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Belief Reliability Distribution Based on Maximum Entropy Principle

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ABSTRACT Belief reliability is a new reliability metric based on the uncertainty theory, which aims to measure system performance incorporating the influences from design margin, aleatory uncertainty, and epistemic uncertainty. A key point in belief reliability is to determine the belief reliability distribution based on the actual conditions, which, however, could be difficult when available information is limited. This paper proposes an optimal model to determine the belief reliability distribution based on the maximum entropy principle when kth moments of what can be obtained. An estimation method using linear interpolation and a genetic algorithm is subsequently applied to the optimal model. When only the expected value and the variance are available, the optimal results are in accordance with the maximum entropy principle. It could be observed in the sensitivity analysis that the accuracy of the optimal results is a decreasing function of the width of variances and an increasing function of the number of interpolation points. Therefore, researchers could adapt to different widths of variances and requirements of accuracy by adjusting the number of interpolation points. It could be concluded that this new method to acquire belief reliability distribution is important in the application of belief reliability.

INDEX TERMS Belief reliability distribution, maximum entropy principle, uncertain variable, uncertain distribution.

I. INTRODUCTION

With urgent requirements for the accuracy of the products reliability assessment, the treatment of uncertainties has attracted much attention. Generally, uncertainties can be classified into two types, aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty describes the uncertainty inherent in the physical behavior of the system, and epistemic uncertainty is attributable to the lack of data and information. Probabilistic method can successfully deal with the aleatory uncertainty however it has obvious drawbacks on the treatment of epistemic uncertainty. In 2007, Liu [1] founded uncertainty theory to deal with human's subjective uncertainty by belief degree mathematically and in 2010, Liu [2] perfected it based on normality, duality, subadditivity and product axioms. Based on uncertainty theory, Zeng *et al.* [3] defined belief reliability as the uncertainty measure of the system to perform specific functions within given time under given operating conditions. Zeng *et al.* [4] developed an evaluation method for component belief reliability, which incorporates the impacts from design margin, aleatory uncertainty and epistemic uncertainty. The issue of quantifying the effect from epistemic uncertainty is addressed using a method, which is established based on the performance of engineering activities related to reduce epistemic uncertainties [5], [6]. However, it is still challenging to widely employ belief reliability in reliability engineering due to the scant methods to acquire belief reliability distributions.

Belief reliability distribution is inherently the uncertainty distribution applied in belief reliability. Researchers have explored several methods to get uncertainty distributions. Liu [2] designed uncertain statistics as a methodology for collecting and interpreting experts' experimental data by uncertainty theory and then proposed a questionnaire survey for collecting expert's experimental data. Chen and Ralescu [7] employed uncertain statistics to estimate the travel distance between Beijing and Tianjin and proposed B-spline interpolation to fit a continuous uncertainty distribution.

Gao and Yao [8] designed a procedure of the Delphi method for determining the uncertainty distribution. Both the B-spline interpolation and the Delphi method can adapt to the cases where uncertainty distributions are unknown. When the form of an uncertainty distribution is certain, Liu [2] utilized the principle of the least squares to estimate the parameters of the uncertainty distribution and Wang and Peng [9] proposed a method of moments for calculating the unknown parameters of the uncertainty distribution.

However, in practice, only partial information about an uncertain variable is available and there are infinite numbers of uncertainty distributions that are in accordance with the given information. In such cases, the existing methods cannot determine its uncertainty distribution.

The entropy is a measurement of the degree of uncertainty. For random cases, Jaynes [10] suggested choosing the distribution which has the maximum entropy. In uncertainty theory, Liu [11] proposed the definition of uncertainty entropy resulting from information deficiency to provide a quantitative measurement for the degree of uncertainty of uncertain variables. Chen and Dai [12] proved the maximum entropy principle when the expected value and the variance are finite. This paper will investigate the maximum entropy method and propose an optimal model to estimate belief reliability distribution based on the maximum entropy principle when k-th moments can be obtained.

The paper is structured as follows. Some basic concepts on uncertainty theory will be introduced in Section 2. Subsequently, basic definitions on belief reliability and belief reliability distribution will be provided and a model based on the maximum entropy principle will be proposed to estimate belief reliability distribution in Section 3. The estimation to the proposed model will be discussed with linear interpolation and genetic algorithm (GA) in Section 4. The proposed model will be verified in Section 5 and a sensitivity analysis will be conducted on the number of interpolation points and the width of variances in the same section. The conclusions on belief reliability distribution based on the maximum entropy principle will be discussed in Section 6.

II. PRELIMINARIES

Uncertainty theory was founded by Liu [1] in 2007 and refined by Liu [2] in 2010. Following that, uncertain process [13], uncertain differential equations [13], uncertain calculus [11] and uncertain programming [14] were proposed. Uncertainty theory has been successfully applied in various areas, including finance [15], reliability [8] and graph [16]. Some basic concepts in uncertainty theory will be stated in this section.

Let Γ be a nonempty set, and let $\mathcal{L}$ be a σ-algebra over Γ. Each element Λ in Γ is called an event. Liu [1] defined an uncertain measure by the following axioms:

Axiom 1 (Normality Axiom): $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.

Axiom 2 (Duality Axiom): $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^C\} = 1$ for any event Λ.

Axiom 3 (Subadditivity Axiom): For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \leq \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}, \tag{1}$$

where $\bigcup_{i=1}^{\infty}\Lambda_i$ is the union of Λ_i, $i = 1, 2, \cdots$.

Furthermore, Liu [11] defined a product uncertain measure by the fourth axiom:

$$\mathcal{M}\left\{\prod_{i=1}^{\infty}\Lambda_i\right\} = \bigwedge_{i=1}^{\infty}\mathcal{M}_i\{\Lambda_i\} \tag{2}$$

where $\mathcal{L}_i$ are σ-algebras over Γ_i, Λ_i are arbitrarily chosen events from $\mathcal{L}_i$ for $i = 1, 2, \cdots$, respectively, and $\prod_{i=1}^{\infty}\Lambda_i$ is the intersection of Λ_i, $i = 1, 2, \cdots$.

Definition 1 (See Liu [1]): Let Γ be a nonempty set, let $\mathcal{L}$ be a σ-algebra over Γ, and let $\mathcal{M}$ be an uncertain measure. Then the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

Definition 2 (See Liu [1]): An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, γ is the element in Γ, i.e., for any Borel set $\mathcal{B}$ of real numbers, we have

$$\{\xi \in \mathcal{B}\} = \{\gamma \in \Gamma | g(\gamma) \in \mathcal{B}\} \in \mathcal{L}. \tag{3}$$

Definition 3 (See Liu [1]): The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \leq x\} \tag{4}$$

for any real number x.

Example 1: An uncertain variable ξ is called normal variable if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(\mu - x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in \mathcal{R} \tag{5}$$

denoted by $\mathcal{N}(\mu, \sigma)$ where μ and σ are real numbers with $\sigma > 0$.

Definition 4 (See Liu [1]): Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then the inverse function $\Phi^{-1}(\cdot)$ is called the inverse uncertainty distribution of ξ.

Example 2: The inverse uncertainty distribution of normal uncertain variable $\mathcal{N}(\mu, \sigma)$ is

$$\Phi_x^{-1}(\alpha) = \mu + \frac{\sigma\sqrt{3}}{\pi} ln\frac{\alpha}{1-\alpha}, \tag{6}$$

where α is the belief degree.

III. BELIEF RELIABILITY AND ITS DISTRIBUTION MODEL

A. BASIC DEFINITIONS AND EXAMPLES

Definition 5 (Belief Reliability): Let a product state variable ξ be an uncertain variable, and Ξ be the feasible domain of a product state. Then the belief reliability is defined as the uncertain measure that the product state is within the feasible domain, i.e.,

$$R_B = \mathcal{M}\{\xi \in \Xi\}. \tag{7}$$

In ***Definition 5***, the state variable ξ describes the product's behavior, while the feasible domain Ξ is a reflection of failure criteria. In reliability engineering, since the product's behavior and the failure criteria usually vary with time [17], both ξ and Ξ can be relevant to time t. In this case, the belief reliability metric will be a function of t, denoted by $R_B(t)$.

Example 3: The state variable ξ can represent the product failure time T which describes system failure behaviors. The product is regarded be reliable at time t if the failure time is larger than t. Thus, the belief reliability of the product at time t can be obtained by letting $\Xi = [t, +\infty)$, i.e. Ξ is relevant to t and $R_B(t)$ can be calculated by

$$R_B(t) = \mathcal{M}\{T > t\}. \tag{8}$$

Example 4: The state variable ξ can also represent the performance margin m of a product, which describes system operation behaviors. m describes the distance between a performance parameter and the associated failure threshold. Therefore, Ξ should be $(0, +\infty)$ and the belief reliability of the product can be written as

$$R_B = \mathcal{M}\{m > 0\}. \tag{9}$$

If we consider the degradation process of the performance margin, i.e., ξ is relevant to t, $R_B(t)$ will be

$$R_B(t) = \mathcal{M}\{m(t) > 0\}. \tag{10}$$

Definition 6 (Belief Reliability Distribution): Assume that a product state variable ξ is an uncertain variable, then the uncertainty distribution of ξ is defined as ***Belief Reliability Distribution***.

Example 5: When the state variable ξ represents the product failure time T. Then the uncertainty distribution Φ of T is belief reliability distribution.

Example 6: When the state variable ξ represents the product performance margin m. Then belief reliability distribution Ψ will be the uncertain measure of m, denoted as

$$\Psi(x) = \mathcal{M}\{m \le x\}. \tag{11}$$

B. BELIEF RELIABILITY DISTRIBUTION MODEL

The entropy measures the degree of uncertainty while uncertainty entropy serves as a quantitative measurement of the degree of uncertainty of uncertain variables. When only partial information is accessible, such as k-th moments, there are infinite numbers of uncertainty distributions that are consistent with the provided information. Here we employ the maximum entropy principle to ascertain the belief reliability distribution.

Relative symbols and notations are introduced briefly as follows:

ξ: an uncertain variable,

$\Phi(x)$: an uncertainty distribution of ξ,

μ_k: the k-th moment of uncertain variable ξ, $k = 1, 2, 3, \cdots$.

Definition 7 (See Liu [11]): Suppose that ξ is an uncertain variable with uncertainty distribution Φ. Then its entropy is determined by

$$H[\xi] = \int_{-\infty}^{+\infty} S(\Phi(x))dx \tag{12}$$

where $S(t) = -tlnt - (1-t)ln(1-t)$.

Definition 8: (See Liu and Chen [18]): Let ξ be an uncertain variable with uncertainty distribution Φ, and let k be a positive integer. Then the k-th moment of ξ is

$$E[\xi^k] = \int_{-\infty}^{+\infty} x^k d\Phi(x). \tag{13}$$

The optimal model is written as:

$$\begin{cases} \max H[\xi] = \displaystyle\int_{-\infty}^{+\infty} S(\Phi(x))dx \\ s.t. \displaystyle\int_{-\infty}^{+\infty} x^k d\Phi(x) = \mu_k, \quad \text{for } k = 1, 2, 3, \cdots \end{cases} \tag{14}$$

More specifically,

$$\begin{cases} \max H[\xi] = -\displaystyle\int_{-\infty}^{+\infty} \Phi(x)ln(\Phi(x)) \\ \qquad\qquad + (1-\Phi(x))ln(1-\Phi(x))dx \\ s.t. \displaystyle\int_{-\infty}^{+\infty} x^k d\Phi(x) = \mu_k, \quad \text{for } k = 1, 2, 3, \cdots \end{cases} \tag{15}$$

IV. ESTIMATION TO BELIEF RELIABILITY DISTRIBUTION MODEL

This section discusses the estimation to the optimal model, which approximates belief reliability distribution based on the maximum entropy principle. Since the form of the belief reliability distribution is unknown, it is intuitive to apply the discretization method to obtain the approximate solution of the distribution. To obtain these discrete data, GA method is adopted to find the global optimum solution with the constraints on k-th moments. Subsequently, the linear interpolation method is used in this paper to estimate the belief reliability distribution.

Belief reliability distribution is discretized into the form of a piecewise linear function as shown in Eq.(16).

$$\Phi(x) = \begin{cases} 0 & \text{if } x < x_1 \\ \alpha_i + \dfrac{(\alpha_{i+1} - \alpha_i)(x - x_i)}{x_{i+1} - x_i} & \text{if } x_i < x < x_{i+1} \\ 1 & \text{if } x > x_N. \end{cases} \tag{16}$$

Then the Eq.(13) can be written as follows:

$$\mu_k = E[\xi^k] = \sum_{i=1}^{N-1} \frac{(\alpha_{i+1} - \alpha_i)(x_{i+1}^{k+1} - x_i^{k+1})}{(k+1)(x_{i+1} - x_i)}, \quad k = 1, 2, \cdots. \tag{17}$$

When the belief degrees satisfy:

$$\alpha(i) = (2i-1)/2N, \quad i = 1, 2, \cdots, N \tag{18}$$

Eq.(12) can be written as:

$$H = N \times \sum_{i=1}^{N-1} p(i) \times (x_{i+1} - x_i) \tag{19}$$

where $p(i)$ is calculated by the Eq.(20).

$$\begin{aligned} p(i) = & -0.5(\alpha(i+1)^2 ln(\alpha(i+1)) - \alpha(i)^2 ln(\alpha(i))) \\ & + 0.25(\alpha(i+1)^2 - \alpha(i)^2) \\ & - 0.5((1-\alpha(i))^2 ln((1-\alpha(i))) \\ & - (1-\alpha(i+1))^2 ln((1-\alpha(i+1)))) \\ & + 0.25((1-\alpha(i))^2 - (1-\alpha(i))^2) \end{aligned} \tag{20}$$

Thus, the optimal model (15) is a non-linear programming problem as follows:

$$\begin{cases} \max H[\xi] = N \times \sum_{i=1}^{N-1} p(i) \times (x_{i+1} - x_i) \\ s.t. \sum_{i=1}^{N-1} \frac{(\alpha_{i+1} - \alpha_i)(x_{i+1}^{k+1} - x_i^{k+1})}{(k+1)(x_{i+1} - x_i)} - \mu_k = 0, \\ \qquad\qquad\qquad\qquad k = 1, 2, \cdots \\ x_1 < x_2 < \cdots < x_N \end{cases} \tag{21}$$

where $p(i)$ is calculated by Eq.(20).

Then genetic algorithm is applied to solve this non-linear programming problem. Finally, the approximation to the belief reliability distribution can be obtained by linear interpolation methods.

In summary, the estimation to the belief reliability distribution model can be divided into five steps, which is concisely illustrated in the Fig.1.

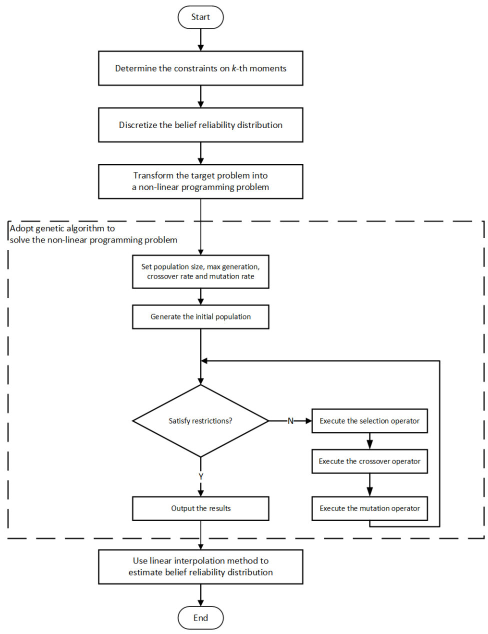

FIGURE 1. Flow chart of the estimation to the belief reliability distribution model.

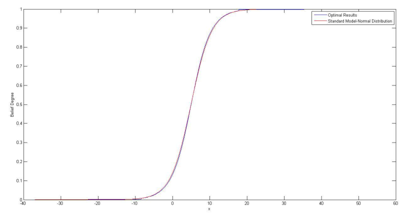

FIGURE 2. Optimal results and the standard model at $\mu = 5, \sigma^2 = 25, N = 500$.

V. MODEL VERIFICATION AND SENSITIVITY ANALYSIS

This section will briefly introduce the maximum entropy principle proved by Chen and Dai [12] and then verify the proposed optimal model based on this principle. Moreover, a sensitivity analysis is conducted on the effect of the width of variances and the number of interpolation points.

A. MODEL VERIFICATION

In uncertainty theory, Chen and Dai [12] proved a theorem called maximum entropy principle when the expected value and variance are known.

Theorem 1 (Maximum Entropy Principle): Let ξ be an uncertain variable with finite expected value μ and variance σ^2. Then

$$H[\xi] \leq \frac{\pi\sigma}{\sqrt{3}} \tag{22}$$

and the equality holds if is a normal uncertain variable with expected value e and variance σ^2 , i.e. $\mathcal{N}(\mu, \sigma)$.

According to the maximum entropy principle theorem, belief reliability distributions based on maximum entropy principle are determined when the 1st and 2nd moments are known.

When the expected value $\mu = 5$, the variance $\sigma^2 = 25$ and the number of interpolation points $N = 500$, the optimal results and the standard model are shown in Fig.2. The red line represents the standard model, normal uncertainty distribution $\mathcal{N}(5, 5)$, and the blue one shows the estimation to the proposed optimal model. As demonstrated in the Fig.2, there is not a great difference between the optimal results and the ideal results, which leads to the conclusion that the estimation by using linear interpolation and GA is effective to the optimal model. Fig.3 shows that absolute errors between the optimal results and standard model. As shown in Fig.3, the absolute errors are no more than 0.015, which also shows there is not a great difference between the optimal results and the ideal results. It could be concluded from the Fig.2 and Fig.3 that the optimal results are consistent with the standard model and the proposed estimating approach is effective to determine belief reliability distribution based on the maximum entropy principle.

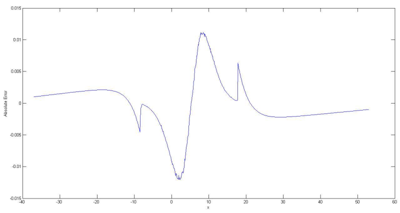

FIGURE 3. **Absolute errors between optimal results and standard model at $\mu = 5, \sigma^2 = 25, N = 500$.**

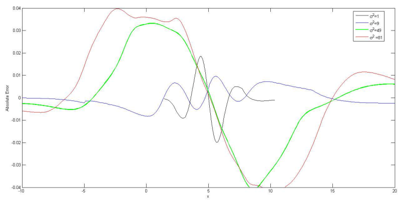

FIGURE 4. **Absolute errors between optimal results and standard model at $\mu = 5, N = 500$.**

B. SENSITIVITY ANALYSIS

As illustrated in Eq.(22), uncertainty entropy H is associated with variances. A sensitivity analysis on the width of variances is conducted to investigate the relationship between the width of variances and the optimal results.

In Fig.4, the four curves show the fluctuation of the absolute errors between optimal results and standard model at $\mu = 5, N = 500$ with variances at 1,9,49,81, respectively. As can be seen from the figure, there is an increasing tendency of the degree of the fluctuations with the width of variances raising. In other words, it could be implied that the accuracy of the optimal results decreases as the variances increase when the expected value and the number of interpolation points keep the same.

Moreover, the number of interpolation points N also has a significant influence on the optimal results. A sensitivity analysis on the number of interpolation points is conducted to explore the connection between the number of interpolation points and the optimal results.

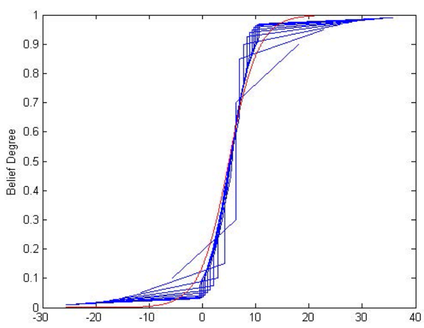

FIGURE 5. **Optimal results and the standard model at at $\mu = 5, \sigma^2 = 25, N = 5:5:50$.**

Fig.5 shows the number of interpolation points from 5 to 50 with the step length 5 when the expected value $\mu = 5$ and the variance $\sigma^2 = 25$ and Fig.6 shows the number of interpolation points from 55 to 100 with the step length 5 when the expected value and the variance keep the same. As shown in Fig.5, the fitting results of linear part are growing better as the number of interpolation is increasing. As shown in Fig.6, the fitting results of non-linear part are approaching the standard model as the number of interpolation points is growing. It could be inferred that the larger the number of interpolation points is, the better the optimal results are.

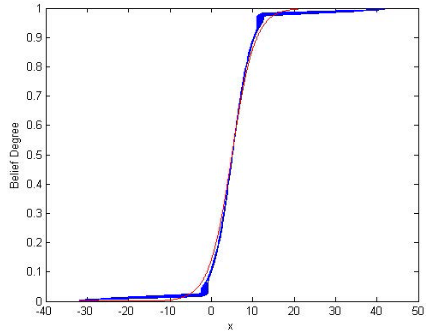

FIGURE 6. **Optimal results and the standard model at $\mu = 5, \sigma^2 = 25, N = 55:5:100$.**

From the above analysis, it could be observed that the accuracy of the optimal results is a decreasing function of the width of variances and an increasing function of the number of interpolation points.

VI. DISCUSSIONS AND CONCLUSIONS

This paper specified the definition of belief reliability and belief reliability distribution, extended the application of maximum entropy principle in uncertainty theory, and proposed an optimal model based on maximum entropy principle to estimate belief reliability distribution and an approach to estimate the optimal model using linear interpolation and genetic algorithm. According to the theorem proved by Chen and Dai [12], the proposed estimating method is effective to determine the belief reliability distribution. The estimating results are sensitive to the width of the variances and the number of interpolation points. Based on the results of the sensitivity analysis, when the number of interpolation points keeps still, the accuracy of the optimal results decreases as the width of the variance increases. In addition, the accuracy of the optimal results is an increasing function of the number of interpolation points when the width of the variance keeps the same. In actual situations, it is possible to obtain more accurate optimal results when we increase the number of interpolation points. Besides, the number of interpolation points also reflects the data density of belief degree according to Eq. (18). Therefore, when we only concentrate on belief degree around 0.5, we could adopt a small number of interpolation points to get satisfying optimal results. By contrast, when we focus on belief degree near to 0 or 1, we have to adopt a large number of interpolation points to obtain reasonable optimal results.

The proposed optimal model to estimate belief reliability distribution based on the maximum entropy principle can be applicable to the cases when k-th moments of an uncertain state variable are available, which is important in the development of belief reliability. Moreover, the proposed approach also provides a new approach to obtain uncertainty distributions in uncertainty theory.

The estimating approach applied is a simple but time-consuming one to obtain optimal results. Therefore, the alternative of the estimation deserves further investigation. Moreover, this paper only considers k-th moments as the constraints of the optimal model. More information could be included in the optimal model to adapt to diverse cases.

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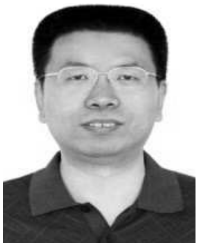

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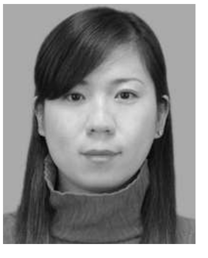

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