, the input-oriented CDEA model[3] for the short-term policy can be written as follows (extension proposed following Fang [42] and Mar-Molinero et al. [88]):

$$\min \theta,$$

$$s.\,t:$$

$$\sum_{j=1}^{J} \lambda_j x_{tj} \le \theta \sum_{j=1}^{J} x_{tj}, \forall t=1, \ldots, T,$$

$$\sum_{j=1}^{J} \lambda_j x_{ntj} \le \sum_{j=1}^{J} x_{ntj}, \forall nt=1, \ldots, NT,$$

$$\sum_{j=1}^{J} \lambda_j x_{ndj} \le \sum_{j=1}^{J} x_{ndj}, \forall nd=1, \ldots, ND,$$

$$\sum_{j=1}^{J} \lambda_j y_{kj} \le \sum_{j=1}^{J} y_j, \forall k=1, \ldots, K,$$

$$\sum_{j=1}^{J} \lambda_j = 1,$$

$$\lambda_j \ge 0,\ \theta\ free \tag{1}$$

where the number of schools operating in the network remains unchanged ( $\sum_{j=1}^{J} \lambda_j=1$) and any teacher can be dismissed. Transferable inputs refer to non-permanent teachers and non-transferable input to those who have permanent contracts. Non-discretionary factors are detailed in Section 4.2.

The CDEA model for the middle-term policy is exactly the same, but leaving free the restriction of the *lambdas* and omitting the restriction of non-transferable inputs as both categories of teachers are adjustable. In this model $n^*$ indicates the optimum number of schools that the network should have to be more efficient ($n \neq n^*$). This implies that the decision maker seeks to reduce both the total number of non-permanent staff to be dismissed and those permanent teachers that can be transferred.

$$\min \theta,$$

$$s.\,t:$$

$$\sum_{j=1}^{J} \lambda_j x_{tj} \le \theta \sum_{j=1}^{J} x_{tj}, \forall t=1, \ldots, T,$$

$$\sum_{j=1}^{J} \lambda_j x_{ndj} \le \sum_{j=1}^{J} x_{ndj}, \forall nd=1, \ldots, ND,$$

$$\sum_{j=1}^{J} \lambda_j y_{kj} \le \sum_{j=1}^{J} y_j, \forall k=1, \ldots, K,$$

$$\sum_{j=1}^{J} \lambda_j = n^*,$$

$$\lambda_j \ge 0,\ \theta\ free \tag{2}$$

Once we obtain this $n^*$, we can decide how to reallocate the resources among the number of schools that make up the new education network.

### 3.2. Looking for poor performance schools to reallocate resources

Once we have applied the CDEA model from Eq. (2) we will know the optimum size of the education network in order for it to be more efficient and comply with the budget constraints imposed by the government. Regarding the objective of the Government, poorest performing schools can be merged with neighboring schools with available capacity. The model from Eq. (2) sets the optimal size of the network and the number of schools that are candidates to merge, but does not reveal which schools these are. A method is needed to find out which schools should be restructured. To do this, and following the nonparametric nature of this paper, DEA can be used since it can easily handle multiple dimensions of performance and is less vulnerable to the misspecification problems that can affect econometric models. However, this approach presents some significant drawbacks [26]: (1) statistical inference is not possible due to its deterministic

[3] We focus on input orientation due to the current environment of budget constraints in the Spanish public education system. The goal is to maintain the level of educational outputs minimizing the input consumption.

nature; (2) it is very sensitive to the presence of outliers and measurement errors in data; and (3) it experiences dimensionality problems due to its slow convergence rates.

In order to overcome these problems, Cazals et al. [19], and Daraio and Simar [30,31,32], introduced the order-$m$ model.[4] Order-$m$ frontier estimators are known to be more robust to outliers, extreme values or noise in the data than the full frontier estimates (DEA or FDH). In essence, this approach does not consider the full set of observations for defining the efficiency score. Instead, it repeatedly considers subsamples of $m$ ($\geq 1$) observations randomly drawn from the sample. Robust measures are obtained by repeating this process $B$ times ($B$ being a large number). For each observation $j$, the robust order-$m$ model is then computed as the average value of the efficiency scores ($\hat{\theta}_m^1,\ldots,\hat{\theta}_m^B$) defined over the $B$ iterations. As outlying observations do not form part of the set $m$ in every draw, the impact of outlying observations on these order-$m$ efficiency scores is effectively mitigated.

Jeong, Park, and Simar [66] state that order-$m$ estimates have attractive properties in that they are consistent and have a fast rate of convergence. In particular, this approach is becoming more popular in the literature on efficiency in education (e.g., [36,106]; among others).

Continuing with the notation introduced in Section 3.1, students transform a set of inputs $x \in \mathbb{R}_+^i$ into heterogeneous outputs $y \in \mathbb{R}_+^k$. In this framework, the production technology is the set of all feasible input-output combinations:

$$\Psi = \{(x, y) \in \mathbb{R}_+^{i+k} \mid x \text{ can produce } y\} \tag{3}$$

In this framework, an observed production unit defines an individual production possibility set which, under the free disposability of inputs and outputs, can be written as:

$$\Psi(x_j, y_j) = \left\{ (x, y) \in \mathbb{R}_+^{i+k} \mid x_j \geq x; y \leq y_j \right\} \tag{4}$$

In order to estimate the relative efficiency of each school, we estimate a frontier following the ideas developed by Farrell [44]. Specifically, the measure of output-oriented[5] efficiency score for a unit operating at the level $(x, y)$ is defined as Eq. (5):

$$\theta(x, y) = \sup\{\theta \mid (x, \theta y) \in \Psi\} \tag{5}$$

Here $\theta(x, y) \geq 1$ represents the proportionate increase of outputs the unit operating at level $(x, y)$ should attain to be considered as being efficient. The efficient frontier corresponds to those points where $\theta(x, y) = 1$. As stated in Section 2, and given that the set $\Psi$ cannot be directly observed, it has to be estimated from a random sample of production units denoted by $\omega = \{(x, y) \in R_+^{i+k} |, j = 1,\ldots,J\}$. Since the pioneering work of Farrell [44], the literature has developed different approaches to achieve this goal. In this paper, we focus on applying an order-$m$ frontier model as it is more robust to outliers, extreme values or noise in the data than the full frontier estimates (DEA or FDH). Therefore, for a given level of inputs $x$ in the interior of the support of $X$, consider $m$ i.i.d. random variables $Y_j$, $j = 1, \ldots, m$, we define the set:

$$\Psi_m(x) = \left\{ (x', y) \in \mathbb{R}_+^{i+k} \mid x' \leq x, i = 1, \ldots, m \right\} \tag{6}$$

Then, the output oriented order-$m$ efficiency score ($\hat{\theta}_m$) can be obtained from Eq. (7) as follows:

$$\hat{\theta}_m(x, y) = \sup \theta \mid (x, \theta y) \in \Psi_m(x) = E\left[ \max_{j=1,\ldots,m} \left\{ \min_{k=1,\ldots,K} \left( \frac{y_j^k}{y^k} \right) \right\} \mid X \leq x \right] \tag{7}$$

This estimator compares the efficiency of a *DMU* with the $m$ potential observations that have a production larger than or equal to $y$. As $m$ increases, the expected order-$m$ estimator tends toward the DEA efficiency score. For reasonable $m$ values, the efficiency score will exhibit a value higher than unity, which indicates that the unit is inefficient. When $\hat{\theta}_m < 1$, the *DMU* under evaluation is labeled as super-efficient, since the order-$m$ frontier exhibits lower levels of outputs than the unit under analysis [31].

### 3.3. Dealing with environmental factors

Following with our empirical analysis, we need to bear in mind that a potential source of inefficiency is the impact of exogenous or environmental factors denoted as $x_{nd} \in \mathbb{R}^{nd}$. An evaluation of schools' performance should explicitly include this information to ensure that the efficiency score finally assigned to the school truly reflects the portion of the production process for which the unit itself is responsible [91].

There are different ways of incorporating the effect of environmental factors into the production process to estimate efficiency scores (see Fried et al. [46] for an overview). Although traditional approaches such as the two-step method [101,90] are popular and widely employed, the specific literature devoted to environmental factors and their influence on efficiency has advanced significantly with the development of a more general and appealing full nonparametric conditional approach, based on the probabilistic definition of the frontier developed by Cazals et al. [19] and Daraio and Simar [30,31,32].

This approach has become very popular in the recent literature on efficiency measurement. Hence, it is possible to find several studies using this approach to measure the efficiency of units operating in a wide range of settings, including the education sector (e.g., [16,25,33,34,35,56]).

The conditional model works with probabilistic formulation and incorporates the environmental impact conditioning the production process to a given value of the non-discretionary factors ($X_{nd} = x_{nd}$). It constructs a boundary representing the reference set in which each unit is compared. This method also avoids the restrictive separability assumption required by traditional approaches in order to provide meaningful results, and does not require specification of the influence of each environmental variable on the efficiency. The set defined before in Eq. (6) can be adapted as follows:

$$\Psi_m\left(x \mid x_{nd}\right) = \left\{ (x', y) \in \mathbb{R}_+^{i+k} \mid x' \leq x, X_{nd} = x_{nd}, i = 1, \ldots, m \right\} \tag{8}$$

Using a probabilistic formulation, the order-$m$ efficiency measure defined above in Eq. (7) has to be adapted to the condition $X_{nd} = x_{nd}$ as follows:

$$\begin{aligned} \hat{\theta}_m(x, y \mid x_{nd}) &= \sup \theta \mid (x, \theta y) \in \Psi_m\left(x \mid x_{nd}\right) \\ &= \sup\left\{ \theta \mid S_Y(\theta y \mid x, x_{nd}) > 0 \right\} \end{aligned} \tag{9}$$

where $S_Y(y|x, x_{nd}) = Prob(Y \geq y) X \leq x, X_{nd} = x_{nd})$

To estimate the conditional model, smoothing techniques are needed in $x_{nd}$. For this purpose, the following kernel estimator of $S_Y(y \mid x, x_{nd})$ is used:

$$\hat{S}_{Y,n}\left(y|x, x_{nd}\right) = \frac{\sum_{j=1}^{n} I(x_j \leq x, y_j \geq y) K_{\hat{h}}(x_{nd}, x_{ndj})}{\sum_{j=1}^{n} I(x_j \leq x) K_{\hat{h}}(x_{nd}, x_{ndj})} \tag{10}$$

[4] See Cazals et al. [19] and Badin and Daraio [7] for a detailed explanation of the method and a discussion about its attractive statistical properties.

[5] In that case, in contrast to what we did with CDEA, we apply an output orientation model. The reason for this change is related to the purpose of each model. In this case, the objective is to know which schools achieve the best results with the available resources to conduct the reallocation process. In other words, our goal is to know which schools obtain the best (and worst) academic results (i.e., for benchmarking purposes) in order to establish which schools are the candidates to be merged.

where $K_{\hat{h}}(\cdot)$ represents the kernel function, $I(\cdot)$ is an indicator function and $h$ is an appropriate bandwidth parameter for this kernel.

As can be seen, this approach relies on the estimation of a nonparametric kernel function to select the appropriate reference partners, and a bandwidth parameter $h$ using a method with some bandwidth choice. In our case, we use a multivariate kernel function (developed by [33]) since we have mixed data in our data set (continuous and ordinal variables). We employ the Epanechnikov kernel function $\left(K_{\hat{h}}(x_{nd}, x_{ndj})=h^{-1}K\left(\frac{x_{nd}, x_{ndj}}{h}\right)\right)$ for continuous variables and the Li and Racine [77] discrete kernel function for ordered variables. To estimate the bandwidth parameters, we follow the data-driven selection approach developed by Badin et al. [8].[6] Subsequently, the conditional order-$m$ estimator $\hat{\theta}_m(x, y \mid x_{nd})$ can be obtained by plugging in $\hat{S}_{Y,n}=(y \mid x, x_{nd})$ in Eq. (9).

The relative conditional order-$m$ efficiency index for each *DMU* indicates which schools are less efficient and, therefore, those that could face mergers if the reallocation process is applied.

## 4. CDEA. A case study in public schools from Catalonia

### 4.1. The case of Catalonia

The main purpose of this paper is to propose a method to assess and re-design a group of schools in a public education network to make additional savings in the education budget. In addition, we aim to reallocate resources through a new composition of the network to optimize the overall efficiency without jeopardizing the level of educational quality. To do so, we develop the CDEA model proposed in Section 3.1 in a specific Spanish region, Catalonia.

Catalonia is a region located in the north-east of Spain. It is a very densely populated and highly industrialized territory, and has led the Spanish industrial sector since the nineteenth century. Its economy is the most important of the regions in Spain, generating more than 18.7% of Spanish GDP. Like the rest of regions, Catalonia has a public education system that belongs to the Spanish public education network. The case of the public education system in Catalonia is especially important for this study because, despite being traditionally one of the richest regions in Spain, it is also suffering severe cuts in education, so much so that the regional Department of Education is closing or merging some schools. For instance, during the academic year 2012/2013 seven schools were closed and plans were made to cut 73 elementary school groups due to lack of resources. In addition, more schools could be added to the list of closures if they do not achieve a minimum of 15 registrations. This is not a new policy, however; this was the second consecutive academic year that the Department of Education had encouraged school closures. During the previous academic year six schools were forced to close and elementary education was suspended in four others.

The underlying question is how the government determined which schools should be closed or merged and why. We wonder whether or not they have followed an objective procedure to determine which schools are the most technically inefficient, or whether the process was simply a random exercise.[7] Our goal is to apply an objective procedure for measuring the overall efficiency of the education network to determine the optimal number of schools in the network for it to be more efficient without losing quality; in other words, to determine how many schools the network should have in order to comply with the cutbacks without damaging students' results. We then determine the most inefficient schools that are candidates to be merged with other nearby schools with similar socio-economic environment.

### 4.2. Data and variables

We use a database from the Evaluation Council of the Education System in Catalonia (*Consell Superior d'Avaluació del Sistema Educatiu de la Generalitat de Catalunya*). The sample includes 161 primary public schools for the academic year 2009-2010, representing a specific territorial educational area in Catalonia: *Vallès Occidental.* Catalonia has ten territorial areas covering neighboring municipalities under the control of a specific group of education inspectors. We selected just one area in order to simplify the simulation of human resource reallocation to neighboring schools.

The relevant unit of observation is the school. We are aware of the importance of having student level data and the problems that aggregation could cause [60]. However, we decided to maintain school level data because it contains information about the geographical location of each school. These data allow us to subsequently reallocate resources to neighboring schools.[8]

To select the variables, we follow the economics of education literature at elementary and primary level. As shown in Table 1, the outputs we use are the grades in homogeneous aptitude tests taken by all students (e.g., [12,13,97,21,85,99]). As our unit of analysis is the school, we consider the sum of the arithmetic mean of the students' grades in the sixth grade general tests conducted in Catalonia ($Y_1$) and the number of students who pass the exams ($Y_2$). In terms of inputs, students usually transform a set of resources into heterogeneous outputs. Most of the studies in the literature distinguish between human, capital and physical resources [59]. In this category, we include the number of permanent teachers ($X_1$) and non-permanent teachers ($X_2$) (e.g., [58,94,91,29,96,72]). Finally, exogenous variables may have different origins as they can be derived from environmental factors (which include the students' personal characteristics and close family environment) or complexity factors (variables reflecting diversity inside the school) [62]. In line with the literature, we include two variables to capture the student's home background.

First, parents' education ($X_{nd1}$) (e.g., [37,86,33]), and second the percentage of unemployed parents ($X_{nd2}$) ([89,37,17,33]). Finally, we incorporate two variables related to the complexity inside the school: the percentage of immigrants ($X_{nd3}$) (e.g., [67,47,99,33]), and annual absenteeism (e.g., [13,20,40]).

Summary statistics are provided in Table 2. As can be seen, schools have an average of 21.660 permanent teachers, while the number of non-permanent teachers falls to 5.220. This information shows that the education system in this education area basically consists of teachers with stable contracts. In addition, students score an average of 67.135 points out of 100 in the sixth grade final tests. At these levels of education most students usually pass, although there are some who do not reach scores of 50. Non-

[6] This approach can be easily adapted to the case of mixed environmental variables. This choice has been applied in previous studies as Cordero et al. [26].

[7] An example retrieved from the news demonstrates the weak criteria followed by the government to close some schools. For instance, a school located in the metropolitan area of Barcelona was closed because of its disreputable location, despite running an educational project that improved students' outcomes by more than 50%.

[8] The possibility of having student level data available should not mean that the implications of the paper would be dramatically different from an analysis of efficiency to reallocate resources. We would like to thank the referees for pointing this out.

**Table 1**
Description of variables.
*Source*: The authors.

| *Type* | *Variable* | | *Description* |
|---|---|---|---|
| Discretionary input | $X_1$ | Permanent teachers | Total number of teachers with a stable contract. |
| | $X_2$ | Non-permanent teachers | Total number of temporary teachers. |
| Non-discretionary factor | $X_{nd1}$ | Educational level | Parents' education (mean). 0. No education 1. Primary education. 2. Secondary Education. 3. Intermediate Professional Training. 4. Baccalaureate (post-compulsory school). 5. Higher Professional Training, 6. Graduate.7. Post-Graduate. 8. PhD. |
| | $X_{nd2}$ | Unemployed | Percentage of unemployed parents. |
| | $X_{nd3}$ | Immigrants | Percentage of non-Spanish students. |
| | $X_{nd4}$ | Annual absenteeism | Percentage of student absences during the academic year (students absent more than 75% of all days). |
| Output | $Y_1$ | Grades | Average test mark obtained by the students in the general sixth grade test. |
| | $Y_2$ | Passed | Total enrolled – repeaters. |

**Table 2**
Summary statistics: Public Schools in Catalonia, 2009/2010.
*Source*: The authors.

| *Variable* | *N* | *Min* | *Q25* | *Mean* | *S.D.* | *Median* | *Q75* | *Max* |
|---|---|---|---|---|---|---|---|---|
| $X_1$ | 161 | 4.000 | 16.000 | 21.660 | 7.384 | 24.000 | 28.000 | 35.000 |
| $X_2$ | 161 | 0.000 | 3.000 | 5.220 | 3.307 | 4.000 | 7.000 | 15.000 |
| $Y_1$ | 161 | 38.281 | 58.725 | 67.135 | 9.166 | 67.442 | 73.842 | 88.001 |
| $Y_2$ | 161 | 45.000 | 238.000 | 359.730 | 119.981 | 414.000 | 451.500 | 595.000 |
| $X_{nd1}$ | 161 | 2.841 | 4.642 | 5.196 | 0.956 | 5.017 | 5.716 | 7.450 |
| $X_{nd2}$ | 161 | 0.052 | 0.113 | 0.147 | 0.051 | 0.139 | 0.173 | 0.326 |
| $X_{nd3}$ | 161 | 0.000 | 0.040 | 0.133 | 0.146 | 0.084 | 0.177 | 0.705 |
| $X_{nd4}$ | 161 | 0.000 | 0.000 | 0.004 | 0.009 | 0.001 | 0.030 | 0.079 |

discretionary factors reveal interesting aspects. First, although a high percentage of parents are unemployed, they usually have a higher professional training qualification, or have at least finished secondary education. This finding mirrors the current high level of unemployment in Spain, despite its skilled population. Second, we find a great variety in the percentage of immigrants. In some schools only 4% of the students are immigrants whereas others have up to 70%. This indicates that our sample contains so-called 'ghetto schools' where a large percentage of immigrants are concentrated. Finally, absenteeism rates are low on average, although there are schools where up to 7% of students leave before the end of the academic year.

Table 3 presents the correlation matrix among input and output variables. We conducted a multicollinearity study to detect possible significant relationship and collinearity problems. Both the Tolerance and VIF tests show values that are not disturbing. In all the cases Tolerance is higher than 0.3 and VIF is lower than 3 (see Belsley et al. [11] for thresholds). Collinearity problems are therefore not an issue in our data.

## 5. Results and discussion

### 5.1. CDEA results

In this section we outline the results obtained using the CDEA models from Eqs. (1) and (2) to estimate the overall efficiency score of our sample. Table 4 presents different models of CDEA by changing the sample size. Each column indicates the model we are using, the number of schools operating, the overall efficiency score and the benchmarked schools. Columns 4 and 5 are the most important ones as they reflect the application of middle- and short-term policies, respectively.

When we apply the short-term policy (Model from Eq. (1)) we obtain the result in column 5. Thus, the overall efficiency of the group is 0.873, showing that we could obtain the same outputs even if we save 12.7% of the transferable inputs by maintaining the number of schools operating. To do this, schools should mimic the structure of schools 126 (61 times), 139 (51 times) and 151 (49 times).

However, according to Mar-Molinero et al. [88] and the current

**Table 3**
Correlation Matrix.
*Source*: The authors.

| *Variable* | $X_1$ | $X_2$ | $Y_1$ | $Y_2$ | $X_{nd1}$ | $X_{nd2}$ | $X_{nd3}$ | $X_{nd4}$ |
|---|---|---|---|---|---|---|---|---|
| $X_1$ | 1 | | | | | | | |
| $X_2$ | −0.063 | 1 | | | | | | |
| $Y_1$ | 0.425*** | 0.206*** | 1 | | | | | |
| $Y_2$ | 0.903*** | 0.243*** | 0.397*** | 1 | | | | |
| $X_{nd1}$ | 0.091 | −0.277 | 0.577*** | 0.132* | 1 | | | |
| $X_{nd2}$ | 0.550 | 0.357 | −0.177** | 0.625*** | −0.504** | 1 | | |
| $X_{nd3}$ | 0.083 | 0.310*** | −0.412*** | −0.138* | −0.587** | 0.232** | 1 | |
| $X_{nd4}$ | −0.154 | −0.095 | −0.355*** | −0.211*** | −0.393** | 0.028 | 0.468** | 1 |

Notes:

*** Below the 1% statistical significance thresholds, respectively.

** Below the 5% statistical significance thresholds, respectively.

* Below the 10% statistical significance thresholds, respectively.

**Table 4**
Results of CDEA models.
*Source*: The authors.

| **Column** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Model** | **(2)** | **(2)** | **(2)** | **(2)** | **(1)** | **(2)** | **(2)** | **(2)** | **(2)** | **(2)** | **(2)** |
| **Schools** | **(0.7)n** | **(0.8)n** | **(0.9)n** | **n*** | **n** | **(1.2)n** | **(1.4)n** | **(1.6)n** | **(1.8)n** | **(2)n** | **(3)n** |
| **Overall Efficiency (θ)** | 0.867 | 0.835 | 0.838 | 0.828 | 0.873 | 0.928 | 0.932 | 0.946 | 0.955 | 0.977 | 0.993 |
| λ | | | | | | | | | | | |
| **35** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 41 |
| **118** | 87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **126** | 26 | 54 | 61 | 58 | 61 | 41 | 50 | 58 | 58 | 58 | 0 |
| **139** | 0 | 44 | 50 | 52 | 51 | 51 | 24 | 0 | 0 | 0 | 0 |
| **148** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 120 | 352 |
| **149** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 58 | 0 | 0 |
| **151** | 0 | 31 | 34 | 36 | 49 | 101 | 68 | 37 | 10 | 0 | 0 |
| **156** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 24 | 89 |
| **161** | 0 | 0 | 0 | 0 | 0 | 0 | 83 | 157 | 164 | 120 | 0 |
| **Σλ** | 113 | 129 | 145 | 146 | 161 | 193 | 225 | 258 | 290 | 322 | 483 |

scenario, there are situations in which the decision maker can change the assignment of inputs by merging the most inefficient units or opening new units. In the current context of budget constraints, the government requires maximum cost reductions, so the most inefficient units could be merged and the non-permanent teachers could be dismissed. To do this, we run the model from Eq. (2) (middle-term policy) by changing the value of $n$ each time (range from $n=(0.7)n$ to $n=(3n)$). Feasible solutions do not exist for $n<(0.7)n$, and $n>(3)n$. This means that it would be impossible to obtain the current output level with fewer than 113 schools.

Although $n=161$ is a feasible solution, it is possible to improve the results if we reduce the number of operational schools. The global minimum ($\theta=0.828$, column 4 of Table 4) is reached when $n^*=146$. This implies that 15 schools should be merged with other neighboring schools in order to improve the overall efficiency of the education network by 17.2%. In this case, the cloned schools would be schools 126 (58 times), 139 (52 times) and 151 (36 times). Thus, we can confirm these schools should be taken as benchmarks for reallocation interventions.

Prior to continuing with the reallocation process it is worth noting some features of the benchmark schools. Most significantly, schools 126, 139 and 151 are large in size and students obtain good marks in the general tests. Furthermore, the families' educational level is high: parents are university graduates, on average. However, the unemployment level is also high. At an internal level, these schools have a very low level of absenteeism and a low percentage of immigrants. These features make schools 126, 139 and 151 good benchmarks for the rest.

An important finding is obtained from this information: if the permanent teachers and students at the 15 schools that are candidates to be merged are reallocated, the remaining 146 schools will be enlarged. Therefore, we demonstrate the existence of increasing returns to scale in this education network. That is, we obtain a new network composition that consists of schools with a higher student-teacher ratio, but without altering students' outcomes. This finding is in line with the strategy announced by the government noted in the introduction and Section 4.1. However, our model is selective in the reallocation of the available resources and budget. Indiscriminate cuts are not made to the entire network; we only penalize the poorest performers.

This management mechanism would introduce internal competition among schools, and those that did not achieve good results would be penalized. The economic theory suggests that competition should improve school performance by providing incentives for their efficiency and effectiveness [2]. Through this process, we have created a performance-based scheme of regulation that introduces incentives and motivates schools to perform effectively.

### 5.2. *Resource reallocation. Simulation results*

This section details the reallocation process according to the middle-term policy we described in Section 1 and the result obtained in Section 5.1 using CDEA (model from Eq. (2) column 4 in Table 4).

Under the middle-term policy only non-permanent teachers can be dismissed and permanent teachers are transferable resources. Following the results of the model from Eq. (2) and in accordance with the objective of the Department of Education, we have to reallocate permanent teachers and students from the 15 merged schools (most inefficient schools) to surviving neighboring schools that have a similar environment. In order to decide which schools are candidates to be merged, we use the conditional and robust order-$m$ model explained in Sections 3.2 and 3.3. Table 5 shows summary statistics of the efficiency estimations. In this case, recall that we estimate the model using an output orientation (see note 5 for further details). Following Daraio and Simar [30,31,32], we estimate the value of $m$ as the level for which the percentage of super-efficient observations decreases only marginally. Indeed, if $m$ is small the probability of drawing the evaluated observation is rather low, and consequently, we will observe more super-efficient observations. In our application, we selected $m=50$.

Focusing on the relative performance of schools controlling for exogenous variables, we note that the average efficiency score is $\hat{\theta}_m=1.172$. Thus, inefficient schools could improve their

**Table 5**
Efficiency estimates.
Source: The authors.

| ***Variable*** | ***N*** | ***Min*** | ***Q25*** | ***Mean*** | ***S.D.*** | ***Median*** | ***Q75*** | ***Max*** | ***Super- Efficient units*** | ***Efficient units*** |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\theta}_m$ | 161 | 0.855 | 0.995 | 1.172 | 0.0904 | 1.151 | 1.206 | 1.402 | 6 (3.723%) | 119 (73.913%) |

**Table 6**
Reallocation process (only permanent teachers).
*Source*: The authors

| DMU | PEER | PEER | | BEFORE | | RECEPTION CAPACITY | | REALLOCATION | | AFTER | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Stud. | Teach. | Stud. | Teach. | Stud. | Teach. | Stud. | Teach. | Stud. | Teach. |
| 9 | 126 | 463 | 27 | 257 | 18 | 206 | 9 | 100 | 6 | 357 | 24 |
| 12 | 139 | 415 | 15 | 391 | 22 | 30 | −7 | 24 | 0 | 415 | 22 |
| 13 | 126 | 463 | 27 | 475 | 28 | −12 | −1 | 0 | 0 | 475 | 28 |
| 17 | 139 | 415 | 15 | 364 | 19 | 51 | −4 | 30 | 0 | 394 | 19 |
| 18 | 151 | 175 | 10 | 138 | 12 | −37 | −2 | 17 | 0 | 155 | 12 |

performance by 17.2%, on average, to achieve the efficiency levels of the best practices. Likewise, six schools have an efficiency score below 1, and can be identified as the best performers in the sample. These super-efficient schools are performing better than the average 50 units they are benchmarked with.

Before explaining the reallocation process, it is worth pointing out the social cost of reallocating resources by merging less efficient schools if this simulation exercise were to be carried out. This cost refers to the reallocation of permanent teachers and students to operating schools. Bearing in mind our assumption that no cost is entailed in transferring permanent teachers and dismissing non-permanent teachers, extra effort was made to minimize these side effects, in an attempt to attenuate the possible associated transportation costs. Teachers and students were therefore reallocated to nearby schools.

The proposed reallocation process is conducted as follows. First, we rank all schools in our sample by efficiency score and select the 15 schools with the lowest performance score. Second, we compare each surviving school with a more efficient peer (benchmark schools 126, 139 and 151) in order to calculate the differences in terms of students and permanent teachers. This procedure gives us the capacity of reception of each school that would be included in the new network. Then, using the schools' geographical coordinates, we calculate the distance (in kilometers) between each candidate for merger and the rest of the sample. Finally, we list the schools by distance, in ascending order, to reallocate students and permanent teachers to the closest schools. This is an iterative and dynamic process, so after reallocating the resources of each school, we proceeded to recalculate the reception capacity of the rest.

Through this process a total of 5,814 students and 380 permanent teachers were reallocated to neighboring schools, considering that nobody should have to travel more than four kilometers. The total number of non-permanent teachers in our sample is 943. The model from Eq. (2) (column 4 in Table 4) establishes that it is possible to save 17.21% of them without losing outputs. This percentage represents 163 teachers and the number of non-permanent teachers from the 15 candidate schools for merger is exactly the same, 163. Therefore, these 163 non-permanent teachers are the candidates for dismissal. Table 6 shows a sample of how to carry out the reallocation process.

As can be seen, the system assigns students and teachers according to the reception capacity of each school compared to the peer (the benchmark). Typically, fewer students than the school can receive are reallocated to account for new students that arrive each academic year. For instance, some schools receive both students and teachers (such as *DMU* 9); others only receive students (such as *DMU*s 12, 17 or 18) and others like *DMU* 13, whose current composition is very close to the peer, receive neither students nor teachers.

We end this section by acknowledging the restrictive nature of the middle-term policy we propose. While it is an efficient cost-saving approach, it is still restrictive as it forces the closure of the worst performing schools. It should be noted that other forms of management could improve the results of the current education network without having to merge any schools. The way to enable these choices lies in transparency and accountability (e.g., Grosskopf and Moutray [54] study the performance of public schools in Chicago based on decentralization policies). An example, already implemented in the United States, is the publication of inspectors' evaluation reports [98].

A further example is the United Kingdom, where the Office for Standards in Education (OFSTED) publishes its annual school inspection reports[9] and the annual statistics for education institutions are also published by the Higher Education Statistics Agency (HESA). This mechanism introduces competition and motivates schools to work effectively without penalizing them. The publication of such reports disciplines schools because they are aware of the consequences of poor performance (parents do not choose them, and they cannot be sustained in the future). Notwithstanding the importance of this issue, the limitations of the Spanish education system prevent us from addressing it in depth.

## 6. Conclusions

The main goal of this paper has been to propose a method to assess and re-design a group of schools in a public education network to make additional savings in the education budget. We also reallocated human resources through a new network composition to optimize overall efficiency without jeopardizing the level of educational quality. We applied the model in a sample of 161 schools from the public education network in Catalonia (Spain). We paid particular attention to different policies the decision maker can apply to improve the overall efficiency of the system and reduce expenses. To this end, we extend the CDEA model developed by Lozano and Villa [79] and Mar-Molinero et al. [88] in order to include the specific treatment of transferable and non-transferable resources, and non-discretionary factors.

The paper presents different input oriented CDEA models in order to obtain efficiency improvements inside the education network in line with the strategy established by the government, and bearing in mind the current economic situation in Spain. We find that the current education network is inefficient. Specifically, under the short-term policy, results from model (1) indicate that without modifying the number of schools operating, the school system could save 12.7% of its transferable resources (non-permanent teachers). However, if we try to optimize the performance of the network (middle-term policy, model (2)) the system should consist of 146 schools instead of 161, which would mean a saving of 17.2% of the resources in the form of candidates for dismissal (non-permanent teachers). The remaining teachers and students could be reallocated to schools with suitable reception capacity, considering distance constraints between them.

One interesting finding emerges when simulating the reallocation of permanent teachers and students from the 15 schools

[9] For further information, see OFSTED's website: http://www.ofsted.gov.uk/

to be merged. This procedure would enlarge the remaining 146 schools, thus proving the existence of increasing returns to scale. By resizing the network in this way, we would achieve a new composition consisting of schools with a higher student-teacher ratio without altering the students' results. This result supports the government strategy; however, our model is selective in the reallocation of available resources and budget. Indiscriminate cuts are not made in all schools since only the worst performers are penalized.

These conclusions might have important implications for management practices. They establish the necessary actions to optimize the network and to optimally redistribute the available budget. Thus, the decision maker would have an objective justification for strengthening the efficient schools and incentives for supporting less efficient units. From the empirical point of view, the extended CDEA we propose overcomes the problem of including non-transferable inputs, an aspect that has not been included in previous research. Finally, this study goes beyond a methodological application of a data set since it proposes an implementation involving a real case, so the applicability of the results is highly illustrative.

Despite the practical implications, the paper has some limitations. First, as noted above, we considered data for only one academic year in the simulation analysis about reallocation. It would be very fruitful to undertake a longitudinal analysis to see how the reallocation and merging process would affect the performance of schools that continue operating after applying this procedure. This would provide additional evidence about the effect of mergers on efficiency. As we noted in the introduction, few empirical papers have addressed this issue in the literature to date (e.g., [104,61,64,87,51,69]). However, there is no real data about reallocations in Catalonia due to the fact that this study is a simulation exercise.

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