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Multi-step wind speed forecasting based on a hybrid forecasting architecture and an improved bat algorithm



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ABSTRACT

As one of the most promising sustainable energy sources, wind energy plays an important role in energy development because of its cleanliness without causing pollution. Generally, wind speed forecasting, which has an essential influence on wind power systems, is regarded as a challenging task. Analyses based on single-step wind speed forecasting have been widely used, but their results are insufficient in ensuring the reliability and controllability of wind power systems. In this paper, a new forecasting architecture based on decomposing algorithms and modified neural networks is successfully developed for multi-step wind speed forecasting. Four different hybrid models are contained in this architecture, and to further improve the forecasting performance, a modified bat algorithm (BA) with the conjugate gradient (CG) method is developed to optimize the initial weights between layers and thresholds of the hidden layer of neural networks. To investigate the forecasting abilities of the four models, the wind speed data collected from four different wind power stations in Penglai, China, were used as a case study. The numerical experiments showed that the hybrid model including the singular spectrum analysis and general regression neural network with CG-BA (SSA-CG-BA-GRNN) achieved the most accurate forecasting results in one-step to three-step wind speed forecasting.

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1. Introduction

As one of the most promising potential renewable energy sources [1], wind energy has attracted the focus of many researchers and scientists [2], and nearly every government across the world has introduced positive policies to support wind energy development [3,4]. In 2015, the global total capacity of wind farms is approximately 432,419 MW, with the 22% growth rate, as shown in Fig. 1 [5]. With the increased proportion of wind energy in whole energy networks, accurate wind speed forecasting results are

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becoming increasingly crucial for managers to schedule the daily power distribution and decrease the reserve capacity. To protect wind power from the breakdown and make sure the success of wind power conversation, accurate forecasting results of wind speed are also required [6]. However, due to the non-stationary and nonlinear fluctuations, wind speed is regarded as one of the hardest weather parameters to predict [7,8].

In recent decades, many methods have been presented for wind speed forecasting, and these methods can be divided into four categories [9]: (a) physical models; (b) statistical models; (c) spatial correlation models; and (d) artificial intelligence models. Physical models which are based on physical parameters, such as topography, temperature and pressure, are usually applied in long term wind speed forecasting [10–12]. Statistical models are built based on the mature statistical equations to get the potential change rule from history data sampling [13–17]. Spatial correlation models mainly consider the spatial relationship of wind speed at different sites. In some situations, it can obtain higher precision [18,19]. With the rapid development of artificial techniques, some artificial intelligence forecasting methods, including artificial neural networks (ANNs) [20–25], fuzzy logic methods [18,26] and support



Abbreviations: ANN, artificial neural network; ARIMA, autoregressive integrated moving average; BA, bat algorithm; CSA, cuckoo search algorithm; CG, conjugate gradient; EA, evolutionary algorithm; EEMD, ensemble empirical mode decomposition; EMD, empirical mode decomposition; FEEMD, fast ensemble empirical mode decomposition; FVD, forecasting validity degree; GA, genetic algorithm; GRNN, general regression neural network; MAE, mean absolute error; MAPE, mean absolute percentage error; MSE, mean square error; PSO, particle swarm optimization; RBFNN, radical basis function neural network; SDA, steepest descent algorithm; SSA, singular spectrum analysis; SVM, support vector machine; WD, wavelet decomposition; WPD, wavelet packet decomposition.

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| Nomenclature | | | | | | |
|--|---|---|--|--|--|--|
| Nomenclat $\alpha \qquad \beta \qquad \beta \qquad d_i^k \qquad d_i \qquad \varepsilon \qquad \phi(d_i) \qquad F_i \qquad -\nabla f(\boldsymbol{x}^{iter}) \qquad \boldsymbol{g}_i^k \qquad h_j \qquad I \qquad IMF_i(t)$ | ture a random vector, with a value between 0 and 1 a random vector, with a value between 0 and 1 the search direction of \mathbf{x}_i at iteration k Euclidian distance a random vector, with a value between 0 and 1 outputs from the hidden layer of RBFNN the fitness function of \mathbf{x}_i gradient of \mathbf{x}^{iter} gradient of \mathbf{x}_i at iteration k correlation coefficient input vector of RBFNN intrinsic mode function | $M \\ N \\ O \\ r \\ R \\ r_n(t) \\ \sigma \\ \sigma_s^i \\ S_s \\ S_w \\ t \\ v^t$ | total number of CG iterations number of generations <i>P</i> output of RBFNN pulse rate of a bat spread parameter <i>n</i> th residue spread parameter marginal standard deviation simple summation weighted summation current iteration number the velocity of x at iteration <i>t</i> | | | |
| $\begin{array}{l} \mathbf{IMF}_{j}\left(t\right)\\ iter\\ iter\\ Iter_{max}\\ K\\ k_{j}^{i}\\ L\\ L\\ m_{i}\\ m_{j}^{j}\\ \lambda_{i}^{iter}\\ \lambda_{k}^{i} \end{array}$ | intrinsic mode function current iteration number maximum number of iterations shape matrix (<i>i</i> , <i>j</i>)th element of the shape matrix <i>K</i> the loudness of a bat center vector <i>j</i> th element of center vector step length step length of x _i at iteration <i>k</i> | v_i^t w w_i X x_b x_t x_i^t x_i^t Y | the velocity of \mathbf{x}_i at iteration t interconnection weight weight of the hidden layer of RBFNN input vector of GRNN the value of \mathbf{x} with the best fitness value in the population training data position of \mathbf{x}_i at iteration t positions of bats output vector of GRNN | | | |

vector machines (SVMs) [27], have been developed for wind speed forecasting.

Meanwhile, to decrease the negative influences that are intrinsic to individual models, many hybrid wind speed forecasting models have been proposed [28–36].

To achieve higher forecasting accuracy, some data-processing algorithms, such as wavelet decomposition (WD) [28], wavelet packet decomposition (WPD) [29], empirical mode decomposition (EMD) [30], the ensemble empirical mode decomposition (EEMD) algorithm [31] and the fast ensemble empirical mode decomposition (FEEMD) algorithm [32], have been employed in ANNs to build hybrid models. The data decomposition, which could reduce the non-stationary feature of the original data, promotes the forecasting performance indirectly.

Moreover, intelligent optimization algorithms including the genetic algorithm (GA) [33], particle swarm optimization (PSO) [34], the evolutionary algorithm (EA) [35], and the cuckoo search algorithm (CSA) [36], are utilized to determine the initial weights and thresholds of ANNs. In 2010, Yang proposed the bat algorithm (BA) [37], which is inspired by the echolocation characteristics of bats with varying pulse rates of emission and loudness. It has been applied to a wide range of optimization applications [38], including image processing [39], classifications [40], scheduling [41], the electricity market [42], energy systems [43] and various other problems. Experiments have shown its promising efficiency for global optimization.

Analyses based on single-step wind speed forecasting have been widely used, while their results are insufficient in ensuring the reliability and controllability of wind power systems. Thus, it is required to build a model to achieve accurate results for multistep wind speed forecasting. Among various ANN models, the radical basis function neural network (RBFNN) and general regression neural network (GRNN) are good choices to achieve high convergence rates and accurate results. In this paper, a hybrid architecture, which contains four hybrid models, with two decomposing algorithms (i.e., FEEMD and singular spectrum analysis (SSA)) which are used to realize the non-stationary wind speed decomposition, and the modified RBFNN and GRNN is proposed for wind speed forecasting. In the modified RBFNN and GRNN, an improved BA, which is on the basis of conjugate gradient (CG) method to improve convergence performance over time and prevent individual bats from entrapment in local optima, is introduced to optimize the initial weights and thresholds of RBFNN and GRNN. The aim of this study is to investigate and enhance the forecasting performance of hybrid model based on signal processing algorithms, intelligent optimization algorithm and artificial neural networks for multi-step accurate wind speed forecasting. To investigate the forecasting abilities of the four models, the wind speed data collected from four different wind power stations in Penglai, China, were used as a case study. The main contributions in this paper are demonstrated as follows.

- (1) The forecasting focus of the forecasting architecture is not only on the single-step forecasting but also on the multistep forecasting. Although the wind speed single-step predictions have been studied widely, to protect the wind power, wind speed single-step forecasting results alone are insufficient, and wind speed multi-step forecasting results are definitely expected, thus the forecasting architecture is aim to enhance the forecasting accuracy of multi-step wind speed forecasting.
- (2) To globally investigate the forecasting performance of different combination of decomposing algorithms and neural networks, a forecasting architecture contains four hybrid models is proposed. In the architecture, four different hybrid forecasting models based on the two most popular decomposing algorithms, an improved optimization algorithm and two neural networks, are investigated and compared (the performance of multi-step forecasting is given special attention in the investigation) with four different sites data for one-step to three-step forecasting to obtain the best one.
- (3) **The speed of local convergence and the accuracy of finding the optimal solution of BA are enhanced**. To improve both the exploration and exploitation capacities and avoid the weakness of the local optima searching ability, the improved BA based on CG is proposed, and to evaluate the improved algorithm, four testing functions are used.



Fig. 1. Top 10 countries of wind power newly increased installed capability in 2015.

- (4) **The forecasting accuracy and stability of RBFNN and GRNN are enhanced**. The improved BA, CG-BA, is employed to select the initial weights and thresholds for the RBFNN and GRNN. According to the experiment results, the forecasting performance of RBFNN and GRNN are directly enhanced with CG-BA.
- (5) To validate the effectiveness of the proposed hybrid forecasting architecture, a number of comparable experiments are provided. Besides the hybrid FEEMD-CG-BA-RBFNN model, the hybrid FEEMD-CG-BA-GRNN model, the hybrid SSA-CG-BA-RBFNN model and the hybrid SSA-CG-BA-GRNN model, the single RBFNN model, the single GRNN model and the single ARIMA (autoregressive integrated moving average) model are also included in the performance comparison to obtain the best combination in the proposed architecture.

The remainder of the paper is organized as follows. Section 2 introduces the hybrid forecasting strategy proposed in this paper. Section 3 presents the wind speed decomposition contained in the hybrid forecasting strategy. Section 4 develops a new improved optimization algorithm. Section 5 proposes four hybrid models. The forecasting results of the proposed hybrid models and comparisons are discussed in Section 6. Finally, Section 7 concludes the paper.

2. Framework of the proposed hybrid architecture

The flowchart of the proposed hybrid architecture in this study is given in Fig. 2. In Fig. 2, the proposed study can be summarized briefly as follows:

- Decompose the original wind speed time series with the FEEMD algorithm and the SSA algorithm into several sub-layers.
- Build the modified neural networks, CG-BA-RBFNN and CG-BA-GRNN to predict each wind speed sub-layers for the one-step, two-step and three-step prediction.
- Summarize the one-step, two-step and three-step predicted results of each sub-layers from FEEMD and SSA to obtain the final results of CG-BA-RBFNN and CG-BA-GRNN.
- Compare the forecasting performance of each model and find the best one. The compared algorithms include four hybrid models, i.e. FEEMD-CG-BA-RBFNN, FEEMD-CG-BA-GRNN, SSA-CG-BA-RBFNN and SSA-CG-BA-GRNN, in the proposed architecture, four comparison hybrid models, i.e. FEEMD-BA-RBFNN, FEEMD-BA-GRNN, SSA-BA-RBFNN and SSA-BA-GRNN and three single models, i.e. RBFNN, GRNN and ARIMA.

3. Wind speed decomposition

In this paper, two decomposing methods, the SSA algorithm and the FEEMD algorithm, are employed to process the original wind speed data. More information about SSA and FEEMD are shown in Appendix A.

4. CG-BA

In this part, CG-BA is proposed and four test functions are employed to evaluate this developed algorithm.



Fig. 2. Structure of hybrid forecasting strategy.

Table 1 Test functions.

| Function name | Modal characteristic | Test function | Variable domain | Global optimum |
|---------------|----------------------|--|---------------------------|-----------------------------|
| Sphere | Unimodal | $f(\mathbf{x}) = \sum_{i=1}^{d} x_i^2$ | $x_i \in [-5.12, 5.12]$ | $f_{\min}(0,0,0\cdots0)=0$ |
| Rosenbrock | Multi-modal | $f(X) = \sum_{i=1}^{d-1} [100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2]$ | $x_i \in [-2.084, 2.084]$ | $f_{\min}(1,1,1\cdots 1)=0$ |
| Rastrigin | Multi-modal | $f(X) = \sum_{i=1}^{d} (x_i^2 - 10(2\pi x_i) + 10)$ | $x_i \in [-5.12, 5.12]$ | $f_{\min}(0,0,0\cdots0)=0$ |
| Schaffer | Multi-modal | $f(X) = \frac{\sin^2 \sqrt{\sum_{l=1}^d x_l^2 - 0.5}}{\left[1 + 0.001 \left(\sum_{l=1}^d x_l^2\right)\right]^2} + 0.5$ | $x_i \in [-5.12, 5.12]$ | $f_{\min}(0,0,0\cdots0)=0$ |

4.1. CG-BA

The bat algorithm is a novel optimization algorithm proposed by Yang [48,49], which was inspired by the echolocation behavior of natural bats in determining their foods. BA not only offers powerful global exploration and exploitation abilities but also has the good ability to find the local optimum.

However, conventional BA continues to suffer from slow convergence during the later period of optimization when it is applied to large-scale and complex problems. To speed up the convergence, a new improved BA based on the CG quasi-Newton method was developed in this work. CG is developed on the basis of the Newton algorithm and the steepest descent algorithm (SDA). Meanwhile, the shortcomings of these two algorithms, slow convergence of SDA and complex computation of Newton algorithm, are overcome with CG [50]. As shown in Fig. 2 Part b, it is used when BA updates solutions in an iteration to find a local optimal solution and thus enhance the local optimization ability and the speed of the local convergence of the whole algorithm.

Let \mathbf{x}^{iter} be the positions of bats in BA, where *iter* represents the current iteration number. Generally, \mathbf{x}^{iter} is input into the fitness

function directly to evaluate the current best value. To improve the local search ability of BA, a CG circulation is added in BA. In this circulation, \mathbf{x}^{iter} will be the initial value to continue searching with the gradient $-\nabla f(\mathbf{x}_{j}^{iter})$ and step length λ_{j}^{iter} . This iterative loop could be presented as

$$\boldsymbol{x}_{j+1}^{iter} = \boldsymbol{x}_j^{iter} + \lambda_j^{iter} \boldsymbol{d}_j^{iter}, \quad (j = 0, 1, \dots, M-1)$$
(1)

where $d_i^{iter} = -\nabla f(\mathbf{x}_i^{iter})$, and *M* is the total number of CG iterations.

After processing the CG circulation, a new position of \mathbf{x}_{M}^{iter} is obtained. \mathbf{x}_{M}^{iter} is input into the fitness function to evaluate a value as the current best result. Then \mathbf{x}_{M}^{iter} is updated to \mathbf{x}_{0}^{iter+1} according to the BA rules [48,49]. Meanwhile, to keep the results from trapping in local optimums, a lot of experiments have been done to select the total iteration number *M*. Finally we find that when *M* is set in the region from 4 to 6, the optimization performance is good. If *M* is less than 4, the rate of convergence may not be enhanced. And if *M* is greater than 6, the results are easily trapped in local optimums. The pseudo code for CG-BA is provided in Algorithm 1.

Algorithm 1. CG-BA

| $x_{n} - \text{the value of } x \text{ with the best fitness value in the population}$ $Perameters:$ $a - a random vector, with a value between 0 and 1. p - a random vector, with a value between 0 and 1. p - a random vector, with a value between 0 and 1. p - a random vector, with a value between 0 and 1. p - a random vector, with a value between 0 and 1. p - the fitness function of x, N - the number of generations P. t - current iteration number. Iter max - the maximum number of iterations of conjugate gradient algorithm. L - the loudness of a bat. T - the pulse rate of a bat. T - the splate rate of a bat. T - the pulse rate of a bat. T - the splate rate of a bat. T - the splate rate of a bat. T - the pulse rate of a bat. T - the splate rate of x, at iteration t. T - the pulse rate of a bat. T - the steplength of x, at iteration t. T - the splate rate of x, at iteration t. T - the steplength of x$ | | Output |
|---|----------|---|
| Parameters:a—a random vector, with a value between 0 and 1. <i>p</i> —a random vector, with a value between 0 and 1. <i>p</i> —a random vector, with a value between 0 and 1. <i>p</i> —a random vector, with a value between 0 and 1. <i>p</i> —a random vector, with a value between 0 and 1. <i>p</i> —the fitness function of x, <i>N</i> —the number of generations <i>P</i> . <i>t</i> —current iteration number. <i>t</i> —current iteration number of iterations. <i>M</i> —the maximum number of iterations. <i>L</i> —the loudness of a bat. <i>vi</i> _—the velocity of x, at iteration <i>t</i> . <i>xi</i> _1—the velocity of x, at iteration <i>t</i> . <i>xi</i> _1—the velocity of x, at iteration <i>k</i> . <i>xi</i> _1—the velocity of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction of <i>x</i> , at iteration <i>k</i> . <i>xi</i> _1—the search direction <i>P</i> (<i>x</i> , <i>i</i> =1, <i>2</i> ,, <i>N</i>) in random positions.*/1 <i>f</i> = FORE ACCH <i>i</i> =1:NDO <i>f</i> = FOR B (<i>C</i> (<i>H</i>) = 1:ND <i>ff</i> = <i>f</i> = <i>f</i> (<i>xi</i> = <i>f</i>) = <i>f</i> (<i>xi</i> | | x_{i} —the value of x with the best fitness value in the population |
| Parameters:a — a random vector, with a value between 0 and 1. p — a random vector, with a value between 0 and 1. p — a random vector, with a value between 0 and 1. F_{i} — the fitness function of x_{i} . N — the number of generations P . r — current iteration number. $Ilter max_max$ —the maximum number of iterations. M — the maximum number of iterations. M — the maximum number of iterations of conjugate gradient algorithm. L — the loudness of a bat. r'_{i} — the velocity of x_{i} at iteration I . x'_{i} — the velocity of x_{i} at iteration k . x'_{i} — the position of x_{i} at iteration k . x'_{i} — the search direction of x_{i} at iteration k . a'_{i} — the steplength of x_{i} at iteration k . a'_{i} — the search direction of x_{i} at iteration k . a'_{i} — the steplength of x_{i} at iteration k . a'_{i} — the steplength of x_{i} at iteration k . i'_{i} — the steplength of x_{i} at iteration k . i'_{i} — the corresponding fitness function F_{i} . 5 END FOR 6 WHILE $<^{J(r)}$ max DO 7 FOR EACH $i=1:N$ DO 8 $y'_{i} = y'_{i}^{-1} + y'_{i}$ 10 $x'_{i} = x'_{i}^{-1} + y'_{i}$ 11 END FOR 12 y''_{i} exclusite gradient algorithm.*/ 13 FOR EACH $i=1:N$ DO 14 $y'_{i} = y'_{i}^{-1} + y'_{i}^{-1}$ 17 $y'_{i} = y'_{i}^{-1} + y'_{i}^{-1}$ 18 $y'_{i} = -g'_{i}^{+1} + y'_{i}^{-1}$ 19 $y'_{i} = -g'_{i}^{+1} + y'_{i}^{-1}$ 10 $y'_{i} = -g'_{i}^{+1} + y'_{i}$ | | |
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| $F_{i} = \text{the fitness function of } x_{i}.$ $N_{i} = \text{the number of generations } P_{i}.$ $I_{=\text{current iteration number.}}$ $I_{er_{max}} = \text{the maximum number of iterations.}$ $M_{i} = \text{the maximum number of iterations of conjugate gradient algorithm.}$ $I_{i} = \text{the loudness of a bat.}$ $v_{i}' = \text{the velocity of } x_{i}$ at iteration $I.$ $x_{i}' = \text{the velocity of } x_{i}$ at iteration $I.$ $g_{i}^{k} = \text{the search direction of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ at iteration $k.$ $\lambda^{k} = \text{the steplength of } x_{i}$ $\lambda^{k} = \frac{1}{k} \cdot x_{i} \cdot x_{i}^{k} \cdot x_{i}^{k}$ | | ε a random vector, with a value between 0 and 1. |
| $N = \text{the number of generations } P.$ $I = -\text{current iteration number:}$ $Iker_{max} = -\text{the maximum number of iterations.}$ $M = \text{the maximum number of iterations.}$ $I = \text{the number of a bat.}$ $r = \text{the pulse rate of a bat.}$ $r'_{i} = \text{the lower of a bat.}$ $r'_{i} = \text{the pulse rate of a transmitter of iteration t.}$ $g_{i}^{k} = \text{the pulse rate of a iteration t.}$ $g_{i}^{k} = \text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = \text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = -\text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = -\text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = -\text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = -\text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = -\text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = -\text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = -\text{the search direction of x_{i} at iteration k.}$ $d_{i}^{k} = -\text{the assearch direction of x_{i} at iteration k.}$ $f \text{ FOR EACH i=1:N DO$ $f = \int e^{k_{i}} e^{k_{i}} = \sqrt{r} \left(\frac{x_{i}^{k}}{x_{i}} \right)^{k_{i}}$ $g^{k_{i}} = \sqrt{r} \left(\frac{x_{i}^{k}}{x_{i}} \right)^{k_{i}}$ $g^{k_{i}} = -g_{i}^{k} + \varphi^{k_{i}} d_{i}^{k_{i}}$ $f = -g_{i}^{k_{i}} - \varphi^{k_{i}} d_{i}^{k_{i}}$ $g^{k_{i}} = -g_{i}^{k_{i}} d_{i}^{k_{i}}$ $g^{k_{i}} = -g_{i}^{k_{i}} d_{i}^{k_{i}}$ $g^{k_{i}} = -g_{i}^{k_{i}} d_{i}^{k_{i}}$ $f = -g_{i}^{k_{i}} + \lambda^{k_{i}} d_{i}^{k_{i}}$ $f = -g_{i}^{k_{i}} + \lambda^{k$ | | F — the fitness function of x_i . |
| $Iter_{max} - the maximum number of iterations. M Iter maximum number of iterations of conjugate gradient algorithm. L—the loudness of a bat. r—the pulse rate of a bat. r—the pulse rate of a bat. right - the velocity of x, at iteration t. x'_ithe search direction of x, at iteration k. d'_ithe search direction of x, at iteration k. \lambda^{i}the search direction of x, at iteration k.\lambda^{i}the search direction of x, at iteration k.\lambda^{i}the search direction of x, at iteration k.1 /*Initialize generation P (xi,i=1,2,,N) in random positions.*/2 /*Initialize t=0.*/3 FOR EACH i=1:N DO4 Evaluate the corresponding fitness function Fi5 END FOR6 WHILE t=1(ter_max DO7 FOR EACH i=1:N DO8 F_{i}=F_{min}+(F_{max}-F_{min})\alpha9 V_{i}=v_{i}^{i-1}+(x_{i}'-x^{*})F_{i}'10 x_{i}'=x_{i}^{i-1}+v_{i}'11 END FOR12 *Use conjugate gradient algorithm.*/13 FOR EACH i=1:N DO14 FOR EACH i=1:N DO15 q^{i-1}=\frac{\ g^{k}\ ^{2}}{\ g^{k-1}\ ^{2}}16 q^{i-1}=\frac{\ g^{k}\ ^{2}}{\ g^{k-1}\ ^{2}}17 q^{i}=-g^{i}+\varphi^{k-1}d^{i-1}18 q^{i}=-g^{i}+\varphi^{k-1}d^{i-1}18 x_{i}^{i}=-g^{i}+x_{i}^{k-1}d^{k-1}19 x_{i}^{i}=x_{i}^{i}+\lambda^{k}d^{i}19 x_{i}^{i}=x_{i}^{i}+\lambda^{k}d^{i}10 END FOR21 FOR EACH i=1:N DO22 END FOR23 ^*Update the current best solution x*.*/24 FOR EACH i=1:N DO25 EVALUATE the current best solution x*.*/26 END FOR27 UE for FOR29 x_{i}=x_{i}=x_{i}^{i}+\lambda^{k}d^{i}$ | | N—the number of generations P . |
| $M = \text{the maximum number of iterations.}$ $M = \text{the maximum number of iterations of conjugate gradient algorithm.}$ $L = \text{the loudness of a bat.}$ $r = \text{the pulse rate of a bat.}$ $r'_{i} = \text{the velocity of } x_{i} at iteration t.$ $x'_{i} = \text{the velocity of } x_{i} at iteration t.$ $g_{i}^{k} = \text{the gradient of } x_{i} at iteration t.$ $g_{i}^{k} = \text{the search direction of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $\lambda^{k} = \text{the steplength of } x_{i} at iteration k.$ $k^{k} = \text{the steplength of } x_{i} at iteration k.$ $k^{k} = \text{the steplength of } x_{i} at iteration k.$ $FOR EACH (i = 1:N DO$ $k^{k} = \frac{x_{i}^{k} + x_{i}^{k} +$ | | <i>t</i> —current iteration number. |
| <i>L</i> —the loadness of a bat. <i>r</i> —the pulse rate of a bat. <i>r</i> —the pulse rate of a bat. <i>r</i> ⁱ —the velocity of <i>x_i</i> at iteration <i>t</i> . <i>x</i> ⁱ ₁ —the position of <i>x_i</i> at iteration <i>k</i> . <i>d</i> ⁱ ₁ —the gradient of <i>x_i</i> at iteration <i>k</i> . <i>d</i> ⁱ ₁ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ —the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ =the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ =the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ =the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ =the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ =the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ =the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ =the search direction of <i>x_i</i> at iteration <i>k</i> . <i>i</i> ⁱ =the search direction <i>x</i> ⁱ = <i>x</i> ⁱ / <i>i</i> ⁱ at <i>x</i> ⁱ = <i>x</i> ⁱ + <i>x</i> ⁱ = <i>x</i> ⁱ + <i>x</i> ⁱ = <i>x</i> | | $Iter_{max}$ —the maximum number of iterations. |
| $r - \text{the pulse rate of a bat.}$ $r - \text{the pulse rate of a bat.}$ $r'_{i} - \text{the pulse rate of a bat.}$ $r'_{i} - \text{the pulse rate of a bat.}$ $r'_{i} - \text{the position of x_{i} at iteration t.}$ $g_{i}^{k} - \text{the gradient of x_{i} at iteration k.}$ $d_{i}^{k} - \text{the search direction of x_{i} at iteration k.}$ $\lambda^{k} - \text{the steplength of x_{i} at iteration k.}$ $\lambda^{k} - \text{the steplength of x_{i} at iteration k.}$ $\lambda^{k} - \text{the steplength of x_{i} at iteration k.}$ $\lambda^{k} - \text{the steplength of x_{i} at iteration k.}$ $\lambda^{k} - \text{the steplength of x_{i} at iteration k.}$ $\lambda^{k} - \text{the steplength of x_{i} at iteration k.}$ $\frac{1}{2} / ^{k} \text{Initialize } t=0.*/$ FOR EACH $t=1:N$ DO $\frac{1}{2} \text{ Evaluate the corresponding fitness function } F_{i}$ END FOR $\frac{1}{2} \text{ FOR EACH } t=1:N \text{ DO}$ $\frac{1}{2} \text{ FOR EACH } t=1:N \text{ DO}$ $\frac{1}{2} \text{ FOR EACH } t=1:N \text{ DO}$ $\frac{1}{2} \left[\frac{1}{2} \text{ Conjugate gradient algorithm.} */$ $\frac{1}{2} \text{ FOR EACH } t=1:N \text{ DO}$ $\frac{1}{2} \left[\frac{1}{2} \text{ Conjugate gradient algorithm.} */$ $\frac{1}{3} \text{ FOR EACH } t=1:N \text{ DO}$ $\frac{1}{4} \begin{bmatrix} \text{FOR EACH } t=1:N \text{ DO} \\ \text{IS} \\ g_{i}^{k-1} = \frac{ g^{k} ^{2}}{ g^{k-1} ^{2}} \\ d_{i}^{k} = -g_{i}^{k} + \varphi^{k-1} d_{i}^{k-1} \\ \lambda^{k} = -(g^{k})^{k} d^{k} \\ \lambda^{k} = \frac{(g^{k})^{k} d^{k}}{(d^{k})^{k}} \\ \frac{1}{2} \text{ END FOR}$ $\frac{1}{2} \text{ FOR EACH } t=1:N \text{ DO}$ $\frac{1}{2} \text{ FOR EACH } t=1:N \text{ DO}$ $\frac{1}{2} \text{ FOR EACH } t=0 \text{ EXD FOR}$ $\frac{1}{2} \text{ FOR EACH } t=0 \text{ EXD FOR}$ $\frac{1}{2} \text{ FOR EACH } t=0 \text{ EXD FOR}$ $\frac{1}{2} \text{ FOR EACH } t=0 \text{ EXD FOR}$ $\frac{1}{2} \text{ FOR EACH } t=0 \text{ EXD FOR}$ $\frac{1}{2} \text{ EXD FOR}$ $\frac{1}{2} \text{ EXD FOR}$ $\frac{1}{2} \text{ EXD FOR}$ | | M—the have maximum number of iterations of conjugate gradient algorithm. |
| $v_{i}^{t} - \text{the velocity of } x_{i} \text{ at iteration } t.$ $x_{i}^{t} - \text{the position of } x_{i} \text{ at iteration } t.$ $g_{i}^{t} - \text{the search direction of } x_{i} \text{ at iteration } k.$ $d_{i}^{k} - \text{the search direction of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $k^{k} - \text{the steplength of } x_{i} \text{ at iteration } k.$ $V^{k} \text{ Intialize } t=0.*/$ $V^{k} \text{ Intialize } t=0.*/$ $V^{k} \text{ Intialize } t=0.*/$ $V^{k} = V^{k} - V^{k}$ $V^{k} = V^{k} - V^{k} - V^{k} + V^{k}$ $V^{k} = V^{k} - V^{k} - V^{k} + V^{k}$ $V^{k} = V^{k} - V^{k} + V^{k}$ $V^{k} = V^{k} - V^{k} + V^{k}$ $V^{k} = V^{k} - V^{k} + V^{k} + V^{k}$ $V^{k} = V^{k} - V^{k} + V^{k} $ | | r—the pulse rate of a bat. |
| $\mathbf{x}_{i}^{r} = \text{the position of } \mathbf{x}_{i} \text{ at iteration } \mathbf{t}.$ $\mathbf{x}_{i}^{r} = -\text{the position of } \mathbf{x}_{i} \text{ at iteration } \mathbf{t}.$ $\mathbf{d}^{k} = -\text{the search direction of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{\lambda}^{k} = -\text{the steplength of } \mathbf{x}_{i} \text{ at iteration } \mathbf{k}.$ $\mathbf{FOR EACH :=1:N DO$ $\mathbf{for EACH :=1:N DO}$ $\mathbf{for EACH :=1:M OO}$ $\mathbf{f}_{k}^{k} = -\mathbf{g}_{k}^{k} + \mathbf{\phi}^{k-1}\mathbf{d}_{k}^{k-1}$ $\mathbf{g}_{k}^{k} = -\mathbf{g}_{k}^{k} + \mathbf{\phi}^{k-1}\mathbf{d}_{k}^{k-1}$ $\mathbf{g}_{k}^{k} = -\mathbf{g}_{k}^{k} + \mathbf{\phi}^{k-1}\mathbf{d}_{k}^{k-1}$ $\mathbf{g}_{k}^{k+1} = \mathbf{x}_{k}^{k} + \mathbf{\lambda}^{k}\mathbf{d}_{k}^{k}$ $\mathbf{for EACH :=1:M DO}$ $for EACH :=$ | | \mathbf{v}_{i}^{t} —the velocity of \mathbf{r}_{i} at iteration t |
| $x_{i} = -\text{the position of } x_{i} \text{ at iteration } k.$ $g_{i}^{k} = -\text{the search direction of } x_{i} \text{ at iteration } k.$ $d_{i}^{k} = -\text{the search direction of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k} = -\frac{(k^{k})^{k}}{(k^{k} - k^{k})}$ $k = -\frac{(k^{k})^{k}}{(k^{k})^{k}}$ $k^{k} = -\frac{(k^{k})^{k}}{(k^{k})^{k}}$ $k^{k-1} = \frac{ ^{k}}{ $ | | \mathbf{r}^{t} the position of \mathbf{r} at iteration t |
| $g_{i}\text{the gradient of } x_{i} \text{ at iteration } k.$ $d_{i}^{k}\text{the search direction of } x_{i} \text{ at iteration } k.$ $\lambda^{k}\text{the steplength of } x_{i} \text{ at iteration } k.$ $\lambda^{k}\text{the steplength of } x_{i} \text{ at iteration } k.$ $1 /*\text{Initialize generation } P(x_{h}i=1,2,,N) \text{ in random positions.*/}$ $2 /*\text{Initialize i=0.*/$ $FOR EACH i=1:N DO$ $4 Evaluate the corresponding fitness function F_{i} FOR EACH i=1:N DO 7 FOR EACH i=1:N DO 8 F_{i}=F_{\min}^{m}+(F_{\max}-F_{\min})\alpha y_{i}^{i} = y_{i}^{i-1} + y_{i}^{i} 10 FOR EACH i=1:N DO 14 FOR EACH i=1:N DO 14 FOR EACH i=1:N DO 15 fOR EACH i=1:N DO 16 g_{i}^{k} = \nabla F(x_{i}^{k}) q_{i}^{k-1} = \frac{ g^{k} ^{2}}{ g^{k-1} ^{2}} 16 g_{i}^{k-1} = -g_{i}^{k} + \varphi^{k-1}d_{i}^{k-1} 18 g_{i}^{k-1} = -g_{i}^{k} + \varphi^{k-1}d_{i}^{k-1} 19 z_{i}^{k+1} = x_{i}^{k} + \lambda^{k}d_{i}^{k} 19 END FOR 21 FOR EACH i = 1:N DO 22 END FOR 23 /^{k}\text{Update the current best solution } x^{*}.*/ 4 FOR EACH i = 1:N DO 24 END FOR 25 END FOR 25 END FOR 26 END FOR 26 END FOR 27 W_{i} = -\frac{e_{i}^{*} T HEN}$ | | x_i — the position of x_i at iteration <i>i</i> . |
| d_{i}^{i} —the search direction of x_{i} at iteration k . λ^{k} —the steplength of x_{i} at iteration k . 1 /*Initialize generation $P(x_{i},i=1,2,,N)$ in random positions.*/ 2 /*Initialize $t=0.*/$ FOR EACH $i=1:N$ DO 4 Evaluate the corresponding fitness function F_{i} END FOR 6 WHILE $t < lter_{max}$ DO 7 FOR EACH $i=1:N$ DO 8 $F_{i}=F_{min}+(F_{max}-F_{min})\alpha$ 9 $v_{i}^{i} = v_{i}^{i-1} + (x_{i}^{i} - x^{*})F_{i}$ 10 $x_{i}^{i} = x_{i}^{i-1} + v_{i}^{i}$ 11 END FOR 12 (*Use conjugate gradient algorithm.*/ 13 FOR EACH $k=1:N$ DO 14 FOR EACH $k=1:N$ DO 15 $\left \begin{array}{c} g_{i}^{k} = \nabla F(x_{i}^{k}) \\ \varphi^{k-1} = \frac{\ g^{k}\ ^{2}}{\ g^{k-1}\ ^{2}} \\ d_{i}^{k} = -g_{i}^{k} + \varphi^{k-1}d_{i}^{k-1} \\ \lambda^{k} = -\frac{(g^{k})^{T} d^{k}}{(d^{k})^{T} Ad^{k}} \\ l^{*}$ Where A is the symmetric positive definite matrix.*/ $x_{i}^{k+1} = x_{i}^{k} + \lambda^{k} d_{i}^{k}$ 21 END FOR 22 END FOR 23 (*Update the current best solution x^{*} .*/ 44 FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_{i} 26 END FOR | | g_i —the gradient of x_i at iteration k. |
| λ^{k} | | d_i^* —the search direction of x_i at iteration k. |
| 1 /*Initialize generation $P(x_i, i=1,2,,N)$ in random positions.*/ 2 /*Initialize $i=0,*/$ 3 FOR EACH $i=1:N$ DO 4 Evaluate the corresponding fitness function F_i 5 END FOR 6 WHILE / <iter.max do<br="">7 FOR EACH $i=1:N$ DO 8 $F_i=F_{min}+(F_{max}-F_{min})\alpha$ 9 $v_i' = v_i'^{-1} + (x_i' - x^*)F_i$ 10 $x_i' = x_i'^{-1} + v_i'$ 11 END FOR 12 *Use conjugate gradient algorithm.*/ 13 FOR EACH $k=1:M$ DO 14 FOR EACH $k=1:M$ DO 15 $g_i^k = \nabla F(x_i^k)$ 16 $g_i^k = \nabla F(x_i^k)$ 16 $g_i^k = -g_i^k + \phi^{k-1}d_i^{k-1}$ 18 $\lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T Ad^k}$ 19 /*Where A is the symmetric positive definite matrix.*/ 20 $x_i^{k+1} = x_i^k + \lambda^k d_i^k$ 21 END FOR 22 END FOR 23 /*Update the current best solution x^*.*/ 44 FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_i 26 Evaluate the corresponding fitness function F_i 27 E $x \in C^*$ THEN</iter.max> | | λ^k —the steplength of x_i at iteration k. |
| $ for each indice generation f(x_i, i^{-1}, z_{i+1},, r_{i}) infinited positions. f for each i = 1: N DO FOR EACH i = 1: N DO for each i = 1: N DO For each i = 1: N DO for each i = 1: N DO For each i = 1: N DO for each i = 1: N DO for each k = 1: N DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for each k = 1: M DO for k = -(k)^{T} d^k d_i^{k} = -g_i^{k} + \varphi^{k-1} d_i^{k-1} for each k = 1: M DO For each k = 1: M DO for k = -(k)^{T} d^k k = -(k)^{T} k + k)^{T} k k = -(k)^{T} k + k)^{T} k + $ | 1 | /*Initialize generation $P(\mathbf{r}, i=1, 2, N)$ in random positions */ |
| 3 FOR EACH <i>i</i> =1: <i>N</i> DO 4 Evaluate the corresponding fitness function F_i 5 END FOR 6 WHILE <i>t</i> < <i>ttr</i> _{max} DO 7 FOR EACH <i>i</i> =1: <i>N</i> DO 8 $F_i = F_{min} + (F_{max} - F_{min})\alpha$ 9 $v_i^t = v_i^{t-1} + (x_i^t - x^*)F_i$ 10 $x_i^t = x_i^{t-1} + v_i^t$ 11 END FOR 12 /*Use conjugate gradient algorithm.*/ FOR EACH <i>k</i> =1: <i>M</i> DO 14 FOR EACH <i>k</i> =1: <i>M</i> DO 15 $g_i^k = \nabla F(x_i^k)$ 16 $g_i^k = -g_i^k + \varphi^{k-1} d_i^{k-1}$ 17 $d_i^k = -g_i^k + \varphi^{k-1} d_i^{k-1}$ 18 $\lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T A d^k}$ 19 (*Where <i>A</i> is the symmetric positive definite matrix.*/ $x_i^{k+1} = x_i^k + \lambda^k d_i^k$ 11 END FOR 22 END FOR 23 /*Update the current best solution x^* .*/ 4 FOR EACH <i>i</i> =1: <i>N</i> DO 25 Evaluate the corresponding fitness function F_i 26 END FOR | 2 | / initialize generation $T(x_i, i-1, 2,, iv)$ in random positions. "/ |
| 4 Evaluate the corresponding fitness function F_i 5 END FOR 6 WHILE $i < liter_{max}$ DO 7 FOR EACH $i=1:N$ DO 8 $ F_i = F_{min} + (F_{max} - F_{min})\alpha$ 9 $ v_i' = v_i'^{-1} + (x_i' - x^*)F_i$ 10 $ x_i' = x_i'^{-1} + v_i'$ 11 END FOR 12 $ *$ Use conjugate gradient algorithm.*/ 13 FOR EACH $k=1:M$ DO 14 FOR EACH $i=1:M$ DO 15 $ g_i^k = \nabla F(x_i^k)$ 16 $ g_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1}$ 18 $ \lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T Ad^k}$ 19 $ (x_i^{k+1} = x_i^k + \lambda^k d_i^k)$ 19 $ FOR$ EACH $i=1:M$ DO 21 END FOR 22 END FOR 23 $ *$ Update the current best solution x^* .*/ 24 FOR EACH $i=1:M$ DO 25 $ Evaluate the corresponding fitness function F_i26 END FOR$ | 3 | FOR EACH <i>i</i> =1: <i>N</i> DO |
| 5 END FOR 6 WHILE $t < t ler_{max}$ DO 7 FOR EACH $i = 1:N$ DO 8 $ F_i = F_{min} + (F_{max} - F_{min})\alpha $ 9 $ v_i' = v_i^{t-1} + (x_i' - x^*)F_i $ 10 $ v_i' = v_i^{t-1} + v_i' $ 11 END FOR 12 $/*$ Use conjugate gradient algorithm.*/ 13 FOR EACH $i = 1:M$ DO 14 FOR EACH $i = 1:M$ DO 15 $ g_i^k = \nabla F(x_i^k) $ 16 $ g_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1} $ 18 $ \lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T A d^k} $ 19 $ X_i^{k+1} = x_i^k + \lambda^k d_i^k $ 21 END FOR 22 END FOR 23 $/*$ Update the current best solution $x^* \cdot x/$ 4 FOR EACH $i = 1:N$ DO 25 $ Evaluate the corresponding fitness function F_i26 END FOR$ | 4 | Evaluate the corresponding fitness function F_i |
| 6 WHILE $l^{<}limstreak}$ DO 7 FOR EACH $i=1:N$ DO 8 $ F_i = F_{\min} + (F_{\max} - F_{\min})\alpha $ 9 $ v_i' = v_i^{t-1} + (x_i' - x^*)F_i $ 10 $ x_i' = x_i^{t-1} + v_i' $ 11 END FOR 12 /*Use conjugate gradient algorithm.*/ 13 FOR EACH $k=1:M$ DO 14 FOR EACH $k=1:M$ DO 15 $ g_i^k = \nabla F(x_i^k) $ 16 $ g_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1} $ 17 $ d_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1} $ 18 $ \lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T Ad^k} $ 19 $ V $ Where A is the symmetric positive definite matrix.*/ 20 $ x_i^{k+1} = x_i^k + \lambda^k d_i^k $ 21 END FOR 22 END FOR 23 /*Update the current best solution $x^* \cdot^*/$ 4 FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_i 26 END FOR | 5 | END FOR |
| $\begin{cases} F_i = F_{\min} + [F_{\max} \cdot F_{\min})\alpha \\ v_i^i = v_i^{i-1} + (x_i^i - x^*)F_i \\ x_i^i = x_i^{i-1} + v_i^i \\ \end{cases}$ $\begin{cases} FOR EACH k=1:M DO \\ FOR EACH k=1:M DO \\ fOR EACH k=1:M DO \\ \end{cases}$ $\begin{cases} g_i^k = \nabla F(x_i^k) \\ \varphi_i^{k-1} = \frac{\ g_i^k\ ^2}{\ g_i^{k-1}\ ^2} \\ d_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1} \\ \lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T Ad^k} \\ \end{cases}$ $\begin{cases} FOR EACH i = x_i^k + \lambda^k d_i^k \\ \beta_i^k = x_i^k + \lambda^k d_i^k \\ \beta_i^k = x_i^k + \lambda^k d_i^k \\ fOR EACH i = 1:N DO \\ fOR EACH i$ | 6 7 | WHILE $t < lter_{max}$ DO |
| 9 10 11 12 13 14 14 15 16 19 19 19 19 10 14 17 18 19 19 19 19 19 19 19 19 19 19 | 8 | $F = F_{min} + (F_{mov} - F_{min})\alpha$ |
| $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | 0 | $\mathbf{v}^{t} - \mathbf{v}^{t-1} + (\mathbf{x}^{t} - \mathbf{x}^{*})F$ |
| 10 $ x_i^{i} = x_i^{i^{-1}} + v_i^{i}$ 11 END FOR 12 /*Use conjugate gradient algorithm.*/ 13 FOR EACH $k=1:M$ DO 14 FOR EACH $i=1:M$ DO 15 $ g_i^k = \nabla F(x_i^k)$ 16 $ g_i^{k-1} = \frac{\ g^k\ ^2}{\ g^{k-1}\ ^2}$ 17 $ d_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1}$ 18 $ \lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T A d^k}$ 19 $ \lambda^k = x_i^k + \lambda^k d_i^k$ 21 END FOR 22 END FOR 23 /*Update the current best solution $x^*.*/$ FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_i 26 END FOR 27 END FOR | , | $(\mathbf{r}_i - \mathbf{r}_i) + (\mathbf{x}_i - \mathbf{x}_i)^T \mathbf{r}_i$ |
| 11 END FOR 12 /*Use conjugate gradient algorithm.*/ 13 FOR EACH $k=1:M$ DO 14 FOR EACH $i=1:N$ DO 15 $g_i^k = \nabla F(x_i^k)$ 16 $g_i^{k-1} = \frac{\ g^k\ ^2}{\ g^{k-1}\ ^2}$ 17 $d_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1}$ 18 $\lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T Ad^k}$ 19 /*Where <i>A</i> is the symmetric positive definite matrix.*/ 20 /*Where <i>A</i> is the symmetric positive definite matrix.*/ 21 END FOR 22 END FOR 23 /*Update the current best solution x^* .*/ 4 FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_i 26 END FOR 27 E E $e \in E^*$ THEN | 10 | $x_i' = x_i'' + v_i'$ |
| 12 <i>F</i> OSE conjugate gradient algorithm.*/ 13 FOR EACH $k=1:M$ DO 14 FOR EACH $i=1:N$ DO 15 $g_i^k = \nabla F(x_i^k)$ 16 $g_i^{k-1} = \frac{\ g^k\ ^2}{\ g^{k-1}\ ^2}$ 17 $d_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1}$ 18 $\lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T Ad^k}$ 19 <i>Line in the symmetric positive definite matrix.*/</i> 20 <i>Line in the symmetric positive definite matrix.*/</i> 21 Line in the symmetric positive definite matrix.*/ 22 END FOR 23 <i>Line in the symmetric positive definite matrix.*/</i> 24 FOR EACH $i=1:N$ DO 25 Line in the symmetric positive definite matrix.*/ 26 Line in the symmetric positive definite matrix. */ 27 Line in the symmetric positive definite matrix. */ 28 Line in the symmetric positive definite matrix. */ | 11 | END FOR |
| 14 14 15 16 16 17 18 19 20 21 END FOR 22 END FOR 23 FOR EACH $i=1:N$ DO $g_i^k = \nabla F(x_i^k)$ $\varphi^{k-1} = \frac{\ g^k\ ^2}{\ g^{k-1}\ ^2}$ $d_i^k = -g_i^k + \varphi^{k-1}d_i^{k-1}$ $\lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T Ad^k}$ /*Where <i>A</i> is the symmetric positive definite matrix.*/ $x_i^{k+1} = x_i^k + \lambda^k d_i^k$ 21 END FOR 23 /*Update the current best solution $x^*.*/$ FOR EACH $i=1:N$ DO 25 EVALUATE TO FOR 27 EVALUATE TO FOR 27 EVALUAT | 12 | /* Use conjugate gradient algorithm.*/ FOR $\mathbf{F} \wedge \mathbf{CH} = 1: M \mathbf{DO}$ |
| 15 16 17 18 19 21 20 21 21 22 END FOR 23 4 $k^{k-1} = \frac{\ g^k\ ^2}{\ g^{k-1}\ ^2}$ $d_i^k = -g_i^k + \phi^{k-1}d_i^{k-1}$ $\lambda^k = -\frac{(g^k)^T d^k}{(d^k)^T Ad^k}$ /*Where <i>A</i> is the symmetric positive definite matrix.*/ $x_i^{k+1} = x_i^k + \lambda^k d_i^k$ END FOR 23 /*Update the current best solution x^* .*/ 24 FOR EACH <i>i</i> =1: <i>N</i> DO 25 Evaluate the corresponding fitness function F_i END FOR 27 HE E $e \in E^*$ THEN | 13 | FOR EACH $i=1:N$ DO |
| 16 16 17 18 19 21 END FOR 23 FOR EACH <i>i</i> =1: <i>N</i> DO 25 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR END | 15 | $\boldsymbol{\varrho}_{\cdot}^{k} = \nabla F(\boldsymbol{x}_{\cdot}^{k})$ |
| 16 17 17 18 19 20 21 END FOR 22 END FOR 23 /*Update the current best solution x^* .*/ 4 FOR EACH <i>i</i> =1: <i>N</i> DO 25 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR END FOR | 10 | |
| 17 17 18 19 19 20 21 END FOR 22 END FOR 23 /*Update the current best solution x^* .*/ 4 FOR EACH <i>i</i> =1: <i>N</i> DO 25 EVD FOR 27 END FOR END FOR EN | 16 | $a^{k-1} = \frac{\ \boldsymbol{g}^{k}\ }{\ \boldsymbol{g}^{k}\ }$ |
| 17 18 19 19 20 21 END FOR 22 END FOR 23 /*Update the current best solution x^* .*/ FOR EACH $i=1:N$ DO 25 EVALUATE THE SUME SUME SUME SUME SUME SUME SUME SUM | 10 | $\left\ \boldsymbol{g}^{\boldsymbol{k} \cdot \boldsymbol{l}} \right\ ^2$ |
| 18 18 19 19 20 21 END FOR 22 END FOR 23 /*Update the current best solution x^* .*/ FOR EACH <i>i</i> =1: <i>N</i> DO 25 EVAluate the corresponding fitness function F_i EVALUATE THE SUME STATES SUME SUME SUME SUMMER SUMME | 17 | $\boldsymbol{d}^{k} = -\boldsymbol{\sigma}^{k} + \boldsymbol{\omega}^{k-1} \boldsymbol{d}^{k-1}$ |
| 18 19 20 21 END FOR 23 f^{*} Update the current best solution x^{*} .*/ FOR EACH $i=1:N$ DO 25 EVAluate the corresponding fitness function F_i 26 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR 27 END FOR END | 17 | $\mathbf{w}_i = \mathbf{y}_i + \mathbf{\psi} \cdot \mathbf{u}_i$ |
| $\begin{array}{c c} & \begin{pmatrix} d^k \end{pmatrix}^T A d^k \\ \hline & \begin{pmatrix} d^k \end{pmatrix}^T A d^k \\ \hline & \begin{pmatrix} k^{k+1} \\ \end{pmatrix}^K \text{Where } A \text{ is the symmetric positive definite matrix.*/} \\ \hline & \begin{pmatrix} x_i^{k+1} \\ x_i^k + \lambda^k d_i^k \\ \end{pmatrix} \\ \hline & \begin{array}{c} \text{END FOR} \\ \text{22} \\ \text{END FOR} \\ \hline & \begin{array}{c} \text{23} \\ \end{pmatrix}^K \text{Update the current best solution } x^* */ \\ \hline & \begin{array}{c} \text{FOR EACH } i=1:N \text{ DO} \\ \hline & \begin{array}{c} \text{Evaluate the corresponding fitness function } F_i \\ \hline & \begin{array}{c} \text{EVD FOR} \\ \hline & \end{array}{\end{array} \end{array} \right \right \right \right \right \right \right $ | 18 | $\lambda^{k} = -\frac{(g^{*}) d^{*}}{d^{*}}$ |
| 19/*Where A is the symmetric positive definite matrix.*/20/*Where A is the symmetric positive definite matrix.*/21END FOR22END FOR23/*Update the current best solution x^* .*/24FOR EACH $i=1:N$ DO25Evaluate the corresponding fitness function F_i 26END FOR27HE $E_i < \in E^*$ THEN | 10 | $\left(d^{k}\right)^{T}Ad^{k}$ |
| 20 20 $x_i^{k+1} = x_i^k + \lambda^k d_i^k$ 21 END FOR 22 END FOR 23 /*Update the current best solution $x^*.*/$ 24 FOR EACH <i>i</i> =1: <i>N</i> DO 25 Evaluate the corresponding fitness function F_i 26 END FOR 27 E E E C E THEN | 19 | /*Where A is the symmetric positive definite matrix.*/ |
| 21 END FOR 22 END FOR 23 /*Update the current best solution $x^*.*/$ 24 FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_i 26 END FOR 27 HE F. $e \in F^*$ THEN | 20 | $\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \lambda^k d_i^k$ |
| 22 END FOR 23 /*Update the current best solution $x^*.*/$ 24 FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_i 26 END FOR 27 HE $F_i < F_i^*$ THEN | 21 | END FOR |
| 23 /*Update the current best solution x^* .*/ 24 FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_i 26 END FOR 27 HE $F_i < F_i^*$ THEN | 22 | END FOR |
| 24 FOR EACH $i=1:N$ DO 25 Evaluate the corresponding fitness function F_i 26 END FOR 27 IF $E_i < E^{T}$ THEN | 23 | /*Update the current best solution x^* .*/ |
| 26 END FOR 27 IF $E < E^*$ THEN | 24 | FOR EACH $i=1:N$ DO Evaluate the corresponding fitness function E |
| $27 \qquad \text{IF } E < E^* \text{ THEN}$ | 25 26 | EVAluate the corresponding fitness function F_i END FOR |
| 27 IF Γ best Γ I HEIN | 27 | IF $F_{besl} < F^*$ THEN |
| $\begin{array}{c c} 28 & \mathbf{FOR} \ \mathbf{EACH} \ i=1:N \ \mathbf{DO} \\ 10 & 10 \\ 10 & $ | 28 | FOR EACH <i>i</i> =1: <i>N</i> DO |
| $\begin{array}{c c} 29 \\ 30 \\ 10$ | 29 | $\mathbf{x}_{new} = \mathbf{x}_{old} + \varepsilon L'$ |
| $\begin{array}{c c} \mathbf{J} \mathbf{I} \mathbf{F} \mathbf{L} < \rho \mathbf{I} \mathbf{I} \mathbf{E} \mathbf{N} \\ \mathbf{J} \mathbf{I}^{t+1} - \beta \mathbf{I}^t \end{array}$ | 30 31 | $\begin{bmatrix} \mathbf{I} \mathbf{F} L < \rho \mathbf{I} \mathbf{\Pi} \mathbf{E} \mathbf{N} \\ I^{t+1} = R I^{t} \end{bmatrix}$ |
| $\begin{bmatrix} L_i & -\mu L_i \\ \mu_i & \mu_i \end{bmatrix}$ | 31 | $\begin{bmatrix} L_i - \rho L_i \\ r^{t+1} = r^0 \begin{bmatrix} 1 & \operatorname{sym}(r, r) \end{bmatrix}$ |
| $\frac{32}{100} \qquad $ | 32 | $\begin{bmatrix} r_i = r_i \ \lfloor 1 - \exp(-\gamma t) \rfloor \end{bmatrix}$ |
| 33 END IF | 33 | END IF |
| 34 END FOK 35 END IF | 54 35 | END FOR |
| 36 <i>iter=iter</i> +1 | 36 | iter=iter+1 |
| 37 END WHILE | 37 | END WHILE |
| 38 RETURN <i>x</i> _{best} | 38 | RETURN xbest |

 Table 2

 The experimental parameters of BA and CG-BA.

| Experimental parameters | BA | CG-BA |
|--|-----------------------------------|--|
| Maximum generation Population size Convergence tolerance Maximum generation of CG | 10,000 100 10 ⁻⁵ | 10,000 100 10 ⁻⁵ 5 |
| | | |

4.2. Test of CG-BA

To evaluate the proposed algorithm, CG-BA, four test functions are employed as shown in Table 1. Sphere function is unimodal, Rosenbrock's function is multimodal, Rastrigin's function is multimodal and Schaffer function is multimodal. The tests of BA and CG-BA on all test functions were performed on an Intel i7-4870 2.50 GHz machine with 16 GB RAM. The experimental parameters of BA and CG-BA are shown in Table 2.

As the test results shown in Table 3, two points can be concluded:

- (a) The max value, min value and average value of iteration of CG-BA are less than the original BA for four test functions. This means the convergence ability of BA has been successfully improved with CG-BA.
- (b) For the Rosenbrock's function, Rastrigin's function and Schaffer function, the convergence rates of BA didn't obtain 1. While for these three test functions, the convergence rates of CG-BA are obtained 1. Thus the optimization performance of the original BA has also enhanced with CG-BA.

Remark. Through the experimental results and above analysis, the optimization ability of the original BA has been successfully enhanced by the proposed CG-BA.

5. Optimization of RBFNN and GRNN

The proposed CG-BA is employed to optimize the initial weights and thresholds for the RBFNN and GRNN.

Table 3

Test results of BA and CG-BA.

| Table | 5 |
|--------|------|
| Experi | imei |

| Experimental | parameter | setting. |
|--------------|-----------|----------|
|--------------|-----------|----------|

| Model | Experimental parameters | Default value |
|-------|---|--|
| GRNN | Neuron number of the input layer Neuron number of the hidden layer Neuron number of the output layer Radial basis function expansion Maximum number of training Training requirement precision | 4 9 1 0.1 to 2.0 1000 0.00002 |
| RBFNN | Neuron number of the input layer Neuron number of the hidden layer Neuron number of the output layer Sample Maximum number of training Training requirement precision | 4 9 1 400 1000 0.00002 |

5.1. RBFNN optimized by CG-BA

This section contains the standard RBFNN and the improved RBFNN that is optimized by CG-BA.

5.1.1. Standard RBFNN

The structure of RBFNN is simple and includes an input layer, a hidden layer and an output layer, as shown in Fig. 2 Part c. The hidden layer is the key part of RBFNN, and its neurons represent RBFNN. More information about RBFNN is shown in Appendix B.

5.1.2. RBFNN optimized by CG-BA

The final results are dependent on the initial random weights and threshold values of an ANN, which will increase the unstable factor in forecasting. In this part of the paper, the CG-BA-RBFNN model is developed, and the proposed optimization algorithm CG-BA is used to optimize the initial weight and threshold of the RBFNN to improve the forecasting performance of RBFNN. The details of the CG-BA-RBFNN is presented as Algorithm 2.

| Test function | Dimension | Algorithm | Max value of iteration | Min value of iteration | Average value of iteration | Convergence rate |
|---------------|-----------|-----------|------------------------|------------------------|----------------------------|------------------|
| Sphere | 10 | BA | 213 | 174 | 198 | 1 |
| - | | CG-BA | 3 | 1 | 1.4 | 1 |
| | 20 | BA | 343 | 151 | 182 | 1 |
| | | CG-BA | 21 | 12 | 17.2 | 1 |
| | 50 | BA | 513 | 398 | 441 | 1 |
| | | CG-BA | 112 | 84 | 98 | 1 |
| Rosenbrock | 2 | BA | - | - | _ | - |
| | | CG-BA | 165 | 79 | 103 | 1 |
| Rastrigin | 10 | BA | 421 | 315 | 369 | 1 |
| | | CG-BA | 197 | 144 | 182 | 1 |
| | 20 | BA | 860 | 731 | 795 | 0.83 |
| | | CG-BA | 528 | 378 | 469 | 1 |
| | 50 | BA | - | - | - | - |
| | | CG-BA | 1342 | 873 | 1128 | 0.96 |
| Schaffer | 2 | BA | 1236 | 981 | 1035 | 0.81 |
| | | CG-BA | 56 | 24 | 43 | 1 |

Table 4

Statistical parameters for the data used in this paper.

| Region | Mean value (m/s) | Std. dev. (m/s) | Maximum value (m/s) | Minimum value (m/s) | Median value (m/s) |
|--------|------------------|-----------------|---------------------|---------------------|--------------------|
| Site 1 | 6.5564 | 2.4147 | 14.3000 | 1.4000 | 6.3000 |
| Site 2 | 5.8237 | 2.1162 | 15.7000 | 0.9000 | 5.6000 |
| Site 3 | 7.8363 | 3.4319 | 18.3000 | 1.0000 | 7.1000 |
| Site 4 | 6.4602 | 2.5402 | 17.2000 | 0.8000 | 6.3000 |

Algorithm 2. CG-BA-RBFNN

Input:

 $x_t^{(0)} = (x^{(0)}(1), x^{(0)}(2), K, x^{(0)}(q))$ -sequence of training wind speed data.

 $x_{\nu}^{(0)} = (x^{(0)}(q+1), x^{(0)}(q+2), \mathbf{K}, x^{(0)}(q+d))$ -sequence of verification wind speed data

Output:

 $\hat{y}_{z}^{(0)} = (\hat{y}^{(0)}(q+1), \hat{y}^{(0)}(q+2), \mathbf{K}, \hat{y}^{(0)}(q+d))$ —the forecasting electrical load data from

RBFNN

Parameters:

- α —a random vector, with a value between 0 and 1.
- ε —a random vector, with a value between 0 and 1.
- β —a random vector, with a value between 0 and 1.
- F_i —the fitness function of x_i .
- N-the number of generations P.

t—current iteration number.

- Itermax—the maximum number of iterations.
- M-the maximum number of iterations of conjugate gradient algorithm.
- L-the loudness of a bat.
- *r*—the pulse rate of a bat.
- x_i —generation *i* (the weight and threshold of the RBFNN)

Fitness function

$$F = \sum_{i=1}^{d} \left| \hat{x}_{v}^{(0)} \left(q+i \right) - \hat{y}_{z}^{(0)} \left(q+d \right) \right|$$

- /*Initialize generation $X(x_i, i=1, 2, ..., N)$, the weight and threshold of the RBFNN) in random 1
- positions.*/
- 2 /*Initialize *t*=0.*/
- **3** FOR EACH *i*=1:*N* DO
- 4 Evaluate the corresponding fitness function F_i
- 5 END FOR
- 6 WHILE *t*<*Iter*_{max} DO

FOR EACH *i*=1:*N* **DO**

8 $F_i = F_{\min} + (F_{\max} - F_{\min})\alpha$

9
10
11
END FOR

$$v_i^t = v_i^{t-1} + (x_i^t - x^*)F_i$$

 $x_i^t = x_i^{t-1} + v_i^t$

| 12 | /*Use conjugate gradient algorithm.*/ | | | | | |
|----|---|--|--|--|--|--|
| 13 | FOR EACH <i>k</i> =1: <i>M</i> DO | | | | | |
| 14 | FOR EACH <i>i</i> =1: <i>N</i> DO | | | | | |
| 15 | $oldsymbol{g}_i^k = abla Fig(oldsymbol{x}_i^kig)$ | | | | | |
| 16 | $arphi^{k-1} = rac{\left\ oldsymbol{g}^k ight\ ^2}{\left\ oldsymbol{g}^{k-I} ight\ ^2}$ | | | | | |
| 17 | $\boldsymbol{d}_i^k = -\boldsymbol{g}_i^k + \varphi^{k-1} \boldsymbol{d}_i^{k-1}$ | | | | | |
| 18 | $\lambda^k = -rac{\left(oldsymbol{g}^k ight)^Toldsymbol{d}^k}{\left(oldsymbol{d}^k ight)^Toldsymbol{A}oldsymbol{d}^k}$ | | | | | |
| 19 | /*Where A is the symmetric positive definite matrix.*/ | | | | | |
| 20 | $oldsymbol{x}_i^{k+1} = oldsymbol{x}_i^k + \lambda^k oldsymbol{d}_i^k$ | | | | | |
| 21 | END FOR | | | | | |
| 22 | END FOR | | | | | |
| 23 | /*Update the current best solution <i>x</i> *.*/ | | | | | |
| 24 | FOR EACH <i>i</i> =1: <i>N</i> DO | | | | | |
| 25 | Evaluate the corresponding fitness function F_i | | | | | |
| 26 | END FOR | | | | | |
| 27 | IF $F_{best} < F^*$ THEN . | | | | | |
| 28 | FOR EACH <i>i</i> =1: <i>N</i> DO | | | | | |
| 29 | $\mathbf{x}_{new} = \mathbf{x}_{old} + \varepsilon L^t$ | | | | | |
| 30 | IF L^{t} > THEN | | | | | |
| 31 | $L_i^{t+1} = \beta L_i^t$ | | | | | |
| 32 | $r_i^{t+1} = r_i^0 \Big[1 - \exp(-\gamma t) \Big]$ | | | | | |
| 33 | END IF | | | | | |
| 34 | END FOR | | | | | |
| 35 | END IF | | | | | |
| 36 | iter = iter + 1 | | | | | |
| 37 | END WHILE | | | | | |
| 38 | RETURN x_b | | | | | |
| 39 | Set the weight and threshold of the RBFNN according to x_b . | | | | | |
| 40 | Use x_t to train the RBFNN and update the weight and threshold of the | | | | | |

41 Input the historical data into RBFNN to obtain the forecasting value \hat{y} .

| (| Training | samp | oles | |) (| Testing s | ample | 95 | |
|---------|-------------------|--------|-----------------|-------------|--------------------|-------------------|--------|-----------------|-------------|
| | Input | 1-step | Outpu 2-step | t 3-step | | Input | 1-step | Outpu 2-step | t 3-step |
| | 1, 2,, 5 | 6 | 7 | 8 | | 1501, 1502,, 1505 | 1506 | 1507 | 1508 |
| samples | 2, 3,, 6 | 7 | 8 | 9 | 8 | 1502, 1503,, 1506 | 1507 | 1508 | 1509 |
| | 3, 4,, 7 | 8 | 9 | 10 | i ja | 1503, 1504,, 1507 | 1508 | 1509 | 1510 |
| | 4, 5,, 8 | 9 | 10 | 11 | 8 | 1504, 1505,, 1508 | 1509 | 1510 | 1511 |
| - S | : | ÷ | ÷ | ÷ | l i <mark>g</mark> | ÷ | : | : | : |
| ~ | 1498, 1499,, 1502 | 1503 | 1504 | 1505 | | 1998, 1999,, 2002 | 2003 | 2004 | 2005 |
| | 1499, 1500,, 1503 | 1504 | 1505 | 1506 | | 1999, 2000,, 2003 | 2004 | 2005 | 2006 |
| ļ | 1500, 1501,, 1504 | 1505 | 1506 | 1507 | | 2000, 2001,, 2004 | 2005 | 2006 | 2007 |

RBFNN.

Fig. 3. Input and output data selection.

5.2. GRNN optimized by CG-BA

This section contains the standard GRNN and the improved GRNN that is optimized by CG-BA.

5.2.1. Standard GRNN

Specht [51] proposed a new type of neural network model named GRNN, which is based on the advantage of a standard sta-

tistical technology known as Kernel regression [52,53]. Four layers, i.e., the input layer, pattern layer, summation layer, and output layer, compose a GRNN, as shown in Fig. 2 Part c. More information about GRNN is shown in Appendix B.

5.2.2. GRNN optimized by CG-BA

This part proposed that the initial weight and threshold of the GRNN is optimized by the proposed optimization algorithm

CG-BA to improve the forecasting performance. The details of the CG-BA-GRNN are presented as Algorithm 3.

Algorithm 3. CG-BA-GRNN

Input:

 $x_t^{(0)} = (x^{(0)}(1), x^{(0)}(2), \mathbf{K}, x^{(0)}(q))$ -sequence of training wind speed data.

 $x_v^{(0)} = (x^{(0)}(q+1), x^{(0)}(q+2), K, x^{(0)}(q+d))$ -sequence of verification wind speed data

Output:

 $\hat{y}_{z}^{(0)} = (\hat{y}^{(0)}(q+1), \hat{y}^{(0)}(q+2), K, \hat{y}^{(0)}(q+d))$ —the forecasting wind speed from GRNN

Parameters:

 α —a random vector, with a value between 0 and 1.

- ε —a random vector, with a value between 0 and 1.
- β —a random vector, with a value between 0 and 1.
- F_i —the fitness function of x_i .
- N—the number of generations P.
- *t*—current iteration number.

Itermax-the maximum number of iterations.

M—the maximum number of iterations of conjugate gradient algorithm.

- L-the loudness of a bat.
- *r*—the pulse rate of a bat.

 x_i —generation *i* (the weight and threshold of the GRNN)

Fitness function

$$F = \sum_{i=1}^{d} \left| \hat{x}_{v}^{(0)} \left(q+i \right) - \hat{y}_{z}^{(0)} \left(q+d \right) \right|$$

- /*Initialize generation $X(x_{i},i=1,2,...,N)$, the weight and threshold of the GRNN) in random
- 1 positions.*/
- 2 /*Initialize *t*=0.*/
- 3 FOR EACH *i*=1:*N* DO
- 4 Evaluate the corresponding fitness function F_i
- 5 END FOR
- 6 WHILE *t*<*Iter*_{max} DO
- **FOR EACH** *i*=1:*N* **DO**
- 8 $F_i = F_{\min} + (F_{\max} F_{\min})\alpha$

9
10

$$v_i^t = v_i^{t-1} + (x_i^t - x^*)F_i$$

 $x_i^t = x_i^{t-1} + v_i^t$

11 END FOR

12 /*Use conjugate gradient algorithm.*/

| 13 | | FOR EACH k=1:M DO |
|----|-----|---|
| 14 | | FOR EACH <i>i</i> =1: <i>N</i> DO |
| 15 | | $oldsymbol{g}_i^k = abla Fig(oldsymbol{x}_i^kig)$ |
| 16 | | ${{arphi}^{k-1}} = rac{{{{\left\ {{m{g}}^{k}} ight\ }^2}}}{{{{\left\ {{m{g}}^{k-l}} ight\ }^2}}}$ |
| 17 | | $oldsymbol{d}_i^k = -oldsymbol{g}_i^k + arphi^{k-1}oldsymbol{d}_i^{k-1}$ |
| 18 | | $\lambda^{k} = -\frac{\left(\boldsymbol{g}^{k}\right)^{T} \boldsymbol{d}^{k}}{\left(\boldsymbol{d}^{k}\right)^{T} \boldsymbol{A} \boldsymbol{d}^{k}}$ |
| 19 | | /*Where A is the symmetric positive definite matrix.*/ |
| 20 | | $x_i^{k+1} = x_i^k + \lambda^k d_i^k$ |
| 21 | | END FOR |
| 22 | | END FOR |
| 23 | | /*Update the current best solution <i>x</i> *.*/ |
| 24 | | FOR EACH <i>i</i> =1: <i>N</i> DO |
| 25 | | Evaluate the corresponding fitness function F_i |
| 26 | | END FOR |
| 27 | | IF $F_{best} < F^*$ THEN . |
| 28 | | FOR EACH <i>i</i> =1: <i>N</i> DO |
| 29 | | $\mathbf{x}_{new} = \mathbf{x}_{old} + \mathbf{\varepsilon} L^t$ |
| 30 | | IF L^{t} > THEN |
| 31 | | $L_i^{t+1} = \beta L_i^t$ |
| 32 | | $r_i^{t+1} = r_i^0 \left[1 - \exp(-\gamma t) \right]$ |
| 33 | | END IF |
| 34 | | END FOR |
| 35 | | END IF |
| 36 | | iter = iter + 1 |
| 37 | EN | D WHILE |
| 38 | RE | TURN x_b |
| 39 | Set | the weight and threshold of the GRNN according to x_b . |
| 40 | Use | \mathbf{x}_t to train the GRNN and update the weight and threshold of the GRNN. |
| 41 | Inp | ut the historical data into GRNN to obtain the forecasting value \hat{y} . |

6. Forecasting experiment

In this part, the experiments were divided into three parts, *Experiment I, Experiment II* and *Experiment III*. In *Experiment I*, the wind speed multi-step forecasting results and comparisons of the hybrid FEEMD-CG-BA-RBFNN model, the hybrid FEEMD-CG-BA-GRNN model, the hybrid SSA-CG-BA-GRNN model are given. In *Experiment II*, the performance of the hybrid FEEMD-CG-BA-RBFNN model, the hybrid FEEMD-CG-BA-GRNN model are compared with FEEMD-BA-RBFNN, FEEMD-BA-GRNN, SSA-BA-RBFNN, SSA-BA-GRNN, single RBFNN, single GRNN and ARIMA. In *Experiment III*, the DM-test is used to evaluate the performance of each forecasting model. To confirm the universality of the proposed model, *Experiment I, Experiment II* and *Experiment III* are validated at four different sites.

Four data sites in Penglai region have been selected with the latitude from 120°43'N to 120°47'N and longitude from 37°50'E to 37°37'E. The data sites are in a mountain and hilly area near the sea, and its altitude ranges from 100 m to 240 m. The rated power of WTG (wind power generator) is 1500 kW. The mean annual temperature, humidity and air pressure in this region are

11.9 °C, 65% and 1012.7 hPa, respectively. Statistical parameters for the data used in this paper are shown in Table 4.

All algorithms are operated on the following platform: MATLAB R2012a on Windows 8 with 2.50 GHz Intel Core i7 4870HQ 64-bit and 16 GB of RAM. The experimental parameters are shown in Table 5. Meanwhile, considering randomness factors and ensuring that the final results are reliable and independent of the initial weights, we carry out each experiment 50 times and then take the average value. The input layer of all the ANNs is constructed with four neurons. Hecht–Nelson method [54] is employed to determine the node number of the hidden layer. When the node number of the input layer is n, the node number of the hidden layer is 2n + 1.

6.1. Accuracy estimating indexes

To learn the global traits of the models, three metric parameters are taken: the MAE (mean absolute error), the MAPE (mean absolute percentage error) and the MSE (mean square error). MAE is the average absolute forecast error of n times forecast results. Because the prediction error may be positive and negative, it cannot reflect the level of error; this problem can be avoided by using MAE. MSE is the average of the prediction error squares, which can

Forecasting performance of four proposed hybrid models.

| | | SSA-CG-E | SSA-CG-BA-RBFNN | | SSA-CG-E | SSA-CG-BA-GRNN | | | FEEMD-CG-BA-RBFNN | | | FEEMD-CG-BA-GRNN | | |
|--------|----------|----------|-----------------|--------|----------|----------------|--------|--------|-------------------|--------|--------|------------------|--------|--|
| | | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | |
| Site 1 | MAE | 0.0906 | 0.1783 | 0.2799 | 0.0698 | 0.1621 | 0.2334 | 0.1047 | 0.1869 | 0.2963 | 0.0755 | 0.1661 | 0.2581 | |
| | MSE | 0.0091 | 0.0353 | 0.0871 | 0.0054 | 0.0291 | 0.0605 | 0.0122 | 0.0389 | 0.0977 | 0.0063 | 0.0306 | 0.0740 | |
| | MAPE (%) | 1.6510 | 3.2489 | 5.1002 | 1.2720 | 2.9528 | 4.2507 | 1.9053 | 3.4029 | 5.3953 | 1.3752 | 3.0261 | 4.7022 | |
| Site 2 | MAE | 0.1011 | 0.2155 | 0.3121 | 0.0773 | 0.1767 | 0.2483 | 0.0948 | 0.2024 | 0.3379 | 0.0862 | 0.1881 | 0.2861 | |
| | MSE | 0.0122 | 0.0555 | 0.1163 | 0.0071 | 0.0373 | 0.0738 | 0.0107 | 0.0489 | 0.1364 | 0.0089 | 0.0422 | 0.0977 | |
| | MAPE (%) | 1.5737 | 3.3484 | 4.8506 | 1.2016 | 2.7491 | 3.8576 | 1.4738 | 3.1483 | 5.2544 | 1.3405 | 2.9228 | 4.4539 | |
| Site 3 | MAE | 0.0938 | 0.2195 | 0.3241 | 0.0851 | 0.1755 | 0.2587 | 0.0955 | 0.2127 | 0.3472 | 0.0903 | 0.2088 | 0.3121 | |
| | MSE | 0.0105 | 0.0574 | 0.1252 | 0.0086 | 0.0367 | 0.0795 | 0.0108 | 0.0538 | 0.1435 | 0.0097 | 0.0519 | 0.1159 | |
| | MAPE (%) | 1.3249 | 3.0974 | 4.5735 | 1.2031 | 2.4749 | 3.6545 | 1.3478 | 3.0035 | 4.8984 | 1.2746 | 2.9477 | 4.4049 | |
| Site 4 | MAE | 0.1001 | 0.2193 | 0.3633 | 0.0919 | 0.2031 | 0.2846 | 0.1042 | 0.2251 | 0.3645 | 0.0975 | 0.2215 | 0.2903 | |
| | MSE | 0.0114 | 0.0548 | 0.1502 | 0.0096 | 0.0469 | 0.0922 | 0.0123 | 0.0577 | 0.1513 | 0.0108 | 0.0559 | 0.0959 | |
| | MAPE (%) | 1.3448 | 2.9471 | 4.8832 | 1.2347 | 2.7263 | 3.8254 | 1.4008 | 3.0241 | 4.8963 | 1.3099 | 2.9763 | 4.3003 | |



Fig. 4. Forecasting results of four proposed hybrid models.

evaluate the change of the prediction model; the smaller the MSE value is, the better the prediction model is. MAPE is a measure of the accuracy of the prediction method for use in the performance evaluation and comparison in statistics. The detailed equations of these three error indexes are given in Appendix C.

6.2. Experiment I

The data from four wind power stations in Penglai, China are used as test data in this experiment, we chose 1500 history data for training and 500 data for testing as shown in Fig. 3. The

multi-step forecasting results of SSA-CG-BA-RBFNN, SSA-CG-BA-GRNN, FEEMD-CG-BA-RBFNN and FEEMD-CG-BA-GRNN are shown in Table 6 and Fig. 4. The detailed multi-step promoting percentages of the hybrid models of the four sites are shown in Table 7 and Fig. 5.

Table 6 and Fig. 4 indicate the following:

(a) For Site 1, when the forecasting is 1-step, SSA-CG-BA-GRNN has the highest accuracy forecasting results with a 1.2720% MAPE value. The second-highest to fourth-highest accurate models are FEEMD-CG-BA-GRNN,

| Table 7 |
|---|
| Improvement percentages of four proposed hybrid models. |

| | | SSA-CG-BA-RBFNN vs. SSA-CG-BA-GRNN | | | SSA-CG-BA- RBFNN | RBFNN vs. FEEM | D-CG-BA- | SSA-CG-BA-RBFNN vs. FEEMD-CG-BA- GRNN | | | |
|--------|-----------------------|------------------------------------|-----------------------|----------|---------------------|-----------------------|----------|--|------------------------|---------|--|
| | | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | |
| Site 1 | ζ _{ΜΑΕ} (%) | -22.958 | -9.086 | -16.613 | 15.563 | 4.823 | 5.859 | -16.667 | -6.842 | -7.788 | |
| | ζ _{ΜSE} (%) | -40.659 | -17.564 | -30.540 | 34.066 | 10.198 | 12.170 | -30.769 | -13.314 | -15.040 | |
| | ζ _{ΜΑΡΕ} (%) | -22.956 | -9.114 | -16.656 | 15.403 | 4.740 | 5.786 | -16.705 | -6.858 | -7.804 | |
| Site 2 | ζ _{МАЕ} (%) | -23.541 | -18.005 | -20.442 | -6.231 | -6.079 | 8.267 | -14.738 | -12.715 | -8.331 | |
| | ζ _{МЅЕ} (%) | -41.803 | -32.793 | -36.543 | -12.295 | -11.892 | 17.283 | -27.049 | -23.964 | -15.993 | |
| | ζ _{МАРЕ} (%) | -23.645 | -17.898 | -20.472 | -6.348 | -5.976 | 8.325 | -14.819 | -12.711 | -8.178 | |
| Site 3 | ξ _{ΜΑΕ} (%) | -9.275 | -20.046 | -20.179 | 1.812 | -3.098 | 7.127 | -3.731 | -4.875 | -3.703 | |
| | ξ _{ΜSE} (%) | -18.095 | -36.063 | -36.502 | 2.857 | -6.272 | 14.617 | -7.619 | -9.582 | -7.428 | |
| | ζ _{ΜΑΡΕ} (%) | -9.193 | -20.098 | -20.094 | 1.728 | -3.032 | 7.104 | -3.797 | -4.833 | -3.686 | |
| Site 4 | ξ _{ΜΑΕ} (%) | -8.192 | -7.387 | -21.663 | 4.096 | 2.645 | 0.330 | -2.597 | 1.003 | -20.094 | |
| | ξ _{ΜSE} (%) | -15.789 | -14.416 | -38.615 | 7.895 | 5.292 | 0.732 | -5.263 | 2.007 | -36.152 | |
| | ζ _{ΜΑΡΕ} (%) | -8.187 | -7.492 | -21.662 | 4.164 | 2.613 | 0.268 | -2.595 | 0.991 | -11.937 | |
| | | SSA-CG-BA- RBFNN | GRNN <i>vs.</i> FEEMI | D-CG-BA- | SSA-CG-BA- GRNN | GRNN <i>vs.</i> FEEMI | D-CG-BA- | FEEMD-CG- BA-GRNN | BA-RBFNN <i>vs.</i> FE | EMD-CG- | |
| | | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | |
| Site 1 | ξ _{ΜΑΕ} (%) | 50.000 | 15.299 | 26.949 | 8.166 | 2.468 | 10.583 | -27.889 | -11.129 | -12.892 | |
| | ξ _{ΜSE} (%) | 125.926 | 33.677 | 61.488 | 16.667 | 5.155 | 22.314 | -48.361 | -21.337 | -24.258 | |
| | ξ _{ΜΑΡΕ} (%) | 49.788 | 15.243 | 26.927 | 8.113 | 2.482 | 10.622 | -27.822 | -11.073 | -12.846 | |
| Site 2 | ζ _{ΜΑΕ} (%) | 22.639 | 14.544 | 36.085 | 11.514 | 6.452 | 15.224 | -9.072 | -7.065 | -15.330 | |
| | ζ _{ΜSE} (%) | 50.704 | 31.099 | 84.824 | 25.352 | 13.137 | 32.385 | -16.822 | -13.701 | -28.372 | |
| | ζ _{ΜΑΡΕ} (%) | 22.653 | 14.521 | 36.209 | 11.560 | 6.318 | 15.458 | -9.045 | -7.163 | -15.235 | |
| Site 3 | ξ _{ΜΑΕ} (%) | 12.221 | 21.197 | 34.210 | 6.110 | 18.974 | 20.642 | -5.445 | -1.834 | -10.109 | |
| | ξ _{ΜSE} (%) | 25.581 | 46.594 | 80.503 | 12.791 | 41.417 | 45.786 | -10.185 | -3.532 | -19.233 | |
| | ξ _{ΜΑΡΕ} (%) | 12.027 | 21.358 | 34.037 | 5.943 | 19.104 | 20.534 | -5.431 | -1.858 | -10.075 | |
| Site 4 | ζ _{МАЕ} (%) | 13.384 | 10.832 | 28.074 | 6.094 | 9.060 | 2.003 | -6.430 | -1.599 | -20.357 | |
| | ζ _{МЅЕ} (%) | 28.125 | 23.028 | 64.100 | 12.500 | 19.190 | 4.013 | -12.195 | -3.120 | -36.616 | |
| | ζ _{МАРЕ} (%) | 13.453 | 10.923 | 27.994 | 6.091 | 9.170 | 12.414 | -6.489 | -1.581 | -12.172 | |

SSA-CG-BA-RBFNN and FEEMD-CG-BA-RBFNN with MAPE values of 1.3752%, 1.6510% and 1.9053%, respectively. When the forecasting is 2-step, SSA-CG-BA-GRNN has the most accurate forecasting results with a MAPE value of 2.9528%. According to the MAPE value, FEEMD-CG-BA-GRNN is the second most accurate model, SSA-CG-BA-RBFNN is the third most accurate model and FEEMD-CG-BA-RBFNN is the fourth most accurate model with MAPE values of 3.0261%, 3.2489% and 3.4029%, respectively. When the forecasting is 3-step, SSA-CG-BA-GRNN is still the most accurate forecasting model among the proposed four hybrid models.

- (b) For Site 2, SSA-CG-BA-GRNN has the most accurate forecasting results among the 1.2720%, 2.7491% and 3.8576%, respectively. When the forecasting is 2-step, FEEMD-CG-BA-RBFNN is more accurate than SSA-CG-BA-RBFNN. In the three-step forecasting, the precision of the hybrid models is ranked from high to low as SSA-CG-BA-GRNN, FEEMD-CG-BA-GRNN, SSA-CG-BA-RBFNN and FEEMD-CG-BA-RBFNN.
- (c) SSA-CG-BA-GRNN is the most accurate model for one-step to three-step forecasting among the four hybrid models in Site 3. The CG-BA-RBFNN with the SSA decomposition algorithm is more precise than CG-BA-RBFNN with the FEEMD algorithm.
- (d) SSA-CG-BA-GRNN is still the most accurate forecasting model from one-step forecasting to three-step forecasting among all of the proposed models for the data from Site 4. However, for this site, the two-step forecasting results of SSA-CG-BA-RBFNN are more accurate than FEEMD-CG-BA-GRNN.

Table 7 and Fig. 5 shows the following:

- (a) In the one-step predictions, the ξ_{MAPE} (%) value indicates that, from four sites, the MAPE value of SSA-CG-BA-RBFNN, FEEMD-CG-BA-GRNN and FEEMD-CG-BA-RBFNN are decreased with SSA-CG-BA-GRNN with -22.956, -23.645, -9.193 and -8.187; 8.113, 11.560, 5.943 and 6.091; -16.705, -14.819, -3.797 and -2.595, respectively.
- (b) In the two-step and three-step predictions, SSA-CG-BA-GRNN also decreases the MAPE value based on SSA-CG-BA-RBFNN, FEEMD-CG-BA-GRNN and FEEMD-CG-BA-RBFNN.
- (c) In the one-step predictions, the ξ_{MAPE} (%) value illustrates that from four sites, FEEMD-CG-BA-GRNN decreases 27.822%, 9.045%, 5.431% and 6.489% MAPE values based on FEEMD-CG-BA-RBFNN. In the two-step predictions, FEEMD-CG-BA-GRNN decreases 11.073%, 7.163%, 1.858% and 1.581% MAPE values based on FEEMD-CG-BA-RBFNN. 12.846%, 15.235%, 10.075% and 12.172% MAPE values are decreased with FEEMD-CG-BA-GRNN based on FEEMD-CG-BA-RBFNN in the three-step predictions, respectively.

Remark. By comparing the four proposed hybrid models, the SSA-CG-BA-GRNN hybrid model has the most accurate forecasting results. Comparisons of FEEMD-CG-BA-GRNN with FEEMD-CG-BA-RBFNN and SSA-CG-BA-GRNN with SSA-CG-BA-RBFNN could conclude that the forecasting ability of GRNN is stronger than that of RBFNN. Comparisons of SSA-CG-BA-GRNN with FEEMD-CG-BA-GRNN and SSA-CG-BA-RBFNN with FEEMD-CG-BA-GRNN could reveal that the hybrid models combined with SSA are more accurate than the hybrid models combined with FEEMD.



Fig. 5. Promoting percentages of four hybrid models.

 Table 8

 Forecasting performance of SSA-BA-RBFNN, SSA-BA-GRNN, FEEMD-BA-RBFNN and FEEMD-BA-GRNN.

| | | SSA-BA-RBFNN | | SSA-BA-GRNN | | | FEEMD-BA-RBFNN | | | FEEMD-BA-GRNN | | | |
|--------|----------|--------------|--------|-------------|--------|--------|----------------|--------|--------|---------------|--------|--------|--------|
| | | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step |
| Site 1 | MAE | 0.0971 | 0.1855 | 0.2886 | 0.0825 | 0.1665 | 0.2838 | 0.1628 | 0.2298 | 0.3498 | 0.0962 | 0.1846 | 0.4383 |
| | MSE | 0.0104 | 0.0383 | 0.0926 | 0.0076 | 0.0309 | 0.0728 | 0.0138 | 0.0461 | 0.1034 | 0.0103 | 0.0379 | 0.0732 |
| | MAPE (%) | 1.7693 | 3.3822 | 5.2634 | 1.5041 | 3.0375 | 4.7621 | 2.2752 | 3.7619 | 5.5838 | 1.7552 | 3.3672 | 5.3457 |
| Site 2 | MAE | 0.1138 | 0.2387 | 0.3335 | 0.1081 | 0.1902 | 0.2638 | 0.1067 | 0.2225 | 0.3333 | 0.1049 | 0.2215 | 0.2961 |
| | MSE | 0.0154 | 0.0581 | 0.1329 | 0.0139 | 0.0432 | 0.0831 | 0.0636 | 0.0892 | 0.1526 | 0.0131 | 0.0586 | 0.1311 |
| | MAPE (%) | 1.7698 | 3.6455 | 5.1838 | 1.6799 | 2.9566 | 4.1009 | 1.7593 | 3.5575 | 5.5822 | 1.6307 | 3.4444 | 5.2921 |
| Site 3 | MAE | 0.1161 | 0.2403 | 0.3621 | 0.1072 | 0.2197 | 0.2901 | 0.1272 | 0.2446 | 0.3746 | 0.1207 | 0.2438 | 0.3274 |
| | MSE | 0.0161 | 0.0687 | 0.1559 | 0.0137 | 0.0575 | 0.1001 | 0.0193 | 0.0713 | 0.1671 | 0.0174 | 0.0708 | 0.1325 |
| | MAPE (%) | 1.6391 | 3.3909 | 5.1124 | 1.5125 | 3.1007 | 4.0939 | 1.7951 | 3.4511 | 5.2852 | 1.7036 | 3.4409 | 4.7401 |
| Site 4 | MAE | 0.1273 | 0.2421 | 0.3844 | 0.1221 | 0.2364 | 0.3149 | 0.1385 | 0.2576 | 0.3933 | 0.1304 | 0.2447 | 0.3112 |
| | MSE | 0.0184 | 0.0667 | 0.1683 | 0.0169 | 0.0636 | 0.1128 | 0.0218 | 0.0756 | 0.1761 | 0.0193 | 0.0684 | 0.1303 |
| | MAPE (%) | 1.7106 | 3.2516 | 5.1642 | 1.6411 | 3.1774 | 4.2304 | 1.8609 | 3.4617 | 5.2849 | 1.7519 | 3.2871 | 4.9806 |

6.3. Experiment II

This experiment is divided into two parts. The first part illustrates the multi-step forecasting results of SSA-BA-RBFNN, SSA-BA-GRNN, FEEMD-BA-RBFNN and FEEMD-BA-GRNN (as shown in Table 8 and Fig. 6) and the detailed multi-step promoting percentages, using the data from four sites, to evaluate the efficiency of the developed optimization algorithm CG-BA (as shown in Table 9 and Fig. 6). In the second part, five single models, ELM, SVM, RBFNN, GRNN and ARIMA, are employed in multi-step prediction (as shown in Tables 10 and 11). The detailed multi-step promoting percentages of SSA-CG-BA-RBFNN, SSA-CG-BA-GRNN, FEEMD-CG-BA-RBFNN and FEEMD-CG-BA-GRNN by RBFNN, GRNN and ARIMA of four sites are shown in Tables 12 and 13.

Table 8 shows the following:

- (a) SSA-BA-GRNN achieves the highest accuracy in one-step prediction to three-step prediction based on the data from four sites.
- (b) FEEMD-BA-GRNN is ranked as the second most accurate model among the four models listed in Table 5, except for two-step prediction at Site 4.



Fig. 6. Forecasting results and promoting percentages of SSA-BA-RBFNN, FEEMD-BA-RBFNN, SSA-BA-GRNN and FEEMD-BA-GRNN.

Improvement percentages between four proposed hybrid models and SSA-BA-RBFNN, SSA-BA-GRNN, FEEMD-BA-RBFNN and FEEMD-BA-GRNN.

| | | SSA-CG-BA-RBFNN vs. SSA-BA- RBFNN | | SSA-CG-BA GRNN | A-CG-BA-GRNN <i>vs.</i> SSA-BA- NN | | | FEEMD-CG-BA-RBFNN vs. FEEMD-BA-RBFNN | | | FEEMD-CG-BA-GRNN vs. FEEMD- BA-GRNN | | |
|--------|-----------------------|--------------------------------------|---------|-------------------|---------------------------------------|---------|---------|---|---------|---------|--|---------|---------|
| | | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step |
| Site 1 | ξ _{ΜΑΕ} (%) | -6.694 | -3.881 | -3.015 | -15.394 | -2.643 | -17.759 | -35.688 | -18.668 | -15.294 | -21.517 | -10.021 | -41.113 |
| | ξ _{ΜSE} (%) | -12.534 | -7.833 | -5.940 | -28.947 | -5.825 | -16.896 | -11.594 | -15.618 | -5.513 | -38.834 | -19.261 | -1.0928 |
| | ξ _{ΜΑΡΕ} (%) | -6.686 | -3.941 | -3.101 | -15.431 | -2.788 | -10.739 | -16.257 | -9.543 | -3.376 | -21.649 | -10.131 | -12.037 |
| Site 2 | ξ _{ΜΑΕ} (%) | -11.159 | -9.719 | -6.417 | -28.492 | -7.098 | -5.876 | -11.153 | -9.034 | -1.380 | -17.826 | -15.079 | -3.3772 |
| | ζ _{ΜSE} (%) | -20.779 | -4.475 | -12.491 | -48.921 | -13.657 | -11.191 | -83.176 | -45.179 | -10.616 | -32.061 | -27.986 | -25.476 |
| | ζ _{ΜΑΡΕ} (%) | -11.081 | -8.150 | -6.428 | -28.472 | -7.018 | -5.933 | -16.228 | -11.503 | -5.872 | -17.796 | -15.143 | -15.838 |
| Site 3 | ξ _{ΜΑΕ} (%) | -19.207 | -8.656 | -10.494 | -20.615 | -20.118 | -10.823 | -24.921 | -13.041 | -7.315 | -25.186 | -14.356 | -4.6731 |
| | ξ _{ΜSE} (%) | -34.782 | -16.448 | -19.692 | -37.226 | -36.173 | -20.579 | -44.041 | -24.544 | -14.123 | -44.253 | -26.694 | -12.528 |
| | ξ _{ΜΑΡΕ} (%) | -19.169 | -8.656 | -10.541 | -20.456 | -20.182 | -10.733 | -24.917 | -12.969 | -7.319 | -25.182 | -14.333 | -7.0716 |
| Site 4 | ξ _{ΜΑΕ} (%) | -21.366 | -9.418 | -5.489 | -24.733 | -14.086 | -9.622 | -24.763 | -12.616 | -7.323 | -25.231 | -9.481 | -6.7159 |
| | ξ _{ΜSE} (%) | -38.043 | -17.841 | -10.754 | -43.195 | -26.258 | -18.262 | -43.578 | -23.677 | -14.082 | -44.041 | -18.274 | -26.401 |
| | ζ _{ΜΑΡΕ} (%) | -21.384 | -9.365 | -5.441 | -24.763 | -14.197 | -9.574 | -24.724 | -12.641 | -7.353 | -25.229 | -9.455 | -13.659 |

Table 10

Forecasting performance of RBFNN, GRNN and ARIMA.

| | | RBFNN | | | GRNN | | | ARIMA | | |
|--------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step |
| Site 1 | MAE | 0.2523 | 0.3171 | 0.4377 | 0.1861 | 0.2947 | 0.3979 | 0.2912 | 0.4418 | 0.6071 |
| | MSE | 0.0707 | 0.1117 | 0.2131 | 0.0385 | 0.0966 | 0.1761 | 0.0943 | 0.2169 | 0.4095 |
| | MAPE (%) | 4.6003 | 5.7811 | 7.9794 | 3.3943 | 5.3733 | 7.2559 | 5.3109 | 8.0568 | 11.068 |
| Site 2 | MAE | 0.2938 | 0.3846 | 0.5073 | 0.2229 | 0.3395 | 0.4707 | 0.3431 | 0.5241 | 0.7085 |
| | MSE | 0.1031 | 0.1767 | 0.3073 | 0.0594 | 0.1377 | 0.2646 | 0.1405 | 0.3279 | 0.5996 |
| | MAPE (%) | 4.5687 | 5.9792 | 7.8871 | 3.4642 | 5.2785 | 7.3189 | 5.3339 | 8.1481 | 11.013 |
| Site 3 | MAE | 0.3245 | 0.4228 | 0.5418 | 0.2424 | 0.3901 | 0.4853 | 0.3981 | 0.5769 | 0.7797 |
| | MSE | 0.1253 | 0.2128 | 0.3494 | 0.0699 | 0.1811 | 0.2804 | 0.1886 | 0.3961 | 0.7233 |
| | MAPE (%) | 4.5794 | 5.9651 | 7.6461 | 3.4238 | 5.5041 | 6.8491 | 5.6167 | 8.1408 | 11.006 |
| Site 4 | MAE | 0.3223 | 0.4522 | 0.5823 | 0.2571 | 0.4079 | 0.5157 | 0.4049 | 0.5858 | 0.8383 |
| | MSE | 0.1183 | 0.2328 | 0.3861 | 0.0753 | 0.1894 | 0.3028 | 0.1867 | 0.3906 | 0.8006 |
| | MAPE (%) | 4.3311 | 6.0751 | 7.8223 | 3.4524 | 5.4806 | 6.9291 | 5.4401 | 7.8699 | 11.261 |

Table 11

Forecasting performance of ELM and SVM.

| | | ELM | | | SVM | | |
|--------|----------|--------|--------|--------|--------|--------|--------|
| | | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step |
| Site 1 | MAE | 0.1774 | 0.2756 | 0.4566 | 0.2614 | 0.3015 | 0.4118 |
| | MSE | 0.0381 | 0.0905 | 0.1645 | 0.0495 | 0.1166 | 0.1849 |
| | MAPE (%) | 3.1044 | 5.0019 | 6.9985 | 3.6498 | 5.7654 | 7.6811 |
| Site 2 | MAE | 0.1956 | 0.3141 | 0.4415 | 0.2415 | 0.3124 | 0.4845 |
| | MSE | 0.0428 | 0.1124 | 0.2561 | 0.0469 | 0.1244 | 0.3146 |
| | MAPE (%) | 3.2107 | 5.0163 | 6.9875 | 3.7105 | 5.8647 | 7.6447 |
| Site 3 | MAE | 0.2014 | 0.3421 | 0.4685 | 0.2768 | 0.3216 | 0.4975 |
| | MSE | 0.0419 | 0.1031 | 0.2541 | 0.0684 | 0.1467 | 0.3017 |
| | MAPE (%) | 3.1964 | 5.1651 | 6.9541 | 3.8004 | 5.9451 | 7.1847 |
| Site 4 | MAE | 0.2051 | 0.3518 | 0.5251 | 0.2617 | 0.5131 | 0.5347 |
| | MSE | 0.0423 | 0.1179 | 0.2741 | 0.0751 | 0.2015 | 0.3241 |
| | MAPE (%) | 3.1553 | 5.1618 | 6.9144 | 3.8117 | 5.9614 | 7.1874 |

(c) The forecasting accuracy of BA-RBFNN with the decomposition algorithms SSA and FEEMD is lower than that of BA-GRNN with the decomposition algorithms SSA and FEEMD, mostly.

Table 9 illustrates the following:

- (a) In the one-step to three-step predictions, SSA-BA-RBFNN decreases the MAPE values from four sites based on SSA-CG-BA-RBFNN are 6.6862%, 3.9412% and 3.1007%;11.081%, 8.1497% and 6.4277%; 19.169%, 8.6555% and 10.541% and 21.384%, 9.3646% and 5.4413%, respectively.
- (b) For the data from four sites, SSA-BA-GRNN decreases the MAPE values in the one-step to three-step predictions on the basis of SSA-CG-BA-GRNN are 15.431%, 2.7884% and 10.739%;28.472%, 7.0181% and 5.9328%; 20.456%, 20.182% and 10.733% and 24.763%, 14.197% and 9.5735%, respectively.
- (c) The ξ_{MAPE} (%) values presents that from four sites in the onestep to three-step predictions, the MAPE values are decreased with 16.257%, 9.5431% and 3.3758%; 16.228%, 11.5025% and 5.8722%; 24.917%, 12.969% and 7.3186% and 24.724%, 12.641% and 7.3531% with FEEMD-CG-BA-RBFNN based on FEEMD-BA-RBFNN, respectively.

 Table 12

 Improvement percentages between four proposed hybrid models and RBFNN and GRNN.

| | | SSA-CG-BA-RBFNN vs. RBFNN | | | FEEMD-CG-BA- RBFNN vs. RBFNN | | | SSA-CG-BA-GRNN vs. GRNN | | | FEEMD-CG-BA-GRNN vs. GRNN | | |
|-----------|--|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| | | 1-step | 2-step | 3-step |
| Site 1 | ξ _{MAE} (%) ξ _{MSE} (%) ^ξ MAPE (%) | -105.408 -323.076 -105.591 | -65.2832 -173.654 -65.3882 | -42.1579 -102.181 -42.2669 | -166.619 -612.963 -166.848 | -81.8014 -231.959 -81.9731 | -70.4799 -191.074 -70.6989 | -77.7459 -215.573 -78.1504 | -57.6779 -148.329 -57.9035 | -34.2896 -80.2457 -34.4856 | -146.491 -511.111 -146.822 | -77.4232 -215.686 -77.5652 | -54.1651 -137.973 -54.3086 |
| Site 2 | ξ _{MAE} (%) ξ _{MSE} (%) ξ _{MAPE} (%) | -120.474 -386.885 -120.131 | -57.541 -148.108 -57.642 | -50.817 -127.515 -50.887 | -188.357 -736.619 -188.299 | -92.134 -269.169 -92.008 | -89.569 -258.537 -89.727 | -135.127 -455.141 -135.052 | -67.737 -181.595 -67.662 | -39.302 -93.988 -39.291 | -158.585 -567.416 -158.426 | -80.489 -226.303 -80.597 | -64.523 -170.829 -64.326 |
| Site 3 | ξ _{MAE} (%) ξ _{MSE} (%) ^ξ MAPE (%) | -158.422 -565.714 -158.419 | -77.722 -215.505 -77.701 | -49.738 -123.961 -49.756 | -184.841 -712.791 -184.582 | -122.279 -393.461 -122.397 | -87.592 -252.704 -87.416 | -153.822 -547.222 -154.029 | -83.404 -236.617 -83.256 | -39.775 -95.401 -39.823 | -168.439 -620.619 -168.618 | -86.830 -248.941 -86.725 | -55.495 -141.933 -55.488 |
| Site 4 | ξ _{MAE} (%) ξ _{MSE} (%) ^ξ MAPE (%) | -156.843 -560.526 -156.722 | -86.001 -245.621 -85.966 | -41.949 -101.597 -41.897 | -179.761 -684.375 -179.614 | -100.837 -303.838 -101.027 | -81.202 -228.416 -81.134 | -146.737 -512.195 -146.459 | -81.208 -228.249 -81.231 | -41.482 -100.132 -41.517 | -163.692 -597.222 -163.562 | -84.154 -238.819 -84.141 | -77.644 -215.746 -61.131 |

 Table 13

 Improvement percentages between four proposed hybrid models and ARIMA.

| | | SSA-CG-BA-RBFNN vs. ARIMA | | FEEMD-CO ARIMA | FEEMD-CG-BA- RBFNN <i>vs.</i> ARIMA | | | SSA-CG-BA-GRNN vs. ARIMA | | | FEEMD-CG-BA-GRNN vs. ARIMA | | |
|--------|-----------------------|---------------------------|---------|-------------------|--|---------|---------|--------------------------|---------|---------|----------------------------|---------|---------|
| | | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step | 1-step | 2-step | 3-step |
| Site 1 | ξ _{ΜΑΕ} (%) | 221.412 | 147.784 | 116.899 | 317.192 | 172.548 | 160.111 | 178.128 | 136.383 | 104.893 | 285.695 | 165.984 | 135.218 |
| | ξ _{ΜSE} (%) | 936.263 | 514.447 | 370.149 | 1646.29 | 645.361 | 576.859 | 672.951 | 457.583 | 319.142 | 1396.82 | 608.823 | 453.378 |
| | ξ _{ΜΑΡΕ} (%) | 221.677 | 147.985 | 117.011 | 317.523 | 172.853 | 160.381 | 178.743 | 136.762 | 105.141 | 286.191 | 166.243 | 135.379 |
| Site 2 | ξ _{ΜΑΕ} (%) | 239.367 | 143.201 | 127.011 | 343.855 | 196.604 | 185.341 | 261.919 | 158.942 | 109.677 | 298.027 | 178.628 | 147.641 |
| | ξ _{ΜSE} (%) | 1051.63 | 490.810 | 415.563 | 1878.87 | 779.089 | 712.466 | 1213.08 | 570.552 | 339.589 | 1478.65 | 677.014 | 513.715 |
| | ξ _{ΜΑΡΕ} (%) | 238.940 | 143.343 | 127.044 | 343.899 | 196.392 | 185.488 | 261.914 | 158.809 | 109.595 | 297.903 | 178.777 | 147.266 |
| Site 3 | ξ _{ΜΑΕ} (%) | 324.413 | 162.824 | 140.573 | 367.803 | 228.718 | 201.392 | 316.858 | 171.227 | 124.568 | 340.863 | 176.293 | 149.823 |
| | ξ _{MSE} (%) | 1696.19 | 590.069 | 477.715 | 2093.02 | 979.292 | 809.811 | 1646.29 | 636.245 | 404.041 | 1844.33 | 663.198 | 524.072 |
| | ξ _{MAPE} (%) | 323.933 | 162.826 | 140.647 | 366.852 | 228.935 | 201.163 | 316.731 | 171.043 | 124.685 | 340.663 | 176.174 | 149.858 |
| Site 4 | ξ _{ΜΑΕ} (%) | 304.495 | 167.122 | 130.746 | 340.588 | 188.429 | 194.554 | 288.579 | 160.239 | 129.986 | 315.282 | 164.469 | 188.771 |
| | ξ _{MSE} (%) | 1537.71 | 612.773 | 433.023 | 1844.79 | 732.836 | 768.329 | 1417.88 | 576.949 | 429.147 | 1628.71 | 598.747 | 734.827 |
| | ξ _{MAPE} (%) | 304.528 | 167.038 | 130.607 | 340.601 | 188.666 | 194.374 | 288.357 | 160.239 | 129.992 | 315.306 | 164.418 | 161.865 |

(d) In the one-step to three-step predictions, the MAPE values from four sites of by FEEMD-BA-GRNN are decreased 21.649%, 10.131% and 12.037%; 17.796%, 15.143% and 15.838%; 25.182%, 14.333% and 7.0716% and 25.229%, 9.4551% and 13.659% with FEEMD-CG-BA-GRNN, respectively.

Remark. By comparing SSA-CG-BA-RBFNN, SSA-CG-BA-GRNN, FEEMD-CG-BA-RBFNN and FEEMD-CG-BA-GRNN with SSA-BA-RBFNN, SSA-BA-GRNN, FEEMD-BA-RBFNN and FEEMD-BA-GRNN, the performance of the proposed optimization algorithm CG-BA is better than that of the original BA.

Tables 10–13 indicate the following:

- (a) GRNN and ARIMA are the most and least accurate forecasting models, respectively, among RBFNN, GRNN and ARIMA in one-step to three-step prediction of the data from four sites.
- (b) The four proposed models are more accurate than ELM and SVM. ELM is more accurate than RBFNN, GRNN, ARIMA and SVM.
- (c) Based on the one-step to three-step prediction of RBFNN, from four sites, SSA-CG-BA-RBFNN decreases 105.591%, 65.3882% and 42.2669%; 120.131%, 57.6424% and

50.8865%; 158.419%, 77.7006% and 49.7562%; 156.722%, 85.9658% and 41.8967% MAPE values, respectively. And FEEMD-CG-BA-RBFNN decreases 166.848%, 81.9731% and 70.6989%; 188.299%, 92.0083% and 89.7268%; 184.582%, 122.397% and 87.4155%; 179.614%, 101.027% and 81.1342% MAPE values, respectively.

- (d) In the one-step to three-step prediction, the MAPE promoted percentages from four sites of SSA-CG-BA-GRNN and FEEMD-CG-BA-GRNN by GRNN are -78.1504%, -57.9035% and -34.4856% and -146.822%, -77.5652% and -54.3086%; -135.052%, -67.6619% and -39.2909% and -158.426%, -80.5974% and -64.3257%; -154.029%, -83.2562% and -39.8232% and -168.618%, -86.7252% and -55.4882%; -146.459%, -81.2308% and -41.5171% and -163.562%, -84.1414% and -61.13065%, respectively.
- (e) For Site 1, the MAPE promoted percentages, in the one-step to three-step prediction, from four sites of ARIMA with SSA-CG-BA-RBFNN, FEEMD-CG-BA-RBFNN, SSA-CG-BA-GRNN and FEEMD-CG-BA-GRNN are -1221.677%, -147.985% and -117.011%; -317.523%, -172.853% and -160.381%; -178.743%, -136.762% and -105.141%; -286.191%, -166.243% and -135.379%, respectively.
- (f) For Site 2, the MAPE promoted percentages, in the one-step to three-step prediction, from four sites of ARIMA with SSA-CG-BA-RBFNN, FEEMD-CG-BA-RBFNN, SSA-CG-BA-GRNN

Forecasting results of the persistence prediction test.

| No. | Actual value (m/s) | Forecasting value (m/s) | MAPE (%) | No. | Actual value (m/s) | Forecasting value (m/s) | MAPE (%) |
|-----|--------------------|-------------------------|----------|-----|--------------------|-------------------------|----------|
| 1 | 3.8 | 3.8418 | 1.0988 | 51 | 4.5 | 4.4465 | 1.1881 |
| 2 | 3.8 | 3.7554 | 1.1742 | 52 | 4.8 | 4.7456 | 1.1330 |
| 3 | 3.8 | 3.7536 | 1.2200 | 53 | 4.6 | 4.6508 | 1.1038 |
| 4 | 3.9 | 3.8576 | 1.0867 | 54 | 4.1 | 4.0504 | 1.2091 |
| 5 | 4.1 | 4.1469 | 1.1439 | 55 | 4.3 | 4.3519 | 1.2079 |
| 6 | 3.7 | 3.6552 | 1.2109 | 56 | 4.4 | 4.3468 | 1.2095 |
| 7 | 3.1 | 3.1386 | 1.2440 | 57 | 4.3 | 4.2474 | 1.2242 |
| 8 | 3.2 | 3.1592 | 1.2737 | 58 | 4.2 | 4.1463 | 1.2787 |
| 9 | 2.2 | 2.1748 | 1.1450 | 59 | 4.4 | 4.4485 | 1.1012 |
| 10 | 2.1 | 2.1252 | 1.2022 | 60 | 4.3 | 4.3470 | 1.0927 |
| 11 | 2.2 | 2.1744 | 1.1647 | 61 | 5.2 | 5.2608 | 1.1697 |
| 12 | 2 | 2.0227 | 1.1333 | 62 | 6.2 | 6.2764 | 1.2327 |
| 13 | 2.5 | 2.5284 | 1.1362 | 63 | 6.4 | 6.3210 | 1.2344 |
| 14 | 2.4 | 2.4285 | 1.1854 | 64 | 6.2 | 6.1210 | 1.2745 |
| 15 | 2.3 | 2.3289 | 1.2551 | 65 | 5.4 | 5.4598 | 1.1078 |
| 16 | 2.8 | 2.7645 | 1.2687 | 66 | 4.2 | 4.1539 | 1.0988 |
| 17 | 3.5 | 3.4555 | 1.2715 | 67 | 4.1 | 4.0514 | 1.1861 |
| 18 | 4 | 4.0486 | 1.2152 | 68 | 4.4 | 4.3482 | 1.1770 |
| 19 | 4.2 | 4.2510 | 1.2144 | 69 | 5.4 | 5.4656 | 1.2143 |
| 20 | 3.8 | 3.7584 | 1.0936 | 70 | 5.9 | 5.8301 | 1.1840 |
| 21 | 2.8 | 2.8315 | 1.1248 | 71 | 6 | 6.0666 | 1.1100 |
| 22 | 2.8 | 2.7650 | 1.2489 | 72 | 6 | 5.9321 | 1.1324 |
| 23 | 3.1 | 3.1383 | 1.2361 | 73 | 6.4 | 6.4788 | 1.2310 |
| 24 | 3.1 | 3.0665 | 1.0813 | 74 | 5.9 | 5.9689 | 1.1685 |
| 25 | 3.6 | 3.5583 | 1.1574 | 75 | 6.3 | 6.2274 | 1.1518 |
| 26 | 5.6 | 5.5395 | 1.0802 | 76 | 6.1 | 6.0293 | 1.1589 |
| 27 | 5 | 5.0582 | 1.1649 | 77 | 6 | 5.9268 | 1.2208 |
| 28 | 4.8 | 4.8592 | 1.2340 | 78 | 6.1 | 6.1661 | 1.0839 |
| 29 | 4.6 | 4.6569 | 1.2369 | 79 | 6.3 | 6.3734 | 1.1649 |
| 30 | 3.4 | 3.4370 | 1.0872 | 80 | 6.4 | 6.4716 | 1.1194 |
| 31 | 4.3 | 4.3526 | 1.2244 | 81 | 6.8 | 6.7207 | 1.1660 |
| 32 | 4.6 | 4.6511 | 1.1105 | 82 | 7.1 | 7.0178 | 1.1582 |
| 33 | 4.3 | 4.3517 | 1.2015 | 83 | 7.1 | 7.0177 | 1.1594 |
| 34 | 4.4 | 4.4540 | 1.2277 | 84 | 6.9 | 6.8151 | 1.2310 |
| 35 | 4.3 | 4.3543 | 1.2635 | 85 | 6.2 | 6.2696 | 1.1232 |
| 36 | 3.4 | 3.4419 | 1.2331 | 86 | 5.7 | 5.6276 | 1.2699 |
| 37 | 2.4 | 2.4273 | 1.1375 | 87 | 5.7 | 5.7692 | 1.2143 |
| 38 | 2.3 | 2.3275 | 1.1952 | 88 | 5.6 | 5.6698 | 1.2467 |
| 39 | 2.6 | 2.5691 | 1.1893 | 89 | 5.7 | 5.6365 | 1.1135 |
| 40 | 2.8 | 2.8338 | 1.2089 | 90 | 5.8 | 5.7259 | 1.2780 |
| 41 | 2.6 | 2.5684 | 1.2158 | 91 | 5.6 | 5.5296 | 1.2569 |
| 42 | 2.7 | 2.6657 | 1.2690 | 92 | 5.9 | 5.8345 | 1.1110 |
| 43 | 2.8 | 2.8342 | 1.2219 | 93 | 5.8 | 5.8674 | 1.1614 |
| 44 | 3.6 | 3.6397 | 1.1039 | 94 | 5.6 | 5.5303 | 1.2451 |
| 45 | 4.1 | 4.0520 | 1.1700 | 95 | 5.3 | 5.2394 | 1.1437 |
| 46 | 4.6 | 4.6558 | 1.2124 | 96 | 5.2 | 5.1429 | 1.0980 |
| 47 | 4.4 | 4.3494 | 1.1500 | 97 | 5.3 | 5.3587 | 1.1073 |
| 48 | 3.9 | 3.8546 | 1.1632 | 98 | 5.4 | 5.3363 | 1.1790 |
| 49 | 4.1 | 4.0489 | 1.2466 | 99 | 5.4 | 5.4637 | 1.1790 |
| 50 | 3.7 | 3.7445 | 1.2027 | 100 | 4.8 | 4.8524 | 1.0910 |

and FEEMD-CG-BA-GRNN are -238.940%, -143.343% and -127.044%; -343.899%, -196.392% and -185.488%; -261.914%, -158.809% and -109.595% and -297.903%, -178.777% and -147.266%, respectively.

- (g) The MAPE promoted percentages, for Site 3 in the one-step to three-step prediction, from four sites of ARIMA with SSA-CG-BA-RBFNN, FEEMD-CG-BA-RBFNN, SSA-CG-BA-GRNN and FEEMD-CG-BA-GRNN are -323.933%, -162.826% and -140.647%; -366.852%, -228.935% and -201.163%; -316.731%, -171.043% and -124.685% and -340.663%, -176.174% and -149.858%, respectively.
- (h) In the one-step to three-step prediction for Site 4, the MAPE promoted percentages from four sites of ARIMA with SSA-CG-BA-RBFNN, FEEMD-CG-BA-RBFNN, SSA-CG-BA-GRNN and FEEMD-CG-BA-GRNN are -304.528%, -167.038% and -130.607%; -340.601%, -188.666% and -194.374%; -288.357%, -160.239% and -129.992%; -315.306%, -164.418% and -161.865%, respectively.

Remark. By comparing SSA-CG-BA-RBFNN, SSA-CG-BA-GRNN, FEEMD-CG-BA-RBFNN and FEEMD-CG-BA-GRNN with ELM, SVM, RBFNN, GRNN and ARIMA, the forecasting performance of the proposed four hybrid models are better than that of the single models.

6.4. Experiment III: Persistence prediction test

To evaluate the proposed model, in this part a persistence prediction test is employed. In this test, the proposed model is used to output 100 continuous data. The forecasting results presented in Table 14 show that SSA-CG-BA-GRNN could always keep a high forecasting performance in this test.

6.5. Experiment IV: Diebold-Mariano (DM)-test and forecasting validity degree (FVD)

The DM test and FVD are conducted to further evaluate the levels of accuracy achieved by the proposed hybrid models (as shown in Fig. 7).



Fig. 7. Process of hybrid forecasting strategy and DM test results.

DM test which is a comparison test focusing on predictive accuracy, could be used to compare the forecasting performance of the proposed hybrid model with others. For more details, one can refer [55], which provides a complete description of the DM test theory.

FVD can be measured not only by the square sum of forecasting error but also by the mean and mean squared deviation of the forecasting accuracy. It is a useful tool to evaluate the forecasting accuracy of the model. For more details, one can refer [56], which provides a complete description of the DM test theory.

The results given in Table 15 indicate the following:

- (a) SSA-CG-BA-RBFNN is more accurate than RBFNN and ARIMA at the 10% significance level, more accurate than SSA-BA-RBFNN, FEEMD-BA-RBFNN, SSA-BA-GRNN, FEEMD-BA-GRNN and GRNN at the 5% significance level, and more accurate than FEEMD-CG-BA-RBFNN at the 1% significance level.
- (b) FEEMD-CG-BA-RBFNN is more accurate than RBFNN, GRNN and ARIMA at the 10% significance level and more accurate than SSA-BA-RBFNN, FEEMD-BA-RBFNN, SSA-BA-GRNN and FEEMD-BA-GRNN at the 5% significance level.
- (c) SSA-CG-BA-GRNN is the most accurate model among these models. It is more accurate than SSA-CG-BA-RBFNN, FEEMD-CG-BA-RBFNN and FEEMD-CG-BA-GRNN at the 1%

significance level, more accurate than SSA-BA-RBFNN, FEEMD-BA-RBFNN, SSA-BA-GRNN and FEEMD-CG-BA-GRNN at the 5% significance level, and more accurate than RBFNN, GRNN and ARIMA at the 10% significance level.

(d) FEEMD-CG-BA-GRNN is more accurate than SSA-CG-BA-RBFNN and FEEMD-CG-BA-RBFNN at the 1% significance level, more accurate than SSA-BA-RBFNN, FEEMD-BA-RBFNN, SSA-BA-GRNN and FEEMD-CG-BA-GRNN at the 5% significance level, and more accurate than RBFNN, GRNN and ARIMA at the 10% significance level.

FVD is measured to evaluate the forecasting accuracy of the hybrid models and the other six comparison models. A more accurate forecasting model leads to a larger FVD value. The results presented in Table 16 show that the FVD value for the SSA-CG-BA-GRNN model is larger than those of the comparison models.

Remark. Based on the results from the above two methods, the forecasting performance of SSA-CG-BA-GRNN has been globally evaluated. From the results of the DM test and FVD, one can see that SSA-CG-BA-GRNN is the most accurate forecasting model among the proposed forecasting architecture for multi-step wind speed forecasting. As shown in Table 17, SSA-CG-BA-GRNN is more

Results for the DM test.

| | SSA-CG-BA-RBFNN |
|--|---|
| DM-test FEEMD-CG-BA-RBFNN SSA-CG-BA-GRNN FEEMD-CG-BA-GRNN SSA-BA-RBFNN FEEMD-BA-RBFNN SSA-BA-GRNN FEEMD-BA-GRNN RBFNN GRNN ARIMA | 1.773275 [°] 1.298743 1.480342 2.092264 [°] 2.043212 ^{°*} 1.999822 ^{°*} 1.970851 ^{°*} 3.673214 ^{°**} 2.570125 ^{°*} 5.662563 ^{°**} |
| SSA-CG-BA-RBFNN SSA-CG-BA-GRNN FEEMD-CG-BA-GRNN SSA-BA-RBFNN FEEMD-BA-RBFNN SSA-BA-GRNN FEEMD-BA-GRNN RBFNN GRNN ARIMA | 1.577545 1.290934 1.400953 1.992563** 2.001252** 1.900823** 1.980123** 3.356216*** 2.790325*** 5.467216*** SSA-CG-BA-GRNN |
| SSA-CG-BA-RBFNN FEEMD-CG-BA-RBFNN FEEMD-CG-BA-GRNN SSA-BA-RBFNN FEEMD-BA-RBFNN SSA-BA-GRNN FEEMD-BA-GRNN RBFNN GRNN ARIMA | 1.933213 [°] 1.956331 [°] 1.763224 [°] 2.456424 ^{°*} 2.473563 ^{•*} 2.345621 ^{•*} 2.578142 ^{•**} 3.809438 ^{•**} 3.155621 ^{•**} 5.779335 ^{•**} FEEMD-CG-BA-GRNN |
| SSA-CG-BA-RBFNN FEEMD-CG-BA-RBFNN SSA-CG-BA-GRNN SSA-BA-RBFNN FEEMD-BA-RBFNN SSA-BA-GRNN FEEMD-BA-GRNN RBFNN GRNN ARIMA | 1.893781 [°] 1.960031 [°] 1.602371 2.244341 ^{°°} 2.389023 ^{°°} 2.109824 ^{°°} 2.100231 ^{°°} 3.673214 ^{°°°} 5.662563 ^{°°°} |

1% significance level.

^{**} 5% significance level.

** 10% significance level.

| Table 1 | 6 | |
|---------|-----|------|
| Results | for | FVD. |

| Model | FVD | | |
|-------------------|---------|---------|---------|
| | 1-step | 2-step | 3-step |
| SSA-CG-BA-RBFNN | 98.5264 | 96.8396 | 95.1481 |
| FEEMD-CG-BA-RBFNN | 98.4681 | 96.8553 | 94.8889 |
| SSA-CG-BA-GRNN | 98.7722 | 97.2742 | 96.1030 |
| FEEMD-CG-BA-GRNN | 98.6750 | 97.0318 | 95.5347 |
| SSA-BA-RBFNN | 98.2778 | 96.5825 | 94.8191 |
| FEEMD-BA-RBFNN | 98.0774 | 96.4420 | 94.5660 |
| SSA-BA-GRNN | 98.4156 | 96.9320 | 95.7032 |
| FEEMD-BA-GRNN | 98.2897 | 96.6151 | 94.9104 |
| RBFNN | 95.4801 | 94.0499 | 92.1663 |
| GRNN | 96.5663 | 94.5909 | 92.9118 |
| ARIMA | 94.5746 | 91.9461 | 88.9130 |

The best results are formatted in bold.

Table 17

Total computation time of each model.

| Model | CPU time (s) |
|-------------------|--------------|
| SSA-CG-BA-RBFNN | 38.6145 |
| FEEMD-CG-BA-RBFNN | 39.1547 |
| FEEMD-CG-BA-GRNN | 31.0245 |
| SSA-CG-BA-GRNN | 28.6414 |
| SSA-BA-RBFNN | 23.6554 |
| FEEMD-BA-RBFNN | 24.0046 |
| SSA-BA-GRNN | 22.0894 |
| FEEMD-BA-GRNN | 23.1663 |
| RBFNN | 16.2461 |
| GRNN | 14.2541 |
| ARIMA | 8.9564 |
| ELM | 12.6791 |
| SVM | 18.4989 |
| | |

efficient than FEEMD-CG-BA-GRNN, SSA-CG-BA-RBFNN and FEEMD-CG-BA-RBFNN. The relation between the wind speed and wind power generation could be expressed as the following equation:

$$P_{a} = \left\{ \frac{\exp[-(v_{c}/c)^{k}] - \exp[-(v_{r}/c)^{k}]}{(v_{r}/c)^{k} - (v_{c}/c)^{k}} - \exp[-(v_{f}/c)^{k}] \right\} \times P_{r}$$
(2)

where P_a is the average power output of the wind turbine (kW), P_r is the rated electrical power of the wind turbine (kW), v_c is the cut-in wind speed (m/s), v_f is the cut-off wind speed (m/s), v_r is the nominal wind speed (m/s), and *c* is the Weibull scale parameter (m/s). It is observed that the accurate wind speed forecasting plays an important role in the wind power generation.

7. Conclusion

As one of the most promising potential renewable energies, wind energy has been a focus of many scientists and researchers and supported by almost every government across the world. To integrate wind energy into the power system, it is important to forecast wind power generation. Wind speed is affected by various environmental factors, so wind speed data present high fluctuations, autocorrelation and stochastic volatility, and it is difficult to forecast wind speed using a single model. In this paper, four hybrid models based on two decomposition algorithms, SSA and FEEMD, and two neural networks, RBFNN and GRNN, are proposed for multi-step wind speed forecasting. Meanwhile, to improve the performance of the neural networks, a new improved BA algorithm, CG-BA, based on CG is proposed to optimize the initial weights and thresholds of neural networks. Based on a series of forecasting results, the DM test and FVD, the following can be concluded: (a) the hybrid SSA-CG-BA-GRNN model is the most accurate model among the four proposed models in multi-step wind speed forecasting; (b) the decomposition algorithm SSA is better than FEEMD in this study; (c) the performance of the single model GRNN is more accurate than RBFNN and ARIMA.

Thus, the proposed SSA-CG-BA-GRNN model, which has the highest precision, is a promising model for use in the future. This hybrid model can also be applied in many other fields, such as tourism demand forecasting, product sales forecasting, power load forecasting, and traffic flow forecasting.

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Appendix A

A.1. SSA and FEEMD

SSA is a novel nonparametric method, which is employed in the analysis of time series and combines multivariate statistic and probability theory, and it is often used for identifying and extracting periodic, quasi-periodic and oscillatory components from the primal data [28]. Standard SSA performs four steps, which include embedding, singular value decomposition, grouping and diagonal averaging. However, the first two steps are also called the time series decomposition, and steps three and four are known as the reconstruction. For more details, one can refer [28,44], which provides a complete description of the SSA theory.

FEEMD is an extension of the empirical mode decomposition [45] and ensemble empirical mode decomposition techniques [46]. The fast ensemble empirical mode decomposition technique is a time-domain decomposing method, which can convert a group of time series into multiple empirical modes, named as the intrinsic mode functions (IMFs). y(t) is the time series, it can be decomposed and expressed using the following formula:

$$y(t) = \sum_{j=1}^{n} \mathbf{IMF}_{j}(t) + r_{n}(t)$$
(A1)

where $\mathbf{IMF}_j(t)$, j = 1, 2, ..., n, is the intrinsic mode function (i.e., local oscillation) based on empirical mode decomposition and $r_n(t)$ is the *n*th residue (i.e., local trend). For more details, one can refer [47], which provides a complete description of the SSA theory.

Appendix **B**

B.1. Standard RBFNN

Definition 1. Cluster centers were composed of elements m_i^j (j = 1 - n) from the center vector m_i , which is in the input space.

Definition 2. With elements I_j (j = 1 - n), the distance measure is used to determine how far the center vector m^i is from an input vector I. The popular distance measure is the Euclidian distance, defined as

$$d_{i} = \sqrt{\sum_{j=1}^{n} k_{j}^{i} (I_{j} - m_{j}^{i})^{2}}$$
(B1)

where k_j^i is the (i, j)th element of the shape matrix K, defined as the inverse of the covariance matrix:

$$k_j^i = \frac{h_j^i}{\left(\sigma_j^i\right)^2} \tag{B2}$$

where h_j^i is the correlation coefficient, and σ_j^i represents the marginal standard deviation.

Definition 3. A transfer function transforms the Euclidian summation d_j (i = 1 - m) and gives an output for each node. The output generated by the hidden layer was from the input layer via a distance measure of Eq. (B1) and a transfer function. A weighted sum of the outputs of $\phi(d_i)$ from the hidden layer processed the output of the network, i.e.,

$$0 = w_0 + \sum_{i=1}^{m} w_i \phi(d_i)$$
(B3)

Appendix B2 Standard GRNN.

Definition 1. The input layer accepts information and also stores an input vector *X*, whose dimension *m* equals the number of input layer neurons. The pattern layer is then fed the data that comes from the input neurons of the input layer. A nonlinear transformation, which transforms the input space into the pattern space, was used by the pattern layer. The neurons of the pattern layer can remember the relation between the input neuron and the proper response of the pattern layer, in which the number of neurons is equal to the number of training samples *n*. The pattern Gaussian function of p_i is expressed as

$$p_i = \exp\left[-\frac{(X - X_i)^T (X - X_i)}{2\sigma^2}\right]$$
 (*i* = 1, 2, ..., *n*) (B4)

where σ represents the spread parameter, and *X* is the network's input variable. In the pattern layer, *X_i* is a specific training sample of neuron *i*.

Definition 2. S_s and S_w are the two summations of the summation layer. The simple summation S_s is used to calculate the arithmetic sum from the outputs that belong to the pattern layer, and *i* is the interconnection weight of the simple summation. The weighted summation S_w is used to calculate the weighted sum from the outputs that belong to the pattern layer, and *w* is the interconnection weight of the weighted summation. The transfer functions can be described by Eqs. (B4) and (B5):

$$S_s = \sum_{t=1}^{n} p_t, \quad t = 1 \dots n \tag{B5}$$

$$S_{wt} = \sum_{t=1}^{\infty} w_t p_t, \quad t = 1 \dots n$$
(B6)

where w_t is the weight of pattern neuron t that is connected to the summation layer.

Definition 3. In the output layer, the number of neurons is equal to the dimension k of the output vector Y. In the summation layer, after the summations, the output absorbs the neurons, and the output Y of the output neurons can be computed as

$$\hat{Y}_o = S_s / S_{wo}, \quad o = 1 \dots k \tag{B7}$$

If the training set is given, the spread parameter R is the only parameter that must be confirmed.

Appendix C

C.1. Accuracy estimating indexes

The detailed equations of MAE MSE and MAPE are given in Table C1.

| Table | C1 | |
|-------|--------|--------|
| Three | metric | rules. |

| Metric | Definition | Equation |
|--------|--|---|
| MAE | The average absolute forecast error of <i>n</i> times forecast results | $MAE = \frac{1}{N} \sum_{n=1}^{N} y_n - \hat{y}_n $ |
| MSE | The average of the prediction error squares | $MSE = \frac{1}{N} \sum_{n=1}^{N} (y_n - \hat{y}_n)^2$ |
| MAPE | The average of absolute error | $MAPE = \frac{1}{N} \sum_{n=1}^{N} \left \frac{y_n - \hat{y}_n}{y_n} \right \times 100\%$ |

Where y_n and \hat{y}_n denote the actual value and predicted value, respectively, of the *n*th data for the performance estimate, and *N* is the length of the dataset to compare and evaluate.

Additionally, to obtain the detailed promoting percentages when comparing two forecasting, i.e. model 1 and model 2, three percentage error indexes are also defined as follows:

$$\zeta_{MAE} = \frac{MAE_2 - MAE_1}{MAE_1} \times 100\%$$
(C1)

$$\xi_{MSE} = \frac{MSE_2 - MSE_1}{MSE_1} \times 100\% \tag{C2}$$

$$\xi_{MAPE} = \frac{MAPE_2 - MAPE_1}{MAPE_1} \times 100\%$$
(C3)

The negative value of ξ_{MAE} (%) means model 2 decreases $|\xi_{MAE}|$ % MAE value based on model 1, the positive value of ξ_{MAE} (%) means model 2 increases $|\xi_{MAE}|$ % MAE value based on model 1. So do ξ_{MSE} (%) and ξ_{MAPE} (%).

References

- Yesilbudak M, Sagiroglu S, Colak I. A new approach to very short term wind speed prediction using k-nearest neighbor classification. Energy Convers Manage 2013;69(69):77–86.
- [2] Meng A, Ge J, Yin H, Chen S. Wind speed forecasting based on wavelet packet decomposition and artificial neural networks trained by crisscross optimization algorithm. Energy Convers Manage 2016:114:75–88.
- [3] Ozay C, Celiktas MS. Statistical analysis of wind speed using two-parameter Weibull distribution in Alaçatı region. Energy Convers Manage 2016;121:49–54.
- [4] Liu H, Tian HQ, Li YF, Zhang L. Comparison of four Adaboost algorithm based artificial neural networks in wind speed predictions. Energy Convers Manage 2015;92(92):67–81.
- [5] Zhao ZY, Hu J, Zuo J. Performance of wind power industry development in China: a DiamondModel study. Renew Energy 2009;34(12):2883–91.
- [6] Zhang C, Wei H, Zhao X, Liu T, Zhang K. A Gaussian process regression based hybrid approach for short-term wind speed prediction. Energy Convers Manage 2016;126:1084–92.
- [7] Liu H, Tian H, Li Y. Four wind speed multi-step forecasting models using extreme learning machines and signal decomposing algorithms. Energy Convers Manage 2015;100:16–22.
- [8] Zhou J, Jing S, Gong L. Fine tuning support vector machines for short-term wind speed forecasting. Energy Convers Manage 2011;52(4):1990–8.
- [9] Lei M, Shiyan L, Chuanwen J, Hongling L, Yan Z. A review on the forecasting of wind speed and generated power. Renew Sustain Energy Rev 2009;13 (4):915–20.
- [10] Landberg L. Short-term prediction of the power production from wind farms. J Wind Eng Ind Aerodyn 1999;80:207–20.
- [11] Alexiadis MC, Dokopoulos PS, Sahsamanoglou HS, Manousaridis IM. Short term forecasting of wind speed and related electrical power. Sol Energy 1998;63 (1):61–8.
- [12] Negnevitsky M, Potter CW. Innovative short-term wind generation prediction techniques. In: Proceedings of the power systems conference and exposition. p. 60–5.
- [13] Riahy GH, Abedi M. Short term wind speed forecasting for wind turbine applications using linear prediction method. Renew Energy 2008;33(1):35–41.
- [14] Ma L, Luan SY, Jiang CW, Liu HL, Zhang Y. A review on the forecasting of wind speed and generated power. Renew Sustain Energy Rev 2009;13:915–20.
- [15] Xiao L, Wang J, Yang X, Xiao L. A hybrid model based on data preprocessing for electrical power forecasting. Int J Electr Power Energy Syst 2015;64 (64):311–27.
- [16] Lydia M, Kumar SS, Selvakumar AI, Kumar GEP. Linear and non-linear autoregressive models for short-term wind speed forecasting. Energy Convers Manage 2016;112:115–24.
- [17] Noorollahi Y, Jokar MA, Kalhor A. Using artificial neural networks for temporal and spatial wind speed forecasting in Iran. Energy Convers Manage 2016;115:17–25.
- [18] Barbounis TG, Theocharis JB. A locally recurrent fuzzy neural network with application to the wind speed prediction using spatial correlation. Neurocomputing 2007;70(7/9):1525–42.
- [19] Focken U, Lange M, Moonnich K, Waldl H-P, Georg Beyer H, Luig A. Short-term prediction of the aggregated power output of wind farms – a statistical analysis of the reduction of the prediction error by spatial smoothing effects. J Wind Eng Ind Aerodyn 2002;90(3):231–46.
- [20] Niu M, Sun S, Wu J, Yu L, Wang J. An innovative integrated model using the singular spectrum analysis and nonlinear multi-layer perceptron network optimized by hybrid intelligent algorithm for short-term load forecasting. Appl Math Model 2016;40(5–6):4079–93.
- [21] Xiao L, Shao W, Wang C, Zhang K, Lu H. Research and application of a hybrid model based on multi-objective optimization for electrical load forecasting. Appl Energy 2016;180(C):213–33.

- [22] Su Z, Wang J, Lu H, Zhao G. A new hybrid model optimized by an intelligent optimization algorithm for wind speed forecasting. Energy Convers Manage 2014;85(9):443–52.
- [23] Monfared M, Rastegar H, Kojabadi HM. A new strategy for wind speed forecasting using artificial intelligent methods. Renew Energy 2009;34:845–8.
 [24] Cadenas E, Rivera W. Short term wind speed forecasting in La Venta, Oaxaca,
- México, using artificial neural networks. Renew Energy 2009;34(1):274–8. [25] Xiao L, Shao W, Liang T, Wang C. A combined model based on multiple
- seasonal patterns and modified firefly algorithm for electrical load forecasting.
 Appl Energy 2016;167:135–53.
 [26] Sfetsos A. A comparison of various forecasting techniques applied to mean
- hourly wind speed time series. Renew Energy 2000;21(1):23–35.
- [27] Mohandes MA, Halawani TO, Rehman S, Hussain AA. Support vector machines for wind speed prediction. Renew Energy 2004;29:939–47.
- [28] Chen N, Qian Z, Meng X. Multi-step wind speed forecasting based on wavelet and gaussian processes. Math Probl Eng 2013.
- [29] Liu H, Tian H-Q, Pan D-F, Li Y-F. Forecasting models for wind speed using wavelet, wavelet packet, time series and Artificial Neural Networks. Appl Energy 2013;107:191–208.
- [30] Liu H, Chen C, Tian H-Q, Li Y-F. A hybrid model for wind speed prediction using empirical mode decomposition and artificial neural networks. Renew Energy 2012;48:545–56.
- [31] Wang Y, Wang S, Zhang N. A novel wind speed forecasting method based on ensemble empirical mode decomposition and GA-BP neural network. Presented at the IEEE power and energy society general meeting; 2013.
- [32] Liu H, Tian H-Q, Li Y-F. Comparison of new hybrid FEEMD-MLP, FEEMD-ANFIS, Wavelet Packet-MLP and Wavelet Packet-ANFIS for wind speed predictions. Energy Convers Manage 2014;89:1–11.
- [33] Liu H, Tian H-Q, Chen C, Li Y-F. An experimental investigation of two WaveletMLP hybrid frameworks for wind speed prediction using GA and PSO optimization. Int J Electr Power Energy Syst 2013;52(1):161–73.
- [34] Poitras G, Cormier G. Wind speed prediction for a target station using neural networks and particle swarm optimization. Wind Eng 2011;35(3):369–80.
- [35] Carro-Calvo L, Salcedo-Sanz S, Prieto L, Kirchner-Bossi N, Portilla-Figueras A, Jiménez-Fernández S. Wind speed reconstruction from synoptic pressure patterns using an evolutionary algorithm. Appl Energy 2012;89(1):347–54.
- [36] Xiao L, Wang J, Hou R, Wu J. A combined model based on data pre-analysis and weight coefficients optimization for electrical load forecasting. Energy 2015;82:524–49.
- [37] Yang XS. A new metaheuristic bat-inspired algorithm. Nature inspired cooperative strategies for optimization (NICSO 2010). Studies in computational intelligence, vol. 284. Berlin: Springer Verlag; 2010. p. 65–74.
- [38] Yang XS. Bat algorithm: literature review and applications. Int J Bio-Inspired Comput 2013;5(3):141–9.
- [39] Du ZY, Liu B. Image matching using a bat algorithm with mutation. Appl Mech Mater 2012;203:88–93.
- [40] Mishra S, Shaw K, Mishra D. A new meta-heuristic bat inspired classification approach for microarray data. Procedia Technol 2012;4:802–6.
- [41] Musikapun P, Pongcharoen P. Solving multi-stage multi-machine multiproduct scheduling problem using bat algorithm. Second international conference on management and artificial intelligence (IPEDR), vol. 35. Singapore: IACSIT Press; 2012. p. 98–102.
- [42] Niknam T, Sharifinia S, Abaraghooee RA. A new enhanced bat-inspired algorithm for finding linear supply function equilibrium of GENCOs in the competitive electricity market. Energy Convers Manage 2013;76:1015–28.
- [43] Sambariya DK, Prasad R. Robust tuning of power system stabilizer for small signal stability enhancement using metaheuristic bat algorithm. Int J Electr Power Energy Syst 2014;61:229–38.
- [44] Skittides C, Früh WG. Wind forecasting using Principal Component Analysis. Renew Energy 2014;69(September):365-74.
- [45] Huang NE, Liu HH. The empirical mode decomposition and the hilbert spectrum for nonlinear and non-stationary time series analysis. Proc Roy Soc A Math Phys Eng Sci 1971;454:903.
- [46] Wu Z, Huang NE. Ensemble empirical mode decomposition: a noise-assisted data analysis method. Adv Adapt Data Anal 2009;1(01):1–41.
- [47] Wang YH, Yeh CH, Young HWV, Hu K, Lo MT. On the computational complexity of the empirical mode decomposition algorithm. Physica A 2014;400(2):159–67.
- [48] Yang XS. A new metaheuristic bat-inspired algorithm. Nature inspired cooperative strategies for optimization (NICSO), studies in computational intelligence, vol. 284. Springer; 2010. p. 65–74.
- [**49**] Yang XS. Bat algorithm for multiobjective optimization. Int J Bio-Inspired Comput 2011;3(5):267–74.
- [50] Shen F, Chao J, Zhao J. Forecasting exchange rate using deep belief networks and conjugate gradient method. Neurocomputing 2015;167:243–53.
- [51] Guo XT, Zhu Y. Evolutionary neural networks based on genetic algorithms. J Tsinghua Univ (Sci Technol) 2000;40(10):116–9.
- [52] Specht DF. A general regression neural network. IEEE Trans Neural Networks 1991;2(6):568–76.
- [53] Polat O, Yildirim T. Hand geometry identification without feature extraction by general regression neural network. Expert Syst Appl 2008;34:845–9.
- [54] Hecht-Nielsen SR. Kolmogorov's mapping neural network existence theorem. In: IEEE joint conf on neural networks, New York, USA, vol. 3; 1987. p. 11–4.
- [55] Diebold FX, Mariano R. Comparing predictive accuracy. J Bus Econ Stat 1995;13:253–63.
- [56] Chen HY, Hou DP. Research on superior combination forecasting model based on forecasting effective measure. J Univ Sci Technol China 2002;2:006.